Uncovering Subjective Models from Survey Expectations

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Abstract

Expectations about different macroeconomic aspects correlate with each other. I perform a structural test in framework of noisy information model and show that individual forms their expectations on multiple macroeconomic variables jointly rather than independently, thus causing these expectations to be correlated with each other. In particular, they have a subjective model about the economy. They believe economic conditions will be worse during episode with extensive inflation news, even if there's only mild inflation, causing their average expectation on inflation to co-move with that of unemployment and business condition. To alleviate the concern of possible mis-specification of linear noisy information model, I then propose an innovative generic learning model that can cover a large class of expectation formation models, including those are standard in the literature. The effect of signals on expectational variables is estimated with Recurrent Neural Network. I found again realized inflation increase household's perceived future unemployment rate change whereas actual unemployment rate hike will lower their expected inflation. The pessimistic effect of inflation is not because of household's belief on interest rate and is particularly strong after late 1990s. These patterns call for explanations on how agents form beliefs on interactions between macroeconomic variables that are different from the actual structure of data. They also suggest Central Bank should use inflation-related expectation management policy with cautious, as such policy may induce pessimistic responses among households.

Keywords: Expectation Formation, Noisy Information Model, Recurrent Neural Network, Survey Data

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1 Introduction

The last decade has seen a growing literature documenting behaviour that deviates from Full Information Rational Expectation (FIRE) theories. Most of these studies focus on inflation expectation (e.g. *Coibion and Gorodnichenko 2015, Malmendier and Nagel 2015* etc), or expectation on a single economic variable [1]. However there are few papers examine the link between expectations on different macroeconomics variables, despite the fact that in classical macroeconomics model cross-correlations between these variables are particularly important.

These correlations can offer us new insights on reasons of behaviours that deviate from FIRE. To be specific: (1) if agent forms their expectations on various aspects of macroeconomy using a collections of information they have, they may or may not understand how macroeconomic variables interact with each other in a way that is consistent with complicated modern macroeconomics model. In other words, they have a different "model" in mind. Then even if they have full information, they will form expectations that are different from FIRE benchmark; (2) if they do have the same "model" in mind, noisy information environment will induce correlations between expectations, which are absent in standard models under FIRE assumption.

Both these two possibilities are important to policy maker as current policy serves as signals to economic agents. For example, if an agent believes inflation is a signal of possible economic downturn, moving inflation expectation up in Zero Lower Bound (ZLB) episode may have additional contractionary effect than suggested in *Eggerson and Woodford (2003).* And indeed there are evidence suggesting inflation has a negative impact on household's consumption behaviour, especially in the ZLB episode[2].

In this paper, I examine correlations across expectations on a wide scope of macroeconomic variables, using Michigan Survey of Consumers (MSC) and Survey of Consumer Expectations (SCE). I find: (1) Agents believe higher unemployment rate and worse economic conditions are more likely to happen together with high inflation, a feature that is neither seen in realized data nor in Survey of Professional Forecast (SPF), and at the same time inconsistent with standard New Keynesian Model. (2) We also use the perceived news measure documented by MSC to show news heard or believed by consumers have significant yet different impact on their expectations. For example, agents heard of news about inflation are likely to believe in not only higher inflation, but also worse economic condition in the future; whereas bad news about labor market only affect agents' belief on unemployment condition. (3) The fraction of households report on particular type of perceived news is time-varying and heterogeneous across demographic groups.[3]

These new data patterns are hard to be explained by learning models where agents make inference about a single variable of interest. I then modify the noisy information framework to allow agents jointly form their expectations across different macroeconomic variables. The model then contains both interactions between the perceived hidden state variables and signal extraction problem, and I propose a test for the hypothesis that agents form expectations jointly rather than independently, on top of the test for deviation from FIRE as proposed by *Coibion and Gorodnichenko (2012).* As the correlation between expectations are most pronounced in consumer's expectation rather than professionals, I focus on two surveys of consumer expectations and find significant evidence for joint expectation formation.

I then take on the question of developing a model of agent forming expectation jointly using a large set of public and individual specific signals available to them. I employ an innovative

[1]For example *Barsky and Sims (2011), Doms and Morin (2004)* look at consumers' expectation on economic condition only.

[2]For example, *Bachmann et al.(2015)* and *Burke and Ozdagli (2013)* found expected inflation has negative impact, if any, on durable good consumption attitude for US consumers, and this impact is even more negative during Zero Lower Bound (ZLB) episode. *Coibion et. al. (2019)* found similar results from a field experiment in Netherland.

[3]We confirm and add more empirical patterns to the recent studies by *Bhandari et. al. (2018)* and *Kamdar (2019).*

generic learning model using Recurrent Neural Network(RNN) approach to model agent's learning and expectation formation behaviour. This method is different from the standard approaches used in the empirical literature using survey expectations, in which researchers typically assume linear Gaussian structures and specific ways of making statistical inference used by agents[4].

The need of using RNN comes from the fact that standard approaches are not designed to deal with the high-dimensional problem that I plan to deal with: the number of signals I consider are on the level of hundreds. Furthermore, the way signals affect household's expectation may be non-linear for multiple reasons, depending on what structural assumptions the researcher makes. For example, in the learning with experience model from *Malmendier and Nagel (2015)*, current realizations on inflation has non-linear impacts on expected inflation even though the authors assumed an AR process of inflation itself. Because it affects agents' posterior beliefs on the parameters, which is later multiplied with current inflation again. The non-linearity brought in by RNN is not restricted to function form itself, another important dimension it allows is the interaction between signals. For example, prices at gas pumps may not affect agent's expectation as much in good times than in bad times. This state-dependent response is not available in linear models, however it's particularly important in my analysis given the time-dependency and heterogeneity of the correlation I found from survey data.

It is important to note that, apart from the reasons above, using RNN rather than standard learning models is less restrictive. The RNN is robust to a wide range of true models agents may have employed to form their expectations. These models include the most commonly used ones: linear or extended Kalman Filters, Least Square Learning and Hidden Markov Model. It is also designed to work with sparse system, which can appear in expectation formation context if inattention behaviours are present. I will discuss the robustness of RNN approach to specific learning models using simulation in **Section 5.2**.

The results from RNN then can give us robust results on what signals affect households' expectations on various macroeconomic indicators significantly and how. The generic learning model based on RNN can also recover dynamic effects of signals on expectational variables without pre-specified latent variable structure and functional forms of agents' expectation formation process.

The paper is organized as follows: **Section 2** summarizes the literature related to this paper. **Section 3** organizes the empirical findings on cross-correlation between expectations on different variables. **Section 4** performs the test of joint expectation formation under the noisy information model. **Section 5** summarizes the structure and results of RNN and **Section 6** concludes.

2 Literature and Contribution

Two papers also examine the cross-correlations between household's expectations. In *Bhandari, Borovicka and Ho (2018)*, the authors explore the correlation between households' expected inflation and unemployment lookout. They also find that agents are making association between high inflation and bad economy condition in the future. My empirical findings confirm their results and add the time-varying pattern and heterogeneity of this correlation. I also explore a bigger set of expectational variables other than inflation and unemployment. They then explain these stylized facts with robustness preference with an uncertainty shock and focused more on the General Equilibrium consequences of the shock. Whereas I focus more on learning and expectation formation part where agent's expectations are responding to consequences of economic activities rather than their model mis-specification concerns.

[4]For example, noisy information model assumes agent perceived law of motion to be linear Gaussian and agent makes inferences of hidden states using Baynes Rule with conjugate priors to ensure stationarity and a clean reduced form representation; where as Constant Gain Learning first proposed by *Honkapohja and Evens (2001)* and its variations assume agents update beliefs about parameters in a linear model by running an OLS recursively.

My paper is more close to *Kamdar (2019)*, in which the author explored the same set of correlations and proposed a rational inattention model in which agent optimally chooses a signal as linear combination of fundamentals. My paper first serves as an empirical test for their model as after choosing the signal, agents face a signal extraction problem similar to that from noisy information model. The results suggest a signal as in *Kamdar (2019)* alone is not enough to explain the observed stylized facts in survey data. I then turn to explore what are the possible signals that trigger agents to jointly adjust their beliefs on various economic variables, and how demographic heterogeneity affects the impact of these signals.

While contributing to the new literature on cross-correlations between expectational variables, this paper also contributes to the empirical literature on imperfect or rigid information. This literature considers structure from noisy-information model[5] or information rigidity model[6] and perform tests using the implied structure of forecasting error and forecast revisions. However most of these tests focus on a single expectational variable. For example, *Coibion and Gorodnichenko (2012,2015), Andrade and Le Bihan (2013)* and many other researches found evidence of deviation from FIRE in a set of expectational variables, but in a framework that individual form expectation on these variables separately. In my paper I extend these tests to allow for interactions between perceived states as well as signals that are believed to contain information about multiple hidden states. I found significant evidence in support of joint expectation formation and this implies the results from single variable frameworks may suffer from omitting variable problems. These implications will be discussed in Section 4.

Despite most of the empirical work on expectation formation has been done using aggregate level data. There is also a growing literature in learning and behavioural macroeconomics using micro-level survey data. My paper also contributes to this literature on explaining the heterogeneity in agents' expectations. In this literature various models are used. For example *Bordalo et. al. (2019)* combined the standard noisy information model with diagnostic expectation formation model[7] to explain the over-reaction of individual forecast in SPF and the under-reaction in consensus expectations. *Malmendier and Nagel (2015)* instead employed a Least Square Learning model to capture the heterogeneity in inflation expectation from MSC. In this paper I use an innovative RNN approach to avoid making strong assumptions about agents' information set and statistical properties of their expectation formation behaviour. In stead the features of various sequential learning models will be captured by the RNN due to its flexibility in approximating non-linear mappings and state-dependency between input(signals) and output(expectations).

The last strand of literature this paper relates to is the empirical study on how various information sources affect household expectation. This literature dated back to *Carroll(2003)* , in which the author shows that information flows from SPF to household expectation. In the follow-up work, *Ehrmann, Pfajfar and Santoro (2015)* found news media reports on inflation lower the bias of household's inflation expectation. Apart from inflation expectation, *Dorms and Morin (2004)* found volume and tones of news media reports also affect consumer's sentiment about future. My paper explores how public signals including news media, public statistics, professionals' opinions as well as Central Bank communication affect households' expectations on a broad scale of variables. The flexibility of RNN allows for sparse responses to the rich set of information as well as state dependent and asymmetric impacts on expectations. The dynamic learning framework also allows me to explore the persistent effects of specific signals and assess the their relative importance.

[5]Woodford (2002) and Sims(2003)

[6]Mankiw and Reis (2002)

[7]Bordalo et. al. (2018)

3 Cross-correlation between Expectational Variables

In this section I show the cross-correlation structure across various expectational variables. For data on these expectations I use Reuters/Michigan Survey of Consumers (MSC) and Survey of Consumer Expectation (SCE) from Federal Reserve Bank of New York. As MSC is available for a longer period and has a wider range of questions on household expectation, I use it as the baseline results and show it is robust to the use of SCE over the same period when it's available.

3.1 Aggregate Time Series

I first show the simultaneous correlation across consensus expectations from MSC, all the expectational variables are average of individual expectations within the quarter. Table 1 summarizes the cross correlation (the simultaneous one) between these expectational variables.

Table 1: Correlation MCS: 1978q1-2017q4

	(1)	(2)	(3)	(4)	(5)	(6)
(1) inflation ($E\pi_{t+4,t}$)	1.00	0.30^{***}	-0.11	-0.42^{***}	0.21^{***}	-0.50^{***}
(2) unemp change ($E\Delta un_{t+4,t}$)		1.00	-0.37^{***}	-0.64^{***}	0.03	-0.26^{***}
(3) interest rate change ($E\Delta i_{t+4,t}$)			1.00	0.38^{***}	0.16^{**}	0.04
(4) Busi Condition change ($E\Delta y_{t+4,t}$)				1.00	0.5^{***}	0.77^{***}
(5) nominal income change ($E\Delta W_{t+4,t}$)					1.00	0.62^{***}
(6) real income change ($E\Delta w_{t+4,t}$)						1.00

*** means significant at 1%,** means 5 % and * means 10%, data in use are quarterly from MSC.

Table 1 shows that most of the consensus(mean) expectations on different variables are highly correlated, which is not surprising itself as the corresponding macroeconomic variables show systematic co-movement as well. However, the positive correlation between expected inflation and expected unemployment increase suggests agents believe future inflation will occur together with unemployment rate increase. This is consistent with the negative correlation between expected inflation and better business condition in a year. These results are also documented by *Bhandari, Borovicka and Ho (2018)* and *Kamdar (2019)* and they conclude these correlations are caused by consumer's sentiment that affect all aspects of their expectations. [8]

However there can be various different reasons for the cross-correlation I observe in expectational variables, it doesn't necessarily mean agents are forming expectations jointly. If agents are making predictions using adaptive learning or rational expectation models, we will also see a cross-correlation structure of their expectations and it should be similar to that of realized variables. I then use the real time statistics from St. Louis FED to construct the real-time counterpart of Table

[8] There are some other interesting findings in these cross-correlations. For example, I also find agents expect interest rate fall together with unemployment increase, which may suggest they understand the role of monetary policy in stimulating economy, as found in *Dräger et. al. (2016)*. I also find agents understand the idea of real versus nominal income as they expect inflation go up together with an increase of their nominal income, on the other hand they do believe the nominal income will increase less, so that their real wage will eventually fall.

1[9]. Table 2 reports the same cross-correlation structure of the realized macroeconomic variables.

Table 2: Correlation FRED: 1978q1-2017q4

	(1)	(2)	(3)	(4)	(5)	(6)
(1) CPI	1.00	0.09	0.39^{***}	-0.03	0.63^{***}	-0.28^{***}
(2) Δun		1.00	-0.52^{***}	-0.79^{***}	-0.53^{***}	0.08
(3) ΔFFR			1.00	0.43^{***}	0.55^{**}	-0.18
(4) $\Delta RGDP$				1.00	0.61^{***}	0.12
(5) ΔW					1.00	0.16^{**}
(6) Δw						1.00

*** *means significant at 1%,* ** *means 5 % and* * *means 10%, data in use are quarterly from MSC.*

Table 2 shows that as measured by real time data, inflation is not significantly correlated with either change of unemployment or economic growth, as opposed to the case with expectational variables. Meanwhile the actual data also suggests unemployment change is not correlated with real income dynamics, this may be due to the fact nominal wage and price level decrease together in episodes of recessions which leaves the real income to be relative stable across good and bad times. However this link seems to be missing in expectational data as well. These patterns suggest the cross correlation structure of expectational variables is hard to be reconciled with rational expectation or adaptive learning models, as both suggest expectations should be closely linked with realized data, so that expectational variables should have similar correlation structure as realized ones.

Another interesting fact one may notice is the correlation between inflation and unemployment change is around 0 instead of being negative. This seems to be contradicting the existence of Phillips Curve relationship between inflation and unemployment rate. One explanation for this question is the correlation between inflation and unemployment is time varying, if one starts the sample for post 1981 period, excluding the very end of the stagflation episode in the 1970s, the Phillips Curve-like negative correlation will show up more significantly. However, I want to point out this correlation I document is not directly comparable to a Phillips Curve relation because I'm using year to year unemployment rate change rather than a gap that measures economic slackness which is typically used in modern Phillips Curve analysis.

Through the comparison between Table 1 and 2, it is clear that the cross correlation structure of households' expectation differs from that of the realized data. And these differences lie mainly on the three key variables in modern macroeconomics model: inflation, unemployment and interest rate. Furthermore, as the aggregate survey data other than expected inflation are constructed from categorical responses thus not directly comparable with its real time counter-part. I then follow *Bhandari, Borovicka and Ho (2018)* and *Mankiw et al. (2003)* to impute the expectational series[10].I confirmed that the same cross-correlation structure remains for imputed series, and robust to use

[9]I use year to year change of Consumer Price Index for all unban consumers as inflation; Federal Funds Rate for interest rate; real GDP growth for Business Condition; compensation for employee received for nominal income and real compensation per hour as real income. For comparability everything except inflation are year to year change.

[10]The imputation method involves use of SPF. As the CPI series from SPF starts from 1981 and it doesn't contain expectation on income, I can only obtain imputed series for change of unemployment rate, interest rate and real GDP growth.

of SCE and monthly data. Meanwhile similar correlation doesn't appear in SPF. These results and the imputation approach are discussed in **Appendix A**.

3.2 Individual-level Cross-correlation

In this section, I examine whether individual respondents in household surveys make similar association. This will help me to understand whether the patterns in aggregate level data have a micro-level foundation or they are mainly coming from aggregation process. Various former literature suggests properties of consensus expectation may differ from those of individual expectations[11].

In the noisy information context related to this paper, if agents separately form expectation on various subjects, their individual forecasts may not exhibit similar correlation as seen in consensus forecasts. The aggregate correlation then is induced by a time-specific common bias in signals of these economic variables. To rule out this possibility, I estimate a fixed effect model (1) with time dummies using panels dimension of MSC and SCE. I focus on expected unemployment change as well as interest rate change, and report their conditional correlations with expected inflation.

$$E_{i,t}\pi_{t+12,t} = \beta_0 + \beta_1 E_{i,t}un_{t+12,t} + \beta_2 E_{i,t}i_{t+12,t} + \theta X_{i,t} + D_t + \mu_i + \epsilon_{i,t} \tag{1}$$

In the fixed effect model (1), $X_{i,t}$ includes controls such as expectations on other subjects and social economic status, when using SCE I also include individual information such as employment status and individual uncertainty about inflation forecasts. [12] Table 3 reports the results from panel data.

Table 3: FE Panel Regression

coef	MSC 1980m7-2017m12	coef	SCE 2013m6-2018m7
Unemployment up	0.30***	$\hat{\beta}_1$	0.012***
	(0.05)		(0.0018)
Unemployment down	−0.22***		
	(0.05)		
Interest rate up	0.53***	$\hat{\beta}_1$	−0.002
	(0.06)		(0.0014)
Interest rate down	−0.49***		
	(0.04)		
FE	Y		Y
Time dummy	Y		Y

Table 3 column 1 shows that for MSC, an agent that expects unemployment to go up will predict inflation to be 0.3% higher on average than one that believes unemployment be stable; and 0.52% higher than one that believes unemployment rate will fall. Meanwhile the standard deviation of expected inflation across this episode is 1.17%, and the standard deviation of CPI is around 2.19%. These results are comparable to those from *Kamdar (2019)*, where the author estimates a similar fixed effect model but only on correlation between expected inflation and unemployment

[11] For instance, *Coibion and Gorodnichienko (2015)* suggests the predictability of forecasting error from forecast revision is an emergence property of aggregation across individuals and may not be seen at individual level; *Bordalo et. al. (2019)* documents over-reaction of inflation expectation to new information on individual level, in contrary to under-reaction typically found with consensus expectations.

[12] When using MSC, the expected unemployment and interest rate change are categorical variables, and I construct dummies that stand for "increase" or "decrease" for each of these variables. In SCE those variables are reported percentage points for the likelihood of corresponding variable to increase.

change, without controlling for other expectational variables. The estimate shown in column 2 from SCE is consistent with that from MSC: if agent expects there is a 22% chance (which is the standard deviation of the variable) unemployment rate will increase in 12 months, he will also expect inflation to be 0.22% higher. It's worth noting the controls of fixed and time effect means the positive correlation between unemployment and inflation is not due to a common time varying bias, which should have been captured by the time fixed effect; and is not due to the effect of "pessimistic individuals" which is taken out by individual fixed effects.

4 Test of Joint Expectation Formation

From last section we see significant reduced-form cross-correlations between households' expectational variables. Although for most macroeconomic aspects households are having cross correlations consistent with the realized data, indicating an understanding of major macroeconomic comovements, yet they constantly believe economic performance will be worse when there is concern about future inflation. This stylized effect is specific to household expectations and exists on both individual and aggregate level. And it's not due to time specific or individual specific factors. This distinction between expectation and reality gives rise to the possibility of a joint learning model. However, it is not sufficient to distinguish such a model from one where agent learns about each variable independently. For this reason, I develop a test on joint expectation formation under the framework of noisy information model that is most commonly used in the empirical literature with survey data on expectations.

The noisy information model has long history dated back to *Lucas (1972)* and the recent version was proposed by *Woodford (2003), Sims (2003)*. It is then widely adopted for tests on information friction and deviation from FIRE assumptions. For example, *Coibion and Gorodnichienko (2012)* shows existence of imperfect(noisy) information implies predictability of forecasting errors and provides evidence of imperfect information using consensus expectations of consumers, professionals and policy makers; similarly *Andrade and Le Bihan (2013)* provides evidence in support of information friction in ECB professional forecasts. More recently, researchers have focused on estimating implied structure of noisy information model with individual level data[13].

However all of these empirical tests are assuming for each variable agent tries to predict, the filtering and updating process is done independently from other variables the same agent wants to predict. This serves as an extra assumption when agents try to predict more than one outcomes of the future at the same time, which is usually the case in daily life and in survey environment. I call such a model "joint expectation formation" model, in which agent form expectations on multiple variables using the same set of information. In this section I follow the baseline noisy information model in the literature and allow for joint expectation formation and test whether household surveys indicate agents form expectation jointly rather than independently.

Consider the Actual Law of Motion(ALM) takes the form of state-space representation of multiple macroeconomic variables $\boldsymbol{L}_{t+1,t}$ as in (2). and agents observe noisy signals on these variables, the observation equation is given by (3).

$$\boldsymbol{L}_{t+1,t} = A\boldsymbol{L}_{t,t-1} + w_{t+1,t} \tag{2}$$

$$\boldsymbol{s}^i_t = G\boldsymbol{L}_{t,t-1} + v^i_t + \eta_t \tag{3}$$

Agents face four different channels of imperfect information: 1. the correct list of state variables $\boldsymbol{L}$ that they need to perceive; 2. the function form of ALM (linear in this case); 3. the

[13]For inflation expectation of US consumers, see *Ryngaet (2017)*; for expectations on various subjects of SPF, see *Bordalo et. al. (2019)*.

correct structural parameters in (2) and (3); 4. observability of $\boldsymbol{L}_{t,t-1}$. Most of noisy information framework assumes the only source of imperfect information comes from not observing $\boldsymbol{L}_{t,t-1}$ perfectly. Recently researchers also consider the possibility of mis-specified parameters in the context of forecasting inflation, and found households use a different persistence parameter than ALM in predicting inflation[14].

In this section, I follow the existing literature and assume the two sources of imperfect information are imperfect observation of $\boldsymbol{L}_{t,t-1}$ and possibly mis-specified parameters. Specifically, I allow agents to have a subjective model, which is possibly mis-specified, where they use $\hat{A}$ and $\hat{G}$ in place of $\boldsymbol{A}$ and $\boldsymbol{G}$. Their Perceived Law of Motion (PLM) then can be expressed as:

$$\begin{aligned}\boldsymbol{L}_{t+1,t} &= \hat{A}\boldsymbol{L}_{t,t-1} + w_{t+1,t} \\ \boldsymbol{s}^i_t &= \hat{G}\boldsymbol{L}_{t,t-1} + v^i_t + \eta_t\end{aligned} \tag{4}$$

It is obvious that $\hat{A}$ represents households' subjective model about the economy. In the single-variable expectation formation context, this usually means agents mis-perceived the persistence of state variable. In a joint expectation formation model, a $\hat{A}$ that is different from A suggests agent believe in cross-correlation between macroeconomic variables that is different from actual data or models economists use. Intuitively, either $\hat{A}$ or $\hat{G}$ can then help to explain the difference of cross-correlation structure between survey expectations and actual data. Furthermore, they stands for different reasons why we see such a discrepancy: a mis-specified model of economy from $\hat{A}$, or a noisy environment represented by $\hat{G}$. The test I propose in this section will shed lights on these two mechanisms, I leave the discussion to Section 4.3.

In the joint learning model I also allow for an individual specific noise v^i_t as well as a time specific one η_t, both of which follow a normal distribution with mean zero. The individual noise is independent across agent and time and the time specific noise is not auto correlated and independent with structural shock $w_{t+1,t}$. Each element in v^i_t, η_t and $w_{t+1,t}$ are also assumed to be independent with each other for simplicity. Adding a time specific noise doesn't change the nature of individual signal extraction problem, the only difference it makes is to allow for imprecise signal after aggregation at each time point.

The agents then update their beliefs upon observing s^i_t and form expectation according to a linear Kalman Filter as described in (5), where K is the stationary Kalman Gain, $\boldsymbol{L}^i_{t+h,t-s|t}$ stands for the mean of agent i's expectation for $\boldsymbol{L}_{t+h,t-s}$ formed at time t. For derivation of standard Kalman Filter please see **Appendix B**.

$$\begin{aligned}\boldsymbol{L}^i_{t+1,t|t} &= \hat{A}\boldsymbol{L}^i_{t,t-1|t} \\ &= \hat{A}(\boldsymbol{L}^i_{t,t-1|t-1} + K(\boldsymbol{s}^i_t - \hat{G}\boldsymbol{L}^i_{t,t-1|t-1}))\end{aligned} \tag{5}$$

Extend this formula to a forecast at horizon of h rather than 1, and replace the observation equation into (5) we get:

$$\begin{aligned}\boldsymbol{L}^i_{t+h,t|t} \equiv \sum_{j=0}^{h-1} \boldsymbol{L}^i_{t+h-j,t+h-j-1|t} &= \sum_{j=0}^{h-1} \hat{A}^j \boldsymbol{L}^i_{t+1,t|t} \\ &= \sum_{j=0}^{h-1} \hat{A}^j [\hat{A}(I - K\hat{G})\boldsymbol{L}^i_{t,t-1|t-1} + \hat{A}KG\boldsymbol{L}_{t,t-1} + \hat{A}Kv^i_t + \hat{A}K\eta_t]\end{aligned} \tag{6}$$

The above equation means that if there is no imperfect information, then $v^i_t + \eta_t = 0$ and G is identity matrix. These imply K is just identity matrix and we go back to the full information rational

[14] For example, see *Ryngaet (2017)*

expectation case, where coefficients on all elements in $\boldsymbol{L}^i_{t,t-1|t-1}$ be 0. With information friction there should be additional dependence on lag forecasts after controlling for current realization. The tests on forecasting error then follows from the derivation, for example, in the case of $h = 1$:

$$\begin{aligned}
FE^i_{t+1,t|t} &\equiv \boldsymbol{L}_{t+1,t} - \boldsymbol{L}^i_{t+1,t|t} \\
&= A\boldsymbol{L}_{t,t-1} - [\hat{A}(I - K\hat{G})\boldsymbol{L}^i_{t,t-1|t-1} + \hat{A}KG\boldsymbol{L}_{t,t-1} + \hat{A}K(v^i_t + \eta_t)] + w_{t+1,t} \\
&= \hat{A}(I - K\hat{G})(\boldsymbol{L}_{t,t-1} - \boldsymbol{L}^i_{t,t-1|t-1}) + \underbrace{(A - \hat{A}KG - \hat{A}(I - K\hat{G}))}_{M}\boldsymbol{L}_{t,t-1} + w_{t+1,t} - \hat{A}K(v^i_t + \eta_t) \\
&= \hat{A}(I - K\hat{G})FE^i_{t,t-1|t-1} + M\boldsymbol{L}_{t,t-1} + w_{t+1,t} - \hat{A}K(v^i_t + \eta_t) \qquad (7)
\end{aligned}$$

Averaging across agents i at each time t we get an aggregate test on forecasting errors as $w_{t+1,t}$ and η_t are independent with $w_{t-h+1,t-h}$ and η_{t-h} for any $h = 1, 2, ...$

The standard single-variable noisy information model used in the empirical literature then features: (1) $\hat{A}$ is diagonal with persistence parameters on the diagonal; (2) $\hat{G}$ is diagonal and usually identity matrix with same dimension as number of hidden variables. If all three of the previous assumptions hold, then the Kalman Gain K is a diagonal matrix, this formulation collapses to the single-variable noisy information model on each variable in $\boldsymbol{L}$ and one can perform the forecasting error tests separately for each variable, which is done in *Coibion and Gorodnichenko (2012)* and *Andrade and Le Bihan (2013).*

Joint expectation formation occurs when either $\hat{A}$ or $\hat{G}$ is not diagonal, or neither are diagonal. Off-diagonal elements of $\hat{A}$ means cross-correlation between state variables $\boldsymbol{L}_{t+1,t}$, and those in $\hat{G}$ matrix means the existence of signals that are jointly created by multiple state variables[15]. The test then is simply regressing current forecast errors of each macro variable on lag forecast errors of itself (own-lag) and other expectational variables (other-lags) and test whether the estimators are significantly different from zero. This test is available conditional on presence of information friction, there are three sets of possible results: (1) If estimates on own-lag and some other-lags are all significantly different from zero, it suggests a joint expectation formation model is used; (2) if estimates on own-lag is significant yet other-lags are not, it suggests single-variable noisy information model is used; (3) if estimates on all lagged forecasting errors are insignificant, it supports full information rational expectation framework. In **Appendix B** I include experiment with simulated data to illustrate the three cases and the corresponding test results.

4.1 Test Results from MSC

I then perform the test for joint expectation as described above, using consensus expectation from MSC. I focus on two variables to be forecast: inflation and unemployment rate change.

One complication to perform the test is that it requires unemployment rate change in comparable levels as the realized data, whereas the data in MSC on unemployment expectation is categorical. I perform the same transformation as mentioned in **Section 3** following *Bhandari, Borovicka and Ho (2018)* and *Mankiw et al. (2003)*, the details are included in **Appendix A**. It is worth noting here the assumptions essential to recover unemployment expectation is the predicted unemployment change follows normal distribution with a constant variance across time. These assumptions are particularly plausible in the framework of noisy information model with stationary Kalman Filter, as the posterior distribution of forecasted variables are normally distributed and stationarity guarantees a time-invariant posterior variance.

[15]Notice here the structure of *Kamdar (2019) after signal selection is a special case of G being non-diagonal. In her model the signal is a linear combination of unemployment rate and inflation. The test in this section provides empirical evidence on possibility of G being non-diagonal, but is not sufficient to justify her structure as the only possibility.*

Another modification I need to do is rewriting (7) using the forecast errors at time horizon 4, corresponding to the year ahead expectation asked in MSC. This can be done using (6) and the fact $\boldsymbol{L}_{t+h,t} \equiv \sum_{j=0}^{h-1} \boldsymbol{L}_{t+1+j,t+j}$, and the implied estimatable model becomes:

$$
\begin{aligned}
FE_{t+4,t|t} = &\hat{W}\hat{A}(I-K\hat{G})\hat{W}^{-1}FE_{t+3,t-1|t-1} + (I-\hat{W}\hat{A}(I-K\hat{G})\hat{W}^{-1})\boldsymbol{L}_{t+3,t-1} \\
&-(\hat{W}\hat{A}KG+I)\boldsymbol{L}_{t,t-1} + A\boldsymbol{L}_{t+3,t+2} + w_{t+4,t+3} - \hat{W}\hat{A}K\eta_t
\end{aligned}
\tag{8}
$$

Where $\hat{W} = I + \hat{A} + \hat{A}^2 + \hat{A}^3$, the fact that $\hat{A}$ is stationary guarantee $\hat{W}$ to be invertible. The full derivation is included in **Appendix B**.

In equation (8), the matrix $WA(I-KG)W^{-1}$ is consistently estimated because the two components of the error term are uncorrelated with all the regressors. The $w_{t+4,t+3}$ is unpredictable error happening after $t+3$, thus uncorrelated with forecasting errors up to $t+3$ as well as any variable realized before $t+4$. The noise attached to public signal η_t is realized at time t thus not correlate with forecast error with information set at time $t-1$, and here I have to assume there is not feedback effect of η_t on realized macroeconomic variables after time t through general equilibrium so that η_t is uncorrelated with any variable(except for expectational ones) realized beyond time t[16].

I can then test for joint expectation formation of inflation and unemployment change using MSC. The vector of forecast errors

$$
FE_{t+4,t|t} = \begin{pmatrix} CPI_{t+4,t} - E\pi_{t+4,t|t} \\ dun_{t+4,t} - Edun_{t+4,t|t} \end{pmatrix} \equiv \begin{pmatrix} fe^{\pi}_{t+4,t|t} \\ fe^{un}_{t+4,t|t} \end{pmatrix}
$$

I then estimate the following regression with OLS:

$$
fe^{\pi}_{t+4,t|t} = \beta^{\pi}_0 + \beta^{\pi}_1 fe^{\pi}_{t+3,t-1|t-1} + \beta^{\pi}_2 fe^{un}_{t+3,t-1|t-1} + \Theta^{\pi}\boldsymbol{X}_{t+3,t-1} + \epsilon^{\pi}_{t+4,t+3} \tag{9}
$$

$$
fe^{un}_{t+4,t|t} = \beta^{un}_0 + \beta^{un}_1 fe^{\pi}_{t+3,t-1|t-1} + \beta^{un}_2 fe^{un}_{t+3,t-1|t-1} + \Theta^{un}\boldsymbol{X}_{t+3,t-1} + \epsilon^{un}_{t+4,t+3} \tag{10}
$$

In (9) and (10), the vector $\boldsymbol{X}_{t+3,t-1}$ corresponds to the controls needed for realized unemployment change and inflation as derived in (8). The ϵ's collects the error terms in (8) that is orthogonal to regressors. If there is no information friction, we should expect the estimates $\beta^{\pi}_1 = \beta^{un}_2 = 1$; if agents are forming expectation on inflation and unemployment separately, one should see $\beta^{\pi}_2 = \beta^{un}_1 = 0$. Table 4 summarizes the results from MSC, Newey-West HAC standard errors are reported in the brackets:

[16] Notice v^i_t disappeared as we derive the consensus expectation, this is because the idiosyncratic noise has mean zero at each time point.

Table 4: Aggregate Test on Joint Expectation, MSC, 1981-2017

	Single-variable Learning	Joint Learning		
	1981-2017	1981-2017	1981-2004	2004-2017
β_1^π	0.61***	0.59***	0.62***	0.56***
	(0.058)	(0.066)	(0.067)	(0.138)
β_2^{un}	0.52***	0.55***	0.62***	0.51***
	(0.089)	(0.088)	(0.130)	(0.099)
β_2^π		-0.19*	-0.2	0.01
		(0.11)	(0.13)	(0.23)
β_1^{un}		0.08**	0.08	0.20***
		(0.033)	(0.059)	(0.077)
Observations	141	141	89	52

* **** means significant at 1%,** means 5 % and * means 10%, data in use are quarterly from MSC.*

The first column in Table 4 is the case when single-variable learning is assumed, where agent applies Kalman Filters on inflation and unemployment change independently. This is equivalent to estimating (9) and (10) restricting $\beta_2^\pi = \beta_1^{un} = 0$ and omitting corresponding controls. Column 2 to 4 are results from estimating (9) and (10) without restrictions for different samples. The estimates of β_1^π and β_2^{un} are significantly positive implies there is information friction so that agent is not fully adjusting their forecast errors when new information arrives, as found in *Coibion and Gorodnichenko (2012)* and other researchers. The key finding for this paper is that the estimates of β_2^π and β_1^{un} are also significantly different from zero for the baseline period 1981-2017. This implies the off-diagonal elements of $\hat{W}\hat{A}(I - K\hat{G})\hat{W}^{-1}$ are non-zero, which rejects the single-variable learning model. The comparison between column 1 and 2 suggests restricting the off-diagonal elements to be zero will induce bias in estimating β_1^π and β_2^{un}, which usually have close link to measure of information rigidity.

These results show strong evidence in support of joint expectation formation instead of single-variable learning. The straight forward implication of joint expectation formation is that agents are taking into account the link between inflation and unemployment when they are learning to predict the future from signals. The forecast error of one variable then is predictable by lag forecast error of another variable because now noise of one variable will affect the forecast of another as well. More importantly, this predictability can come from two distinctive sources of the noisy information structure: the off-diagonal elements in $\hat{A}$ and $\hat{G}$.

Structure of Economy from $\hat{A}$: The first is related to agents' subjective model of economy (2). If there is correlation between state variables so that the off-diagonal elements of A are non-zero, then a positive forecast error of unemployment now means agents make additional mistakes in predicting inflation as compared to the single-variable case, where the errors only come from past mistakes and current noise in signal.

Mixed Signal from $\hat{G}$: Another possibility comes from signal extraction problem. Suppose there is no correlation between state variables, however signals are jointly created by multiple state variables, in other words, $\hat{G}$ is non-diagonal. Then when agents observe such a signal they will adjust beliefs on all state variables related to this signal. In this case they update their beliefs even more cautiously on one variable and mistakenly adjust their beliefs on another one, which induces the predictability we see in the test.

For example, the signal extraction part of inattention model proposed in *Kamdar (2019)* is a special case of $\hat{G}$ being non-diagonal. In her model there is only one signal that is a linear combination of unemployment and inflation. The test I performed can be seen as a check of her information friction. For her model to explain the positive correlation observed in expectational data, she needs both weights on unemployment and inflation be positive. However, such a parametrization implies negative values on β_2^{π} and β_1^{un}, which is inconsistent with findings from Table 4. Because if signal is created in such a way, when inflation go up agents will believe in both higher inflation and higher unemployment rate in the future. The former results in under-prediction of actual inflation whereas the latter implies over-prediction of unemployment.

This means the correlation we are interested in is not explainable solely by the structure of observational equation (3). The estimates in Table 4 then suggest agents believe current unemployment depends on lag inflation in the state space equation (2). And the fact that such a correlation doesn't exist in realized data suggests the dependency agents believe in is a mis-specification of the model they use[17].

Asymmetric Dependency on Lag Errors: Table 4 also shows there is asymmetry in estimates on the lag forecast errors. If this comes from non-diagonal A used by agents, it implies agents believe future inflation depends negatively on current unemployment and the opposite for future unemployment change. If it's due to mixed signals, the signs on β_2^{π} and β_1^{un} should be the same though they may have different values because the weights on state variables that create a signal matters for how much information agent can extract from that signal[18]. This is again inconsistent with my findings in Table 4, as estimate β_1^{un} is significantly positive whereas for β_2^{π} it's usually negative if not zero.

All these evidence from estimation seem to be in line with the friction of agents' belief on economic structure $\hat{A}$, rather than mixed signals that contain information on multiple variables. I then provide more evidence from survey data on testable implications of these two frictions, to show that data is in favour of subjective model friction.

4.2 Perceived News and Expectation

One key distinction between the two frictions is the response of expectational variable to news. With the mixed signal friction agents typically can't distinguish between news about inflation or unemployment and unlabelled bad news will affect both inflation and unemployment expectation positively. If there are signals specific to one subject, it will only affect the expectation on such subject. Whereas friction on subjective model $\hat{A}$ suggests agents can distinguish between different signals, and according to the estimates in Table 4, those signals on inflation will move both inflation and unemployment forecasts up whereas news about unemployment will only increase unemployment forecast, with negative or no impact on inflation forecast.

To examine these implications I use the perceived news measures from MSC as in *Dorms and Morrin (2004)*, *Pfajfar and Santoro (2013)* and *Lamla and Maag (2012)*. This variable includes a label on what kind of news agent has heard of in recent 3 months. The description of these variables are included in **Appendix C**.

In presence of only mixed signal friction, suppose these labels on news heard truthfully reflect agents' understanding of the content, we may expect different news have impact only on expectational variable of the same subject. If we believe agents still cannot distinguish the content of this news and they randomly pick a label in reporting, we should expect both expectations adjust in response of receiving such news, as long as it's unfavourable. Both these are different from implication of subjective model friction: it suggests we should observe news on inflation has impact on both expectations on unemployment rate as well as inflation itself, whereas unemployment news will only affect unemployment forecasts. We can test these implications using micro-level data from MSC.

I first split the samples into subgroups conditional on news agents heard of. I focus for now on only news about inflation, employment and interest rate, favourable or unfavourable. For every

[17] I estimate matrix A with realized data over the same period, the off-diagonal elements are insignificant from 0 and the estimate on persistence are both around 0.7. This means matrix A is indeed diagonal and agents may have used a mis-specified non-diagonal A when they form expectations.

[18] May need a formal proof or discussion here.

group, I compute the percentage deviation of expectations on inflation, unemployment change and interest rate change[19] from their means of all the survey participants at each time point, to eliminate the the time specific effect in each expectational variable. I then take the average of this deviation across time, conditional on news they have heard. Figure 1 shows two matrices for the deviations of these conditional expectations.

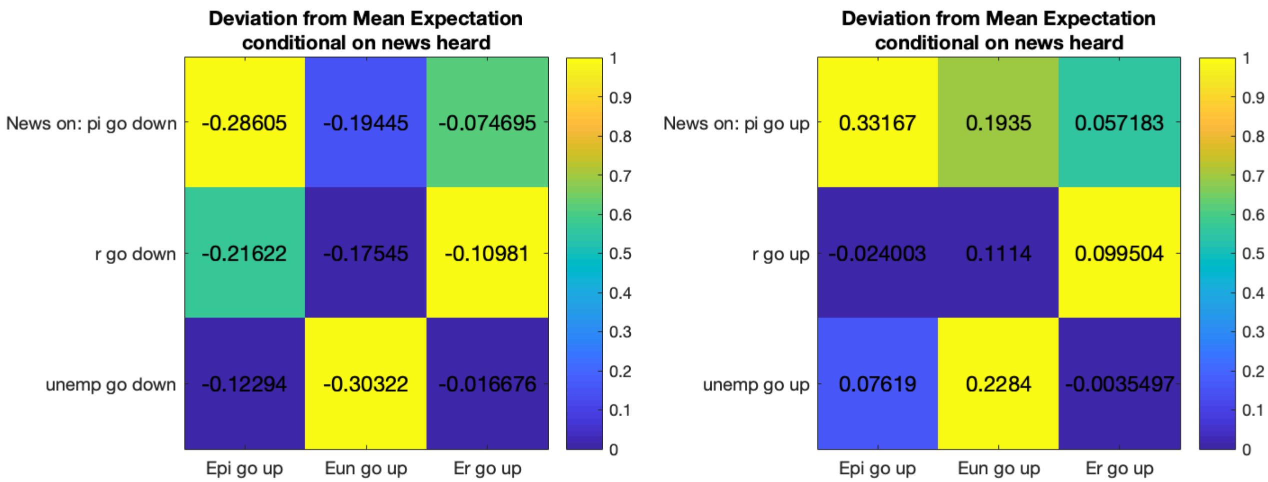

Figure 1: On y-axis is the news heard for each subgroup, on x-axis is expectation under examination. The left panel are responses of receiving good/favorable news, the right are those of bad/unfavorable news.

To interpret Figure 1, consider the top left corner in the left panel(matrix), -0.286 means for a person who has heard of inflation being lower, he/she also reports expected inflation 28.6% lower than the average at the time(the unconditional time mean across that cross-section). The color of boxes inside each panel is normalized vertically: the most yellow box means agents with that type of news have the highest deviation in absolute value, whereas the most dark blue one has the lowest. For example, the first column in the left panel means agents heard of inflation being low have inflation expectation further lower than those with interest rate and employment news.

Figure 1 shows that news have the biggest impact on the variable it is labelled with. Furthermore, inflation news has big impacts on all three expectational variables when comparing to other news, especially when it's news on high inflation (in right panel). For agents with news on high inflation, they report 33% higher in expected inflation, 19% higher in unemployment change and 5.7% higher in interest rate expectation. However, we also see similar response to unfavourable employment news, though with smaller impact. This is due to the fact I haven't controlled for individual fixed effect. As news are self-reported, it is possible pessimistic agents pay attention to all kinds of bad news and also more likely to form worse expectations than average. Then when I condition on agents with bad employment news, they have higher inflation expectation not because of the news, but the fact they almost always expect higher inflation than average.

To control for this fixed effect, I consider the likelihood each agent increase his/her expectation upon receiving different news, similar to *Pfajfar and Santoro (2013)*. I use the two-wave panel available for MSC and compute the fraction of agents who adjust their expectations upwards or downwards, conditional on receiving news in the second period. The likelihood of adjusting expectation is reflected in two ways: (1) agents with specific news are more likely to adjust expectation upwards comparing to others; (2) agents with specific news are less likely to adjust expectation

[19] In **Appendix C** the same experiment with more expectational variables are available, here for ease to read I only report the three key expectations.

downwards. To capture both these two ways I sum up these two types of likelihood difference between agents with specific news and others. Figure 2 shows the results:

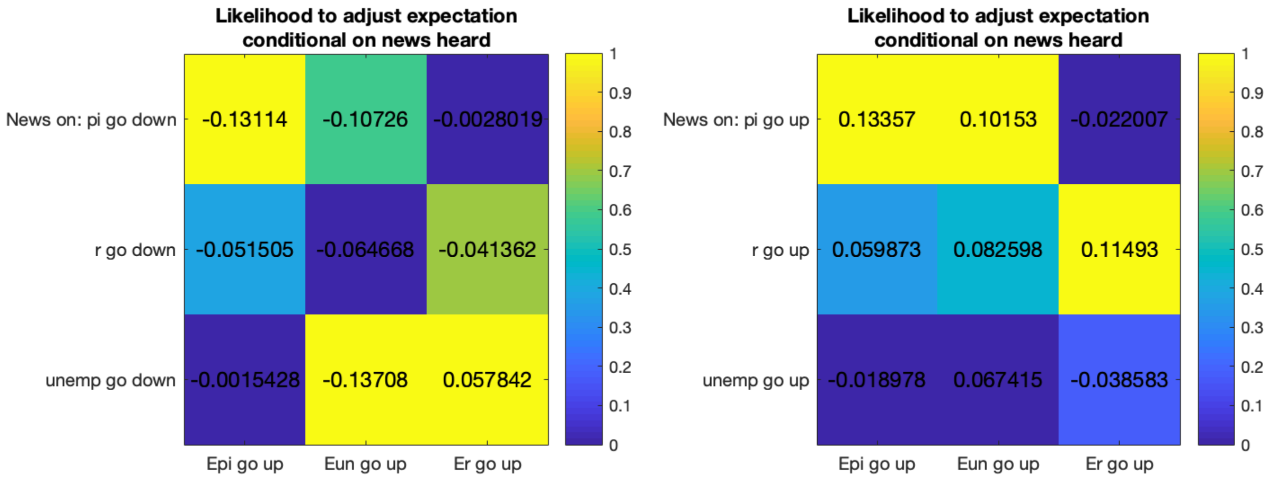

Figure 2: On y-axis is the news heard for each subgroup, on x-axis is expectation under examination. The left panel are responses of receiving good/favorable news, the right are those of bad/unfavorable news.

The two panels are organized in the same way as Figure 1, except for the interpretation of values inside boxes differ. Now it stands for the difference of likelihood in adjusting expectations between agents with specific news and those who don't hear of such news. Now for an agent that hears of news about high inflation, he/she has 13% higher chance to adjust his/her inflation expectation upwards, and 10% higher chance to believe in higher unemployment rate in the future, the opposite is true for those who hear of news on low inflation. However employment news barely has any impact on inflation expectation now and in presence of unfavorable employment news, agents are more likely to adjust inflation forecast downwards.

Finally, I perform a panel regression controlling for individual as well as fixed effect, the parameters of interest are dummy variables on what kind of news agent receives. This can be seen as a compliment result for the previous ones:

Table 5: FE Panel

	(1)	(2)
news on:	$E\pi$	Eun
high inflation	0.50^{***}	0.065^{***}
	(0.09)	(0.011)
low inflation	-0.33^{***}	-0.065^{***}
	(0.10)	(0.016)
employment unfavorable	0.007	0.11^{***}
	(0.052)	(0.007)
employment favorable	-0.08	-0.15^{***}
	(0.057)	(0.009)
Observations	163233	162369
R^2	0.68	0.69

Table 5 suggests hearing news on high (low) inflation increase reported expected inflation by about 0.5% (0.33%) and increase the probability to believe unemployment rate will rise (fall) by 6.5%. However news about employment only has significant impact on unemployment expectation but not on inflation expectation. Furthermore, the individual level impact of inflation and employment news seem to transmit into consensus expectation perfectly through aggregation. In Figure 3 I plot the mean of each year for consensus expectations on inflation and unemployment, conditional on hearing inflation news or not.

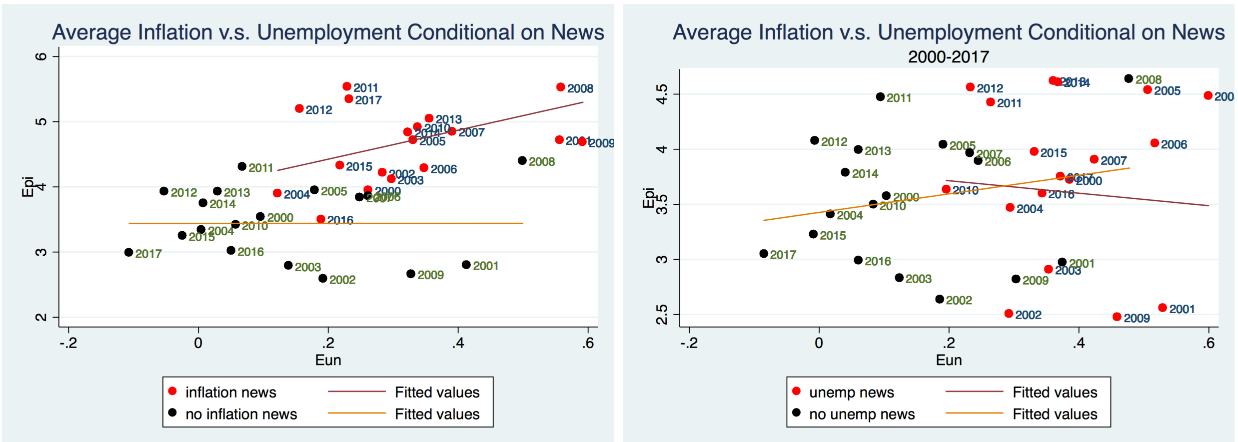

Figure 3

In Figure 3 the red dots are consensus expectation over the year conditional on hearing inflation (top panel) or unemployment (bottom panel) news and black dots are that of people without those news. It is clear that agents with inflation news have both higher inflation and higher unemployment expectation comparing to those who didn't hear such news. Whereas unemployment news only shift unemployment expectation to the right[20]. Moreover we clearly see the positive correlation between the two expectations across time for those agents with inflation news. Once we take these people out, the black dots present no correlation at all.

These findings together strongly support the subjective model friction and stand against the mixed signal friction.

4.3 Discussion of Results

I showed in a noisy information framework, there are strong evidence in support of joint expectation formation rather than single variable learning. These evidence suggest agents believe current inflation has a positive impact on future unemployment status whereas current unemployment status barely contributes (or in a weakly negative way) to future inflation. And an alternative explanation through mixed signals is less likely to be true. This is supported by the fact agents' expectations have asymmetric responses to their self-reported news on different subjects. However there are still questions remaining to be answered:

Mis-specification of Subjective Model: At the beginning of Section 4 I proposed 4 channels of imperfect information. The joint expectation formation model assume only 2 of the 4 information frictions are present. However the other 2 forms of mis-specification that agents have in their subjective models may as well be true. My analysis so far is not robust to the presence of such information frictions.

One key message from the test in this section is that when agents use the same set of information to predict multiple variables, they are forming beliefs on state variables that we as econometrician do not see. In the test I expand the one-state model to a two-state scenario and show that missing

[20]For data from 1984-1999 the same pattern persist, I'll include them in Appendix C.

states may induce bias in estimates and more importantly miss the key correlations that has policy implications. For example, if agents indeed believe lag inflation contributes to bad economic performance in the future, an inflation-tolerating policy may have contractionary effect rather than suggested in *Eggerson and Woodford (2003).* In fact many researchers have found negative impact of expected inflation on household consumption[21].

The missing state problem is not unique to the single-variable noisy information model. It is common to any learning model with pre-specified state space. For example, the two-variable model I estimated in this paper may suffer from missing states as well, because agents are predicting more variables other than inflation and unemployment rate. Furthermore, the state variable they are making inferences on may not be macroeconomic outcomes. The state variable could be shocks about future as suggested in *Barsky and Sims (2012)*, or structural parameters as proposed by *Malmendier and Nagel (2015)* and *Honkapohja and Evens (2001)*[22].

Lack of information about signals: I have shown agents form expectation jointly rather than independently. However what kind of information makes their beliefs to co-move remain unclear. One problem (also advantage in terms of simplicity) of the noisy information model is it depends heavily on assumption of linearity and structure of observational equation so that signals can be replaced by current aggregate variables.

However, when agents face a large set of signals, how they perceive the content of information from these signals matter. The perceived news measure is an outcome of signal extraction rather than the signal itself. It cannot tell us any information about what kind of signals agents are exposed to and how it transmit into their expectations. For example, we may see agents expect worse economic condition in response of current inflation hike. This alone tells us nothing about the effect of more media exposure on inflation targeting. As agents may not infer any information from news media, the observed expectation response comes from the local prices of grocery stores or prices at the gas pump. In the context of noisy information model, this is because inflation affects both signals and we cannot distinguish which one agent is responding to.

These unanswered questions all point to bigger problems about learning models: the pre-specified structure may limit the capacity of a model to capture important correlations between expectational variables and lack of information about signals make it hard to understand what type of information is likely to affect agents' expectations. These weaknesses restrict us from exploring heterogeneity in the way agents interpret information and its relation to dispersion in expectational data. To tackle these problems we need a more flexible learning framework that start from actual information available to households.

5 Generic Learning Mode: An RNN Approach

In this section, I propose an innovative generic learning model based on Recurrent Neural Network(RNN). The goal of such a model is to solve the problems discussed before and be able to learn what types of information affect agent expectations jointly and how.

5.1 Agent's Problem

Consider the same information structure as in Section 4: agents observe a set of noisy signals $\boldsymbol{s}_t$ and they use their subjective model to form expectation about future. Then follow from notations in Section 4, I denote the expectational variable agents forming expectation over is $\boldsymbol{L}_{t+1}$[23], and agents

[21] See for example *Coibion et. al. (2019)* and *Bachmann et al.(2015)*.

[22] In papers with Least Square Learning, agents learn about structural or reduced-form parameters by running OLS recursively. In this case the parameters can be treated as hidden states as well.

[23] To save notations I drop the step t, however generally speaking this could be h step expectations agents form, and it can be over any object $\boldsymbol{L}$.

observe signals that they believe containing information about $\boldsymbol{L}_{t+1}$. These information includes public signals, which are realization of macro-variables $\boldsymbol{X}_\tau$ for $\tau = t, t-1, ..., 0$, as well as private signals $\boldsymbol{s}^i_t$. There are some noise around the public signals agents see, denoted as $\epsilon_{i,t}$. The most general form of agents' expectation formation model then can be written as:

$$\boldsymbol{L}^i_{t+1|t} \equiv \hat{\mathbb{E}}(\boldsymbol{L}_{t+1}|\boldsymbol{X}_t, \boldsymbol{s}^i_t, \epsilon_{i,t}, \boldsymbol{X}_{t-1}, \boldsymbol{s}^i_{t-1}, \epsilon_{i,t-1}...) \tag{11}$$

The $\hat{\mathbb{E}}$ means agents may form expectation using subjective beliefs. Instead of assuming the full structure of agents' knowledge as usually done in the learning literature, I will make two assumptions that are much less restrictive:

Assumption 1. *Agent forms expectation through two steps: updating and forecasting. In updating step, agent forms a finite dimensional latent variable $\Theta_{i,t}$, which follows a first order markov process:*

$$\Theta_{i,t} = H(\Theta_{i,t-1}, \boldsymbol{X}_t, \boldsymbol{s}^i_t, \epsilon_{i,t}) \tag{12}$$

In the forecasting step, they use $\Theta_{i,t}$ to form expectation:

$$\boldsymbol{L}^i_{t+1|t} = F(\Theta_{i,t}) \tag{13}$$

It is worth noting that the structure in assumption 1 covers a large class of learning models existing in the literature. This includes the noisy information model I discussed before as well as other information rigidity models. Take the joint expectation formation model I presented in Section 4 as an example, it is a special case of the structure defined in assumption 1. The latent variable $\Theta_{i,t}$ will be the perceived current states in Kalman Filter $\boldsymbol{L}^i_{t|t}$. And this generic structure is also robust to the other 2 types of model misspecification I discussed in Section 4.3, as here I do not need to specify the exact structure of $\Theta_{i,t}$

I then describe the second assumption on the unobserved noise around public signal:

Assumption 2. *The idiosyncratic noise on public signal, $\epsilon_{i,t}$ is i.i.d across individual and time. And it is orthogonal to past and future public and private signals:*

$$\mathbb{E}(\epsilon_{i,t}\boldsymbol{X}_\tau) = \mathbb{E}(\epsilon_{i,t}\boldsymbol{s}^i_\tau) = \mathbb{E}(\epsilon_{i,t}\epsilon_{i,\tau}) = \boldsymbol{0} \quad \forall i, t, \tau$$

Assumption 2 is a common assumption made in noisy information and other learning models on unobserved noise present in signals.

5.2 Economist's Problem

Given the generic learning model presented before, I allow agents to form expectation according to their observed signals, without putting structural assumptions on the dimensionality on $\Theta_{i,t}$ as well as functional forms on $F(.)$ and $H(.)$. For this reason such a model allows for all four possible information frictions I discussed in Section 4, meanwhile it makes estimating the effect of signals a much harder problem.To illustrate how to tackle this problem, it's helpful to described as economist how I can estimate the effects of signals given that agents are forming expectation generic learning model presented above.

As econometrician, I observe signals that agents observe $\{\boldsymbol{X}_\tau, \boldsymbol{s}^i_\tau\}^t_{\tau=0}$ and their expectations $\boldsymbol{L}^i_{t+1|t}$. The objective of interest can be estimated by approximating the conditional expectation function $f_x(\{\boldsymbol{X}_\tau, \boldsymbol{s}^i_\tau\})$ defined below:

$$
\begin{aligned}
l_{t+1|t} &\equiv f_x(\{\boldsymbol{X}_\tau, \boldsymbol{s}^i_\tau\}^t_{\tau=0}) \\
&= \mathbb{E}[\boldsymbol{L}^i_{t+1|t} | \{\boldsymbol{X}_\tau, \boldsymbol{s}^i_\tau\}^t_{\tau=0}] \\
&= \mathbb{E}[F(\Theta_{i,t}) | \{\boldsymbol{X}_\tau, \boldsymbol{s}^i_\tau\}^t_{\tau=0}] \\
&= \mathbb{E}_{\{\epsilon_{i,\tau}\}^t_{\tau=0}}[F(H(\Theta_{i,t-1}, \boldsymbol{X}_t, \boldsymbol{s}^i_t, \epsilon_{i,t})) | \{\boldsymbol{X}_\tau, \boldsymbol{s}^i_\tau\}^t_{\tau=0}]
\end{aligned} \tag{14}
$$

The difficulties here are: (1) $F(.)$ and $H(.)$ are unknown; (2) dimensionality of $\Theta_{i,t}$ is unknown. I solve this problem by approximating (14) with a Recurrent Neural Network(RNN). RNN is a universal functional approximator constructed using nested logistic or partial linear functions, according to the Universal Functional Approximation Theorem, the following results hold[24].

Theorem 1. *For any dynamic system described in* (12) *and* (13)*, with assumptions 1 and 2 hold. There exists finite dimensional $\hat{\Theta}_{i,t} \in \mathbb{R}^d$, continuous function $\hat{f} : \mathbb{R}^d \to \mathbb{R}^l$ and measurable function $\hat{h} : \mathbb{R}^s \times \mathbb{R}^d \to \mathbb{R}^d$ s.t. the conditional expectation function described in* (14) *can be described as dynamic system:*

$$
\begin{aligned}
l_{t+1|t} &= \hat{f}(\hat{\Theta}_{i,t}) \\
\hat{\Theta}_{i,t} &= \hat{h}(\hat{\Theta}_{i,t-1}, X_t, S_{i,t})
\end{aligned} \tag{15}
$$

With first stage approximation of $l_{t+1|t} \equiv f_x(\{\boldsymbol{X}_\tau, \boldsymbol{s}^i_\tau\}^t_{\tau=0})$, one can estimate the average marginal effect of a specific signal x_t by computing:

$$
\frac{\partial l_{t+1|t}}{\partial x_t} \approx lim_{\delta \to 0} \frac{f_x(\boldsymbol{X}_t + \delta, \{\boldsymbol{X}_\tau, \boldsymbol{s}^i_\tau\}^{t-1}_{\tau=0}) - f_x(\boldsymbol{X}_t, \{\boldsymbol{X}_\tau, \boldsymbol{s}^i_\tau\}^{t-1}_{\tau=0})}{\delta} \tag{16}
$$

The structure I proposed above can offer an estimate for effect of public signal on expectational variables that is robust to lots of existing learning models, including noisy or rigid information models with all 4 types of information friction in Section 4.Furthermore, such a structure is also robust to potential non-linearity in learning process. Non-linearity can occur for many different reasons. It may be due to non-linear structure of state variables agents believe in, or it can come from learning process endogenously even if agents believe in linear structures. For example, in *Malmendier and Nagel (2015)*, the signals(current realized inflation in their case) affect expected inflation non-linearly because it enters the linear law of motion as well as the learned persistence parameter.

I illustrate the performance of RNN when either noisy information model or learning with experience model is the true model with Monte Carlo simulations. These results can be found in Appendix D.

5.3 LSTM with Survey Data

I then use actual data to train LSTM. Table 8 describes data I consider to use on input and output variables. Table 8 summarizes the data we use for LSTM.

[24] According to the Universal Functional Approximation Theorem (See *Hornik et. al. (1989), Barron (1994)*), a single layer feedforward neural network can approximate any continuous function.

Layer:	Description	Source
Output	**Expectations** on: inflation, unemployment change, real income interest rate, business condition (past and future) purchasing attitude for durable, car, home (now)	Michigan Survey
Input	**SPF** on: CPI, UNEMP, RGDP, TBILL Anxious Index	Professional Forecast
Input	**Official Stats** on: CPI, UNEMP, FFR, RGDP Stock market index, Consumption, Invest Gov Spending, Real Oil, Gas Price	FRED
Input	**News Volume (and Tones)** on: Inflation, Unemployment Stock market, GDP, interest rate, monetary/fiscal policy (major coverage media: NYT, Washington Post etc.)	Lexis-Nexus, Factiva Newslibrary

Table 6: Data available

The set of results I show here are using consensus expectations from MSC, where I consider an average person is learning and forming expectations among various macroeconomic variables. In these results I include official statistics as well as corresponding expectations from professional forecasts as signals that are available to the average household. I'm now training a much bigger panel of expectations with synthetic agents constructed from MSC as well as the panel from Survey of Consumer Expectations. These results will soon become available.

5.3.1 Approximated Conditional Expectation Function

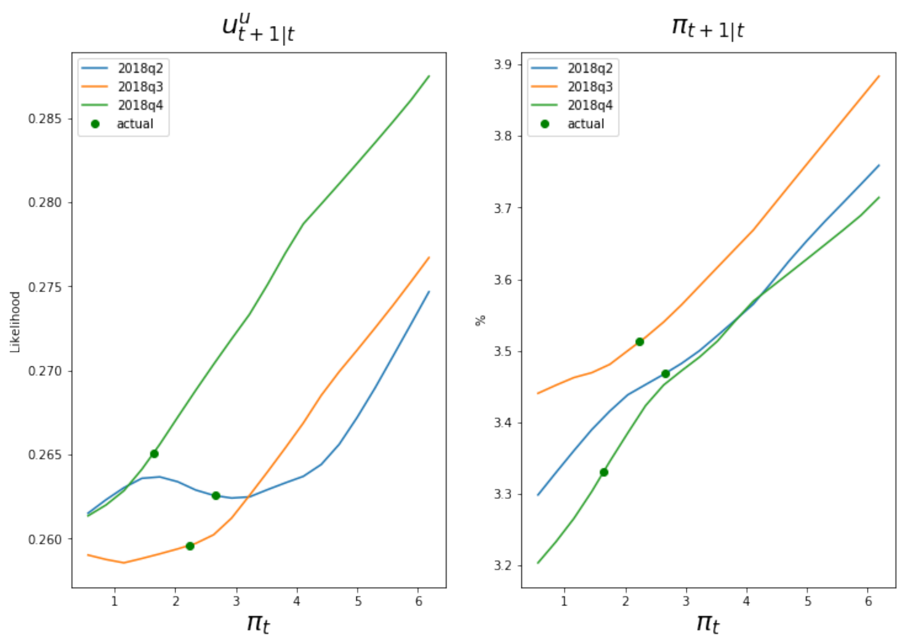

Figure 4: Conditional Expectation Function of household's expectation as function of inflation signal. The green line as CEF in 2018 quarter 4, orange line in 2018 quarter 3 and blue line in 2018 quarter 2. The green dot on each curve is the observation at that point of time

I first plot the approximated conditional expectation function (hence-force CEF). I focus on the response of expected inflation and expected unemployment rate change. In Figure 4 I plot the CEF of expected unemployment change and expected inflation on inflation signal. As the RNN allows for state-dependent response, the CEF for the same variables may be different at different point of time. These differences come from different histories of past signals at each point of time. I plot the green line as CEF in 2018 quarter 4, orange line in 2018 quarter 3 and blue line in 2018 quarter 2. The green dot on each curve is the observation at that point of time.

It is clear in Figure 4 that higher inflation observed will make agents adjust both their beliefs about future inflation and unemployment rate upwards. On contrary, Figure 5 shows that actual unemployment rate increase moves expectation on unemployment upwards whereas it moves inflation expectation in an indeterminate way. For example, in Figure 4, the green line suggests that in 2018 quarter 4, if inflation increases from 2% to 3%, there will be on average 0.5% more agents believe unemployment rate will go up in the future and inflation will go up by around 10 basis point(one eighth of the standard deviation for expected inflation). And in Figure 5, the green line shows in 2018 quarter 4, if change of unemployment rate increases from 0% to 1%, agents are 1% more likely to believe in unemployment rate will go up (about 10% of the standard deviation of expected unemployment),whereas expected inflation barely moves by increasing around 1 basis point.

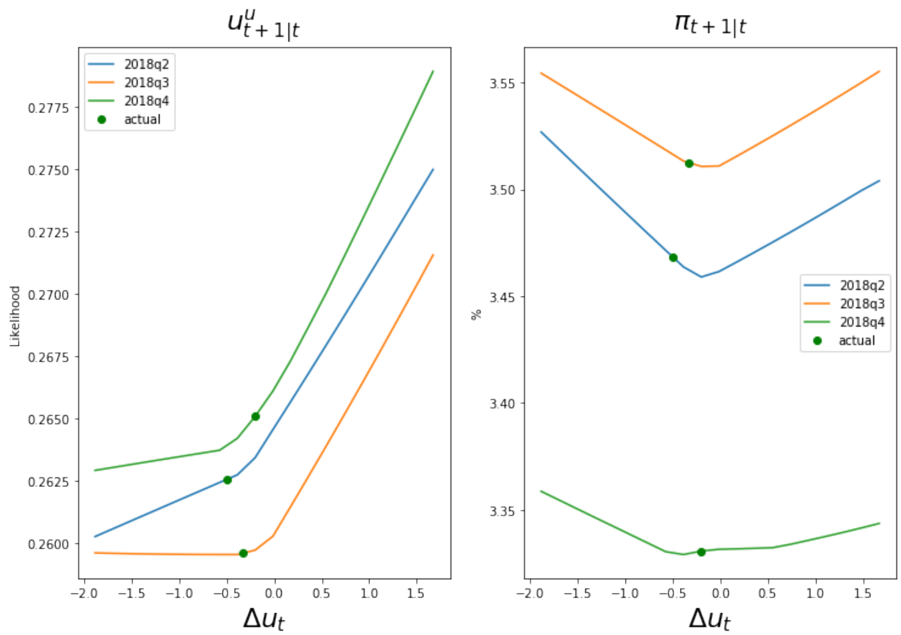

Figure 5: Conditional Expectation Function of household's expectations as function of unemployment signal. The green line as CEF in 2018 quarter 4, orange line in 2018 quarter 3 and blue line in 2018 quarter 2. The green dot on each curve is the observation at that point of time

5.3.2 State-dependent Marginal Effects

I then compute the marginal effect given the approximated conditional expectation function following (16). One can think of the procedure as computing the slopes for each of the curves in Figure 4 and 5, at the green point(actual data). I then represent the slope at each point of time across the entire sample: 1981 to 2018. One strength of RNN is that it can detect change of marginal effects across different internal states. Figure 6 below illustrates the state-dependent marginal effects of realized inflation on three expectational variables:

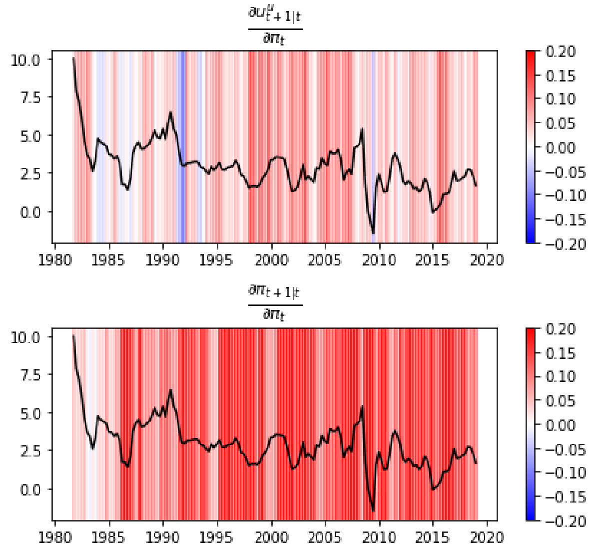

Figure 6: State-dependent marginal effect of inflation signal. The top left panel is marginal effect on expected unemployment change; the top right panel is marginal effect on perceived interest rate change and the left bottom panel is expected inflation change

The solid line in each panel is the actual series of inflation (signal). Each color bar represents the sign of marginal effect at that point of time: red means positive and blue means negative. The darker color within each color code stands for the bigger magnitude of marginal effect. The magnitude of marginal effect is normalized by standard deviation. For example, 0.2 means one standard deviation change of the signal is associated with 0.2 standard deviation change of the corresponding expectational variable.

It is clear from Figure 6 that positive inflation drives up expected unemployment conditions, with most periods in the top left panel being red-colored. It is also worth noting that such pessimistic effect of inflation is not due solely due to agents believing in interest rate responding more than one on one to inflation, which should show up as red color in top right panel. However the pale color after 2009 in top right panel suggests agent seems to understand within zero lower bound episode interest rate is not excessively responsive to inflation any more. This finding exclude one explanation discussed in *Bachmann et. al (2015)* that the agents may not understand the policy in zero lower bound well enough to create pessimistic responses to inflation.

I then plot the same state-dependent marginal effect of unemployment rate signal in Figure 7. Not surprisingly, unemployment rate going up will make agents believe future unemployment rate to go up as well in most of the time. However, the bottom left panel also suggests that unemployment signals actually have negative, or negligible (due to the pale color) effect on expected inflation.

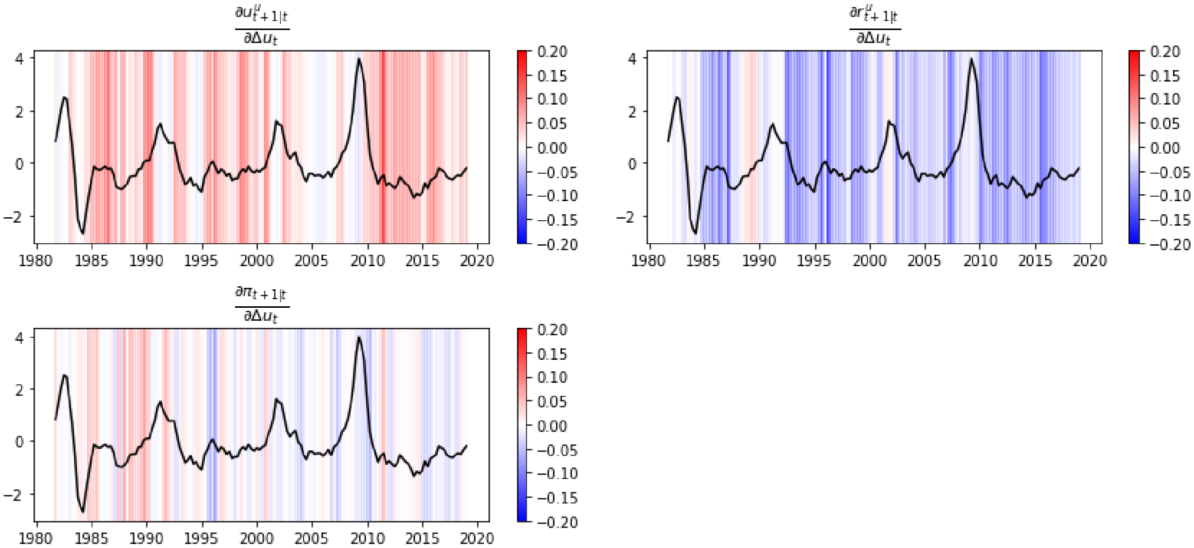

Figure 7: State-dependent marginal effect of unemployment rate signal. The top left panel is marginal effect on expected unemployment change; the top right panel is marginal effect on perceived interest rate change and the left bottom panel is expected inflation change

All these findings are consistent with results from the joint learning test before as well as the empirical results using perceived news. Inflation and unemployment rate seem to have asymmetric cross-effects on expected inflation and unemployment conditions. Realized inflation itself will not only increase agents' expected inflation but also make them believe worse employment status in the future, whereas unemployment rate doesn't have such impact on inflation. These results then are more consistent with the explanation that agents may have a different model of the economy, which features a different cross-correlation structure in $\hat{A}$.

6 Conclusion

In this paper I examine a large set of expectational variables from US household surveys and find these expectations correlate with each other. Some of these correlations are inconsistent with either corresponding realized data or economic theories, and are unique to household surveys. Specifically, agents predict high inflation together with worse economic performances including higher unemployment rate and weaker growth.

These patterns are hard to be explained by standard single-variable noisy information model. In stead I propose a joint expectation formation model and a simple test to distinguish it from standard single variable models. I then show survey data strongly support the idea agents form expectations on various subjects jointly rather than independently. Furthermore I find the cross-correlation in household survey expectations is due to agents' beliefs in the structure of the economy rather than mixed signals generated by multiple state variables.

However the new model alone is silent about what news or signals affect agents expectations and how, it may also suffer model-mispecification problem about agents' expectation formation process. I then propose a generic learning model and estimate the marginal effects of public signals with an innovative RNN approach. I show the RNN can be used for purpose other than prediction: it can approximate dynamic effects of a wide range of commonly used learning models. I then train LSTM using household surveys and a large set of signals to let it learn how to form expectations and provide brand new insights on household's expectation formation behaviours.

The findings of this paper have important implications on households' behaviours in response to their expectations and Central Bank Communication. Multiple researchers have found negative responses of household's consumption attitudes to their inflation expectations. This paper shows inflation specific news makes agents believe economic condition in general will be worse. The pre-cautionary motive and anticipated income decrease can generate the negative response of consumption. For Central Bank Communication, signals on current or future inflation is likely to create pessimistic beliefs on economic performance among households. Furthermore this is only true for inflation signals, clear information on employment and economic growth will have little impact on inflation in return. I will be able to offer more insights once the panel version of RNN model is trained and evaluated.

Appendix A Data Appendix

A.1 Data Description

SCE: SCE run by New York Fed started in June 2013. It is a nationally representative, internet-based rotating panel of about 1300 household heads, each stay in the panel for 12 months. The survey is month by month and in each month new respondents are drawn to match various demographic targets[25]. The survey contains a richer set of questions comparing to existing surveys about consumer expectations, including individual's employment status and different characteristics of the household. The panel feature of SCE allows me to control for individual fixed effect that could induce spurious correlation between different perceptions and to follow individual along time which is important to capture learning behaviour.

MSC: The monthly component Michigan Survey of Consumers started from 1978[26]. I will use the aggregate component of MSC as well as the cross-sectional archive as a complement part to the SCE. So far most of the literature using aggregate or micro-level data are utilizing this dataset.

A.2 Aggregate Survey Forecast and Real-time Data

To first illustrate the difference between the survey expectation and realized data, Figure 11 plots raw data on average expectation from MSC with realized data for inflation, unemployment rate change and real GDP growth. All real time series are change from a year ago, as the corresponding expectation series are one-year-forward forecasts.

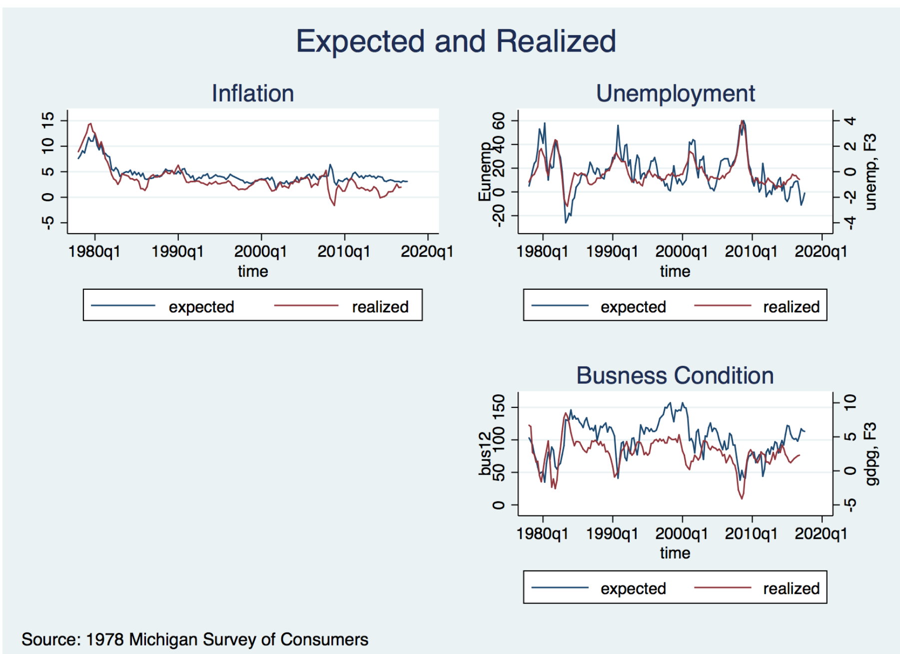

Figure 8: Survey expectation from MSC against realized data. All macro data are changes from a year ago, survey expectations are one-year-forward forecasts. Unemployment and business condition expectations are aggregated from categorical data. Positive number (over a hundred) means more people believes unemployment (business condition) will increase (be better) in the future.

[25] For details of SCE see *Armantier et al. (2016)*

[26] Quarterly data starts earlier from 1960 but with a lot of dimensions missing.

A.3 Recover Survey Mean from Categorical Data

From the cross-sectional dataset of MSC, I can acquire information on the fraction of respondents with different answers. Denote f_t^u as fraction of responses that are "increase" and f_t^d as "decrease". Assume for each period of t, there is a cross-section of answers formed by individuals about the change of the asked subject (unemployment rate or business condition and price). And assume this measure follows a normal distribution with mean μ_t and variance σ_t^2.

Assumption 3. *At each period t, survey respondent i forms a belief $x_{i,t}$ that indicates the change of asked variable x, this belief follows a normal distribution:*

$$x_{i,t} \sim N(\mu_t, \sigma_t^2)$$

Then suppose the agents have a common scale in answering the categorical question: If $x_{i,t}$ is close to some level b, then he will consider the subject will barely change; if $x_{i,t}$ is much bigger than b, he will answer increase, otherwise answer decrease.

$$category_{i,t} = \begin{cases} increase & x_{it} > b + a \\ decrease & x_{it} < b - a \\ same & x_{it} \in [-a + b, b + a] \end{cases}$$

Then the fraction of answer "increase", denoted as f_t^u, and "decrease" , denoted f_t^d, will directly follow from normality:

$$f_t^d = \Phi\left(\frac{b - a - \mu_t}{\sigma_t}\right) \tag{17}$$

$$f_t^u = 1 - \Phi\left(\frac{a + b - \mu_t}{\sigma_t}\right) \tag{18}$$

The items I want to recover is μ_t, which is the corresponding average change of the asked subject a year from now. This can be computed using:

$$\sigma_t = \frac{2a}{\Phi^{-1}(1 - f_t^u) - \Phi^{-1}(f_t^d)} \tag{19}$$

$$\mu_t = a + b - \sigma_t \Phi^{-1}(1 - f_t^u) \tag{20}$$

From (19) and (20), compute the average across time we have:

$$\hat{\sigma} = 1/T \sum_t^T \sigma_t = 1/T \sum_t^T \frac{2a}{\Phi^{-1}(1 - f_t^u) - \Phi^{-1}(f_t^d)} \tag{21}$$

$$\hat{\mu} = 1/T \sum_t^T \mu_t = 1/T(a + b - \sigma_t \Phi^{-1}(1 - f_t^u)) \tag{22}$$

As in MSC there is no information on $\hat{\sigma}$ and $\hat{\mu}$, I use the time-series mean of the data from Survey of Professional Forecast (SPF) on comparable questions to approximate those from MSC[27]. Following *Bhandari, Borovicka and Ho (2018)* I assume the ratio of time-series average between inflation expectation and other expectation in MSC equals to its counterpart in SPF:

[27]For unemployment rate change, I use the average difference between projected unemployment rate at $t + 3$ and the historical data at $t - 1$ which is the last information available to the economist. For real GDP growth I use the real GDP growth projection for the next four quarters after $t - 1$.

Assumption 4. *For the variable x asked in the survey:*

$$\hat{\sigma}_x^{MCS} = \frac{1/T\sum_t^T \sigma_{E\pi,t}^{MCS}}{1/T\sum_t^T \sigma_{E\pi,t}^{SPF}} \times 1/T\sum_t^T \sigma_{x,t}^{MCS}$$

And

$$\hat{\mu}_x^{MCS} = \frac{1/T\sum_t^T \mu_{E\pi,t}^{MCS}}{1/T\sum_t^T \mu_{E\pi,t}^{SPF}} \times 1/T\sum_t^T \mu_{x,t}^{MCS}$$

Then from (21) and (22) and Assumption 2 I can back out a and b, and with (20) I can recover $\mu_{x,t}$ for the expectational variable x.

Recovered series: To test whether the above method is plausible, I use the cross-sectional data of MSC for inflation expectation to construct categorical variable using different ranges $1\% - 2\%$, $3\% - 4\%$ and $4\% - 5\%$ for answers to be "stay the same". Then I use the proposed method to recover the $\mu_{\pi,t}$ and compare it with the actual average of expected inflation. Figure 12 plots the recovered mean and the actual mean.

Figure 9: Recovered Expected Inflation v.s. Actual

Figure 12 shows that the recovered data is actually quite close to the actual mean expectation, with correlation of 0.93, 0.95 and 0.91 respectively. Figure 13 shows the recovered data on unemployment change and real GDP growth (economy condition change) comparing to actual data.

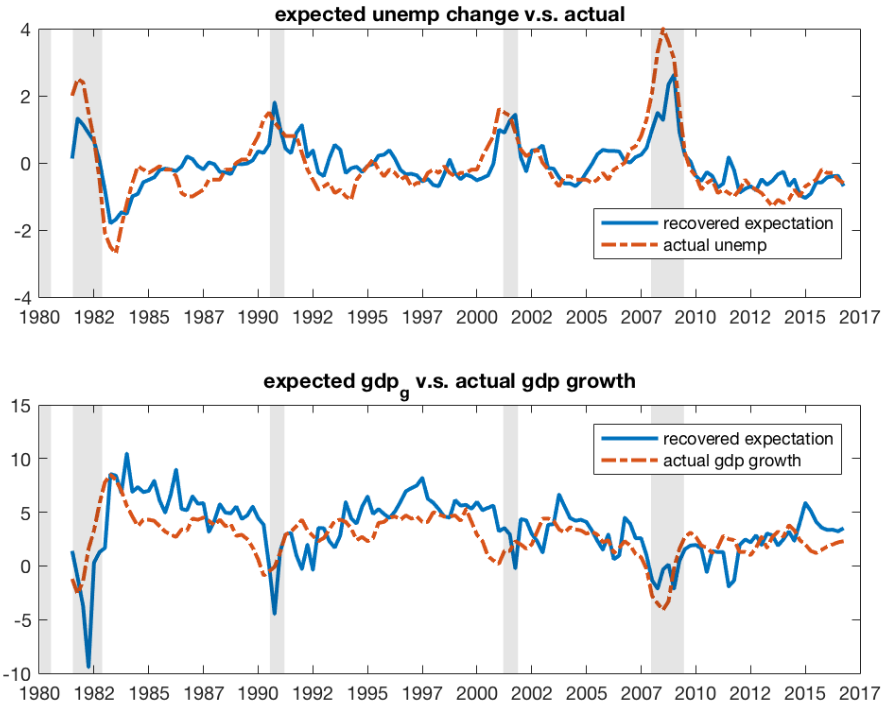

Figure 10: Recovered expected series v.s. realized data. Data from 1981q3 to 2017q4 due to availability of quarterly SPF on CPI inflation.

A.4 Cross-Correlation with recovered data and SPF

In Table 9, I report the same cross-correlation exercise using the imputed data as mentioned before. I also include the cross correlation structure for the same set of expectational variable from SPF and SCE for comparison.

Panel A of Table 9 shows using recovered data from MSC starting from 1981, we still see the stark positive association between expected inflation and worse economic performance(both unemployment increase and business condition worsen). Whereas in SPF we cannot find such a correlation. The cross correlation structure of SPF is very similar to that of the realized data, suggesting the correlation between inflation expectation and the projection of future economic condition is not an artifact of expectation formation in general, but rather a unique feature of household expectation. On the other hand, expectational variables seem to impose a common negative correlation between inflation and interest rate change. This may be because economic agents focus more on the stabilizing effect of monetary policy on future inflation rather than the contemporaneous response of interest rate to possible inflation. Panel B illustrates the cross correlation structure of household is robust to use of monthly data, a more recent time period, and other data source(SCE).

Appendix B Derivation of Noisy Information Model

B.1 Basic stationary Kalman Filter

Consider the ALM and observational equation as in (2) and (3), where $w_{t+1,t}$, v^i_t and η_t are independent normally distributed:

$$w_{t+1,t} \sim N(\mathbf{0}, Q) \quad v^i_t \sim N(\mathbf{0}, R_1) \quad \eta_t \sim N(\mathbf{0}, R_1)$$

Table 7: Correlation: Recovered MSC, SPF, Realized Data and SCE

	Panel A: quarterly 1981q3-2017q4			Panel B: monthly 2013m6-2017m12	
Correlation of:	MSC	SPF	Real time	MSC	SCE
$E\pi, E\Delta un$	0.14^{*}	0.03	-0.00	0.36^{***}	0.32^{***}
$E\pi, E\Delta i$	-0.06	-0.41^{***}	0.08	-0.42^{***}	-0.33^{***}
$E\pi, E\Delta y$	-0.25^{***}	-0.01	0.08	-	-
$E\Delta un, E\Delta i$	-0.35^{***}	-0.36^{***}	-0.63^{***}	-0.46^{***}	-0.18
$E\Delta un, E\Delta y$	-0.63^{***}	-0.79^{***}	-0.78^{***}	-	-
$E\Delta i, E\Delta y$	0.36^{***}	0.44^{***}	0.49^{***}	-	-

*** means significant at 1%,** means 5 % and * means 10%, data in use are quarterly from MSC.

Denote $R = R_1 + R_2$, and the perceived value of $\boldsymbol{L}_{t,t-1}$ for individual i at time t as $\boldsymbol{L}^i_{t,t-1|t}$. At each time t the Filtering process is:

$$\boldsymbol{L}^i_{t,t-1|t} = A\boldsymbol{L}^i_{t,t-1|t} = \boldsymbol{L}^i_{t,t-1|t-1} + K_t(\boldsymbol{s}^i_t - G\boldsymbol{L}^i_{t,t-1|t-1}) \tag{23}$$

The Kalman Filter is given by:

$$K_t = \Sigma_{t|t-1}G'(G\Sigma_{t|t-1}G' + R)^{-1}$$

$$\Sigma_{t+1|t} = A\Sigma_{t|t-1}A' - AK_tG\Sigma_{t|t-1}A' + Q$$

With common beliefs on structural parameters A, G, Q and R, and A given by (2) is stationary. Then $\Sigma_{t+1|t}$ converges to Σ is stationary. The stationary Kalman Gain is neither individual specific nor time specific, denoting as K. Then the expectation is given by:

$$\boldsymbol{L}^i_{t+1,t|t} = A(\boldsymbol{L}^i_{t,t-1|t-1} + K(\boldsymbol{s}^i_t - G\boldsymbol{L}^i_{t,t-1|t-1}))$$

B.2 Derivation of Year-ahead Forecasting Error Rule

Consider the year-ahead consensus forecast $\boldsymbol{L}^c_{t+4,t|t}$ and year-ahead realization $\boldsymbol{L}_{t+4,t}$, using ALM (2) we have:

$$\boldsymbol{L}_{t+4,t} \equiv \sum_{j=1}^{4} \boldsymbol{L}_{t+j,t+j-1} = A\boldsymbol{L}_{t+3,t-1} + \sum_{j=1}^{4} w_{t+j,t+j-1} \tag{24}$$

The consensus expectation from (6) when $h = 4$ we have:

$$\boldsymbol{L}^c_{t+4,t|t} = (A^3 + A^2 + A + I)[A(I - KG)\boldsymbol{L}^c_{t,t-1|t-1} + AKG\boldsymbol{L}_{t,t-1} + AK\eta_t] \tag{25}$$

Meanwhile from (23) and ALM we know:

$$\boldsymbol{L}^c_{t+3,t-1|t-1} = \sum_{j=0}^{3} \boldsymbol{L}^c_{t+j,t+j-1|t-1} = (A^3 + A^2 + A + I)\boldsymbol{L}^c_{t,t-1|t-1}$$

Denote $W = (A^3 + A^2 + A + I)$ and stationarity of A guarantees W is invertible. Plug above equation into (25) we have:

$$\boldsymbol{L}^c_{t+4,t|t} = W[A(I-KG)W^{-1}\boldsymbol{L}^c_{t+3,t-1|t-1} + AKG\boldsymbol{L}_{t,t-1} + AK\eta_t]$$

Now write the forecasting error $FE_{t+4,t|t}$ as defined:

$$\begin{aligned}
FE_{t+4,t|t} \equiv \boldsymbol{L}_{t+4,t} - \boldsymbol{L}^c_{t+4,t|t} &= A\boldsymbol{L}_{t+3,t-1} + \sum_{j=1}^{4} w_{t+j,t+j-1} - \boldsymbol{L}^c_{t+4,t|t} \\
&= WA(I-KG)W^{-1}FE_{t+3,t-1|t-1} + (A - WA(I-KG)W^{-1})\boldsymbol{L}_{t+3,t-1} \\
&- WAKG\boldsymbol{L}_{t,t-1} - WAK\eta_t + \sum_{j=1}^{4} w_{t+j,t+j-1} \\
&= WA(I-KG)W^{-1}FE_{t+3,t-1|t-1} + (A - WA(I-KG)W^{-1})\boldsymbol{L}_{t+3,t-1} \\
&- WAKG\boldsymbol{L}_{t,t-1} + \boldsymbol{L}_{t+3,t} - A\boldsymbol{L}_{t+2,t-1} - WAK\eta_t + w_{t+4,t+3} \\
&= WA(I-KG)W^{-1}FE_{t+3,t-1|t-1} + (I - WA(I-KG)W^{-1})\boldsymbol{L}_{t+3,t-1} \\
&- (I + WAKG)\boldsymbol{L}_{t,t-1} + A\boldsymbol{L}_{t+3,t+2} - WAK\eta_t + w_{t+4,t+3}
\end{aligned} \tag{26}$$

The last equation follows from the fact:

$$\boldsymbol{L}_{t+3,t-1} = \boldsymbol{L}_{t+3,t+2} + \boldsymbol{L}_{t+2,t+1} + \boldsymbol{L}_{t+1,t} + \boldsymbol{L}_{t,t-1} = \boldsymbol{L}_{t+2,t-1} + \boldsymbol{L}_{t+3,t+2}$$

B.3 Simulation Results

To be completed.

Appendix C News Measure from MSC

C.1 Description

The news measures from MSC are usually referred as "perceived news" as the question asked in the survey is:

A6. During the last few months, have you heard of any favorable or unfavorable changes in business conditions?

A6a. What did you hear?

The news reported in this question should be considered as self-reported information, it may contain both public and private information heard by the surveyee. The content of news is described by the surveyee and then categorized into 80 different categories. In Figure 14 I plot the share of surveyees that report hearing any news. And Figure 15 depicts the fraction of agents hearing news about unemployment and inflation conditional on hearing any news.

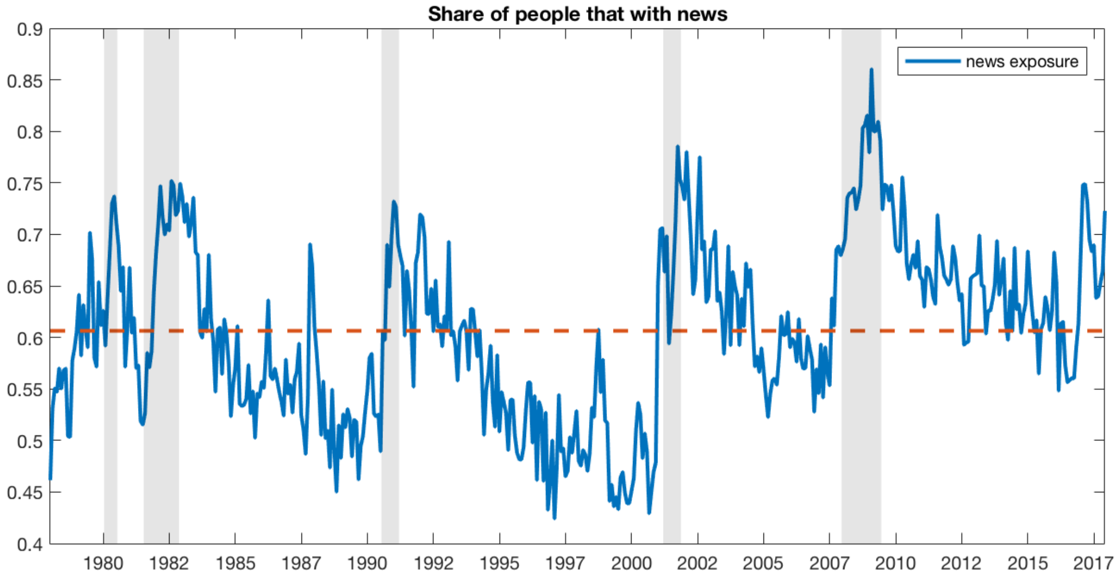

Figure 11: Share of people that report hearing of news

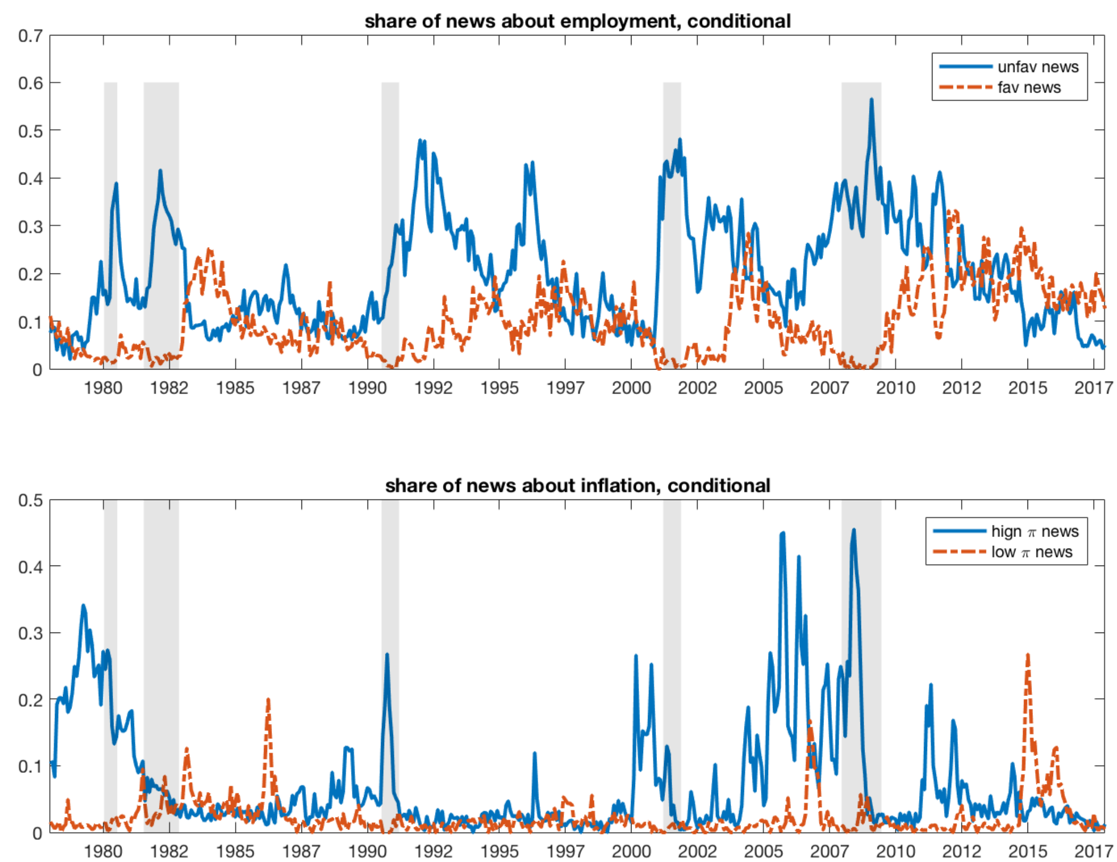

Figure 12: Share of people that report hearing news on employment or inflation, conditional on hearing news. In top panel, the blue line is fraction with unfavourable news on employment and red dash line is fraction with favourable news. In bottom panel, blue line is fraction with news on higher inflation.

On average there are more than 60% agents report they have heard some news about the economy, and the fraction is comoving with business cycle, peaking in each recessions. Among this news about unemployment and inflation accounts for more than 40% on average, peaking at about 80% in the recent recession. And there is an asymmetry in tones of news: the blue curve is almost always above red ones, which suggests agents report to hear of bad news more often than good ones. At first pass it seems agents are making distinctions in labelling news about inflation and employment. Figure 16 plots the specific news against realized data, the news heard is highly comoving with corresponding macroeconomic variable. And the news on inflation is also highly correlated with real oil price (0.51) which indicates households' inflation expectations are sensitive to gas prices, as various researchers have suggested.

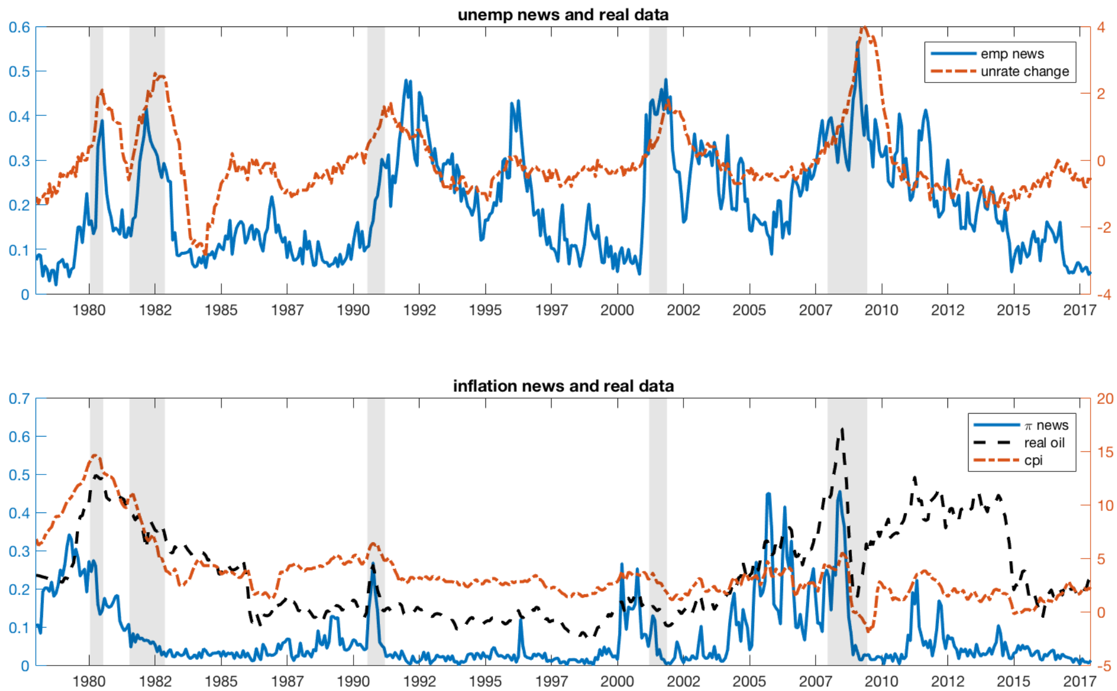

Figure 13: News heard with actual data. In both panels blue lines are fraction of news on employment or inflation, red dash lines are corresponding actual data. In the bottom panel the black dotted line is real oil price obtained from FRED.

C.2 Extra Figures

Figure 17 and 18 are similar matrices to Figure 1 from **Section 4.2**, with more news categories and expectational variables as response variables. Figure 17 are deviation of expectational variables from their unconditional mean, conditional on hearing unfavorable news. Figure 18 are the same exercise conditional on hearing good news.

Figure 14: On y-axis is the news heard for each subgroup, on x-axis is expectation under examination. This figure are responses of receiving bad/unfavorable news.

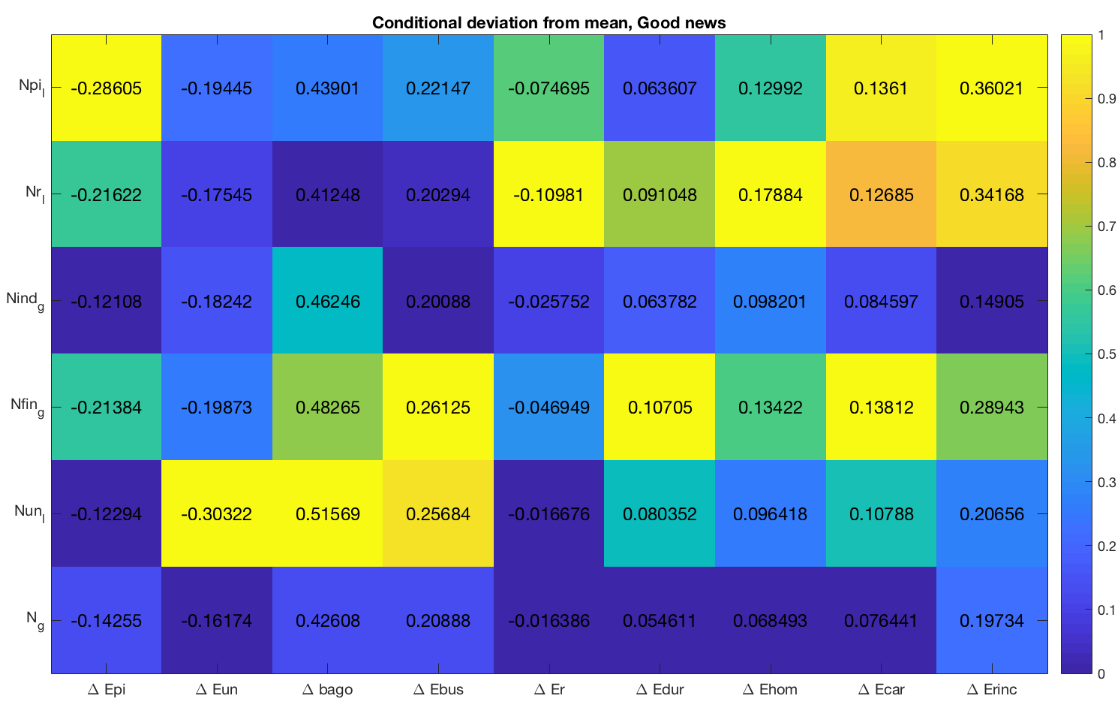

Figure 15: On y-axis is the news heard for each subgroup, on x-axis is expectation under examination. This figure are responses of receiving good/favorable news.

Figure 19 and 20 are similar matrices to Figure 2 from **Section 4.2**, with more news categories and expectational variables as response variables. Figure 19 is for unfavorable news, Figure 20 is for favorable news.

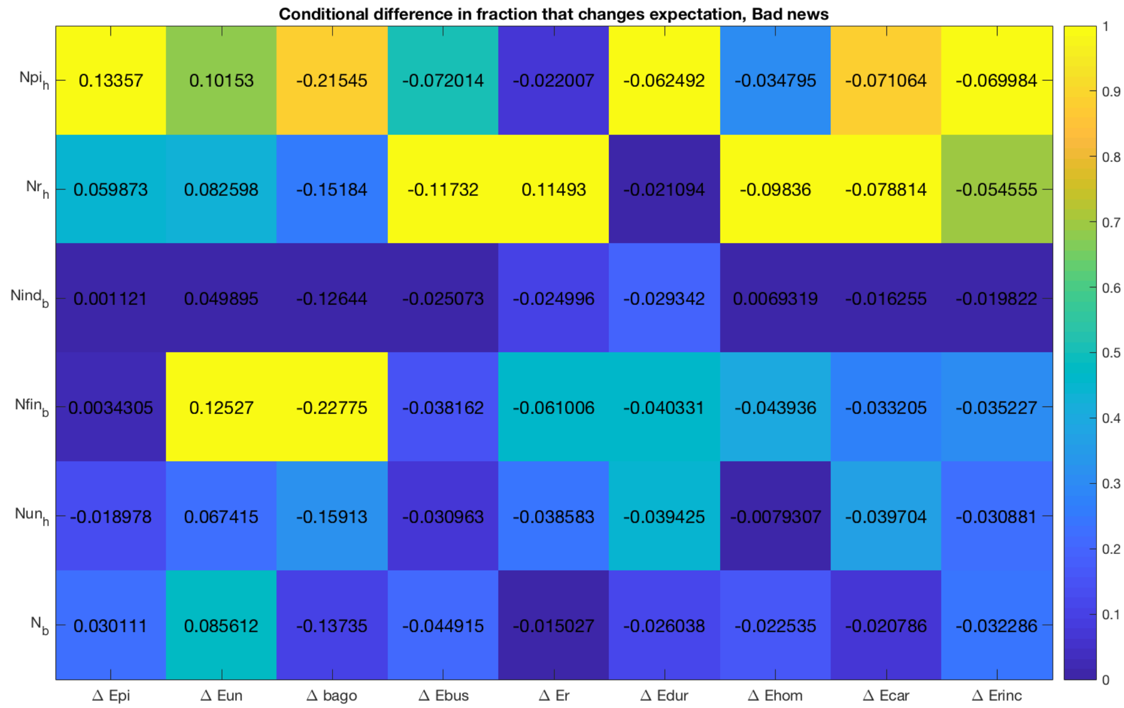

Figure 16: On y-axis is the news heard for each subgroup, on x-axis is expectation under examination. This figure are responses of receiving bad/unfavorable news.

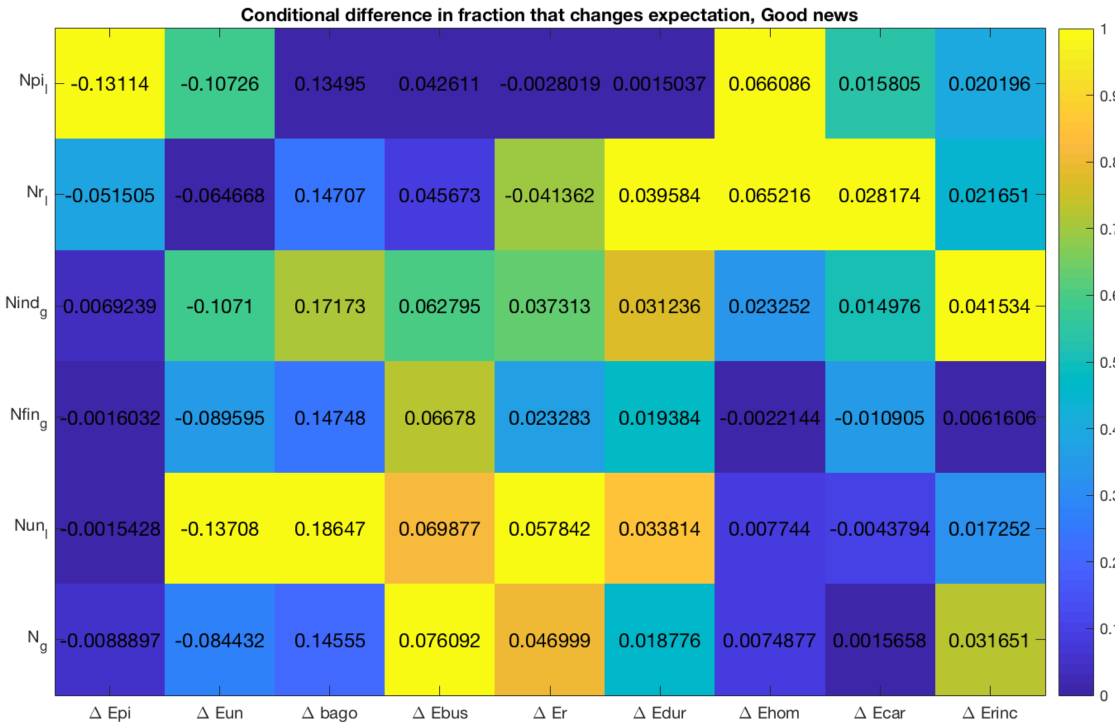

Figure 17: On y-axis is the news heard for each subgroup, on x-axis is expectation under examination. This figure are responses of receiving good/favorable news.

Appendix D Performance of RNN on various learning models

To assess the performance of RNN and show it can consistently recover the mapping between input signals and expectation variables generated by different learning models, I perform tests using simulated data on expected inflation and compare the performance of an LSTM (state-of-art RNN) with OLS and the ground truth. Throughout the tests I consider a slight non-linear ALM, for a single time series π_t, but the results are not sensitive to this form:

$$\pi_t = \rho_1 \pi_{t-1} + \rho_2 f(\pi_{t-1}, \pi_{t-2}, z_t) + e_t$$

Where

$$f(\pi_{t-1}, \pi_{t-2}, z_t) = \frac{\pi_{t-1}^2}{1+\pi_{t-1}} + \rho_3 \pi_{t-2} z_t^2 \quad e_t \sim N(0,1)$$

D.1 Model 1: Malmendier and Nagel (2015)

The first model I considered is from *Malmendier and Nagel (2015)*, it's a learning with experience model. The model is an "ordinary least square learning" model with decreasing gain $\gamma_{t,s}$, where s stands for cohort. Agent from cohort s will run OLS at every period and apply a decreasing weight to the newly available data. With this learning scheme agent perceives different values for parameters in their perceived law of motion and form expectation accordingly. Following their set up, the DGP of expectation data is:

$$\pi_{t+1} = b_0 + b_1 \pi_t + \eta_{t+1} \quad \text{(PLM)}$$

$$b_t = b_{t-1} + \gamma_t R_t^{-1} \boldsymbol{X}_{t-1}(\pi_t - b'_{t-1}\boldsymbol{X}_{t-1})$$
$$R_t = R_{t-1} + \gamma_t(\boldsymbol{X}_{t-1}\boldsymbol{X}'_{t-1} - R_{t-1})$$
$$\gamma_t = \frac{\theta}{t}$$
$$\boldsymbol{X}_t = [1 \quad \pi_t]' \quad b = [b_0 \quad b_1]'$$

For simplicity I dropped cohort s, as in simulation I only consider one guy that born at time 0. Then they form their expectation at time t:

$$E_t\pi_{t+1} = b'_t\boldsymbol{X}_t \tag{27}$$

Now suppose the agent is learning with the above set-up. As an observer we see: $\boldsymbol{X}_t, E_t\pi_{t+1}$ up to each time t, and the hidden variables $\boldsymbol{l}_t$ are b_t, R_t, γ_t. We also don't know the function form that connects the hidden variables, observables and expectation variables.

The recursive mapping from observables (and previous hidden variables) to hidden variables $h(.)$ then can be given by:

$$\gamma_t \equiv h_1(\boldsymbol{X}_t, \boldsymbol{X}_{t-1}, ..., \boldsymbol{l}_{t-1}) = \frac{\gamma_{t-1} + \theta}{\gamma_{t-1}\theta}$$
$$R_t \equiv h_2(\boldsymbol{X}_t, \boldsymbol{X}_{t-1}, ..., \boldsymbol{l}_{t-1}) = R_{t-1} + \gamma_t(\boldsymbol{X}_{t-1}\boldsymbol{X}'_{t-1} - R_{t-1})$$
$$b_t \equiv h_3(\boldsymbol{X}_t, \boldsymbol{X}_{t-1}, ..., \boldsymbol{l}_{t-1}) = b_{t-1} + \gamma_t R_t^{-1}\boldsymbol{X}_{t-1}(\pi_t - b'_{t-1}\boldsymbol{X}_{t-1})$$

Then the expectation formation model $f(.)$ is given by:

$$E_t\pi_{t+1} \equiv f(\boldsymbol{X}_t, \boldsymbol{X}_{t-1}, ..., \boldsymbol{l}_t) = b'_t\boldsymbol{X}_t$$

Notice now I have shown the learning by experience model can be written as the system I described in **Section 5.1** and this specific model has the following properties:

1. Hidden states are learned parameters by agents, they are non-linear functions of observables, and at time t the hidden states b_t and R_t have long and varying time dependency on observables;

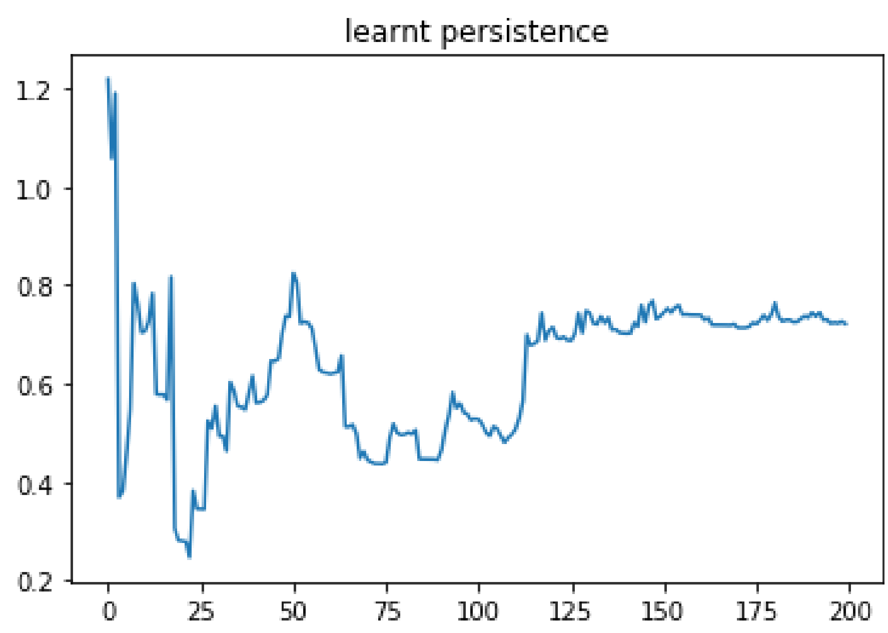

Figure 18

2. The response of expectation to new signal $\boldsymbol{X}_t$ is non-linear and time-varying, the non-linearity exists even though the PLM is linear. It comes from the learning part as it contributes to form belief on b_t.

The LSTM I use have one recurrent layer with 32 ReLu neurons and it is connected to a fully connected dense layer with 64 ReLu neurons. The LSTM will capture both these two properties that are otherwise difficult to be recovered, without knowing the exact learning procedure of the agent. Specifically, point 1 will be captured by the recurrent layer that keeps a high-dimensional representation of the hidden variables, and point 2 will be captured by the dense layers after the recurrent ones that can approximate flexible function forms.

I simulate the expectational data according to the previous learning model and use LSTM to learn the expectation formation model.

Out-of-sample Prediction: First we consider the simple task: how well the LSTM predicts the expectation on test set (80 period), I train the model with random starting weights for 30 times and report the mean squared error of the prediction and compare it with an OLS, with the same set of variables as predictors.

Table 8: Performance of LSTM v.s. OLS2

Distance to:	Training Set	Test Set			
LSTM baseline	0.07	0.15			
OLS2	0.08	0.2			
Stability of LSTM: (MSE)	*mean*	*median*	*min*	*max*	*std*
	0.15	0.12	0.04	0.6	0.11

* This is run with different random initial weights, but fixed initial states of learning.

It shows the LSTM results are quite stable in terms of different initial weights used, and it learns to generate expectation on the unseen test sets pretty well. With the best result (remember the initial weights are tuning hyper parameter, so picking the result with best performance is like tuning the initial weights), it reduces the MSE for around 5 times comparing to OLS. This is very impressive given the fact that when using the test set that is further down in time, the expectation formation process is closer to a linear rule.

Marginal Effect of Input: One crucial test for LSTM is that whether it can capture the non-linearity in expectation formation process. Given this special DGP in *Malmendier and Nagel(2015)* (hence force M-N), the policy function is quite complicated: it's non-linear and time-varying. This means: (1) at a fixed episode time t, current realization of inflation affect expectation non-linearly; (2) at different point of time this non-linearity is different. The first point is associated with non-linearity of NN, the second is associated with time-dependency introduced by recursive neurons.

I plot the policy function at an early date $t = 50$ and $t = 150$, the orange line is ground truth, blue one is recovered by LSTM, the $t = 50$ one is in training set, $t = 150$ is in test set.

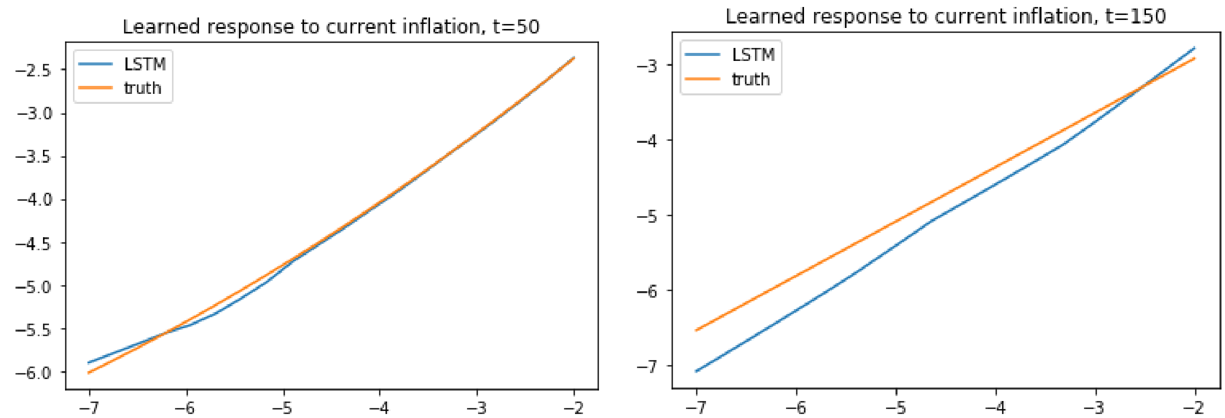

Figure 19

The two graphs show that indeed the response of expectation to current inflation is non-linear and it is particularly so in early periods. This illustrate point (1) mentioned above. For point (2), we can plot the implied slope of expectation to current inflation π_t, at different point of time. This is approximately the "learned persistent" in Figure 7. This shows one of the most important benefit of using RNN: it can capture time dependent responses. In this example the time-dependency comes from the learned persistent parameter b_t by agent, which is unobservable to econometrician but it is a non-linear function of past data available X_t. Given different learned values of b_t along time, for the same X_t observed by agent, he will adjust his expectation differently. This difference is depicted in Figure 7. The orange line is b_t that is generated by the true learning with experience model I presented before and the blue curve are the time-varying marginal effect of X_t that is recovered by LSTM, it captures the time-dependency of true model quite well. On the other hand, if we plot the same marginal effect for OLS it will be a flat line with no time variation at all.

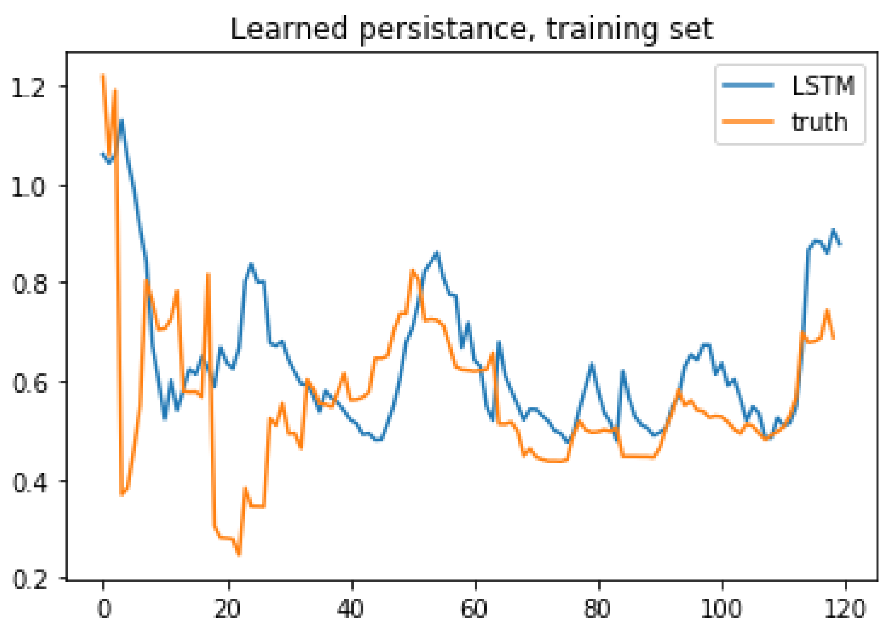

Figure 20

Neglected Variable z_t: As we described the actual law of motion, there is a variable z_t that affects the actual inflation. However in forming expectations, agents didn't use this piece of information. Now if one uses LSTM to predict actual inflation, the effect of z_t will be picked up by the model. However it shouldn't have any impact on expectational variables as it is neglected by agents. This is similar to the sparsity/ inattention models, for some reasons agents may choose to or occasionally miss some important information, we then expect LSTM to "make the same mistake" as the agent does.

The following is the policy function for changes of this z_t variable in the test set at time $t = 130, 150$ and 170, all are virtually with zero slope.

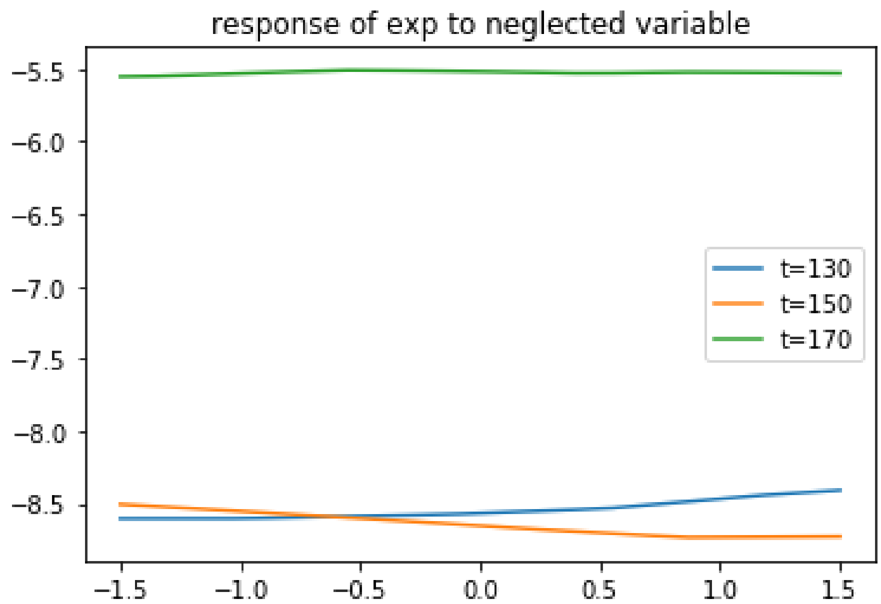

Figure 21

The mean slope across all the time period is -0.04 with standard error 0.04, which is not significant from 0. This can be compared to the OLS results that yield significant positive 0.16 on z_t.

This means in presence of sparsity or inattention, LSTM model will not pick up the signals that are not used by agents, even if it affects the actual inflation process.

D.2 Model 2: Kalman Filter with Hidden State

I then turn to a noisy information model similar to that was discussed in **Section 4**. This is to show LSTM is not only appropriate to learn expectation formations that feature recursively learned parameters, it can also help to learn models with fixed parameter, but the latent variables are recursive learned by solving signal extraction problem.

Suppose with the same ALM, instead of learning with experience, the agent believes in a simple PLM with a latent factor L_t:

$$\pi_t = \phi\pi_{t-1} + \beta L_t + \epsilon_t$$

$$L_t = \rho L_{t-1} + \eta_t$$

The observational equations are:

$$s_t = gL_t + v_t^i$$

$$\pi_t = \pi_t$$

Then agent observes a vector $[s_t, \pi_t]$ at each time t and update their belief about the latent variable $\hat{L}_{t|t}$. With the latent variable they form expectation for next period:

$$\begin{bmatrix} E_t \pi_{t+1} \\ E_t L_{t+1} \end{bmatrix} \equiv X_{t+1|t} = A(X_{t|t-1} + K(O_t - GX_{t|t-1}))$$

Now as an outsider, we cannot observe the perceived latent variable $E_{t-1}L_t$, this is then the hidden state that will be handled by LSTM. In this case we can use all the observables at time t as input of LSTM:

$$\boldsymbol{X}_t = \begin{bmatrix} \pi_t \\ s_t \\ \pi_{t|t-1} \end{bmatrix}$$

And the hidden variable $l_t = E_{t-1}L_t$. The "hidden state" $\boldsymbol{h}_t$ in LSTM then is:

$$\boldsymbol{h}_t = \begin{bmatrix} \pi_t \\ s_t \\ \pi_{t|t-1} \\ E_t L_{t+1} \end{bmatrix}$$

The mapping $h(.)$ then is defined by the linear Kalman Filter relation.

Notice, in the simplest single variable Kalman Filter model, h_t is usually omitted, this is less of an issue if there is very weak dependency between L_t and π_t, which is β in our formulation. However this will have substantial impact if β is big. I will compare the performance of an OLS which is the same as single variable KF model that neglects the hidden state l_t, in the case $\beta = 0.9$.

Out-of-sample Prediction: Similar to that from section 5.2.1, I summarize the performance of OLS, LSTM in terms of MSE on test set in Table 7. The LSTM outperforms OLS in all the Training, Validation and Test sets, by decreasing MSE by at least 4 times. This is because in OLS one doesn't observe one of the latent variable $E_t L_{t+1}$ thus assuming there is only one latent variable $E_t \pi_{t+1}$. This is similar to the case of single-variable learning model when the true data generating process is instead joint-expectation formation, where the researcher using single-variable models simply neglects another latent variable that is correlated with regressors thus having omitting variable bias in estimators.

However the RNN will try to capture the extra latent variable through the recurrent layer. Because the extra latent variable describes nothing but the correct dependency of outcome variable, $E_t \pi_{t+1}$, on past signals s_{t-s} and π_{t-s}. The RNN will directly try to approximate this dependency, whereas OLS captures such dependency by the mis-specified structure.

Table 9: Performance of LSTM v.s. OLS2

Distance to:	Training Set	Validation Set	Test Set
LSTM baseline	0.019	0.028	0.032
OLS2	0.18	0.13	0.12

* This is run with different random initial weights, but fixed initial states of learning.

Impulse Response: The intuition above can also be verified if we look at the implied response of $E_t \pi_{t+1}$ to one-time 1 standard deviation change on signal s_t. In Figure 10, x-axis are periods after the change in s_t. The blue line is the ground truth of response along time for such a signal, the green line is that response as predicted by LSTM and orange line depicts the same response as implied by OLS. The response of OLS clearly under-estimate the persistence of the response. This is because it missed the persistence that comes from the extra latent variable L_t, whereas LSTM capture this time-dependency correctly.

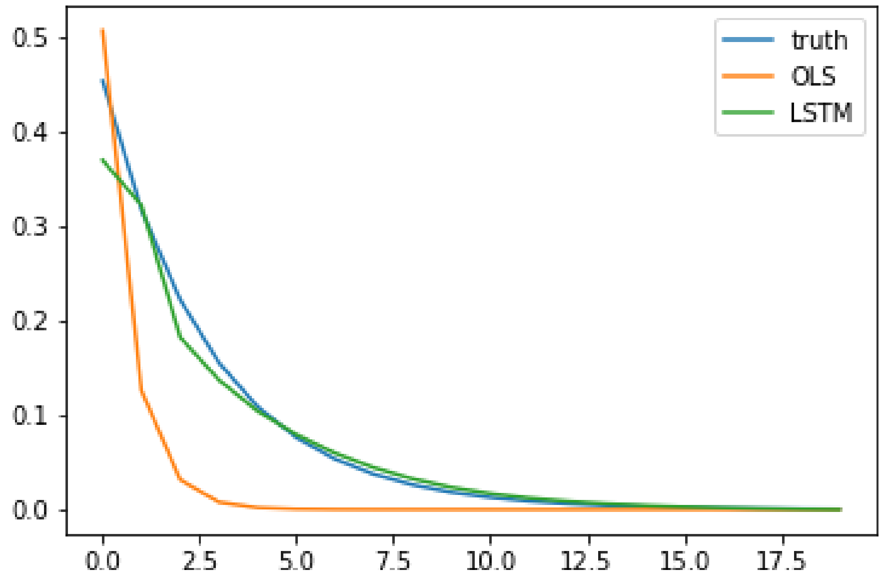

Figure 22

With these two examples, I showed LSTM/RNN out-performs mis-specified linear models when dealing with different classes of learning models. Its advantages in performance include and go beyond more accurate out-of-sample predictions. It can capture state-dependent and non-linear marginal effects of signals as well as the dynamic responses well, and is robust to missing latent variable problem I discussed in previous sections. These properties make LSTM/RNN especially suitable to explore the link between expectational variables and signals.

References

[1] Philippe Andrade and Hervé Le Bihan. Inattentive professional forecasters. *Journal of Monetary Economics*, 60(8):967–982, 2013.

[2] Robert Barsky and Lutz Kilian. Do We Really Know that Oil Caused the Great Stagflation? A Monetary Alternative. 2001.

[3] Jess Benhabib and Chetan Dave. Learning, large deviations and rare events. *Review of Economic Dynamics*, 17(3):367–382, 2014.

[4] Anmol Bhandari and Paul Ho. Survey data and subjective beliefs in business cycle models âĹŮ. (22225), 2017.

[5] Mary A Burke and Ali Ozdagli. Household Inflation Expectations and Consumer Spending: Evidence from Panel Data. *Working Paper Series (Federal Reserve Bank of Boston)*, 13(25):1–43, 2014.

[6] Carlos Capistrán and Allan Timmerman. Disagreement and Biasses in Inflation Expectations. *Banco de México, Documento de Investigación*, 41(2006-07), 2006.

[7] By Christopher D Carroll, Jeffrey C Fuhrer, and David W Wilcox. Does Consumer Sentiment Forecast Household Spending ? If So , Why ? 84(5):1397–1408, 2016.

[8] Carlos Carvalho and Fernanda Nechio. Do people undestand monetary policy? 66:108–123, 2012.

[9] R Clarida, J Gali, and M Gertler. Monetary policy rules and macroeconomic stability: Evidence and some theory. *Quarterly Journal of Economics*, 115(1):147–180, 2000.

[10] Olivier Coibion and Yuriy Gorodnichenko. What Can Survey Forecasts Tell Us about Information Rigidities? *Journal of Political Economy*, 120(1):116–159, 2012.

[11] Isabel Correia, Emmanuel Farhi, Juan Pablo Nicolini, and Pedro Teles. Unconventional Fiscal Policy. *American Economic Review*, 103(4):1172–1211, 2013.

[12] Richard K Crump, Stefano Eusepi, Andrea Tambalotti, and Giorgio Topa. Subjective Intertemporal Substitution. *SSRN Electronic Journal*, (July), 2015.

[13] Francesco D'Acunto, Daniel Hoang, and Michael Weber. The effect of unconventional fiscal policy on consumption expenditure. *CESifo DICE Report*, 15(1):9–11, 2017.

[14] Du Du and Redouane Elkamhi. Learning About Rare Economic Disasters and Asset Prices. (852):1–48, 2012.

[15] M Ehramnn and D Pfajafar. Consumer attitudes and the epidemiology of inflation expectations. 2014.

[16] Cross-sectional Evidence, Rüdiger Bachmann, Tim O Berg, and Eric R Sims. Inflation Expectations and Readiness to Spend-skripsi.pdf. 7(1):1–35, 2012.

[17] Hibiki Ichiue and Shusaku Nishiguchi. Inflation expectations and consumer spending at the zero bound: Micro evidence. *Economic Inquiry*, 53(2):1086–1107, 2015.

[18] Guy Laroque, Gaetano Gaballo, John Leahy, Jennifer Lao, Stephen Morris, Heraklis Polemarchakis, Edouard Schaal, Martin Schneider, Karl Shell, Michal Lukasz Szkup, Laura Veldkamp, and Michael Woodford. Sentiments and Aggregate Demand Fluctuations. (314), 2014.

[19] Carlos Madeira and Basit Zafar. Heterogeneous Inflation Expectations and Learning. *Journal of Money, Credit and Banking*, 47(5):867–896, 2015.

[20] Ulrike Malmendier and Stefan Nagel. Learning from inflation experiences. *Quarterly Journal of Economics*, 131(1):53–87, 2016.

[21] Ulrike Malmendier and Leslie Sheng Shen. Scarred Consumption. *National Bureau of Economic Research Working Paper Series*, No. 24696, 2018.

[22] N. Gregory Mankiw and Ricardo Reis. Imperfect information and aggregate supply. *Handbook of Monetary Economics*, 3(C):183–229, 2010.

[23] By Albert Marcet and Juan P Nicolini. Recurrent Hyperinflations and Learning. 93(5):1476–1498, 2003.

[24] Fabio Milani. Expectations, learning and macroeconomic persistence. *Journal of Monetary Economics*, 54(7):2065–2082, 2007.

[25] Giorgio Topa Olivier Armantier Wilbert van der Klaauw, and Basit Zafar. An Overview of the Survey of Consumer Expectations. (800), 2016.

[26] Athanasios Orphanides. Monetary-Policy Rules and the Great Inflation. *American Economic Review*, 92(2), 2002.

[27] Athanasios Orphanides. Monetary Policy Rules , Macroeconomic Stability , and Inflation : A View from the Trenches. *Journal of Money , Credit and Banking*, 36(2):151–175, 2004.

[28] Athanasios Orphanides and John Williams. *Imperfect Knowledge, Inflation Expectations, and Monetary Policy*. Number December. 2003.

[29] Athanasios Orphanides and John C. Williams. Inflation scares and forecast-based monetary policy. *Review of Economic Dynamics*, 8(2 SPEC. ISS.):498–527, 2005.

[30] Damjan Pfajfar and Emiliano Santoro. Heterogeneity, learning and information stickiness in inflation expectations. *Journal of Economic Behavior and Organization*, 75(3):426–444, 2010.

[31] Stefano Eusepi; Bruce Presto. Expectations , Learning , and Business Cycle Fluctuations. *American Economic Review*, 101(6):2844–2872, 2011.

[32] Savitar Sundaresan. Emergency Preparedness: Rare Events and the Persistence of Uncertainty. *Working Paper*, pages 1–42, 2015.

[33] Thomas A . Lubik and Frank Schorfheide. Testing for Indeterminacy : An Application to U . S . Monetary Policy. *American Economic Review*, 94(1):190–217, 2004.

[34] Michael Woodford and Gauti Eggertsson. The Zero Bound on Interest Rates and Optimal. *International Monetary Fund*, 2003.