

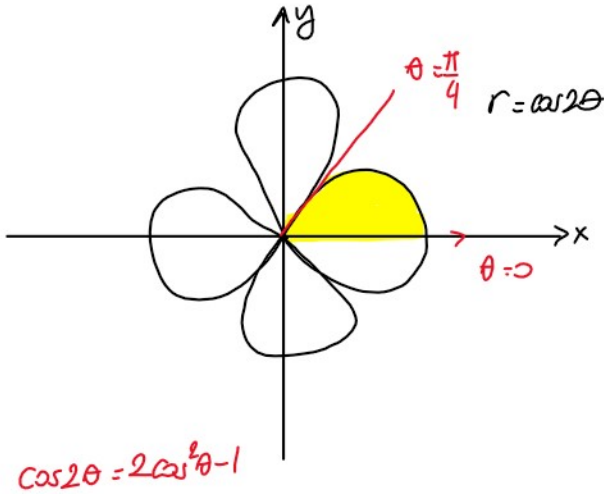
Kutupsal Koordinatlar (Sorular)

15 Mayıs 2022 Pazar 21:31



Kutupsal
Koordinatlar...

Soru $r = \cos 2\theta$ eğrisi tarafından sınırlanan bölgenin alanını bulunuz.



$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\cos^2 2\theta = \frac{1 + \cos 4\theta}{2}$$

$$\frac{A}{8} = \frac{1}{2} \int_0^{\pi/4} r^2 d\theta$$

$$\frac{A}{8} = \frac{1}{2} \int_0^{\pi/4} (\cos 2\theta)^2 d\theta$$

$$A = 4 \int_0^{\pi/4} \cos^2 2\theta d\theta$$

$$A = 4 \int_0^{\pi/4} \left(\frac{1 + \cos 4\theta}{2} \right) d\theta$$

$$A = 2 \left(\theta + \frac{\sin 4\theta}{4} \right) \Big|_0^{\pi/4}$$

$$A = 2 \left(\left(\frac{\pi}{4} + \frac{\sin \pi}{4} \right) - \left(0 + \frac{\sin 0}{4} \right) \right)$$

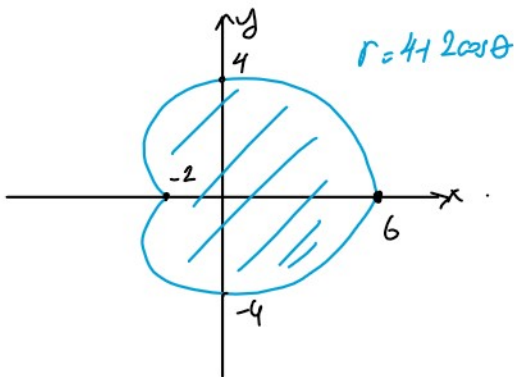
$$A = \frac{\pi}{2} \text{ birim}^2$$

$$a + b \cos \theta \Rightarrow \frac{a}{b} = 1 \rightarrow \text{Kardiyoid}$$

$$a + b \cos \theta, \frac{a}{b} > 1 \rightarrow \text{Simgen}$$

$$r = 4 + 2\cos\theta$$

Soru $r = 4 + \cos 2\theta$ eğrisi tarafından sınırlanan bölgenin alanını bulunuz.



$$A = \frac{1}{2} \int_0^{2\pi} r^2 d\theta$$

$$A = 2 \cdot \frac{1}{2} \int_0^{\pi} (4 + 2\cos\theta)^2 d\theta$$

$$A = \int_0^{\pi} (16 + 16\cos\theta + 4\cos^2\theta) d\theta$$

$$A = \int_0^{\pi} \left(16 + 16\cos\theta + 4 \left(\frac{1 + \cos 2\theta}{2} \right) \right) d\theta$$

$$A = \int_0^{\pi} (18 + 16\cos\theta + 2\cos 2\theta) d\theta$$

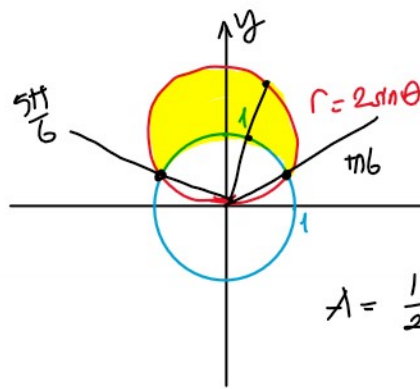
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$$A = \int_0^{\pi} (18 + 16 \cos \theta + 2 \cos 2\theta) d\theta$$

$$A = \left(18\theta + 16 \sin \theta + \sin 2\theta \right) \Big|_0^{\pi}$$

$$\underline{\underline{A = 18\pi \text{ } \text{m}^2}}$$

Soru $r = 1$ çemberinin dışında $r = 2 \sin \theta$ çemberinin içinde kalan bölgenin alanını bulunuz.



$$2 \sin \theta = 1$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \quad \theta = \frac{5\pi}{6}$$

$$A = \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} ((2 \sin \theta)^2 - (1)^2) d\theta$$

$$= \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4 \sin^2 \theta - 1) d\theta$$

$$= \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(4 \cdot \left(\frac{1 - \cos 2\theta}{2} \right) - 1 \right) d\theta$$

$$= \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (1 - 2 \cos 2\theta) d\theta$$

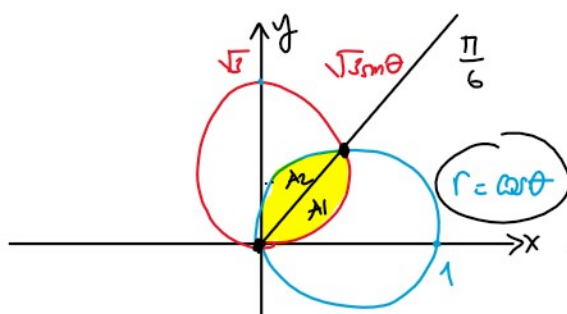
$$A = \frac{1}{2} \left(\theta - \sin 2\theta \right) \Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = \frac{2\pi + \sqrt{27}}{6} \text{ } \text{m}^2$$

$$= \frac{\pi + \sqrt{3}}{3} \text{ } \text{m}^2$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

Soru $r = \cos \theta$ ve $r = \sqrt{3} \sin \theta$ çemberlerinin her ikisinde içinde kalan bölgenin alanını bulunuz.

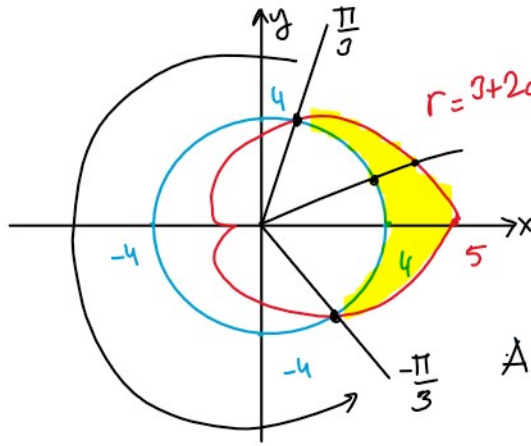


$$\begin{aligned}\sqrt{3} \sin \theta &= \cos \theta \\ \tan \theta &= \frac{1}{\sqrt{3}} \\ \theta &= \frac{\pi}{6}\end{aligned}$$

$$\begin{aligned}A &= A_1 + A_2 \\ A &= \frac{1}{2} \int_0^{\frac{\pi}{6}} (\sqrt{3} \sin \theta)^2 d\theta + \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\cos \theta)^2 d\theta\end{aligned}$$

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Soru $r = 3 + 2 \cos \theta$ limaçonunun içinde $r = 4$ çemberinin dışında kalan bölgenin alanını bulunuz.



$$r = 3 + 2 \cos \theta$$

$$3 + 2 \cos \theta = 4$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, \quad \theta = \frac{5\pi}{3}$$

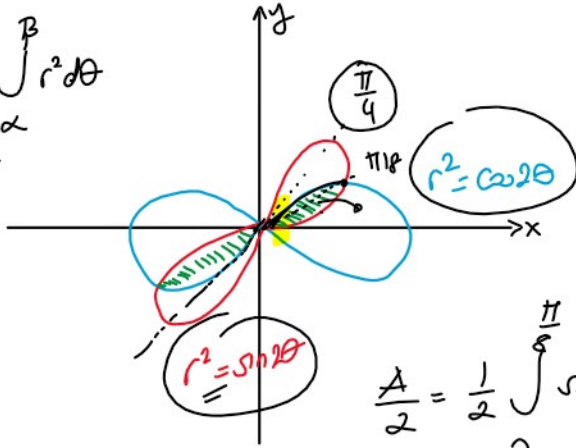
$$A = \frac{1}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} ((3 + 2 \cos \theta)^2 - 4^2) d\theta$$

$$A = 2 \cdot \frac{1}{2} \int_0^{\pi/3} ((3 + 2 \cos \theta)^2 - 16) d\theta$$

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Soru $r^2 = \cos 2\theta$ ve $r^2 = \sin 2\theta$ leminiskatlarının arasında kalan bölgenin alanını bulunuz.

$$A = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$$



$$\cos 2\theta = \sin 2\theta$$

$$\tan 2\theta = 1$$

$$2\theta = \frac{\pi}{4}$$

$$\theta = \frac{\pi}{8}$$

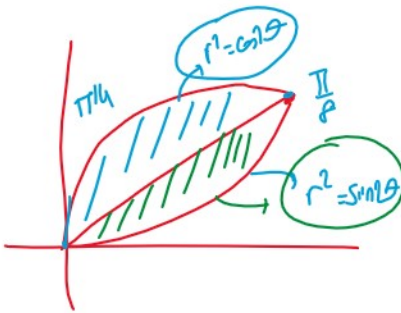
$$\frac{A}{2} = \frac{1}{2} \int_0^{\frac{\pi}{8}} \sin 2\theta d\theta + \frac{1}{2} \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \cos 2\theta d\theta$$

$$A = \frac{1}{2} \left(-\frac{\cos 2\theta}{2} \Big|_0^{\frac{\pi}{8}} + \frac{1}{2} \left(\frac{\sin 2\theta}{2} \Big|_{\frac{\pi}{8}}^{\frac{\pi}{4}} \right) \right)$$

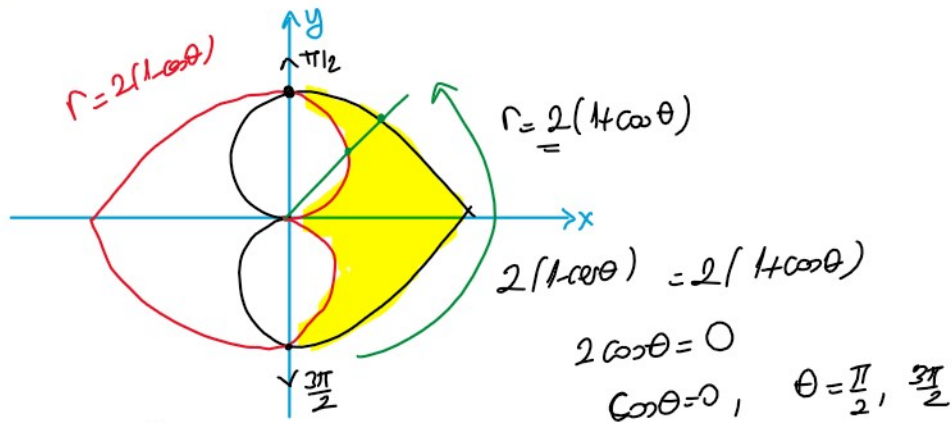
$$\frac{A}{2} = \frac{1}{4} \left(-\cos \frac{\pi}{4} + \cos 0 \right) + \frac{1}{4} \left(\sin \frac{\pi}{2} - \sin \frac{\pi}{4} \right)$$

$$\frac{A}{2} = \frac{1}{4} \left(1 - \frac{\sqrt{2}}{2} \right) + \frac{1}{4} \left(1 - \frac{\sqrt{2}}{2} \right)$$

$$A = \frac{2 - \sqrt{2}}{2} \text{ birim}^2$$



Soru $r = 2(1 + \cos \theta)$ kardioidinin içinde $r = 2(1 - \cos \theta)$ kardioidinin dışında kalan bölgenin alanını bulunuz.



$$\begin{aligned}
 A &= \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[(2(1 + \cos \theta))^2 - (2(1 - \cos \theta))^2 \right] d\theta \\
 &= 2 \cdot \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left((4 + 8\cos \theta + 4\cos^2 \theta) - (4 - 8\cos \theta + 4\cos^2 \theta) \right) d\theta \\
 &= 16 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos \theta d\theta = 16 \left(\sin \theta \right)_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 16 \left(\sin \frac{\pi}{2} - \sin \left(-\frac{\pi}{2}\right) \right) = \underline{\underline{16 \text{ birim}^2}}
 \end{aligned}$$

Soru $r = \sin^3 \frac{\theta}{3}$ eğrisinin $0 \leq \theta \leq 3\pi$ aralığında kalan parçasının uzunluğunu bulunuz.

$$l = \int_{\alpha}^{\beta} \sqrt{r^2 + (r')^2} d\theta$$

$$r = \sin^3 \frac{\theta}{3}$$

$$r' = \cancel{3} \sin^2 \frac{\theta}{3} \cdot \cos \frac{\theta}{3} \cdot \frac{1}{\cancel{3}} = \sin^2 \frac{\theta}{3} \cdot \cos \frac{\theta}{3}$$

$$r^2 + (r')^2 = \sin^6 \frac{\theta}{3} + \sin^4 \frac{\theta}{3} \cdot \cos^2 \frac{\theta}{3} = \sin^4 \frac{\theta}{3} \left(\sin^2 \frac{\theta}{3} + \cos^2 \frac{\theta}{3} \right) = \sin^4 \frac{\theta}{3}$$

$$l = \int_0^{3\pi} \sqrt{\sin^4 \frac{\theta}{3}} d\theta = \int_0^{3\pi} \sin^2 \frac{\theta}{3} d\theta$$

$$= \frac{1}{2} \int_0^{3\pi} (1 - \cos \frac{2\theta}{3}) d\theta$$

$$= \frac{1}{2} \left(\theta - \frac{3}{2} \sin \frac{2\theta}{3} \right) \Big|_0^{3\pi}$$

$$= \frac{1}{2} \left[\left(3\pi - \frac{3}{2} \sin \frac{6\pi}{3} \right) - \left(0 - \frac{3}{2} \sin \frac{0}{3} \right) \right]$$

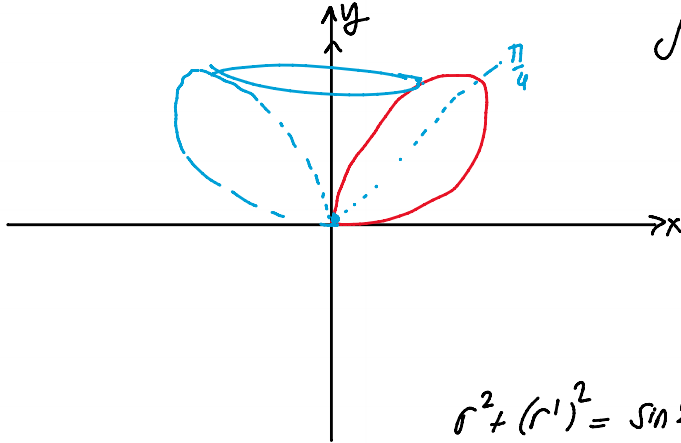
$$\boxed{l = \frac{3\pi}{2} \text{ br}}$$

$$\cos 2\theta = 1 - 2\sin^2 \theta$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\sin^2 \frac{\theta}{3} = \frac{1 - \cos \frac{2\theta}{3}}{2}$$

Soru $r^2 = \sin 2\theta$ leminiskatının y ekseninin sağda kalan kısmının y eksenini etrafında döndürülmesiyle oluşan şeklin yüzey alanını bulunuz.



$$S = 2\pi \int_0^{\pi/4} r \cos \theta \cdot \sqrt{r^2 + (r')^2} d\theta$$

$$r = \sqrt{\sin 2\theta}$$

$$r' = \frac{1}{2} \frac{2 \cos 2\theta}{\sqrt{\sin 2\theta}} = \frac{\cos 2\theta}{\sqrt{\sin 2\theta}}$$

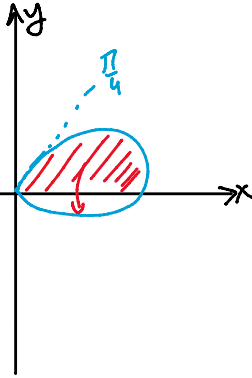
$$r^2 + (r')^2 = \sin 2\theta + \frac{\cos^2 2\theta}{\sin 2\theta} = \frac{\sin^2 2\theta + \cos^2 2\theta}{\sin 2\theta} = \frac{1}{\sin 2\theta}$$

$$S = 2\pi \int_0^{\pi/4} \cancel{\sqrt{\sin 2\theta}} \cdot \cos \theta \cdot \sqrt{\frac{1}{\cancel{\sin 2\theta}}} d\theta$$

$$= 2\pi \int_0^{\pi/4} \cos \theta d\theta = 2\pi \left(\sin \theta \right)_0^{\pi/4} = 2\pi \left(\sin \frac{\pi}{4} - \sin 0 \right) = 2\pi \cdot \frac{1}{2} = \pi$$

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Soru. $r = \sqrt{\cos 2\theta}$ eğrisinin $0 \leq \theta \leq \frac{\pi}{4}$ arasında kalan kısmının x eksenini etrafında döndürülmesiyle oluşan şeklin yüzey alanını bulunuz.



$$S = 2\pi \int_0^{\pi/4} r \sin \theta \cdot \sqrt{r^2 + (r')^2} d\theta$$

$$= 2\pi \int_0^{\pi/4} \sqrt{\cos 2\theta} \cdot \sin \theta \cdot \sqrt{r^2 + (r')^2} d\theta$$

Böyle