Soru Çözümü 3

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SORU

$$\int \frac{1}{\sec\theta + \tan\theta}\,d\theta = \int \frac{1}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}\,d\theta = \int \frac{1}{\frac{1+\sin\theta}{\cos\theta}}\,d\theta$$

$$= \int \frac{\cos\theta}{1+\sin\theta}\,d\theta$$

$1+\sin\theta = u,\quad \cos\theta\, d\theta = du$

$$= \int \frac{du}{u} = \ln|u| + c$$

$$= \underline{\underline{\ln(1+\sin\theta) + c}}$$

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$$\int \frac{dx}{x\sqrt{3+x^2}} = \operatorname{arcsec}\left(\frac{x}{\sqrt{3}}\right) + c \quad \checkmark$$

$3 \to (\sqrt{3})^2$

SORU

$$\int (\csc x - \sec x)(\sin x + \cos x)\,dx$$

$$= \int \left(\frac{1}{\underset{\cos x}{\sin x}} - \frac{1}{\underset{\sin x}{\cos x}}\right)(\sin x + \cos x)\,dx = \int \frac{(\cos x - \sin x).(\sin x + \cos x)}{\cos x \sin x}\,dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\cos x . \sin x}\,dx$$

$$= \int \frac{\cos 2x}{\frac{\sin 2x}{2}}\,dx = 2\int \frac{\cos 2x}{\sin 2x}\,dx$$

$$= \int \frac{\cos 2x}{\frac{\sin 2x}{2}} dx = 2\int \frac{\cos 2x}{\sin 2x} dx$$

$\sin 2x = t$

$2\cos 2x\, dx = dt$

$$= \int \frac{dt}{t}$$

$$= \ln t + c$$

$$= \ln(\sin 2x) + c$$

SORU

$$\int \frac{6\, dy}{\sqrt{y}(1+y)}$$

$y = t^2$

$dy = 2t\, dt$

$$= 12\int \frac{\cancel{t}\, dt}{\cancel{t}(1+t^2)} = 12\int \frac{dt}{1+t^2} = 12 \operatorname{arctg} t + c = 12 \operatorname{arctg}(\sqrt{y}) + c$$

SORU

$$\int ((x^2 - 1)(x+1))^{-2/3}\, dx$$

integralinin aşağıdaki değişken dönüşümlerinden herhangi biriyle hesaplanabileceğini gösterin.

× **a.** $u = 1/(x+1)$

b. $u = ((x-1)/(x+1))^k$ $\quad k = 1, 1/2, 1/3, -1/3, -2/3$ ve -1 için

c. $u = \tan^{-1} x$

d. $u = \tan^{-1}\sqrt{x}$ $\qquad$ **e.** $u = \tan^{-1}((x-1)/2)$

f. $u = \cos^{-1} x$ $\qquad$ **g.** $u = \cosh^{-1} x$

$$\int ((x-1)(x+1)(x+1))^{-2/3} dx = \int ((x-1)(x+1)^2)^{-2/3} dx$$

$$\frac{1}{x+1} = t, \quad x+1 = \frac{1}{t}, \quad dx = -\frac{dt}{t^2}, \quad x = \frac{1}{t} - 1 = \frac{1-t}{t}$$

$$x - 1 = \frac{1}{t} - 2 = \frac{1-2t}{t}$$

$$= \int \left(\left(\frac{1-2t}{t}\right)\cdot\left(\frac{1}{t^2}\right)\right)^{-2/3} \cdot -\frac{dt}{t^2}$$

$$= -\int \frac{(1-2t)^{-2/3}}{\ldots} \cdot \frac{dt}{\ldots} = -\int (1-2t)^{-2/3}\, dt$$

$$= -\int \frac{(1-2t)^{-2/3}}{\cancel{t^2}} \cdot \frac{dt}{\cancel{t^2}} \qquad = -\int (1-2t)^{-2/3}\, dt$$

$1-2t = u$

$-2\,dt = du \Rightarrow dt = -\frac{du}{2}$

$$= \frac{1}{2}\int u^{-2/3}\, du$$

$$= \frac{1}{2}\left(\frac{u^{1/3}}{\frac{1}{3}}\right) + c = \frac{3}{2} u^{\frac{1}{3}} + c$$

$$= \frac{3}{2}(1-2t)^{\frac{1}{3}} + c$$

$$= \frac{3}{2}\left(1 - \frac{2}{x+1}\right)^{\frac{1}{3}} + c$$

SORU

$$\int e^{\sqrt{3s+9}}\, ds$$

$3s+9 = t^2, \quad 3ds = 2t\,dt$

$ds = \frac{2}{3} t\,dt$

$$= \frac{2}{3}\int e^{t}\, t\, dt$$

$$\int u\,dv = u.v - \int v\,du$$

$t = u, \quad e^t dt = dv$

$dt = du, \quad e^t = v$

$$= \frac{2}{3}\left(te^t - \int e^t dt\right) + c = \frac{2}{3}e^t(t-1) + c = \frac{2}{3}e^{\sqrt{3s+9}}\left(\sqrt{3s+9}-1\right) + c$$

SORU

$$\int \ln(x + x^2)\, dx$$

$\ln(x+x^2) = u, \qquad dx = dv$

$\frac{2x+1}{x^2+x}\, dx = du, \qquad x = v$

$$\int \ln(x+x^2)\, dx = x\ln(x+x^2) - \int \frac{\cancel{x}(2x+1)}{x^2+x}\, dx \rightarrow (\cancel{x}\cdot(x+1))$$

$$\int \frac{2x+1}{x+1}\, dx = ?$$

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$$\begin{array}{r|l} 2x+1 & x+1 \\ \hline -\ 2x+2 & 2 \\ \hline -1 & \end{array}$$

$2 - \frac{1}{x+1}$

$= \int \left(2 - \frac{1}{x+1}\right) dx = 2x - \ln(x+1) + c$

$\int \ln(x+x^2)\, dx = x\ln(x+x^2) - 2x + \ln(x+1) + c$

SORU

$$\int \sin(\ln x)\, dx$$

$\ln x = t$

$\frac{dx}{x} = dt \quad , \quad dx = x\,dt$

$dx = e^t dt$

$= \int \sin t \cdot e^t \, dt$

SORU

$$\int \frac{2x+1}{x^2 - 7x + 12} dx$$

SORU

$$\int \frac{y^4 + y^2 - 1}{y^3 + y} dy$$

SORU

$$\int \frac{8\,dx}{(4x^2 + 1)^2}$$