KARNAUGH MAPS (K-Map)

1). first order K-Map

dec	x	$F = \bar{x}$
0	0	1
1	1	0

x	f
0	1
1	0

$f_{sop} = \bar{x}$ $\qquad$ $f_{sop} = \sum m(0)$

$f_{pos} = \bar{x}$ $\qquad$ $f_{pos} = \prod M(1)$

Algebra $\qquad$ Numeric

2) second order K - Map

Dec	A B	f
0	0 0	1
1	0 1	0
2	1 0	0
3	1 1	1

$\bar{A}\bar{B}$ $\bar{A}B$

A \ B	0	1
0	1 (0)	0 (1)
1	0 (2)	1 (3)

f

$A\bar{B}$ AB

B \ A	0	1
0	1 (0)	0 (2)
1	0 (1)	1 (3)

f

$$f_{SOP} = \bar{A}\bar{B} + AB$$

$$f_{POS} = (\bar{A}+B)(A+\bar{B})$$

$$f_{SOP} = \Sigma m(0,3)$$

$$f_{POS} = \Pi M(1,2)$$

Third - order. K - Map

A B C	f
0 0 0	0
0 0 1	1
0 1 0	1
0 1 1	0
1 0 0	0
1 0 1	1
1 1 0	1
1 1 1	0

$\bar{A}\bar{B}C$ $\bar{A}B\bar{C}$

A \ BC	00	01	11	10
0	0 (0)	1 (1)	0 (3)	1 (2)
1	0 (4)	1 (5)	0 (7)	1 (6)

→ GRAY CODE

$A\bar{B}C$ $AB\bar{C}$ f

AB \ C	0	1
00	0 (0)	1 (1)
01	1 (2)	0 (3)
11	1 (6)	0 (7)
10	0 (4)	1 (5)

$$f_{SOP} = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}C + AB\bar{C}$$

$$f_{POS} = (A+B+C)\cdot(A+\bar{B}+\bar{C})\cdot(\bar{A}+B+C)\cdot(\bar{A}+\bar{B}+\bar{C})$$

$$f_{SOP} = \Sigma m(1,2,5,6)$$

$$f_{POS} = \Pi M(0,3,4,7)$$

fo·

fourth - order K - Map.

Dec	ABCD	f
0	0000	0
1	0001	1
2	0010	1
3	0011	0
4	0100	0
5	0101	0
6	0110	1
7	0111	1
8	1000	1
9	1001	0
10	1010	0
11	1011	0
12	1100	1
13	1101	1
14	1110	0
15	1111	1

AB \ CD	00	01	11	10
00	0 (0)	1 (1)	0 (3)	1 (2)
01	0 (4)	0 (5)	1 (7)	1 (6)
11	1 (12)	1 (13)	1 (15)	0 (14)
10	1 (8)	0 (9)	0 (11)	0 (10)

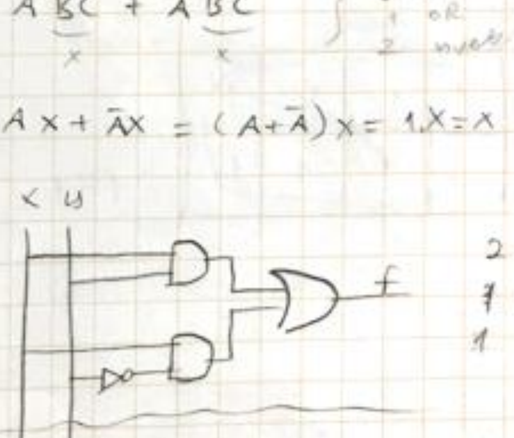
CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

K- MAP FUNCTION REDUCTION = MINIMIZATION

Logic Adjacencies / lojik komşuluk

$A\underbrace{\bar{B}C}_{x} + \bar{A}\underbrace{\bar{B}C}_{x}$ } 2 AND, 1 OR, 2 inverter / 11 input $\quad xy + x\bar{y} = x.(\underbrace{y+\bar{y}}_{1}) = x.1 = x$

$Ax + \bar{A}x = (A+\bar{A})x = 1.x = x = \bar{B}C$ → 1 AND, 1 inverter / 3 input



2 AND
1 OR
1 inverter
+7 input (Line) = ... Gates / inputs

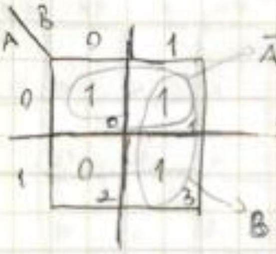
→ 1

Cost criteria

Gates / inputs.

0'lar ile 0'lar komşudur
1'ler ile birbirleri komşudur

Logic Adjacent

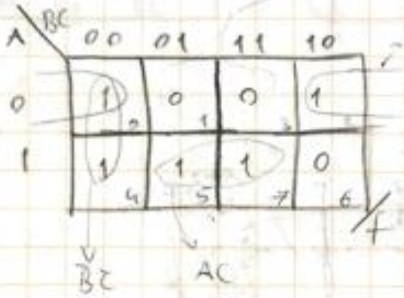


$$f_{sop} = \bar{A}\bar{B} + \bar{A}B + AB$$

$$= \bar{A}\bar{B} + (\bar{A}+A)\cdot B = \bar{A}\bar{B} + B$$

$$= (\bar{A}+B)\cdot(\overbrace{B+\bar{B}}^{1}) = \bar{A}+B$$

$$f_{sop} = \bar{A} + B$$

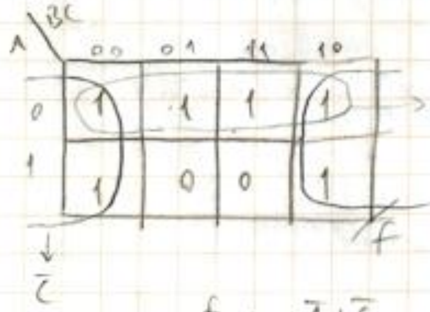


$$f_{sop} = \bar{A}\bar{C} + \bar{B}\bar{C} + AC$$

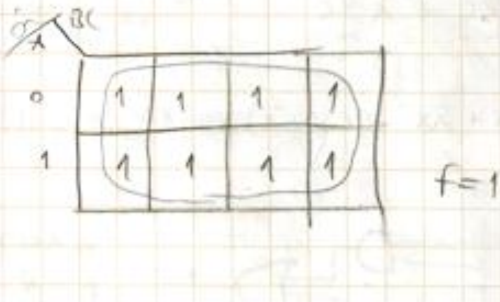
2 şekilde kullanıyoruz ve hepsini alıyoruz

$$f_{pos} = (A+\bar{C})\cdot(\bar{A}+\bar{B}+C)$$

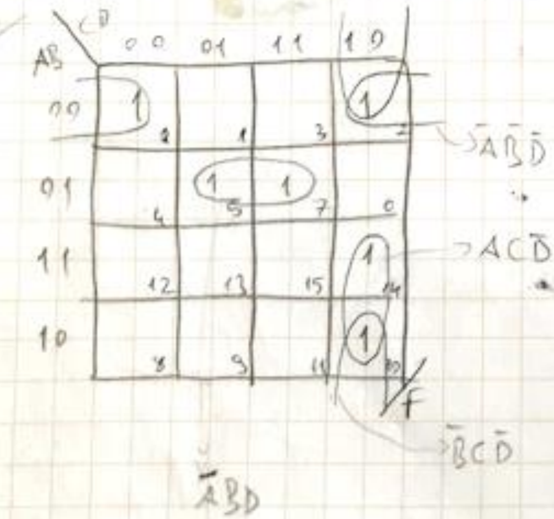
2^n sayısında hücreler komşu olabilir.. yeni 6 hücre 5 hücre 3 hücre komşu olmaz

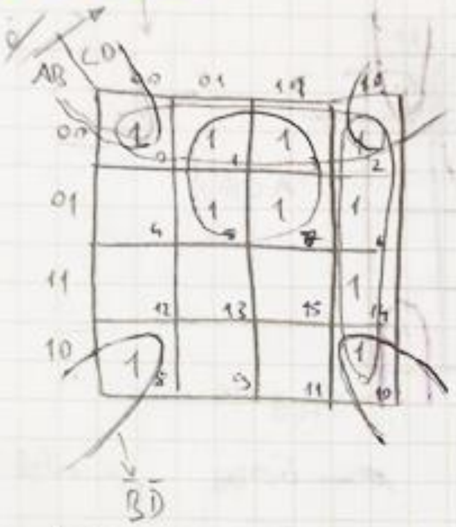


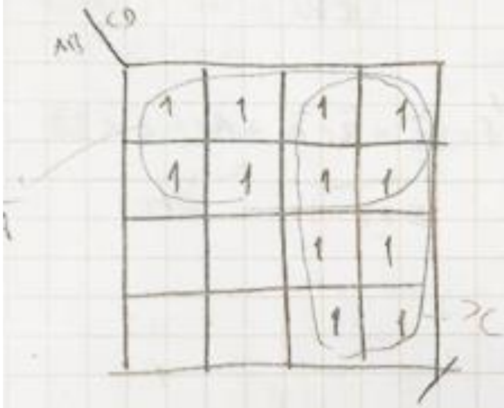
$$f_{sop} = \bar{A} + \bar{C}$$

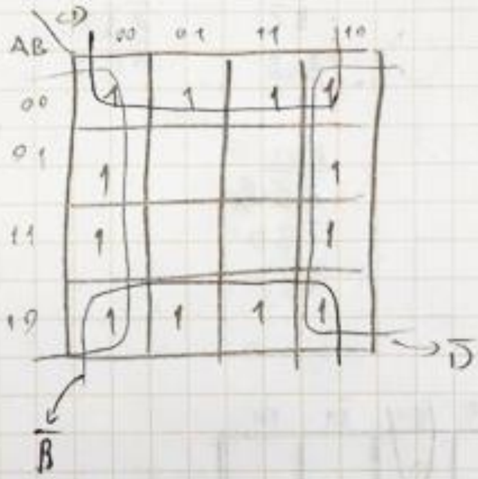


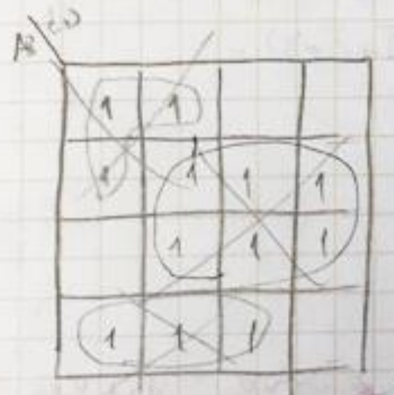
$$f = 1$$









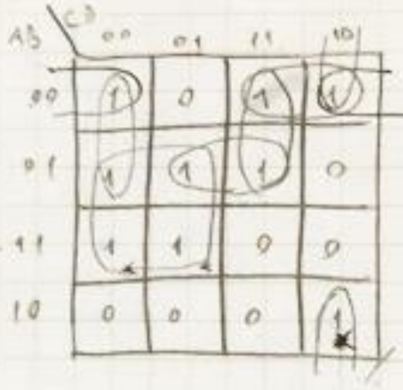


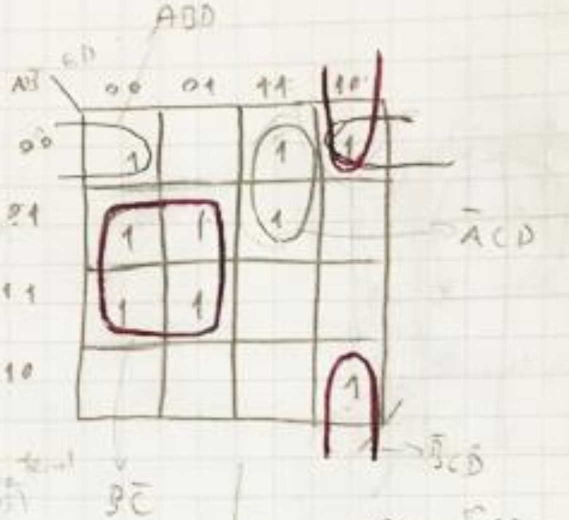
Prime Implicant (PI)

Essential Prime Implicants (EPI)

Optional Prime Implicants (OPI)

Redundant " " (RPI)

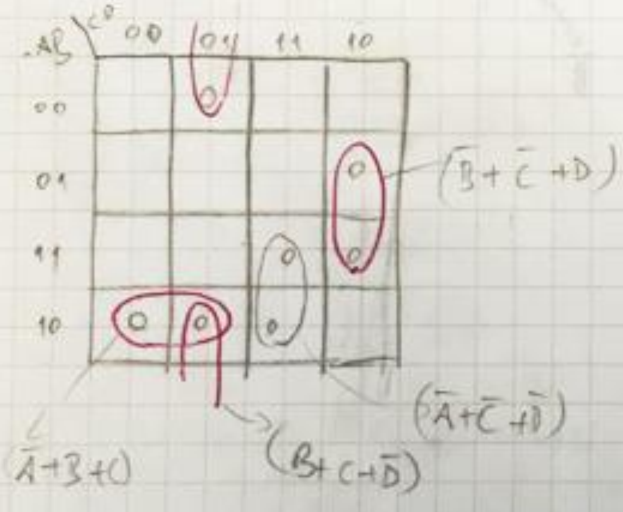




PI	EPI	OPI
$B\bar{C}$	$B\bar{C}$	$\bar{A}\bar{B}\bar{D}$
$\bar{A}\bar{C}\bar{D}$	$\bar{B}C\bar{D}$	$\bar{A}CD$
$\bar{A}\bar{B}\bar{D}$		
$\bar{A}BD$	**RPI**	
$\bar{A}CD$	$\bar{A}\bar{C}\bar{D}$	
$\bar{A}\bar{B}C$	$\bar{A}BD$	
$\bar{B}C\bar{D}$	$\bar{A}\bar{B}C$	

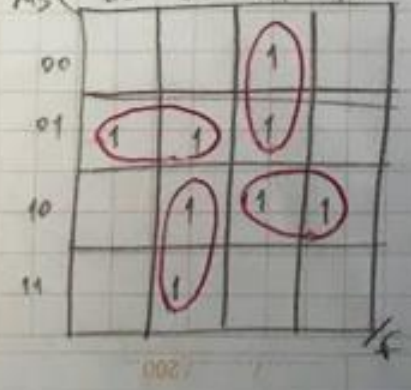
* önce gerekli (EPI) bi yapıyoruz

$$f = \underbrace{B\bar{C} + \bar{B}C\bar{D}}_{EPI} + \underbrace{\bar{A}CD + \bar{A}\bar{B}\bar{D}}_{OPI}$$



$$f_{pos} = (\bar{A}+B+C).(B+C+\bar{D}).(\bar{A}+\bar{C}+\bar{D}).(\bar{B}+\bar{C}+D)$$

* önce Mutlaka önce gereklilerden başlıyoruz sonra yaparken de Maksimum kapsulu istisnai gitte diyoruz



Incompletely specified functions
(DON'T CARE)

a/

$f(A,B,C) = \Sigma m(0,1,5,7) + \phi(2,6)$

$g(A,B,C,D) = \Pi M(0,1,4,6,8,14,15) \cdot \phi(2,3,9)$

A \ BC	00	01	11	10
0	1	1	0	Ø
1	Ø	1	1	0

$\bar{B}$ AC

GRAY CODE

$f_{sop} = \bar{B} + AC$

$(A+\bar{B})$

A \ BC	00	01	11	10
0	1	1	0	Ø
1	Ø	1	1	0

$(\bar{B}+C)$

$f_{pos} = (A+\bar{B}) \cdot (\bar{B}+C)$

g (SOP)

AB \ CD	00	01	11	10
00	0	0	Ø	Ø
01	0	1	1	0
11	1	1	0	0
10	0	Ø	1	1

$(A+D)$

$(\bar{A}+\bar{B}+\bar{C})$

$(B+C)$

$f_{pos} = (B+C)(A+D) \cdot (\bar{A}+\bar{B}+\bar{C})$

AB \ CD	00	01	11	10
00	0	0	0	Ø
01	0	1	1	0
11	1	1	0	0
10	0	Ø	1	1

$\bar{A}BD$

$(\bar{B} \cdot C)$

$AB\bar{C}$

$f_{sop} = AB\bar{C} + \bar{B}C + \bar{A}BD$