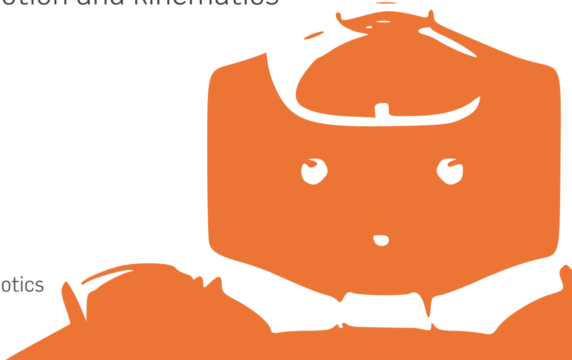
You can download the sources of this presentation here:
github.com/severin-lemaignan/module-mobile-and-humanoid-robots

ROBOTICS WITH PLYMOUTH UNIVERSITY

ROCO318
Mobile and Humanoid Robots
Part 4 - Wheeled locomotion and kinematics



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PART 4 – WHEELED LOCOMOTION AND KINEMATICS

For further reading, see:

- (Siciliano and Katib, 2008) or chapter 2 from Siegwart and Nourbakhsh (2004)

WHEELED ROBOTS

MOBILE ROBOTS WITH WHEELS

Wheels are the most appropriate solution for most applications.

$\rightarrow$ The robots are faster and consume less energy.

Three wheels are ***sufficient*** and ***guarantee stability***.

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Without a flexible suspension, only three wheels would make contact with the ground, all other wheels would slip or not contact the ground.

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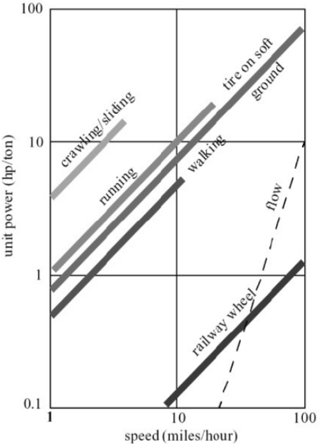
Without a flexible suspension, only three wheels would make contact with the ground, all other wheels would slip or not contact the ground. A large number of wheel types exist

- Selection of wheel type depends on the application.

EFFICIENCY OF ROLLING

Wheels on rails are almost 100 times more efficient than human walking.

- Robot walking is about 15 times less efficient than human walking.



THREE WHEEL TYPES

- Standard wheel (including the caster wheel).
- Omnidirectional wheel (aka Swedish or Mecanum wheel).
- Ball or spherical wheel.

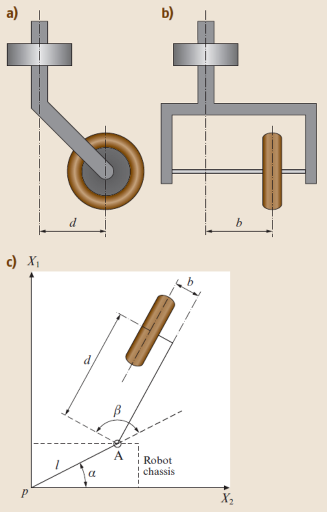
TYPE 1: STANDARD WHEEL

Has two offsets *d* and *b*.
The wheel allows steering or not?

- Can the wheel's orientation be changed or not?

Is the wheel orientation steered or not?

- Active steering means that a motor changes the wheel's orientation, passive means that the wheel just follows.



TYPE 1: STANDARD WHEEL

Passive fixed wheel

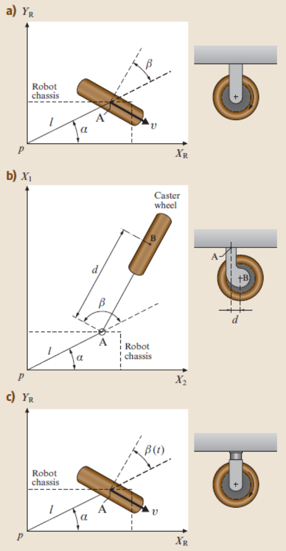
- No motors.

Passive or active caster wheel

- The wheel is off-center.
- In the active case, the driving and steering motion is controlled by two motors.

Active wheel that can be oriented

- This requires two motors, one for driving and one for steering.



TYPE 1: STANDARD WHEEL

Examples

Passive fixed wheel

Passive caster wheel

TYPE 1: STANDARD WHEEL

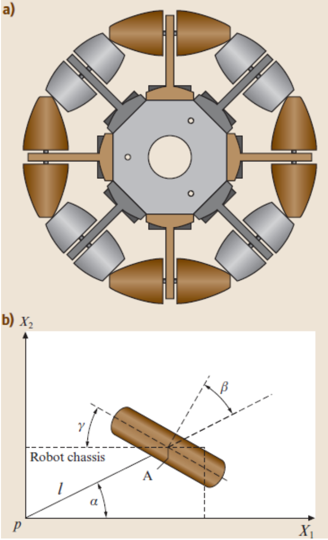
Standard wheels are most used, but have a **non-holonomic velocity constraint**.

- Which just means that a wheel can only roll in the direction of the wheel and cannot slide sideways.
- This constrains what a robot (or vehicle) can do.

TYPE 2: OMNIDIRECTIONAL WHEEL

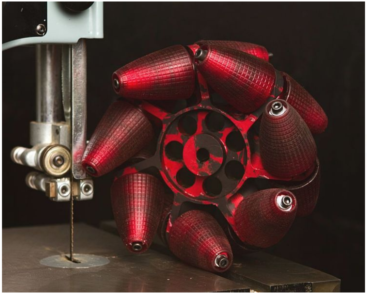
Omnidirectional wheel (= Swedish or Mecanum wheel).

- Three degrees of freedom; rotation around the (motorized) wheel axle, around the rollers and around the contact point.



TYPE 2: OMNIDIRECTIONAL WHEEL

- The wheel is designed to let a vehicle move in any direction.
- Rollers are mounted at 0 or 45 angles to a line along the plane of the wheel.

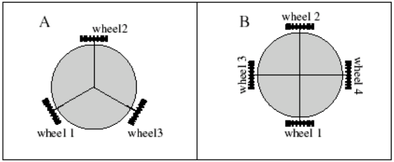


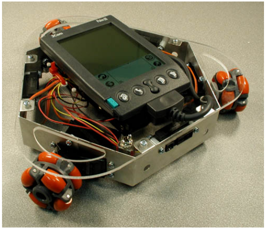
Source: Wikipedia

TYPE 2: SWEDISH WHEEL

Several possible configurations

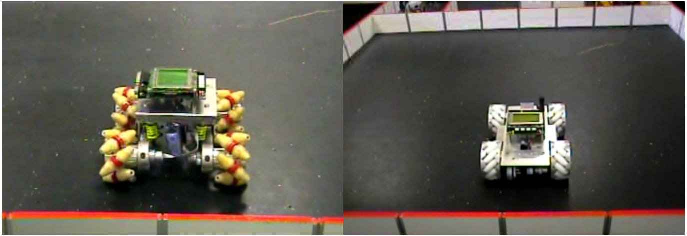
- Three motors, and three wheels.
- Four motors and four wheels.





TYPE 2: SWEDISH WHEEL

- Four wheels and four motors.



- Airtrax lift truck (video)

CHARACTERISTICS OF WHEELED ROBOTS AND VEHICLES

Stability of a vehicle is guaranteed with 3 wheels

- centre of gravity must stay within the triangle formed by the ground contact point of the wheels.

Stability is improved by 4 and more wheels

- however, this arrangements are **hyperstatic** and require a flexible suspension system.

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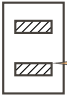
Combining actuation and steering on one wheel makes the design complex and adds additional errors for odometry.

DIFFERENT ARRANGEMENTS OF WHEELS I

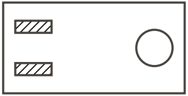
Two wheels

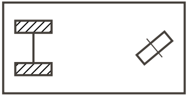


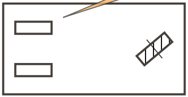
Hatched: powered wheels

Three wheels

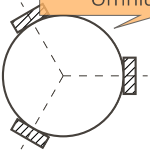
Empty: free wheel

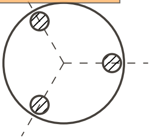


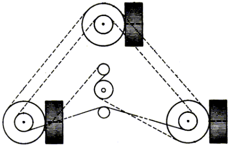




Omnidirectional drive



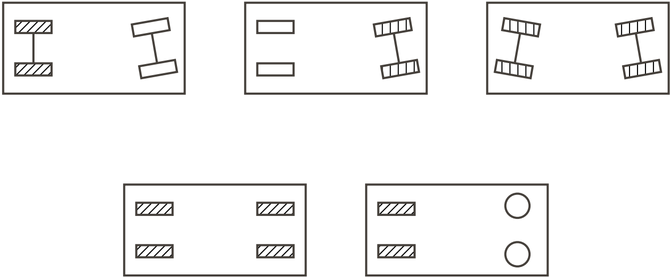




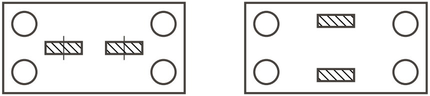
Sy
aka

DIFFERENT ARRANGEMENTS OF WHEELS II

Four wheels



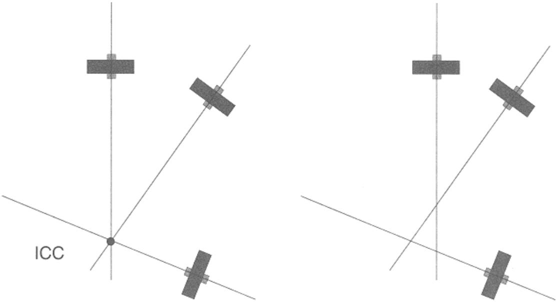
Six wheels



INSTANTANEOUS CENTRE OF CURVATURE (ICC)

In order for all wheels of a vehicle to **roll** without slipping, the axes of all wheels need to intersect in one point.

This point is called the ICC.



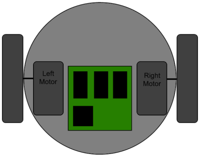
(a)

(b)

DIFFERENTIAL DRIVE CONFIGURATION

Most used wheeled robot configuration

- Cheap (two motor on two drive wheels, one support wheel).
- Motors do driving and steering together.
- Excellent steering behaviour.
- Turn on the spot.
- However, relatively poor straight forward driving and cannot carry heavy weights (the supporting wheels is critical here).



Source: cognitoware

SOME EXAMPLES

Source: kadtronix

Segway

Pioneer

SYNCHRO DRIVE

All wheels are actuated synchronously by one motor

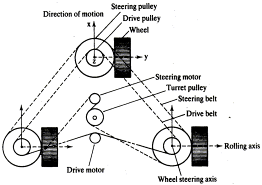
- defines the speed of the vehicle

All wheels steered synchronously by a second motor

- sets the heading of the vehicle

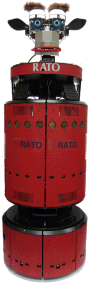
The orientation in space of the robot frame will always remain the same

- It is therefore not possible to control the orientation of the robot frame.



SYNCHRO DRIVE

- Only for indoor robots.
- Very precise and excellent dead reckoning capabilities.
- Example: PR2 from Willow Garage, B12, B21 robots by Real World Interfaces, popular in the research labs.



STEPPING / WALKING WITH WHEELS

- SpaceCat, and micro-rover for Mars, developed by Mecanex Sa and EPFL for the European Space Agency (ESA)

SHRIMP, A MOBILE ROBOT WITH EXCELLENT CLIMBING ABILITIES

Objective

- Passive locomotion concept for rough terrain

Results: The Shrimp

6 wheels

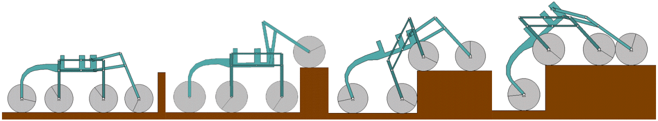
- one fixed wheel in the rear
- two boogies on each side
- one front wheel with spring suspension

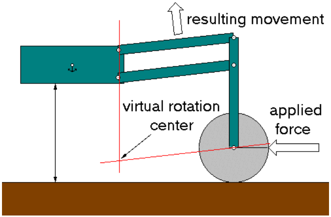
robot measures around 60 cm in length and 20 cm in height

highly stable in rough terrain

overcomes obstacles up to 2 times its wheel diameter

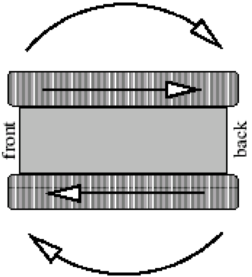
SHRIMP

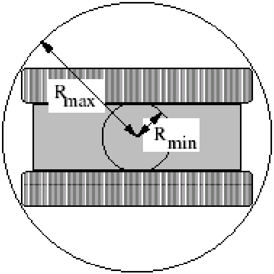



resulting movement
virtual rotation center
applied force

TRACKED DRIVE

- Same principle as differential drive, but instead of wheels the robot has tracks.
- Good grip, low slippage. Excellent for rough terrain.
- Very unpredictable rotation.





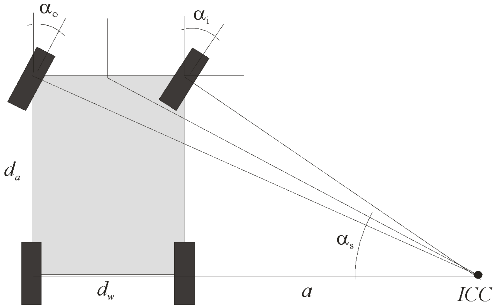
ACKERMAN DRIVE

Four wheels, just as a car.

Front wheels for steering.

When turning, all wheels turn at different speeds.

- Is resolved for the back wheels by a differential gear.



ACKERMAN DRIVE

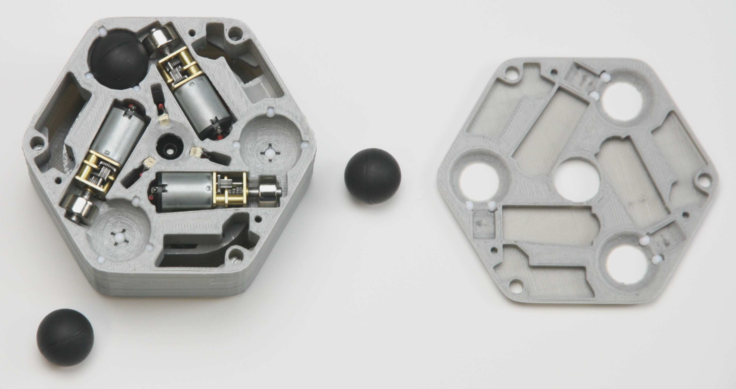
- Front wheels need different steering angle, to avoid slippage. Given by...

$$a = \frac{d_a}{tan(a_i)} - d_w$$

$$a_i = tan^{-1}(\frac{d_a}{a})$$

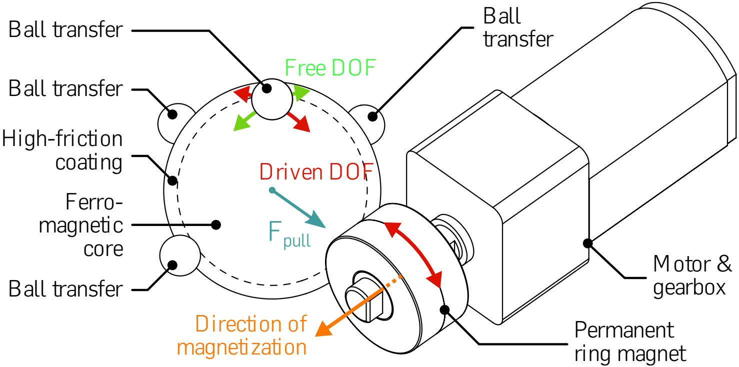
$$tan(a_s) = \frac{d_a}{a + \frac{d_w}{2}}$$

BALLDRIVE



Source: *Özgür et al., Permanent Magnet-Assisted Omnidirectional Ball Drive*

BALLDRIVE



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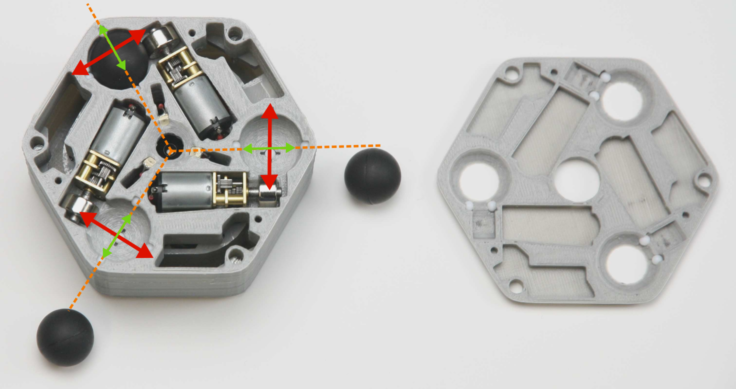

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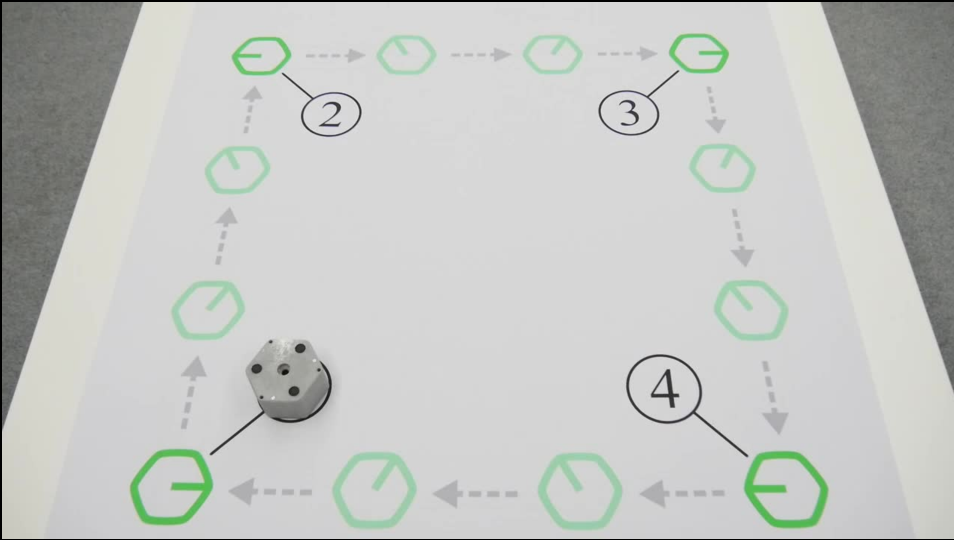
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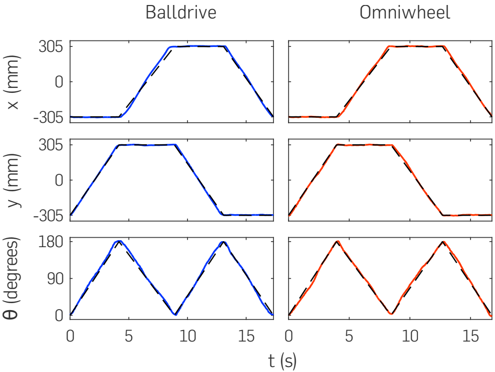
BALLDRIVE VS. OMNIWHEELS



Source: Özgür et al., Permanent Magnet-Assisted Omnidirectional Ball Drive

KINEMATICS

KINEMATICS VS DYNAMICS

Kinematics describes the motion of objects or systems (combination of objects), but does not take forces into account. No mass, friction, gravity, momentum, ...

Dynamics take forces into account.

- Weightlifting robot, using dynamics to overcome the weight limitation.

Kinematics is simpler than dynamics. This module only considers kinematics.

MOBILE ROBOT KINEMATICS

Aim: description of mechanical behavior of the robot for ***control***

Similar to robot manipulator kinematics

However, a mobile robot can move unbounded with respect to its environment

- There is no direct way to measure the robot's position
- Position must be integrated over time
- Leads to inaccuracies of the position (motion) estimate $\rightarrow$ a major challenge in mobile robotics

FORWARD AND INVERSE KINEMATICS

Forward kinematics

Calculate the position, orientation and speed of robot as a function of its wheel angles and speeds.

- Example 1: inverse kinematics in a robot arm tells you the position of the robot hand in function of the joint angles.
- Example 2: when you know the speed of the left and right wheel of a differential drive robot, you know the velocity of the robot body.

FORWARD AND INVERSE KINEMATICS

Inverse kinematics

Calculate the wheel angles and speeds to achieve a certain robot speed or robot position and orientation.

- Example 1: if the robot hand needs to be in position (x,y,z), what joint angles should the robot arm have?
- Example 2: if the you want the robot to drive at v m/s and rotate at rad/s, what should the wheels do?

Inverse Kinematics is harder because there are most often several (sometimes infinite) solutions to the problem.

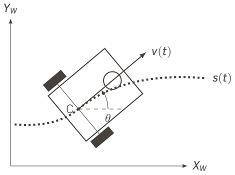
- For example: you can reach a point with a robot hand with any number of joint angles.

A KINEMATICS MODEL

Goal: establish the robot speed $\dot{\xi} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$ as a function of the wheel speeds $\dot{\varphi}_i$, steering angles β_i, steering speeds $\dot{\beta}_i$ and the geometric parameters of the robot (***configuration coordinates***).

Forward kinematics:

$$\dot{\xi} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(\dot{\varphi}_1, \ldots, \dot{\varphi}_n, \beta_1, \ldots, \beta_m, \dot{\beta}_1, \ldots, \dot{\beta}_m)$$



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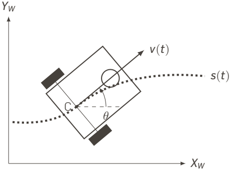


$$\dot{\xi} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(\dot{\varphi}_1, \ldots, \dot{\varphi}_n, \beta_1, \ldots, \beta_m, \dot{\beta}_1, \ldots, \dot{\beta}_m)$$

Inverse kinematics:

$$\begin{bmatrix} \dot{\varphi}_1 & \ldots & \dot{\varphi}_n & \beta_1 & \ldots & \beta_m & \dot{\beta}_1 & \ldots & \dot{\beta}_m \end{bmatrix}^T = f(\dot{x}, \dot{y}, \dot{\theta})$$

Note that $\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = f(\varphi_1, \ldots, \varphi_n, \beta_1, \ldots, \beta_m)$ does not make sense.

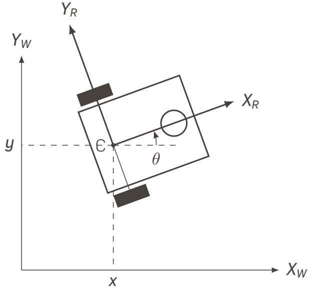
Why?

REPRESENTING ROBOT POSITION

Representing the robot within an arbitrary world frame

- World frame: X_W, Y_W
- Robot frame: X_R, Y_R
- Robot position: $\xi_W = [x \quad y \quad \theta]^T$

Mapping between the two frames:



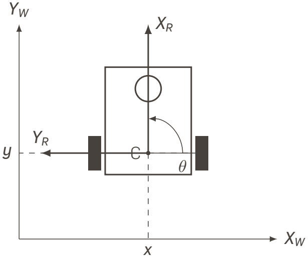
$$\dot{\xi}_R = R(\theta) \cdot \dot{\xi}_W = R(\theta) \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

$$R(\theta) = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

EXAMPLE: ROBOT ALIGNED WITH Y_W

$$R(\theta) = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\theta = 90^\circ = \frac{\pi}{2}$$



$$\dot{\xi}_R = R(\frac{\pi}{2}) \cdot \dot{\xi}_W = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$

KINEMATICS OF A WHEELED ROBOT

The kinematics of a robot can be understood by looking at the kinematics of the individual wheels.

But we need to assume some stringent constraints

- Movement on a horizontal plane
- Point contact of the wheels
- Wheels not deformable
- Pure rolling
- No slipping, skidding or sliding
- No friction for rotation around contact point
- Steering axes orthogonal to the surface
- Wheels connected by rigid frame (chassis)

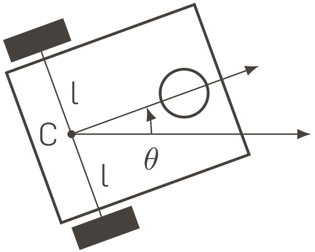
If these constraints are not met, our calculations will be off.

DIFFERENTIAL DRIVE

FORWARD KINEMATICS OF DIFFERENTIAL DRIVE (1)

Differential drive robot

- Two wheels with radius r
- Distance between wheels is $2 \cdot l$
- Orientation wrt inertial basis is θ
- Turning speed of wheels is $\dot{\varphi}_1, \dot{\varphi}_2$

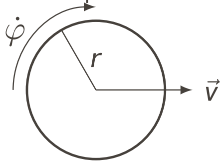


Robot's overall speed in global reference frame:

$$\dot{\xi}_W = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(l, r, \theta, \dot{\varphi}_1, \dot{\varphi}_2)$$

FORWARD KINEMATICS OF DIFFERENTIAL DRIVE (2)

The speed of a wheel over ground is

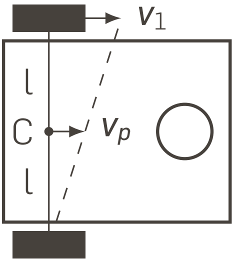


$$\dot{\varphi}[rad/s] \cdot r[m] = v[m/s]$$

Consider one wheel turning, the other wheel stationary.

The robot will pivot around the other wheel and the instantaneous contribution of the rotating wheel will be that point ***p*** has a speed v_p.

$$v_p = \frac{1}{2} \cdot v_1 = \frac{1}{2} \cdot \dot{\varphi}_1 \cdot r$$



FORWARD KINEMATICS OF DIFFERENTIAL DRIVE (3)

The contribution of both wheels can be added together. So, when both wheel spin, the robot's mid point moves at

$$\boldsymbol{v_p} = \frac{1}{2} \cdot \dot{\varphi}_1 \cdot \boldsymbol{r} + \frac{1}{2} \cdot \dot{\varphi}_2 \cdot \boldsymbol{r}$$

- In particular, if wheel 1 has speed $\dot{\varphi}_1 = \boldsymbol{a}$ and wheel 2 $\dot{\varphi}_2 = -\boldsymbol{a}$ then the robot will turn on the spot and have no translational speed: $\boldsymbol{v_p} = 0$.

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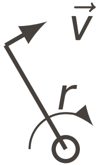
The speed in the robot reference frame is:

$$\dot{x}_R = v_p$$
$$\dot{y}_R = 0$$

FORWARD KINEMATICS OF DIFFERENTIAL DRIVE (4)

The rotational component $\dot{\theta}_R$ of $\dot{\xi}_R$ can be computed by adding the instantaneous contribution of each wheel to the robots rotational velocity.



$$\omega[rad/s] = \frac{v[m/s]}{l[m]}$$

For the left wheel and for the right wheel:

$$\omega_1 = \frac{\dot{\varphi}_1 \cdot r}{2l}; \qquad \omega_2 = -\frac{\dot{\varphi}_2 \cdot r}{2l};$$

Rotational velocity of robot after adding these two:

$$\dot{\theta} = \frac{r}{2l} \cdot (\dot{\varphi}_1 - \dot{\varphi}_2)$$

FORWARD KINEMATICS OF DIFFERENTIAL DRIVE (5)

Written in matrix form this is:

$$\dot{\xi}_R = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} r \cdot (\dot{\varphi}_1 + \dot{\varphi}_2) \\ 0 \\ \frac{r}{2l} \cdot (\dot{\varphi}_1 - \dot{\varphi}_2) \end{bmatrix}$$

This can be remapped to the world frame by:

$$\dot{\xi}_W = R(\theta)^{-1} \cdot \dot{\xi}_R$$

PRACTICAL APPROACH

Sampling interval I for wheel encoders. Encoders report wheel increment N_L and N_R.

Assume that:

$$c_m = \frac{2\pi r}{n \cdot C_e}$$

- r = wheel radius.
- C_e = encoder resolution (in pulses per revolution).
- n = gear ratio between motor and wheel.
- c_m = conversion factor from encoder ticks to wheel displacement.

PRACTICAL APPROACH

Incremental travel distance at time ***i*** for left and right wheel is:

$$\Delta U_{L,i} = c_m \cdot N_{L,i}$$
$$\Delta U_{R,i} = c_m \cdot N_{R,i}$$

Incremental linear displacement for robot's centerpoint ***C*** is:

$$\Delta U_i = \frac{\Delta U_{L,i} + \Delta U_{R,i}}{2}$$

PRACTICAL APPROACH

Incremental change of orientation is

$$\Delta\theta_i = \frac{\Delta U_{L,i} - \Delta U_{R,i}}{b}$$

with $b = 2l$ the wheelbase, measured between the contact points of the wheels with the ground.

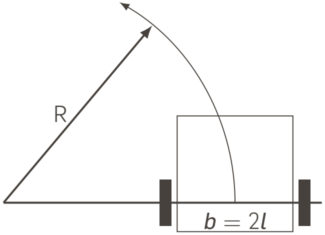
The robots new state is:

$$\theta_i = \theta_{i-1} + \Delta\theta_i$$
$$x_i = x_{i-1} + \Delta U_i \cdot cos(\theta_i)$$
$$y_i = y_{i-1} + \Delta U_i \cdot sin(\theta_i)$$

PRACTICAL APPROACH

Rotation radius *R* is:

$$\omega = \frac{v_L}{R - l} = \frac{v_R}{R + l}$$
$$\Rightarrow R = l \cdot \frac{\Delta U_R + \Delta U_L}{\Delta U_R - \Delta U_L}$$



DIFFERENTIAL DRIVE

Driving behaviour of differential wheel configuration is not very good.

It has poor “open loop” driving behaviour.

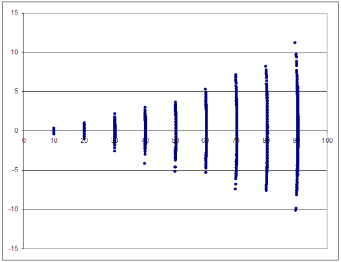
Dead reckoning (keeping track of the robot’s position using the number of rotations of wheels) is hard.

- Errors build up, as dead reckoning is integrative: small errors in parameters or encoder measurements becomes amplified over time.

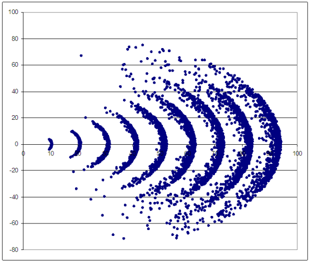
DEAD RECKONING: SIMULATION (1)

- Straight driving.
- Varying noise (i.e. variation on reported distance a wheel has rolled).

Noise = 0.1%

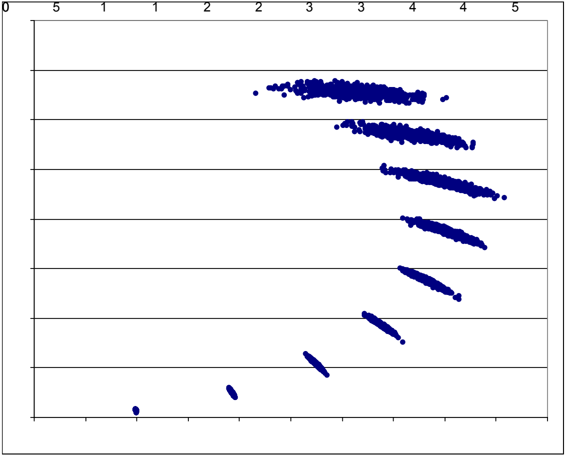


Noise = 1%



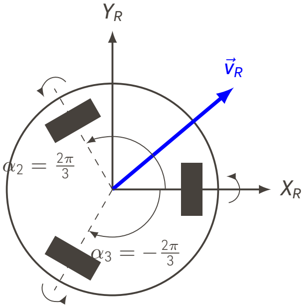
DEAD RECKONING: SIMULATION (2)

- Robot taking a turn (left wheel speed = 1, right wheel speed = 1.005).

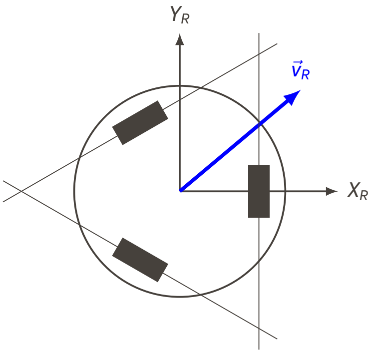


OMNIDIRECTIONAL ROBOT

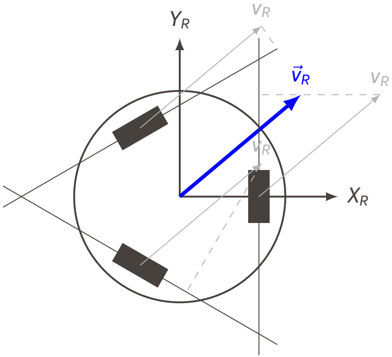
KINEMATICS OF AN OMNIDIRECTIONAL ROBOT: ROBOT FRAME



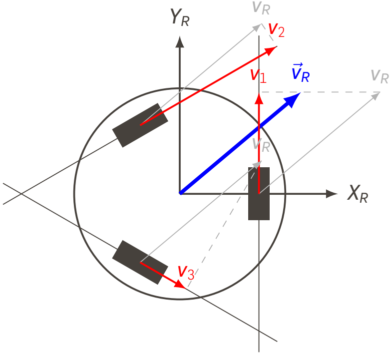
KINEMATICS OF AN OMNIDIRECTIONAL ROBOT: ROBOT FRAME



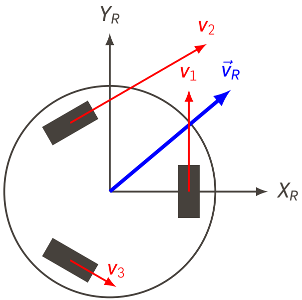
KINEMATICS OF AN OMNIDIRECTIONAL ROBOT: ROBOT FRAME



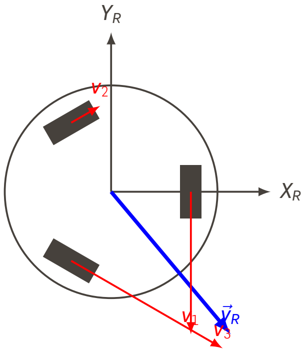
KINEMATICS OF AN OMNIDIRECTIONAL ROBOT: ROBOT FRAME



KINEMATICS OF AN OMNIDIRECTIONAL ROBOT: ROBOT FRAME



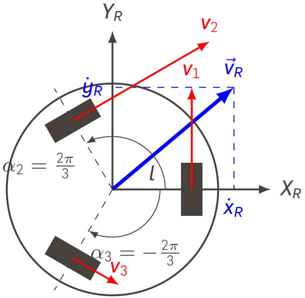
KINEMATICS OF AN OMNIDIRECTIONAL ROBOT: ROBOT FRAME



KINEMATICS OF AN OMNIDIRECTIONAL ROBOT: ROBOT FRAME

$$v_i = \dot{\varphi}_i \cdot r$$
$$= -\dot{x}_R \cdot sin(\alpha_i) + \dot{y}_R \cdot cos(\alpha_i)$$

with r the wheels radius and $\dot{\varphi}_i$ the angular velocity of each wheel.

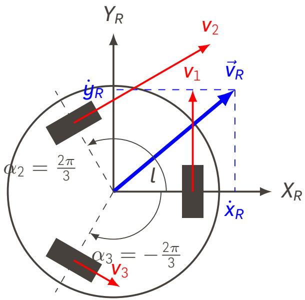


KINEMATICS OF AN OMNIDIRECTIONAL ROBOT: ROBOT FRAME

velocity component due to the robot's rotation

$$v_i = \dot{\varphi}_i \cdot r$$
$$= -\dot{x}_R \cdot sin(\alpha_i) + \dot{y}_R \cdot cos(\alpha_i) + \boxed{l \cdot \dot{\theta}}$$

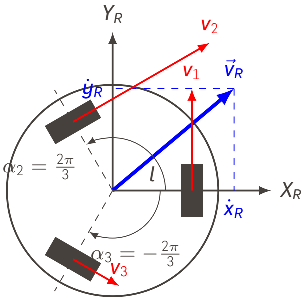
with r the wheels radius and $\dot{\varphi}_i$ the angular velocity of each wheel. l is the distance between the wheels' contact points with the ground and the centre.



KINEMATICS OF AN OMNIDIRECTIONAL ROBOT: ROBOT FRAME

$$v_i = \dot{\varphi}_i \cdot r$$
$$= -\dot{x}_R \cdot sin(\alpha_i) + \dot{y}_R \cdot cos(\alpha_i) + l \cdot \dot{\theta}$$

$$\dot{\varphi} \cdot r = \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\varphi}_3 \end{bmatrix} \cdot r$$
$$= \begin{bmatrix} -sin\alpha_1 & cos\alpha_1 & l \\ -sin\alpha_2 & cos\alpha_2 & l \\ -sin\alpha_3 & cos\alpha_3 & l \end{bmatrix} \cdot \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta} \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 1 & l \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & l \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & l \end{bmatrix} \cdot \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta} \end{bmatrix}$$



⇒ we can determine the wheels angular velocities to obtain a specific robot velocity.

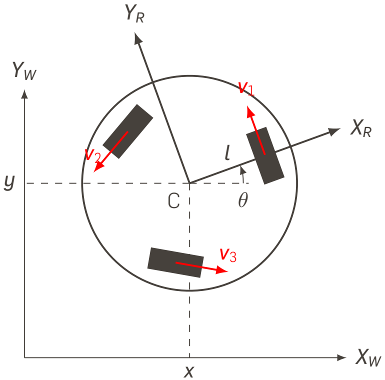
KINEMATICS OF AN OMNIDRIVE ROBOT: WORLD FRAME

This can be remapped to the world frame by:

$$\dot{\xi}_W = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

$$= R(\theta)^{-1} \cdot \dot{\xi}_R$$

$$= \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta} \end{bmatrix}$$



KINEMATICS OF AN OMNIDRIVE ROBOT: WORLD FRAME

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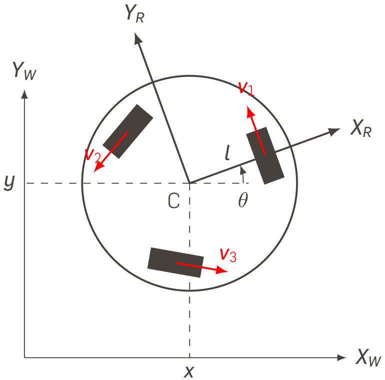

$$\Rightarrow \dot{\xi}_W = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 & 1 & l \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & l \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & l \end{bmatrix}^{-1} \cdot \begin{bmatrix} r \cdot \dot{\varphi}_1 \\ r \cdot \dot{\varphi}_2 \\ r \cdot \dot{\varphi}_3 \end{bmatrix}$$

That's all, folks!

Questions:
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Slides:
github.com/severin-lemaignan/module-mobile-and-humanoid-robots