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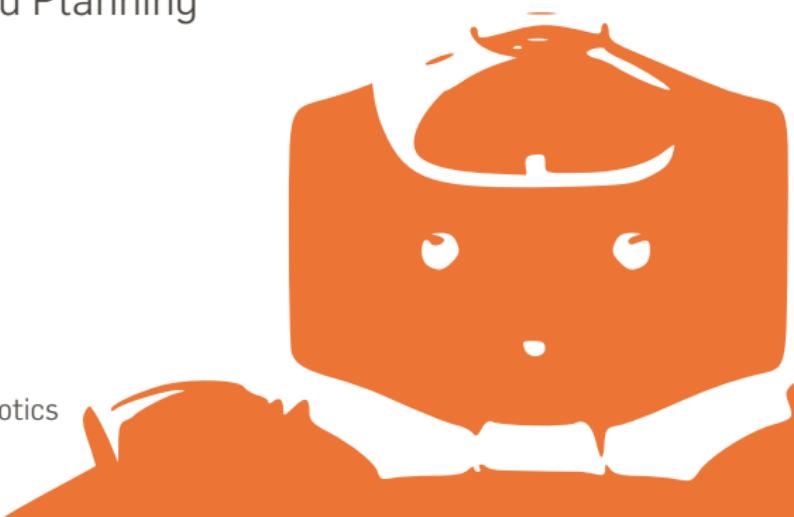
**ROBOTICS  
WITH  
PLYMOUTH  
UNIVERSITY**

ROC0318

Mobile and Humanoid Robots

Part 6 – Localisation and Planning

Centre for Neural Systems and Robotics  
**Plymouth University**





# POSE AND NAVIGATION

## Pose maintenance

- Pose (= position and orientation) maintenance is keeping an as precise as possible estimate of the robot's pose.

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## Planning: getting from A to B

- A cognitive skill: given partial knowledge of the environment, a goal and sensor readings, decide what to do to reach the goal as efficiently and reliably as possible.
- Different from inverse kinematics: IK *locally* maps a target state (position and orientation) in a global space (e.g. Cartesian space) to the robot's internal space (joint space, or generally *configuration space*)
- → IK work on a small scale (< meter) while planning/navigation works on larger scale (room, building, roadmap, ...)

## PLANNING AND NAVIGATION – TRADITIONAL APPROACH

Today's industrial robots can operate **without any intelligence** because their environment is static and very structured.

In mobile robotics, cognition and reasoning is primarily of geometric nature, such as picking a safe path or determining where to go next.

→ already been largely explored in literature for cases in which complete information about the current situation and the environment exists (e.g. **travelling sales man problem** or **route inspection problem**).

# KIVA SYSTEMS: BEACON BASED LOCALISATION

<http://www.youtube.com/watch?v=lWsMdN7HMuA>



# MONTE-CARLO LOCALISATION



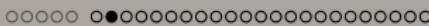
# MONTE CARLO METHODS

## Repeated random sampling to compute results

Used when it is difficult to calculate exact results:

- the system is complex and calculating it through is too computationally expensive,
- or, too many variables and it is impossible to calculate all possibilities (*combinatorial explosion*)





# MONTE CARLO METHODS

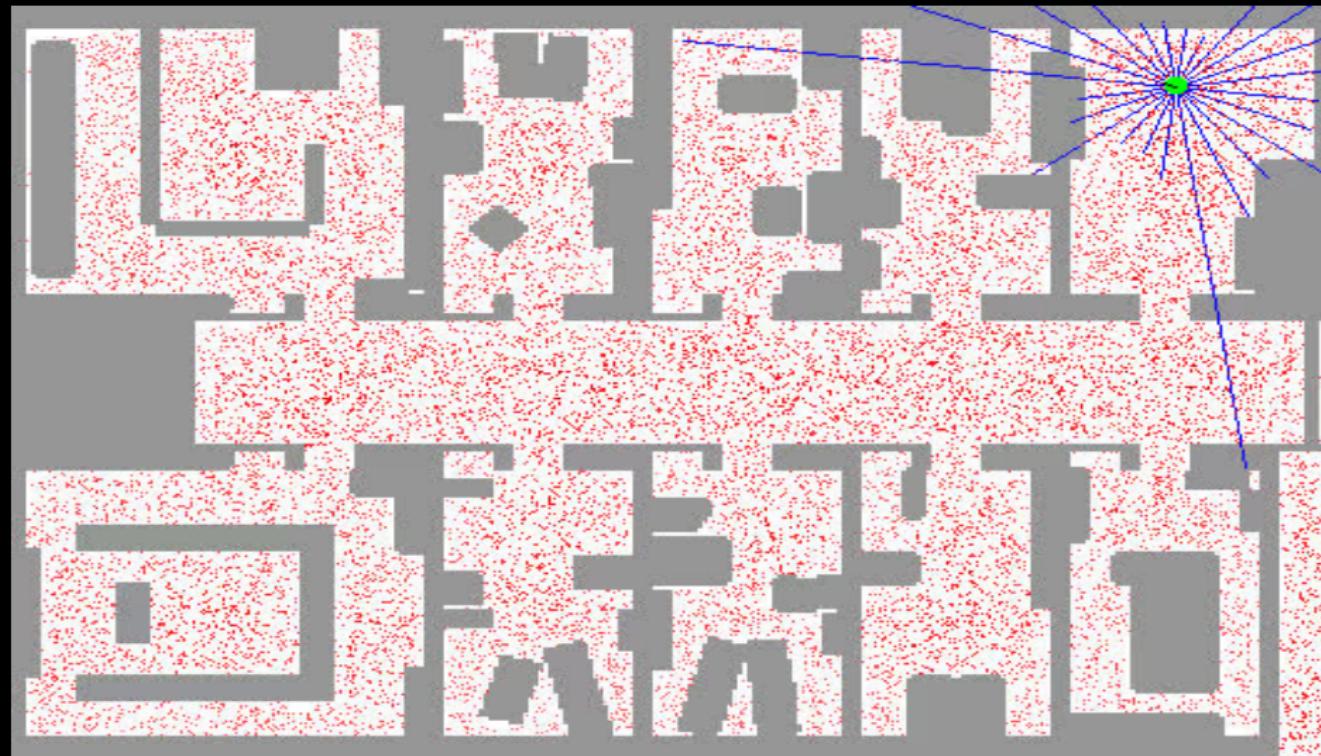
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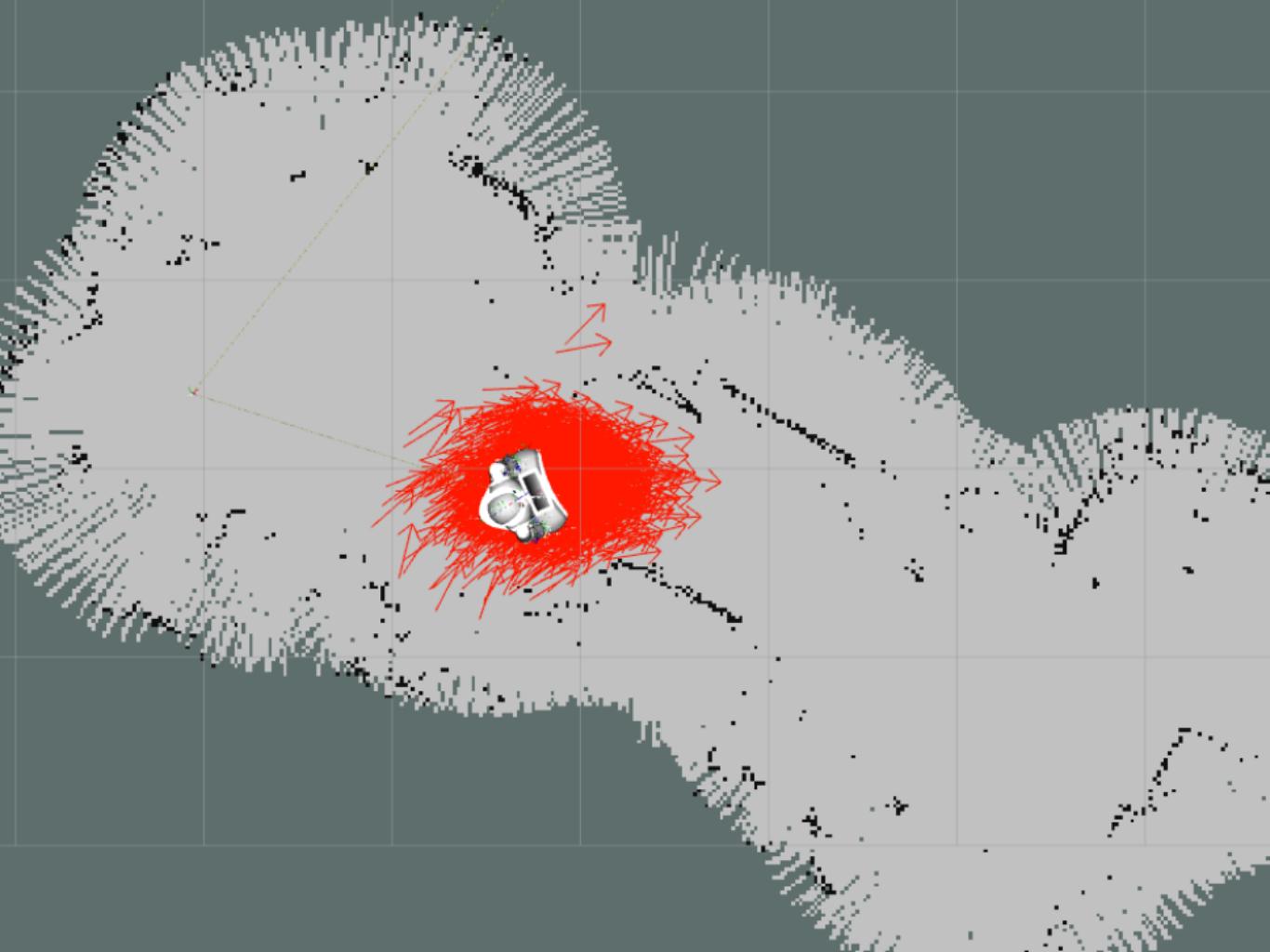
Used when it is difficult to calculate exact results:

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Monte Carlo *localisation* is MC methods applied to robot localisation.





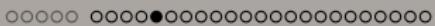
## MONTE-CARLO LOCALISATION – INTRODUCTION

- Monte-Carlo Localisation (MCL) is a **particle filter**, which uses a set  $S_t$  of  $N$  particles at time  $t$ .

$$S_t = \{s_t^i \quad | \quad i = 1 \dots N\}$$

$$s^i = \{\mathbf{x}^i, w^i\} \text{ with } \mathbf{x}^i = \begin{bmatrix} x^i \\ y^i \\ \theta^i \end{bmatrix}$$

- Each particle  $s^i$  contains the pose of the particle  $\mathbf{x}^i$  and a weight  $w^i$ .



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- For small maps,  $N$  is in the hundreds, for larger maps you can have thousands of particles.



## MONTE-CARLO LOCALISATION – INITIALISATION

The  $N$  particles are distributed on the map

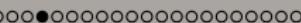
- If the robot's position is not known, they are distributed randomly across the map with a probability  $p = \frac{1}{N}$  (*uniform distribution*).



## MONTE-CARLO LOCALISATION – INITIALISATION

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- If the robot's position is not known, they are distributed randomly across the map with a probability  $p = \frac{1}{N}$  (*uniform distribution*).
- If the robot's position is available, the initial particle distribution could be Gaussian around the robot (*normal distribution*).



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Then , Monte Carlo Localisation runs through three steps:

1. Prediction phase (when the robot takes an action)
2. Update phase (when sensor data comes in)
3. Resampling phase

## MONTE-CARLO LOCALISATION - PREDICTION PHASE

The prediction phase **essentially moves all particles** along with the robot.

Example: if the robot moves 10cm right and turn 10° counterclockwise, then all particles are updated:

$$s_t^i = \{\{x_{t-1}^i + 0.1, y_{t-1}^i, \theta_{t-1}^i - 10^\circ\}, w_{t-1}^i\}$$

Note, the weight is not changed



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This allows for a **motion model** to be implemented, by adding some noise.

$$s_t^1 = \{\{x_{t-1}^1 + 0.11, y_{t-1}^1, \theta_{t-1}^1 - 10.3^\circ\}, w_{t-1}^1\}$$

$$s_t^2 = \{\{x_{t-1}^2 + 0.09, y_{t-1}^2, \theta_{t-1}^2 - 10.2^\circ\}, w_{t-1}^2\}$$

...

## MONTE-CARLO LOCALISATION - PREDICTION PHASE

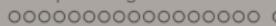
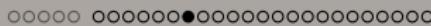
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This allows for a **motion model** to be implemented, by adding some noise.

This motion model should be based on the kinematics of your robot. e.g. for a differential drive robot you can have a good idea of how imprecise it is.



## MONTE-CARLO LOCALISATION - PREDICTION PHASE

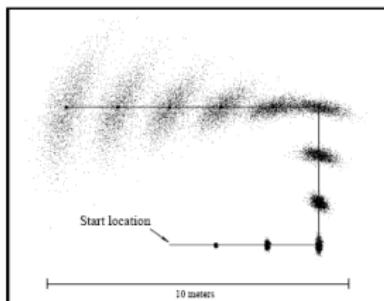
The prediction phase **essentially moves all particles** along with the robot.

Formally, if the robot performs an action  $a$ , all  $N$  particles are updated as follows:

$$\mathbf{x}_t^i = p(\mathbf{x}_t^i | \mathbf{x}_{t-1}^i, a)$$

Motion model: what is the probability of ending up in the location of particle  $s_t$  after doing action  $a$  at the location of particle  $s_{t-1}$

Example: repeated application of the prediction rule leads to gradual loss of pose accuracy:



## MONTE-CARLO LOCALISATION – UPDATE PHASE

- The robot gets **new sensor data** in, all  $N$  particles updated as follows:

$$w_t^i = \alpha \cdot p(z_t | \mathbf{x}_t^i) \cdot w_t^i$$

Normalisation constant: keeps the sum of  $w^i$  equal to 1

Sensor model: what is the probability of seeing sensor reading  $z_t$  at position  $\mathbf{x}_t$

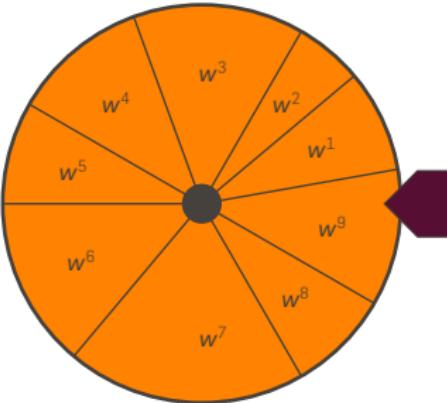
- The normalising constant  $\alpha$  is calculated afterwards to make

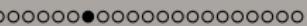
$$\sum_{i=1}^N w^i = 1$$

## MONTE-CARLO LOCALISATION – RESAMPLING PHASE

Generate a new set of particles by drawing randomly from the current set. Likelihood is determined by the weights values.

You can think of drawing particles as a roulette wheel selection. Particles with a higher weight have a higher chance of being selected into the new set. Also, a particle can be selected **more than once**.





## MONTE-CARLO LOCALISATION – RESAMPLING PHASE

Generate a new set of particles by drawing randomly from the current set. Likelihood is determined by the weights values.

In addition, it is useful to add a **small number of uniformly distributed particles** to the set of particles.

This helps the algorithm recover when it loses track of the robot's position.

This is known as the "*kidnapped robot*" problem, and occurs when the robot loses track of its position, due to a malfunction or power outage.

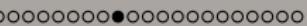


## PSEUDOCODE

```
# initialise all N particles
for i in range(1, N):
    S[i].x = random(x_max)
    S[i].y = random(y_max)
    S[i].θ = random(2 * π)
    S[i].w = 1/N
```

# PSEUDOCODE

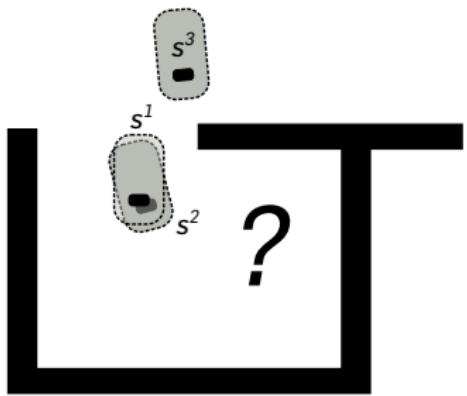
```
while True:  
    # 1- prediction phase  
    x, y, θ = estimate_robot_position()  
    for i in range(1,N):  
        move_particle(S[i], x, y, θ) # adds noise as well -> motion model  
  
    # 2- update phase (z: sensor readings, m: map)  
    n = 0  
    for i in range(1,N):  
        S[i].w = likelihood(z, (S[i].x, S[i].y, S[i].θ), m) * S[i].w  
        n = n + S[i].w  
    # normalise  
    for i in range(1,N):  
        S[i].w = S[i].w/n  
  
    # 3- resample phase  
    S1 = best_particles() # particles with high S.w  
    S2 = worst_particles() # particles with low S.w  
    S = S - S2 + S1 # effectively duplicates good particles  
    # optionally, add a few new random particles  
    for i in range(1, N): # reset the weights  
        S[i].w = 1/N
```

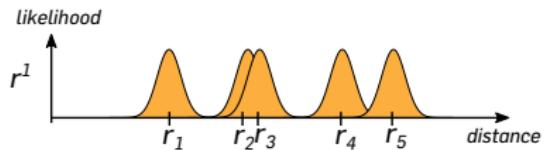
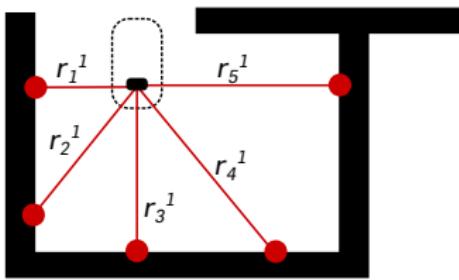


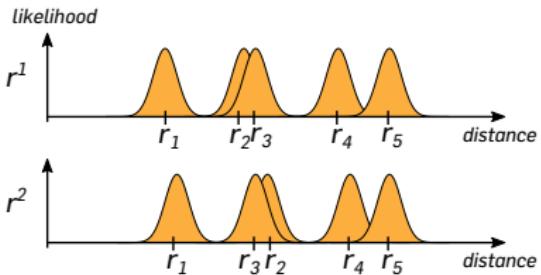
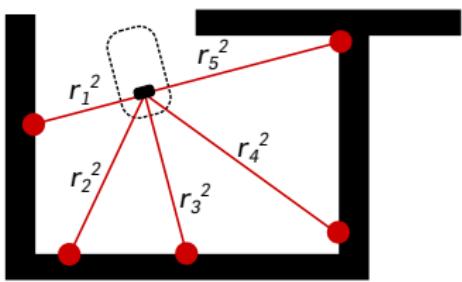
## LIKELIHOOD FUNCTION

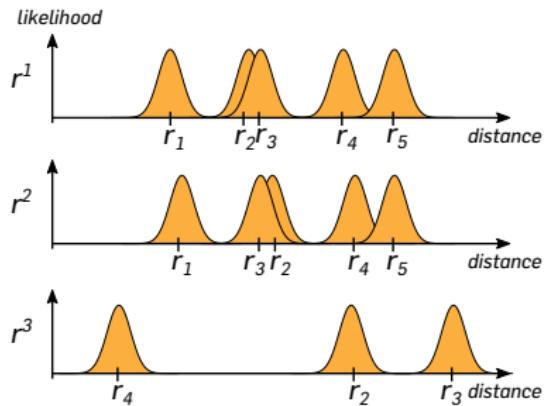
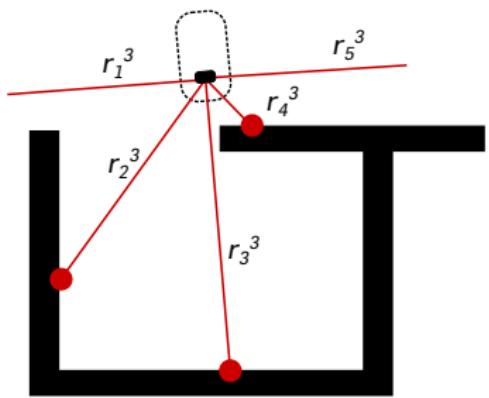
How is `likelihood(z, x, m)` implemented?

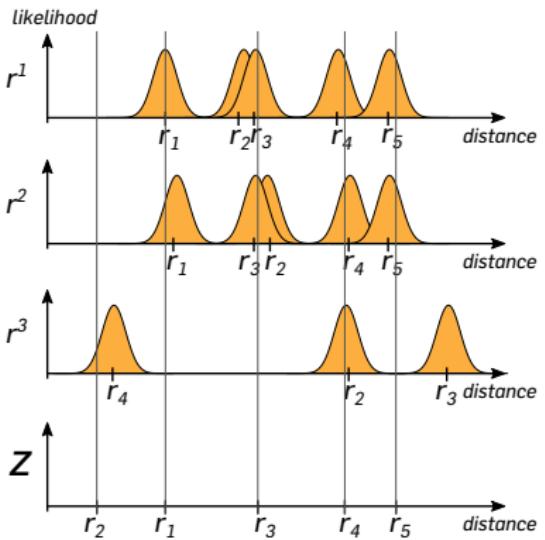
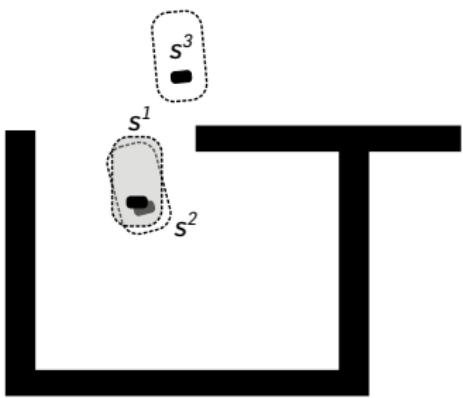
- $z$ : the actual range measurements (sensor reading)
- $x$ : the candidate position of the robot (particle location)
- $m$ : the map

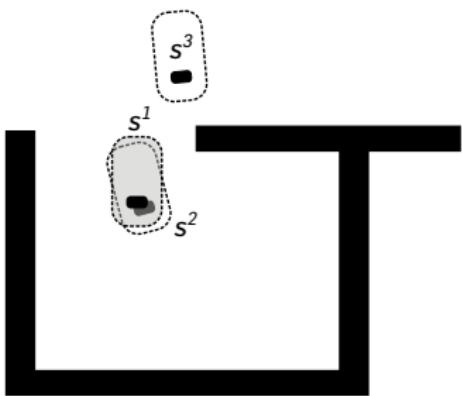




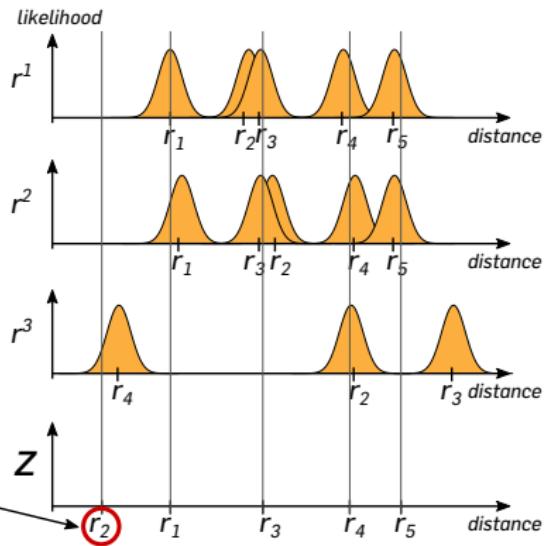








*short reading?  
someone walking in front of the robot?*



# LIKELIHOOD FUNCTION

```
def likelihood(z, x, m):

    p = 1 # likelihood

    for obs_range in z:
        map_range = raycast(x, bearing(z), m)

        pz = 0.0

        # good, but noisy, hit -> normal distribution
        err = obs_range - map_range
        pz += z_hit * exp(-err2 / (2 * sigma_hit2))

        # + other types of measurement errors:
        #     - likelihood of a short reading from unexpected objects
        #     - likelihood of failures (black surface...)
        #     - likelihood of random measurements (reflections...)
        #
        # => pz += p_short + p_failure + p_rand

    p = p * pz

return p
```



## POSITION ESTIMATE

The position estimate at time  $t$  is the mean of the weighted particles locations, i.e. the centre of the weighted particle cloud.

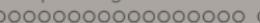
$$\bar{\mathbf{x}}_t = \sum_{i=1}^N w_t^i \cdot \mathbf{x}_t^i$$



## GLOBAL VS. CONTINUOUS LOCALISATION

The  $N$  particles are distributed on the map

- If the robot's position is not known, they are distributed randomly across the map with a probability  $p = \frac{1}{N}$  (*uniform distribution*). **Global localisation**.
- If the robot's position is available, the initial particle distribution could be Gaussian around the robot (*normal distribution*). **Continuous localisation**  $\equiv$  **Tracking**.



## MONTE-CARLO LOCALISATION – SOME PROPERTIES

- Light on memory and computational resources.

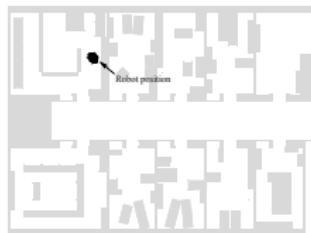
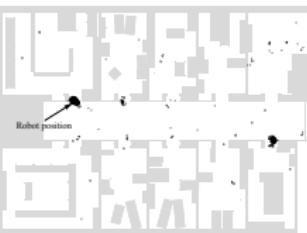
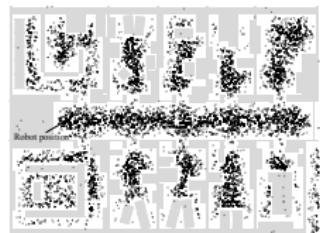
MCL draws inspiration from *Markov Localisation*, but Markov Localisation uses much more memory as it needs to keep track of belief at each location of the map even the ones that have a low belief. This also results in a computationally expensive algorithm.

- “Any time” algorithm: you can interrupt the algorithm and still get an estimate out.
- Powerful, yet efficient.
- Easy to implement.

# MONTE-CARLO LOCALISATION – SOME PROPERTIES

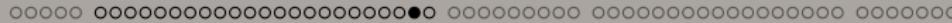
Compared to Kalman filtering:

- Kalman filtering does not work for multi-modal distributions of the position estimate (*data association problem*)
- Kalman filtering assume linear sensors/motion models
- Kalman filtering is proved to be optimal



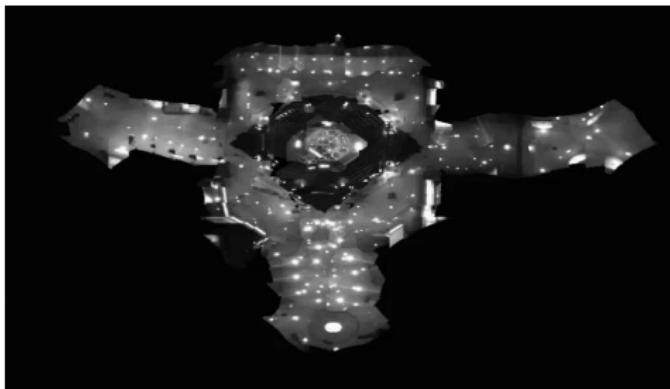
Example of bi-modal distribution arising from the symmetry of the environment

Source: *Fox et al., Monte Carlo Localization: Efficient Position Estimation for Mobile Robots*



## EXAMPLE: MUSEUM POSE MAINTENANCE

Monte Carlo using the ceiling of the Smithsonian museum



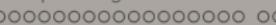
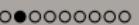
## MONTE-CARLO LOCALISATION – FURTHER READING

- Thrun, Burgard, Fox, *Probabilistic Robotics*
- Dellaert, Fox, Burgard, Thrun, Monte Carlo Localization for Mobile Robots, *IEEE International Conference on Robotics and Automation (ICRA99)*, May, 1999.

Videos of examples of MCL

- [http://www.youtube.com/watch?v=uU\\_1c\\_CxB1g](http://www.youtube.com/watch?v=uU_1c_CxB1g)
- <http://www.youtube.com/watch?v=7K8dZwqBSSA>
- <https://www.youtube.com/watch?v=oKUYj1FWzN4>
- <https://www.youtube.com/watch?v=lCXv4yOcwf8>

SLAM



## WHAT IS SIMULTANEOUS LOCALISATION AND MAPPING?

Given an unknown environment and vehicle pose:

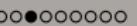
- Move through the environment
- Estimate the robot pose
- Generate a map of environmental features



## WHAT IS SIMULTANEOUS LOCALISATION AND MAPPING?

Given an unknown environment and vehicle pose:

- Move through the environment
  - Estimate the robot pose
  - Generate a map of environmental features
1. Use the robot pose estimate to improve the map landmark position estimates
  2. Use the landmark estimates to improve the robot pose estimate
  3. Repeat



# PROBABILISTIC BASIS OF SLAM

Given:

- Robot control signal  $u_k$  (or measurement → odometry)
- A set of feature observations  $z_k$  (sensor measurements)

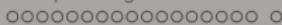
Estimate:

- Map of landmarks  $M_{k+1}$
- Robot Pose  $V_{k+1}$

Sources of Error:

- Control signal
- Motion model
- Sensor model

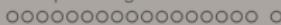
Source: *Jack Collier*



## PROBABILISTIC BASIS OF SLAM

$$p(V_{k+1}, M_{k+1} | z_k, u_k)$$

⇒ estimate the joint probability of  $V_{k+1}$  and  $M_{k+1}$  conditioned on  $z_k$  and  $u_k$ .



## PROBABILISTIC BASIS OF SLAM

$$p(V_{k+1}, M_{k+1} | z_k, u_k) = \\ \eta p(z_{k+1} | V_{k+1}, M_{k+1}) \int p(V_{k+1} | V_k, u_k) p(V_k, M_k | z_k, u_k) dV_k$$

Source: [Jack Collier](#)

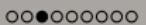


## PROBABILISTIC BASIS OF SLAM

$$p(V_{k+1}, M_{k+1} | z_k, u_k) =$$

$$\eta p(z_{k+1} | V_{k+1}, M_{k+1}) \int p(V_{k+1} | V_k, u_k) \boxed{p(V_k, M_k | z_k, u_k)} \, dv_k$$

Prior state and covariance estimates from the last filter iteration



## PROBABILISTIC BASIS OF SLAM

$$p(V_{k+1}, M_{k+1} | z_k, u_k) = \\ \eta p(z_{k+1} | V_{k+1}, M_{k+1}) \int p(V_{k+1} | V_k, u_k) p(V_k, M_k | z_k, u_k) dv_k$$

Probabilistic motion model estimates the new vehicle pose covariance estimates from the prior estimate and the control



## PROBABILISTIC BASIS OF SLAM

$$p(V_{k+1}, M_{k+1} | z_k, u_k) =$$

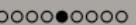
$$\eta p(z_{k+1} | V_{k+1}, M_{k+1}) \int p(V_{k+1} | V_k, u_k) p(V_k, M_k | z_k, u_k) dV_k$$

Measurement model gives  
the expected value of  
the feature observations



## SLAM FAMILIES

- *Kalman filtering*: **Extended KF (EKF) SLAM** + a few others
- *particle filtering*: Rao-Blackwellized particle filter  
**(FastSLAM)**

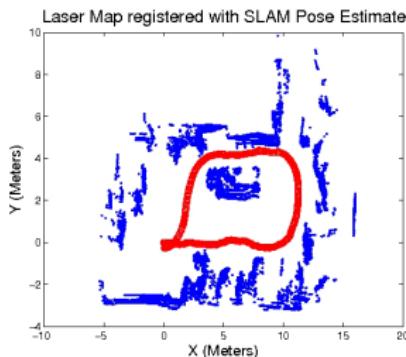
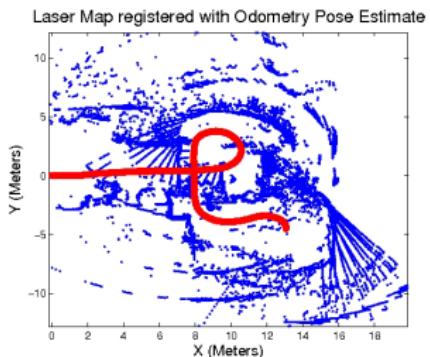


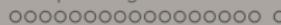
# WHY IS SLAM HARD?

## Chicken and egg problem

If there is no map, how the robot localise itself. If the robot doesn't know its location, how can it build up a map?

Odometry is not to be trusted:

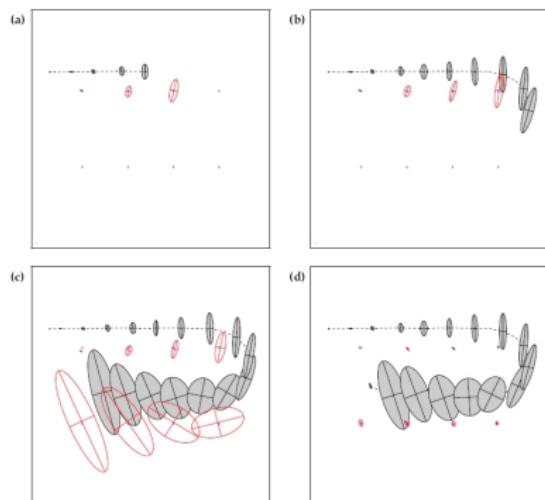


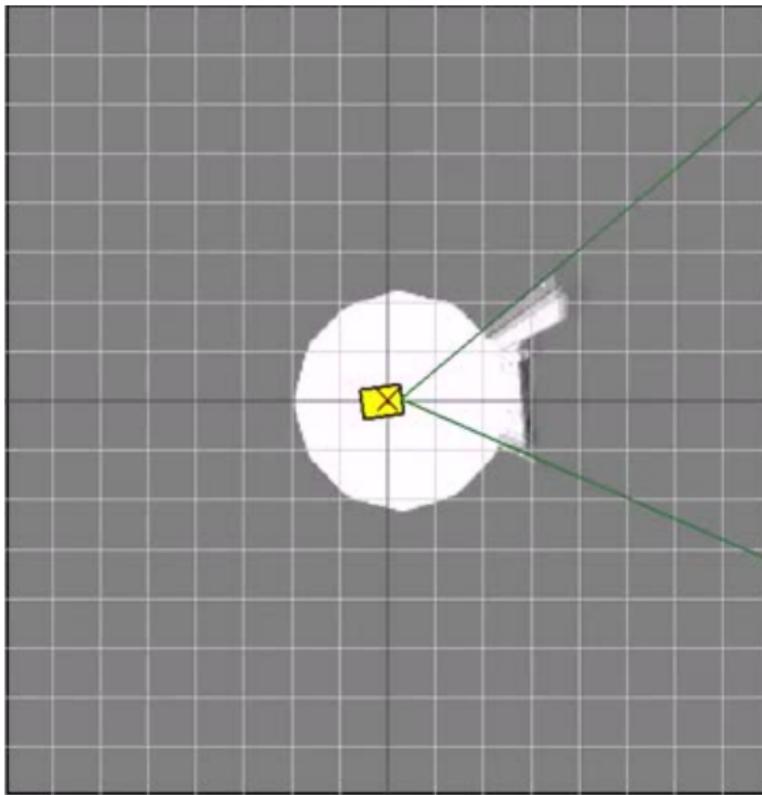


# WHY IS SLAM HARD?

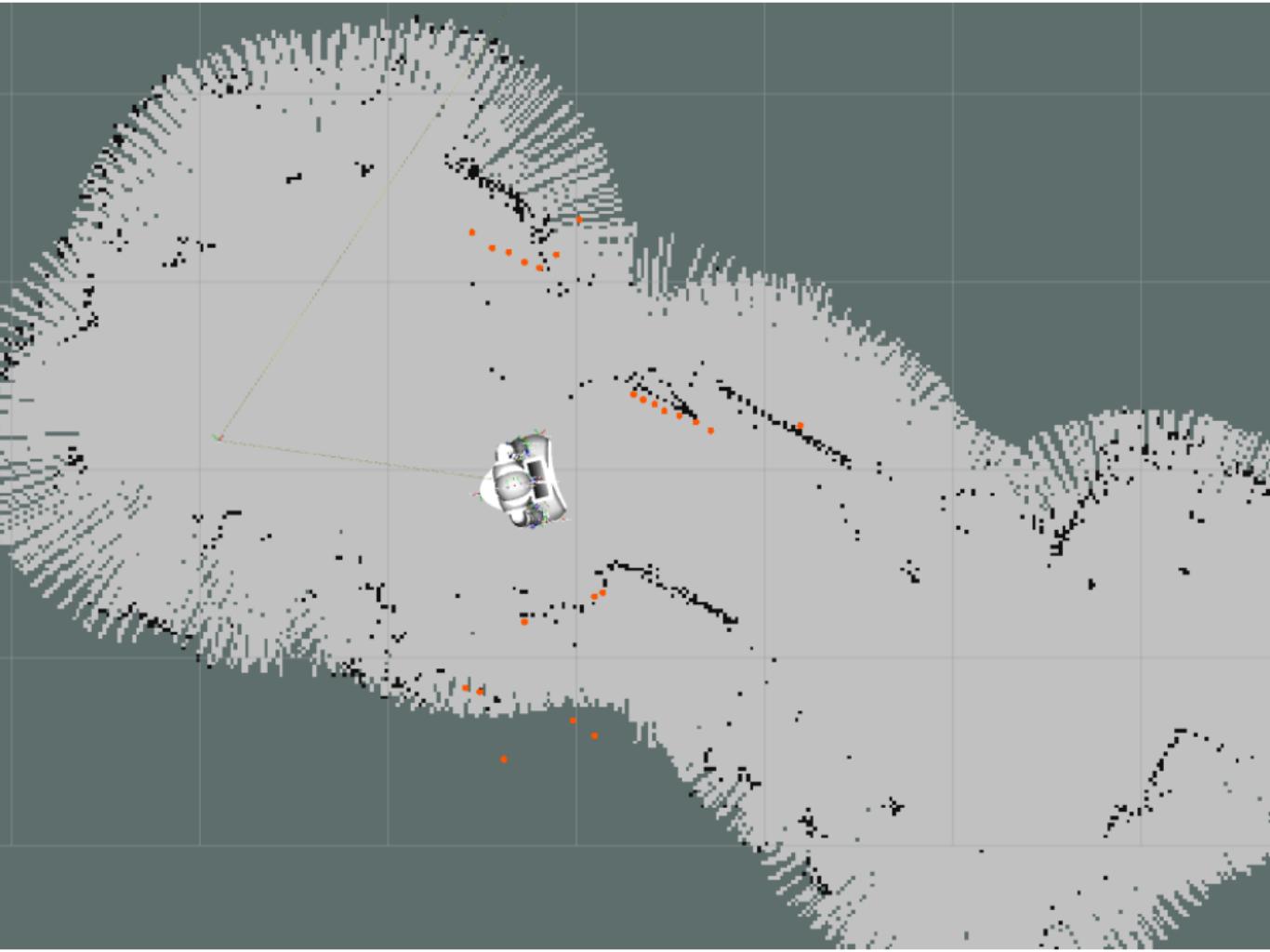
## Loop closure

- Even with the best SLAM algorithm, pose uncertainty will increase as the vehicle moves
- This pose uncertainty means that landmark locations further from the map origin have a higher uncertainty
- Revisiting a previously observed landmarks significantly reduces uncertainty in robot and landmark pose estimates**



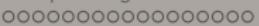
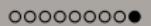


*Source: Robert Sim, University of British Columbia*





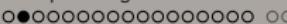
2x



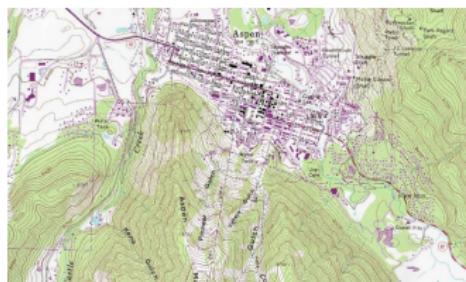
## SLAM – FURTHER READING

- Take a MSc Robotics next year!
- Jack Collier introduction (!! ) to SLAM
- Thrun, Burgard, Fox, *Probabilistic Robotics*

# PATH PLANNING



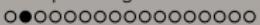
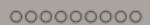
# METRIC VS TOPOLOGICAL MAPS



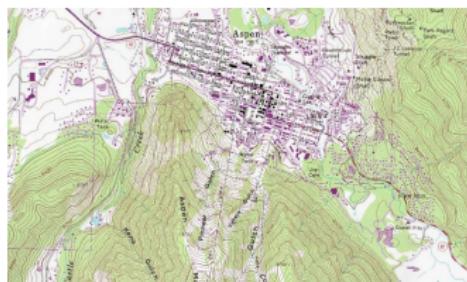
A metric map considers a two-dimensional space in which it places objects. The objects are placed with precise coordinates. This representation is convenient and natural, but *sensitive to noise* and *difficult to calculate the distances precisely*.



A topological map only considers places and relations between them. Often, the distances between places are stored. The map is typically a **graph**, in which the nodes corresponds to places and edges correspond to the paths.



# METRIC VS TOPOLOGICAL MAPS

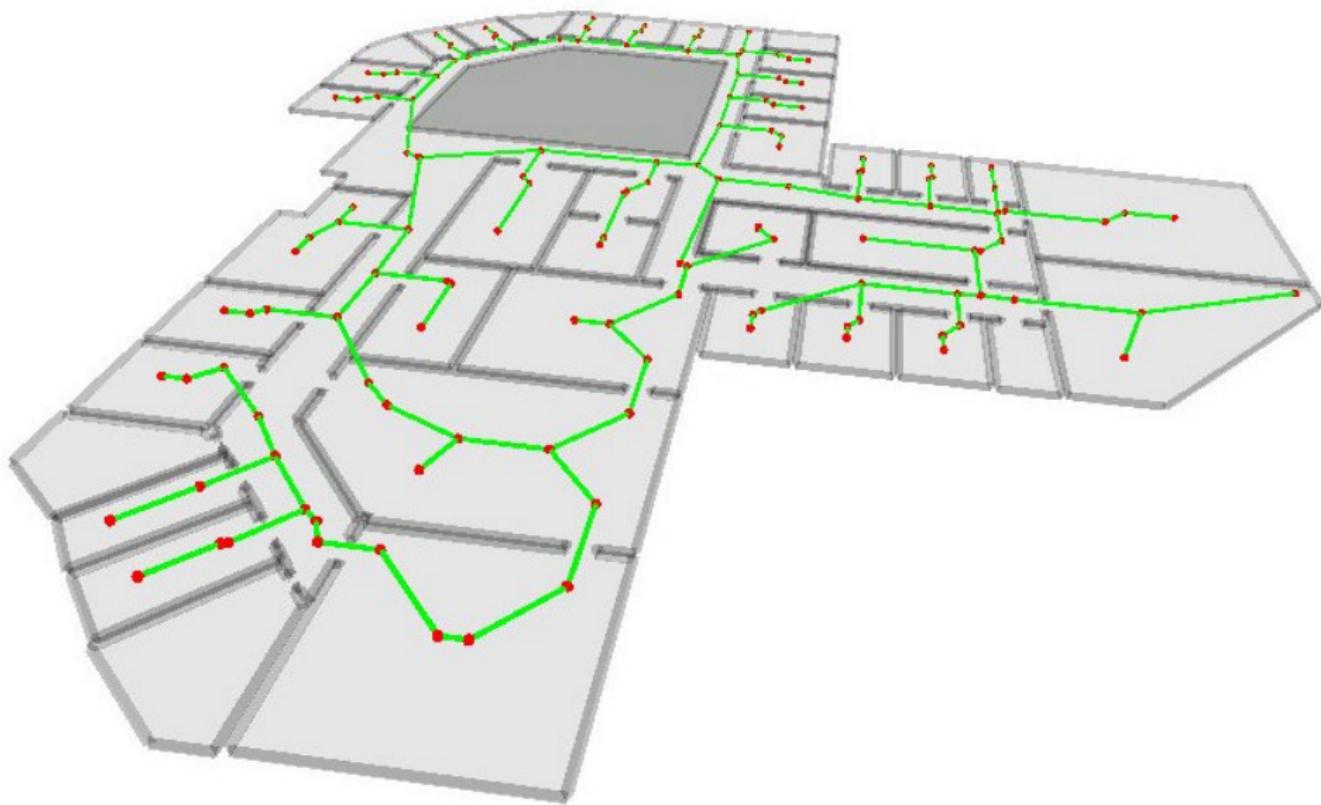


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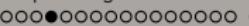
**Cell decomposition** can be used to transform a metric map into a topological one. More on that in a moment.



A topological map only considers places and relations between them. Often, the distances between places are stored. The map is typically a **graph**, in which the nodes corresponds to places and edges correspond to the paths.

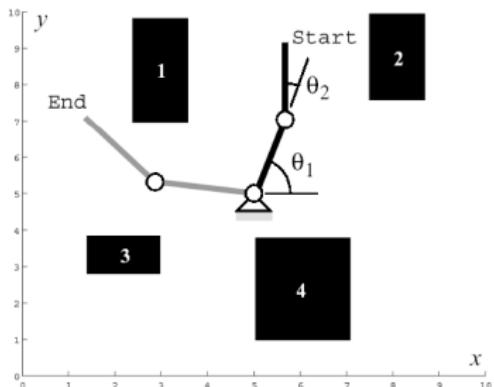


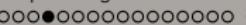




## PATH PLANNING: CONFIGURATION SPACE

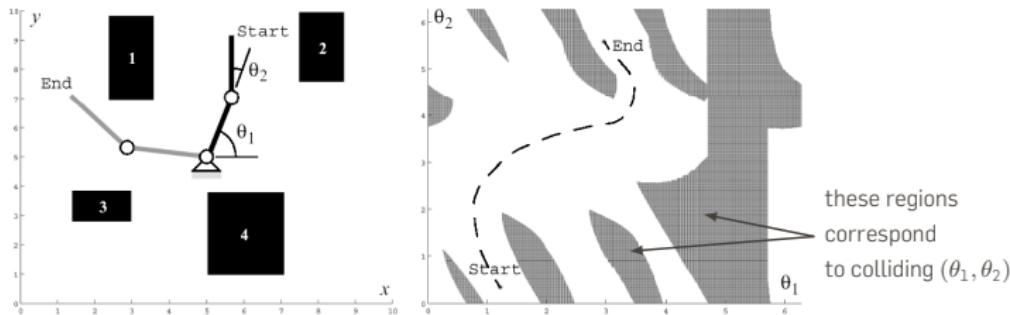
The state or configuration  $q$  of a robot can be described with  $k$  values  $q_i$ . For example, for a two-link planar robot,  $\theta_1$  and  $\theta_2$ :



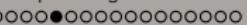


## PATH PLANNING: CONFIGURATION SPACE

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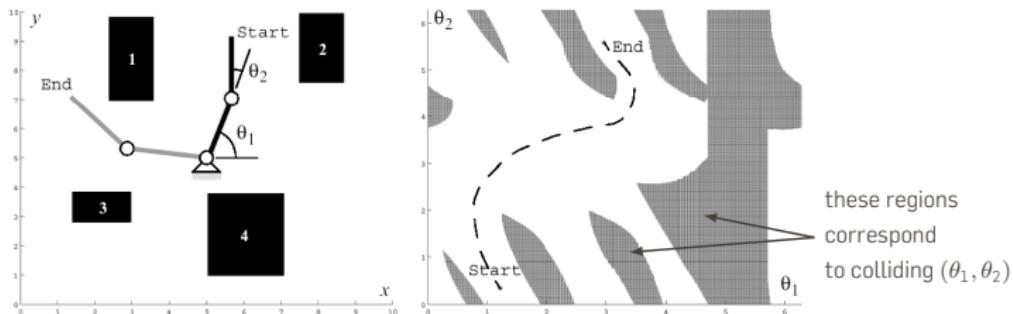


Each  $q$  is a point in a  $k$ -dimensional space called the **configuration space  $C$**  of the robot.



## PATH PLANNING: CONFIGURATION SPACE

The state or configuration  $q$  of a robot can be described with  $k$  values  $q_i$ . For example, for a two-link planar robot,  $\theta_1$  and  $\theta_2$ :



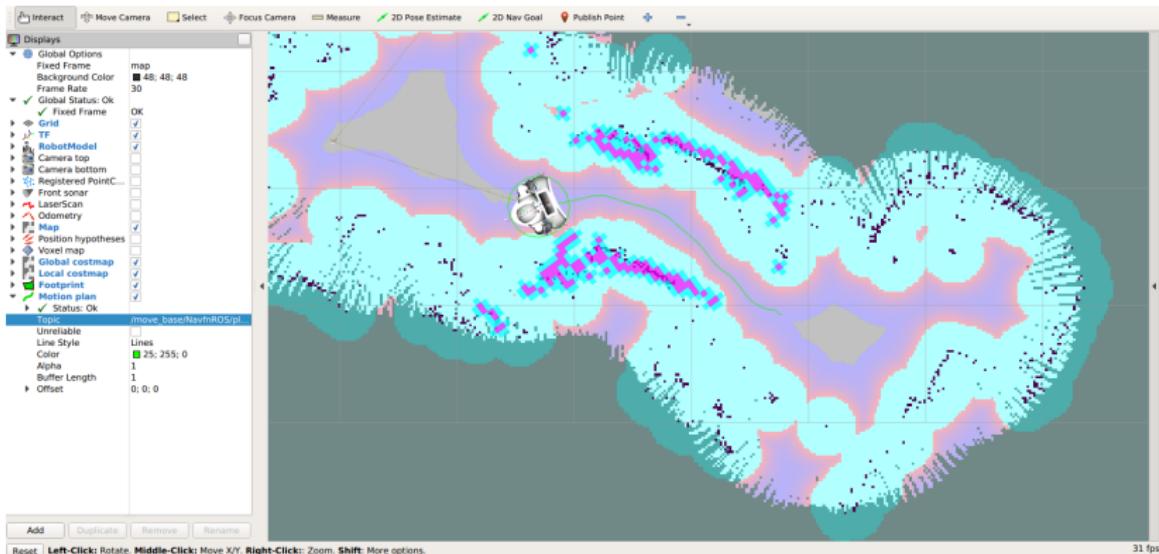
Each  $q$  is a point in a  $k$ -dimensional space called the **configuration space  $C$**  of the robot.

We can map as well obstacles to **configuration space obstacles  $O$** .

The **free space** (where the robot is safe to move) is then  $F = C - O$ .



# GLOBAL VS LOCAL PATH PLANNING



## Global path planning

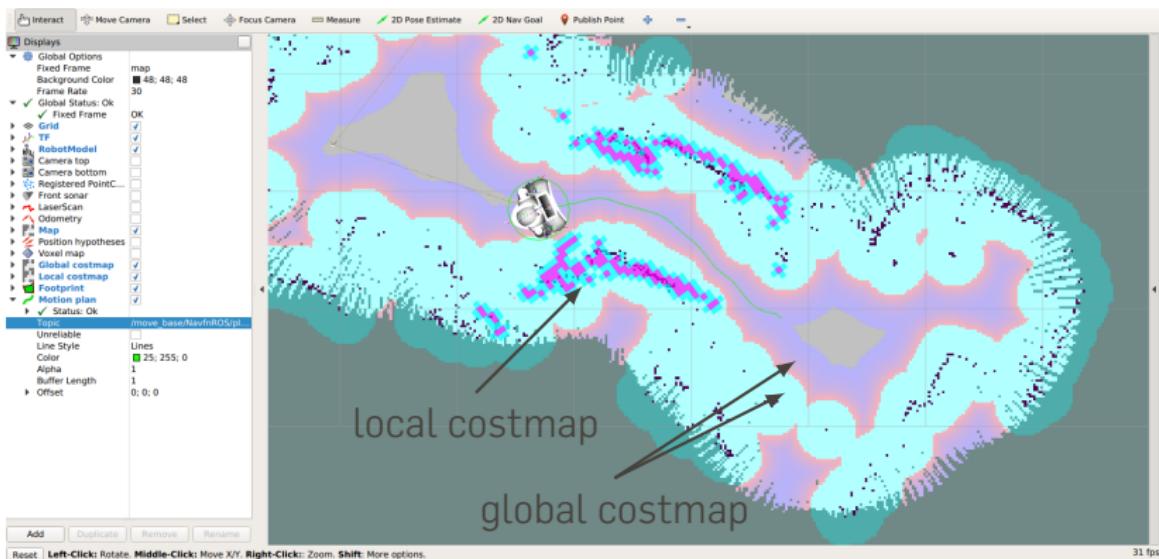
- long distances (i.e. large map, slower calculations)
- static environment

## Local path planning

- short horizon
- dynamic environment (use of sensors)



# GLOBAL VS LOCAL PATH PLANNING

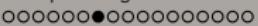


**Global path planning**

→ global (static) **costmap**

**Local path planning**

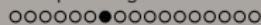
→ local (dynamic) **costmap**



# GLOBAL PATH PLANNING

## *Assumptions:*

- A good enough map of the environment is available for navigation (topological or metric or a mixture between both).
- The robot knows where it is on the map (using for example GPS or Monte Carlo localisation).



# GLOBAL PATH PLANNING

## *Assumptions:*

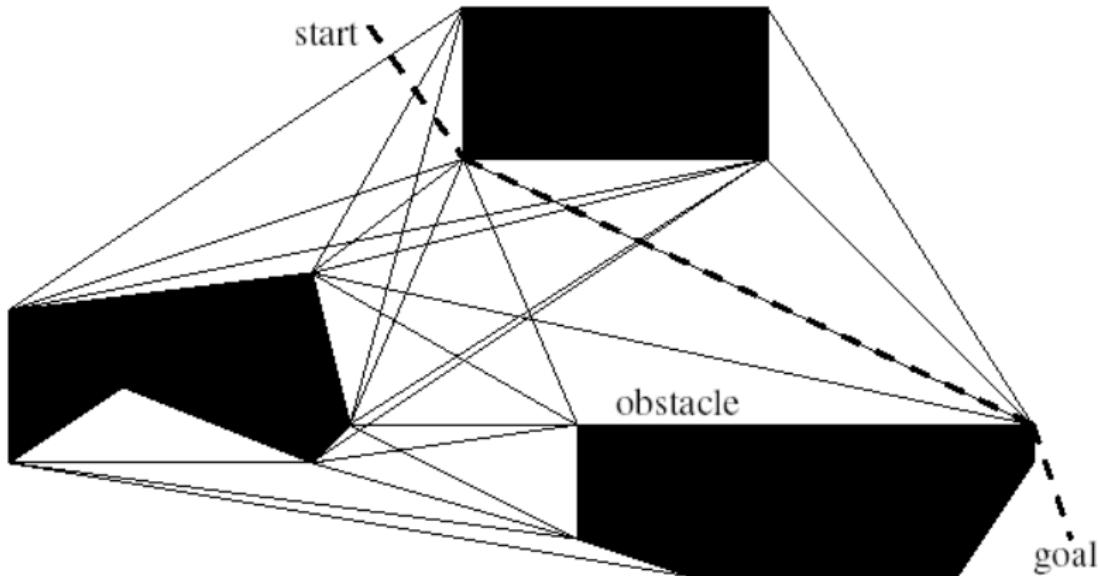
- A good enough map of the environment is available for navigation (topological or metric or a mixture between both).
- The robot knows where it is on the map (using for example GPS or Monte Carlo localisation).

*First step:* Build a representation of the environment using a **road-map (graph)**, **cells** or a **potential field**. The resulting *discrete* locations or cells allow then to use standard planning algorithms.

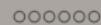
## Possible algorithms:

- Visibility graph
- Voronoi diagram
- Cell decomposition → connectivity graph
- Potential field

## GRAPHS: VISIBILITY GRAPH

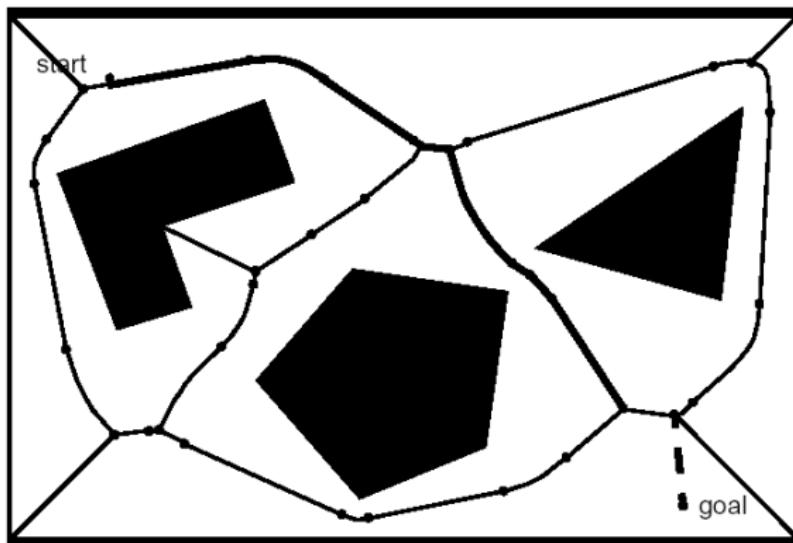


Edges between nodes are the lines of sight from each corner to an obstacle to every other visible corner.

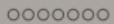
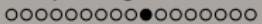


## GRAPHS: VORONOI GRAPHS

Voronoi graph: distance to obstacles from every nodes is maximal.



(by moving along the edges of a Voronoi graph, the robot ensures it is always as far as possible from obstacles)



## GRAPHS: CELL DECOMPOSITION

Divide space into simple, connected regions called **cells**.

Determine which open cells are adjacent and construct a **connectivity graph**.

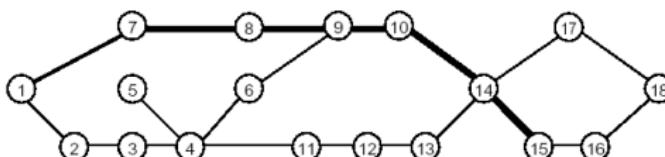
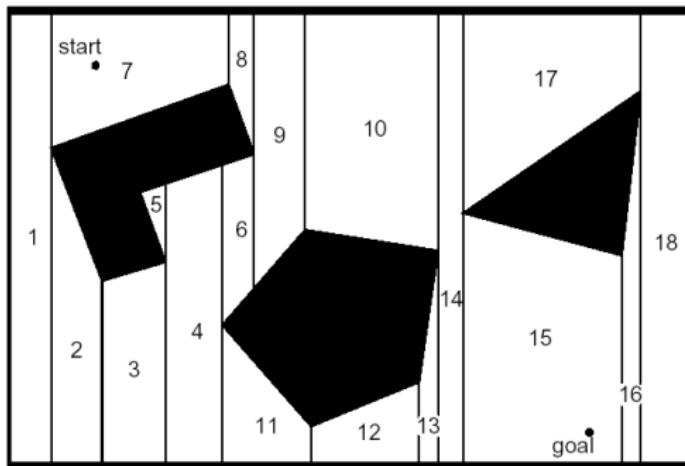
Find cells in which the initial and goal configuration (state) lie and search for a path in the connectivity graph to join them.

From the sequence of cells found with an appropriate search algorithm, compute a path within each cell.

e.g. passing through the midpoints of cell boundaries or by sequence of wall following movements.

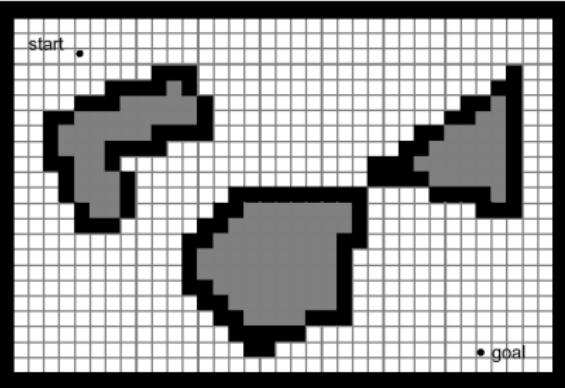
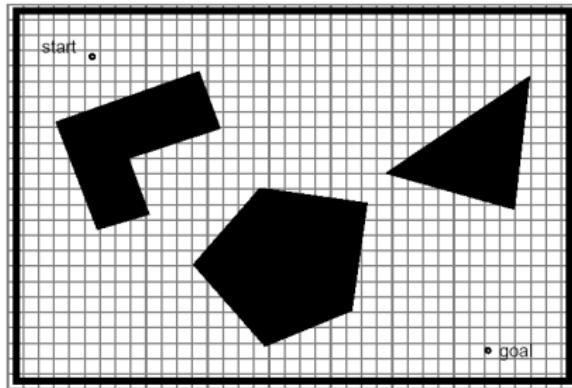
## GRAPHS: CELL DECOMPOSITION

Example: exact cell decomposition



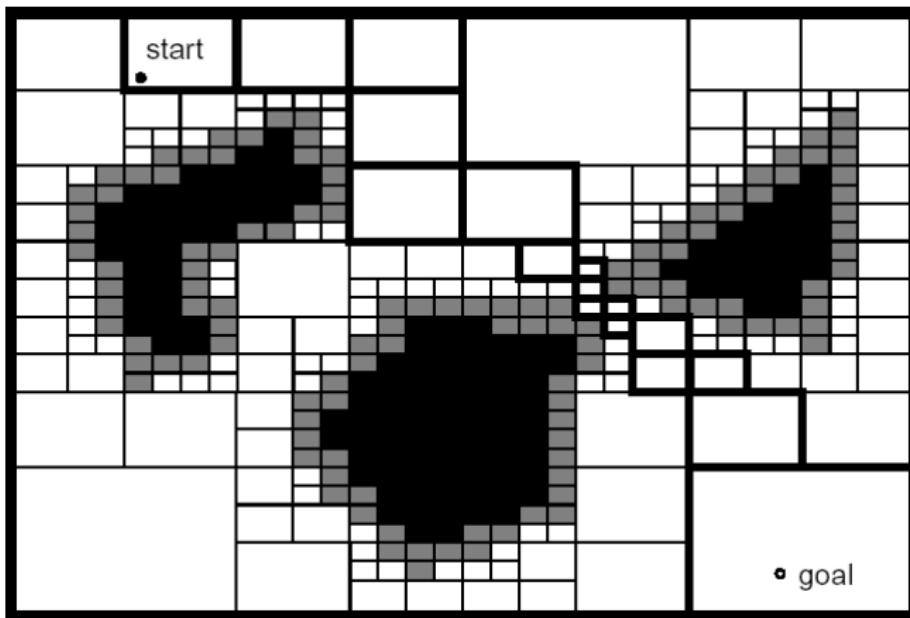
## GRAPHS: CELL DECOMPOSITION

Example: approximate cell decomposition



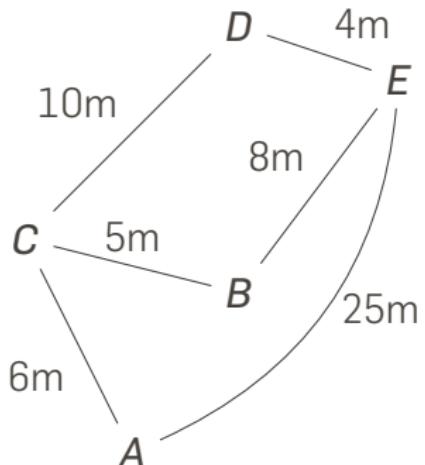
## GRAPHS: CELL DECOMPOSITION

Adaptive cell decomposition: changes the size of each cell according to the detail needed.

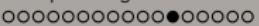


## REPRESENTING GRAPHS

- How is a graph represented in a computer? Graphical representation is intuitive, but not straightforward to implement in a program.
- Graphs can be represented as a matrix:



	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>
<b>A</b>	0	-	6	-	25
<b>B</b>	-	0	5	-	8
<b>C</b>	6	5	0	10	-
<b>D</b>	-	-	10	0	4
<b>E</b>	25	8	-	4	0



## SEARCHING GRAPHS

Graphs can be searched, for example to find the shortest path between two nodes.

Many different search algorithms exist

- Breadth first
- Depth first
- A\*
- Dijkstra's shortest path

## DIJKSTRA'S ALGORITHM: ILLUSTRATION



*Source: Wikipedia*

Dijkstra search between start (red) and goal (green) positions.

Note how Dijkstra is expanding its search out from the starting position, without knowledge about which nodes could bring it closer to the goal.

# SEARCHING GRAPHS - DIJKSTRA

```
def dijkstra(graph, weights, start_node):

    dist = {} # maps nodes to distance to 'start_node'
    previous = {} # needed to reconstruct shortest path

    for node in graph:
        dist[node] = math.inf # initial dist. from 'start_node' to 'node'

    dist[start_node] = 0

    Q = graph.nodes() # unvisited nodes
    while not Q.empty():
        u = pop_min(Q) # remove best node

        for v in u.neighbours: # iterate over nodes connected to u
            if dist[u] + weights(u, v) < dist[v]: # new shorter path to v!
                dist[v] = dist[u] + weights(u, v)
                previous[v] = u

    return dist, previous
```



## SEARCHING GRAPHS – DIJKSTRA

Reading out shortest path from the start node to goal `end_node`:

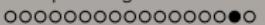
---

```
path = []
node = end_node # goal
while node in previous:
    path = [node] + path # append at the front of our path
    node = previous[node]
```

---

Returns the shortest path (if there are more than one, only one is returned).

Returns empty if no path exists.



# THE COST OF SEARCH

Maps can be huge

- For example, 10,000  $m^2$  shopping mall ( $\sim$  Marshmills' Sainsburys), with a 10x10cm map resolution, requires  $10^{10} = 10$  billion nodes on the graph!

Search time can be problematic:

- Dijkstra has a complexity of  $O(N + V^2)$ . For example, for a map with  $10^7$  nodes and  $10^8$  vertices, it can take  $10^7 + 10^{2 \times 8} \sim 10^{16}$  calculations to find a shortest path between two points.
- Some implementations are much more efficient. A\* search is at worst the same as Dijkstra, but can be polynomial in the number of nodes  $O(N^k)$  when a good heuristic is used. For example, if you can choose between two nodes to explore, take the node closer to the goal.

## A\* ALGORITHM - ILLUSTRATION

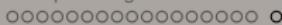
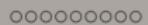


*Source: Wikipedia*

A\* search for finding path between start and goal position.

A\* uses a **heuristic** (here the distance to the goal) to guide the exploration (first expand to nodes closer to the goal). This gives a chance of finding the solution in less time.

# POTENTIAL FIELD PATH PLANNING



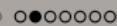
## POTENTIAL FIELD PATH PLANNING

Robot is treated as a *point under the influence* of an artificial potential field.

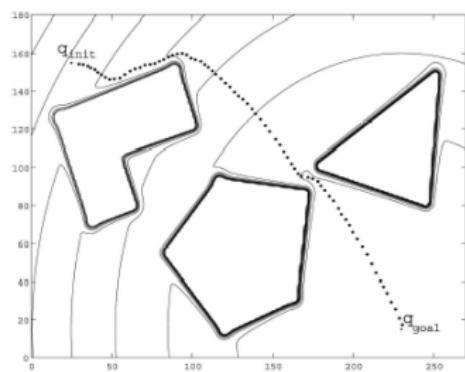
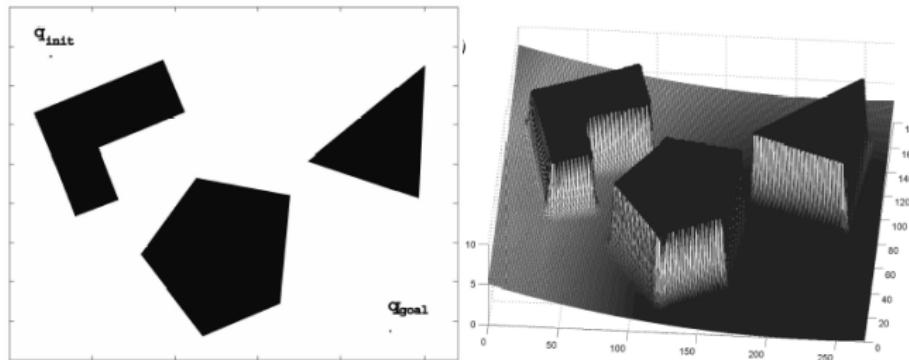
- Generated robot movement is similar to a ball rolling down the hill
- Goal generates attractive force
- Obstacle are repulsive forces

Needed:

- Location of robot
- Location of obstacles
- ⇒ a metric map is well suited



# POTENTIAL FIELD PATH PLANNING

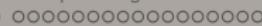




## POTENTIAL FIELD GENERATION

Generation of potential field function  $U(q)$

- attracting (goal) and repulsing (obstacle) fields
- summing up the fields
- functions must be **differentiable**



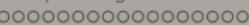
## POTENTIAL FIELD GENERATION

Generation of potential field function  $U(q)$

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Generate artificial force field  $F(q)$

$$F(q) = -\nabla U(q) = -\nabla U_{att}(q) - \nabla U_{rep}(q) = \begin{bmatrix} \frac{\partial U}{\partial x} \\ \frac{\partial U}{\partial y} \end{bmatrix}$$



## POTENTIAL FIELD GENERATION

Generation of potential field function  $U(q)$

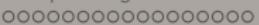
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$$F(q) = -\nabla U(q) = -\nabla U_{att}(q) - \nabla U_{rep}(q) = \begin{bmatrix} \frac{\partial U}{\partial x} \\ \frac{\partial U}{\partial y} \end{bmatrix}$$

Set robot speed ( $v_x, v_y$ ) proportional to the force  $F(q)$  generated by the field

- the force field drives the robot to the goal
- if robot is assumed to be a point mass



# POTENTIAL FIELD PATH PLANNING

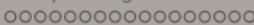
## Attractive potential field

- For example, a parabolic function representing the Euclidean distance  $\rho = ||q - q_{goal}||$  to the goal:

$$U_{att}(q) = \frac{1}{2} k_{att} \cdot \rho_{goal}^2(q)$$

- Attracting force converges linearly towards 0 (goal)

$$\begin{aligned} F_{att}(q) &= -\nabla U_{att}(q) \\ &= -k_{att} \cdot \rho_{goal}(q) \nabla \rho_{goal}(q) \\ &= -k_{att} \cdot (q - q_{goal}) \end{aligned}$$



# POTENTIAL FIELD PATH PLANNING

Repulsing potential field

Should generate a barrier around all the obstacle

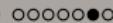
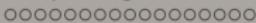
- strong if close to the obstacle
- no influence if far from the obstacle:

distance to  
obstacle

$$U_{rep}(q) = \begin{cases} \frac{1}{2}k_{rep}\left(\frac{1}{\rho(q)} - \frac{1}{\rho_0}\right)^2 & \text{if } \rho(q) < \rho_0 \\ 0 & \text{if } \rho(q) \geq \rho_0 \end{cases}$$

- field is positive or zero and *tends to infinity* as q gets closer to the object:

$$F_{rep}(q) = -\nabla U_{rep}(q) = \begin{cases} k_{rep}\left(\frac{1}{\rho(q)} - \frac{1}{\rho_0}\right) \cdot \frac{1}{\rho(q)^2} & \text{if } \rho(q) < \rho_0 \\ 0 & \text{if } \rho(q) \geq \rho_0 \end{cases}$$



## POTENTIAL FIELD PATH PLANNING

Notes:

- **Local minima problem** exists: robot can get stuck in places on the map where the potential field has a local dip.
- problem is getting more complex if the robot is not considered as a point mass
- If objects are convex there exists situations where several minimal distances exist → can result in oscillations
- Robot path oscillates, e.g. in narrow passages.

Videos:

- <http://www.youtube.com/watch?v=DxzRYYMjxKY>
- <http://www.youtube.com/watch?v=JnPfu7xHNt4>

# EXTENDED POTENTIAL FIELD PATH PLANNING

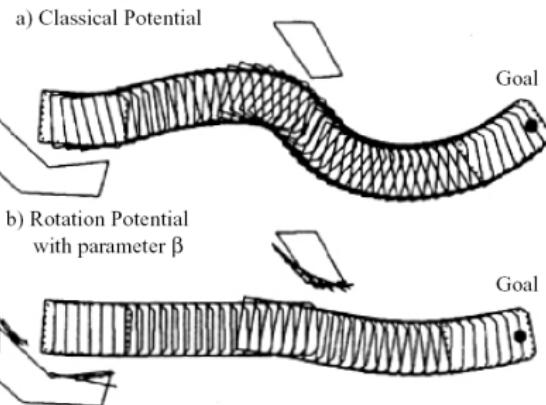
Additionally a **rotation potential field** and a **task potential field** are introduced.

## Rotation potential field

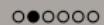
- force is also a function of robots orientation to the obstacle

## Task potential field

- filters out the obstacles that should not influence the robots movements, i.e. only the obstacles in front of the robot are considered

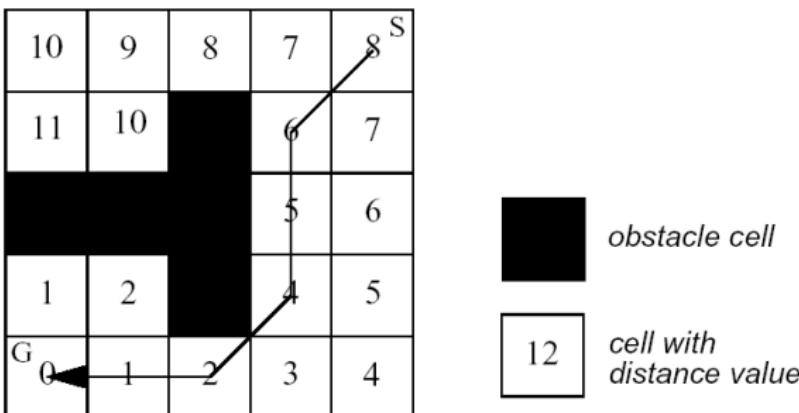


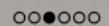
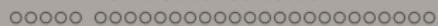
# GRASSFIRE ALGORITHM



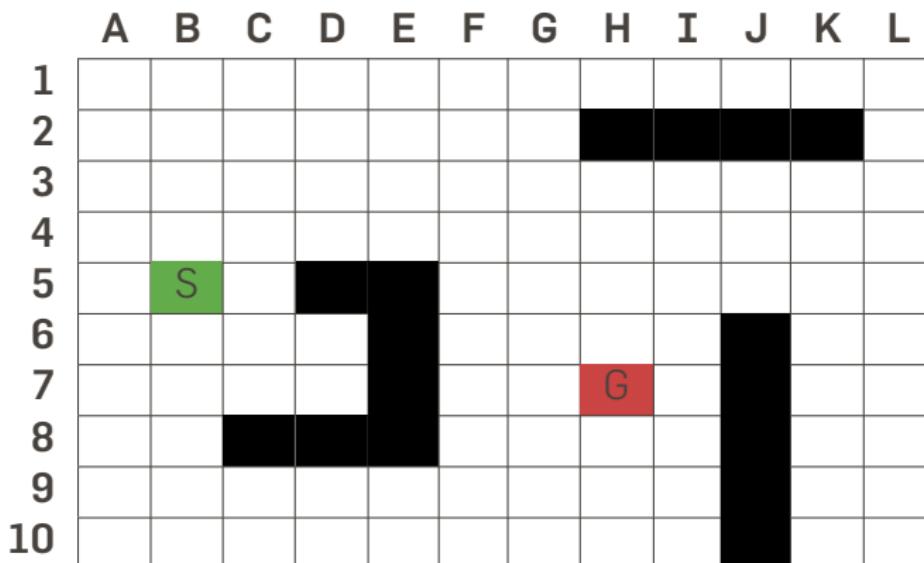
## GRASSFIRE ALGORITHM

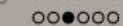
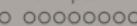
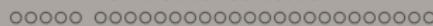
- Starting from a metric map, a **wave front** is expanded around the goal.
- Goal is reached by travelling on descending values.





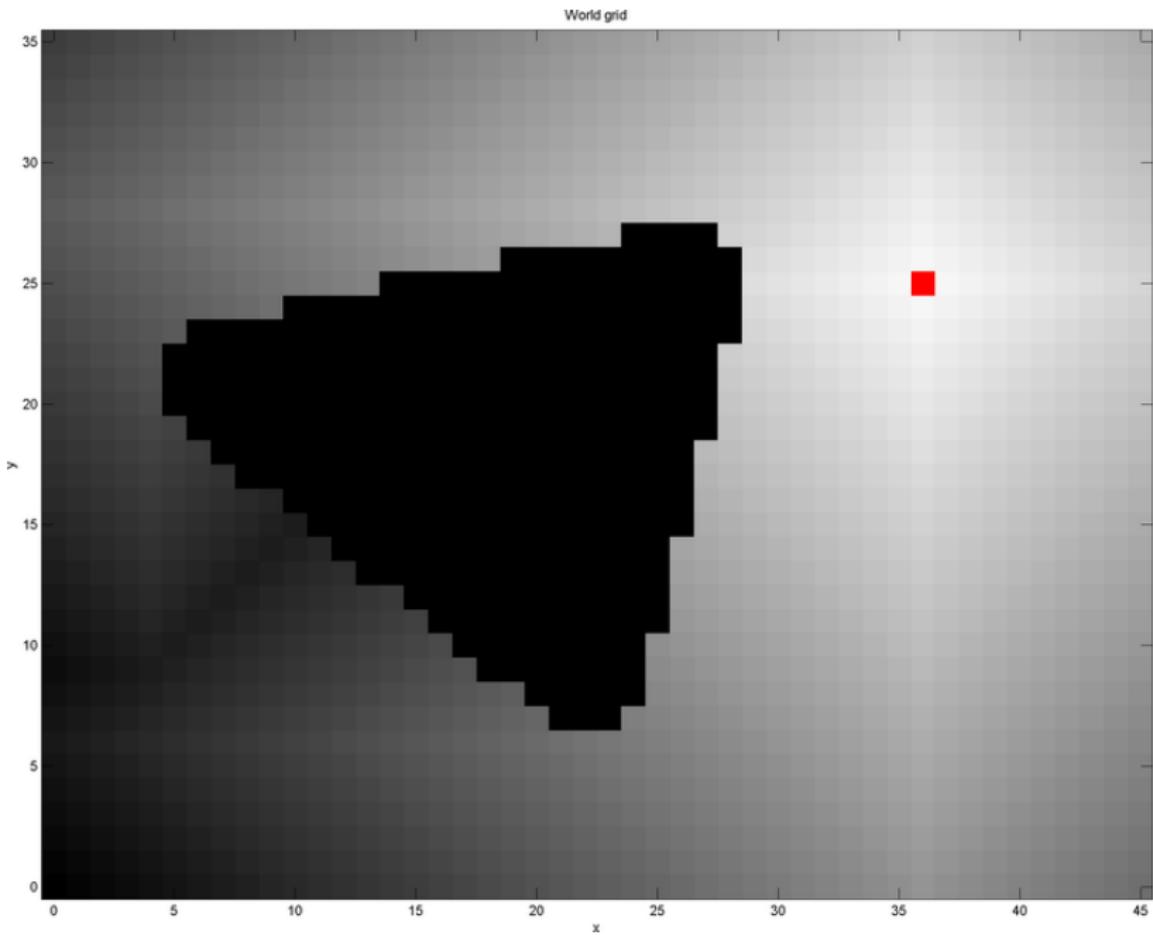
# GRASSFIRE ALGORITHM





# GRASSFIRE ALGORITHM

	A	B	C	D	E	F	G	H	I	J	K	L
1	13	12	11	10	9	8	7	8	9	10	11	10
2	12	11	10	9	8	7	6					9
3	11	10	9	8	7	6	5	4	5	6	7	8
4	10	9	8	7	6	5	4	3	4	5	6	7
5	11	S	9			4	3	2	3	4	5	6
6	12	11	10	11		3	2	1	2		6	7
7	11	10	11	12		2	1	G	1		7	8
8	10	9				3	2	1	2		8	9
9	9	8	7	6	5	4	3	2	3		9	10
10	10	9	8	7	6	5	4	3	4		10	11





## GRASSFIRE ALGORITHM

---

```
def grassfire(M, goal):
    Q.push(goal)
    M[goal] = 0 # set goal value to 0

    while not Q.empty(): # loop until map filled
        a = Q.pop()

        for n in neighbours(a):
            if not n in M and not is_obstacle(n):
                Q.push(n)
                M[n] = M[a] + 1

    return M
```

---

Q is a **queue data structure**, a first-in first-out (fifo) list. The queue keeps track of which locations on the map still need to be visited.



That's all, folks!

Questions:

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Slides:

[github.com/severin-lemaignan/module-mobile-and-humanoid-robots](https://github.com/severin-lemaignan/module-mobile-and-humanoid-robots)