Soru Çözümü 2

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SORU

$$\int \frac{dx}{x-\sqrt{x}} = \int \frac{2t\,dt}{t^2-t} = \int \frac{2\cancel{t}}{\cancel{t}(t-1)}dt = 2\ln(t-1)+c$$

$$= 2\ln(\sqrt{x}-1)+c$$

$x=t^2$

$dx=2t\,dt$

SORU

$$\int \frac{dx}{\sqrt{x}(\sqrt{x}+1)} = \int \frac{2du}{u} = 2\ln u + c = 2\ln(\sqrt{x}+1)+c$$

$\sqrt{x}+1=u$

$\frac{1}{2\sqrt{x}}dx=du$

$\frac{dx}{\sqrt{x}}=2du$

SORU

$$\int \frac{2\,dx}{x\sqrt{1-4\ln^2 x}} = \int \frac{2.\,dt}{2.\sqrt{1-t^2}} = \int \frac{dt}{\sqrt{1-t^2}} = \arcsin t + c$$

$$= \arcsin(2\ln x)+c$$

$2\ln x = t$

$\frac{2dx}{x}=dt$

$\frac{dx}{x}=\frac{dt}{2}$

SORU

$$\int \frac{6\,dx}{x\sqrt{25x^2-1}}$$

$\frac{1}{x}=t \quad , \quad x=\frac{1}{t} \quad , \quad dx=-\frac{dt}{t^2}$

$$= 6\int \frac{-\frac{dt}{t^2}}{\quad} = 6\int \frac{-\frac{dt}{\cancel{t^2}}}{\quad} = -6\int \frac{dt}{\quad}$$

$$= 6\int \frac{-\frac{dt}{t^2}}{\frac{1}{t}\sqrt{\frac{25}{t^2}-1}} = 6\int \frac{-\frac{dt}{\cancel{t^2}}}{\frac{1}{\cancel{t}}\frac{\sqrt{25-t^2}}{\cancel{t}}} = -6\int \frac{dt}{\sqrt{25-t^2}}$$

$$\frac{25-t^2}{t^2}$$

$$= -6 \arcsin\left(\frac{t}{5}\right) + c$$

$$= -6 \arcsin\left(\frac{1}{5x}\right) + c$$

SORU

$$\int \frac{dx}{e^x + e^{-x}} = \int \frac{dx}{e^x + \frac{1}{e^x}} = \int \frac{dx}{\frac{e^{2x}+1}{e^x}} = \int \frac{e^x dx}{e^{2x}+1}$$

$e^x = t$

$e^x dx = dt$

$$= \int \frac{dt}{t^2+1}$$

$$= \operatorname{arctg} t + c$$

$$= \operatorname{arctg}(e^x) + c$$

SORU

$$\int \frac{\ln x \, dx}{x + 4x\ln^2 x} = \int \frac{\ln x \, dx}{x(1+4\ln^2 x)}$$

$2\ln x = u$

$2\frac{dx}{x} = du \Rightarrow \frac{dx}{x} = \frac{1}{2}du$

$$= \int \frac{u}{2} \cdot \frac{du}{2(1+u^2)} = \frac{1}{4}\int \frac{u\,du}{1+u^2}$$

$1+u^2 = t$

$2u\,du = dt$

$u\,du = \frac{dt}{2}$

$$= \frac{1}{4}\cdot\frac{1}{2}\int \frac{dt}{t} = \frac{1}{8}\ln t + c = \frac{1}{8}\ln(1+u^2) + c = \frac{1}{8}\ln(1+4\ln^2 x) + c$$

SORU

$$\int \frac{dy}{\sqrt{e^{2y}-1}} = \int \frac{dt}{t\sqrt{t^2-1}} = \operatorname{arcsec} t + c$$

$= \operatorname{arcsec}(e^y) + c$ //

$e^y = t \quad , \quad e^y dy = dt$

$dy = \frac{dt}{e^y}$

$dy = \frac{dt}{t}$

SORU

$$\int (\sec x + \cot x)^2\, dx$$

$$= \int (\sec^2 x + 2\sec x \cot x + \cot^2 x)\,dx = \overbrace{\int \sec^2 x\,dx}^{\tan x} + \int 2\sec x \cot x\,dx + \int \cot^2 x\,dx$$

$$= 2\,\frac{1}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\sin x}\,dx$$

$$= \frac{2}{\sin x}$$

$$+ \int \frac{2}{\sin x}\,dx$$

$\tan\frac{x}{2} = t \quad , \quad dx = \frac{2dt}{1+t^2} \quad , \quad \sin x = \frac{2t}{1+t^2}$

$$\int \frac{2}{\sin x}\,dx = \int \frac{2 \cdot 2dt}{(\cancel{1+t^2}) \cdot \frac{\cancel{2}t}{\cancel{1+t^2}}} = 2\int \frac{dt}{t} = 2\ln t + c = 2\ln\left(\tan\frac{x}{2}\right) + c$$

$$\int \cot^2 x\,dx = ?$$

$(\tan x)' = 1 + \tan^2 x$

$(\cot x)' = -(1 + \cot^2 x)$

$$\int (\cot^2 x + 1 - 1)\,dx = \int (\cot^2 x + 1)\,dx - \int dx$$

$$= -\cot x - x + C$$

$$\int (\sec x + \cot x)^2\,dx = \tan x + 2\ln\left(\tan\frac{x}{2}\right) - \cot x - x + C$$ //

SORU

$$\int \csc x \sin 3x\, dx = \int \frac{\sin 3x}{\sin x}\,dx = \int \frac{\sin(2x+x)}{\sin x}\,dx$$

$$= \int \frac{\sin 2x \cos x + \cos 2x \sin x}{\sin x}\,dx$$

$$= \int \frac{2\cancel{\sin x}\cos x \cos x + \cos 2x \cancel{\sin x}}{\cancel{\sin x}}\,dx$$

$$= \int \frac{\ldots}{\cancel{\sin x}} dx$$

$$= \int (2\cos^2 x + \cos 2x)\, dx$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$= \int \left[2\left(\frac{1+\cos 2x}{2} \right) + \cos 2x \right] dx$$

$$= \int (1 + 2\cos 2x)\, dx = x + 2 \cdot \frac{\sin 2x}{2} + C = x + \sin 2x + C$$

SORU

$$\int \frac{4t^3 - t^2 + 16t}{t^2 + 4}\, dt$$

$$\begin{array}{r|l} 4t^3 - t^2 + 16t & t^2+4 \\ \hline -\;4t^3 + 16t & 4t - 1 \\ -t^2 & \\ -t^2 - 4 & \\ \hline 4 & \end{array}$$

$$\frac{4t^3 - t^2 + 16t}{t^2+4} = (4t-1) + \frac{4}{t^2+4}$$

$$= \int (4t-1)\, dt + \int \frac{4\, dt}{t^2+4} = 2t^2 - t + 4 \cdot \frac{1}{2} \operatorname{arctg} \frac{t}{2} + C$$

$$= 2t^2 - t + 2 \operatorname{arctg} \frac{t}{2} + C$$

SORU

$$\int \frac{x + 2\sqrt{x-1}}{2x\sqrt{x-1}}\, dx = \int \frac{\cancel{x}}{2\cancel{x}\sqrt{x-1}} dx + \int \frac{\cancel{2}\cancel{\sqrt{x-1}}}{\cancel{2}x\cancel{\sqrt{x-1}}} dx$$

$$= \frac{1}{2} \int \frac{dx}{\sqrt{x-1}} + \int \frac{dx}{x}$$

$x - 1 = t^2$

$dx = 2t\,dt$

$$\frac{1}{\cancel{2}} \int \frac{\cancel{2}\cancel{t}\, dt}{\cancel{t}}$$

$$= t + C$$

$$= \sqrt{x-1} + \ln x + C$$

SORU

$$\int \frac{dt}{\sqrt{-t^2 + 4t - 3}}$$

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SORU

$$\int \frac{dx}{(x+1)\sqrt{x^2+2x+1-1}}$$

$x+1=t \quad , \quad dx=dt$

$$= \int \frac{dt}{t\sqrt{t^2-1}} = \operatorname{arcsec} t + c$$

$$= \operatorname{arcsec}(x+1) + c$$