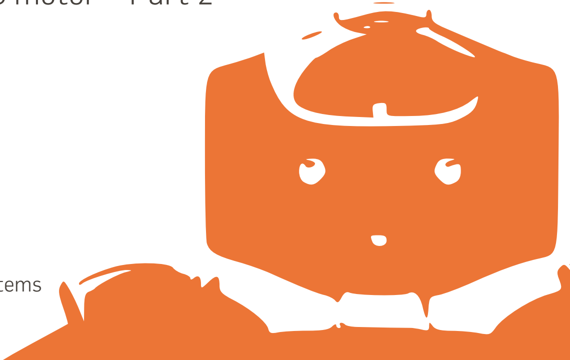
You can download the sources of this presentation here:
github.com/severin-lemaignan/module-introduction-sensors-actuators

ROBOTICS WITH PLYMOUTH UNIVERSITY

ROCO222
Intro to Sensors and Actuators
Electromagnetism & DC motor – Part 2



Séverin Lemaignan

Centre for Robotics and Neural Systems
Plymouth University

LAST WEEK'S PROGRAMMING CHALLENGE

Code a `trumpsays` in Python:

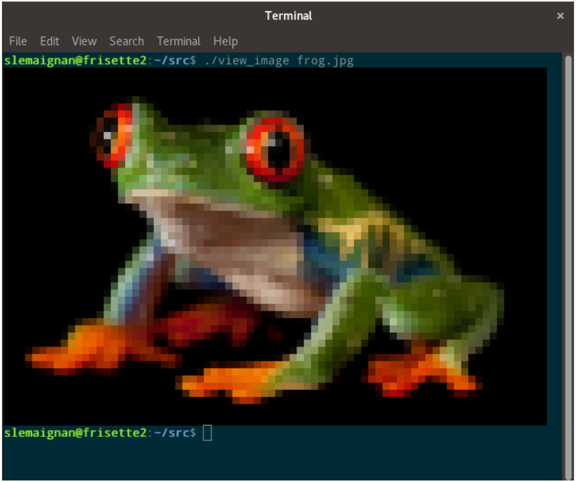
```
$ trumpsays "Being nice to Rocket Man has failed."
   ,------._      ________________________
  /         :    /                        \
 /  ,-__-.  :   | Being nice to Rocket Man |
; ,' _   _`-|  /  has failed.              |
|/  - ||-   | / __________________________/
|\   /_\    |---
:|         |;
 \    O    ;
  \        ;
   `--+---'
```

```python
#! /usr/bin/python

import sys
text = sys.argv[1] # retrieve the first command-line parameter

def go(line):
    """ move the console cursor to the start of a balloon's line.
    Uses ANSI escape codes: https://en.wikipedia.org/wiki/ANSI_escape_code
    """
    sys.stdout.write("\033[u\033[" + str(9-line) + "A\033[G\033[17C")

balloon_length = 24
trump = """
   ,------._
  /         :    ________________________
 /  ,-__-.  :   /                        \\
; ,' _   _`-|  /
|/  - ||-   | /
|\   /_\    |---
:|         |;
 \    O    ;
  \        ;
   `--+---'
"""
print(trump)
print("\033[s") # save the cursor position (bottom of the screen)
```

```
go(0)

line = 0
current_length = 0 # holds the nb of characters printed on current line
for word in text.split(" "): # iterate over each of the words

    # do we need a line break?
    if current_length + len(word) >= balloon_length:
        line += 1
        go(line)
        current_length = 0

    current_length += len(word) + 1
    sys.stdout.write(word + " ") # print the word, without final line break

# Draw the balloon contours
for i in range(line+1):
    go(i)
    sys.stdout.write("\033[2D|\033[" + str(balloon_length + 2) + "C|")

go(line + 1)
sys.stdout.write("\033[D\\" + "_" * balloon_length + "/")

print("\033[u") # restore the position of the cursor, to the bottom
```

THIS WEEK'S CHALLENGE

Console-based image viewer



TIPS FOR THIS WEEK'S CHALLENGE

```python
# how to get the pixels of an image?
from PIL import Image
im = Image.open('image.jpg')
rgb_im = im.convert('RGB')
r, g, b = rgb_im.getpixel((1, 1))


# use ANSI escape code to set the adequate foreground/background color
# see https://en.wikipedia.org/wiki/ANSI_escape_code#24-bit

# use in a clever way the Unicode character U+2580, 'Upper Half Block'
# to increase the resolution of your viewer
```

SO FAR IN ELECTROMAGNETISM…

RECAP OF LAST LECTURE

- Ampère's law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot I$
- Faraday's law of induction: $\mathcal{E} = -N \cdot \frac{d\Phi_B}{dt}$ (with $\Phi_B = B \cdot A \cdot cos(\theta)$ iff $\vec{B}$ is uniform)
- Lenz law: (the minus in Faraday's law)
- Lorentz law: $\vec{F} = \vec{I} \cdot L \times \vec{B}$

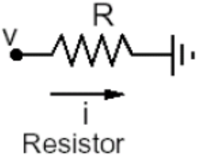
TODAY'S OBJECTIVES

- Understand and know how to characterise **electrical inductance**
- Describe a motor from **its equivalent circuit**
- **Relate a motor's angular velocity to voltage and torque**
- Model the motor's **dynamics** and use **Laplace transforms** to solve the motor differential equations
- Know how to read and interpret a **motor datasheet**
- Undertand the working principle of a **brushless motor** and how it compares to brushed DC motors

INDUCTANCE

ELECTRICAL RESISTANCE

Resists the flow of current



$$v = iR$$

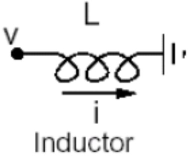
Where

i is the current in A

v is voltage in V

R is the resistance in Ω

ELECTRICAL INDUCTANCE



$$v = -L\frac{di}{dt}$$

Where

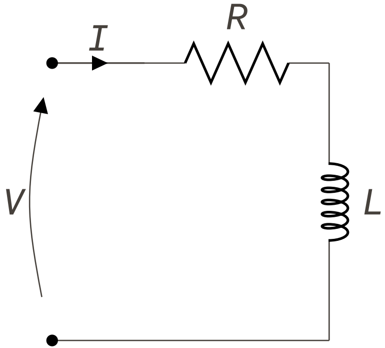
i is the current in A

v is voltage in V

L is the inductance in H

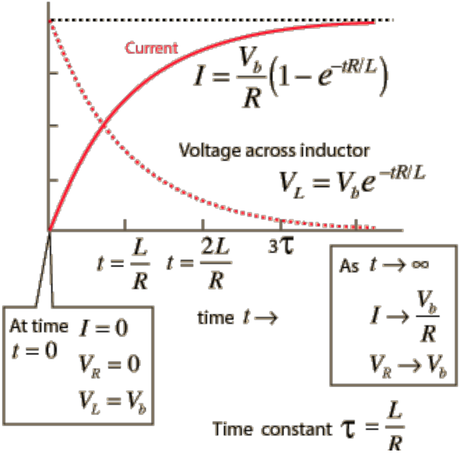
CURRENT IN AN LR CIRCUIT

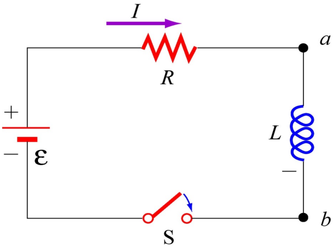
$$V = V_R + V_L$$
$$V_R = I \cdot R$$
$$V_L = L \cdot \frac{dI}{dt}$$
$$\Rightarrow V = I \cdot R + L \cdot \frac{dI}{dt}$$



This is a first-order linear differential equation.

CURRENT IN AN LR CIRCUIT





- We need to consider this behaviors wherever we switch inductive circuits and coils

DEFINITION OF INDUCTANCE

- Arising from Faraday's law

$$\varepsilon_L = -N \frac{d\Phi_B}{dt}$$

- Inductance L may be defined in terms of the EMF generated to oppose a given change in current:

$$EMF = -L \frac{di}{dt}$$

therefore

$$\varepsilon_L = -N \frac{d\Phi_B}{dt} = -L \frac{di}{dt}$$

- Integrating both sides w.r.t time $N\Phi_B = Li$

- For any inductor

$$L = \frac{N\Phi_B}{i}$$

INDUCTANCE OF A SOLENOID

- For a fixed area and changing current, Faraday's law becomes

$$EMF = -N\frac{d\Phi}{dt} \qquad = -NA\frac{dB}{dt}$$

- Since the magnetic field of a solenoid is

$$B = \frac{\mu NI}{l}$$

- Then the EMF is approximated by

$$EMF = -NA\frac{d}{dt}\left(\frac{\mu NI}{l}\right) \qquad = -\frac{\mu N^2 A}{l}\frac{dI}{dt}$$

- From one suitable property of inductance

$$EMF = -L\frac{dI}{dt}$$

- By inspection we obtain

$$L = \frac{\mu N^2 A}{l}$$

- Alternatively:
- From definition of inductance

$$L = \frac{N\Phi_B}{I}$$

- Substitute flux ϕ=BA

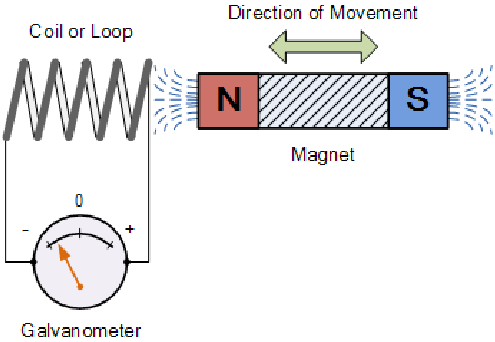
$$\Rightarrow L = \frac{N(\frac{\mu NI}{l}A)}{i}$$

$$\Rightarrow L = \frac{\mu N^2 A}{l}$$

- Where
- N=number of turns
- l=length
- A=cross sectional area
- R = toroid radius to centerline

CONSEQUENCE FOR MOTORS

Moving a magnet in a coil induces current:



Faraday's law of induction: $\mathcal{E} = -\frac{d\Phi_B}{dt}$

CONSEQUENCE FOR MOTORS

If a coil consists of N loops with the same area, the total induced EMF in the coil is given by:

$$\mathcal{E} = -N \cdot \frac{d\Phi_B}{dt}$$

In a uniform magnetic field, the induced EMF can be expressed as:

$$\mathcal{E} = -N \cdot \frac{d}{dt}(BAcos(\theta)) = -N \cdot B \cdot A \frac{dcos(\theta)}{dt} = -N \cdot B \cdot A \cdot sin(\theta) \cdot \dot{\theta}$$

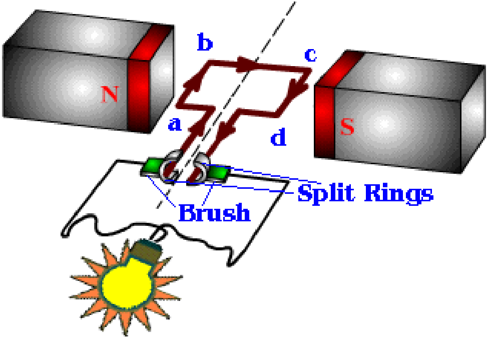
With enough coils,

$$\mathcal{E} = -N \cdot B \cdot A \cdot \dot{\theta} = -K_e \dot{\theta}$$

K_e is the **back-EMF** constant of the motor.

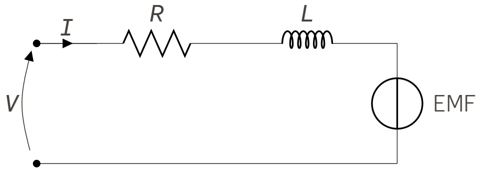
VOLTAGE IN A MOTOR

ELECTRICAL GENERATOR



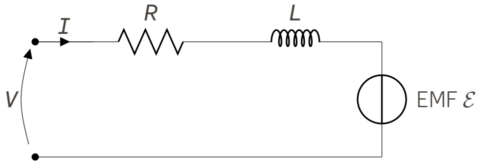
- Speed of rotation affects voltage generated
- So how fast will a motor rotate with applied voltage V?
- Lets consider the equivalent circuit for a DC motor

DC MOTOR EQUIVALENT CIRCUIT



- V is the applied voltage, I is the drawn current
- Resistance R arises from the coil and the brushes
- Inductance L arises from the coil
- Back EMF arises from rotation of the coil in the magnetic field created by the stator magnets

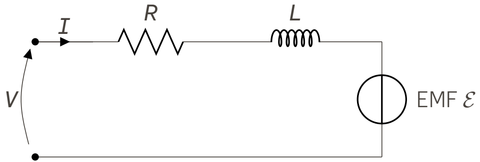
VOLTAGE IN A DC MOTOR



Kirchhoff's voltage law: voltage across resistor, inductance and back EMF balance applied voltage

$$
\begin{aligned}
V &= I \cdot R + L \cdot \frac{dI}{dt} + \mathcal{E} \\
&= I \cdot R + L \cdot \frac{dI}{dt} - K_e \cdot \dot{\theta}
\end{aligned}
$$

VOLTAGE IN A DC MOTOR



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&= I \cdot R + L \cdot \frac{dI}{dt} - K_e \cdot \dot{\theta}
\end{aligned}
$$

How to know I?

CONSERVATION OF ENERGY!

Conservation of energy means electrical power in = mechanical power out + losses

$P_{el} = V \cdot I$

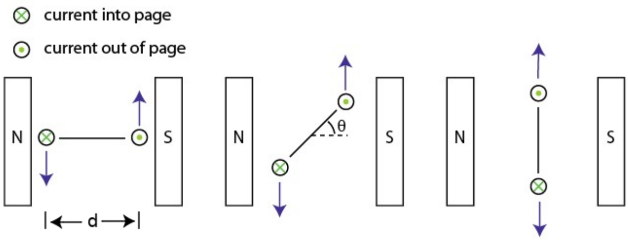
$P_{mech} = \tau \cdot \dot{\theta}$

$P_r = R \cdot I^2$

where:
τ = torque;
$\dot{\theta}$ = angular velocity;
I = input current;
V = applied voltage;
R = windings resistance

$$P_{el} = P_{mech} + P_r$$

REMEMBER TORQUE



If current I flows through a coil of depth L and width d with N turns, with magnetic field B, the coil pivots and generates a torque τ given by:

$$\tau = N \cdot 2(\frac{d}{2}) \cdot F \cdot cos(\theta) = N \cdot d \cdot B \cdot I \cdot L \cdot cos(\theta)$$

REMEMBER TORQUE

With enough coils, we can remove the $\boldsymbol{cos}(\theta)$ term:

$$\tau = N \cdot d \cdot B \cdot I \cdot L = K_\tau \cdot I$$

K_τ is the motor's torque constant.

PUTTING IT ALL TOGETHER

- $V = -K_e \cdot \dot{\theta} + I \cdot R$
- $\tau = K_\tau \cdot I$

V = applied voltage
$\dot{\theta}$ = motor angular velocity
I = motor current
R = motor resistance
τ = torque, including frictional losses

PUTTING IT ALL TOGETHER

- $V = -K_e \cdot \dot{\theta} + I \cdot R$
- $\tau = K_\tau \cdot I$

V = applied voltage
$\dot{\theta}$ = motor angular velocity
I = motor current
R = motor resistance
τ = torque, including frictional losses

- Electrical power in: $V \cdot I = -K_e \cdot \dot{\theta} \cdot I + I^2 \cdot R$
- Mechanical power out: $\tau \cdot \dot{\theta} = K_\tau \cdot I \cdot \dot{\theta}$
- Losses: $I^2 \cdot R$

PUTTING IT ALL TOGETHER

- $V = -K_e \cdot \dot{\theta} + I \cdot R$
- $\tau = K_\tau \cdot I$

V = applied voltage
$\dot{\theta}$ = motor angular velocity
I = motor current
R = motor resistance
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- Electrical power in: $V \cdot I = -K_e \cdot \dot{\theta} \cdot I + I^2 \cdot R$
- Mechanical power out: $\tau \cdot \dot{\theta} = K_\tau \cdot I \cdot \dot{\theta}$
- Losses: $I^2 \cdot R$

Conservation of energy means electrical power in = mechanical power out + losses.

This is true if and only if $K_e = K_\tau$.

PUTTING IT ALL TOGETHER

- $V = -K_e \cdot \dot{\theta} + I \cdot R$
- $\tau = K_\tau \cdot I$

V = applied voltage
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- Electrical power in: $V \cdot I = -K_e \cdot \dot{\theta} \cdot I + I^2 \cdot R$
- Mechanical power out: $\tau \cdot \dot{\theta} = K_\tau \cdot I \cdot \dot{\theta}$
- Losses: $I^2 \cdot R$

Conservation of energy means electrical power in = mechanical power out + losses.

This is true if and only if $K_e = K_\tau$.

In a DC motor, the torque constant and back-EMF constants are equal.

BACK TO THE VOLTAGE

$$V = K_e \cdot \dot{\theta} + I \cdot R$$

$$\tau = K_\tau \cdot I \Rightarrow I = \frac{\tau}{K_\tau}$$

$$\Rightarrow V = \frac{\tau}{K_\tau} \cdot R + K_e \cdot \dot{\theta}$$

$$\Rightarrow \dot{\theta} = \frac{V}{K} - R \cdot \frac{\tau}{K^2}$$

BACK TO THE VOLTAGE

$$V = K_e \cdot \dot{\theta} + I \cdot R$$

$$\tau = K_\tau \cdot I \Rightarrow I = \frac{\tau}{K_\tau}$$

Speed is a linear function of both voltage...

$$\Rightarrow V = \frac{\tau}{K_\tau} \cdot R + K_e \cdot \dot{\theta}$$

$$\Rightarrow \dot{\theta} = \frac{V}{K} - R \cdot \frac{\tau}{K^2}$$

...and torque

MOTOR CONSTANTS

$$\mathcal{E} = -N \cdot B \cdot A \cdot \dot{\theta} = -K_e \cdot \dot{\theta}$$
$$\Rightarrow K_e = N \cdot B \cdot A$$

$$\tau = N \cdot d \cdot B \cdot I \cdot L = K_\tau \cdot I$$
$$\Rightarrow K_\tau = \underbrace{N}_{\text{turns per coil}} \cdot \underbrace{d}_{\text{coil width}} \cdot \underbrace{B}_{\text{permanent magnetic field}} \cdot \underbrace{L}_{\text{coil length}}$$

MOTOR CONSTANTS

$$\mathcal{E} = -N \cdot B \cdot A \cdot \dot{\theta} = -K_e \cdot \dot{\theta}$$
$$\Rightarrow K_e = N \cdot B \cdot A$$

$$\tau = N \cdot d \cdot B \cdot I \cdot L = K_\tau \cdot I$$
$$\Rightarrow K_\tau = N \cdot d \cdot B \cdot L$$

turns per coil

coil width

permanent magnetic field

coil length

$$K_e = K_\tau$$
$$\Rightarrow N \cdot B \cdot A = N \cdot d \cdot B \cdot L$$

HOW TO CHOOSE THE MOTOR CONSTANTS?

$$V = I \cdot R + L \cdot \frac{dI}{dt} - K_e \cdot \dot{\theta}$$

$$\mathcal{E} = -K_e \cdot \dot{\theta}$$

$$\tau = K_\tau \cdot I$$

If you pick a $K_\tau = K_e$ **too high**, you 'run out of voltage' and **can not achieve the speed you need**.

If you pick a $K_\tau = K_e$ **too low**, the current needed to achieve the torque you need will be higher than necessary.

Source: stackexchange

HOW TO CHOOSE THE MOTOR CONSTANTS?

$$V = I \cdot R + L \cdot \frac{dI}{dt} - K_e \cdot \dot{\theta}$$
$$\mathcal{E} = -K_e \cdot \dot{\theta}$$
$$\tau = K_\tau \cdot I$$

Rule of thumb with DC motor selection: **pick a motor with a K_τ and back-EMF constant K_e such that the supply voltage you have available is well-matched with the back-EMF at your maximum speed**.

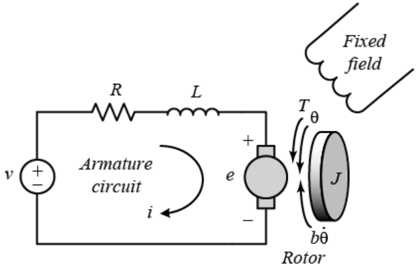
Source: stackexchange

DC MOTOR DYNAMICS

DC MOTOR DYNAMICS

So far, we have been considering the motor in **steady state**: the current, the voltage, the angular velocity do not change over time.

What about the dynamics?



DC MOTOR DYNAMICS

So far, we have been considering the motor in **steady state**: the current, the voltage, the angular velocity do not change over time.

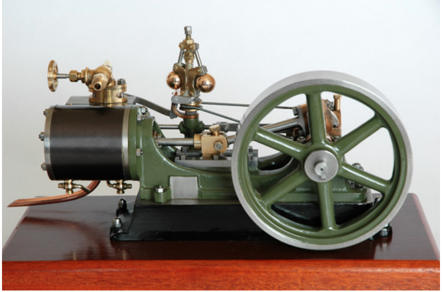
What about the dynamics?

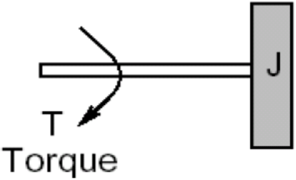
Notation:

- $I \to i(t)$
- $V \to v(t)$
- $\theta \to \theta(t)$

MOMENT OF INERTIA

Resists with opposing torque proportional to angular acceleration

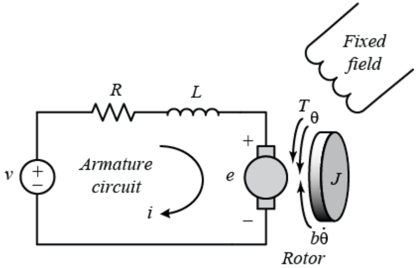




$$\tau = J \cdot \frac{d^2\theta}{dt^2}$$

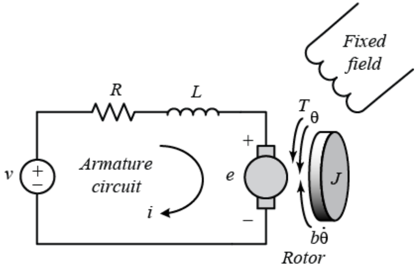
where
τ is the torque, in $N \cdot m$
J is the moment of inertia, in $kg \cdot m^2$

DC MOTOR DYNAMICS



- R = Armature resistance (in ohms Ω)
- L = Armature inductance (in Henrys H)
- J = Moment of inertia for the motor rotor ($kg \cdot m^2$)
- b = Motor viscous friction constant (in $N \cdot m \cdot s$)
- K_t = Motor torque constant (in $\frac{N \cdot m}{A}$)
- K_e = Electromotive force constant (in $\frac{V \cdot rad^{-1}}{sec}$)

DC MOTOR DYNAMICS



Motor torque τ_m is given by $\tau_m = K_\tau \cdot i(t)$

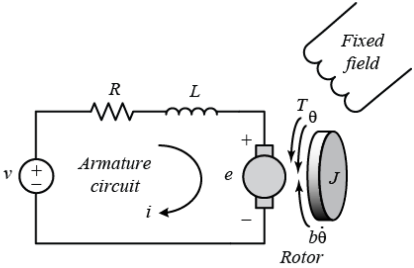
Mechanical resisting torque τ_r is given by $\tau_r = b \cdot \dot{\theta} + J \cdot \ddot{\theta}$

Under no load, $\tau_m = \tau_r$

Therefore:

$$K_\tau \cdot i(t) = b \cdot \dot{\theta} + J \cdot \ddot{\theta}$$

DC MOTOR DYNAMICS



Kirchhoff's voltage law: voltage across resistor, inductance and back EMF balance applied voltage

$$v(t) = i(t) \cdot R + L \cdot \frac{di}{dt} - K_e \cdot \dot{\theta}$$

DIFFERENTIAL EQUATION OF A DC MOTOR

$$K \cdot i(t) = b \cdot \dot{\theta} + J \cdot \ddot{\theta}$$

$$v(t) = i(t) \cdot R + L \cdot \frac{di}{dt} - K \cdot \dot{\theta}$$

DIFFERENTIAL EQUATION OF A DC MOTOR

$$K \cdot i(t) = b \cdot \dot{\theta} + J \cdot \ddot{\theta}$$

$$v(t) = i(t) \cdot R + L \cdot \frac{di}{dt} - K \cdot \dot{\theta}$$

$$\Rightarrow v(t) = (\frac{b}{K} - K) \cdot \dot{\theta} + \frac{J}{K} \cdot \ddot{\theta} + L \frac{d(\frac{b}{K} \cdot \dot{\theta} + \frac{J}{K} \cdot \ddot{\theta})}{dt}$$

DIFFERENTIAL EQUATION OF A DC MOTOR

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$$\Rightarrow v(t) = (\frac{b}{K} - K) \cdot \dot{\theta} + (\frac{J}{K} + \frac{L \cdot b}{K}) \cdot \ddot{\theta} + \frac{L \cdot J}{K} \cdot \dddot{\theta}$$

We can integrate 3 times, or...

LAPLACE TRANSFORM

The Laplace transform is a linear operator that maps a function $f(t)$ to $F(s)$ in the frequency domain.

Specifically:

$$F(s) = \mathcal{L}\{f\}(s) = \mathcal{L}\{f(t)\} = \int_0^{\text{inf}} f(t)e^{-st}dt$$

where $s = \sigma + i\omega$ (the complex number frequency parameter)

Go from a function of a ***real*** variable (here time t) to a complex function of a complex variable (frequency, s).

LAPLACE TRANSFORM

Why bother?

Often **simplifies the process of analyzing the behavior of the system**.

For example, Laplace transformation from the time domain to the frequency domain **transforms differential equations into algebraic equations**.

OPERATIONS USEFUL FOR SOLVING DIFFERENTIAL EQUATIONS

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

$$\mathcal{L}\{\int_0^t f(t)dt\} = \frac{F(s)}{s}$$

See Wikipedia for more properties.

SOLUTION USING LAPLACE TRANSFORMATIONS

Taking Laplace transforms of the differential equations that describe the motor mechanical dynamics:

$$K \cdot i(t) = b \cdot \dot{\theta} + J \cdot \ddot{\theta}$$

becomes:

$$K \cdot I(s) = b \cdot s \cdot \theta(s) + J \cdot s^2 \cdot \theta(s)$$

$$I(s) = \frac{s \cdot (b + J \cdot s) \cdot \theta(s)}{K}$$

SOLUTION USING LAPLACE TRANSFORMATIONS

Taking Laplace transforms of the differential equations that describe the motor voltages:

$$v(t) = i(t) \cdot R + L \cdot \frac{di}{dt} + K \cdot \dot{\theta}$$

becomes:

$$V(s) = I(s) \cdot (R + L \cdot s) + K \cdot s \cdot \theta(s)$$

SOLUTION USING LAPLACE TRANSFORMATIONS

Substituting:

$$I(s) = \frac{s \cdot (b + J \cdot s) \cdot \theta(s)}{K}$$

into:

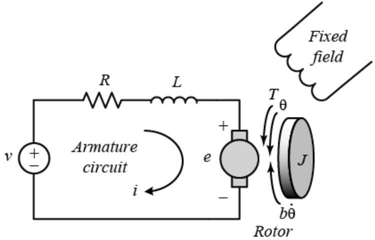
$$V(s) = I(s) \cdot (R + L \cdot s) + K \cdot s \cdot \theta(s)$$

by eliminating current $I(s)$ gives:

$$\theta(s) = \frac{K}{s \cdot ((J \cdot s + b) \cdot (L \cdot s + R) + K^2)} \cdot V(s)$$

TRANSFER FUNCTION

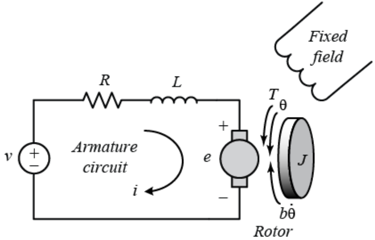
Result of a transfer response output position for a DC electric motor given its input voltage



$$\theta(s) = \frac{K}{s \cdot ((J \cdot s + b) \cdot (L \cdot s + R) + K^2)} \cdot V(s)$$

TRANSFER FUNCTION

Result of a transfer response output position for a DC electric motor given its input voltage



$$\theta(s) = \frac{K}{s \cdot ((J \cdot s + b) \cdot (L \cdot s + R) + K^2)} \cdot V(s)$$

We can differentiate this expression to get the transfer function for speed

$$\dot{\theta}(s) = \frac{K}{(J \cdot s + b) \cdot (L \cdot s + R) + K^2} \cdot V(s)$$

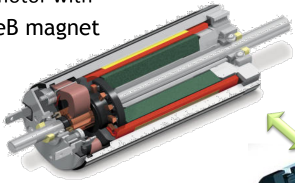
10 min break

WRAP-UP ON DC MOTORS: ANOTHER LOOK AT DATASHEETS

MAXON DC MOTOR VARIANTS

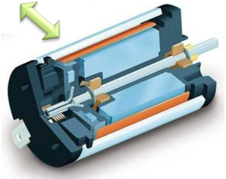
Lots of choice in choosing a motor

RE motor with
NdFeB magnet



ball
bearing

sintered
sleeve bearing

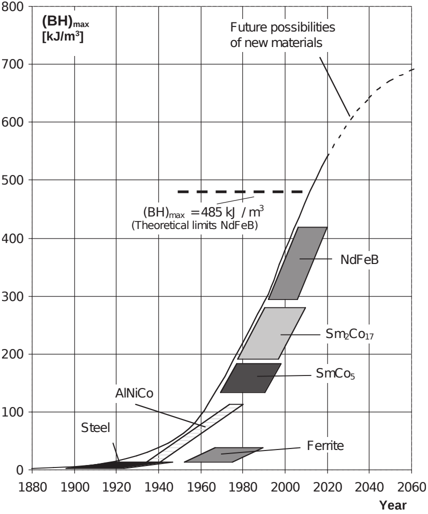


A-max motor with
AlNiCo magnet

precious
metal
brushes

graphite
brushes

DEVELOPMENT OF PERMANENT MAGNETS



Source: vacuumschmelze

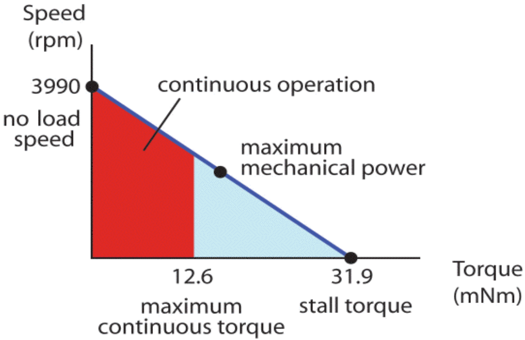
UNDERSTANDING MOTOR DATASHEETS

- Examine DC motor datasheets
- Selecting a motor to suit the job

	Motor Data (provisional)		
	Values at nominal voltage		
1	Nominal voltage	V	9
2	No load speed	rpm	2850
3	No load current	mA	49.7
4	Nominal speed	rpm	2610
5	Nominal torque (max. continuous torque)	mNm	87.8
6	Nominal current (max. continuous current)	A	2.96
7	Stall torque	mNm	873
8	Starting current	A	29
9	Max. efficiency	%	92
	Characteristics		
10	Terminal resistance	Ω	0.311
11	Terminal inductance	mH	0.0824
12	Torque constant	mNm/A	30.2
13	Speed constant	rpm/V	317
14	Speed / torque gradient	rpm/mNm	3.27
15	Mechanical time constant	ms	4.85
16	Rotor inertia	gcm^2	142

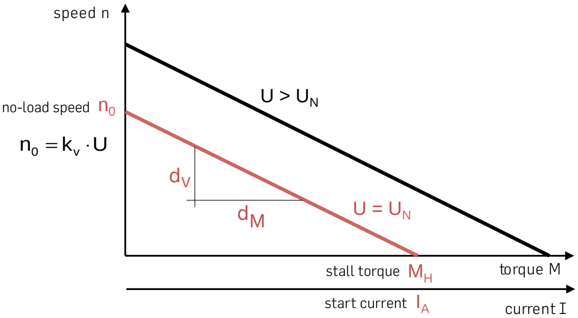
DC MOTOR DATA

Plot of motor rotational speed against torque



Power = Speed (Rads^{-1}) x Torque (NM)
Cannot specify a motor simply on its power.
Require a certain torque at a certain speed.
Motor may be powerful enough but not have the torque available at required speed

SPEED-TORQUE CHARACTERISTIC

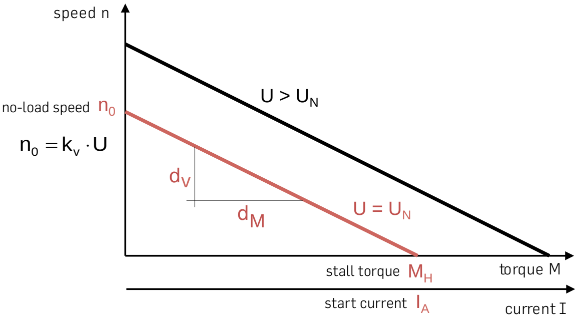


Remember:

$$\dot{\theta} = \frac{V}{K} - R \cdot \frac{\tau}{K^2}$$

$$\tau = K \cdot I$$

SPEED-TORQUE CHARACTERISTIC



Remember:

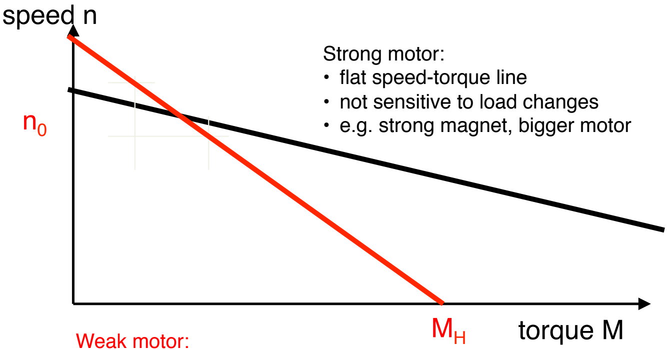
$$\dot{\theta} = \frac{V}{K} - R \cdot \frac{\tau}{K^2}$$
$$\tau = K \cdot I$$

Influence of voltage:

- no-load speed n_0 proportional to applied voltage
- stall torque M_H proportional to the applied voltage
- increasing voltage shifts the speed-torque line upwards
- speed-torque gradient $\frac{d_v}{d_M}$ unaffected

SPEED-TORQUE CHARACTERISTIC

Speed reduced as output motor torque is increased

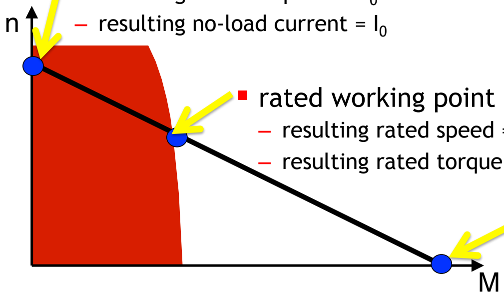


Strong motor:
- flat speed-torque line
- not sensitive to load changes
- e.g. strong magnet, bigger motor

Weak motor:
- steep speed-torque line
- sensitive to load changes
- e.g. weak magnet, smaller motor

OPERATING POINTS 1/2

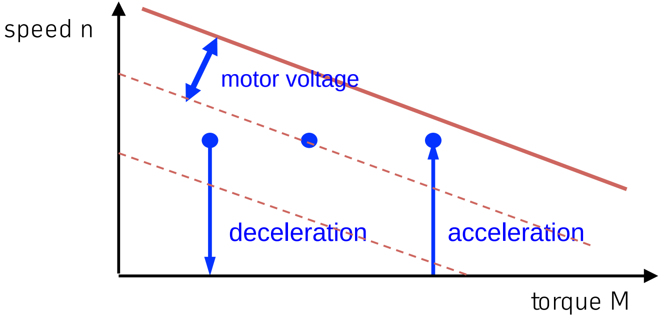
- No-load operating point
  - resulting no-load speed = n_0
  - resulting no-load current = I_0



- rated working point
  - resulting rated speed = n_N
  - resulting rated torque = M_N
- Motor at stall
  - resulting stall torque = M_H
  - resulting starting current = I_A

OPERATING POINTS 2/2

- **Load operating points** are defined by the applications. They are characterised by a load speed n_L at a given load torque M_L
- **Motor operating points** lie on the speed-torque line: select the voltage accordingly.



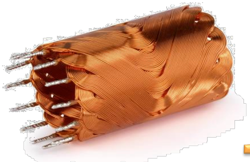
EFFECT OF CHANGING THE WINDINGS

$$\dot{\theta} = \frac{V}{K} - R \cdot \frac{\tau}{K^2}$$

...what would be the effect?

- on max speed?
- on start current/stall torque?

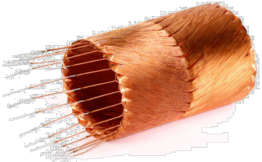
EFFECT OF CHANGING THE WINDINGS

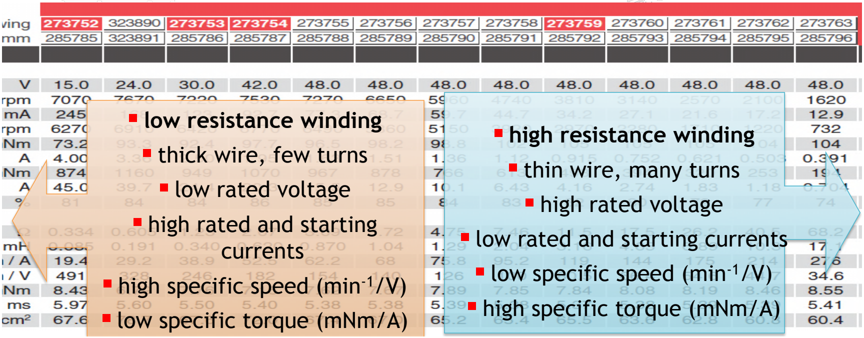


$$\dot{\theta} = \frac{V}{K} - R \cdot \frac{\tau}{K^2}$$

resistance increases from left to right

R





- low resistance winding
  - thick wire, few turns
  - low rated voltage
  - high rated and starting currents
  - high specific speed (min^{-1}/V)
  - low specific torque (mNm/A)

- high resistance winding
  - thin wire, many turns
  - high rated voltage
  - low rated and starting currents
  - low specific speed (min^{-1}/V)
  - high specific torque (mNm/A)

NOMINAL MOTOR CHARACTERISTICS

- Nominal voltage (V) – operating voltage of motor
- No load speed (rpm) – rotational speed at operating voltage of motor with no load
- No load current (mA) – current taken at operating voltage of motor with no load
- Nominal speed (rpm) – rotational speed at operating voltage of motor with no load
- Nominal torque (max. continuous torque) (mNm) – max torque
- Nominal current (max. continuous current) (A) - max continuous current that can be passed through motor
- Stall torque (mNm) – torque generated at operating voltage of motor when motor is held stationary
- Starting current (A) – current draw when motor starts when operating voltage of motor applied
- Max. efficiency (%) – energy efficiency of the motor

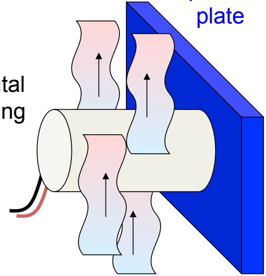
LIST OF MAIN MOTOR PARAMETERS

- Terminal resistance (phase to phase for EC) (Ω)
- Terminal inductance (phase to phase for EC) (mH)
- Torque constant (mNm/A)
- Speed constant (rpm/V)
- Speed / torque gradient (rpm/mNm)
- Mechanical time constant (ms)
- Rotor inertia (gcm^2)

MOTOR THERMAL CONSIDERATIONS

We must take motor heating and thermal limits into account

- Depend strongly on mounting conditions
- Standard mounting:



- Heating and cooling
  - Thermal resistance housing-ambient R_{th2}
  - Thermal resistance winding-housing R_{th1}
  - Thermal time constant of winding t_{thW}
  - Thermal time constant of motor t_{thS}
- Temperature limits
  - Ambient temperature range
  - Max. winding temperature T_{max}

INFLUENCE OF TEMPERATURE ON MOTOR OPERATION

Temperature coefficients

Resistivity ρ for Cu	+ 0.39 % per K
Flux density B for AlNiCo	- 0.02 % per K
Flux density B for Ferrite	- 0.2 % per K
Flux density B for NdFeB	- 0.1 % per K

As temperature goes up

Resistance goes up

Desirable magnetic properties go down

For example: RE motor

If ΔT = + 50K

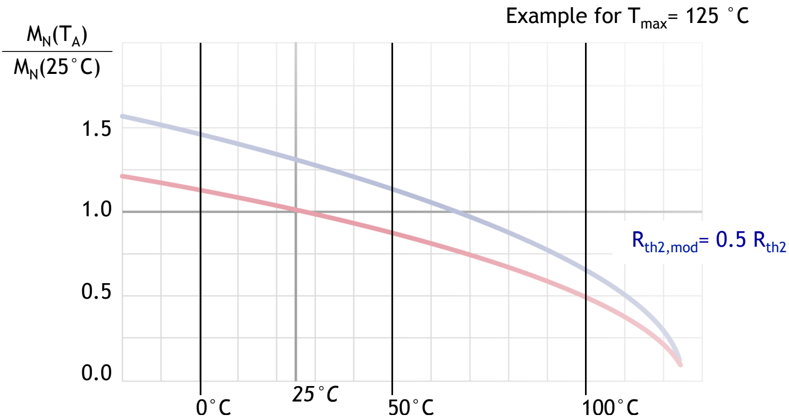
R: + 19.5 %

K_n:+ 5 % (no-load speed)

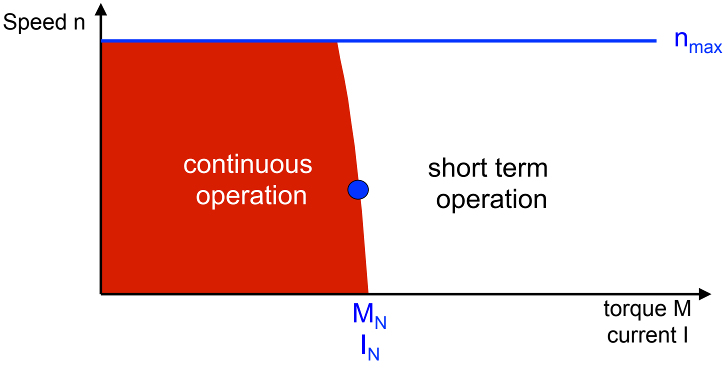
k_M: - 5 % (more current!)

Stall torque M_H : - 22 %

PERMISSIBLE TORQUE AND TEMPERATURE



MOTOR LIMITS: OPERATION RANGES



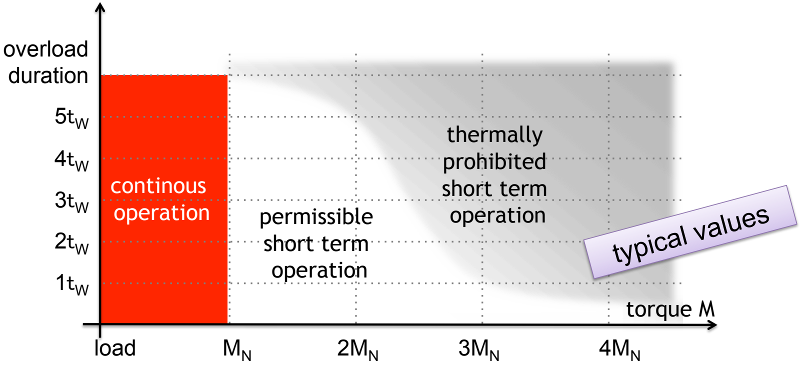
- higher ambient temperature
- heat accumulation

- lower ambient temperature
- good heat dissipation

SHORT-TERM OVERLOAD OPERATION

Motor may be overloaded for a short time and repeatedly

- Limit: max. permissible winding temperature
- Depends on thermal time constant of winding tW and amount of overload

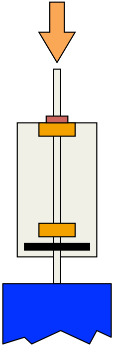


LIST OF MAIN THERMAL MOTOR PARAMETERS

- Thermal resistance housing-ambient (K/W)
- Thermal resistance winding-housing (K/W)
- Thermal time constant winding (s)
- Thermal time constant motor (s)
- Ambient temperature (°C)
- Max. permissible winding temperature (°C)

MECHANICAL MOTOR PARAMETERS

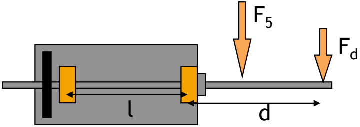
Maximum speed and mechanical properties of motor bearings



axial press fit force
(shaft supported)

- Max. permissible speed
  - Limited by bearing life considerations (EC)
  - Limited by relative speed between collector and brushes (DC)

- Axial and radial play
  - suppressed by a preload
- Axial and radial bearing load
  - Dynamic: in operation
  - Static: at stall



LIST OF MAIN MECHANICAL MOTOR PARAMETERS

Max. permissible speed (rpm)

Axial play (mm)

Radial play (mm)

Max. axial load (dynamic) (N)

Max. force for press fits (static) (N)

(static, shaft supported) (N)

Max. radial loading, 5 mm from flange (N)

OTHER SPECIFICATIONS

Rates power (W)
Life expectancy (h)
Weight of motor (g)
Direction of rotation
Number of pole pairs
Number of commutator segments
Commutator material
Insulation class
Protection class
Armature details
Magnet system
Bearing type
Housing

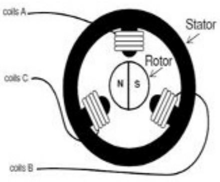
BRUSHLESS DC MOTORS

PROBLEMS OF MECHANICAL COMMUTATION

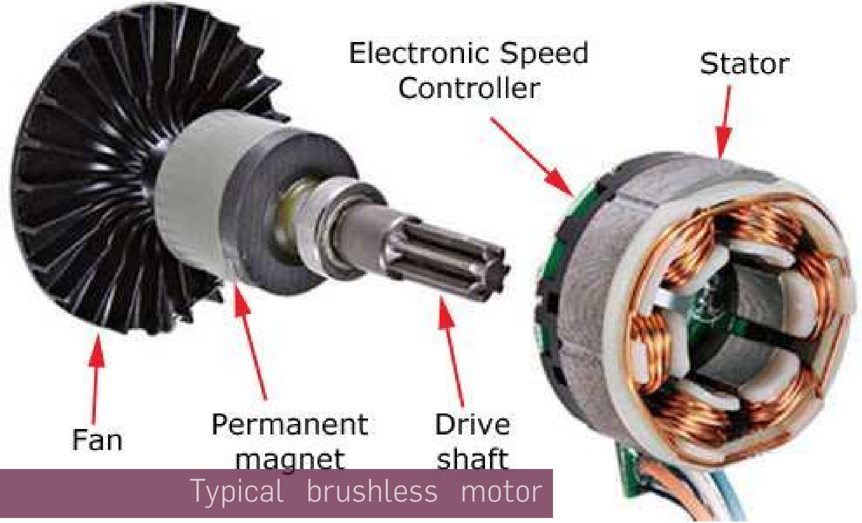
- Can get potential difference across commutator segments
- Commutation shorts out the commutator segments
- Arcing and sparkling at the brushes
- Brushless electronic switching solves this issue

BRUSHLESS DC MOTOR

- looks like DC brushed motor turned inside out
- commutation is performed electronically to eliminate brushes → **electronic commutation, EC**
- the stator generally consists of several coils
- current flow in the stator coils creates magnetic field
- this forces the permanent magnet rotor to spin
- continuous rotation by switching on current in the stator → **sequenced magnetic field**
- brushless motors **require a controller** that perform the commutation

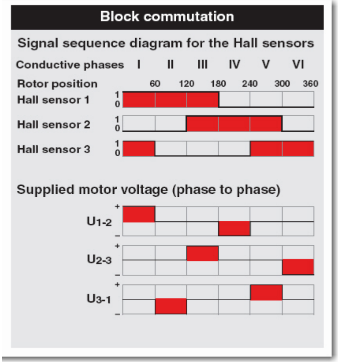


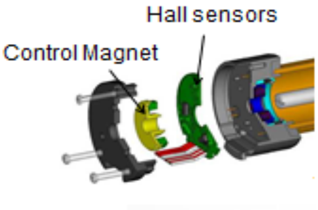
1 pair of poles



Typical brushless motor

HOW DO BRUSHLESS MOTORS WORK?



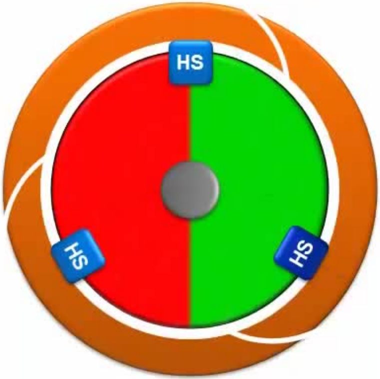


- Electronic commutation is used to switch current in the stator could so that the rotor is forced to rotate

- There is often a control magnet is in line with the poles of the large magnet in the motor to identify rotor angle so that the controller can switch current into the appropriate coils
- As it turns Hall sensors are stimulated by the magnetic flux.
- The Hall sensors are used to tell the controller what the orientation is of the magnet with respect to the three winding phases.
- Current in the stator coils is turned on and off in sequence creating motion from pole to pole.

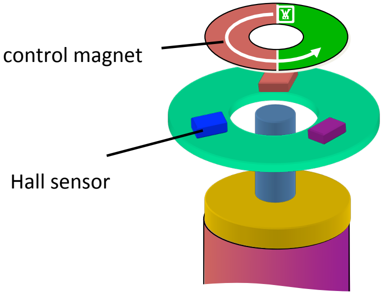
This animation shows the block commutation sequence of a brushless DC motor.

On the right there is a schematic cross section of a maxon EC motor.

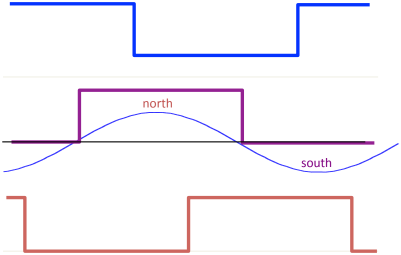


BLOCK COMMUTATION

Rotor position from Hall sensor signals



EC-max and EC flat:
Power magnet is probed directly

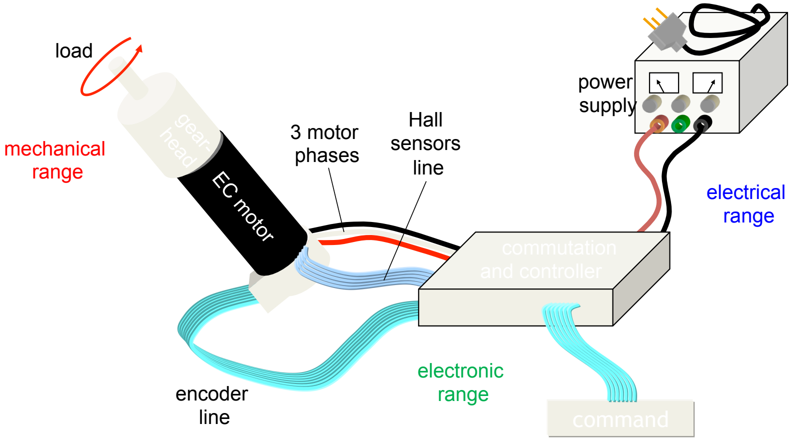


1	1	0	0	0	1	1
0	1	1	1	0	0	0
0	0	0	1	1	1	0

0° 60° 120° 180° 240° 300° 360°

rotation angle

COMPONENTS OF AN EC DRIVE SYSTEM

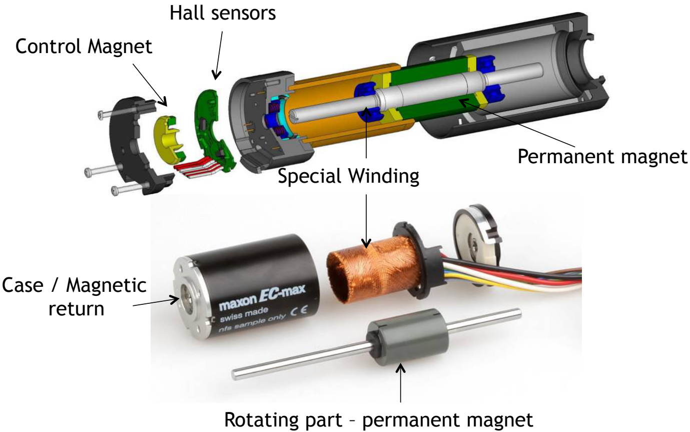


EC motors - overview

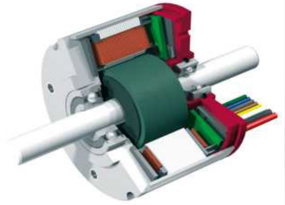
BRUSHLESS MOTOR FOR RC AIRCRAFT

- Product Name : Brushless Motor;Model : A2212-6;KV : 2200RPM/V
- Fit for Battery : 2-3 Li-Poly;Fit for ESC : 30A;Shaft Size : 3 x 12mm/ 0.1" x 0.5"(D*L)
- Motor Part Size : 25 x 28mm/ 1" x 1.1"(L*D);Mounted Screw Hole Diameter : 2.5mm/0.1";Screw Hole Centre Distance : 13 x 13mm/0.5" x 0.5"(L*W)
- Cable Length : 55mm / 2.2";Material : Metal, Electronic Parts;Color : Silver Tone, Brass Tone
- Weight : 59g;Package Content : 1 x Brushless Motor w Prop Adapter

CONSTRUCTION OF A EC BRUSHLESS MOTOR



MAXON EC FLAT BRUSHLESS MOTOR



Multi pole motor

Flat design gives more torque as the flux is acting further from the centre of rotation

ADVANTAGES AND DISADVANTAGES OF EC

Brushed DC motors

- Mechanical commutation
- Need periodic brush maintenance
- Power losses in brushes
- Sparking
- Can have noisy operation
- Linear torque characteristic at lower speeds
- Change direction by changing voltage polarity
- Controller not always needed

EC motors

- Electronic commutation
- Low or no maintenance
- Less power loss
- No sparking
- Quieter operation
- More linear torque characteristic
- Change direction by changing switching sequence
- Always needs drive controller circuitry
- Requires sensors
- Higher reliability & efficiency
- Stator on outside – better for heat dissipation
- Longer life
- More expensive

That's all, folks!

Questions:

Portland Square B316 or **severin.lemaignan@plymouth.ac.uk**

Slides:

github.com/severin-lemaignan/module-introduction-sensors-actuators

...or the DLE!