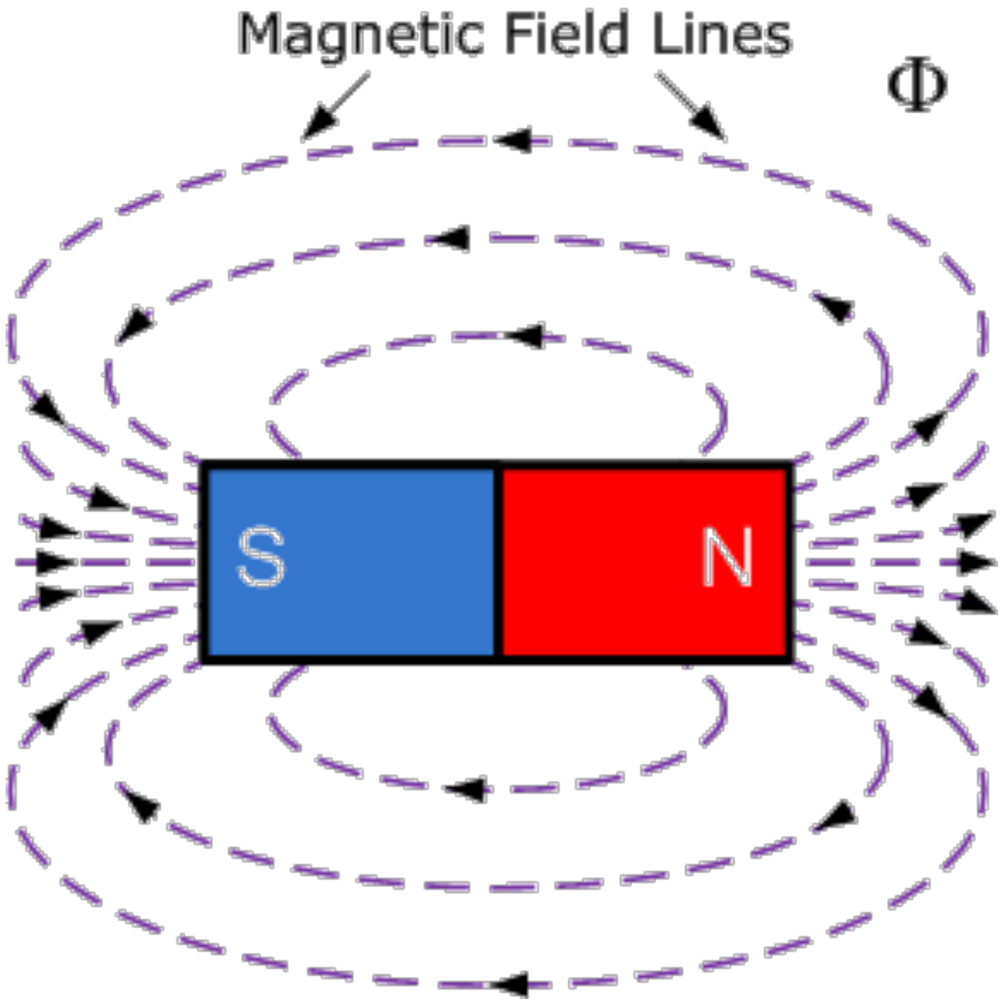
ROCO222: Intro to sensors and actuators

Lecture 2

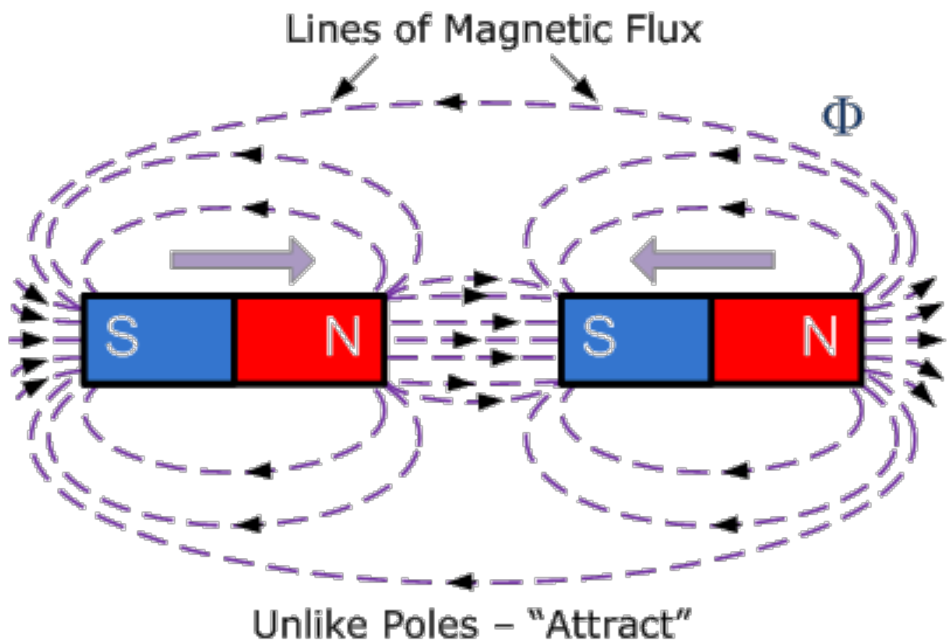
Electromagnetism

Lines of field from a bar magnets

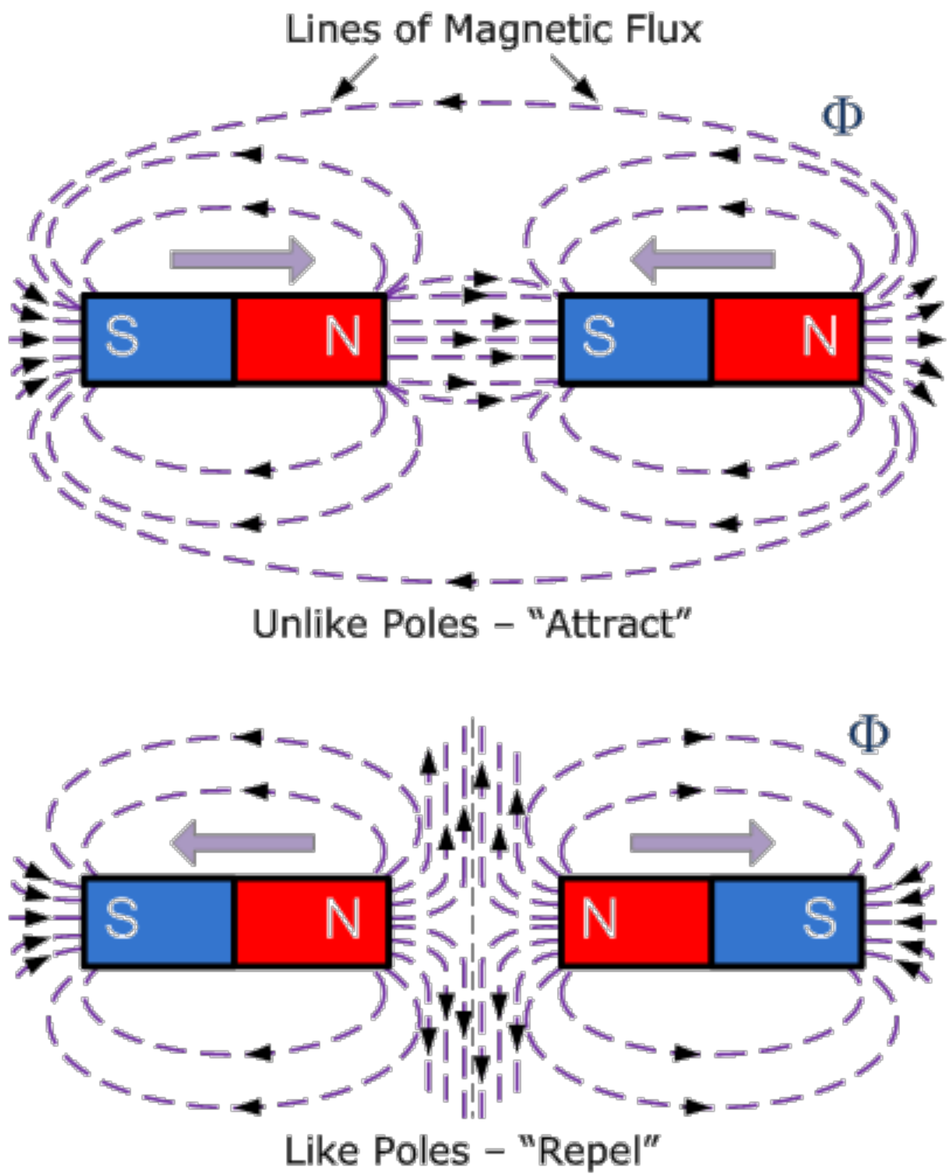


Magnetic field goes north to south

Like polls repel and unlike poles attract



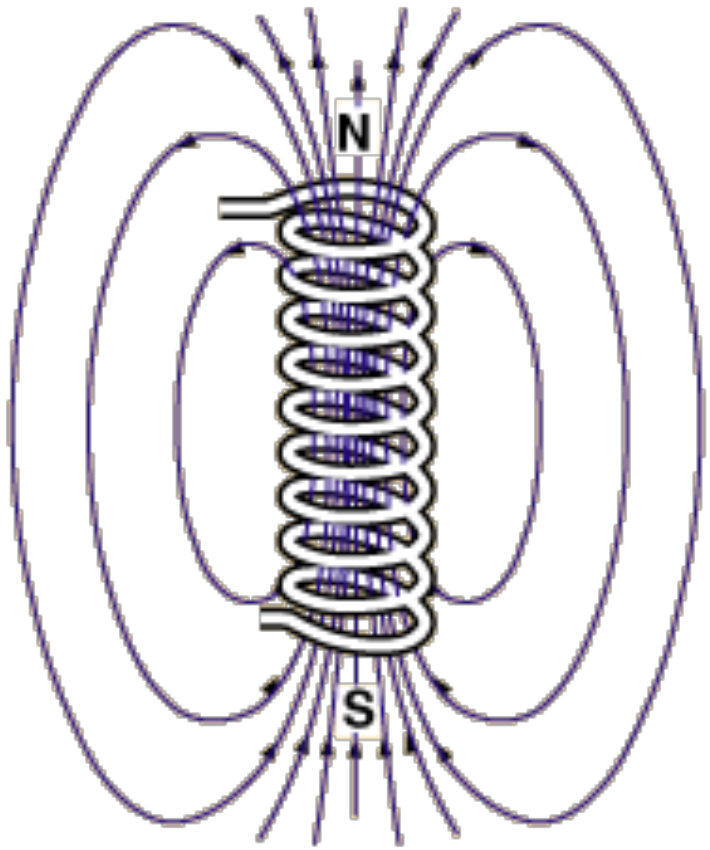
Like polls repel and unlike poles attract



Solenoid and Bar magnet have similar lines of force

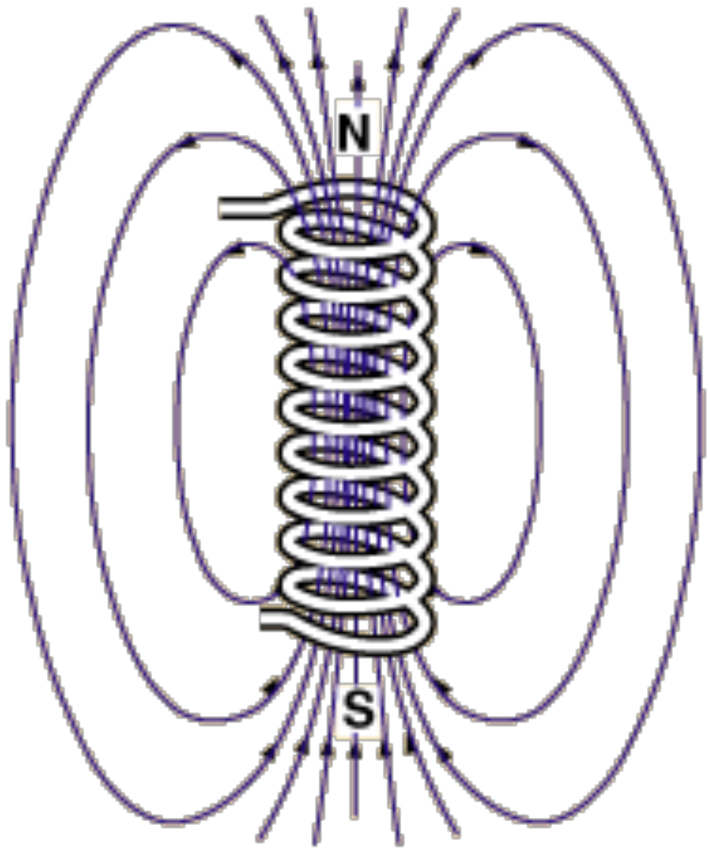
Solenoid

Solenoid and Bar magnet have similar lines of force

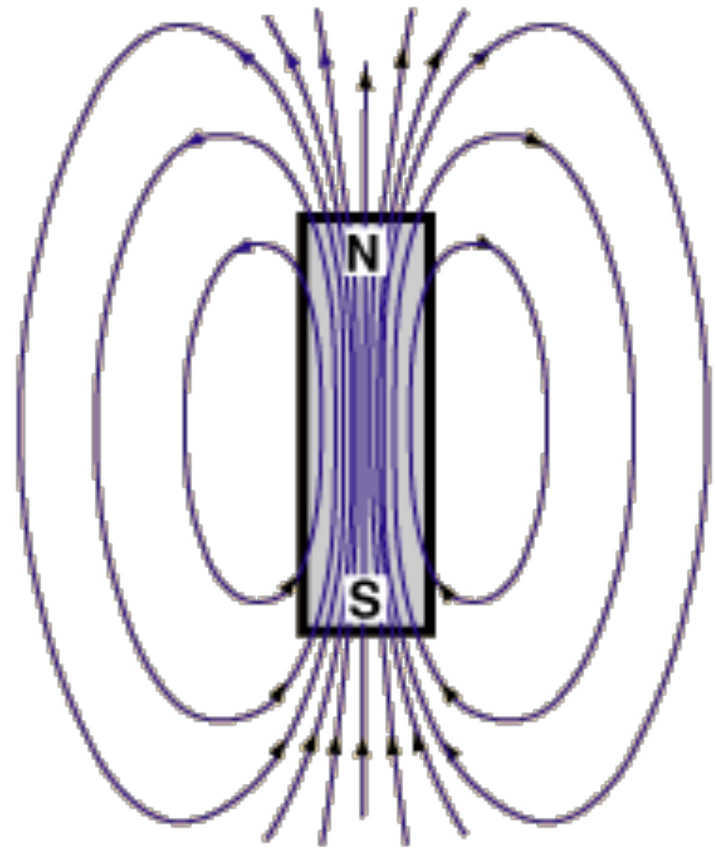


Solenoid

Solenoid and Bar magnet have similar lines of force

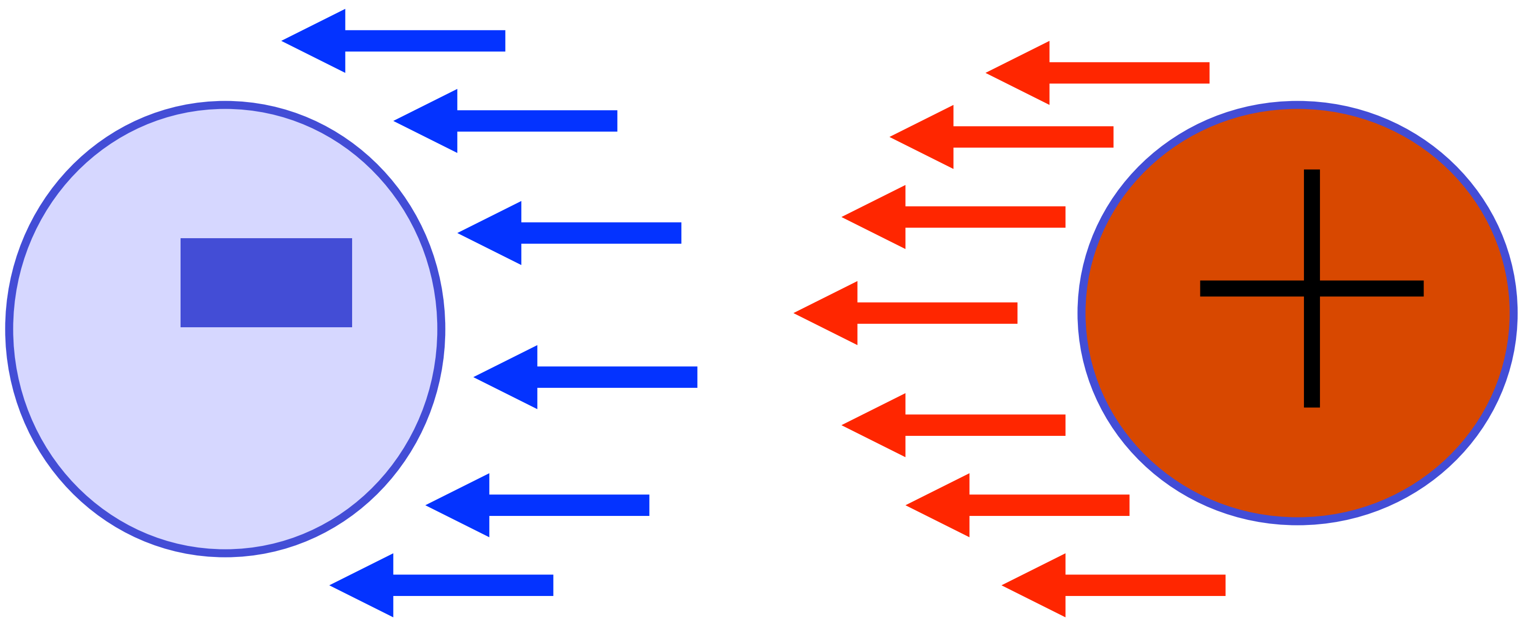


Solenoid



Bar magnet

Direction of conventional current



- Charge flows from positive to negative

Magnetic field strength H and flux density B

Magnetic field strength H is a vector quantity
Magnetic flux density B is is also a vector quantity

In free space the magnetic field strength H is related to the flux density B by the relationship

$B = \mu_0 H$ Where μ_0 is the permeability of free space

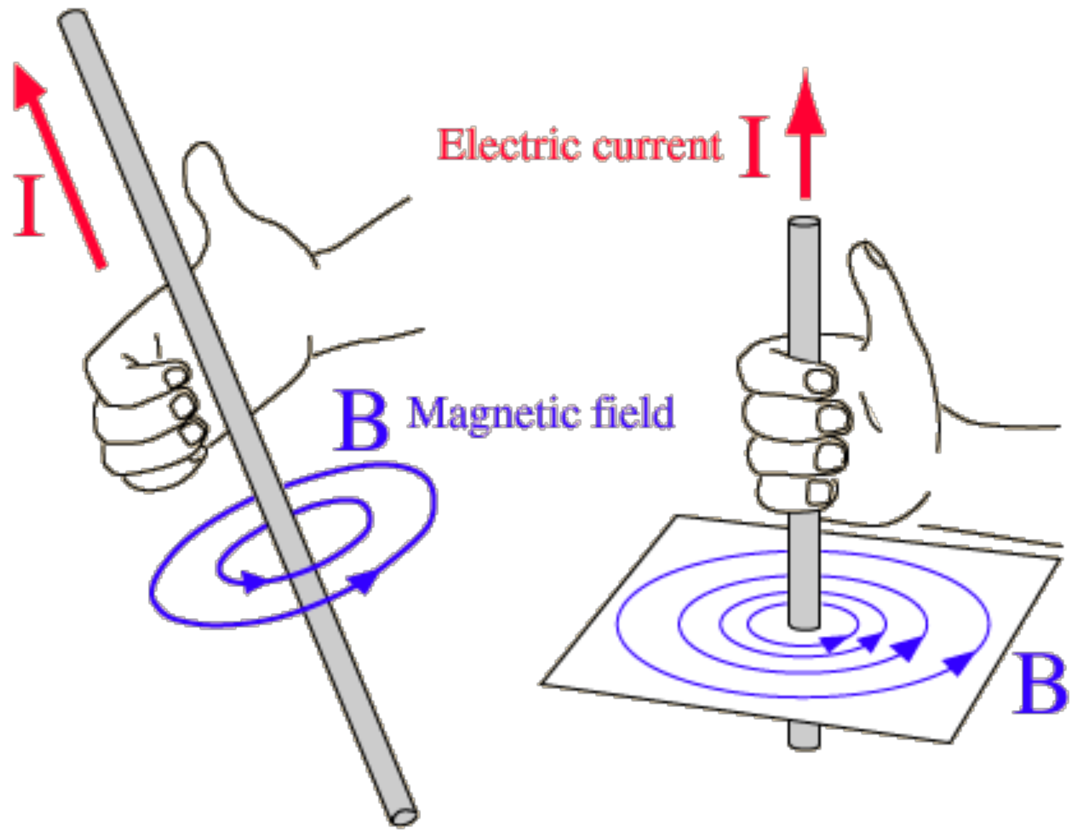
Within a magnetic material, within its linear regime, when B is not too high and does not saturate, the magnetic field strength H is related to the flux density B by the relationship

$B = \mu H$ Where μ is permeability of the material and

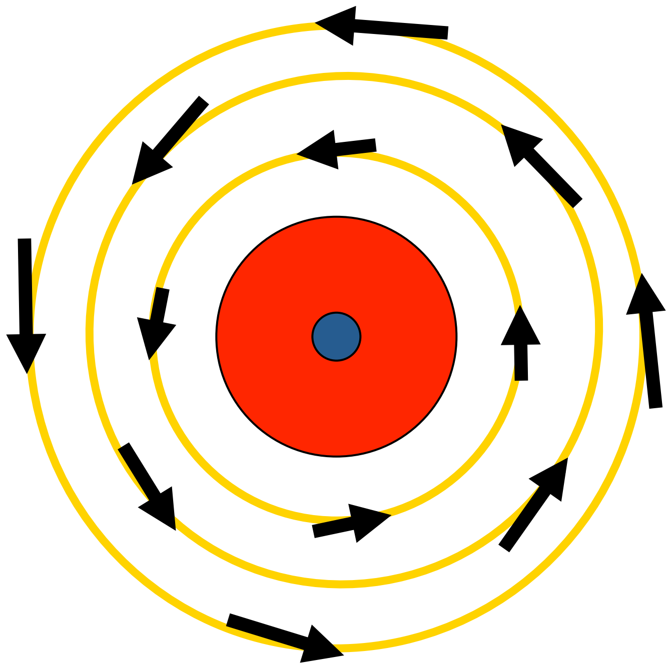
$\mu = \mu_r \mu_0$ Where μ_r is relative permeability of the material

Field around a current carrying wire

- Moving electric charge produces magnetic fields
- Ampère's right hand screw rule

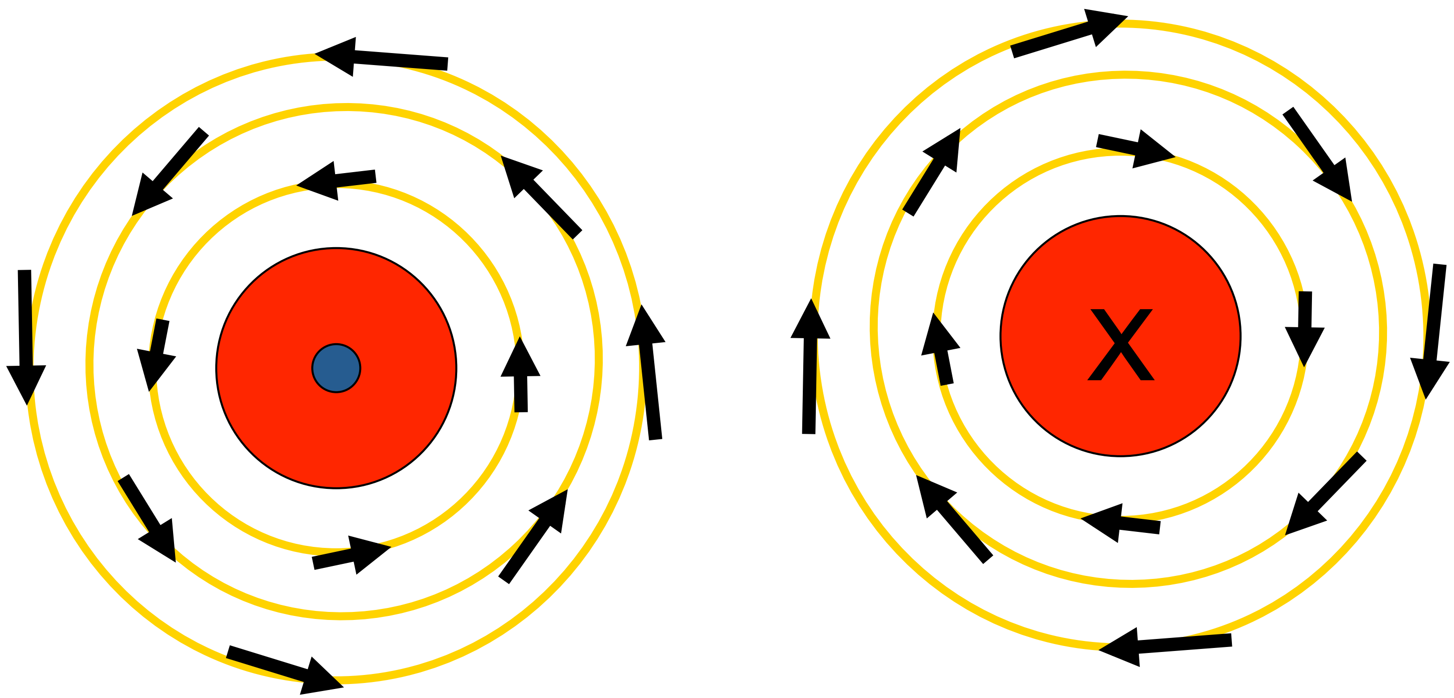


The direction of induced magnetic field



Current coming towards you

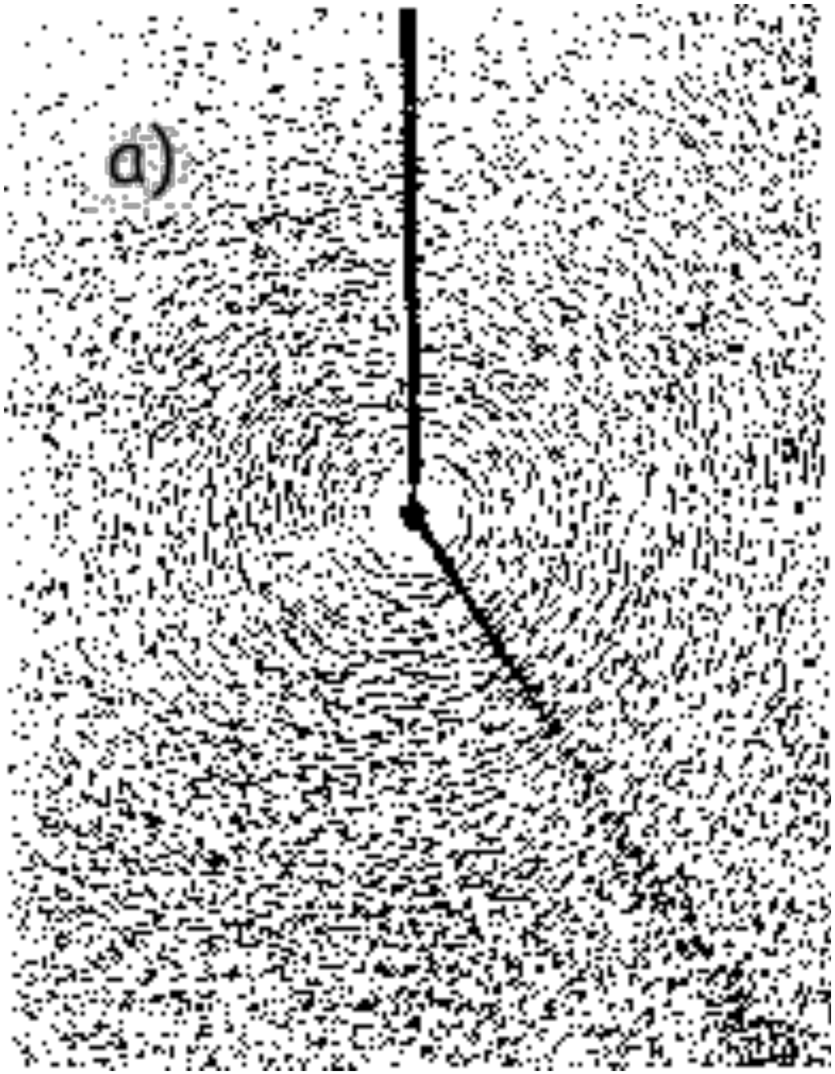
The direction of induced magnetic field



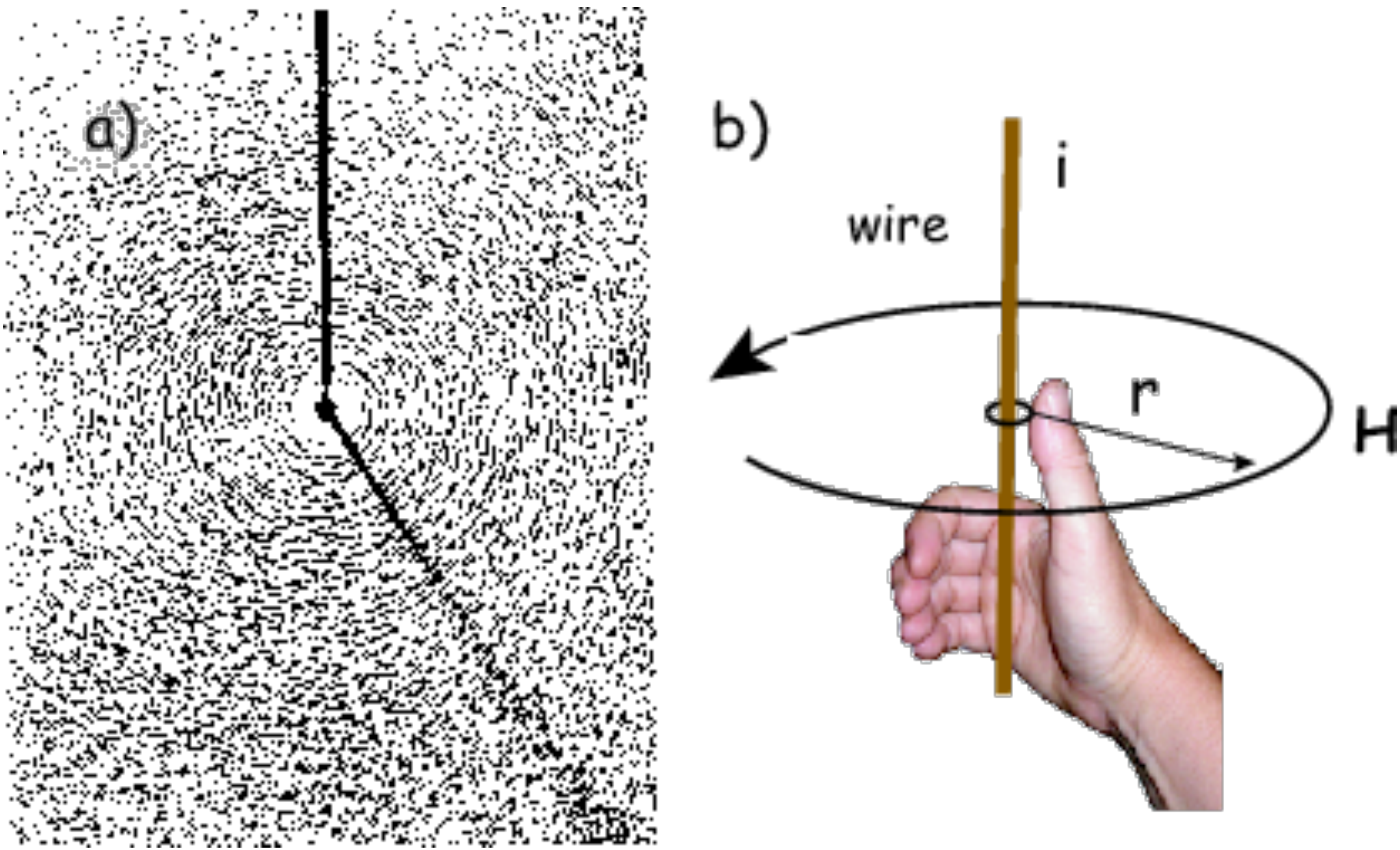
Current coming towards you

Current moving away from you

Iron filings can show field around wire

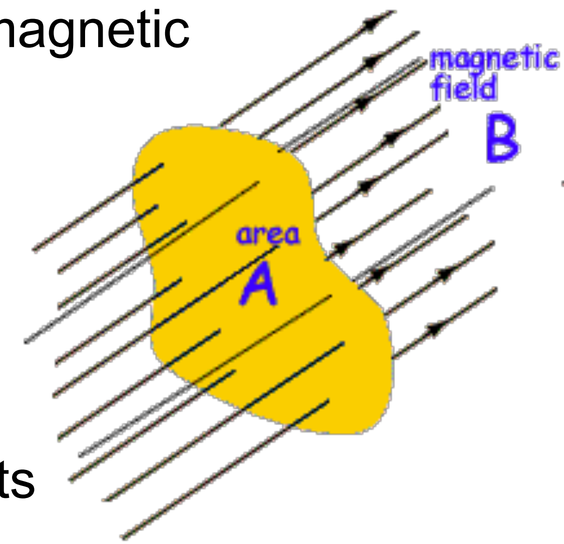


Iron filings can show field around wire



Flux of magnetic field

- We sometimes need to calculate the scalar amount of magnetic field that passes through a loop
- We define a magnetic flux as

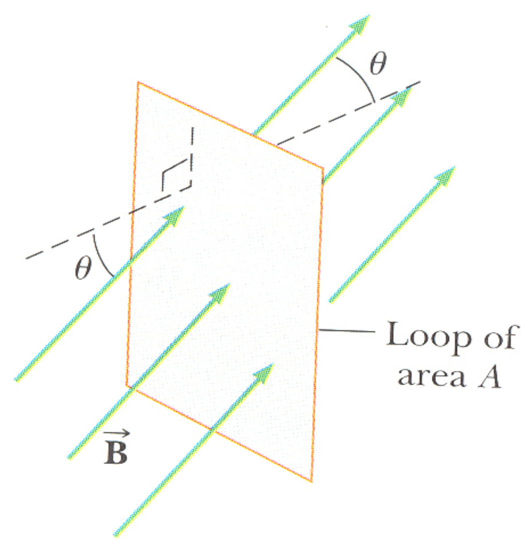


$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

- Where B is flux density, A is area and Φ has the SI units of the weber (Wb): NB 1 weber = 1 Tm^2

- In a uniform magnetic field, the magnetic flux passing through a loop of area A can be expressed as

$$\Phi_B = BA\cos\theta$$



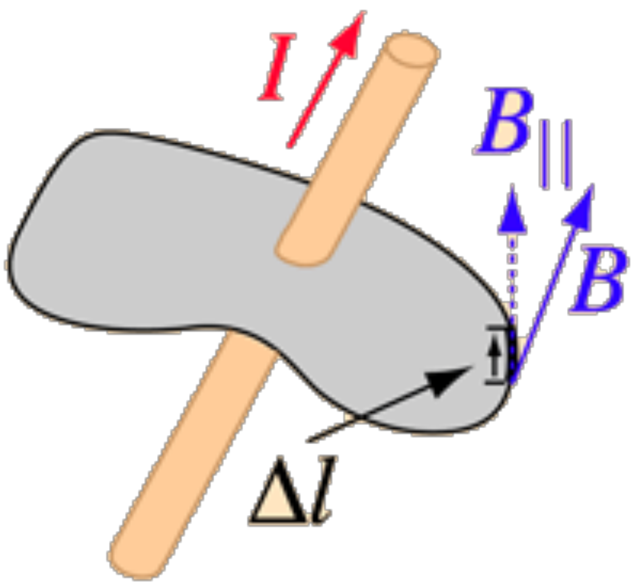
When the area is perpendicular to the flux density then the angle θ=0. In this case the total magnetic flux passing through a region A is given by the relationship

$$\Phi_B = BA$$

Ampere's Law

Ampere's Law states that:

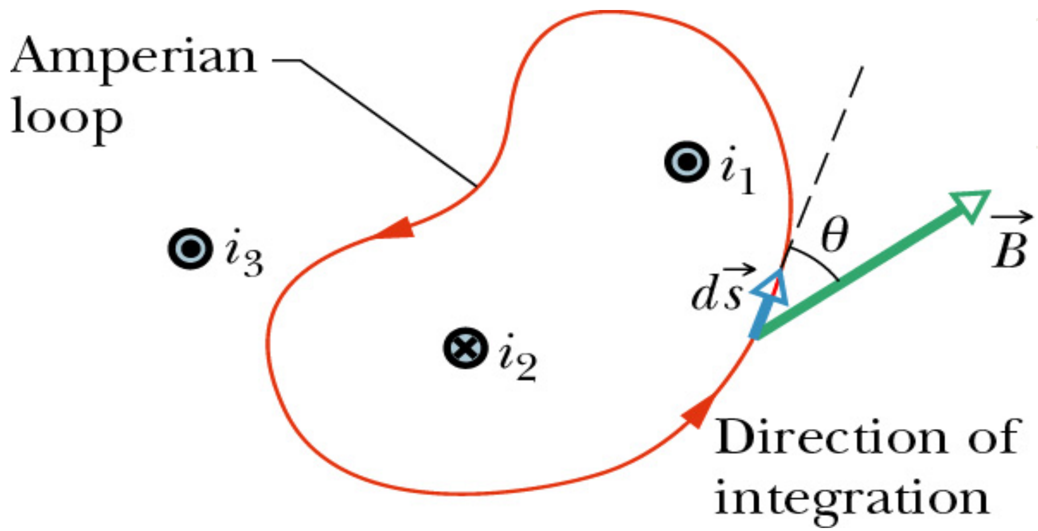
For any closed loop path, the sum of the length elements times the magnetic flux density in the direction of the length element is equal to the permeability times the electric current enclosed in the loop



$$\sum \vec{B}.d\vec{l} = \mu_0 I$$

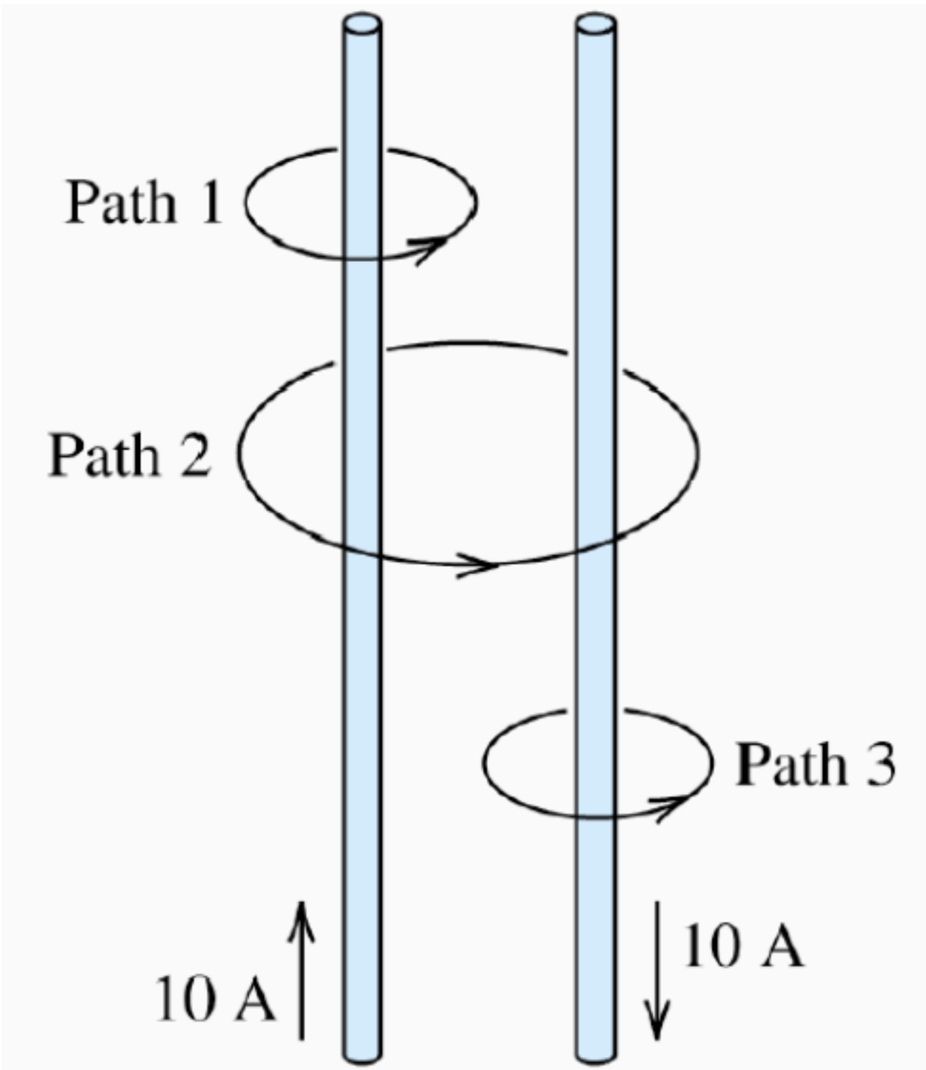
Ampere's Law

- Ampere's law can also be written using an integral
- In both cases this is a linear relationship

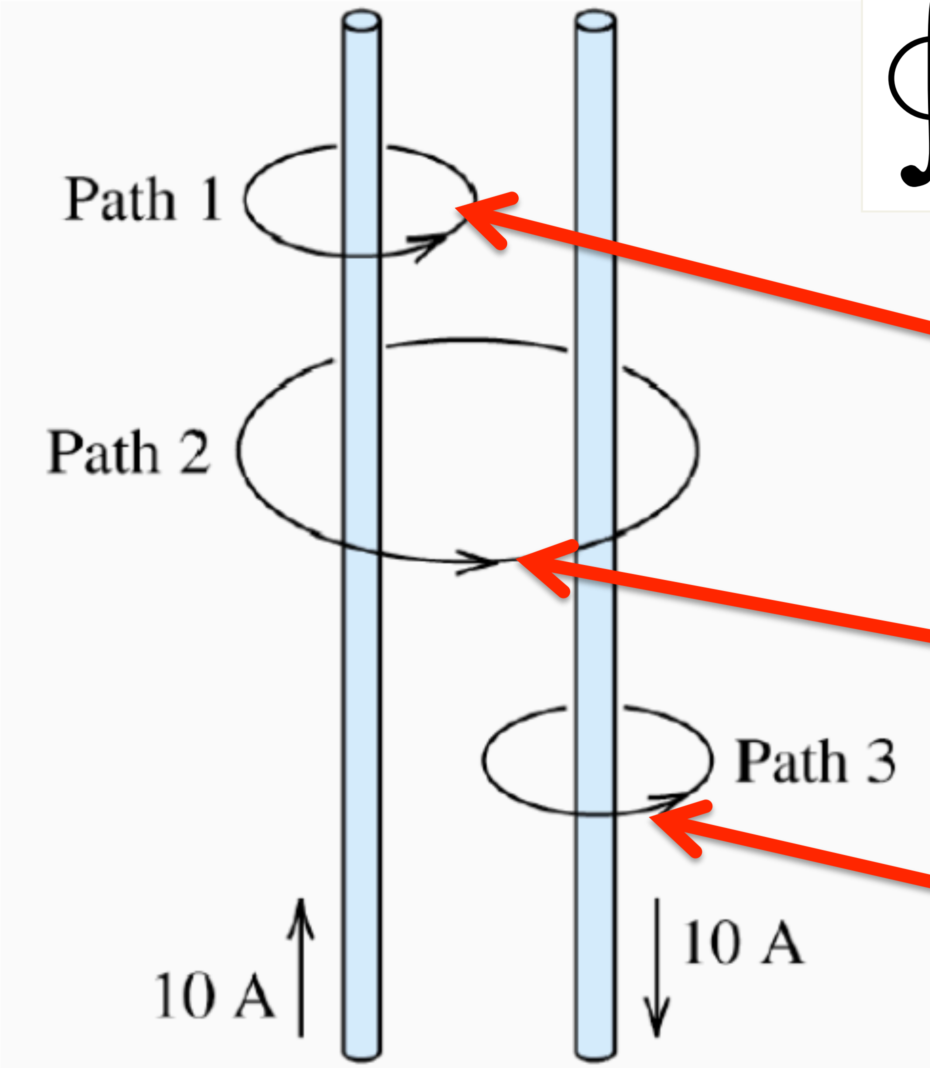


$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

Simple example of Ampere's Law



Simple example of Ampere's Law



$$\oint \vec{H} \cdot d\vec{s} = \sum i_{enc}$$

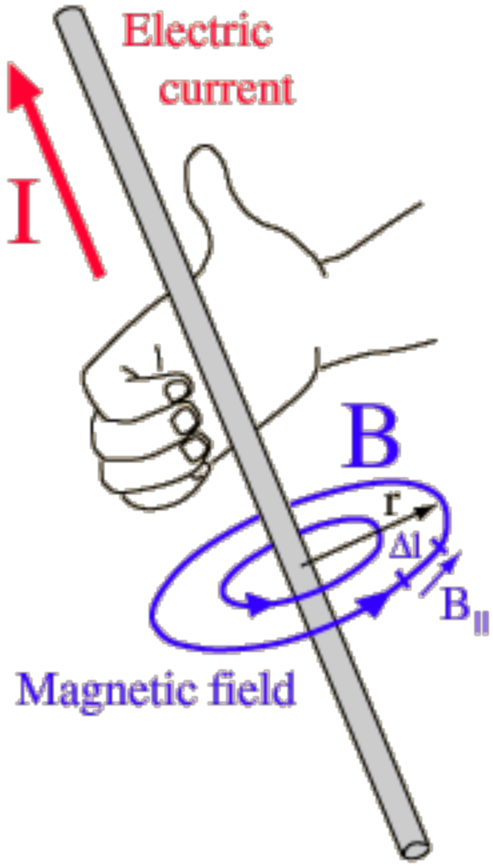
$$\sum i_{enc} = 10A$$

$$\sum i_{enc} = 0A$$

$$\sum i_{enc} = -10A$$

Ampere's Law for as current in straight wire

- For current flowing in a straight wire, magnetic field will encircle the wire
- We can use Amperes law to compute the value of B

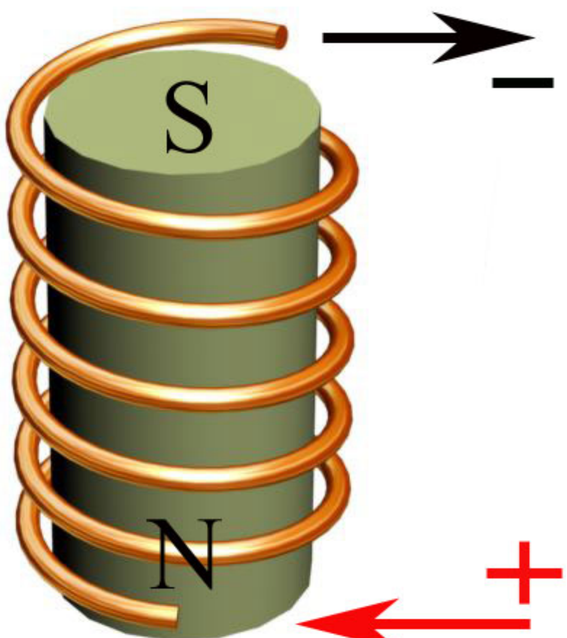


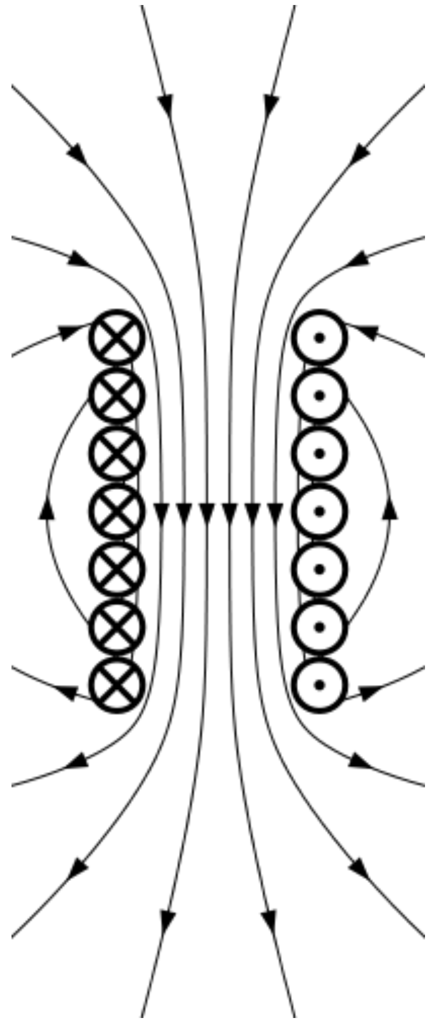
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

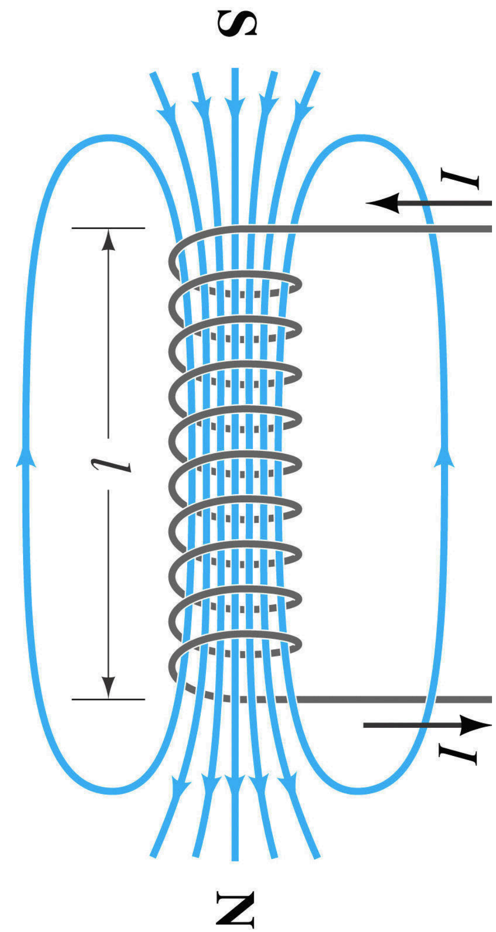
$$B.2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic field due to current flow in a solenoid coil

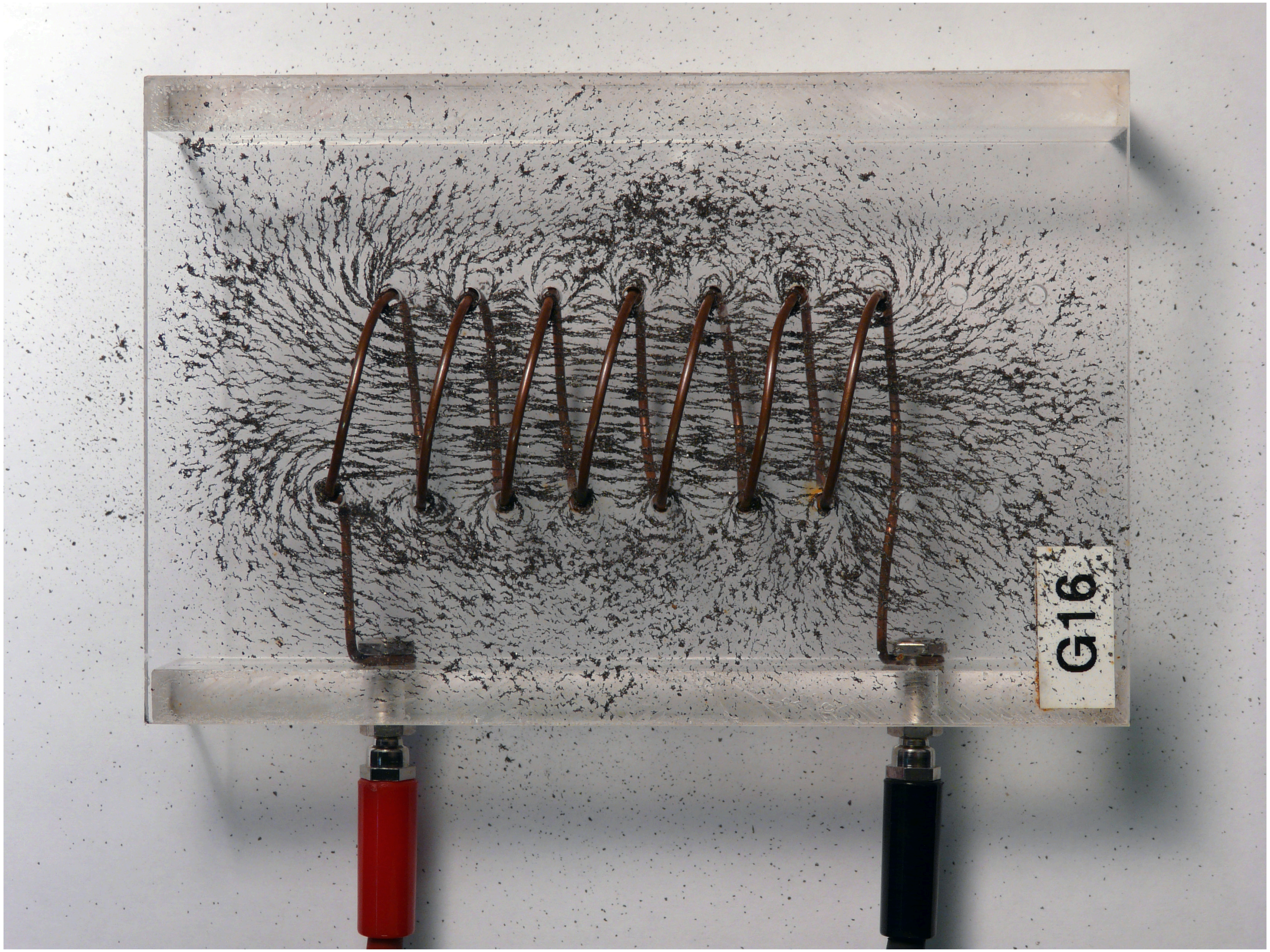






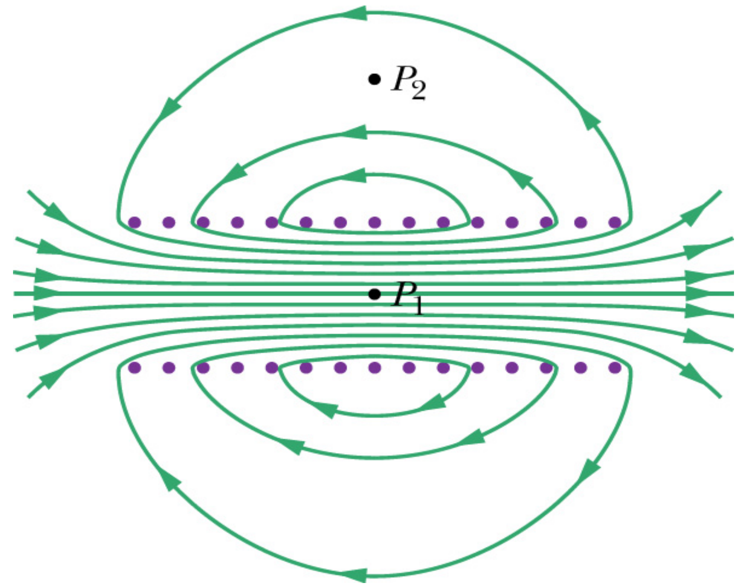
- A solenoid is a linear coil of wire
- A solenoid can be with or without a core
- The field direction depends on current direction and which way coil is wound

Iron filings show field inside solenoid



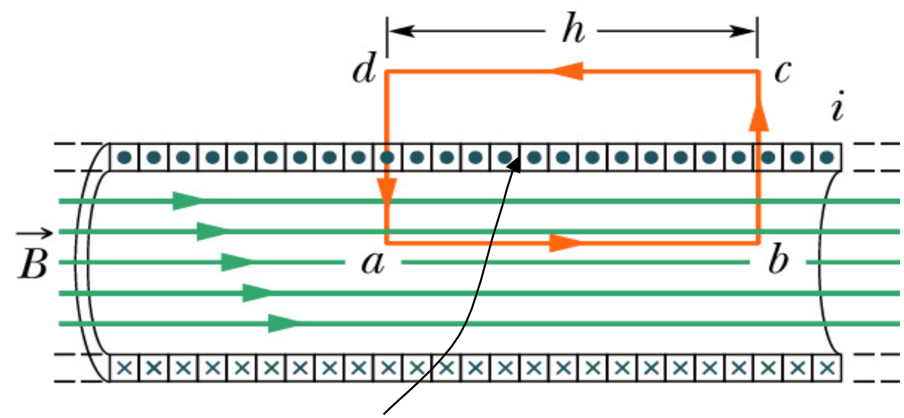
Solenoid field

- The actual field looks more like this:



- Use will use an approximation that the field is constant inside the solenoid and zero outside

- We characterize the windings in terms of number of turns per unit length, n.



only section that has non-zero contribution

Solenoid field

- We can use Ampère law to compute magnetic flux density B
- Assume each turn carries current i, so total current over length h is inh.
- Assume field is constant inside the solenoid and zero outside
- Therefore only central parallel region makes a contribution to B

From Ampere's law $$\boxed{\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}}$$

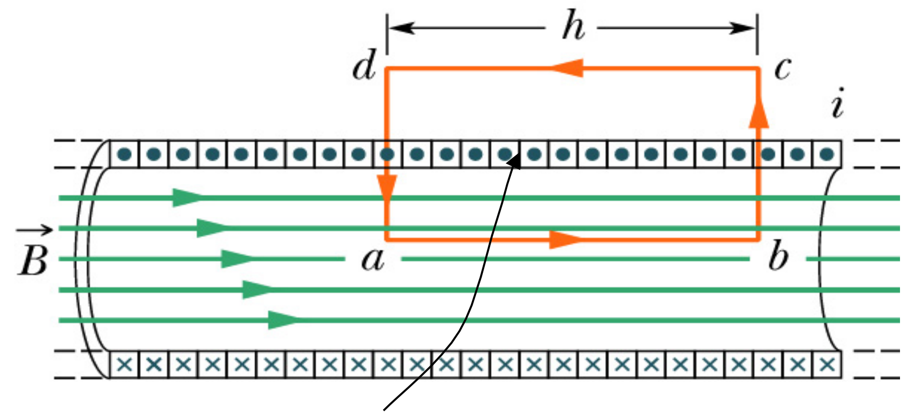
LHS gives $$\oint \vec{B} \cdot d\vec{s} = Bh$$

RHS gives $$\oint \vec{B} \cdot d\vec{s} = \mu_0 inh$$

Therefore $$\boxed{Bh = \mu_0 inh}$$

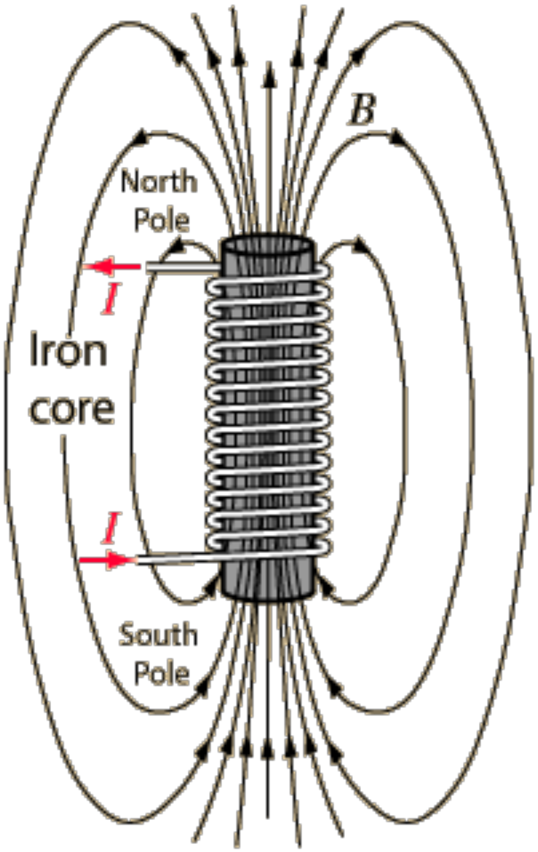
So $$\boxed{B = \mu_0 in} \boxed{= \mu_0 i \frac{N}{l}}$$

Where N=turns, l=length



only section that has non-zero contribution

Electromagnet core material



A ferromagnetic core makes a huge difference

$$B = \mu_r \mu_0 i n$$

Where $\mu_0 = 4\pi \times 10^{-7}$

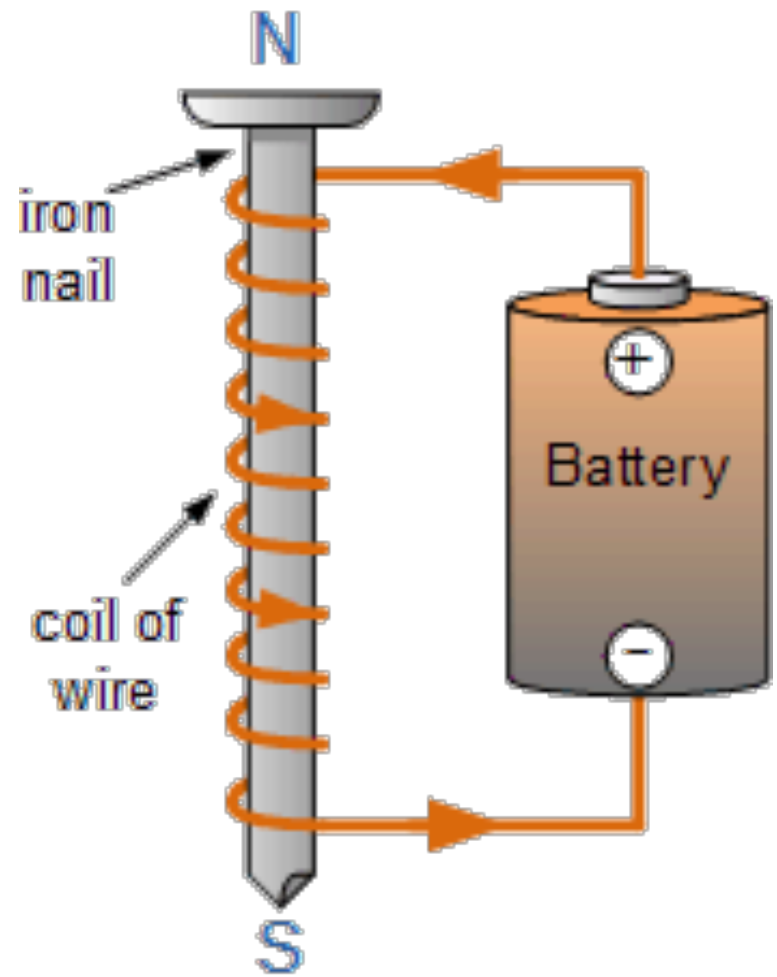
Some representative relative permeabilities:

magnetic iron 200
nickel 100
permalloy....................................8,000
(78.5% nickel, 21.5% iron)
mumetal....................................20,000
(75% nickel, 2% chromium,
5% copper, 18% iron)

at a magnetic flux density of 0.002 W/m^2

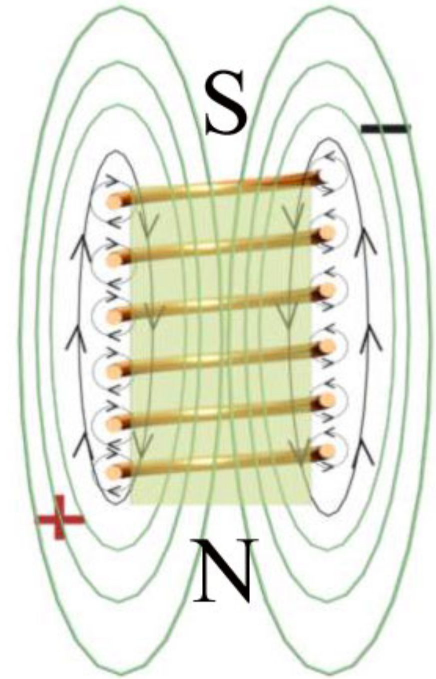
Electromagnet using a nail

- Its easy to build a simple electromagnet
- Can just wrap a coil around a nail
- What determines the steady state current flow?



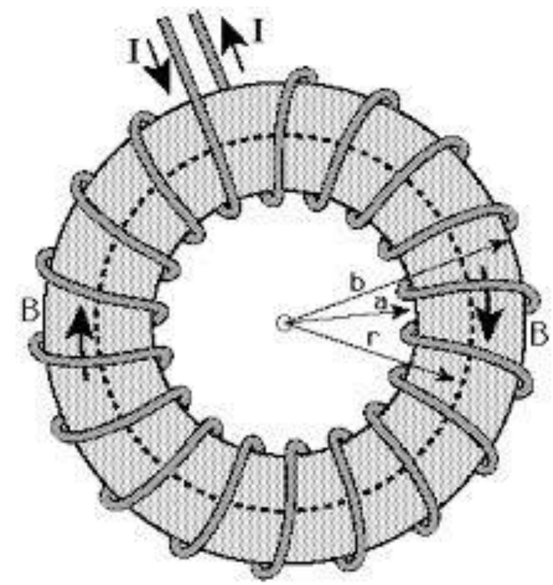
Toroid coil

- The field of the solenoid emerges at both ends, and spreads apart (and reduces) at the ends

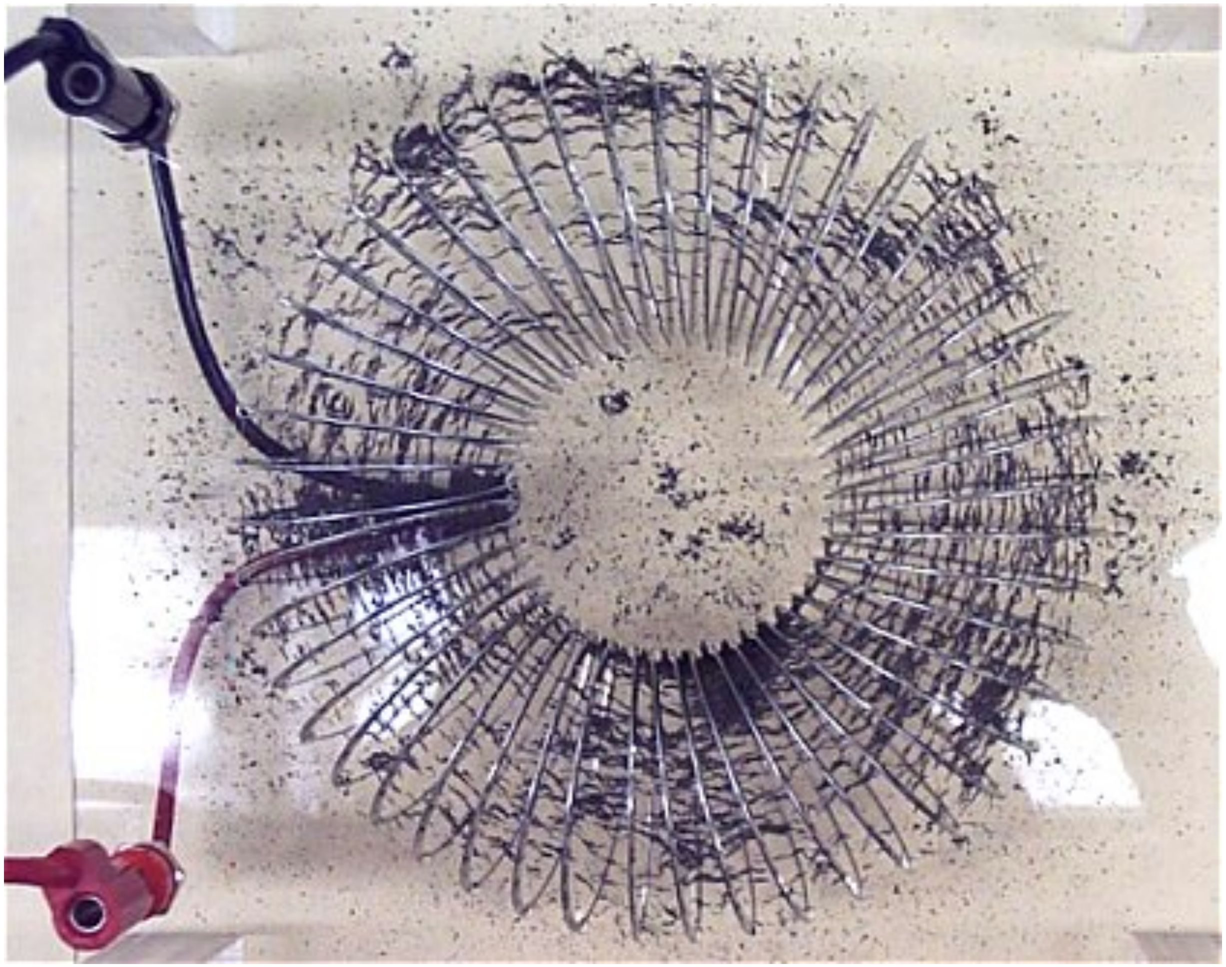


- If we wrap our coil around like a doughnut, so that it has no ends, this is called a toroid

- Now field has no ends, but wraps uniformly around in a circle

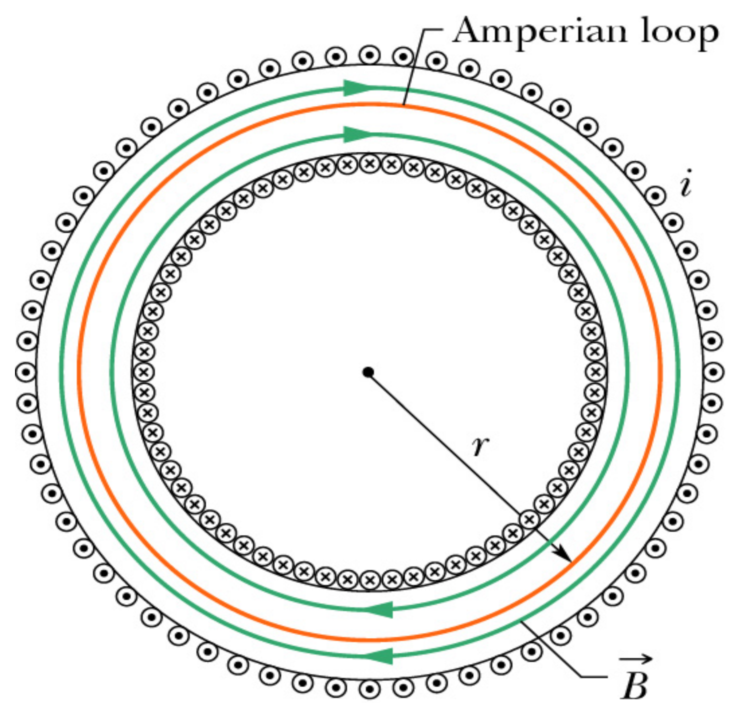


Iron filings show field inside toroid



B inside toroid coil

- To find B inside we draw an Amperian loop parallel to the field, with radius r.
- For coil with a total of N turns
- Amperian loop encloses current Ni.



From Ampere's law $$\boxed{\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}}$$

LHS gives $$\oint \vec{B} \cdot d\vec{s} = B.2\pi r$$

RHS gives $$\oint \vec{B} \cdot d\vec{s} = \mu_0 Ni$$

$$B.2\pi r = \mu_0 Ni \quad \boxed{\Rightarrow B = \frac{\mu_0 iN}{2\pi\ r}}$$

- Flux density inside toroid

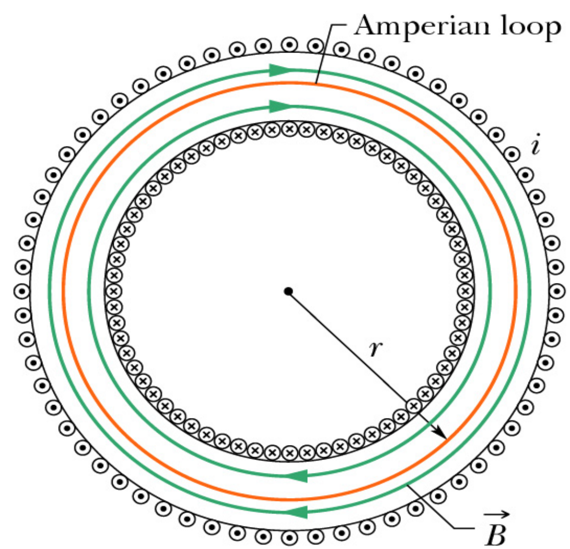
B outside toroid coil

- The magnetic field inside a Toroid is

$$B = \frac{\mu_0 i N}{2\pi r}$$

- Using an Amperian loop, it can be been that the magnetic field outside is zero

- To use a toroid as a low frequency AC transformer laminated soft iron or silicon steel good choice for the core



(b)

- Magnetically soft so little hysteresis, large relative permeability, laminated to prevent eddy currents

- Adding silicon to the iron increase of the resistivity of the metal, further reducing eddy currents still further

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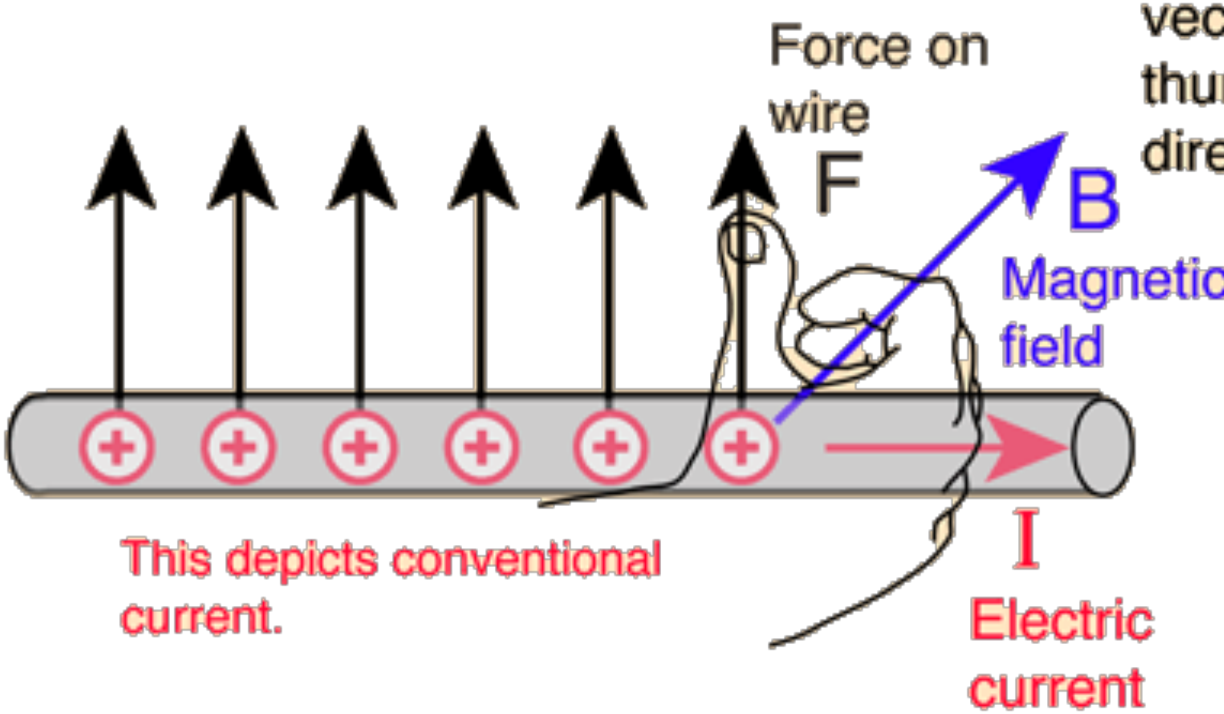
Lecture 2

Conductors in magnetic fields

Magnetic force on wire



Curl fingers as if rotating vector I into vector B. The thumb is then in the direction of the force F

$$\vec{F} = \vec{I}L \, x \, \vec{B}$$

Force on straight wire of length L

Magnetic force between two wires

- Wire carrying a current in a magnetic field feels a force
- When there are two parallel wires carrying current, the magnetic field from one causes a force on the other
- When the currents are parallel, the two wires are pulled together
- When the currents are anti-parallel, the two wires are forced apart

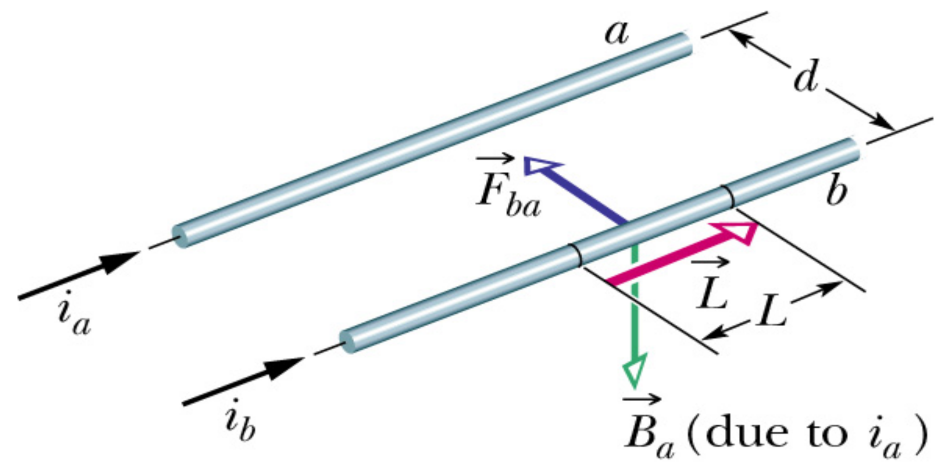
- Force on wire b due to wire a given by:

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a$$

From before, flux density due to current flowing in wire given by

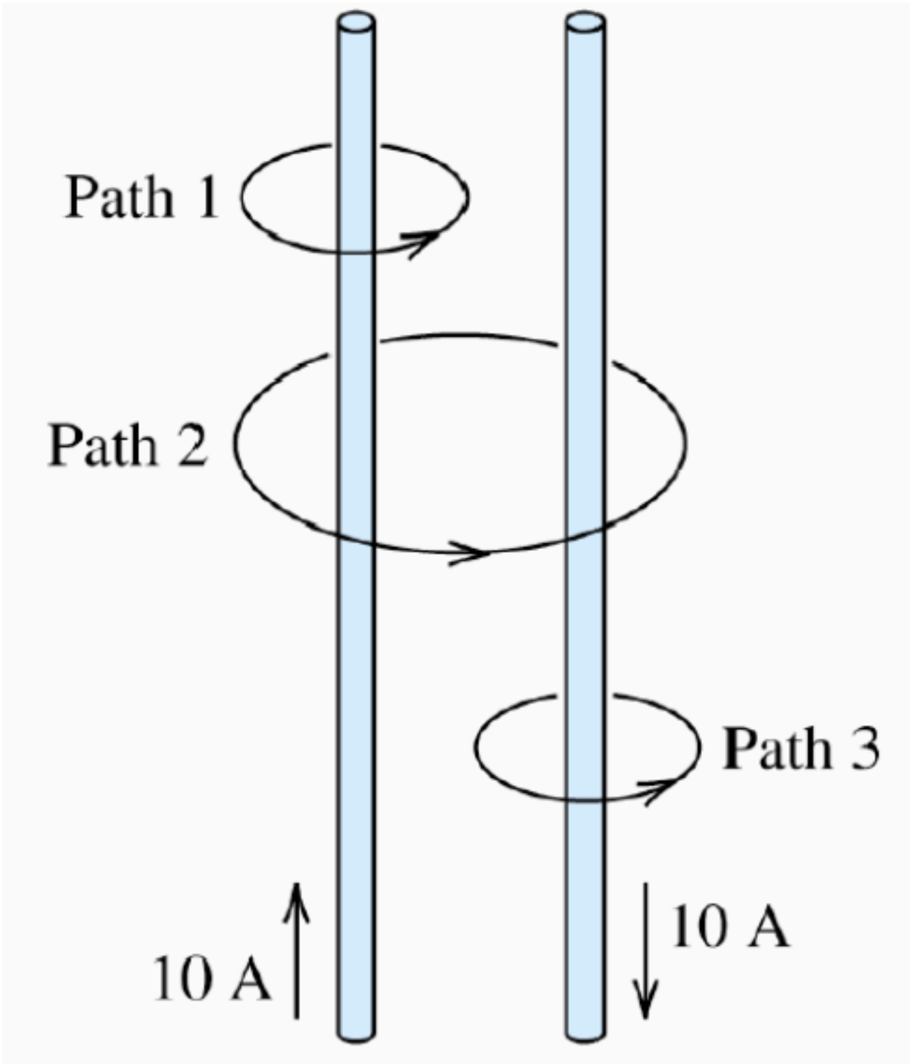
$$B = \frac{\mu_0 i_a}{2\pi\ d}$$

$$\boxed{\Rightarrow F_{ba} = \frac{\mu_0 i_a i_b L}{2\pi d}}$$



Is the force between two parallel currents

Magnetic force between two wires



Find the force between these two wires if they are 1 m long and separated by 0.1 m:

$$B_1(0.1m) = \frac{\mu_0 I_1}{2\pi r}$$

$$= \frac{(4\pi x 10^{-7})(10A)}{2\pi(0.1m)}$$

$$= 20\mu T$$

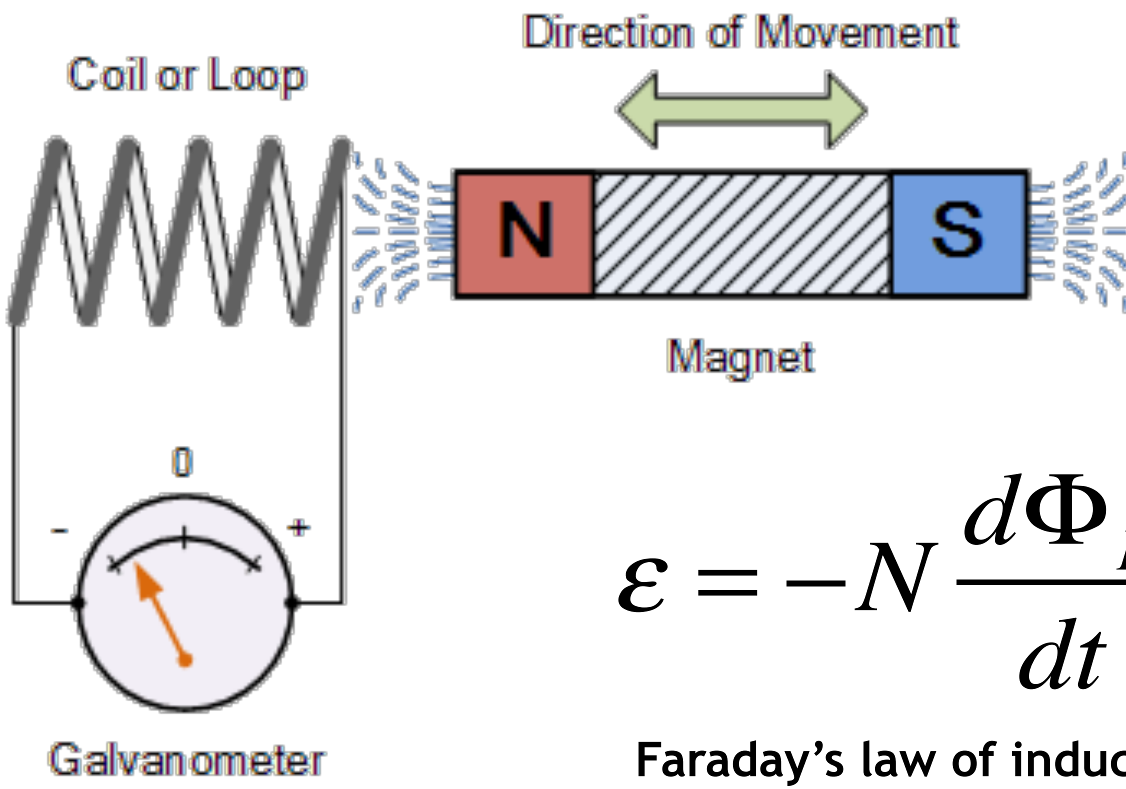
F = BIL

$$= (10A)(1m)(20\mu T)$$

$$= 200\mu N \ repulsive$$

Electromagnetic induction

- Moving a magnet in a coil will induce current



$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

Faraday's law of induction

Faraday's law of induction

- The magnitude of the EMF induced in a conducting loop is equal to the rate at which the magnetic flux through that loop changes with time

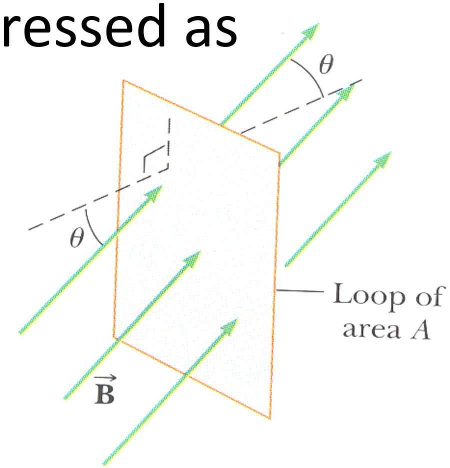
$$\varepsilon = -\frac{d\Phi_B}{dt}$$

- If a coil consists of N loops with the same area, the total induced EMF in the coil is given by

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

- In uniform magnetic field, the induced EMF can be expressed as

$$\varepsilon = -\frac{d}{dt}(BA\cos\theta)$$



- An EMF can be induced in several ways

  The magnitude of B can change with time

  The area enclosed by the loop can change with time

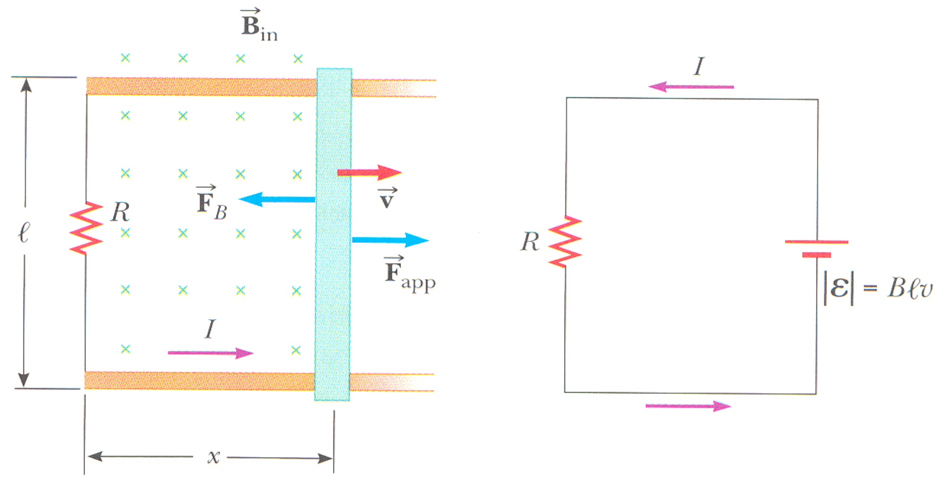
  The angle between B and the normal to the loop can change with time

  Any combination of the above can occur

Power used moving conduction in magnetic field B

- A conducting bar of length l sliding along two fixed parallel conducting rails.
- This induces current I
- The magnetic flux passing through the loop is

$$\Phi_B = Blx$$



- Using Faraday's law, we have

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d}{dt}(Blx) = Bl\frac{dx}{dt} = Blv$$

Also we have from Ohms law

$$I = \frac{\varepsilon}{R} = \frac{Blv}{R}$$

Therefore substitution gives

$$P = I^2 R = \left(\frac{Blv}{R}\right)^2 R = \frac{B^2 l^2 v^2}{R}$$

- The change in energy in the system must also equal to the transfer of energy into the system by work

- If we are moving with constant velocity $F_{app} = F_B = IlB$

Power supplied by the applied force is

$$P = F_{app} v = (IlB)v$$

Substituting I $\Rightarrow P = \dfrac{B^2 l^2 v^2}{R}$

Both the same!

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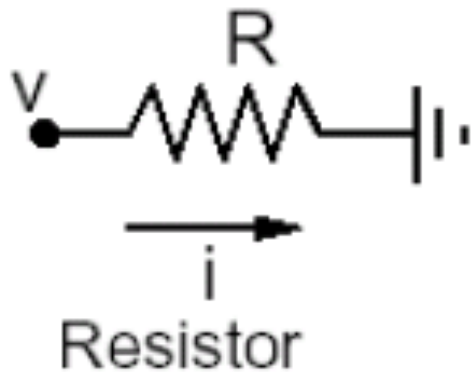
Lecture 2

Inductance

Electrical resistance

Resists the flow of current



$$v = iR$$

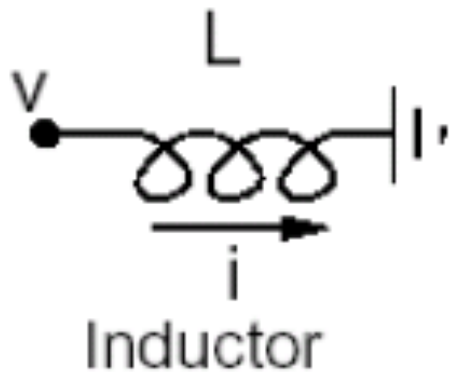
Where

i is the current in A

v is voltage in V

R is the resistance in Ω

Electrical inductance



$$v = -L\frac{di}{dt}$$

Where

i is the current in A

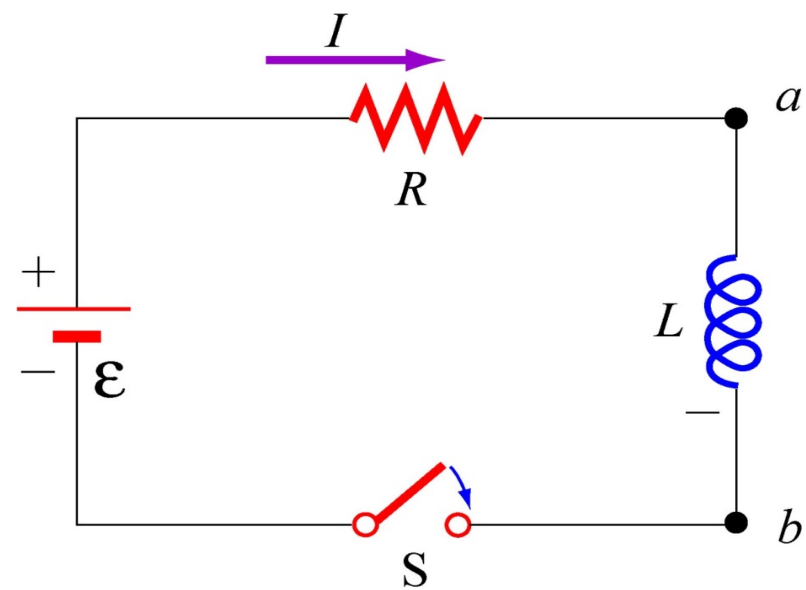
v is voltage in V

L is the inductance in H

Current in an LR circuit

$$V_B = V_R + V_L$$

Adding up voltages



$$V_R = IR$$

Ohms law

$$V_L = L\frac{dI}{dt}$$

Voltage proportional to rate of change of current

$$V_B = IR + L\frac{dI}{dt}$$

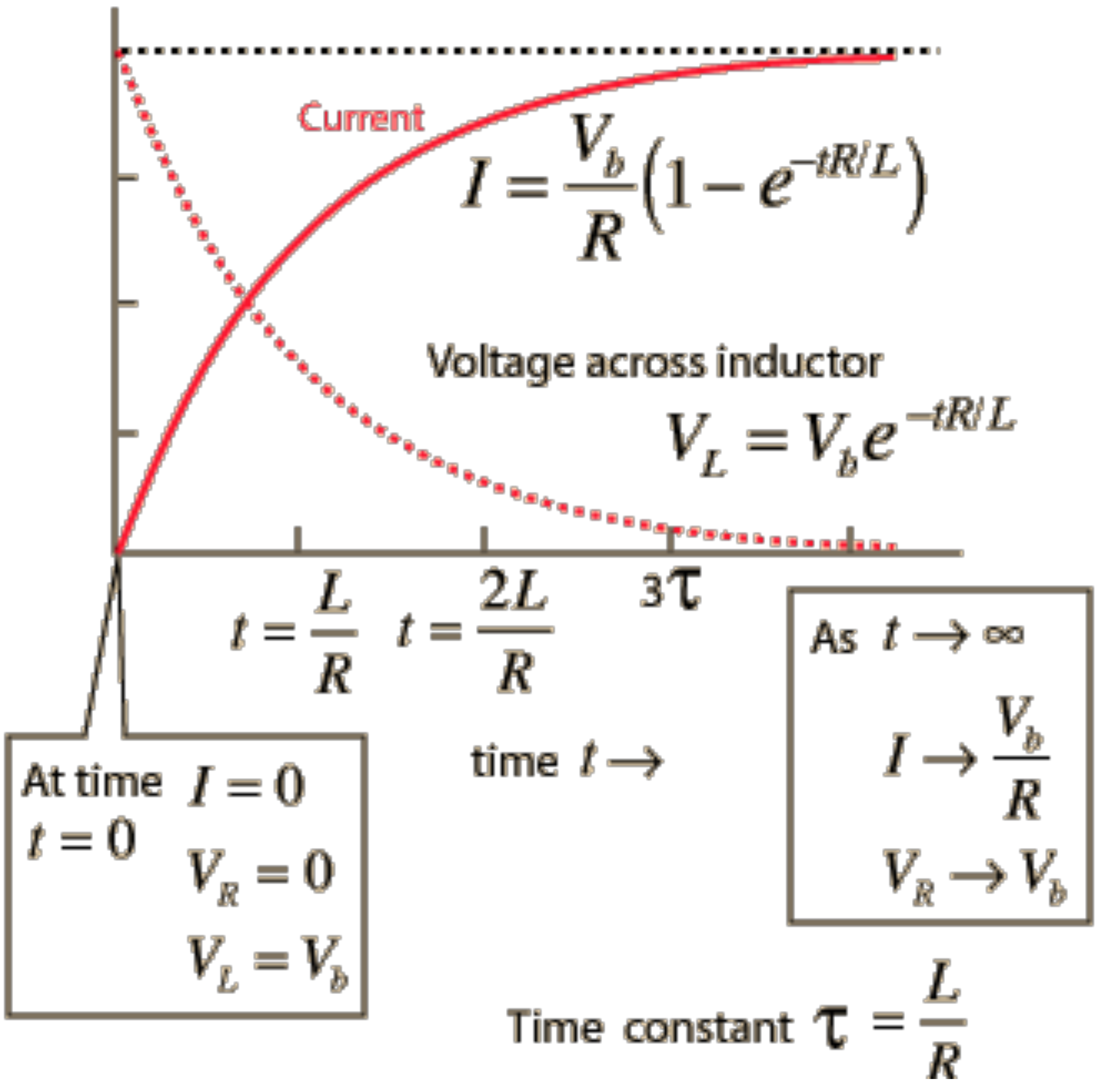
Substituted values

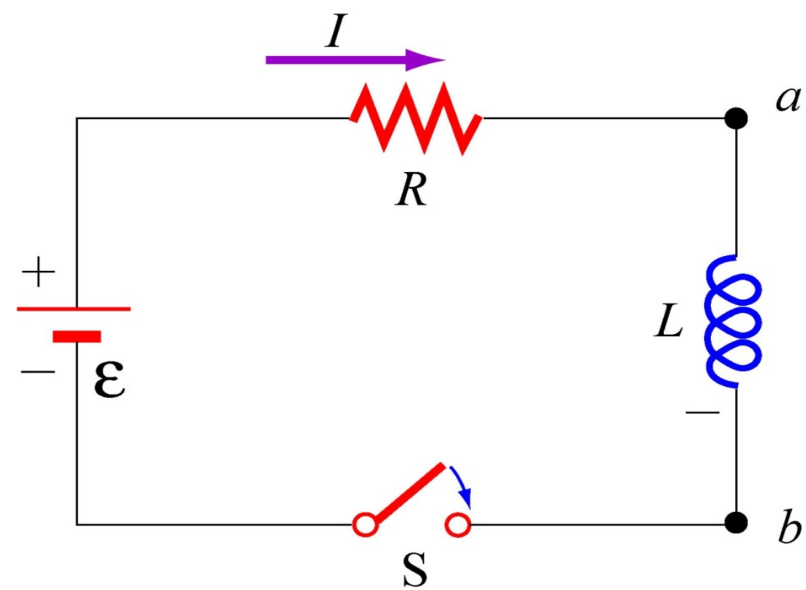
$$\frac{dI}{dt} = \frac{R}{L}\left(\frac{V_S}{R} - I\right)$$

Rearranged

This is a first order linear differential equation

Current in an LR circuit





- We need to consider this behaviors wherever we switch inductive circuits and coils

Definition of inductance

- Arising from Faraday's law

$$\varepsilon_L = -N\frac{d\Phi_B}{dt}$$

- Inductance L may be defined in terms of the EMF generated to oppose a given change in current:

$$\boxed{EMF = -L\frac{di}{dt}}$$

therefore

$$\boxed{\varepsilon_L = -N\frac{d\Phi_B}{dt} = -L\frac{di}{dt}}$$

- Integrating both sides w.r.t time

$$\boxed{N\Phi_B = Li}$$

- For any inductor

$$\boxed{L = \frac{N\Phi_B}{i}}$$

Inductance of a solenoid

- For a fixed area and changing current, Faraday's law becomes

$$EMF = -N\frac{d\Phi}{dt} \qquad = -NA\frac{dB}{dt}$$

- Since the magnetic field of a solenoid is

$$B = \frac{\mu NI}{l}$$

- Then the EMF is approximated by

$$EMF = -NA\frac{d}{dt}\left(\frac{\mu NI}{l}\right) \qquad = -\frac{\mu N^2 A}{l}\frac{dI}{dt}$$

- From one suitable property of inductance

$$EMF = -L\frac{dI}{dt}$$

- By inspection we obtain

$$L = \frac{\mu N^2 A}{l}$$

- Alternatively:
- From definition of inductance

$$L = \frac{N\Phi_B}{I}$$

- Substitute flux ϕ=BA

$$\Rightarrow L = \frac{N(\frac{\mu NI}{l}A)}{i}$$

$$\Rightarrow L = \frac{\mu N^2 A}{l}$$

- Where
- N=number of turns
- l=length
- A=cross sectional area
- R = toroid radius to centerline

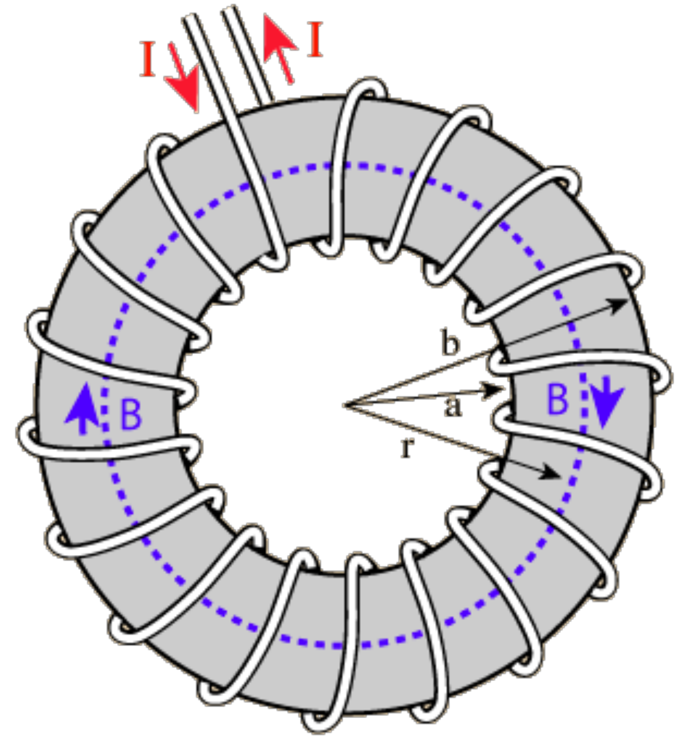
Inductance of a toroid

- Again, for a fixed area and changing current, Faraday's law becomes

$$EMF = -N\frac{d\Phi}{dt} \quad = -NA\frac{dB}{dt}$$

- Since the magnetic field of a toroid is

$$\boxed{B = \frac{\mu I N}{2\pi\ r}}$$



- Then the EMF is approximated by

$$EMF = -NA\frac{d}{dt}\left(\frac{\mu I N}{2\pi\ r}\right) \qquad = -\frac{\mu N^2 A}{2\pi\ r}\frac{dI}{dt}$$

- From the definition of inductance

$$EMF = -L\frac{dI}{dt}$$

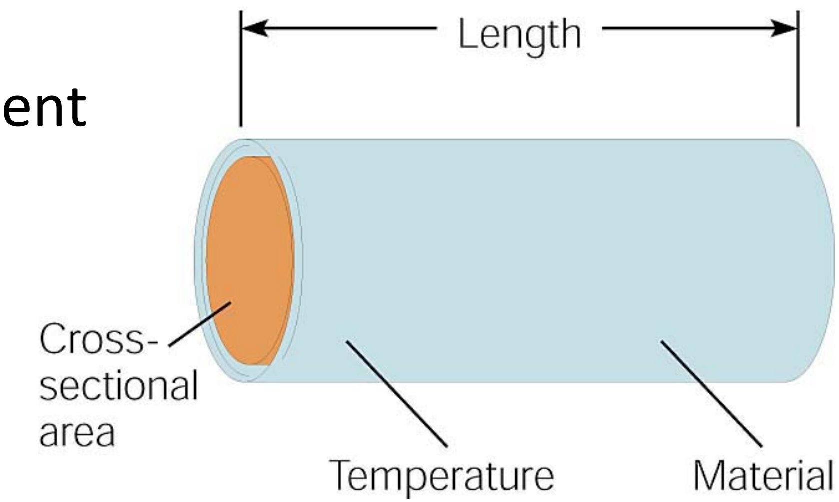
- By inspection we obtain

$$L = \frac{\mu N^2 A}{2\pi r}$$

- Where
- N=number of turns
- A=cross sectional area
- R = toroid radius to centerline

Resistance of a coil

- Transient current response is affected by inductance
- However steady state current in electromagnet depends on resistance
- Ohm's Law: Resistance = Voltage / Current



$$R = \frac{\rho L}{A}$$

Where
L = length
A =area
P = resistivity

Material	Resistivity ρ (ohm m)	
Copper	1.68	$x10^{-8}$
Copper, annealed	1.72	$x10^{-8}$
Aluminum	2.65	$x10^{-8}$
Tungsten	5.6	$x10^{-8}$
Iron	9.71	$x10^{-}$
Nichrome (Ni,Fe,Cr alloy)	100	$x10^{-8}$

Diameter of standard wire gauges (SWG)

	Diameter	
SWG	inches	mm
7/0	0.500	12.700
6/0	0.464	11.786
5/0	0.432	10.973
4/0	0.400	10.160
3/0	0.372	9.449
2/0	0.348	8.839
1/0	0.324	8.236
1	0.300	7.620
2	0.276	7.010
3	0.252	6.401
4	0.232	5.893
5	0.212	5.385
6	0.192	4.877
7	0.176	4.470
8	0.160	4.064
9	0.144	3.658
10	0.128	3.251
11	0.116	2.946
12	0.104	2.642

	Diameter	
SWG	inches	mm
13	0.092	2.337
14	0.080	2.032
15	0.072	1.829
16	0.064	1.626
17	0.056	1.422
18	0.048	1.219
19	0.040	1.016
20	0.036	0.914
21	0.032	0.813
22	0.028	0.711
23	0.024	0.610
24	0.022	0.559
25	0.020	0.508
26	0.018	0.457
27	0.0164	0.417
28	0.0148	0.376
29	0.0136	0.345
30	0.0124	0.315
31	0.0116	0.295

	Diameter	
SWG	inches	mm
32	0.0108	0.274
33	0.0100	0.254
34	0.0092	0.234
35	0.0084	0.213
36	0.0076	0.193
37	0.0068	0.173
38	0.006	0.152
39	0.0052	0.132
40	0.0048	0.122
41	0.0044	0.112
42	0.004	0.102
43	0.0036	0.091
44	0.0032	0.081
45	0.0028	0.071
46	0.0024	0.061
47	0.002	0.051
48	0.0016	0.041
49	0.0012	0.030
50	0.001	0.025

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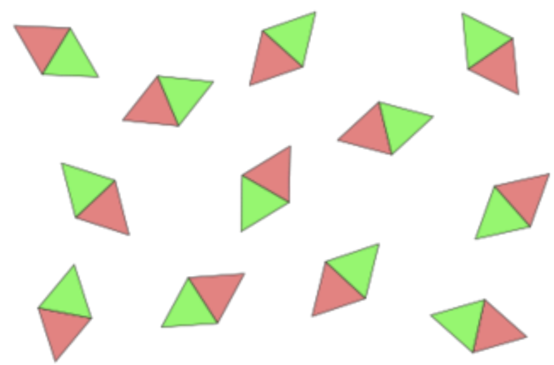
Lecture 2

Magnetic materials

Types of magnetism: Paramagnetism

- Paramagnetism
  - Have a small positive susceptibility to magnetic fields
  - Tendency of magnetic dipoles to align with an external magnetic field
  - Magnetization is proportional to the applied magnetic field



Dipoles - field absent

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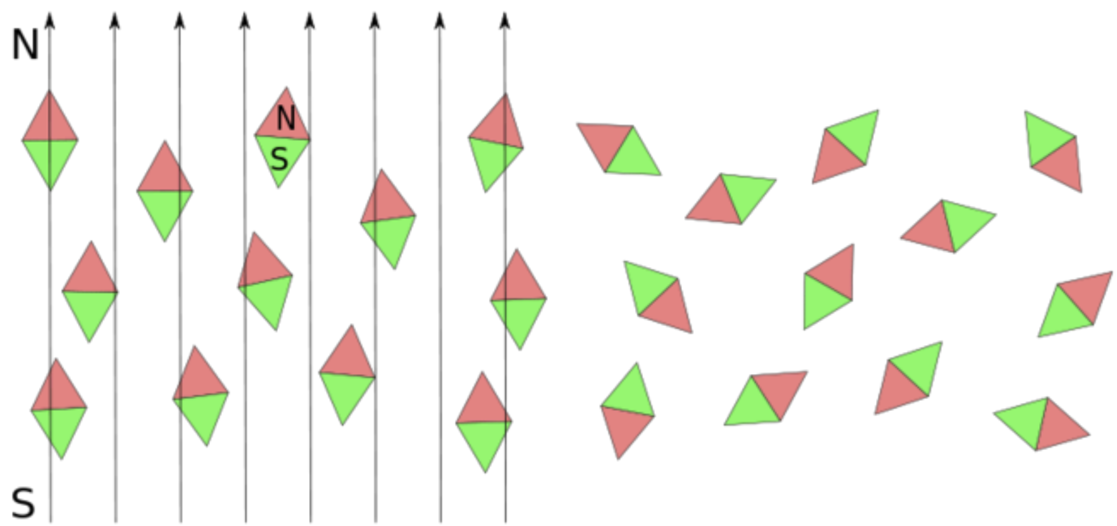
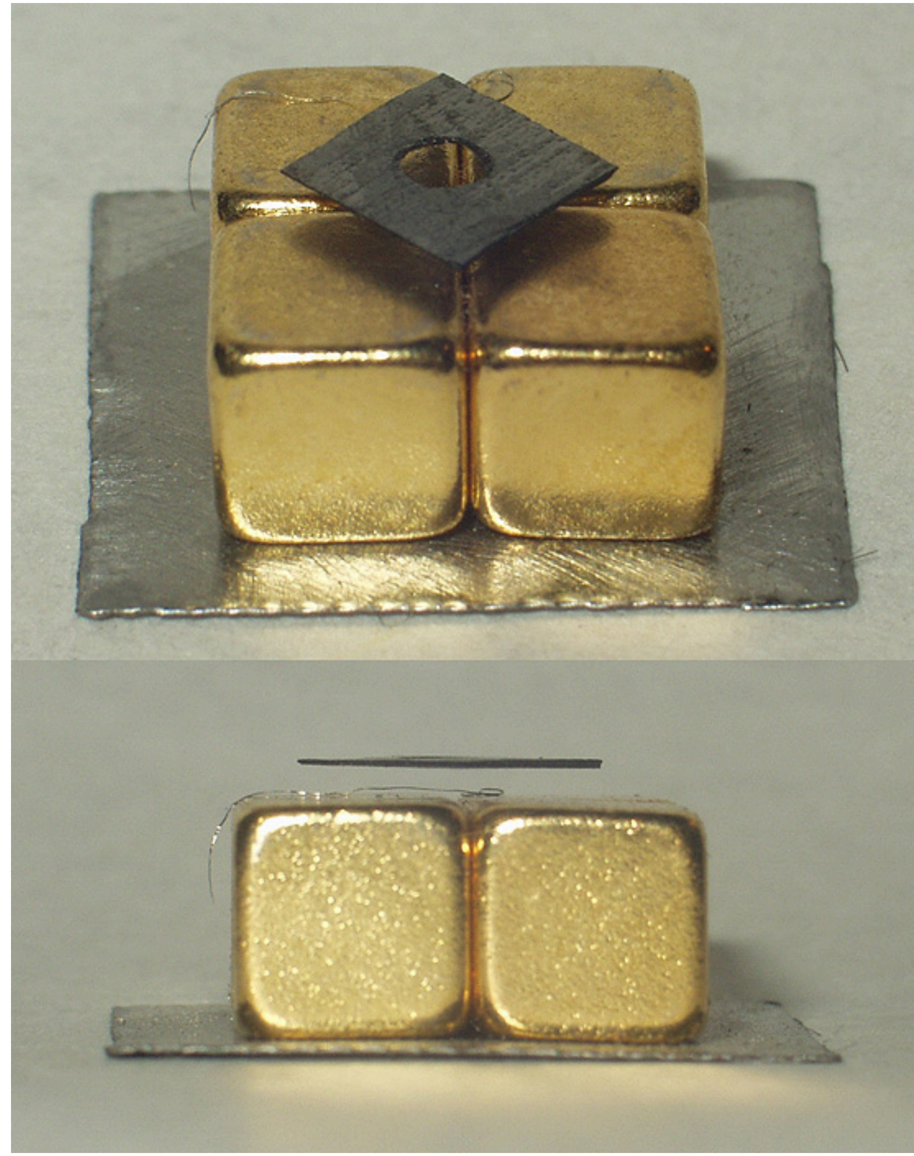


Dipoles - field present **Dipoles - field absent**

Types of magnetism: Diamagnetism

Diamagnetism

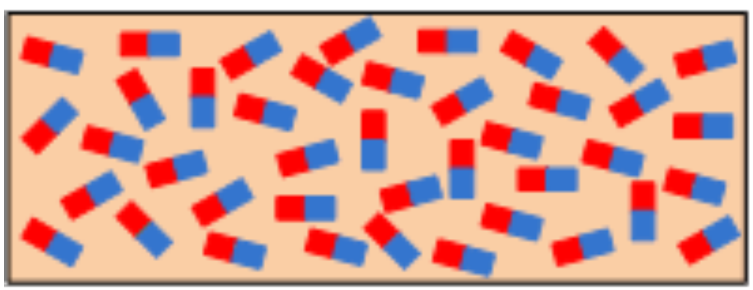
- Induced magnetic field in a direction opposite to an externally applied magnetic field
- Tend to oppose applied field



- Pyrolytic graphite will levitate over a permanent magnet array

Types of magnetism: Ferromagnetism

- Ferromagnetism
  - Unpaired electron spins to line up parallel with each other in a region called a domain
  - Mechanism by which ferromagnetic materials form permanent magnets
  - This is the type of magnetics useful for actuation



Loose and Random Magnetic Domains

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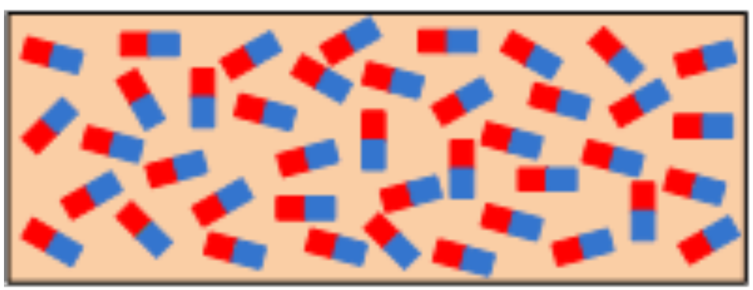
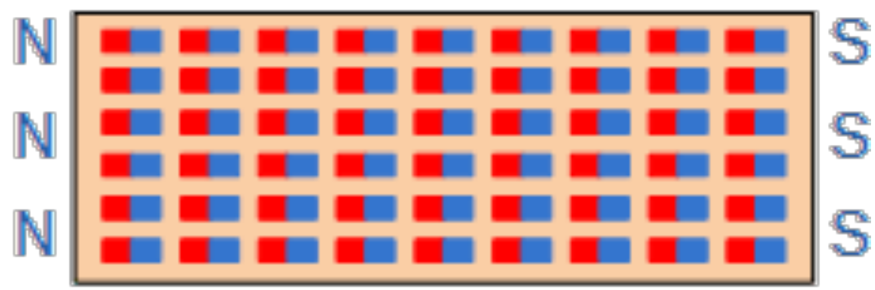


Loose and Random
Magnetic Domains



Effect of Magnetization
Domains Lined-up in Series

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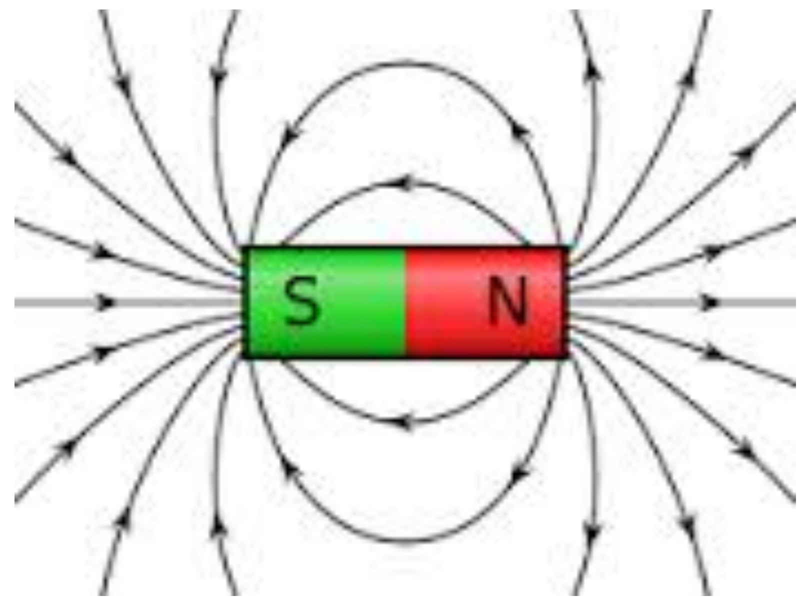
Lecture 2

Permanent magnets

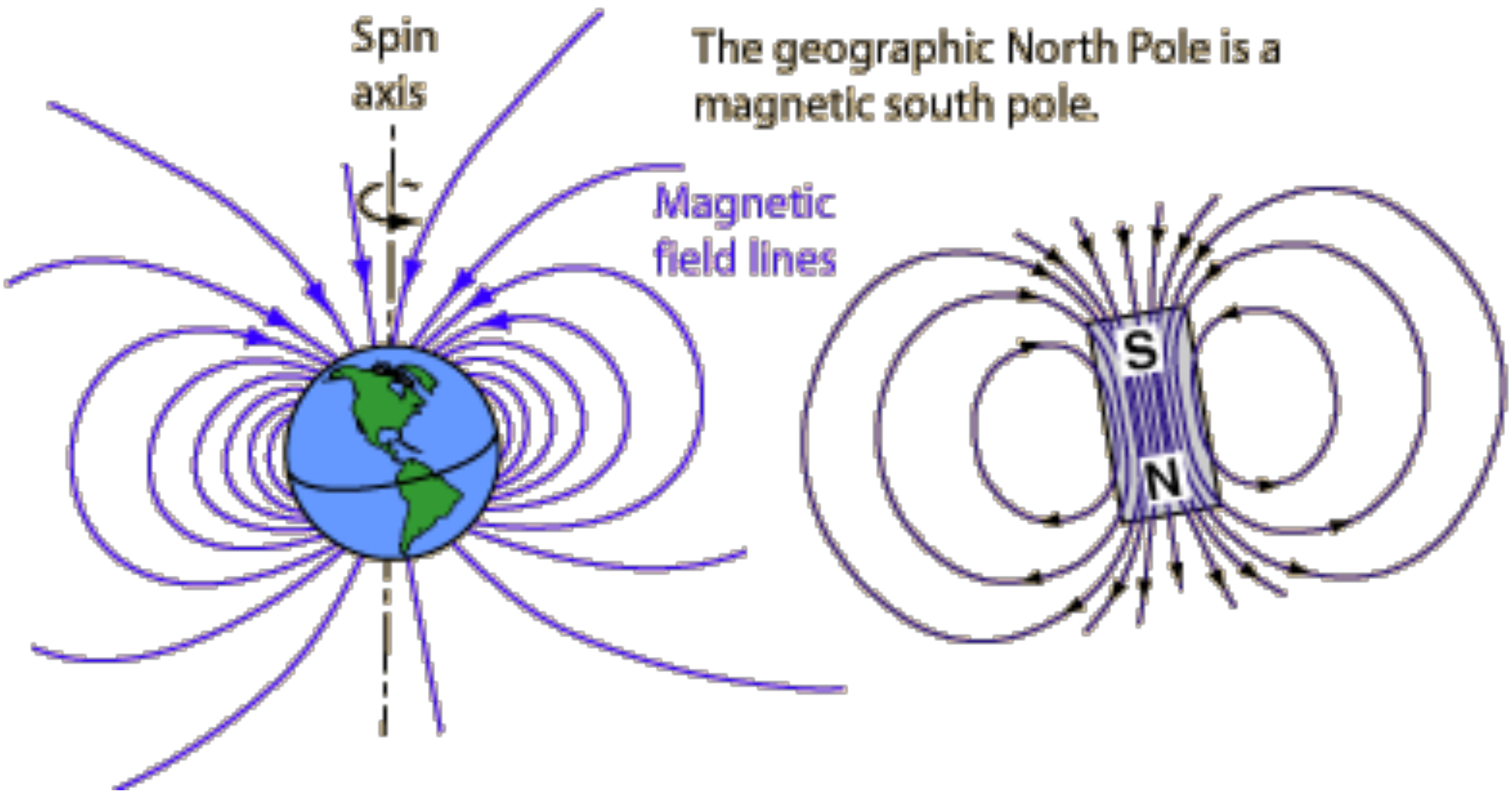
Permanent sources of magnetism

- Lodestone occurs naturally
- Permanent magnets – rare earth, alnico, etc.



Earth’s magnetic field

- The earth also generates an magnetic field



Ferrite magnets

- Used in applications where cost needs to be as low as possible
- Dark graphite grey color
- Reasonable resistance to demagnetization
- Operate at temperatures up to 180 degrees C.
- Considerably weaker than Neodymium magnet

Alinico (AlNiCo)

- Made from aluminium, nickel and cobalt
- Less brittle than most rare-earth magnets
- Largely been replaced by stronger rare-earth magnets
- Still commonly used in the manufacturing of sensors, guitar pickups, loudspeakers

Samarium cobalt (SmCo)

- First types of rare-earth magnets
- Commercially available for almost 30 years
- Operate in high temperatures –
- Good for industrial manufacturing
- Superior resistance to corrosion.
- Brittle and can be chipped and cracked easily
- Unsuitable for applications that require repetitive direct impact to the surface of the magnet.
- Should be recessed into a hole or groove to protect the magnet from impact

Neodymium magnets(NdFeB)

- Developed in the mid 1980s
- Most powerful of all permanent magnets
- Used where the strongest magnetic force is required in smallest volume
- Capable of lifting in excess of 1,000 times their own weight
- Emit deep magnetic fields
- Hard and brittle
- Protective plating needed to avoid rusting

Neodymium – very powerful

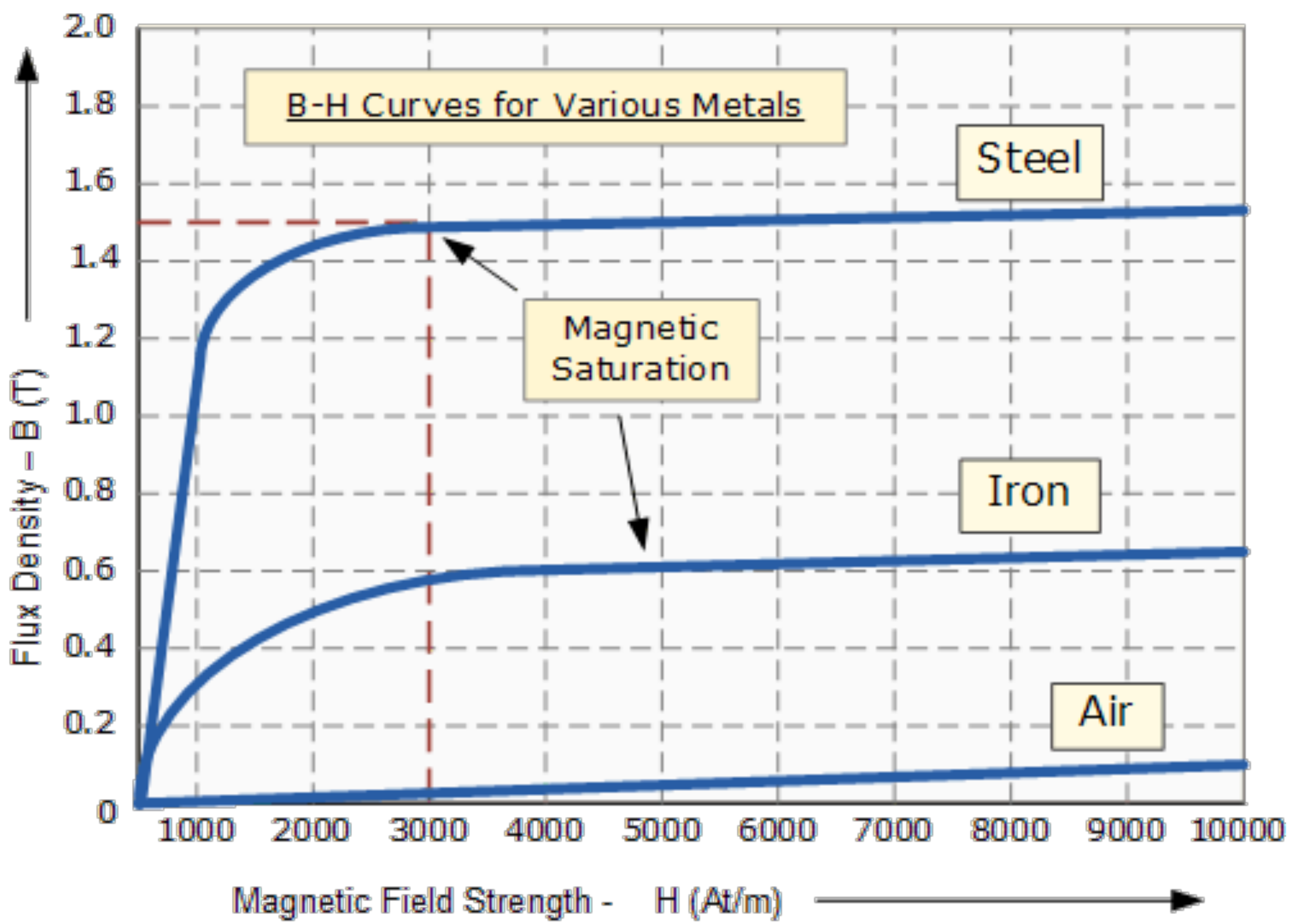
- Large magnets can be dangerous
- May shatter if two allowed to slam together
- Handle with care

$$d = 0.012\,\text{meters}$$

$$B = 0.5\,\text{Tesla}$$

$$h = 0.019\,\text{meters}$$

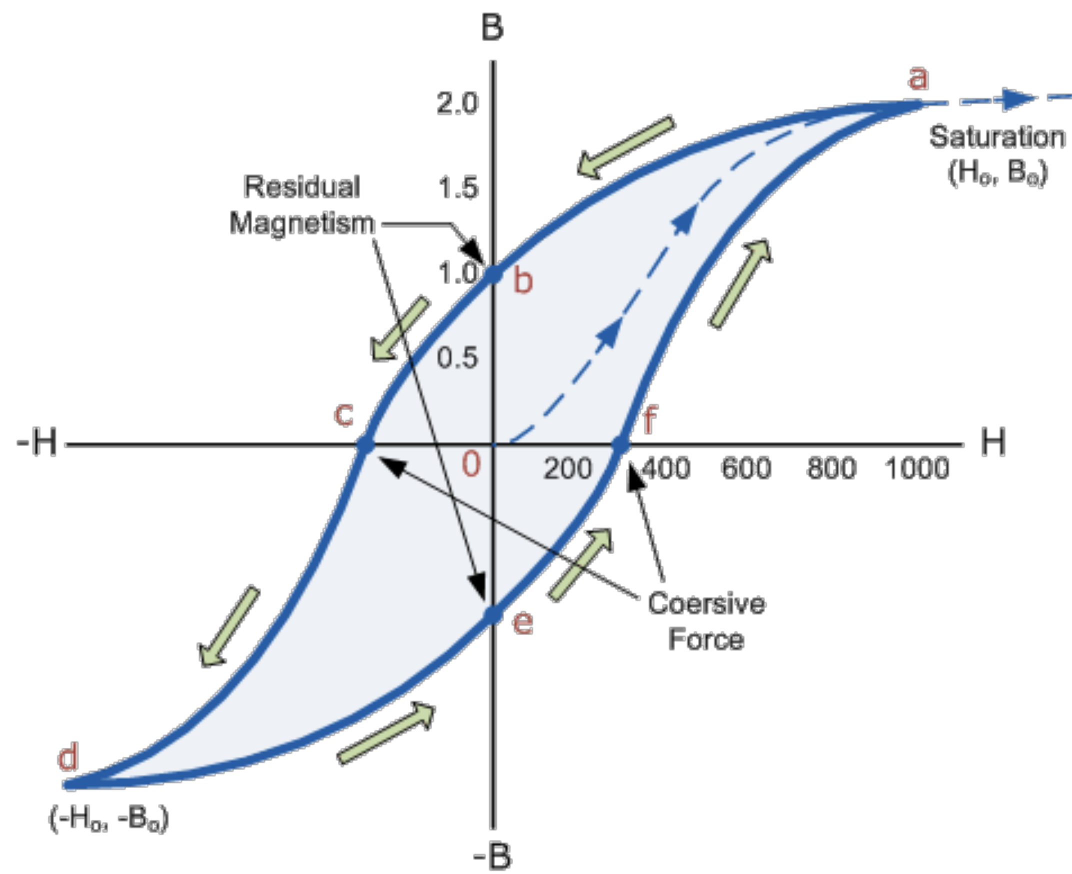
Magnetization or B-H Curve of ferromagnetic material



Magnetic hysteresis loop

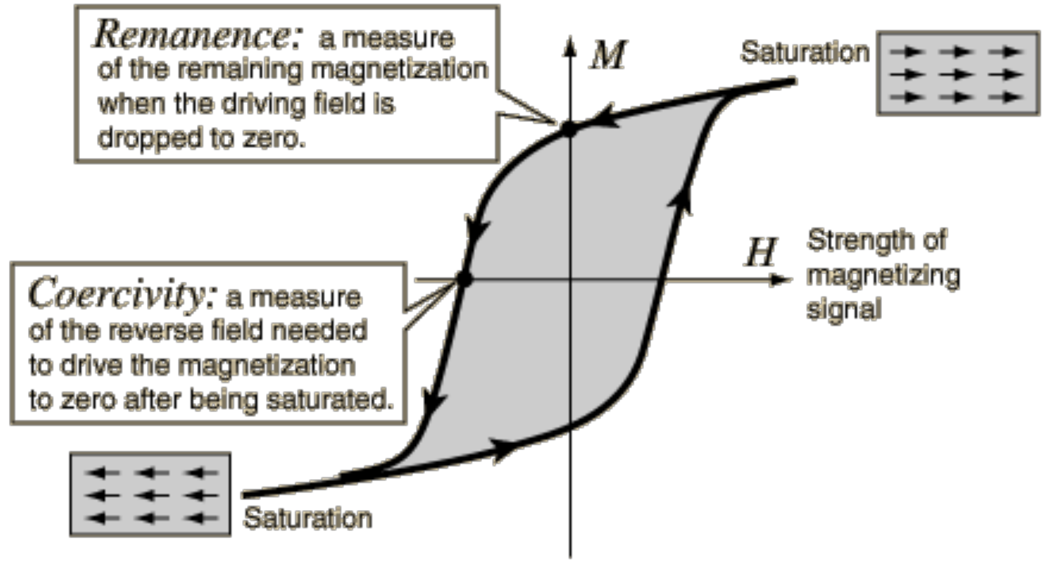
- When magnetic field H is applied to magnetic material the value of the magnetic flux density B does not follow linearly, but rather depends on the part history of the applied field H.
- Initial application of H will magnetize the material and the path starts at the origin. Thereafter reducing H will cause a reduction of B but along a higher path and when H reaches zero B will represent the remnant magnetism

- It will be then necessary to apply a negative field H to achieve a zero value of B.
- Therefore as B is plotted against H that goes up down and then changes sign, the characteristic does not consist of a straight-line passing through the origin but rather takes the form of a loop. The larger the area of the loop the larger the hysteresis of the magnetic material.



Important characteristic of permanent magnets

- Coercivity– determines how easy to demagnetize by applying an magnetic field
- Remanence – the value of B remaining after the magnetization process
- Energy product BH – relates to how much material requires to generate required flux density B



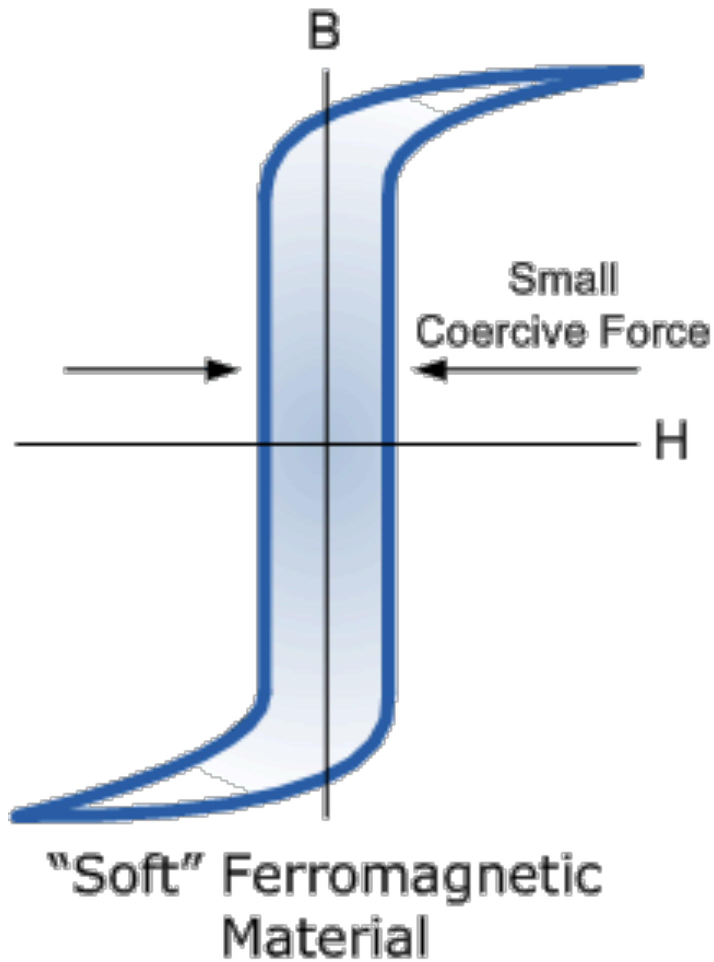
- Operating temperature – at which the magnet can operate satisfactorily
- Direction of magnetization – the location of the North and South poles
- Demagnetization temperature – determined by Curie temperature
- Mechanical robustness – how brittle or tough material is (cf Neodymium versus AlNiCo)
- Chemical stability in environment – sometimes need plating or epoxy coating
- Mechanical dimensions and geometry – bar, rod, square, arc, etc. Choose appropriately for the application

Hard and soft magnetic materials

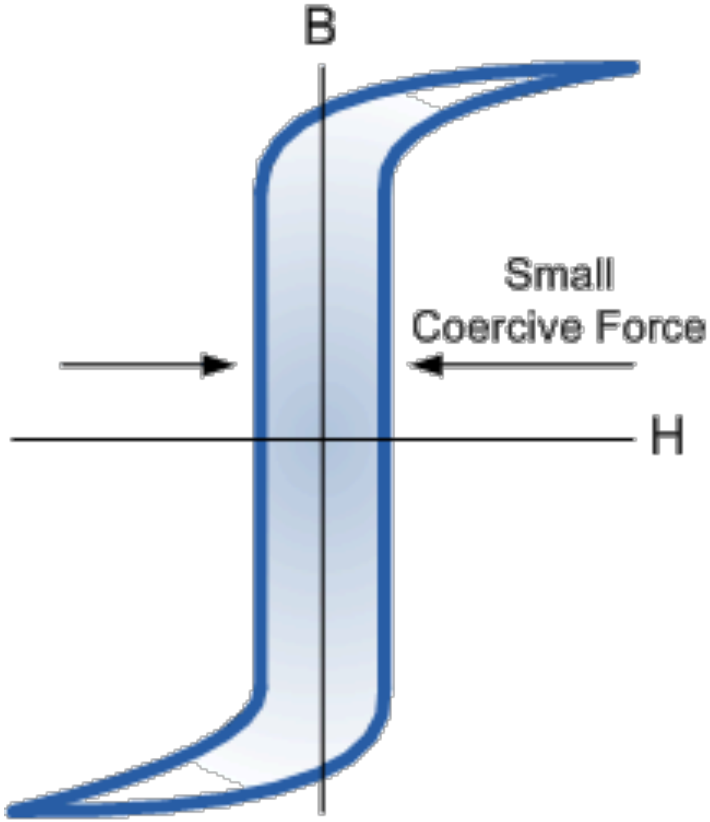
- An example of a hard magnetic material is AlNiCo.
- Exhibit high hysteresis and therefore retain their magnetism after being magnetized
- Can be used to make permanent magnets.
- An application could be the magnet for a loudspeaker.

- Soft iron is soft magnetic material. Is has a high relative permeability so it increases the flux density from a coil wrapped around it.
- Low magnetic hysteresis and therefore does not remain magnetism when the magnetic field it removed.
- Good choice form the core of a lifting electromagnet that can be switched on and off

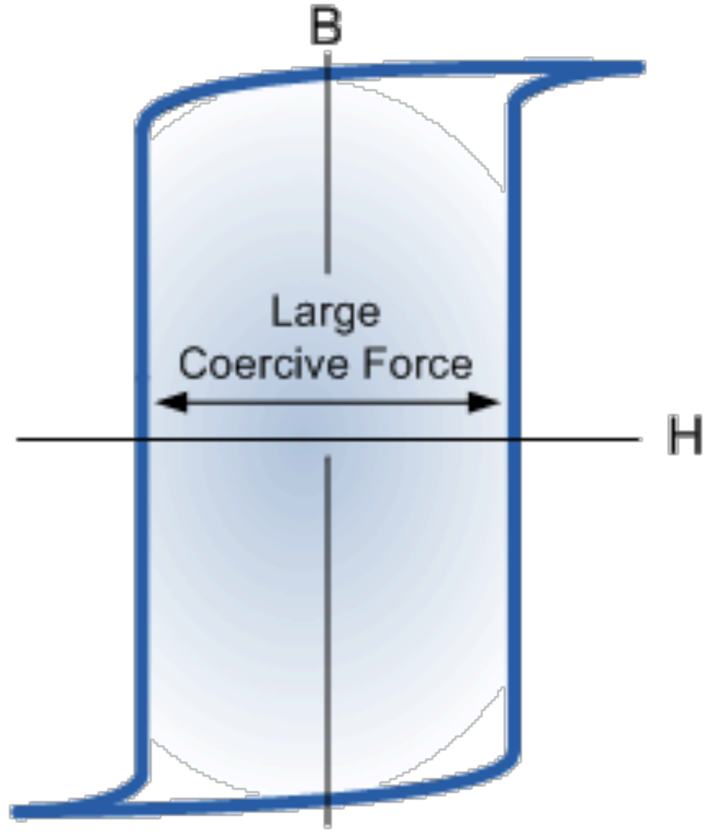
Magnetic hysteresis loops for soft and hard materials



Magnetic hysteresis loops for soft and hard materials



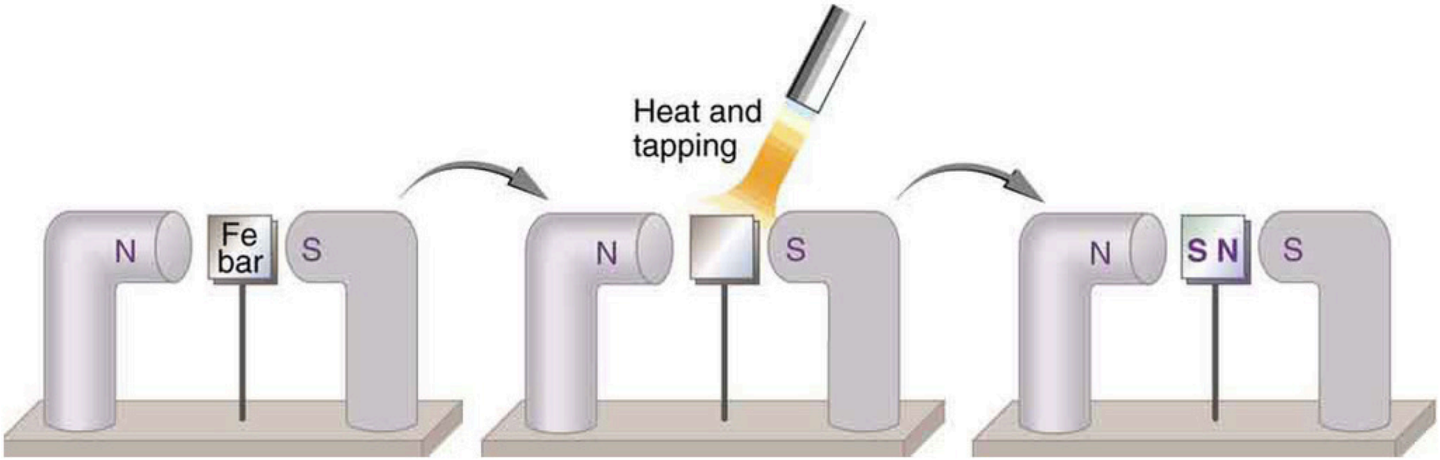
"Soft" Ferromagnetic
Material



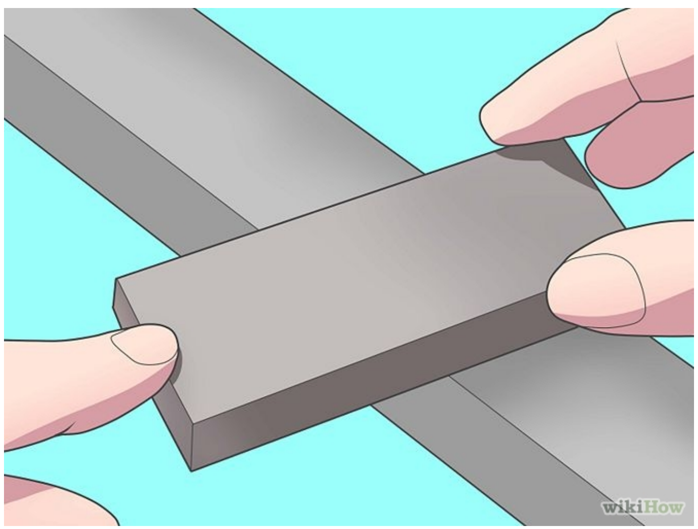
"Hard" Ferromagnetic
Material

Making a permanent magnet

- There are various ways to magnetize ferromagnetic materials



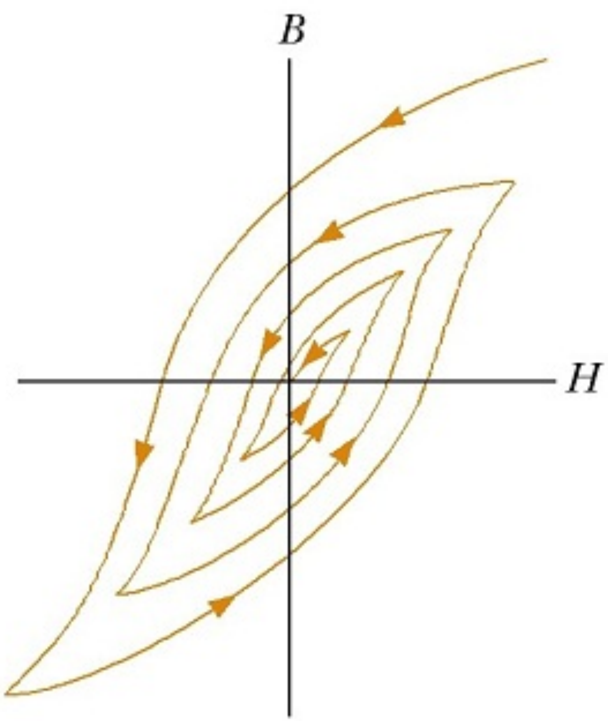
- Apply a magnetic field
- Tapping and heating/cooling will help here too



- Stroke material with permanent magnet

Demagnetizers

- Use AC driven coil
- Amplitude of current dies off slowly
- Therefore applied field slowly decays too
- BH curve therefore spiral inwards around origin
- This removes any residual magnetism



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