Biçimsel Diller ve Otomata Teorisi

Sunu VI
Kleene Kuramı

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Kleene's Theorem

- Any language that can be defined by regular expression, or finite automaton, or transition graph can be defined by all three methods

- Proved in 1956. The most important and fundamental result in the theory of finite automata

Kleene's Theorem

- We wish to show that the set of ZAPS, the set of ZEPS, and the set of ZIPS are all the same
- We need three parts
  - Part I, we shall show that all ZAPS are ZEPS
  - Part II, we shall show that all ZEPS are ZIPS
  - Part III, we shall show that all ZIPS are ZAPS
- [ZAPS $\subset$ ZEPS $\subset$ ZIPS $\subset$ ZAPS]$\equiv$[ZAPS = ZEPS = ZIPS]

Proof

- Part 1: Every language that can be defined by a FA can also be defined by a TG
- Part 2: Every language that can be defined by a TG can also be defined by a RE
- Part 3: Every language that can be defined by a RE can also be defined by a FA

Proof – Part 1

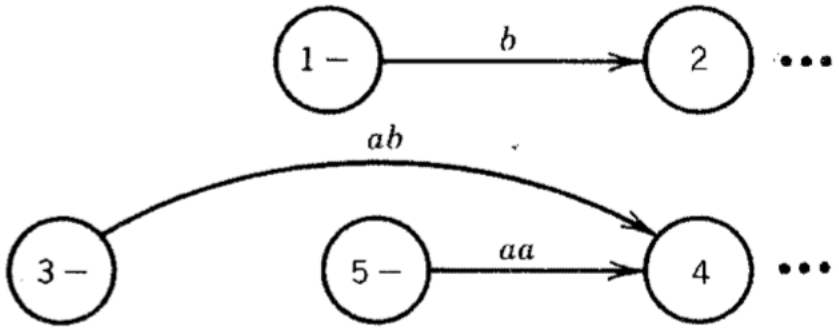
- The easiest part
- Every FA is itself already a TG
  - Any language that has been defined by a FA has already been defined by a TG

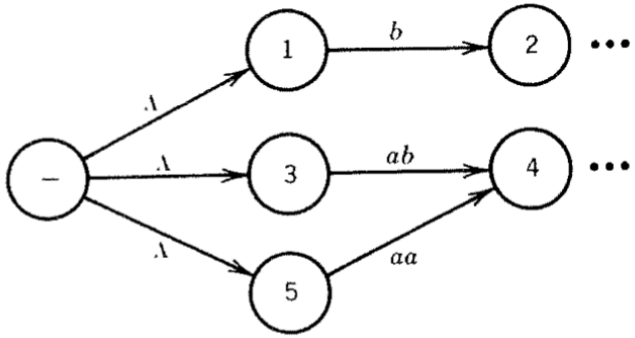
Proof – Part 2

- The proof will be by constructive algorithm
  - We present a procedure that starts out with a TG and ends up with a RE that defines the same language
- To be acceptable, any algorithm must satistfy two criteria
  - It must work for every TG
  - It must guarantee to finish its job in a finite time (number of steps)

Proof – Part 2

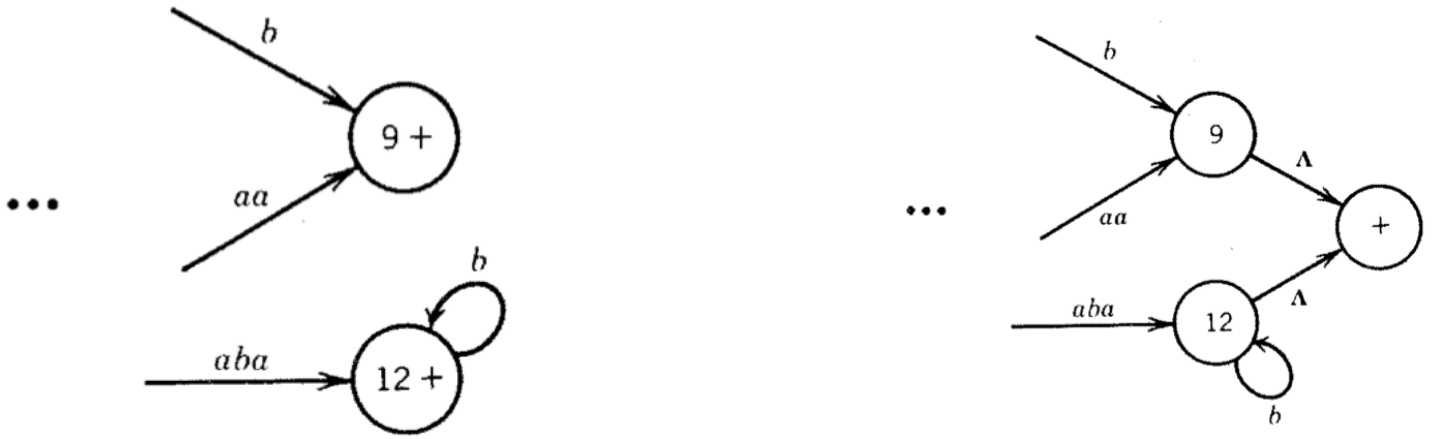
- Let us start with an abstract transition graph *T*
  - *T* may have many start states. We first want to simplify *T* so that it has only one start state
  - Introduce a new state that we label with a minus sign and that we connect to all the previous start states by edges labeled **Λ**
  - Drop the minus signs from the previous start states





Proof – Part 2

- Another simplification we can make is to have a <u>unique final state</u> without changing the language it accepts
  - If *T* had no final states then it accepts no strings and has no language. We need produce no RE other than the null (φ)
  - If *T* has several final states, let us un-final them and introduce a new unique final state labeled with a plus sign. We draw edges from all former final states to the new one each labeled with **Λ**

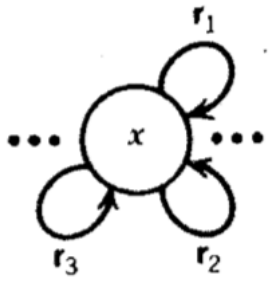


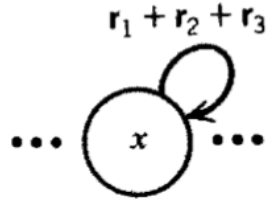
Proof – Part 2

- We shall require that the unique final state be a different state from the unique start state
  - If an old state used to have ± then both signs are removed from it to newly created states
- It should be noted that the new states does not affect the language *T* accepts
  - Any word accepted by *T* is also accepted by the new machine
  - Any word rejected by *T* is also rejected by the new machine

Proof – Part 2

- We are now going to build piece by piece the RE that defines the same language as *T*
  - We will change *T* into a GTG
- Suppose that T has some state x inside it (no – or + state)
  - x has more than one loop
  - We can replace three loops by one loop labeled with a RE



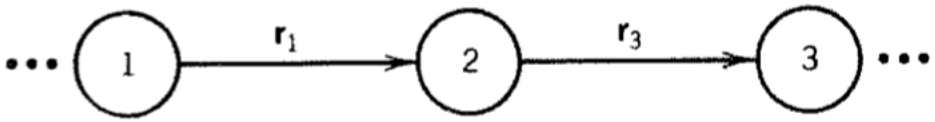


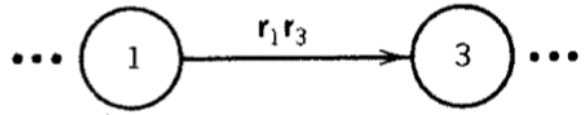
Proof – Part 2

- Similarly, suppose that two states are connected by more than one edge going in the same direction
  - We can replace this with a single edge labeled with a RE

Bypass and State Elimination Operation

- If we have three states in a row connected by edges lebeled with REs (or simple strings)
  - We can eliminate the middleman and go directly from one outer state to the other by a new edge labeled with a RE that is **concatenation** of the previous labels

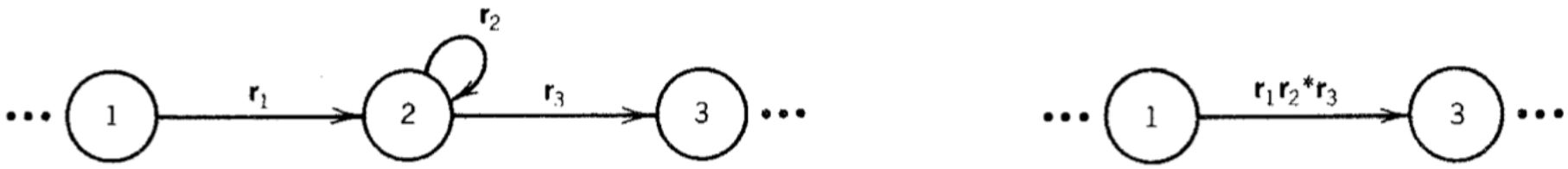




- We do not keep old edges from state 1 to state 2 and state 2 to state 3
  - Unless they are used in paths other than the ones from state 1 to state 3

Bypass and State Elimination Operation

- We can do the bypass operation only as long as state 2 does not have a loop going back to itself
- If state 2 does have a loop, we must use this model



Bypass and State Elimination Operation

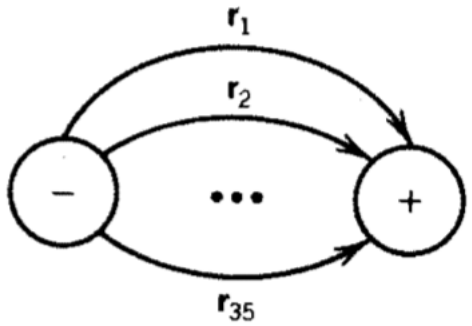
- If state 1 is connected to state 2 and state 2 is connected to more than one other state (e.g. states 3, 4, 5)
  - When we eliminate the edge from state 1 to state 2 we have to add edges that show how to go from state 1 to other states (e.g. states 3, 4, 5)

Bypass and State Elimination Operation

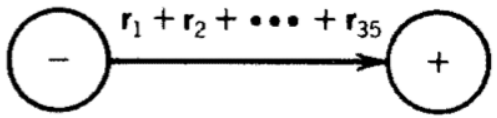
- Every state that leads into state 2 can be made to bypass state 2
  - If state 9 leads into state 2, we can eliminate the edge from state 9 to state 2 by adding edges from state 9 to states 3, 4, and 5 directly
  - We can repeat this operation until nothing leads into state 2
  - We can eliminate state 2 entirely because it will not be part of a path that accepts a language
- Without changing the set of words that *T* accepts, we have eliminated one of its states
  - We can repeat this process until we eliminate all the states from *T* except the unique start and final states

Proof – Part 2

- Finally, we will have this

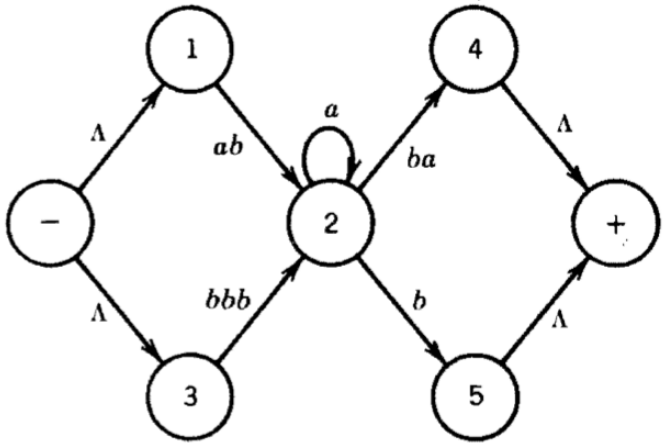


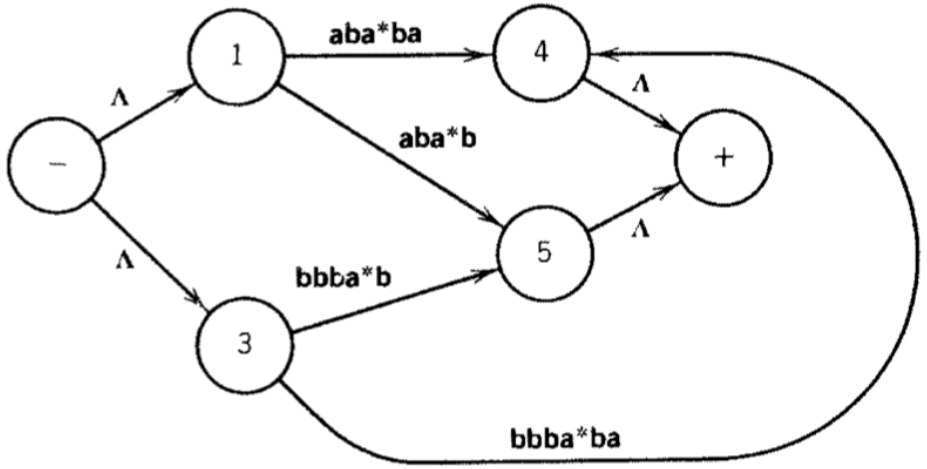
- We can then combine this once more to produce



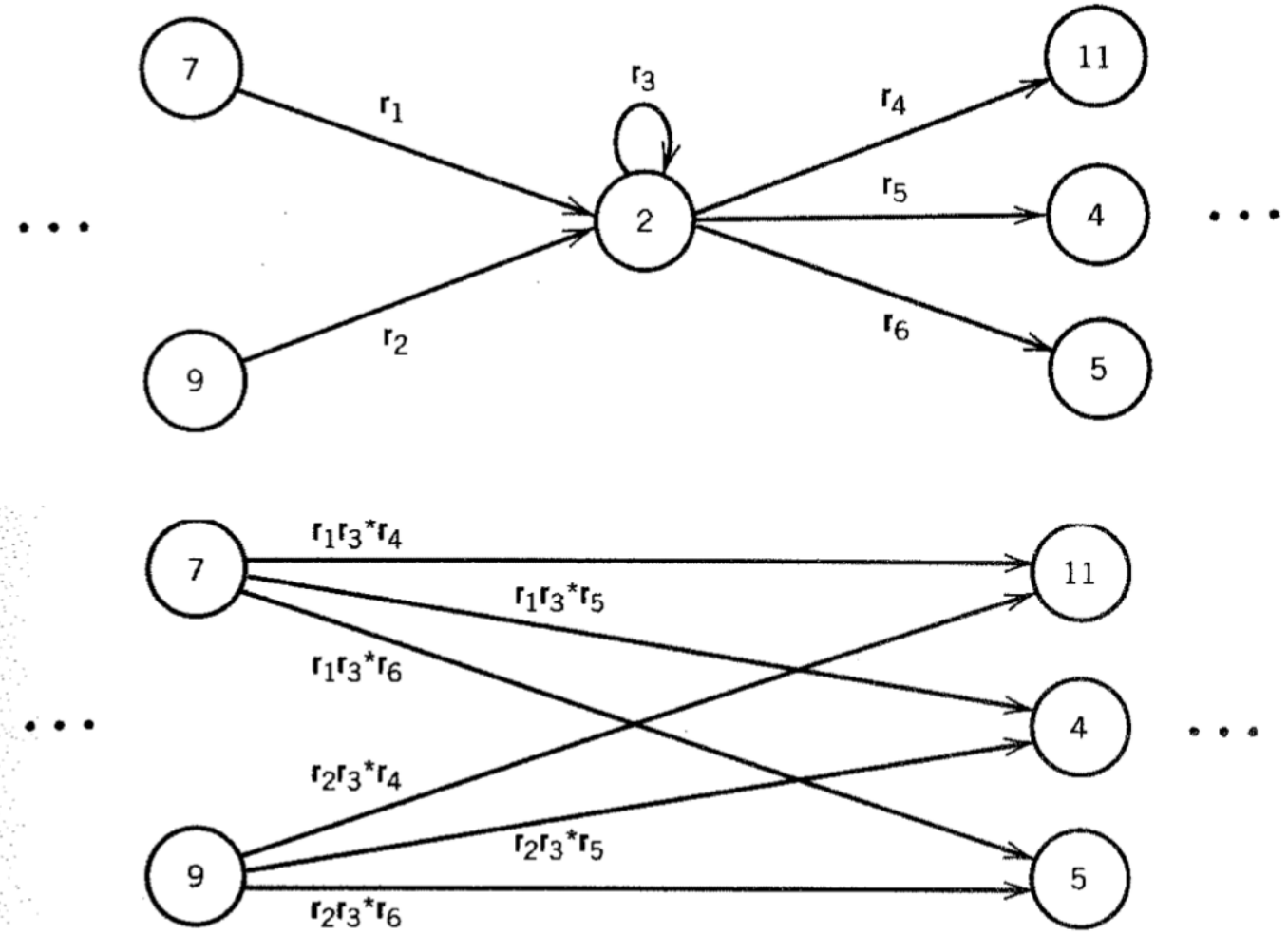
Example

- We can bypass state 2 by introducing a path from
  - State 1 to state 4 labeled aba*ba
  - State 1 to state 5 labeled aba*b
  - State 3 to state 4 labeled bbba*ba
  - State 3 to state 5 labeled bbba*b



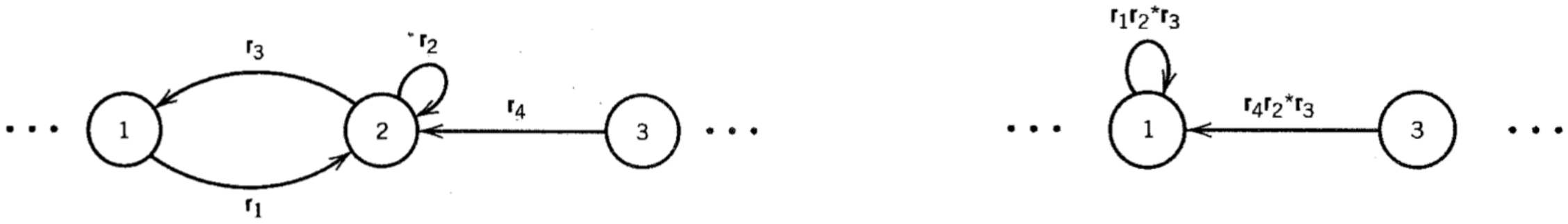


Example

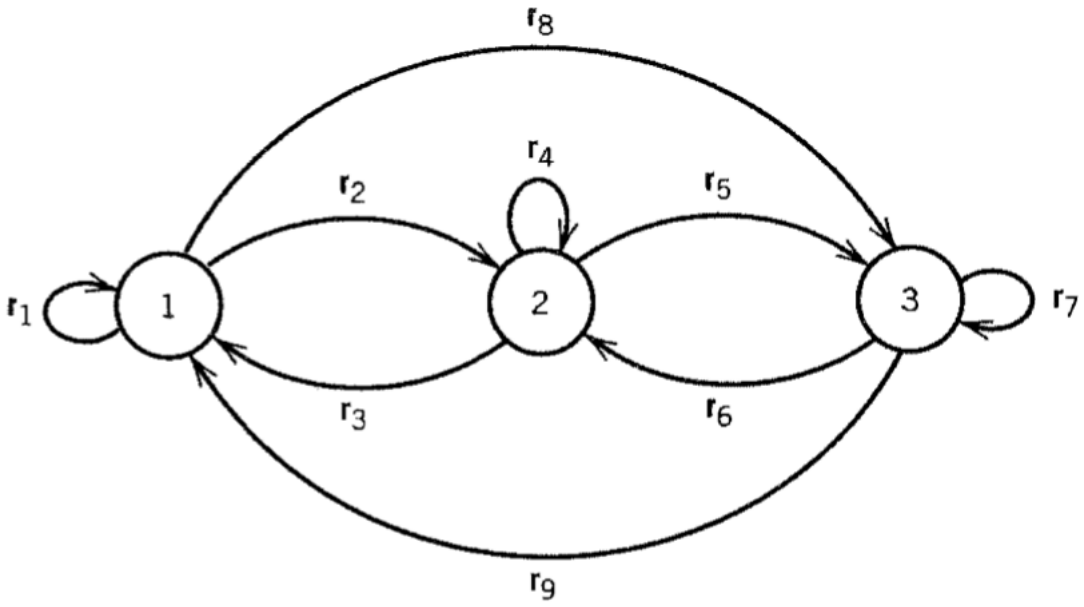

7
9
2
11
4
5
r1
r2
r3
r4
r5
r6
r1r3*r4
r1r3*r5
r1r3*r6
r2r3*r4
r2r3*r5
r2r3*r6

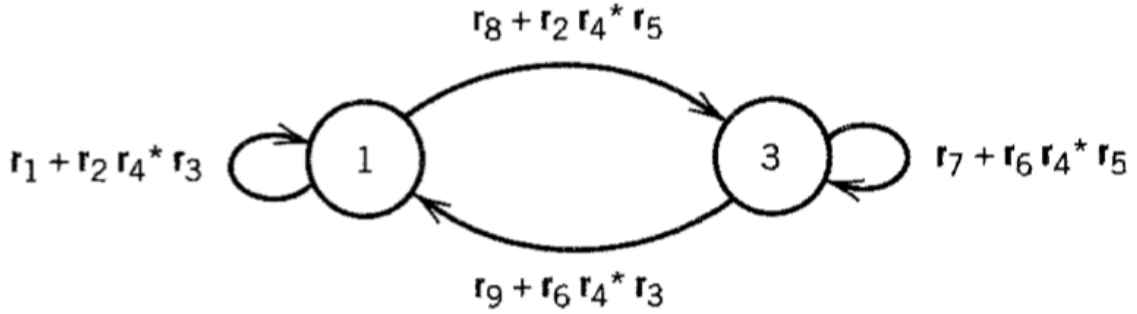
Example

- A special case that we must examine more carefully
- We want to eliminate state 2
  - One of the **source** states to the prospective bypassed state is <u>also</u> a **destination** state from that state



Example





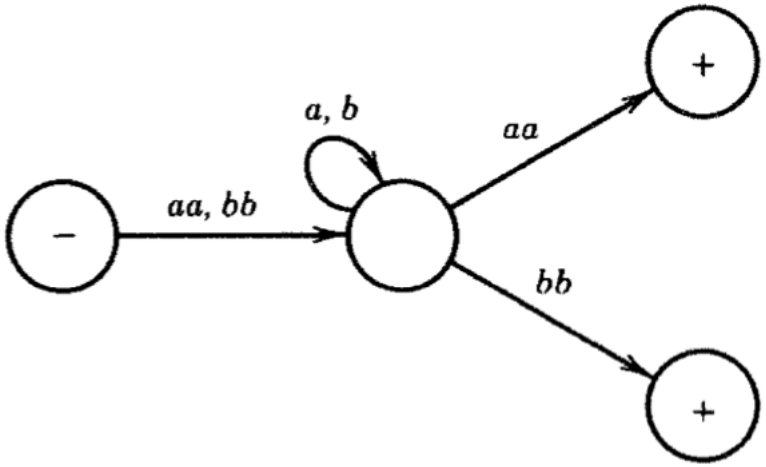
- Whenever we remove an edge or state, we must be sure that we have not destroyed any paths through T or created new paths

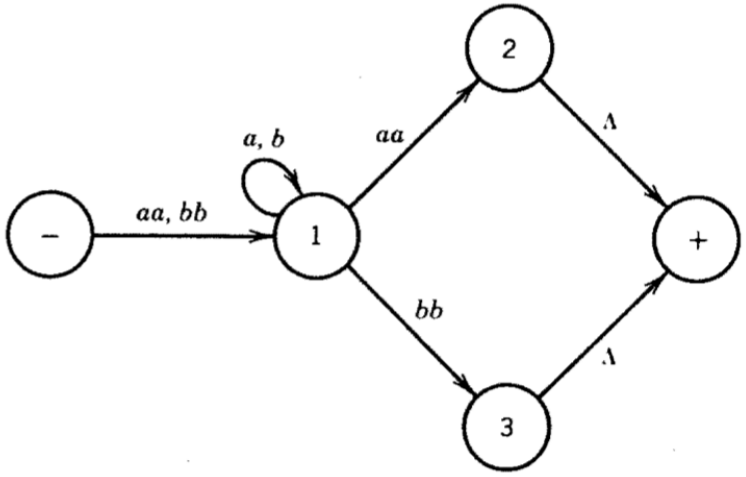
Proof – Part 2

- This algorithm terminates in a finite number of steps
  - T has only finitely many states and one state is eliminated with each iteration of the bypass procedure
- The other important observation is that the method works on all transition graphs
- Therefore, this algorithm provides a satisfactory proof that there is a RE for each TG

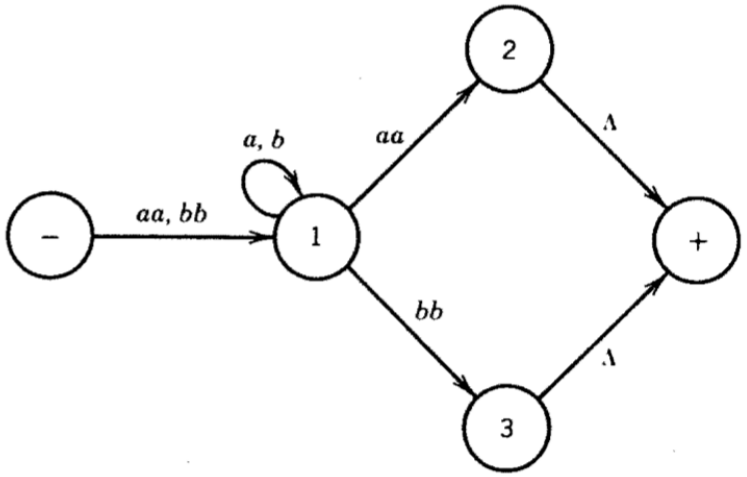
Example

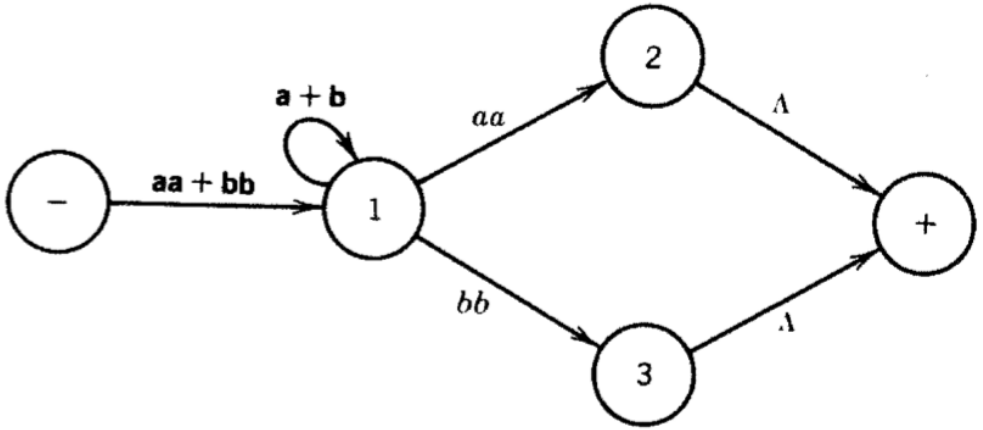
- The TG accepts all words that begin and end with double letters (having at least length 4)





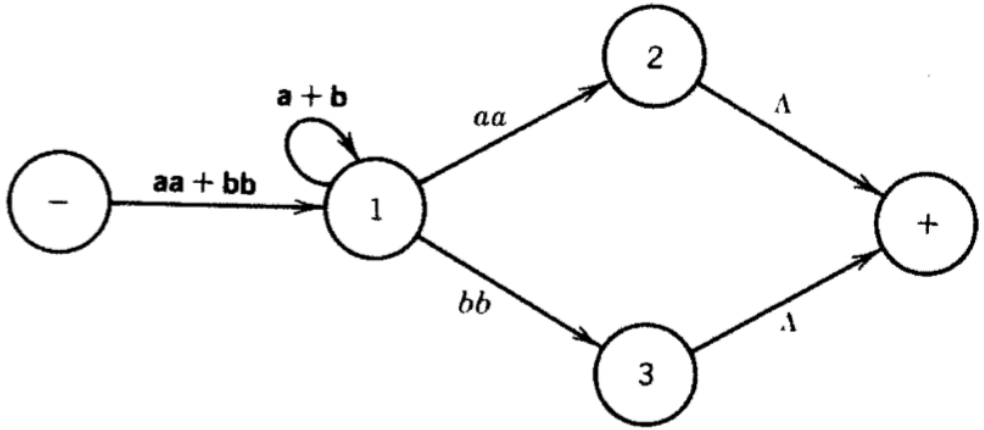
Example, continued

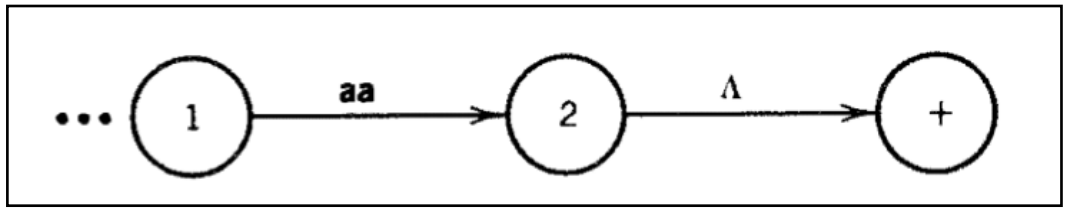


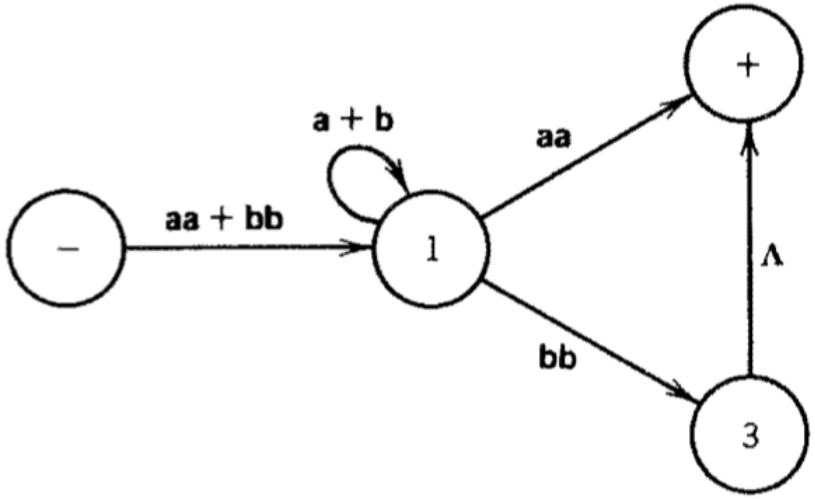


Example, continued

- The algorithm does not tell us which state of the TG we must bypass next. The order of elimination is left up to us
- Let us choose state 2 for elimination

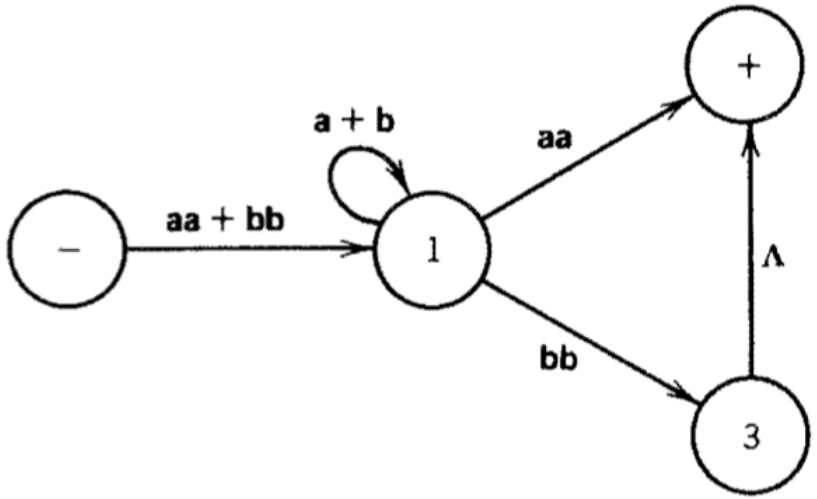


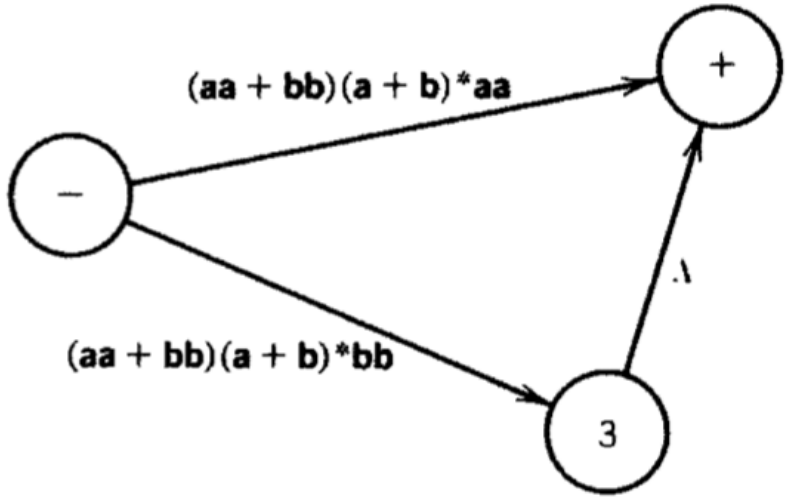




Example, continued

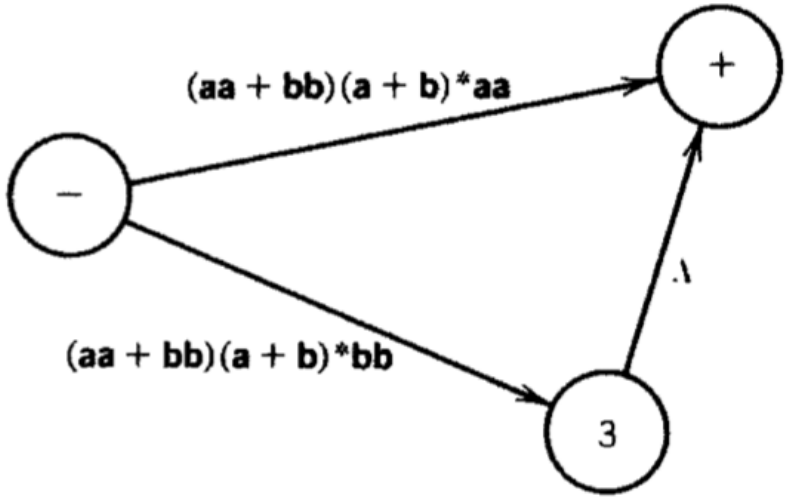
- Bypass state 1 next (before state 3)

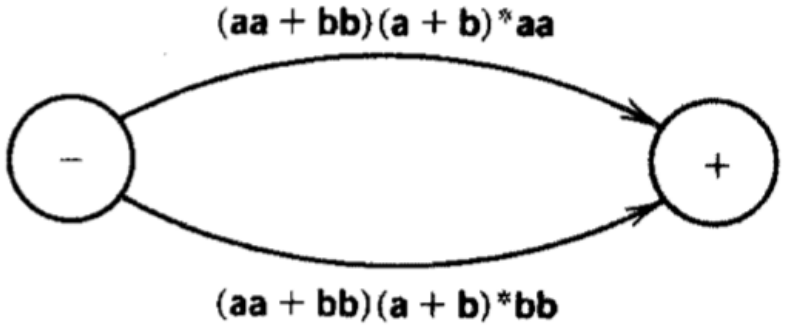




Example, continued

- Now we must eliminate state 3 (this is the only bypassable state left)





- This machine defines the same language as the RE
  - (**aa** + **bb**)(**a** + **b**)*(**aa**) + (**aa** + **bb**)(**a** + **b**)*(**bb**)