ROCO222: Intro to sensors and actuators

Lecture 10

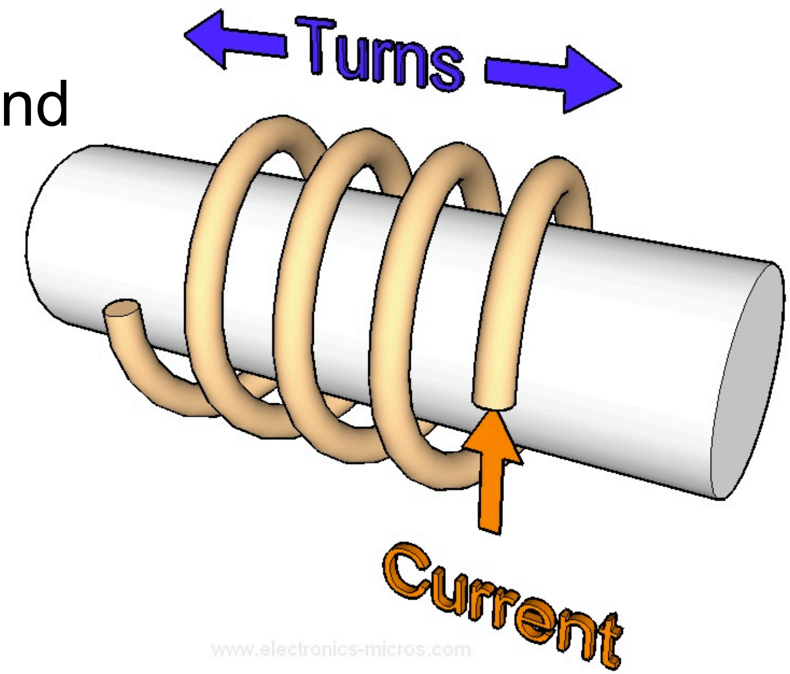
Magnetic circuits

Magnetomotive force F

Magnetomotive force (MMF) is the product of the current passed through a coil, and the number of turns

MMF is analogous to voltage in electric circuit

$F = NI$
where N is the number of turns in the coil and
I is the electric current through the circuit

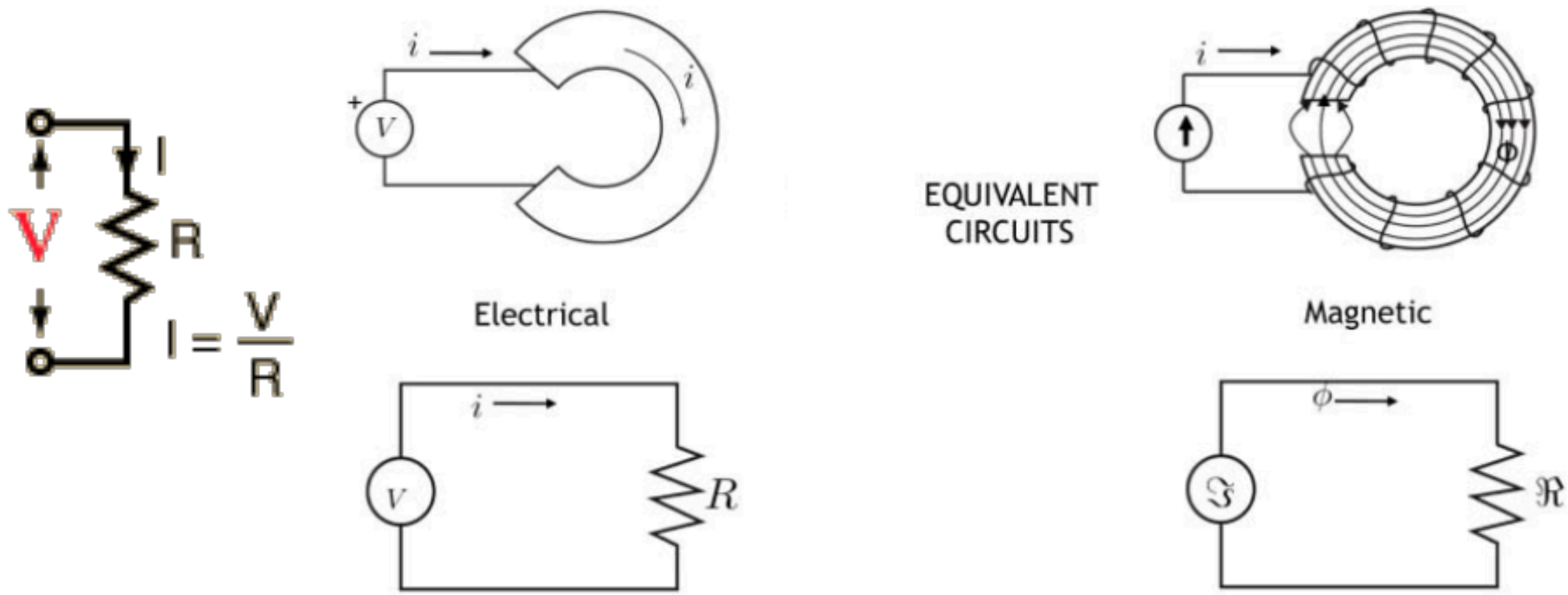


$F = \Phi R$
where Φ is the magnetic flux and
R is the reluctance

analogous to
V=IR in electric circuit

$F = HL$
where H is the magnetizing force (the strength of the magnetizing field) and L is the mean length of a solenoid or the circumference of a toroid

Electrical circuit analogy



Electrical	Magnetic
Voltage v	Magnetomotive Force $\Im = Ni$
Current i	Magnetic Flux ϕ
Resistance R	Reluctance $\Re$
Conductivity $1/\rho$	Permeability μ
Current Density J	Magnetic Flux Density B
Electric Field E	Magnetic Field Intensity H

Magnetic circuit definitions

Magnetomotive Force – mmf

- The "driving force" that causes a magnetic field
- Symbol, F
- Definition, F = NI
- Note too F = HL
- Units, Ampere-turns, (A-t)

Magnetic Field Intensity

- mmf gradient, or mmf per unit length
- Symbol, H
- Definition, H = F/L = NI/L
- Units, (A-t/m)

Magnetic circuit definitions

Flux Density

- The concentration of the lines of force in a magnetic circuit
- Symbol, B
- Definition, $B = \Phi/A$
- Units, (Wb/m^2), or T (Tesla)

Reluctance

- The measure of "opposition" the magnetic circuit offers to the flux
- The analog of Resistance in an electrical circuit
- Symbol, R (or squiggly R)
- Definition, $R = F/\Phi$
- Units, (A-t/Wb)

Reluctance of a magnetic path

- The magnetomotive force of N-turn current carrying coil is

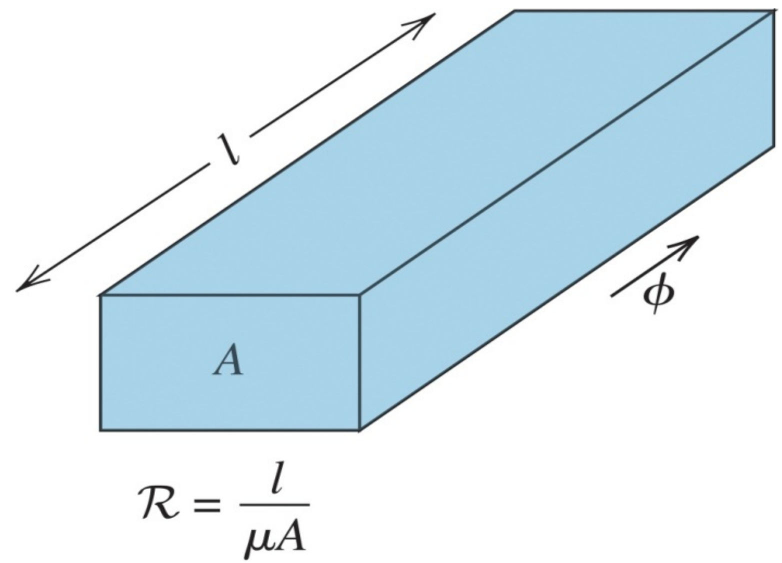
$$F = Ni$$

- Magnetic flux is analogous to current in electrical circuit and is related to F and R in a similar way as Ohm's law

$$F = \Re\Phi$$

- The reluctance R of a magnetic path depends on the mean length *l*, the area *A*, and the permeability *μ* of the material

$$\Re = \frac{l}{\mu A}$$



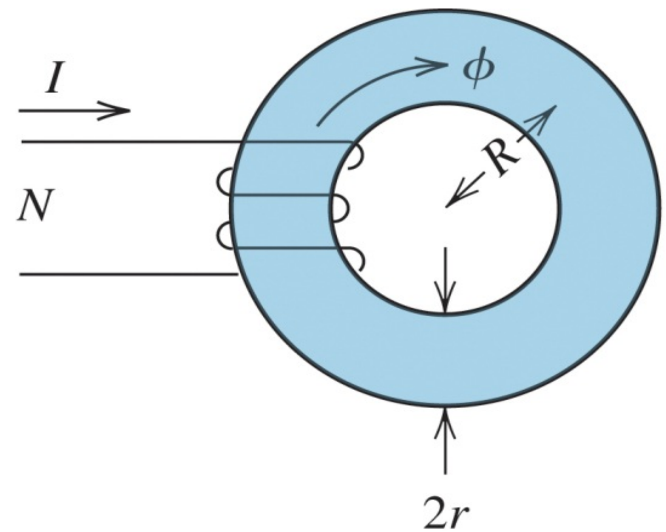
Magnetic circuit

- Consider N turns of wire with current I passing through them
- Therefore MMF F is given by:

$$F = NI \qquad = H\ell \quad \text{Since} \quad B = \mu H$$

$$\Rightarrow \frac{B}{\mu}\ell = NI \quad \text{Since} \quad \Phi = BA$$

$$\Rightarrow \frac{\Phi}{\mu A}\ell = NI \qquad \Rightarrow \Phi = \frac{NI}{\left(\ell / \mu A \right)}$$

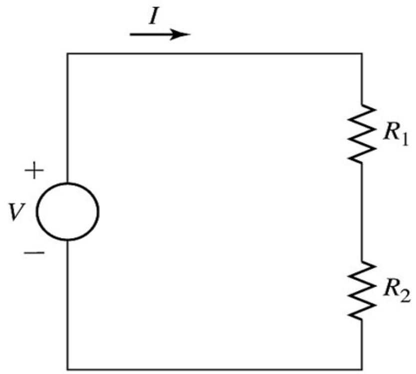


We can write this as

$$\Phi = \frac{\mathrm{NI}}{\Re}$$ where $\Re$ is the *reluctance* of magnetic path

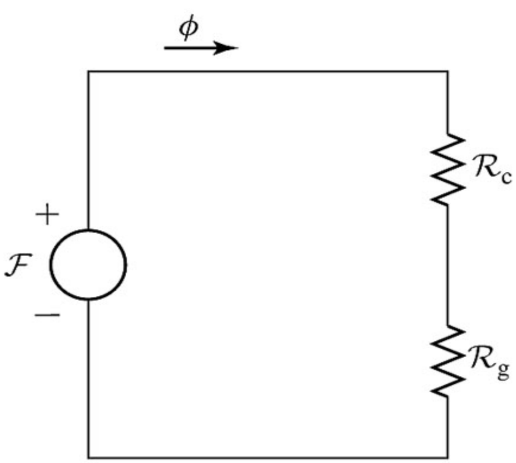
$$\Re = \frac{\ell}{\mu A}$$

Analogy: electric & magnetic circuits



$$I = \frac{V}{(R_1 + R_2)}$$

Electric circuit

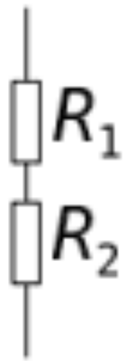


$$\phi = \frac{\mathcal{F}}{(\mathcal{R}_c + \mathcal{R}_g)}$$

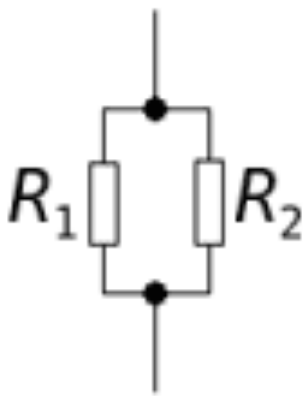
Magnetic circuit

- Magnetic reluctance opposes flux Φ
- MMF drives flux Φ through a magnetic circuit in an analogous fashion to voltage driving current around an electrical circuit that it opposed by its resistance
- Similarly the reluctance of the core represents the opposition to the flow of magnetic flux
- Reluctances of separate sections combine like resistances in in electrical circuits
- In this case we show series resistances and series reluctances

Reluctances combine like resistances



$$R_{TOTAL} = R_1 + R_2 \; . \; . \; . \; R_n$$

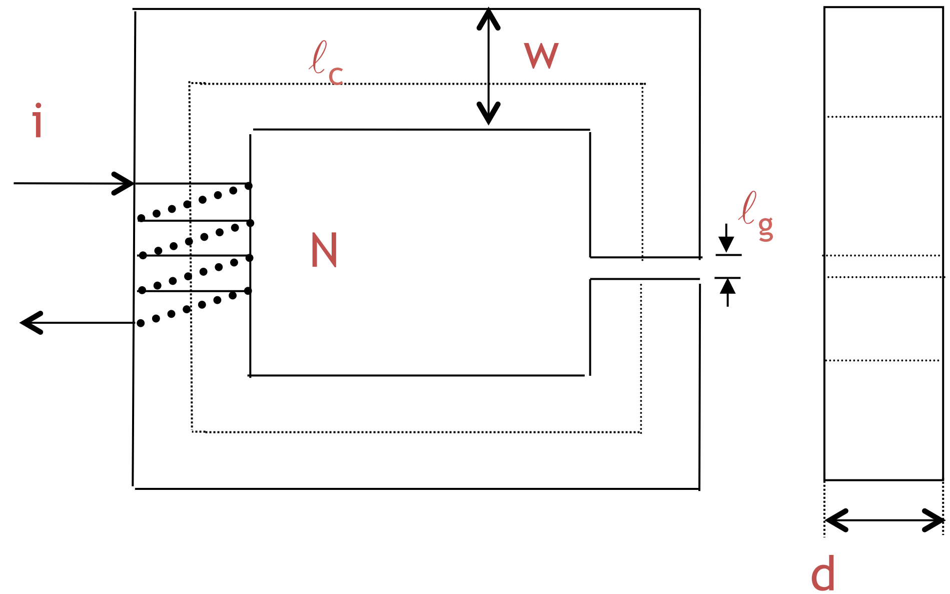


$$\frac{1}{R_{TOTAL}} = \frac{1}{R_1} + \frac{1}{R_2} \; . \; . \; . \; \frac{1}{R_n}$$

Magnetic circuit with air gap

- The magnetic circuit here has a single source of MMF (current flowing thorough coil)) and and two series reluctances arising from 1. the core and 2. the air gap

Reluctance of core $$\Re_c = \frac{l_c}{\mu_c A_c}$$



Reluctance of air gap $$\Re_g = \frac{l_g}{\mu_g A_g}$$

Magnetic flux $$\Phi = \frac{Ni}{\Re_c + \Re_g}$$

MMF $$Ni = H_c l_c + H_g l_g$$

Area of core and airgap $$A_c = A_g = wd$$

Magnetomotive force example 1

- What is the magnetomotive force produced by a coil of 400 turns if the current flowing through the coil is 0.25 A?

- Answer:

$$F = NI$$
$$F = 400 * 0.25$$
$$F = 100\ A$$

Magnetomotive force example 2

- A coil is required to generate a magnetic flux density of 1.2 T in an air gap of length 0.8 mm
- Calculate the magnetic field strength and the MMF F

The relative permeability of the air gap is 1 ($\mu_r = 1$),

Since $B = \mu_0 H$

$$H = B / \mu_0$$

$$H = 1.2 / (4 \pi \times 10{-}7)$$

$$H = 954{,}930 \text{ A/m}$$

The MMF is given by:

$$F = H l$$

Therefore:

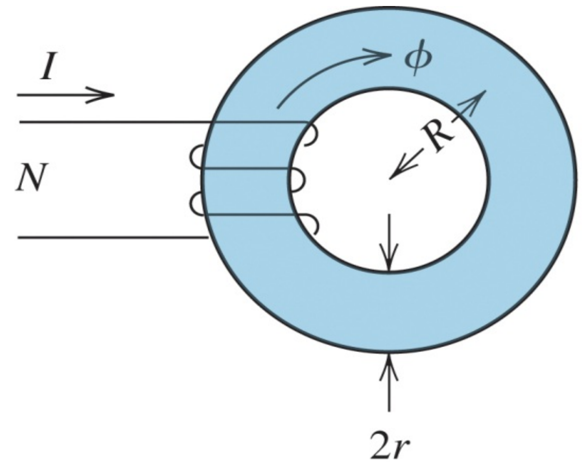
$$F = 954{,}930 \times 0.8 \times 10{-}3 = 763.94 \text{ A}$$

Magnetic circuit for toroid

- The magnetic circuit for the toroid coil can be analyzed to obtain an expression for flux
- In this case there is only a single magnetic path and a single source of MMF
- Magnetomotive force is given by

$$F = NI \qquad = \Re\Phi$$

Where the reluctance is



Coil on toroid core

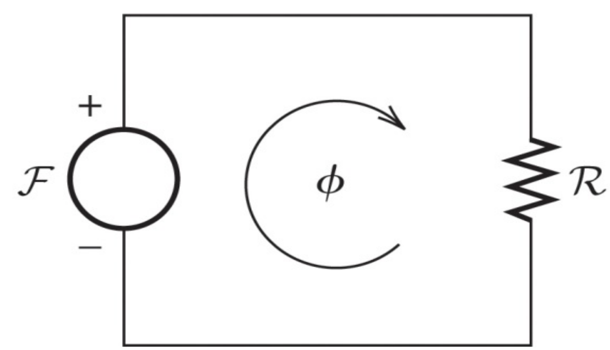
$$\Re = \frac{l}{\mu A} \qquad = \frac{2\pi R}{\mu\pi r^2} \qquad = \frac{2R}{\mu r^2}$$ From 1st equation

$$\Rightarrow NI = \frac{2R}{\mu r^2}\Phi$$

and the magnetic flux is

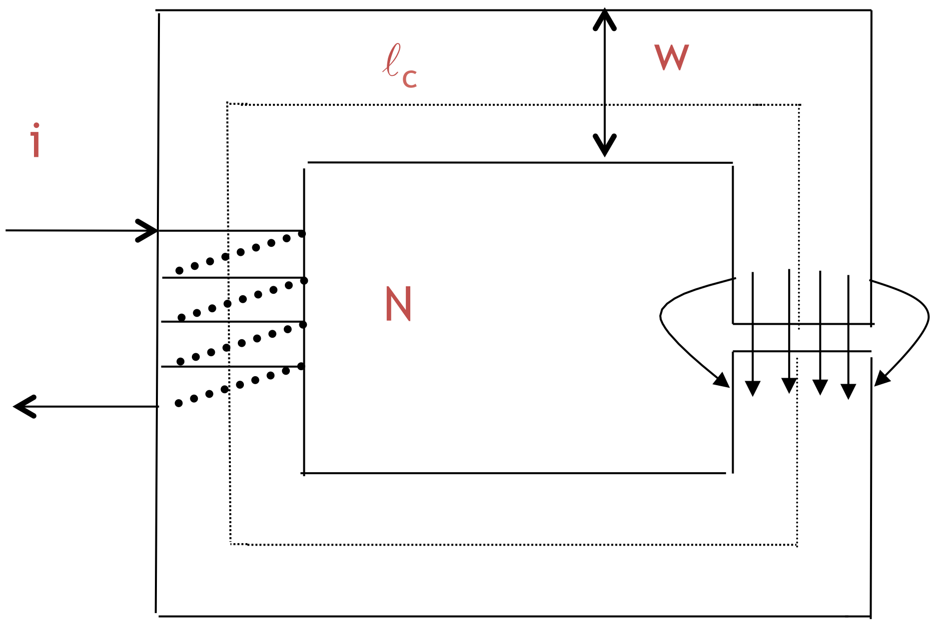
$$\Phi = \frac{NI\mu r^2}{2R}$$

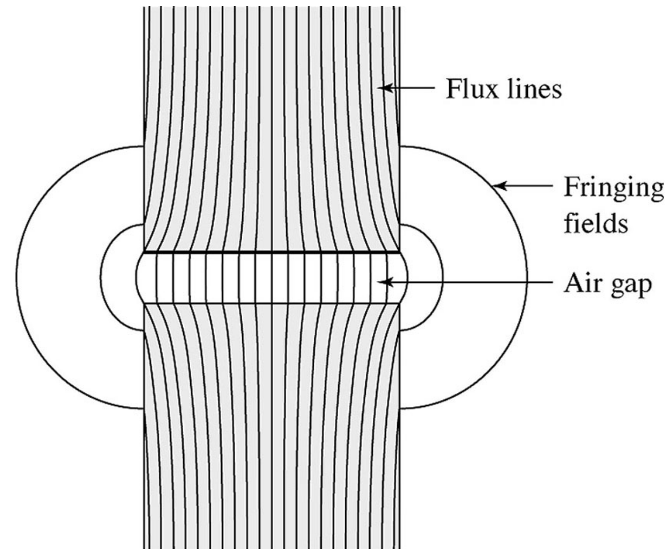
- The magnetic circuit here has a single source of MMF (current flowing thorough coil)) and and a single series reluctances



Magnetic circuit

Accounting for fringing





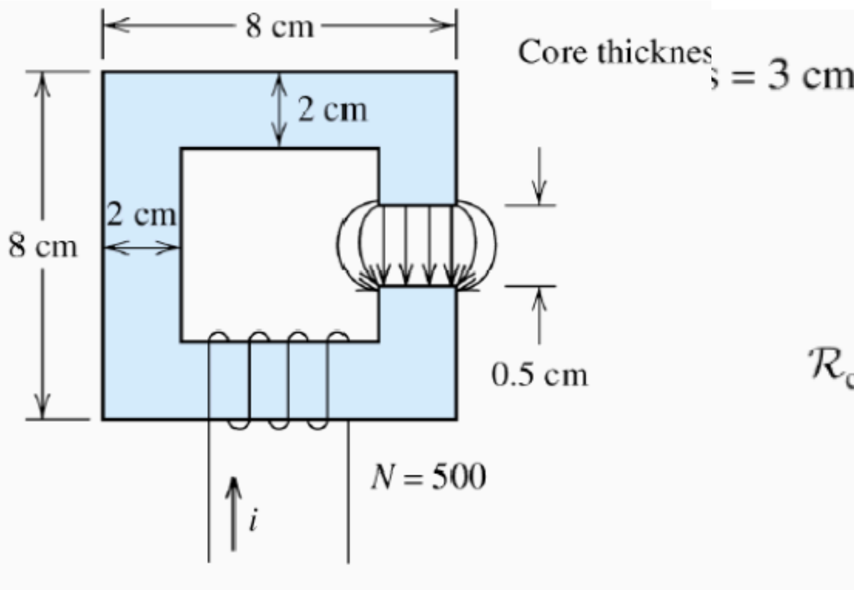
- With large air-gaps, flux tends to leak outside the air gap
- This is called fringing which increases the effective flux area
- One way to approximate this increase is:

$$w_n = w + l_g$$

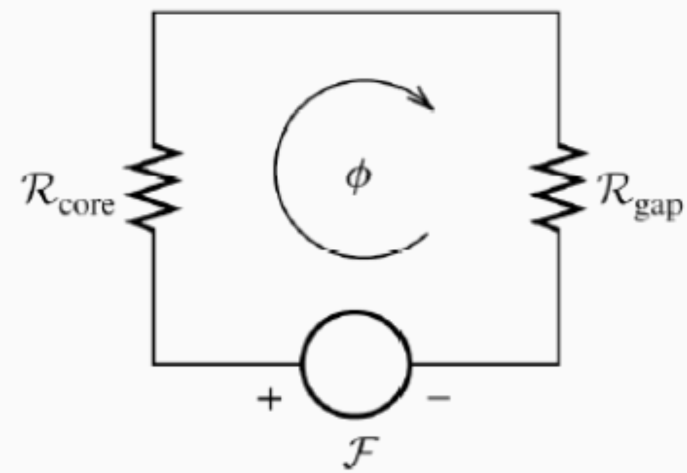
$$d_n = d + l_g$$

$$A_{gn} = w_n d_n$$

Magnetic Circuit with an air gap



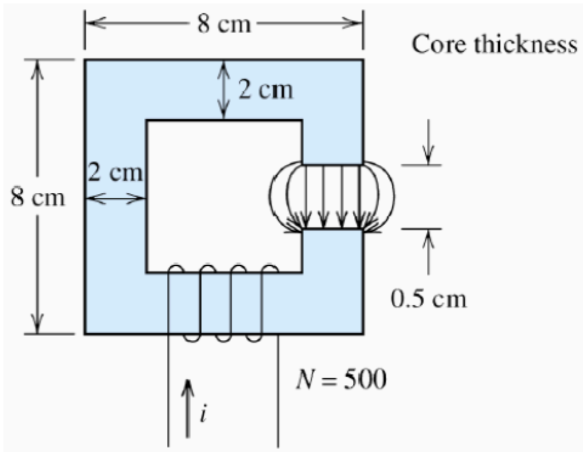
(a) Iron core with an air gap

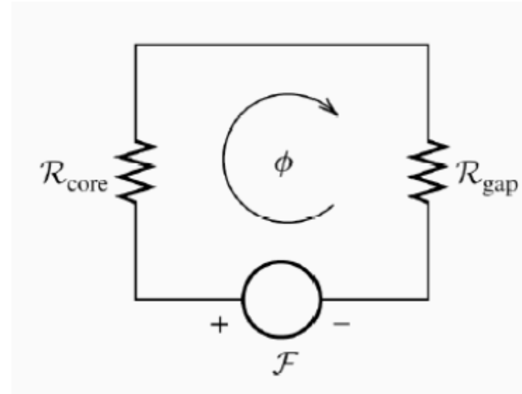


(b) Magnetic circuit

Find the current required to generate a flex density B_{gap} of 0.25T in the air gap

Magnetic Circuit with an air gap





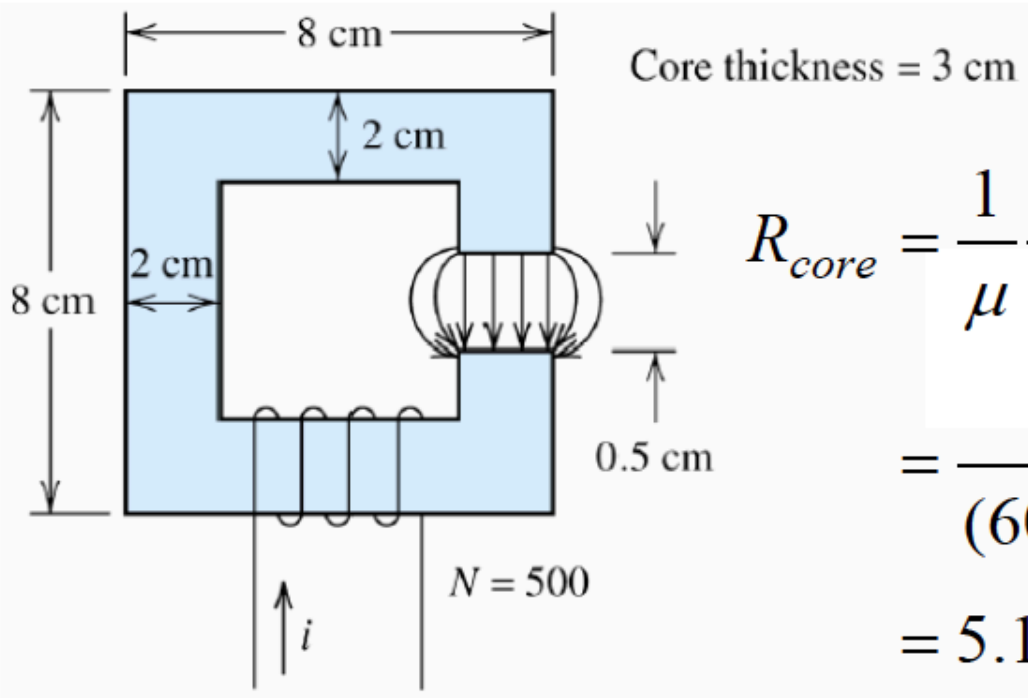
- The magnetic circuit here has a single source of MMF (current flowing thorough coil)) and and two series reluctances arising from 1. The core and 2. The air gap
- Therefore the magnetic circuit diagram is as shown upper RHS:
- The elements have the following values:

$$MMF = F = NI = 500I$$

Reluctance of core is $$\Re_{core} = \frac{Pathlength_{core}}{\mu_{core} Area_{core}}$$

Reluctance of air gap is $$\Re_{core} = \frac{Pathlength_{airgap}}{\mu_{airgap} EffectiveArea_{airgap}}$$

Magnetic circuit with air gap numeric example



$$R_{core} = \frac{1}{\mu}\frac{l}{A} = \frac{1}{\mu_r \mu_0}\frac{(4x6-0.5)cm}{(2cm)(3cm)}$$

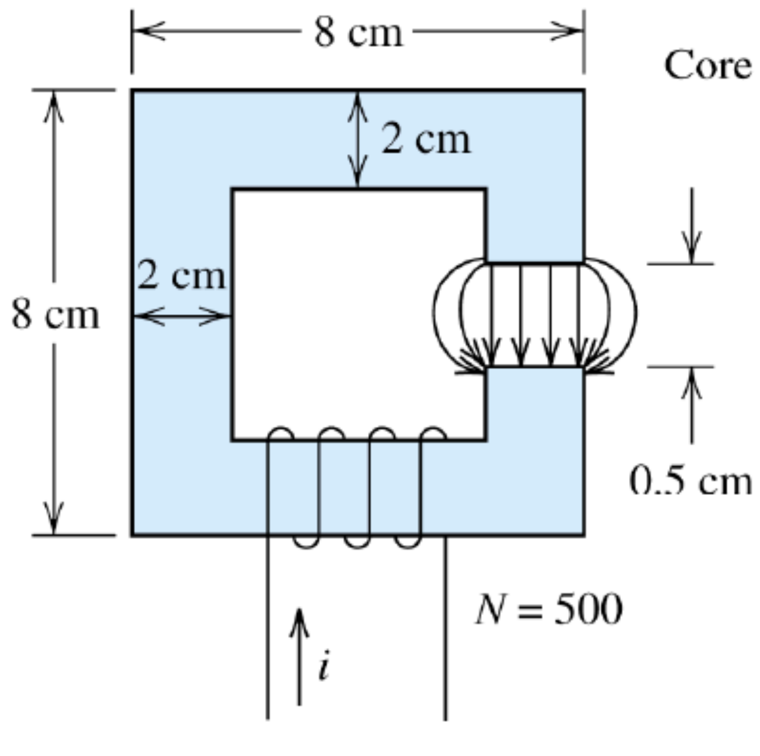
$$= \frac{1}{(6000)(4\pi x10^{-7})}\frac{23.5x10^{-2}\,m}{6x10^{-4}\,m}$$

$$= 5.195x10^{4}$$

$$\mu_r = 6000$$

Magnetic circuit with air gap numeric example

We account for fringing here by adding on 0.5cm to air gap dimensions



Core thickness = 3 cm

$$A_{gap} = (2cm + 0.5cm)x(3cm + 0.5cm)$$

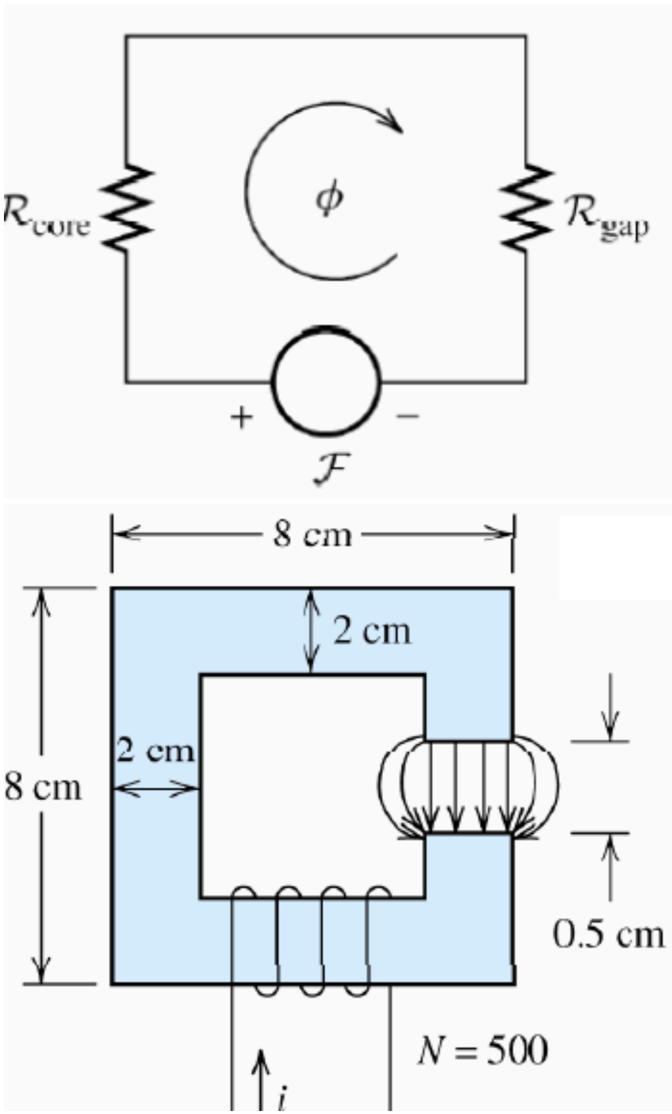
$$= 8.75x10^{-4}\,m^2$$

$$\mu_{gap} \approx \mu_0 = 4\pi x 10^{-7}$$

$$R_{gap} = \frac{1}{4\pi x 10^{-7}} \frac{0.5x10^{-2}\,m}{8.75x10^{-4}\,m^2}$$

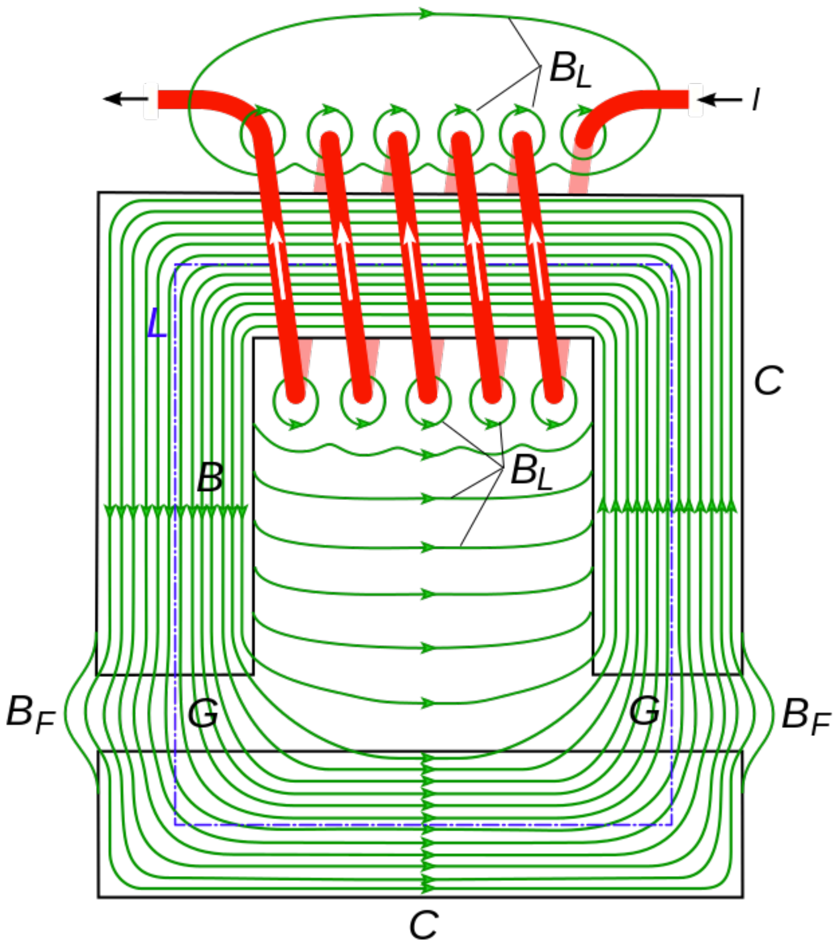
$$= 4.547x10^6$$

Magnetic circuit with air gap numeric example



$$R_{total} = R_{core} + R_{gap}$$
$$= 5.195x10^4 + 4.547x10^6 = 4.600x10^6$$
$$\phi = B_{gap} A_{gap} = (0.25T)(8.75x10^{-4} m^2)$$
$$= 2.188x10^{-4} Wb$$
$$F = \phi R = (2.188x10^{-4})(4.600x10^6)$$
$$= 1006 A\ turns$$
$$= Ni$$
$$i = \frac{F}{N} = \frac{1006 A\ turns}{500\ turns} = 2.012 A$$

Horseshoe magnetic circuit

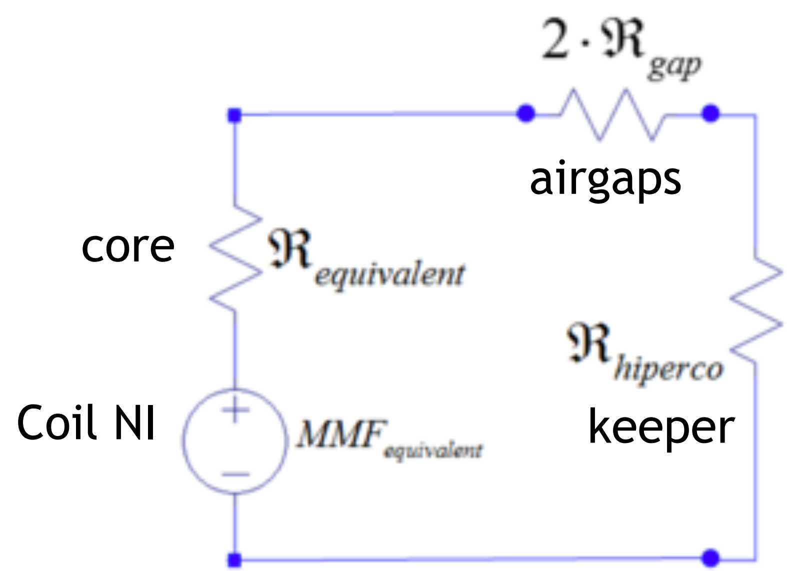


MMF created by a current gives

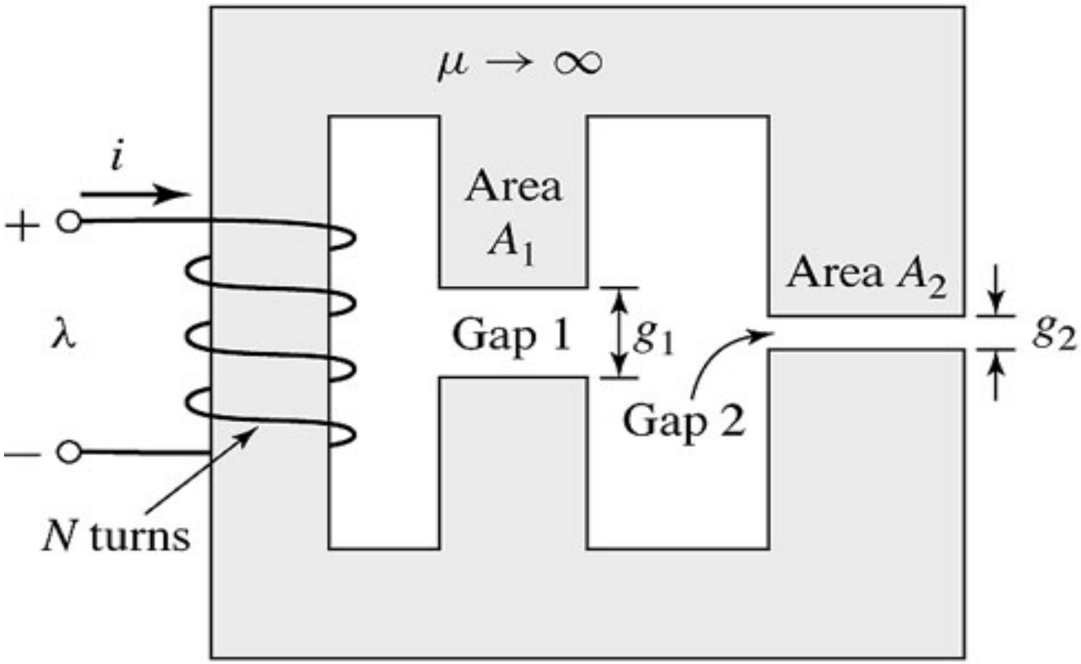
$$NI = H_{\text{core}} L_{\text{core}} + H_{\text{gap}} L_{\text{gap}}$$

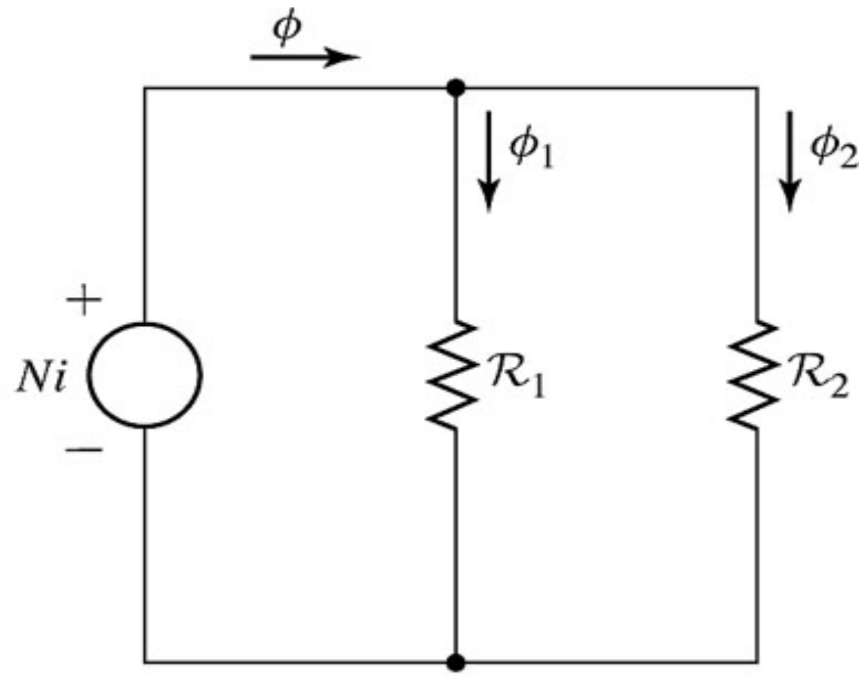
$$NI = B\left(\frac{L_{\text{core}}}{\mu} + \frac{L_{\text{gap}}}{\mu_0}\right)$$

length L_{core} of the magnetic field path is in the core material and length L_{gap} is in air gaps

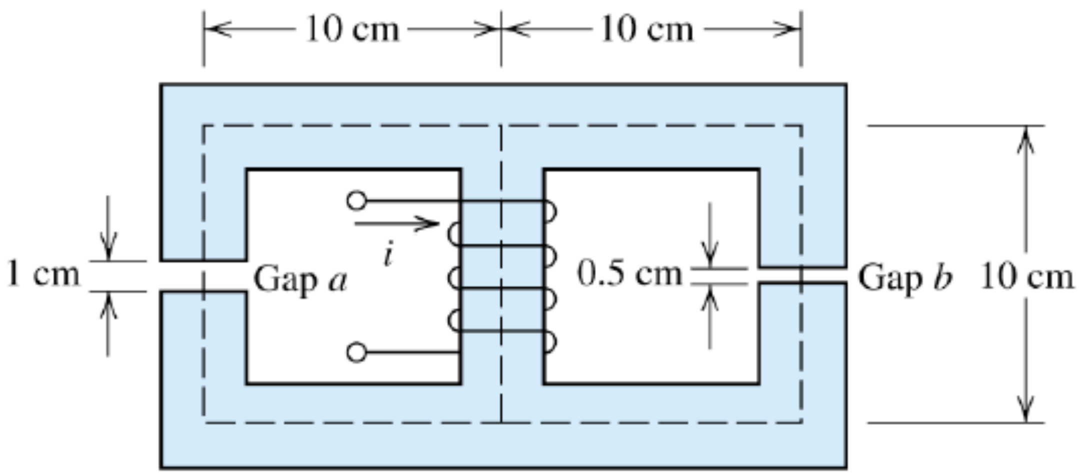


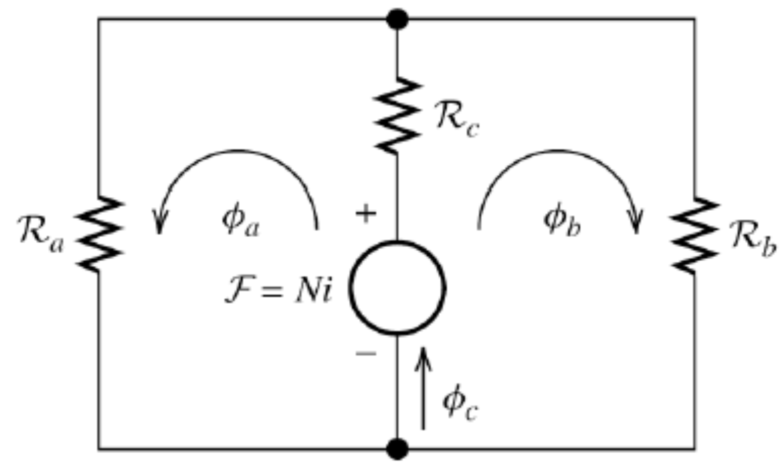
Parallel magnetic circuit and electrical analogy





Parallel and series reluctances in magnetic circuit





ROCO222: Intro to sensors and actuators

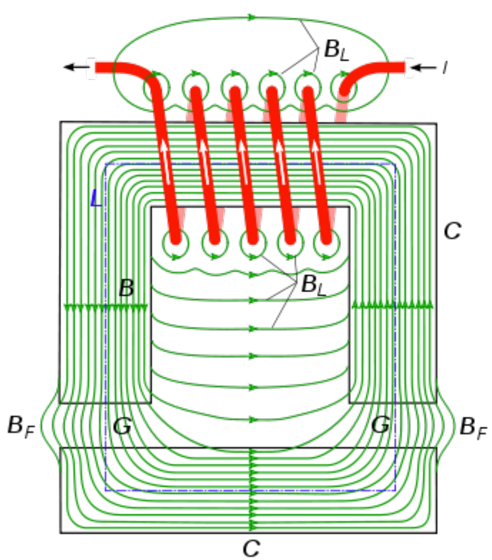
Lecture 10

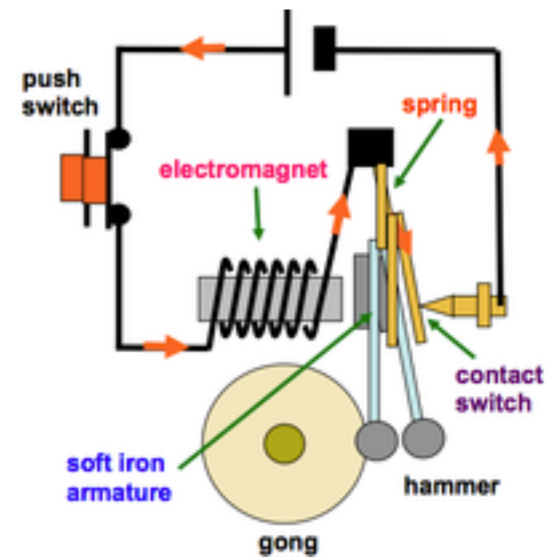
Magnetic force

Electromagnet can generate force

Electromagnets are used for

- Relays
- Doorbells
- lifting magnets, etc
- Force generated can be computed from
  - flux density,
  - gap area
  - the permeability





Force : Derivative of energy with respect to distance

- The force on an object is the negative of the derivative of the potential function U

- This means it is the negative of the slope of the potential energy curve

- Plots of potential functions are valuable aids to visualizing the change of the force in a given region of space

- The negative sign on the derivative shows that if the potential U increases with increasing x, the force will tend to move it toward smaller x to decrease the potential energy

$$F(x) = -\frac{dU}{dx}$$

Remember: Inductance of a solenoid

- For a fixed area and changing current, Faraday's law becomes

$$EMF = -N\frac{d\Phi}{dt} \qquad = -NA\frac{dB}{dt}$$

- Since the magnetic field of a solenoid is

$$B = \frac{\mu NI}{l}$$

- Then the EMF is approximated by

$$EMF = -NA\frac{d}{dt}\left(\frac{\mu NI}{l}\right) \qquad = -\frac{\mu N^2 A}{l}\frac{dI}{dt}$$

- From the definition of inductance

$$EMF = -L\frac{dI}{dt}$$

- By inspection we obtain

$$L = \frac{\mu N^2 A}{l}$$

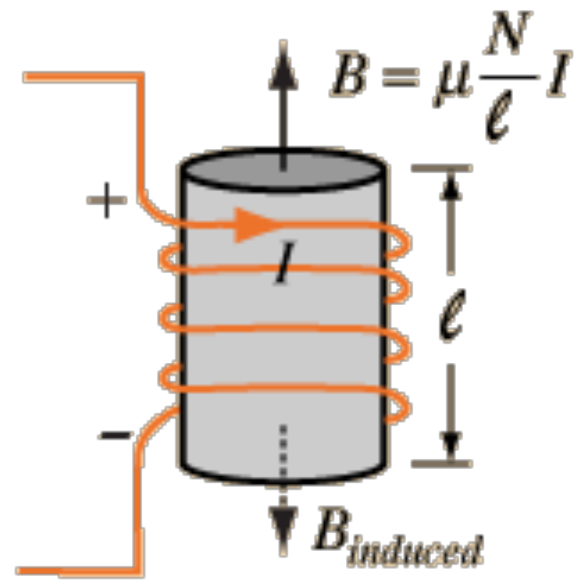
- Where
- N=number of turns
- l=length
- A=cross sectional area
- R = toroid radius to centerline

Energy in a magnetic field

- Consider analysis of the energy E stored in an solenoid type inductor

$$E = \frac{1}{2} L I^2$$

- For a long solenoid $B = \mu \frac{N}{l} I \quad \Rightarrow I = \frac{Bl}{\mu N}$



- Since $L = \mu \frac{N^2 A}{l}$

- The energy density (energy/volume) is therefore

$$\frac{energy}{volume} = \frac{\frac{1}{2} L I^2}{Al} \qquad = \frac{\frac{1}{2} \mu \frac{N^2 A}{l} \left(\frac{Bl}{\mu N} \right)^2}{Al}$$

- So the energy density stored in the magnetic field is

$$\eta = \frac{B^2}{2\mu}$$

Relay closing current

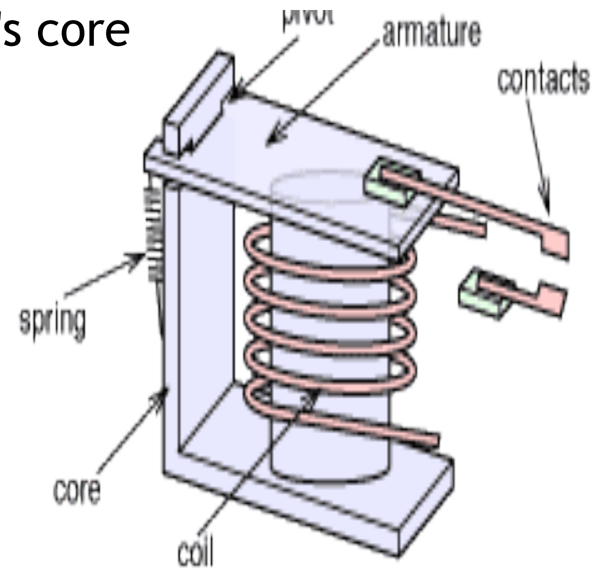
- Energy density in a magnetic field is

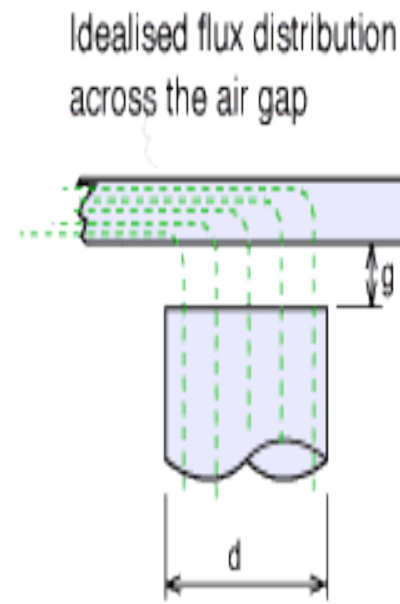
$$\eta = \frac{B^2}{2\mu} \quad \text{joules m}^{-3}$$

- Assuming that the field inside the air gap is uniform the total field energy is obtained by multiplying by the volume of the field V

$$V = gA$$

- Where g is the gap length
- A is the cross sectional area of the coil's core





- The total energy is then

$$W = gA\frac{B^2}{2\mu_0}$$

Relay closing current

To compute the force on the armature we need to calculate the rate of change of energy with gap length

$$W = gA\frac{B^2}{2\mu_0} \qquad \Rightarrow Force = \frac{dW}{dg} = A\frac{B^2}{2\mu_0}$$

Need to find the flux density B

Assume most of the field strength produced by the coil will appear across the air gap between the core and the armature

Ignore the reluctance of the core, pivot and armature.

Now field F is given by

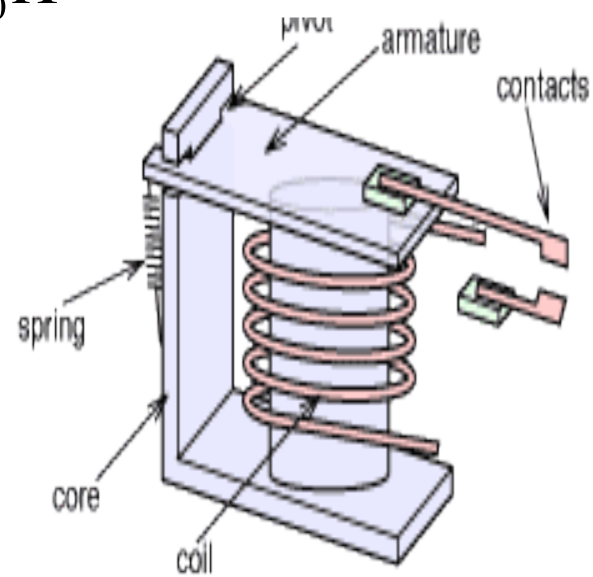
$$F = NI \qquad = Hg \qquad \text{and} \qquad B = \mu_0 H$$

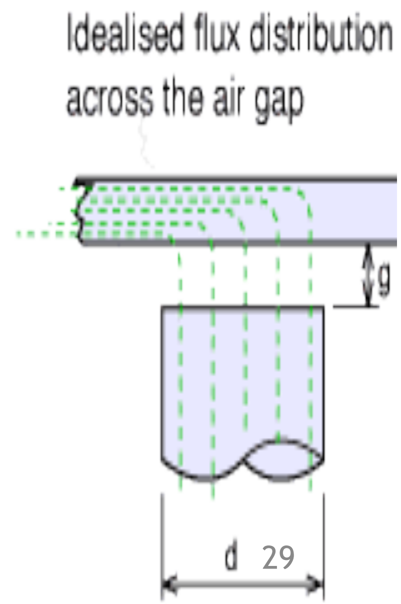
$$\Rightarrow B = \mu_0 \frac{F}{g}$$

Substituting into force equation

$$\Rightarrow Force = A\frac{B^2}{2\mu_0} \quad = A\frac{\left(\mu_0 \dfrac{F}{g}\right)^2}{2\mu_0}$$

$$\Rightarrow Force = A\frac{\mu_0 F^2}{2g^2}$$





Relay closing current example

A relay has a coil of 1200 turns.
The diameter of the coil core is 6 mm and the air gap is 1.8 mm.
The spring exerts a force on the armature of 0.15 N at the part of it opposite the air gap.
The core cross sectional area $A = \pi\,(0.006/2)^2 = 2.83\times10^{-5}\ \mathrm{m}^2$.
What coil current will operate the relay?

Substituting

Force=0.15 (N)
N=1200
$A=2.83\times10^{-5}$ (m^2)
$g=1.8\times10^{-3}$ (m)

$$Force = A\frac{\mu_0 F^2}{2g^2}$$

Into

$$Force = (F)^2\,\mu_0\,A/(2\,g^2)$$

We have

$$0.15 = (1200 \times \underline{I})^2\, 4\pi\times10^{-7} \times 2.83\times10^{-5} / (2 \times (1.8\times10^{-3})^2)$$

Therefore I = 0.138 amps

The flux density will be

$$1200 * 0.138 * 4\pi\times10^{-7} / 1.8\times10^{-3} = 0.116\ \mathrm{T}$$

$$B = \mu_0\frac{F}{g}$$

Horseshoe static magnetic force

- For a closed magnetic circuit of length L and areas A with no air gap

$$F = NI = \Phi\Re$$

Substituting

$$\Phi = BA$$

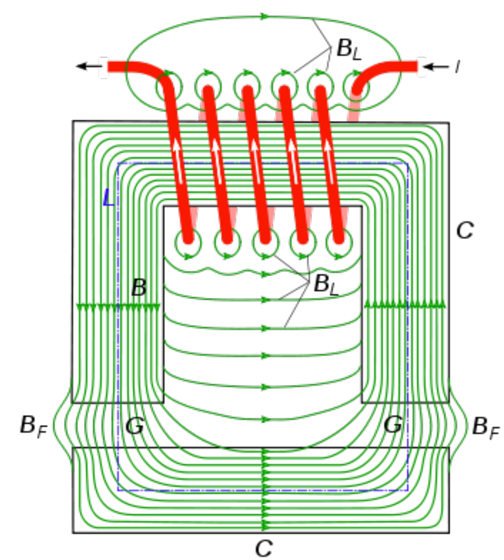
$$\Re = \frac{L}{\mu A}$$

$$NI = BA\frac{L}{\mu A} \quad \Rightarrow B = \mu\frac{N}{L}I$$

- The force exerted by an electromagnet on a section of core material is

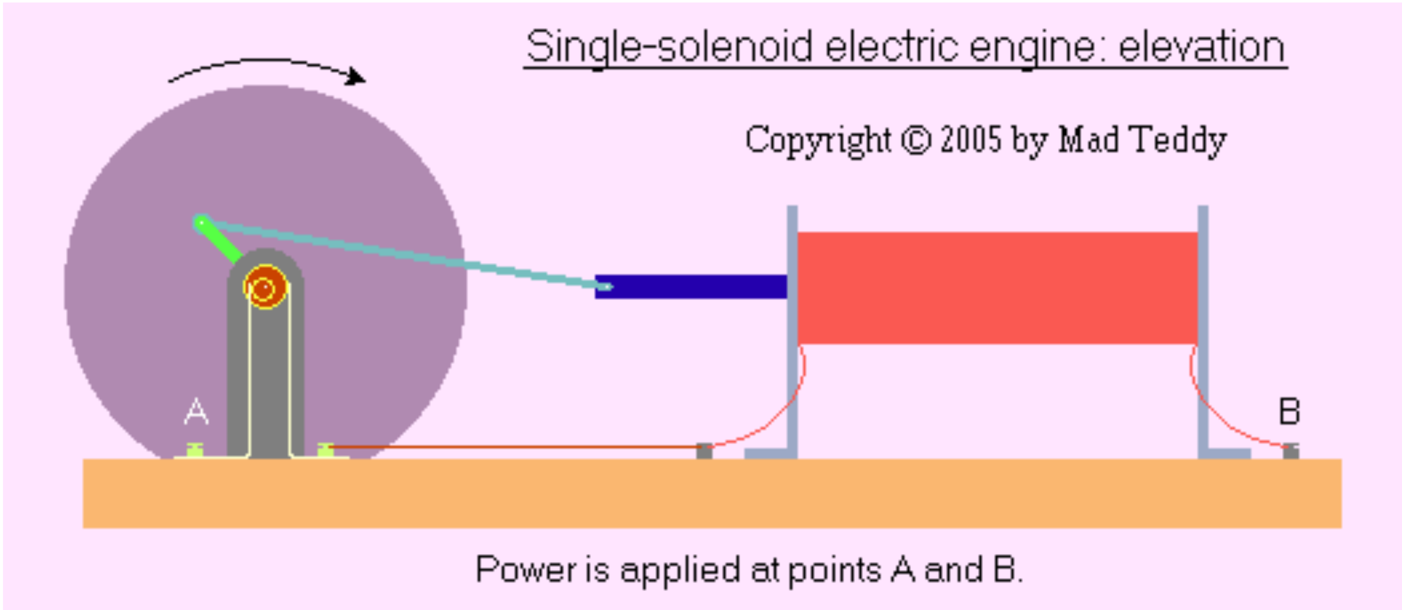
$$Force = \frac{B^2 A}{2\mu_0}$$

- So for horseshoe magnet with a keeper this gives



$$Force = \frac{\left(\mu\frac{N}{L}I\right)^2 A}{2\mu_0} = \frac{\mu^2 N^2 I^2 A}{2\mu_0 L^2}$$

Solenoid motor



- Coil and plunger generates pull force
- Flywheel maintains full cycle movement and retracts the solenoid in second half of cycle since it can only pull and cannot push
- Cam changes linear to rotary motion
- There is also an activation switch or contact so that solenoid current is only provided on the appropriate retraction phase of the cycle