Convergence Across Castes*

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Abstract

India witnessed a sharp wage catch-up by the historically disadvantaged scheduled castes and tribes (SC/STs) towards non-SC/ST levels during the period 1983-2012. We develop a multi-sector, heterogenous agent model where individuals differ in innate ability as well as their caste identity. Castes differ in the costs of schooling and accessing sectoral labor markets which results in caste-based talent misallocations. We show that exogenous productivity growth can explain 72 percent of the observed wage convergence. Endogenous worker re-allocations can explain 39 percent of the overall labor productivity growth in India during this period. Education convergence is the primary driver of the wage convergence in the model. We provide independent evidence in support of this mechanism.

JEL Classification: J6, R2

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Introduction 1

A perennial challenge of managing the development process is to balance the macroeconomic goal of growth with the microeconomic goal of equity. This challenge often come to the fore during periods

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of rapid economic changes in growing economies. An example of this phenomenon is India over the past 30 years. This period has witnessed a rapid takeoff of the Indian economy with average annual growth rates doubling relative to the pre-1991 phase. Did growth lift all groups or were there tradeoffs? What are the mechanisms that linked the two?

We focus on the experience of Scheduled Castes and Scheduled Tribes (SC/STs) – an historically underprivileged section of Indian society.¹ SC/STs experienced a rapid wage catch-up towards non-SC/ST levels between 1983 and 2012 with the mean wage gap shrinking by 10.5 percentage points and the median wage gap declining by 14 percentage points. This wage convergence was accompanied by convergence in education attainment levels, occupation choices, and consumption levels (see Hnatkovska et al. (2012)). The goal of this paper is to assess the importance of growth in inducing the declining economic gaps between non-SC/STs and SC/STs.²

We develop a three-sector model (agriculture, manufacturing and services) of an economy with heterogenous agents. Agents differ along two dimensions. First, agents are different in their innate ability endowment which they all draw from a common ability distribution. Second, individuals in the model belong to one of two castes: non-SC/STs and SC/STs.

Castes in the model differ on two dimensions: (a) the cost of acquiring schooling; and (b) the cost of accessing sectoral labor markets. These cost differences imply that even though the ability distributions of individuals in the two castes are identical, there is a misallocation of ability which generate caste gaps in sectoral employment and wages.

Overall sectoral labor productivity depends both on exogenous productivity as well as worker allocations in education and sectors. Misallocations reduce equilibrium labor productivity while improvements in allocations improve productivity.

We use the model to quantitatively assess the effect of changes in sectoral productivities on schooling and sectoral misallocations during 1983-2012. Specifically, we examine the explanatory power of these productivity changes for the observed decline in the sectoral caste employment gaps, the sectoral caste wage gaps as well as the overall caste wage convergence observed in the data.

The key parameters of the model, including the caste-specific costs of schooling and sectoral labor market access, are calibrated to match the 1983 levels of the sectoral caste employment gaps,

¹SC/STs comprise a list of castes that have been listed in a schedule of the Indian constitution as being historically disadvantaged and consequently eligible for affirmative action programs. Crucially, caste identities are inherited by birth and hence immutable over time. Moreover, the extent of affirmative action reservations for SC/STs have also remained constant over time.

²There has also been sharp convergence in the intergenerational mobility rates in these three indicators (see Hnatkovska et al. (2013)).

sectoral caste wage gaps and the average education levels of the two castes. Our baseline calibration identifies higher schooling costs for SC/STs as the primary cause of the large sectoral caste gaps in employment and wages in $1983.^3$

Armed with the calibrated model for 1983, we conduct a sequence of quantitative experiments to examine the importance of productivity growth. Our experiments yield five key results. First, exogenous sectoral productivity growth during 1983-2012 induces a decrease in the caste wage gap in the model that is 72 percent of the decline in the data. Importantly, while the empirical and analytical focus of the paper is on *declining relative* caste wage gaps, we show that the model also reproduces the observed *increasing absolute* wage gaps between the groups during this period.

The model matches the observed thickening of the tails of the schooling distribution for the two castes during 1983-2012. Since schooling is the key underpinning of economic disparity in the model, we interpret this as evidence of the model's success in reproducing the changing heterogeneity within and across groups in India during this period.

Second, we find that a faster increase in the education attainment rates of SC/STs accounts for most of the wage convergence in the model. Intuitively, real costs of schooling decline with aggregate growth. Since SC/STs start with higher costs of schooling in the model, their schooling costs fall proportionately faster with growth. This sparks the relatively faster increase in SC/ST schooling and wages in the model.

We provide two independent pieces of evidence in support of this mechanism. First, using census data we show that while initial school provisioning was lower in SC/STs dominated villages in India in 1991, school provisioning increased relatively faster in SC/STs dominated villages during 1991-2011. This is suggestive evidence of a faster reduction in schooling costs for SC/STs. Second, we use DFL decompositions of the caste wage gaps to empirically show that education differences account for over 90 percent of the caste wage gaps in both 1983 and 2011-12. Hence, education convergence empirically accounts for most of the wage convergence, as in the model.

Third, the observed sectoral labor productivity growths induce dynamics of sectoral output shares and relative prices in the model that reproduce the patterns in the data. We view these aggregate features of the model as indicative of the model being a good fit to the data.

Fourth, sectoral labor productivity in the model depends on exogenous productivity as well

³Higher education costs for SC/STs might seem counterintuitive to the reader since India has had affirmative action programs for education since 1952. These programs provide reserved seats for SC/STs in colleges and universities. Education costs however depend on much more basic things like provisioning of primary, middle and secondary schools. We show below evidence of systematic under-provisioning of schools in SC/ST dominated geographical units, which provides support for SC/STs facing higher schooling costs despite the affirmative action programs.

as the endogenous sorting of differently endowed workers in terms of their schooling and sectoral employment choices. The caste distortion creates a misallocation of talent and reduces overall labor productivity. Our quantitative exercise finds that the endogenous reduction in caste-based talent misallocation in response to exogenous productivity growth accounts for 45 percent, 37 percent and 11 percent of the overall sectoral labor productivity growth in Agriculture, Manufacturing and Services, respectively. This amounts to 39 percent of the aggregate share-weighted labor productivity growth in the India during 1983-2012.

Fifth, the model allows us to compute the welfare costs of caste distortions. We do this by equalizing both schooling costs and sectoral entry costs across castes. Equalizing all schooling and sectoral entry costs across castes increases average per capita consumption by 10.2 percent in 1983 and 10.3 percent in 2012. Correspondingly, removing all caste distortions results in per capita output rising by 11.4 percent in 1983 and 8.5 percent in 2012. The gains for SC/STS are obviously larger than these overall numbers.

Overall, we interpret our results as suggesting that the rapid growth take-off in India over the past three decades has induced a dramatic narrowing of the historical economic disparities faced by SC/STs. The primary driver of this convergence has been the relatively faster increase in the education attainment rates of SC/STs.⁴

The paper is related to three distinct bodies of work. The first is the work on castes in India and their impact on economic outcomes. Aside from the contributions of Hnatkovska et al. (2012). and Hnatkovska et al. (2013) cited above, notable other contributors to this literature are Banerjee and Knight (1985), Madheswaran and Attewell (2007) and Borooah (2005) who examined the discrimination against SC/STs in labor markets in urban India. On a related theme, Ito (2009) studied labor market discrimination in two Indian states – Bihar and Uttar Pradesh. Exploring the theme of castes as networks, Munshi and Rosenzweig (2006) and Munshi and Rosenzweig (2016) show how caste networks impact labor mobility, education choices and employment.⁵

⁴Our work also evaluates the relative importance of two other features of the Indian economy. We assess the importance of job reservations for SC/STs in India. Our results suggest that this affirmative action policy may have lowered the *levels* of the caste wage gaps but likely did not qualitatively affect the *dynamics* of caste wage gaps between 1983 and 2012. We also find that the process of structural transformation that was also unfolding in the country during this period was important for the caste convergence. The shocks that changed the relative economic shares of the different sectors also changed the sectoral allocation of workers by caste thereby reducing the talent misallocation. Absent this sectoral churn, the misallocations would not have changed similarly.

⁵Another paper that is related to our work is Banerjee and Munshi (2004). They examined the differences between entrants belonging to the incumbent traditional community of *Gounders* in the garment industry in Tirupur in India in the early 1990s relative to entrants from other communities. They found evidence of sharp catch-up of capital and output of outsider firms to the levels of entrants from the Gounder community.

A second literature that is related to our work is the extensive work on structural transformation of countries along the development path wherein countries gradually switch their economic focus from agriculture to non-agricultural sectors. This is a voluminous literature that spans both empirical and theoretical work. Key contributions in this are are Matsuyama (1992), Kongsamut et al. (2001), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008). An excellent overview of this literature can be found in Herrendorf et al. (2014). We differ from this work in our focus on the distributional effects of the transformation.

Our work also relates to the literature that has examined the sources of productivity differences between countries. Two lines of research in this broad tent are closely connected with this paper. The first is the work on misallocation of talent by Hsieh et al. (2019) who analyze the consequences of misallocating talent by gender and race on productivity and growth in the USA. We share their interest in the implications of misallocating labor across sectors due to discrimination or other factors. A second branch of work in this area has focused on the role of occupation selection in accounting for income differences between rural and urban workers (see Young (2013)) and between agricultural and non-agricultural workers (see Lagakos and Waugh (2013)). This list is illustrative rather than being exhaustive.

The next section describes the key facts on caste economic gaps and structural transformation in India. Section 3 presents the model and some analytical results. Section 4 presents the calibration and quantitative results while Section 5 uncovers the main mechanism at play and some independent evidence in its support. In Section 6 we compute thee contribution of declining misallocation to productivity growth while Section 7 discusses our welfare results. Section 8 discusses issues related to affirmative action and structural transformation while the last section concludes.

2 Empirical regularities

Our data comes from different sources. The primary data source is the National Sample Survey (NSS) rounds 38 (1983), 43 (1987-88), 50 (1993-94), 55 (1999-2000), 61 (2003-04), 66 (2009-10) and 68 (2011-12).⁶ The NSS provides household-level data on approximately 600,000 individuals on education, employment, consumption, and wages as well as other social characteristics. We consider individuals between the ages 16-65 belonging to male-headed households who were not enrolled full time in any educational degree or diploma. The sample is restricted to those individuals who

⁶The 68th NSS round is the latest available and released by the Indian government.

provided their 4-digit industry of employment information as well as their education information.⁷ Our focus is on full-time working individuals who are defined as those that worked at least 2.5 days per week. This selection leaves us with a working sample of around 165,000-182,000 individuals, depending on the survey round. The wage data is more limited. This is primarily due to the prevalence of self-employed individuals in rural India who do not report wage income. As a result, the sub-sample with wage data is limited to about 48,000 individuals on average across rounds. Details on the data are contained in the Data Appendix to this paper.

We start by reporting some aggregate facts regarding the education and wage gaps between SC/STs and non-SC/STs since 1983. These facts are extensions of the results reported in Hnatkovska et al. (2012). Figure 1 reports the wage gaps between the castes. Panel (a) shows the mean wage gaps between the groups across the NSS rounds while panel (b) shows the corresponding median gaps. The solid lines depict the unconditional wage gaps while the dashed lines show the wage gaps after controlling for the age characteristics of workers.⁸ Both plots reveal an unambiguous pattern of wage convergence between the two groups since 1983, with the mean wage gap declining by 10.5% and the median gap falling by 14%.

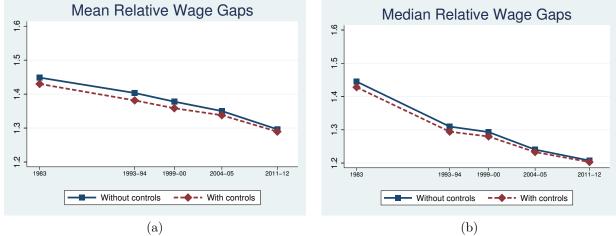


Figure 1: Wage gaps between castes

Notes: Panel (a) of this Figure presents the mean wage gaps between SC/STs and non-SC/STs (expressed as a ratio of non-SCST to SCST) from the 1983 to the 2011-12 NSS rounds. Panel (b) shows the corresponding median wage gaps. The dashed lines in the two panels show the computed wage gaps after controlling for the age characteristics of workers (age, age squared) while the solid lines are the gaps without such controls.

⁷We also consider a narrower sample in which we restrict the sample to only males and find that our results remain robust.

⁸Specifically, to obtain unconditional wage gaps we estimated an OLS regression (for mean) and a RIF regression (for median) of log wages on a constant and an SC/ST dummy. The conditional gaps are computed from the same regression with age and age squared controls.

Next we examine the education patterns of the two groups during this period. Figure 2 shows the relative gaps in the years of education between non-SC/STs and SC/STs. Panel (a) of the Figure shows the gaps for different age cohorts while panel (b) shows the corresponding gaps in the average years of schooling by birth cohorts.

Both panels reveal a pattern of convergence in education attainment rates between the two groups. Panel (a) shows that the education gap for every ten-year age cohort declined over time, suggesting that education attainment rates of SC/STs increased faster than that for non-SC/STs over time.

This impression from Panel (a) is corroborated by Panel (b) of Figure 2 which shows that the relative gap in years of education between the two groups was systematically lower for younger birth cohorts as compared to older birth cohorts. Thus, SC/STs born in later years had education attainment rates that were closer to their non-SC/ST peers. This is especially stark for the cohort born in 1989-1995 where the education gap has virtually disappeared.

The primary takeaway from Figure 2 is that the period 1983-2012 witnessed a sharp convergence of education attainment rates between SC/STs and non-SC/STs. Importantly, we show in Figure 5 below that this narrowing of caste educational gaps unfolded in a backdrop of overall increases in education attainment levels of the workforce across sectors.

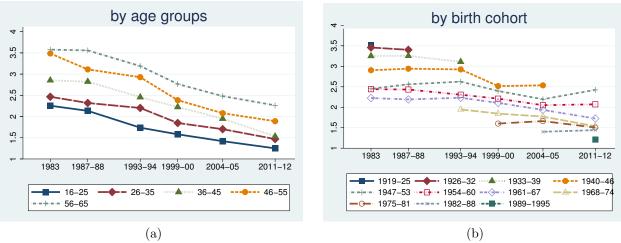


Figure 2: Gaps in years of education: overall and by birth cohorts

Notes: Panel (a) of this Figure shows the unconditional relative gap in average years of education (non-SCST/SCST) across the NSS rounds for different age cohorts while Panel (b) shows the gaps by birth cohorts.

Given the trends in Figures 1 and 2, the natural question to ask is how much of the wage convergence between the two groups is due to convergence in education attainment. Hnatkovska

et al. (2012) examined precisely this question and found that most of the wage convergence is, in fact, due to education convergence.

These trends, while interesting by themselves, raise the logical question about the deeper reasons behind the observed convergence between the groups during this period. While there may have been multiple factors operating simultaneously, in this paper we focus on the two biggest changes that occurred in the Indian economy during this period. First, 1983-2012 was a period of major changes in economic policy in India. There were large scale trade and industrial reforms carried out in the mid-1980s and in the 1990s. Economic growth in India took off from an average of around 3 percent during 1950-1985 to consistently being above 6 percent by the end of the 1990s.

Second, this period was also marked by a structural transformation of the economy. Figure 3 shows that the period 1983-2012 was marked by a gradual contraction in the traditional agricultural sector while the service sector expanded both in terms of its share of output as well as employment (there was an expansion in the manufacturing sector too but much more tepid relative to that of the service sector).⁹

Sectoral Employment Share Sectoral Output Share 9 100 8 8 9 9 4 40 8 20 1983 1993-94 2004-05 1983 2004-05 1993-94 1999-00 2011-12 1987-88 2011-12 1987-88 1999-00 Agri Manuf Serv Manuf Serv Agri (a) (b)

Figure 3: Industry distribution

Notes: Panel (a) of this Figure presents the distribution of workforce across three industry categories for different NSS rounds. Panel (b) presents distribution of output (measured in constant 1980-81 prices) across three industry categories.

1983-2012 was also a period with rapid growth in productivity at the aggregate and sectoral levels. Figure 4 reports labor productivity in each sector. Panel (a) is measured as output per worker computed from the national accounts data, while panel (b) reports the sectoral labor productivity

⁹In order to present the sectoral data facts, we combine 4-digit industry categories in the data into three broad categories: Agriculture, Manufacturing, and Services. See Appendix 10.1 for more details on the industry grouping.

numbers that are reported in the KLEMS dataset for India. All series are normalized by their values in 1983.



Figure 4: Sectoral labor productivity measures

Notes: Panel (a) of this Figure presents labor productivity, measured as GDP (in constant 1980-81 prices) divided by number of workers in each sector. Panel (b) shows the sectoral labor productivity computed from the KLEMS database for India. All series are normalized by their 1983 values.

Figure 4 shows that productivity growth across the three sectors, especially in the non-agricultural sectors, is a feature of both the national income accounts and KLEMS data. Both datasets also reveal a common rank-ordering of sectoral labor productivity growth during 1983-2012: manufacturing grew the fastest, followed by services, with agriculture being the slowest growing sector.

Sectoral education and wages also exhibited rapid but differential growth, with the two rising the fastest in services, followed by manufacturing and agriculture (see Figure 5).

So, how did this overall transformation of the economy affect the two social groups? Figure 6 reports the industry distribution of working individuals among SC/STs and non-SC/STs, and the relative gaps in this distribution. SC/STs were and remain more likely to be employed in agriculture and other farming activities than non-SC/STs. However the gap narrowed somewhat in the last ten years of our sample. The second largest industry of employment for both social groups is services, whose share has risen steadily over time. Interestingly, services also exhibits the sharpest convergence pattern between non-SC/STs and SC/STs. Specifically, the relative gap between non-SC/STs and SC/STs in employment shares in services has shrunk from 60 percent in 1983 to 21 percent in 2012. Manufacturing shows relatively little changes in the employment shares of the two groups over time.

Figures 7 reports the relative gaps in education attainments and median wages between non-

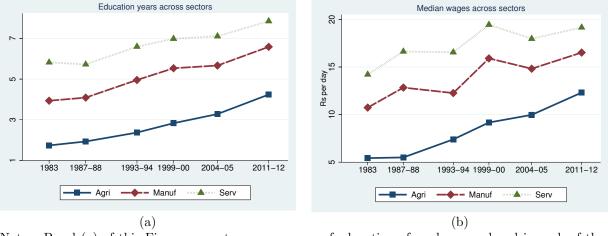


Figure 5: Education and wages by sector

Notes: Panel (a) of this Figure presents average years of education of workers employed in each of the three sectors. Panel (b) reports median wages in the three sectors.

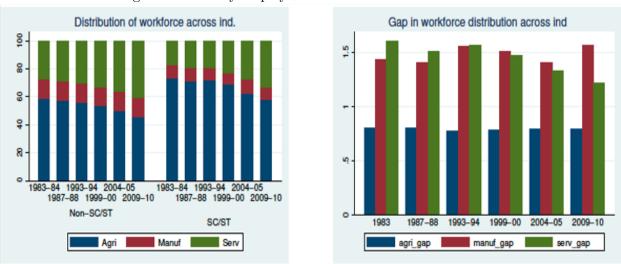


Figure 6: Industry employment distribution across castes

Notes: Panel (a) presents the distribution of workforce across the three industry categories for different NSS rounds. The left set of bars refers to non-SC/STs, while the right set is for SC/STs. Panel (b) presents the ratio of non-SC/STs to SC/STs shares reported in Panel (a) for each industry and year.

SC/STs and SC/STs employed in each sector. The education gaps have narrowed significantly over time between the two caste groups across all sectors. Median wage gaps, on the other hand declined in Services, stayed unchanged in Manufacturing, but widened somewhat in Agriculture.

To summarize the data features documented above, the period 1983-2012 was characterized by high aggregate growth in the economy, rising output per worker in all three sectors and productivity growth across the sectors. Concurrently, there was a gradual transformation of the economy with services becoming a larger share of the economy both in terms of output and employment, while

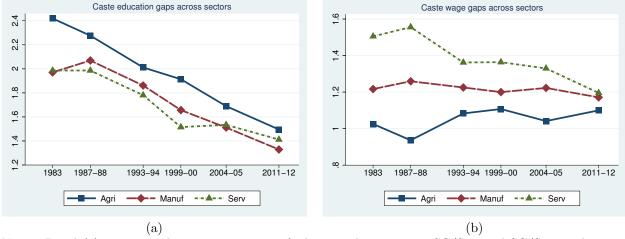


Figure 7: Education and wage gaps between non-SC/STs and SC/STs by sector

Notes: Panel (a) presents relative gap in years of education between non-SC/STs and SC/STs in three sectors. Panel (b) presents the ratio of non-SC/STs median wages to SC/STs median wages in three sectors.

the corresponding agriculture shares shrank.

In terms of the caste distributions, both SC/STs and non-SC/STs exited from agricultural employment during this period. The education gap between the castes declined in all three sectors. Moreover, while wages were converging *overall* between the castes, there were interesting contrasts in the patterns across the sectors. The wage convergence was strong in the service sector. The agricultural sector however saw a divergence in wages between the castes. Interestingly, the wage gaps in the manufacturing sector remained relatively stable over this period.

3 Model

We now ask whether productivity shocks can have a differential impact on the two groups and cause the education and wage gaps between the castes to fall? If so, what are the conditions under which that can happen? Would such an environment also induce sectoral outcomes that are consistent with the facts that we just outlined above, in particular the large changes in the caste labor gap in services?

Consider a one-period lived closed economy that is inhabited by a continuum of agents of measure L. A measure S of these agents belong to caste S (for scheduled castes and tribes or SC/STs) while a measure S belong to caste S for non-SC/ST.

Individuals belonging to different castes will be distinct along two margins: the cost of acquiring

schooling and the cost of accessing sectoral labor markets. We shall elaborate on each of these features below.

An agent i belonging to caste j = n, s maximizes utility from consumption of the final good

$$u\left(c_{ij}\right) = \frac{c_{ij}^{1-\rho}}{1-\rho}$$

Agents produce a final good by combining three intermediate goods using the technology

$$y_{ij} = \left(y_{ij}^a - \bar{y}\right)^{\theta} \left(y_{ij}^m\right)^{\eta} \left(y_{ij}^h\right)^{1-\theta-\eta}$$

where y^k is intermediate good k = a, m, h. In the following we shall refer to the a good as the agricultural good, the m good as the manufacturing good and the h good as the high skill good. \bar{y} is the minimum required level of the a good.

Intermediate goods are acquired by agent i using her income w_i . Specifically, an agent i of caste j = n, s with income w_{ij} chooses y^a, y^m, y^h to maximize production of the final good y subject to the budget constraint

$$p^a y_{ij}^a + p^m y_{ij}^m + p^h y_{ij}^h = w_{ij}$$

The optimal expenditures on intermediate goods by an agent i are 10 :

$$p^{a}y_{ij}^{a} = \theta (w_{ij} - p^{a}\bar{y}) + p^{a}\bar{y}$$
(3.1)

$$p^{m}y_{ij}^{m} = \eta (w_{ij} - p^{a}\bar{y})$$
(3.2)

$$p^{a}y_{ij}^{a} = (1 - \theta - \eta)(w_{ij} - p^{a}\bar{y})$$
(3.3)

Substituting the optimal intermediate goods purchases into the production function for the final good gives

$$y_{ij} = \frac{\theta^{\theta} \eta^{\eta} (1 - \theta - \eta)^{1 - \theta - \eta}}{p^{a\theta} p^{m\eta} p^{h(1 - \theta - \eta)}} (w_{ij} - p^{a} \bar{y})$$

We define the aggregate price index in this economy (the unit cost of producing the final good) as

$$P = \frac{(p^{a})^{\theta}(p^{m})^{\eta}(p^{h})^{1-\theta-\eta}}{\theta^{\theta}\eta^{\eta}(1-\theta-\eta)^{1-\theta-\eta}}$$
(3.4)

¹⁰The detailed derivations of these and other results below are provided in the Appendix that accompanies the paper.

Since we use the final good as the numeraire, with no loss of generality, we set P = 1 throughout the model. Hence, the optimal production of the final good by agent i belonging to caste j = n, s is

$$y_{ij} = w_{ij} - p^a \bar{y} \tag{3.5}$$

The non-homotheticity in production of the final good due to a minimum use of the agricultural good will be one source of structural transformation in the model.

3.1 Ability and Human Capital

Each agent is born with with an endowment of ability a_i and one unit of labor time that is supplied inelastically to the labor market. Ability is drawn from an i.i.d. process that follows the cumulative distribution function G(a), $a \in [\underline{a}, \overline{a}]$. The ability distribution is identical for both castes.

Ability is a productive input in building human capital. Human capital, in turn, determines the agent's labor productivity as well as the cost of accessing sector specific labor markets. Specifically, human capital of an agent i is determined by

$$e_{ij} = a_{ij}q_{ij}^{\chi}, \ \chi \in (0,1)$$
 (3.6)

where q denotes schooling acquired by the agent and χ denotes the schooling elasticity of human capital.

Acquiring human capital is expensive with the cost given by

$$E(q_{ij}) = \lambda_j q_{ij}$$

Note that the marginal cost of education, λ_j , j = n, s is constant and caste specific. This is the first difference between individuals belonging to different castes.

3.2 Human Capital and Sectoral Employment

An agent can work in any of the three sectors conditional on paying the entry costs of accessing those sectors. With no loss of generality, we normalize the entry cost in sector-a to zero. Access to sectors m and h however are costly. Agent i can access sector-k = m, h by spending f_{ij}^k units of the final good. Notice that this specification allows the sectoral entry costs to be caste specific.

In what follows we shall make the following assumptions:

Assumption 1:

$$f_{ij}^{k} = \begin{cases} 0, & k = a; \ j = n, s \\ \phi(\gamma_{j}^{k} - \alpha e_{ij}), & k = m, h; \ j = n, s \end{cases}$$

Assumption 2:
$$\gamma_j^h > \gamma_j^m, \quad j = n, s$$

Assumption 1 says that sectoral entry costs only apply for entry into sectors m and h. The entry costs have two components. The first, γ_j^k , is a fixed cost that is specific to sector and caste. The second component, αe_{ij} , is decreasing in the human capital of the individual but where the marginal effect of human capital on the entry cost is identical across castes. ϕ is a scaling factor that has no qualitative effect on the results but is useful for quantitative purposes.¹¹

Assumption 2 implies that the fixed cost of entry into sector-h is greater than that for entry into sector-m for both castes. This ensures an ability rank order where the highest ability individuals work in sector-h (which is consistent with the evidence on the sectoral distribution of human capital).

The preceding makes clear that there are two fundamental sources of differences across castes: the cost of education λ and the fixed costs of entry into sectors m and h. We shall explore the implications of these differences below.

3.3 Sectoral Production Technologies

The technologies for producing the three goods are all linear in the human capital of the worker. In particular, a worker with ability e_i supplying one unit of labor time to sector a produces

$$y_i^a = Ae_i$$

An m-sector worker with ability e_i produces the manufacturing good m according to

$$y_i^m = Me_i$$

¹¹The second component of the sectoral entry cost, αe_{ij} , is not required for any of the qualitative results on caste gaps that we derive below. However, we allow for this second term, which is independent of caste, to allow for the fact that schooling creates network of connections that is broader than the individual's immediate family and caste connections or networks.

Lastly, an h-sector worker with ability e_i produces the high skill good according to

$$y_i^h = He_i$$

Note that labor supply is inelastic and indivisible. So each worker supplies one unit of labor time to whichever sector she works in.

3.4 Sector and Schooling Choice

The decisions about which sector to work in and what human capital level to acquire are joint in this model since the schooling decision is contingent on the returns to human capital which, in turn, is dependent on the sector of employment of the worker since human capital impacts both the direct returns to work as well as the sectoral entry costs.

3.4.1 The schooling choice

An agent belonging to caste j = n, s who intends to work in sector-a will choose schooling q to maximize to maximize consumption:

$$c_{ij}^a = y_{ij}^a - \lambda_j q_{ij}$$

Similarly, an agent planning to work in sector-m will choose her schooling to maximize

$$c_{ij}^{m} = y_{ij}^{m} - \lambda_{j} q_{ij} - \phi \left(\gamma_{j}^{m} - \alpha a_{ij} q_{ij}^{\chi} \right)$$

while an agent headed for work in sector-h would choose schooling q to maximize

$$c_{ij}^{h} = y_{ij}^{h} - \lambda_{j} q_{ij} - \phi \left(\gamma_{j}^{h} - \alpha a_{ij} q_{ij}^{\chi} \right)$$

where $y_{ij}^k = w_{ij}^k - p^a \bar{y}, k = a, m, h$. w_{ij}^k denotes wages for the individual contingent on the sector that she chooses to work in. These sectoral wages are given by

$$w_{ij}^{k} = \begin{cases} p^{a} A a_{ij} \left(q_{ij}^{a}\right)^{\chi} & \text{if } i \text{ works in } a \\ p^{m} M a_{ij} \left(q_{ij}^{m}\right)^{\chi} & \text{if } i \text{ works in } m \\ p^{h} H a_{ij} \left(q_{ij}^{h}\right)^{\chi} & \text{if } i \text{ works in } h \end{cases}$$

Notice that the schooling choice contingent on working in sector k = a, m, h internalizes the effects of schooling on the sectoral entry costs.

The optimal schooling choices for an agent i belonging to caste j who chooses to work in sector-k = a, m, h are:

$$q_{ij}^a = \left(\frac{\chi a_{ij} p^a A}{\lambda_j}\right)^{\frac{1}{1-\chi}} \tag{3.7}$$

$$q_{ij}^{m} = \left(\frac{\chi a_{ij} \left(p^{m} M + \phi \alpha\right)}{\lambda_{j}}\right)^{\frac{1}{1-\chi}}$$
(3.8)

$$q_{ij}^{h} = \left(\frac{\chi a_{ij} \left(p^{h} H + \phi \alpha\right)}{\lambda_{j}}\right)^{\frac{1}{1-\chi}}$$
(3.9)

The optimal schooling functions above reflect two key features. First, within each sector higher ability agents acquire more schooling and hence, greater human capital. Second, controlling for ability, sectors with higher labor productivity will have workers with greater human capital since schooling is increasing in sectoral productivity.

3.4.2 Sectoral employment choice

The decision regarding the sector of employment is then based on choosing the sector associated with the highest consumption: $\max\left\{c_{ij}^a,c_{ij}^m,c_{ij}^h\right\}$ where c_{ij}^k denotes the consumption of an agent i of caste j working in sector k=a,m,h. Since both schooling and sectoral entry costs are paid out of the household final good, the household budget constraint dictates that $c_{ij}^k=y_{ij}^k-\lambda_jq_{ij}^k-f_{ij}^k$ where y_{ij}^k is given by equation 3.5 and f_{ij}^k is given by Assumption 1.

The sector-specific schooling levels in equations 3.7-3.9 above imply consumption levels for agents contingent on their decisions regarding schooling and sector of employment:

$$c_{ij}^{a} = (1 - \chi) \left(\frac{\chi}{\lambda_{j}}\right)^{\frac{\chi}{1 - \chi}} \left(a_{ij} p^{a} A\right)^{\frac{1}{1 - \chi}} - p^{a} \bar{y}$$

$$(3.10)$$

$$c_{ij}^{m} = (1 - \chi) \left(\frac{\chi}{\lambda_{j}}\right)^{\frac{\chi}{1 - \chi}} \left\{ a_{ij} \left(p^{m} M + \phi \alpha\right)\right\}^{\frac{1}{1 - \chi}} - \phi \gamma_{j}^{m} - p^{a} \bar{y}$$
(3.11)

$$c_{ij}^{h} = (1 - \chi) \left(\frac{\chi}{\lambda_{j}}\right)^{\frac{\chi}{1 - \chi}} \left\{ a_{ij} \left(p^{h} H + \phi \alpha \right) \right\}^{\frac{1}{1 - \chi}} - \phi \gamma_{j}^{h} - p^{a} \bar{y}$$
 (3.12)

As in the schooling decisions, consumption of agents is also increasing in their ability a within each sector. Note that the consumption levels associated with working in each sector are net of the

costs of schooling and sectoral entry costs since those are paid by the agent out of the household final good y_{ij} .

To describe the distribution of agents into the different sectors it is useful to define three ability thresholds:

$$\hat{a}_{j}^{m} = \left[\frac{\phi \gamma_{j}^{m}}{\left(1 - \chi\right) \left(\frac{\chi}{\lambda_{j}}\right)^{\frac{\chi}{1 - \chi}} \left\{ \left(p^{m}M + \phi\alpha\right)^{\frac{1}{1 - \chi}} - \left(p^{a}A\right)^{\frac{1}{1 - \chi}} \right\} \right]^{1 - \chi}$$
(3.13)

$$\hat{a}_{j}^{h} = \left[\frac{\phi \gamma_{j}^{h}}{(1-\chi) \left(\frac{\chi}{\lambda_{j}}\right)^{\frac{\chi}{1-\chi}} \left\{ (p^{h}H + \phi\alpha)^{\frac{1}{1-\chi}} - (p^{a}A)^{\frac{1}{1-\chi}} \right\}} \right]^{1-\chi}$$
(3.14)

$$\tilde{a}_{j}^{h} = \left[\frac{\phi(\gamma_{j}^{h} - \gamma_{j}^{m})}{(1 - \chi) \left(\frac{\chi}{\lambda_{j}}\right)^{\frac{\chi}{1 - \chi}} \left\{ (p^{h}H + \phi\alpha)^{\frac{1}{1 - \chi}} - (p^{m}M + \phi\alpha)^{\frac{1}{1 - \chi}} \right\}} \right]^{1 - \chi}$$
(3.15)

Equation 3.13 defines the threshold ability level \hat{a}^m for which consumption from working in sector-a is the same as consumption from working in sector-m, i.e., $c_{ij}^a = c_{ij}^m$. Hence, an agent with ability \hat{a}^m is indifferent between working in sector-a or sector-a or sector-a and a give the corresponding indifference between sectors-a and a, and between sectors a and a, respectively.

We now make the following assumption to provide greater structure to the cross-sectoral distribution of ability and skills that the model can generate:

Assumption 3: Parameter values guarantee $p^h H + \phi \alpha > p^m M + \phi \alpha > p^a A$

Assumption 3 is necessary (but not sufficient) for there to be a distribution of abilities across all three sectors. This will become clearer in the analysis below.

The thresholds along with Assumptions 1-3 allow a clear pairwise ranking of sectors for each ability type. This is summarized in the following Lemma:

Lemma 3.1. All individuals $i \in caste \ j = n, s$ with ability a_{ij} prefer employment in sector-m to employment in sector-a if $a_{ij} \ge \hat{a}_j^m$; employment in sector-a to sector-a if $a_{ij} \ge \hat{a}_j^h$; and employment in sector-a to sector-a if $a_{ij} \ge \hat{a}_j^h$.

3.4.3 Mapping Abilities to Sectors

How do agents get distributed across sectors in this economy? This depends on the relative rank ordering of the three thresholds \hat{a}_{j}^{m} , \hat{a}_{j}^{h} , and \tilde{a}^{h} . The following lemma is useful for characterizing the different possibilities:

Lemma 3.2. The rank order of the three ability thresholds are

$$\begin{split} \tilde{a}^{h}_{j} < \hat{a}^{h}_{j} < \hat{a}^{m}_{j} & \text{ if } \hat{a}^{h}_{j} = min[\hat{a}^{m}_{j}, \hat{a}^{h}_{j}] \\ \tilde{a}^{h}_{j} > \hat{a}^{h}_{j} > \hat{a}^{m}_{j} & \text{ if } \hat{a}^{h}_{j} = max[\hat{a}^{m}_{j}, \hat{a}^{h}_{j}] \end{split}$$

Proof. See Appendix.

Lemma 3.2 describes the relationship between the three thresholds in the model. Specifically, it says that \tilde{a}_{j}^{h} cannot lie in between \hat{a}_{j}^{m} and \hat{a}_{j}^{h} . Rather, it lies on the same side of \hat{a}_{j}^{m} as \hat{a}_{j}^{h} .

Since the model structure can give rise to $\hat{e}^h \geq \hat{e}^m$, the following Proposition characterizes the mapping of the abilities to sectoral employment under both these cases:

Proposition 3.1. (a) When $\hat{a}_{j}^{h} > \hat{a}_{j}^{m}$, j = n, s, the sectoral distribution of abilities is

$$a_i \in \begin{cases} [\underline{a}_j, \hat{a}_j^m) & : i \in A \\ [\hat{a}_j^m, \tilde{a}_j^h) & : i \in M \\ [\tilde{a}_j^h, \bar{a}_j] & : i \in H \end{cases}$$

b) When $\hat{a}_{j}^{h} < \hat{a}_{j}^{m}$, j = n, s, the sectoral distribution of abilities is

$$a_i \in \begin{cases} [\underline{a}_j, \hat{a}_j^h) & : i \in A \\ [\hat{a}_j^h, \hat{a}_j^m) & : i \in H \\ [\hat{a}_j^m, \bar{a}_j] & : i \in H \end{cases}$$

Proof. (a) When $\hat{a}_j^m < \hat{a}_j^h$, Lemma 3.2 says that we must have $\hat{a}_j^m < \hat{a}_j^h < \tilde{a}_j^h$. The distribution of ability types across the three sectors in this case follows directly from equations 3.13, 3.14, 3.15, and Lemma 3.1. Ability types below \hat{a}_j^m work in sector-a while those in between \hat{a}_j^m and \hat{a}_j^h choose sector-m. For ability types between \hat{a}_j^h and \tilde{a}_j^h , equation 3.15 implies that employment in sector-m is strictly preferred to sector-h. Those with ability above \tilde{a}_j^h choose to work in sector-h, which

follows directly from equation 3.15.

(b) When $\hat{a}_j^h < \hat{a}_j^m$, from Lemma 3.2 we have $\tilde{a}_j^h < \hat{a}_j^h < \hat{a}_j^m$. In this case, the distribution of ability types across sectors follows directly from equations 3.13-3.14 and Lemma 3.1. Ability types below \hat{a}_j^h strictly prefer employment in sector-a to both sectors h and m. For all ability types in caste j = n, s with $a \in [\hat{a}_j^h, \hat{a}_j^m)$, employment in sector-h dominates both sectors a and a. For $a \ge \hat{a}_j^m > \tilde{a}_j^h$, equation 3.13 says that sector-a dominates sector-a while equation 3.15 says that working in sector-a is strictly preferred by these types over sector-a employment.

While the message of Proposition 3.1 is self-explanatory, a comment on part (b), which describes allocations when $\hat{a}_{j}^{h} < \hat{a}_{j}^{m}$, is useful. The ability distribution described in Proposition 3.1 for this case implies that labor from both castes choose employment in either sector-a or sector-h, thereby rendering sector-m empty. This is clearly counterfactual since our data analysis revealed that both castes were employed in all three sectors. In the remainder of the paper we ignore this case and focus exclusively on parameter configurations such that $\hat{a}_{j}^{h} > \hat{a}_{j}^{m}$ for $j = n, s.^{12}$

3.5 Market clearing and Equilibrium

Markets for each good must clear individually. For the intermediate goods, this implies that total production must equal total demand for each good individually:

$$Y^{a} = L \left[s \int_{\underline{a}}^{\overline{a}} y_{is}^{a} dG(a) + n \int_{\underline{a}}^{\overline{a}} y_{in}^{a} dG(a) \right]$$

$$(3.16)$$

$$Y^{m} = L \left[s \int_{a}^{\bar{a}} y_{is}^{m} dG(a) + n \int_{a}^{\bar{a}} y_{in}^{m} dG(a) \right]$$

$$(3.17)$$

$$Y^{h} = L \left[s \int_{\underline{a}}^{\overline{a}} y_{is}^{h} dG(a) + n \int_{\underline{a}}^{\overline{a}} y_{in}^{h} dG(a) \right]$$

$$(3.18)$$

where Y^k denotes total production of intermediate good k = a, m, h. Note that in equations 3.16-3.18, sectoral output of individual i belonging to caste j = n, s whose ability is outside the relevant sectoral ability thresholds given in Proposition 3.1 will be zero.

Total production of the final good must equal the total demand for the final good:

$$C + Q + F = Y = L \left[s \int_{a}^{\bar{a}} y_{is} dG(a) + n \int_{a}^{\bar{a}} y_{in} dG(a) \right]$$
 (3.19)

where Q denotes total costs of schooling by all workers, F denotes the total skill acquisition costs

¹²The case $\hat{a}_j^h = \hat{a}_j^m = \tilde{a}_j^j$ is possible but clearly non-generic. Consequently, we ignore this pathological possibility.

incurred by workers employed in sector m and sector h, while Y denotes total production of the final good by all agents. The market clearing condition for the m good recognizes that part of the use of the good is for acquiring skills.

DEFINITION: The Walrasian equilibrium for this economy is a vector of prices $\{p_m, p_h\}$ and quantities $\{Y^a, Y^m, Y^h, C_s, C_n, Q_s, Q_n, F^m, F^h, \hat{a}_s^m, \hat{a}_n^h, \hat{a}_n^m, \hat{a}_n^h\}$ such that all worker-households satisfy their optimality conditions, budget constraints are satisfied and all markets clear.

3.6 Sectoral Labor and Wage Gaps Between Castes

It is useful at this stage to describe the caste labor gaps and wage gaps in the three sectors since those are a key object of interest. The precise expressions for these gaps depend on the specifics of the underlying distribution from which individuals draw their ability endowment. Throughout the rest of the paper we shall maintain the assumption that the ability distribution is uniform:

Assumption 4: The ability distribution G(a) is uniform on the support $[\underline{a}, \overline{a}]$.

The labor employment gap between caste n and caste s in sector k = a, m, h is the ratio of the fraction of caste n workers employed in sector k to the fraction of caste s workers employed in sector k. Under Assumption 4, these gaps are given by:

$$\Delta s^a = \frac{\hat{a}_n^m - \underline{a}}{\hat{a}_s^m - \underline{a}} \tag{3.20}$$

$$\Delta s^m = \frac{\tilde{a}_n^h - \hat{a}_n^m}{\tilde{a}_s^h - \hat{a}_s^m} \tag{3.21}$$

$$\Delta s^h = \frac{\bar{a} - \tilde{a}_n^h}{\bar{a} - \tilde{a}_s^h} \tag{3.22}$$

To derive the sectoral caste wage gaps from the model, note that the ability thresholds and the sector-contingent schooling choices given by equations 3.7-3.9 imply that the mean sectoral wages of agents belonging to caste j = n, s are

$$\begin{split} w_j^a &= (p^a A)^{\frac{1}{1-\chi}} \left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}} \int_{\underline{a}}^{\hat{a}_j^m} a^{\frac{1}{1-\chi}} \frac{dG(a)}{G(\hat{a}_j^m)} \\ w_j^m &= p^m M \left(p^m M + \phi \alpha\right)^{\frac{\chi}{1-\chi}} \left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}} \int_{\hat{a}_j^m}^{\tilde{a}_j^h} a^{\frac{1}{1-\chi}} \frac{dG(a)}{G(\tilde{a}_j^h) - G(\hat{a}_j^m)} \\ w_j^h &= p^h H \left(p^h H + \phi \alpha\right)^{\frac{\chi}{1-\chi}} \left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}} \int_{\tilde{a}_j^h}^{\bar{a}} a^{\frac{1}{1-\chi}} \frac{dG(a)}{1 - G(\tilde{a}_j^h)} \end{split}$$

Since the caste wage gap in sector k = a, m, h is the ratio of the mean wage of caste n relative to the mean wage of caste s in sector k, the sectoral caste wage gaps under Assumption 4 are given by:

$$\Delta w^{a} = \left(\frac{\lambda_{s}}{\lambda_{n}}\right)^{\frac{\chi}{1-\chi}} \left(\frac{\left(\hat{a}_{n}^{m}\right)^{\frac{1}{1-\chi}+1} - \left(\underline{a}\right)^{\frac{1}{1-\chi}+1}}{\left(\hat{a}_{s}^{m}\right)^{\frac{1}{1-\chi}+1} - \left(\underline{a}\right)^{\frac{1}{1-\chi}+1}}\right) \left(\frac{\hat{a}_{s}^{m} - \underline{a}}{\hat{a}_{n}^{m} - \underline{a}}\right)$$
(3.23)

$$\Delta w^{m} = \left(\frac{\lambda_{s}}{\lambda_{n}}\right)^{\frac{\chi}{1-\chi}} \left(\frac{(\tilde{a}_{n}^{h})^{\frac{1}{1-\chi}+1} - (\hat{a}_{n}^{m})^{\frac{1}{1-\chi}+1}}{(\tilde{a}_{n}^{h})^{\frac{1}{1-\chi}+1} - (\hat{a}_{n}^{m})^{\frac{1}{1-\chi}+1}}\right) \left(\frac{\tilde{a}_{s}^{h} - \hat{a}_{s}^{m}}{\tilde{a}_{n}^{h} - \hat{a}_{n}^{m}}\right)$$
(3.24)

$$\Delta w^{h} = \left(\frac{\lambda_{s}}{\lambda_{n}}\right)^{\frac{\chi}{1-\chi}} \left(\frac{\bar{a}^{\frac{1}{1-\chi}+1} - (\tilde{a}_{n}^{h})^{\frac{1}{1-\chi}+1}}{\bar{a}^{\frac{1}{1-\chi}+1} - (\tilde{a}_{s}^{h})^{\frac{1}{1-\chi}+1}}\right) \left(\frac{\bar{a} - \tilde{a}_{s}^{h}}{\bar{a} - \tilde{a}_{n}^{h}}\right)$$
(3.25)

where the thresholds \hat{a}_{i}^{m} , \tilde{a}_{i}^{h} are given by equations 3.13 and 3.15, respectively.

The wage and labor expressions above make clear that the key variables that determine the sectoral caste gaps in the model are the ability thresholds \hat{a}_{j}^{m} and \tilde{a}_{j}^{h} for j=n,s. The differences in the ability thresholds across the castes, in turn, depend on differences in schooling costs and sectoral entry costs. This follows directly from equations 3.13 and 3.15 which can be used to get

$$\frac{\hat{a}_n^m}{\hat{a}_s^m} = \left(\frac{\lambda_n}{\lambda_s}\right)^\chi \left(\frac{\gamma_n^m}{\gamma_s^m}\right)^{1-\chi} \tag{3.26}$$

$$\frac{\tilde{a}_n^h}{\tilde{a}_s^h} = \left(\frac{\lambda_n}{\lambda_s}\right)^{\chi} \left(\frac{\gamma_n^h - \gamma_n^m}{\gamma_s^h - \gamma_s^m}\right)^{1-\chi} \tag{3.27}$$

These results show that the ability thresholds as well as the education and employment distributions differ across the castes in the model despite members of the two castes drawing from the same ability distribution. These caste gaps arise due to differences in the costs of schooling and the sectoral entry fixed costs which are the only sources of difference across castes in the model.

4 A Quantitative Evaluation

We now turn to a quantitative implementation of the full version of the three-sector model. Specifically, we examine whether a calibrated version of the three sector model can explain the observed caste gap dynamics through the observed macroeconomic growth; and whether the caste education subsidization in India through reservations were crucial for the observed convergence.

The quantitative strategy of this section is to calibrate the model to the mimic the 1983 distribution of education, sectoral employment and sectoral wage of the two castes. Next, we identify the sectoral productivity changes between 1983 and 2012 by matching the change in sectoral labor productivities in the model with the corresponding changes in the sectoral output per unit labor reported in the National Income and Product Accounts data. We then feed the estimated paths of sectoral productivity into the calibrated model. The resulting distributional implications of the model at each date are then compared to the data in order to evaluate the explanatory power of aggregate productivity shocks for the caste wage gap dynamics.

Table 1: Calibration of Key Variables

VARIABLE	Value	VARIABLE	Value
<u>c</u>	0.5	θ	0.46
η	0.15	α	1
\underline{a}	1	\bar{a}	50
$\frac{M}{A}$ 1983	1.2	$\frac{H}{A}$ 1983	1.1
$\stackrel{\scriptstyle 111303}{L}$	1	$\overset{11300}{S}$	0.25

	Calibrated variables									
γ_s^m	20.1360	γ_s^h	299.1381							
$rac{\gamma_n^m}{\gamma_s^m}$	1.036	$\frac{\gamma_n^h\!-\!\gamma_n^m}{\gamma_s^h\!-\!\gamma_s^m}$	1.332							
$\frac{\lambda_s}{\lambda_n}$	1.55	ϕ	0.53							
λ_s	2.53	χ	0.61							

Notes: The table gives the parameters used for calibrating the model. The top panel lists the parameter values that were taken from other studies. The parameters in the bottom panel of the table were picked to match data moments from 1983.

Our focus is on eight key data moments for 1983: the three sectoral caste employment gaps; the three sectoral caste wage gaps; and the two average education levels \bar{q}_n and \bar{q}_s . Our calibration strategy is to match these eight data moments by choosing the following eight parameters: the sectoral entry cost parameters $\left(\gamma_s^m, \gamma_s^h, \frac{\gamma_n^m}{\gamma_s^m}, \frac{\gamma_n^h}{\gamma_s^h}\right)$, the two education cost parameters $\left(\frac{\lambda_n}{\lambda_s}, \lambda_s\right)$, the scaling parameter ϕ and the schooling elasticity of human capital χ .

Table 1 reports the key parameters. The upper panel of the table gives the parameters that were either normalizations or values that were taken from other studies. The numbers in the lower panel are the ones that were calibrated to match the moments of the 1983 caste distribution.

There are two features to note about the calibration parameters in Table 1. First, in order to match the sectoral caste gaps in 1983 the model demands that $\frac{\lambda_s}{\lambda_n} = 1.55$ so that the schooling costs for caste-s are 55 percent higher than that for caste-n. This feature allows the model to match the fact that SC/STs are over-represented in sector-a. The higher cost of schooling limits their access to the non-agricultural sectors.¹³

Second, matching the caste gaps in 1983 also requires the fixed costs of entry into sectors m and h to be lower for the disadvantaged caste-s. Intuitively, given the schooling gap between SC/STs and non-SC/STs, the model would predict counterfactually few SC/STs in the higher skill sectors. To match the sectoral employment distribution then, the model demands lower sectoral access costs for SC/STs. This feature is consistent with the presence of affirmative action programs that provide reservations for SC/STs in public sector jobs, which are mostly in the non-agricultural sectors.

Our quantification strategy is to freeze the calibrated parameters at the 1983 values and recompute the equilibrium by feeding in the identified change in the exogenous sectoral productivities A, M, H between 1983 and 2012. Note that since the model has no intrinsic dynamics, each new level for productivity generates a new equilibrium.

Given the specification of our model, one cannot compute the exogenous sectoral productivities from the sectoral labor productivities reported in the National Income accounts. In the model, agents endogenously acquire human capital through schooling and also choose their sector of employment. This educational and sectoral sorting impacts their productivity. Consequently, sectoral output per unit of sectoral labor would reflect the joint effects of exogenous sectoral productivity, endogenous human capital of the sectoral labor force and the endogenous sectoral sorting by workers. This is true both in the data and the model.

We approach the problem in a hybrid way. We first estimate sectoral productivities in 1983 by running sectoral Mincer wage regressions on five categories of education attainment of workers (primary, middle, secondary, college, diploma/technical) and a constant using the NSS employment/unemployment household survey for 1983. We use the constant in these sectoral wage regressions as our estimates of sectoral productivity in 1983. These numbers for relative sectoral productivities are reported in the top panel of Table 1.

Next, we identify the exogenous sectoral productivity growth between 1983 and 2012 in the data

 $^{^{13}}$ The higher schooling costs for SC/STs reduces the share of SC/ST who transit to the higher skill sectors, thereby raising the average ability of SC/STs in agriculture. However the lower levels of schooling of the SC/STs who remain in Agriculture lowers the labor productivity of SC/ST workers in Agriculture enough to allow the model to simultaneously generate $\Delta s^A < 1$ and $\Delta w^A > 1$.

by using the model. Specifically, to get the growth rates between 1983 and 2012 of the exogenous sectoral productivities A, M, and H, we first fix the calibrated parameters at 1983 level. We then pick the exogenous sectoral productivity growth rates such that the implied growth rates of sectoral output per worker between 1983 and 2012 in the model exactly match the corresponding growth rates in the data.¹⁴ This procedure yields the following sectoral productivity growth rates¹⁵:

$$g_A = 1.1436$$
 $g_M = 2.1421$ $g_H = 2.4068$

Table 2 shows the match between the eight targeted variables and their corresponding data values in 1983. The model clearly matches the rank order and magnitudes of the targeted moments for the sectoral caste gaps in both labor shares and wages gaps. It also does well in matching the mean education levels of the two castes in 1983, though the fit is not quite as precise as that for the six sectoral caste gaps. ¹⁶

How well does the model perform with respect to the non-targeted moments for the two castes in 1983? The bottom panel of Table 2 shows the fit of the model with respect to three non-targeted caste gaps. The first is the one that is the main object of the paper: the overall caste wage gap. The model generates a relative wage premium for non-SC/STs of 34 percent. Relative to the 45 percent non-SC/ST wage premium in the data, we consider the fit to be quite good.

The model allows for heterogeneity both within and across groups. To examine the fit of the model with regard to its predicted heterogeneity, we first fit a Pareto distribution to the years of schooling of agents separately for each caste in the NSS household survey data for 1983. We then do the same to the schooling outcomes in the model and compare the model with the data.

Table 2 reports the Pareto shape parameter estimated in the data and in the model for 1983. The model accurately generates thicker tails for the non-SC/ST education distribution relative to SC/STs. The quantitative fit of the shape parameter is very close for non-SC/STs but somewhat less so for SC/STs. We interpret this as evidence that the model performs well in matching the

$$Ey^k = \frac{p_{83}^k Y^k}{L^k} \quad k = a, m, h$$

where Y^k is given by (3.16)-(3.18), p_{83}^k is price levels at 1983, and L^k is the measure for employment in sector k.

¹⁴In our data analysis the labor productivity is computed as average output per worker for each sector in 1983 prices. It's model counter-part is then:

¹⁵The fact that the estimated agricultural labor productivity growth in India during 1983-2012 is the lowest amongst the three sectors is a pattern that is echoed also in the growth of overall sectoral output per worker during this period. We expand on these sectoral productivities and their implications for sectoral prices in Section 6 below.

¹⁶Schooling in the model is a continuous variable whereas in the data it is in years of schooling. To compare the schooling statistics in the model with the data, we normalize both the model and the data education series by de-meaning them. The statistics reported in Table 2 are computed using these de-meaned series.

Table 2: Data and Model Moments

		19	983	20	012
Variable	Notation	Data	Model	Data	Model
TARGETED	Moments				
Wage Gap Agriculture	Δw^a	1.04	1.04	1.08	1.05
Wage Gap Manufacture	Δw^m	1.20	1.20	1.14	1.20
Wage Gap Service	Δw^h	1.45	1.45	1.33	1.16
Labor Share Gap Agri	Δs^a	0.80	0.85	0.79	0.85
Labor Share Gap Manuf	Δs^m	1.43	1.43	1.57	2.15
Labor Share Gap Serv	Δs^h	1.61	1.60	1.21	1.32
Mean educ SC/ST	$ar{q}_s$	1.81	1.75	4.73	3.78
Mean educ Non-SC/ST	$ar{q}_n$	4.08	3.86	5.78	6.59
Non-Target	ed Momen	TS			
Total wage gap	Δw	1.45	1.34	1.30	1.24
Pareto shape para: Schooling SC/ST	k_s	0.57	0.77	1.33	1.19
Pareto shape para: Schooling Non-SC/ST	k_n	1.12	1.16	1.52	1.58
Notes: The top panel of the table reports the	e sectoral ca	ste gaps	in emplo	yment	

Notes: The top panel of the table reports the sectoral caste gaps in employment and wages with all gaps being the ratio of Non-SC/ST to SC/ST. The bottom panel reports the data and model generated of selected non-targeted moments.

observed schooling heterogeneity in 1983. This is important since schooling heterogeneity is the key for the economic heterogeneity in the model.

Having described the fit of the model to the data moments in 1983, we now examine its dynamic predictions for caste gaps. Table 2 also gives the labor and wage gaps across castes in the model and the data in 2012. The main takeaway from the Table is in the last row. In the data, the wage gap between non-SC/STs and SC/STs declined by 0.15/1.45 or 10.3 percent between 1983 and 2012. The corresponding reduction generated by the model is 7.5 percent. Thus, the baseline model can explain 72 percent of the observed decline in the percentage wage gap.

Underneath the success in reproducing the overall caste wage gap dynamics, the model also has qualitative and quantitative success in generating the observed dynamics of the caste gaps in both sectoral wages and employment shares. Thus, the agricultural wage gap marginally increased during 1983-2012 while the services wage gap decreased, both in the data and in the model. Correspondingly, the model reproduces the relatively unchanged agricultural labor share gap as well as the very sharp decline in the services labor share gap in the data. This last feature is particularly important since, as we showed in the decomposition exercise, the size of the change in the caste labor share gap in services was the largest amongst all the sectoral gaps.

Where the model does not perform well is in matching the dynamics of the labor share and

wage gaps in the manufacturing sector. In the data, the manufacturing labor gap widened by 10 percent while the model generates a 50 percent increase. The model also predicts an unchanged manufacturing wage gap while there was a marginal decrease in this gap in the data.

A key feature of the data is that there was a switch between the relative rank orders of the labor share gaps between manufacturing and services. While services had the largest caste gap in labor shares in 1983, by 2012 it was manufacturing that had the largest caste labor share gap. The model successfully reproduces this switch.

Table 2 also reports the change in the Pareto shape parameter for the schooling distribution of the two castes. Clearly, the model correctly matches the thickening tails of the schooling distribution for both castes, though with slightly more quantitative precision for non-SC/STs. We view this as evidence that sectoral productivity growth can account for a large part of the changes in the distribution of schooling outcomes in India since 1983.

4.1 Relative versus Absolute Convergence

The focal point of our paper is the convergence in relative wages between non-SC/STs and SC/STs. The focus on relative convergence is in keeping with the approach in the literature on inter-group inequalities which typically examines relative gaps between the groups of interest. Our focus on relative gaps is also consistent with the cross-country growth literature which looks at the relative income gaps across countries. We believe that issues of income inequality are best examined through the lens of relative gaps.

There is however, a parallel concern amongst some social scientists and policymakers about absolute inequality. Indeed, this is the reason why researchers sometimes use measures like the absolute Gini coefficient. Our model clearly has predictions for absolute wage convergence between castes. How do these predictions compare with the evidence on the behavior of absolute wage gaps between non-SC/STs and SC/STs during the period 1983-2012? Table 3 shows the change in the relative and absolute caste wage gaps in the data and in the model. We measure the relative caste wage gap at date t as w_{nt}/w_{st} and absolute caste wage gaps as $w_{nt} - w_{st}$. The table reports the percentage change between 1983 and 2012 in these two measures.

As the table shows, the 10.5 percent decrease in the relative caste wage gap during 1983-2012 in the data was accompanied by a 71.3 percent increase in the absolute wage gap between the two groups during the period. Reassuringly, the model reproduces this feature of the data as well by predicting a 96 percent rise in the absolute wage gap.

Table 3: Absolute versus Relative Convergence

	Change	e: 1983-2012
Variable	Data	Model
Relative wage gap	-10.5	-7.5
Absolute wage gap	71.3	96.0

Note: The table reports changes in the relative and absolute wage gaps between non-SC/STs and SC/STs during 1983-2012

We view the results in Table 3 as independent evidence in support of the model since it was not calibrated to target the absolute gaps either in 1983 or in 2012.

4.2 Sectoral prices and quantities

An independent test of the model is how well it fits the aggregate facts on prices and quantities. Table 4 shows the percent change in sectoral prices and quantities in the data and their model counterparts during the period 1983-2012.

Table 4: Sectoral Prices and Quantities

		Percent cha	nge 1983-2012
Variable	Notation	Data	Model
Relative price Agri	p^a	+20.6%	+21.3%
Relative price Manuf	p^m	-8.5%	+21.0%
Relative price Serv	p^h	-6.2%	-26.0%
Output Share Agri	y^a	-75.0%	-19.6%
Output Share Manuf	y^m	+28.6%	-20.0%
Output Share Serv	y^h	+126.7%	+30.8%

Notes: The table reports the percent changes of sectoral prices and quantities in the data and the model.

Two features of the results in Table 4 are noteworthy. First, the model does well in matching the dynamics of the relative prices and quantities of agriculture and services. The predicted dynamics of the agricultural relative price is particularly important in this context. As the Table shows, the relative price of agriculture actually rose during 1983-2012 in India. The model matches this fact. We view this as a particular strength of the model since standard models of structural transformation which generate a declining share of agriculture over time have difficulty in simultaneously generating a rising agricultural relative price. ¹⁷

¹⁷Standard models of structural transformation based on non-homothetic demand for the agricultural good predict

Second, the model encounters difficulties in reproducing the dynamics of the manufacturing sector, both in quantities and prices. It predicts an increase in the relative price of manufacturing and a decrease in its output share. Both are counterfactual. This aspect of the model is similar to its relative underperformance in matching the dynamics of the caste gaps in manufacturing.

5 Mechanism Underlying Convergence

The results above show that the model generates convergence in response to sectoral productivity shocks. What is the mechanism that generates this convergence? We investigate this issue by focusing on the two key margins that determine caste identities in the model. Recall that castes differ in the cost of schooling and the costs of accessing sectoral labor markets. These two costs induce a caste-specific sorting of agents into schooling and sectors which generates caste gaps in sectoral wages and employment. Changes in the caste wage gaps then are the result of differential changes in these caste specific schooling and labor market access costs which alter the schooling and sectoral choices by the two castes.

The two important cost parameters are the schooling cost λ and the entry cost parameter ϕ . Both of these are denominated in terms of the final good, and are *constant*. Hence, growth reduces the *real* costs of access to schooling and sectoral labor markets. The decline in these costs change the schooling and sectoral employment decisions of agents. Consequently, if these costs change at different rates for the two castes, then the sectoral caste gaps in employment and wages would change since the two groups would respond differently in their schooling and employment decisions.

We examine the individual importance of the schooling and labor market frictions by conducting two experiments. First, we scale the entry cost scaling parameter ϕ by the common growth factor. So, in this experiment, schooling costs become smaller due to growth but sectoral entry costs remain invariant. Second, we scale the schooling cost parameters λ_j , j = n, s by the common growth factor while leaving the entry cost parameter unscaled. Hence, in this case, the entry costs fall with growth but schooling costs remain invariant.¹⁸ The results are reported in Table 5.

The column "Scale entry cost" in Table 5 shows the percent changes in the predicted caste

that the relative price of agriculture declines in response to productivity growth since its demand rises less than proportionately with income. Models that generate structural transformation through inelastic elasticity of substitution across sectors predict that resources flow towards the slower growing non-agricultural sectors as their relative prices rise (see Ngai and Pissarides (2007)). But this is counterfactual in the Indian data during 1983-2012 when agriculture was the slowest growing sector.

 $^{^{18} \}text{In these experiments we scale the relevant costs using the common growth factor } \frac{A_{2012}^{\theta} M_{2012}^{\eta} H_{2012}^{1-\theta-\eta}}{A_{1983}^{\theta} M_{1983}^{\eta} H_{1983}^{1-\theta-\eta}}.$

	Scaling costs					
Variable	Data	Baseline		Scale schooling cost		
Δs^a	-1.25%	0.00%	0.00%	0.00%		
Δs^m	9.79%	50.35%	230.07%	36.36%		
Δs^h	-24.84%	-17.50%	-51.88%	5.00%		
Δw^a	3.85%	0.96%	0.00%	0.00%		
Δw^m	-5.00%	0.00%	10.83%	-0.01%		
Δw^h	-8.28%	-20.00%	-27.08%	0.00%		
Δw	-10.34%	-7.46%	-24.63%	5.22%		

Table 5: Schooling and Sectoral Re-sorting

employment and wage gaps in response to the measured productivity growth when entry costs are scaled but schooling cost remain unscaled. Correspondingly, the column "Scale schooling cost" shows the changes in the various caste gaps in response to productivity growth when schooling costs are scaled but sectoral entry cost are not. Relative to the baseline case, the results show that the fall in real schooling costs due to growth is key for generating the convergence in the overall caste wage gap. Indeed, the convergence in the overall mean wage gap is even larger in this case. On the other hand, column "Scaling entry cost" shows that having only entry costs decline while leaving schooling costs unchanged would actually induce an increase in the wage gap.

To understand these results, recall from equations 3.26 and 3.27 that the key determinants of the caste gaps are the ratios of ability thresholds which, in turn, are dependent on the relative costs of schooling and sectoral entry of the two castes. The calibrated schooling costs are proportionately greater for SC/STs in 1983. Hence, growth reduces schooling costs relatively faster for SC/STs. On the other hand, the calibrated sectoral entry costs are proportionately greater for non-SC/STs in 1983. Hence, growth reduces entry costs relatively faster for non-SC/STs.

Scaling entry costs alone prevents growth from reducing the relatively higher entry costs of non-SC/STs while the relatively higher schooling cost for SC/STs continues to decline with growth. Since SC/STs benefit more from this, the predicted wage convergence is greater in this case. On the other hand, scaling schooling costs alone while leaving entry costs unscaled switches the benefits of growth disproportionately towards non-SC/STS. Hence, the wage gap widens in this case. ¹⁹

These results indicate that the key driver of wage convergence in the model is the changing schooling levels of workers in response to the productivity shocks. Absent those improvements in schooling, the caste wage gaps would not only have declined less but may actually have widened.

¹⁹Scaling both costs leaves all caste gaps unchanged since neither threshold changes.

5.1 Evidence: Schooling costs, education and wage gaps

The calibration of the model to match the employment and wage gaps in 1983 required the cost of education to be higher for SC/STs relative to non-SC/STs in 1983. Moreover, we saw in Table 5 that the wage convergence in the calibrated model is due to the faster increase in schooling for SC/STs. This, in turn, is caused by growth causing a relatively faster reduction of real education costs for SC/STs in the model. We now provide independent pieces of evidence in support of these features of the model.

5.1.1 Schooling costs

Are schooling costs higher for SC/STS relative to non-SC/STs? To answer this question, we use Census data from India for 1991 and 2011 to examine the distribution of schools across towns and villages in India.²⁰ Our data comes from the SHRUG open data platform made available by the Development Data lab. Details about the data can be found in Asher et al. (2021).

We use the Census data to examine the differences between non-SC/ST, SC and ST dominated areas in the provisioning of public schools during these years. Note that we separate SCs and STs in these exercises since there is high caste segmentation in rural areas. Hence, separating SCs and STs provides greater clarity to the patterns.

We are especially interested in two questions: (a) were there fewer schools in SC/ST dominated villages and towns relative to non-SC/ST dominated area? (b) did school provisioning increase faster during 1991-2011 in SC/ST dominated areas relative to non-SC/ST areas? If the answers to these two questions are affirmative then it provides indicative evidence that schooling costs were indeed higher for SC/STs but that they also declined at a faster rate than the corresponding schooling costs for non-SC/STs.

Table 6 reports the key statistics on school provisioning. We follow Bailwal and Paul (2021) and define a village or town to be dominated by caste k if the majority of the population in the village or town belongs to caste k where k = Non - SC/ST, SC, ST.

There are two takeaways from Table 6. First, the top panel of the Table shows that in 1991 Non-SC/ST dominated geographic areas had a higher probability of having public schools of all types. School provisioning in SC and ST villages and towns has improved over time but the gap

²⁰Digitized census data for India are available from 1991 onwards. This precludes the evaluation of public school provisioning across Indian villages from 1981, which would have been closer to our household survey data start date of 1983.

	8									
		1991	[2011						
Area Dominance:	\overline{SC}	ST	non-SC/ST	SC	ST	non-SC/ST				
	Р	robability	y of having sch	school in the village or town						
Primary	0.56	0.55	0.71	0.76	0.83	0.84				
Middle	0.09	0.11	0.24	0.31	0.33	0.49				
Secondary	0.04	0.04	0.11	0.11	0.10	0.21				
Fraction of people having school in the village or town										
Primary	90.1%	84.1%	91.4%	95.5%	96.1%	96.1%				
Middle	45.9%	38.2%	52.9%	71.1%	59.6%	72.4%				
Secondary	27.8%	22.6%	33.8%	41.9%	32.6%	46.7%				
Obs	36,243	53,446	306,971	50,037	110,011	423,067				

Table 6: Provisioning of Public Schools

with Non-SC/ST areas still remained in 2011. ²¹

Second, the bottom panel of the Table reports the fraction of the different groups that live in areas that provide the various kinds of schools. As in the top panel, the results show that relative to non-SC/STs, a smaller fraction SCs and STs live in areas which have schools for all three categories of schools. Importantly, the gaps were much smaller relative to 1991 indicating a faster increase in school availability for SCs and STs as compared to non-SC/STs. ²²

We view these results as indicating that (a) relative to non-SC/STs, schooling costs were greater for SC/STs in 1991; and (b) schooling costs declined relatively faster for SC/STs during 1991-2011. We interpret these findings as being supportive evidence for the schooling cost calibration for 1983 as well as their faster decline for SC/STs over time. Recall that the latter is the key mechanism for the wage convergence in the model.²³

5.1.2 Education and wage gaps

We saw above that education re-sorting was the primary driver of the wage convergence in the model. Is there any independent empirical support for this prediction? We delve into this by

²¹We estimate these probabilities by running logit regressions of a binary (1,0) variable indicating availability of school of type j = Primary, Middle, Secondary in the town or village on a constant and dummy variables for SC and ST domination of the area.

²²Note that this measure is not tied to whether a village or town is SC/ST dominated or not. Instead, it directly measures the share of people that have school access where they live.

²³In related work, Bailwal and Paul (2021) examine the distance to the nearest public school from villages in India in 2001 and 2011 and find that (a) the distances to the nearest primary and middle schools are increasing in the village's SC and ST population shares; and (b) the positive correlation between distance to the nearest primary and middle schools and the SC/ST population share of the village declined between 2001 and 2011. While their sample period is different from our paper, nevertheless their finding (a) corroborates our calibration estimate of higher costs of schooling for SC/STs while their finding (b) provides support for a faster decrease in the cost of schooling for SC/STs during the sample.

conducting the DFL decompositions pioneered by DiNardo et al. (1996).

DFL decompositions follow a two-step procedure. It first estimates the wage densities of each group (SC/STs and non-SC/STS for us) separately using kernel density methods. It then constructs an alternative wage density for non-SC/STs by re-weighting the non-SC/ST wage density with the distribution of SC/ST attributes of interest.²⁴ One can then compute a percentile wage gap between the actual and counterfactual wage densities for non-SC/STs. This counterfactual percentile wage gap can then be compared with the percentile wage gap from the actual data to assess the role of the included attributes in accounting for the wage differences between the castes across the entire wage distribution. The closer the counterfactual wage gap to the wage gap in the data, the greater is the explanatory power of the included attributes.

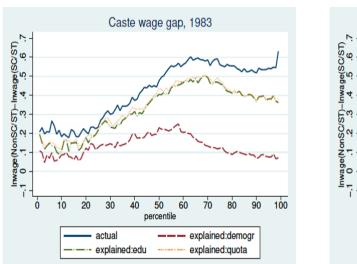


Figure 8: DFL Wage Decomposition

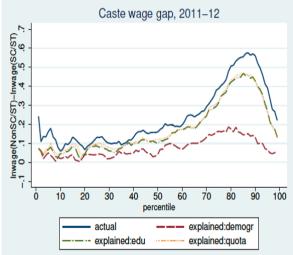


Figure 8 shows the DFL decompositions of the wage densities separately for 1983 and 2011-12 as well as the percentile wage gaps. For both years, we sequentially introduce three SC/ST attributes – demographic variables (like age, gender and location), education, and affirmative action benefits – to re-weight the non-SC/ST wage density. Note that introduction of a new attribute is in addition to the previously included attributes. Thus, in Figure 8, comparing the schedule labelled "explained:edu" with the one labelled "explained:demogr" gives the marginal effect of education relative to just demographics in explaining the caste wage gap.

There are two main takeaways from Figure 8. First, in both 1983 and 2011-12, education

 $^{^{24}}$ It is akin to replacing the X's in a wage regression of one group with the X's of the other group. Additional details regarding the DFL method can be found in DiNardo et al. (1996) and Hnatkovska et al. (2012).

explains over 90 percent of the wage difference between non-SC/STs and SC/STs across the entire distribution. In fact, neither demographics nor affirmative action account for much of the caste wage gap. Second, since the actual percentile wage gap shifted down in 2011-12 relative to 1983 in conjunction with the counterfactual percentile gap given by education, it follows that most of the caste wage convergence in the data is accounted for by education convergence between the groups.

Since the second result is exactly what the model generates, we interpret this as independent empirical evidence that is supportive of the model.

6 Misallocations and Productivity

A key aspect of our model is that labor productivity responds to both exogenous and endogenous factors. The endogenous response arises anytime agents change their schooling and sectoral employment decisions in response to exogenous shocks. This re-sorting changes the human capital of workers as well as the sectoral distribution of the human capital, both of which affect the sectoral and aggregate levels of labor productivity. Put differently, productivity affects talent misallocation but misallocation itself affects productivity in the model. How big is this latter effect? We turn to this question in this section.

We determined the *exogenous* sectoral labor productivity growth rates for 1983-2012 by calibrating them such that the sectoral labor productivity growth rates generated by the model exactly matched the corresponding growth rates in the data. Clearly, the difference between the overall sectoral labor productivity growth and the imputed exogenous sectoral labor productivity growth arises in the model due to endogenous changes in the education and sectoral sorting of workers.

As we saw above, worker re-sorting in the model occurs due to changing costs of schooling and sectoral employment which also change the talent misallocations and caste gaps. We evaluate the quantitative importance of the decreasing caste misallocation for labor productivity by comparing sectoral labor productivity growth rates under three scenarios: (i) sectoral entry costs are scaled to aggregate growth; (ii) schooling costs are scaled to aggregate growth; and (iii) both entry costs and schooling costs are scaled to growth. In each experiment, we hit the model with the imputed exogenous productivity growth rates in the baseline case.

Table 7 reports the sectoral productivity growths under the three scenarios as well as the numbers in the baseline case. Comparing the column "Scale both" with the last column shows that the overall and exogenous sectoral productivity growth rates become identical when both costs

are scaled. In this case there is no change in misallocations as the relative real costs of schooling and sectoral employment remain unchanged for the two castes. Hence, the difference between the exogenous sectoral productivity growth and the overall sectoral productivity growth in the baseline case (column "Baseline") is due to changing misallocations.

The numbers in Table 7 imply that declining misallocations due to endogenous educational and sectoral re-sorting account for 45 percent, 37 percent and 11 percent of overall labor productivity growth in Agriculture, Manufacturing and Services, respectively. If one weights these numbers by the sectoral share parameters θ , η and $1-\theta-\eta$, declining talent misallocation accounts for 39 percent of the labor productivity growth in India during 1983-2012. We view this to be quantitatively important.

Table 7: Changing Misallocations and Productivity Grow	Table 7	Changing	· Misallocations	and Productivity	Growth
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Scaling costs										
Variable	Data	Baseline	Scale entry	Scale educ	Scale Both	Exogenous Growth				
$\frac{Ey_{11}^a}{Ey_{83}^a}$	2.07	2.07	2.10	1.00	1.14	1.14				
$\frac{Ey_{11}^{\widetilde{m}}}{Ey_{83}^{m}}$	3.40	3.40	3.78	1.67	2.14	2.14				
$ \frac{Ey_{11}^a}{Ey_{83}^a} \\ \frac{Ey_{11}^m}{Ey_{83}^m} \\ \frac{Ey_{11}^m}{Ey_{11}^h} \\ \frac{Ey_{11}^h}{Ey_{83}^h} $	2.70	2.70	2.72	1.71	2.41	2.41				

7 Welfare Costs of Caste Distortions

The model that we have outlined has two sources of differences across castes: the costs of schooling and the costs of entry into sectoral labor markets. How expensive are these distortions? How much would SC/ST welfare rise if these distortions were removed? Would non-SC/STs gain as well? What would be the aggregate welfare gains?

In order to interpret the differences across castes in schooling and sectoral entry costs as distortions, we now provide a tax representation of these costs. Specifically, we define:

$$\lambda_s = \lambda_n + \tau_\lambda$$
$$\gamma_s^k = \gamma_n^k + \tau_\gamma^k, \quad k = m, h$$

where τ_{λ} is the tax on schooling and τ_{γ}^{k} , k=m,h is the tax on sectoral entry costs borne by SC/ST agents. Note that since $\gamma_{s}^{k} < \gamma_{n}^{k}$, k=m,h under our calibration in Table 1, $\tau_{\gamma}^{k} < 0$, k=m,h, i.e., affirmative action will act as a subsidy for SC/STs in accessing sectoral labor markets.

Using T_i to denote per capita public expenditure, the government's budget constraint is

$$L\left[s\int_{\underline{a}}^{\overline{a}} T_{i}dG(a_{i}) + n\int_{\underline{a}}^{\overline{a}} T_{i}dG(a_{i})\right] = L\left[s\int_{\underline{a}}^{\overline{a}} \tau_{\lambda}q_{i,s}^{*}dG(a_{i})\right] + L\left[s\int_{\hat{a}_{s}^{m}}^{\tilde{a}_{s}^{h}} \tau_{\gamma}^{m}\phi dG(a_{i}) + s\int_{\tilde{a}_{s}^{h}}^{\overline{a}} \tau_{\gamma}^{h}\phi dG(a_{i})\right]$$

$$(7.28)$$

where $q_{i,s}^*$ stands for the optimal choices of schooling given by equations 3.7-3.9.

This formulation of the cost differences as tax distortions leaves unchanged the production details of the economy since we retain the same calibrated $\lambda_n, \lambda_n, \gamma_n^k, \gamma_s^k$ as in Table 1. The consumption side of the model however does get affected by this reformulation. To see this, note that government expenditure could either be direct consumption by the government or they could be transfers from the government to private citizens. If government expenditures are lump-sum transfers to private agents then consumption of individual i becomes

$$\hat{c}_i = P(w_i - p^a \bar{y}) - \lambda_j q_i^* - E_i^* + T_i$$
(7.29)

where T_i is a lump-sum rebate made to each individual i. E_i^* is the optimal expenditure by agent i on sectoral entry.

When the taxes are consumed by the government instead of being rebated then private consumption is given by

$$\hat{c}_i = P(w_i - p^a \bar{y}) - \lambda_i q_i^* - E_i^* \tag{7.30}$$

To assess the welfare costs of caste distortions, we compare aggregate outcomes under the baseline case with two sets of counterfactual experiments: (a) equal sectoral entry costs for the two castes; and (b) equal schooling and sectoral entry costs. We conduct this comparison both with and without tax rebates. Note that since the two castes draw their ability endowments from the same distribution, equalizing both caste distortions (as in experiment (b) here) would eliminate all caste gaps.

Table 8 reports the results for the case when taxes are not rebated. The last column of the Table ("all equal") in the left panel (1983) shows the effect of removing all caste distortions. As

1983 2012 Variable Baseline γ 's equal all equal Baseline γ 's equal all equal C_s 101.29 98.22 160.1 226.78 223.29 349.0 C_n 160.00 160.27 346.45349.02 349.0 160.1C145.32 144.76 160.1 316.53317.59 349.0 Y_a 134.04 133.72 146.3287.97 288.91 310.6 Y_m 220.68 224.42 241.1 485.35 479.88 515.3 Y_h 293.97 288.55 325.61025.8 1035.57 1127.0 195.87 510.71 Y_f 194.73218.13512.50553.6

Table 8: Welfare Costs of Caste Distortions Under No Rebate

Notes: The table reports average consumption of each caste as well as per capita outputs of the sectoral and final goods under various parameter configurations for schooling and sectoral entry costs when taxes are not rebated to the public.

expected, equalizing all costs equalizes average consumption for both castes since they both draw from the same ability distribution. This translates into an increase in average consumption for SC/STs by 58.8 percent in 1983 and 53.9 percent in 2012. Interestingly, it also marginally raises the average consumption of non-SC/STs in both years. This occurs due to the rise in aggregate output that is induced by the removal of caste distortions. The resultant fall in the relative prices of the intermediate goods benefits non-SC/STs as well.

Aggregate output, Y_f , rises by 11.4 percent in 1983. This is the static gain from removing caste distortions. The corresponding output gain in 2012 is 8.4 percent. The increase in average per capita consumption, C, from removing all caste distortions in this economy is 10.2 percent in 1983 and 10.3 percent in 2012.

How do these estimates change when the caste taxes are rebated back to the public in the form of lump-sum transfers? Table 9 reports the results for average consumption in this case. Since the production side of the economy is unaffected by whether taxes are rebated or not, the output numbers in this case are identical to those in Table 8 above.

As one might expect, the tax rebate raises the average consumption of both castes in the distorted baseline economy relative to the no-rebate case. Outcomes when all distortions are removed however remain identical to those in Table 8. Consequently, the welfare gains for SC/STs from removing all distortions are now smaller in both years. The average per capita consumption gains for SC/STs is 44.3 percent in 1983 and 39.4 percent in 2012.

The interesting feature of the full rebate case is that removal of all distortions now does hurt

		1983		2012			
Variable	Baseline	γ 's equal	all equal	Baseline	γ 's equal	all equal	
C_s	110.94	107.48	160.1	250.33	245.39	349.0	
C_n	169.65	169.53	160.1	370.67	371.12	349.0	
C	154.97	154.01	160.1	340.58	339.69	349.0	

Table 9: Welfare Costs of Caste Distortions Under Lump-Sum Rebates

Notes: The table reports average consumption of each caste as well as per capita outputs of the sectoral and final goods under various parameter configurations for schooling and sectoral entry costs when taxes are fully rebated to the public.

non-SC/STs. Since non-SC/STs receive net positive transfers from SC/STs through the tax rebates under the distorted economy, the removal of all taxes reduces their net income. This effect is strong enough for removal of distortions to cause a reduction in the average consumption of non-SC/STs.

The main takeaway from these results is that there are significant welfare gains from removing caste distortions. These gains are very high for SC/STs who face the burden of the cost distortions in schooling and sectoral entry. Strikingly, in the realistic case of no tax rebates, reforms that remove all caste distortions also raise the welfare of non-SC/STs indicating that the reforms are Pareto improvements.

8 Discussion of Two Other Factors

Our approach to the issue of caste gaps has two features that require some additional discussion. The first is the role of the structural transformation that is built-in to the model. How important is that for the predicted wage convergence? The second is the importance of affirmative action policies for our results since they were constitutionally mandated in India since the early 1950s.

8.1 Caste Gaps and Structural Transformation

The baseline model has three mechanisms that induce structural transformation: (a) non-homothetic production technology; (b) differential sectoral productivity growth; and (c) unscaled schooling and sectoral entry costs. The first two are standard in many models of structural transformation. The last one is more specific to our model.

We examine the importance of macroeconomic structural transformation for the changes in the caste gaps by conducting a quantitative experiment where we remove all the ingredients in the model that induce structural transformation in response to productivity shocks. Thus, we compare the baseline model with one where we impose a common productivity growth on all three sectors (set at the rate of aggregate output growth), set $\bar{y} = 0$, and scale all the costs by making both λ_j , j = n, s and γ_j^k , k = m, h; j = n, s proportional to the growth rate of aggregate output.

Table 10: Role of Structural Transformation

Chang	Changes under common growth and scaled costs 1983-2012								
C	Caste Gaps		Aggr	egate Sectora	l Shares				
Variable	Baseline	$\bar{y} = 0$	Variable	Baseline	$\bar{y} = 0$				
Δs^a	0.15%	0.00%	SL^a	0.68 - 0.71	0.68 - 0.68				
Δs^m	50.23%	0.00%	SL^m	0.19 - 0.12	0.19 - 0.19				
Δs^h	-17.41%	0.00%	SL^h	0.13 - 0.17	0.13 - 0.13				
Δw^a	0.26%	0.00%	SY^a	0.46 - 0.37	0.46 - 0.46				
Δw^m	0.38%	0.00%	SY^m	0.15 - 0.12	0.15 - 0.15				
Δw^h	-19.96%	0.00%	SY^h	0.39 - 0.51	0.39 - 0.39				
Δw	-7.53%	0.00%							

Notes: The left panel of the table gives changes in the caste gaps in sectoral employment and wages between 1983 and 2012 in the baseline case and in the case when $\bar{y} = 0$, common sectoral growth rate, and scaled schooling and sectoral entry costs. The right panel gives the corresponding changes under the baseline case and in the case with $\bar{y} = 0$, common sectoral growth rates and scaled costs. SL^k denotes the employment share of sector k = a, m, h. SY^k denotes the output share of sector k = a, m, h.

Table 10 shows the results both for the baseline case and when we shut down all three forces driving structural transformation. The main takeaway from the Table is that without the conditions that generate structural transformation in the model, productivity changes have no impact on the caste employment and wage gaps. Intuitively, when all the sectors become proportional to aggregate growth, the ability thresholds for the two castes do not respond to changes in productivity since the rewards from switching sectors change at the same rate as the costs schooling and the cost of accessing those sectors.

This result is indicative of the importance of structural transformation in the Indian economy during 1983-2012 for understanding the dynamic evolution of the caste gaps during this period.

8.2 Affirmative Action Policies

The Indian constitution mandates reservations of seats in public institutions of tertiary education, in public sector employment and in political representation for SC/STs. How important were these

reservation policies for the observed caste convergence between 1983 and 2012?

Recall that our calibration of the model for 1983 dictated lower fixed costs of accessing manufacturing and service sector employment for SC/STs. We view these lower sectoral entry costs of SC/STs as the proxy for reservations in the model.

To examine the importance of reservations, we conducting three counterfactual simulations: (a) $\frac{\gamma_n^m}{\gamma_s^m} = 1$; (b) $\frac{\gamma_n^h}{\gamma_s^h} = 1$; and (c) $\frac{\gamma_n^m}{\gamma_s^m} = \frac{\gamma_n^h}{\gamma_s^h} = 1$. In all these experiments we leave γ_n^m and γ_n^h at their baseline levels. In other words, we raise the fixed cost component of sectoral entry costs for SC/STs to non-SC/STs levels in each sector thereby eliminating the advantage of reservations for SC/STs. All the other baseline calibration parameters are left unchanged. Table 11 shows the results.

	Table 11. Role of Amiliative Region									
	1983						2	2012		
Variable	Data	Baseline	γ^m	γ^h	both	Data	Baseline	γ^m	γ^h	both
Δs^a	0.80	0.85	0.84	0.85	0.84	0.79	0.85	0.84	0.85	0.84
Δs^m	1.43	1.43	1.54	0.79	0.84	1.57	2.15	2.54	0.77	0.84
Δs^h	1.61	1.60	1.58	93.37	82.78	1.21	1.33	1.31	3.90	3.84
Δw^a	1.04	1.04	1.01	1.04	1.01	1.08	1.05	1.01	1.05	1.01
Δw^m	1.20	1.20	1.18	1.01	1.00	1.14	1.20	1.18	1.02	1.00
Δw^h	1.45	1.44	1.45	1.26	1.26	1.33	1.16	1.16	1.02	1.02
Δw	1.45	1.34	1.31	1.62	1.58	1.30	1.24	1.22	1.33	1.31

Table 11: Role of Affirmative Action

Notes: The table reports the sectoral caste gaps in employment and wages in 1983 and 2012 in the data and under different special cases of the model. Δs^j , j=a,m,h, is the ratio of the fraction of all of all non-SC/STs working in sector j to the fraction of all SC/STs working in sector j. Δw^j , j=a,m,h is the ratio of the mean non-SC/ST to mean SC/ST wage in sector j. Δw is the ratio of the mean non-SC/ST to mean SC/ST wage.

The left panel of Table 11 shows the effects of equalizing the sectoral entry costs in 1983 while the right panel shows the corresponding effects in 2012. Comparing the column "both" with the "Baseline" column in the Table, one can see that when both sectoral entry costs are equalized, the overall wage gap in 1983 rises to 1.58 from the baseline level of 1.34. For 2012, the model generates an overall wage gap of 1.31 when both entry costs are equalized relative to the baseline of 1.24.

The main takeaway from these "equal costs" counterfactual experiments is that for given productivity levels, removal of reservations for SC/STs induce an increase in the caste wage gap at all dates. However, the dynamic effects of productivity growth remain qualitatively unchanged even without affirmative action. This second fact can be seen by comparing the caste wage gaps when

both costs are equalized in 1983 and 2012. We view this evidence as indicating that affirmative action policies were not the key driver of the observed caste wage convergence during 1983-2012.

9 Conclusion

The paper has examined the role of growth in accounting for the observed convergence in the education, occupation choices and wages of scheduled castes and tribes (SC/STs) in India toward the levels of non-SC/STs during 1983-2012.

We formalized a multi-sector, heterogenous agent model where all individuals draw their innate ability from the same ability distribution. However, the cost of acquiring schooling and the cost of accessing sectoral labor markets are different for individuals belonging to different castes. We examined the aggregate implications of the talent misallocations induced by these two caste based distortions in an environment with exogenous sectoral productivity growth.

Based on quantitative experiments on the model, we estimate that exogenous sectoral labor productivity growth can account for 72 percent of the observed wage convergence between SC/STs and non-SC/STs during the period 1983-2012. Decreasing caste gaps in service sector employment and wages are key for the overall convergence, both in the data and in the model. Importantly, our quantitative model matches the data dynamics of both the relative wage gap, whose narrowing is the focus of the paper, and the absolute caste wage gap, which widened during this period.

Equilibrium labor productivity in the model depends on exogenous productivity as well as the sorting of workers in education and sectoral employment. We find that the exogenous productivity growth induced a re-sorting of workers that reduced the caste-based talent misallocation. The resultant endogenous increase in labor productivity accounted for 44 percent, 37 percent and 11 percent of the overall sectoral labor productivity growth in Agriculture, Manufacturing and Services during 1983-2012. Clearly, the productivity payoffs of the declining misallocations were large.

The model estimates the output costs of caste distortions to be 11.4 percent in 1983 and 8.5 percent in 2012. The corresponding per capita consumption costs of caste distortions are 10.2 percent and 10.3 percent. In the realistic case where distortionary caste taxes are deadweight losses, we find that eliminating caste distortions are Pareto improving: average consumption of both SC/STs and non-SC/STs increase.

The main mechanism driving the caste convergence in the model is SC/ST workers increasing their education levels and switching employment into the higher paying service sector. Absent this re-sorting, the caste wage gap would have actually widened despite the productivity growth. We use census data to show that villages and towns that are dominated by SC/STs enjoyed a faster increase in provisioning of public schools between 1991 and 2001, providing independent evidence in support of the faster decline in the cost of schooling for SC/STs.

We also use household survey data to show that education attainment rates explain over 90 percent of the caste wage gaps in 1983 and 2012, as well as the wage convergence across the distribution during this period. This is independent evidence in support of the model.

Our results suggest that growth during 1983-2012 broke down millenia of caste-based socio-economic disparities.

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10 Appendix

10.1 Data

In the text we group these codes further into three broad industry categories: Ind 1 refers to Agriculture, Hunting, Forestry and Fishing; Ind 2 collects Manufacturing and Mining and Quarrying; Ind 3 refers to all Service industries. These groupings are detailed in Table 12.

Table 12: Industry categories

I., J.,	T. J.	O
Industry code	Industry description	Group
A	Agriculture, Hunting and Forestry	Ind 1
В	Fishing	Ind 1
$^{\mathrm{C}}$	Mining and Quarrying	Ind 2
D	Manufacturing	Ind 2
\mathbf{E}	Electricity, Gas and Water Supply	Ind 3
F	Construction	Ind 3
G	Wholesale and Retail Trade; Repair of Motor Vehicles,	Ind 3
	motorcycles and personal and household goods	
H	Hotels and Restaurants	Ind 3
I	Transport, Storage and Communications	Ind 3
J	Financial Intermediation	Ind 3
K	Real Estate, Renting and Business Activities	Ind 3
L	Public Administration and Defence; Compulsory Social Security	Ind 3
M	Education	Ind 3
N	Health and Social Work	Ind 3
O	Other Community, Social and Personal Service Activities	Ind 3
P	Private Households with Employed Persons	Ind 3
Q	Extra Territorial Organizations and Bodies	Ind 3

10.2 Proofs of Lemmas 3.1 and 3.2

In this section we sketch the proofs of **Lemma 3.1** and **Lemma 3.2** in the main text.²⁵ To ease notation, throughout this section we will use the definition:

$$\Psi_j = (1 - \chi) \left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1 - \chi}}$$

Lemma 3.1 All individuals $i \in caste \ j = n, s$ with ability a_{ij} prefer employment in sector-m to employment in sector-a if $a_{ij} \ge \hat{a}_j^m$; employment in sector-b to sector-a if $a_{ij} \ge \hat{a}_j^b$; and employment in sector-b to s

Proof. The agent will choose the sector that gives the highest c_{ij}^k . It is easy to see that the agent prefers sector a to m if and only if $c_{ij}^a \ge c_{ij}^m$. Similarly, she prefers a to h iff $c_{ij}^a \ge c_{ij}^h$ and m to h if

 $^{^{25}}$ An expanded Appendix with with details of the model solution is available from the authors upon request.

and only if $c_{ij}^m \ge c_{ij}^h$ where c_{ij}^a , c_{ij}^m and c_{ij}^h are given by equations 3.10, 3.11 and 3.12, respectively. We can rewrite these three conditions and define:

$$z_j^m(a_{ij}) \equiv \frac{\phi \gamma_j^m}{a_{ij}^{\frac{1}{1-\chi}}} \ge \Psi_j(p^m M + \phi \alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^a A)^{\frac{1}{1-\chi}}$$
(10.31)

$$z_j^h(a_{ij}) \equiv \frac{\phi \gamma_j^h}{a_{ij}^{\frac{1}{1-\chi}}} \ge \Psi_j(p^h H + \phi \alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^a A)^{\frac{1}{1-\chi}}$$
 (10.32)

$$z_j^h(a_{ij}) - z_j^m(a_{ij}) \equiv \frac{\phi(\gamma_j^h - \gamma_j^m)}{a_{ij}^{\frac{1}{1-\chi}}} \ge \Psi_j(p^h H + \phi \alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^m M + \phi \alpha)^{\frac{1}{1-\chi}}$$
(10.33)

With $0 < \chi < 1$, ϕ , $\gamma_j^k > 0$ and Assumption 2, it is obvious that $z_j^m(a_{ij})$, $z_j^h(a_{ij})$ and $z_j^h(a_{ij}) - z_j^m(a_{ij})$ are strictly decreasing in a_{ij} . Since $p^h H + \phi \alpha > p^m M + \phi \alpha > p^a A$ (Assumption 3), we have:

$$\begin{cases} c_{ij}^a \le c_{ij}^m & \text{iff } a_{ij} \ge \hat{a}_j^m \\ c_{ij}^a \le c_{ij}^h & \text{iff } a_{ij} \ge \hat{a}_j^h \\ c_{ij}^m \le c_{ij}^h & \text{iff } a_{ij} \ge \tilde{a}_j^h \end{cases}$$

Lemma 3.2: The rank order of the three ability thresholds are

$$\begin{split} \tilde{a}^h_j < \hat{a}^h_j < \hat{a}^m_j &\quad \text{if} \quad \hat{a}^h_j = \min[\hat{a}^m_j, \hat{a}^h_j] \\ \tilde{a}^h_j > \hat{a}^h_j > \hat{a}^m_j &\quad \text{if} \quad \hat{a}^h_j = \max[\hat{a}^m_j, \hat{a}^h_j] \end{split}$$

Proof. Consider first the case $\hat{a}^h_j < \hat{a}^m_j$. In this case, suppose $\tilde{a}^h_j > \hat{a}^h_j$. Using the definitions of \hat{a}^h_j and \tilde{a}^h_j from equations 3.14 and 3.15 above, $\tilde{a}^h_j > \hat{a}^h_j$ can be rewritten as

$$\left[\frac{\phi\gamma^{h}}{(1-\chi)\left(\frac{\chi}{\lambda_{j}}\right)^{\frac{\chi}{1-\chi}}\left\{\left(p^{h}H+\phi\alpha\right)^{\frac{1}{1-\chi}}-\left(p^{a}A\right)^{\frac{1}{1-\chi}}\right\}}\right]^{1-\chi} > \left[\frac{\phi\gamma^{m}}{(1-\chi)\left(\frac{\chi}{\lambda_{j}}\right)^{\frac{\chi}{1-\chi}}\left\{\left(p^{m}M+\phi\alpha\right)^{\frac{1}{1-\chi}}-\left(p^{a}A\right)^{\frac{1}{1-\chi}}\right\}}\right]^{1-\chi}$$

But this implies that $\hat{a}^h_j > \hat{a}^m_j$ which is a contradiction. Hence, if $\hat{a}^h_j < \hat{a}^m_j$ then $\tilde{a}^h_j < \hat{a}^h_j < \hat{a}^m_j$.

The other case $\hat{a}_j^h > \hat{e}_j^m$ but $\hat{a}_j^h > \tilde{a}_j^h$ leads to a contradiction by a similar logic. Hence, if $\hat{a}_i^h > \hat{a}_i^m$ then $\tilde{a}_i^h > \hat{a}_i^h > \hat{a}_i^m$.