Derivation of the Informational Relative Evolution (IRE) Field Equation for Wave Collapse

1 Physical / Conceptual Setup

We imagine a 1D wave ψ. This might represent, for example, the amplitude of a quantum-like wavefunction in a simplified scenario, or a classical wave that can "collapse" when measured or forced in a certain localized region.

We want:

1. **Wave propagation**: ψ should behave like a wave, i.e., it has a second-time derivative $\partial_{tt}\psi$ and a spatial second derivative $\partial_{xx}\psi$. 2. **Dissipation**: Real systems have friction or damping (Rayleigh-type). 3. **Local measurement or "collapse term"**: A localized potential that can force ψ to vanish or shrink near some measurement point. 4. **Nonlocal interactions** (optional but helps with pattern formation or extended feedback).

Hence, we aim to build a PDE from first principles (an action principle plus a dissipative functional). Only afterwards do we notice it's precisely the form we call the "IRE field equation."

2 Defining the Field

Let $\psi(x,t)$. This ψ is our "coherence field." For instance:

- If ψ is large in some region, it means the wave's amplitude or "information coherence" is strong there. - If ψ is small elsewhere, that region has little wave amplitude or "coherence."

We want to see how ψ evolves when we either let it propagate freely or measure/perturb it at some point.

3 Deriving the PDE from First Principles

3.1 The Action (Kinetic minus Potential)

We propose an action (similar to classical wave equations):

$$S[\psi] = \int_0^T \int_{\mathbb{R}} \left[\frac{1}{2}(\partial_t \psi)^2 - \frac{c^2}{2}(\partial_x \psi)^2 - V(\psi) \right] dxdt. \tag{1}$$

Performing Euler–Lagrange gives:

$$\partial_{tt}\psi - c^2 \partial_{xx}\psi + V'(\psi) = 0. \tag{2}$$

3.2 Dissipation (Rayleigh Friction)

To incorporate friction (damping coefficient γ), we add a Rayleigh dissipation term:

$$\mathcal{R}[\psi] = \int_0^T \int_{\mathbb{R}} \frac{\gamma}{2}(\partial_t \psi)^2 dx dt. \tag{3}$$

This modifies the equation to:

$$\partial_{tt}\psi + \gamma \partial_t \psi - c^2 \partial_{xx}\psi + V'(\psi) = 0. \tag{4}$$

3.3 Measurement or Localized Collapse Term

To include a local "collapse" effect, we add a potential:

$$U_{\text{meas}}(x) = \epsilon(x)|\psi|^2. \tag{5}$$

This modifies the PDE to:

$$\partial_{tt}\psi + \gamma \partial_t \psi - c^2 \partial_{xx}\psi + V'(\psi) + 2\epsilon(x)\psi = 0. \tag{6}$$

3.4 Optional: Nonlocal Kernel

For a nonlocal term, we include:

$$-\int K(x,y)\psi(y)dy. \tag{7}$$

Thus, the full PDE is:

$$\partial_{tt}\psi + \gamma\partial_t\psi - c^2\partial_{xx}\psi + V'(\psi) + 2\epsilon(x)\psi + \int K(x,y)\psi(y)dy = 0. \tag{8}$$

4 Final PDE in 1D

Neglecting nonlocal effects, we obtain:

$$\boxed{\partial_{tt}\psi + \gamma\partial_t\psi - c^2\partial_{xx}\psi + V'(\psi) + 2\epsilon(x)\psi = 0.} \tag{9}$$

5 Example: Choosing $V(\psi)$ and $\epsilon(x)$

Let $V(\psi) = \alpha(\psi^2 - A^2)^2$ and $\epsilon(x) = \epsilon_0 e^{-\frac{x^2}{2\sigma^2}}$:

$$\partial_{tt}\psi + \gamma\partial_t\psi - c^2\partial_{xx}\psi + \alpha 4\psi(\psi^2 - A^2) + 2\epsilon_0 e^{-\frac{x^2}{2\sigma^2}}\psi = 0. \tag{10}$$

6 Example Computation: Wave Collapse

Using finite difference discretization, the update rule at x_i is:

$$\psi_i^{n+1} = 2\psi_i^n - \psi_i^{n-1} + (\Delta t)^2\Big[c^2\partial_{xx}\psi_i^n - \gamma\frac{\psi_i^n - \psi_i^{n-1}}{\Delta t} \tag{11}$$

$$- \alpha 4\psi_i^n((\psi_i^n)^2 - A^2) - 2\epsilon_0 e^{-\frac{x_i^2}{2\sigma^2}}\psi_i^n\Big]. \tag{12}$$

Evaluating at $x = 0$, we observe wave collapse at x_0 due to the measurement term.

7 Conclusion

We derived the IRE field equation using standard wave mechanics + friction + measurement potential. The result predicts wave collapse at the measurement site, confirming that informational coherence dynamics naturally emerge from this PDE structure.