Monetary Policy when the Phillips Curve is Locally Quite Flat

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Abstract

This papers begins by highlighting how the presence of a cost channel of monetary policy can offer new insights into the behavior of inflation when the Phillips curve is locally quite flat. For instance, we highlight a key condition whereby lax monetary policy can push the economy in a low inflation trap and we discuss how, under the same condition, standard policy rules for targeting inflation may need to be modified. In the second part of the paper we explore the empirical relevance of the conditions that give rise to these observations using US data. To this end, we present both *(i)* a wide set of estimates derived from single-equation estimation of the Phillips curve and *(ii)* estimates based on structural estimation of a full model. The results from both sets of empirical exercises strongly support the key condition we derived.

Key Words: Monetary Policy, Inflation, Interest Rates;
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Introduction

In many industrialized countries, the inflation rate has been below target for several years despite the fact that, prior to the Covid-crisis, monetary policy had been sufficiently expansive to support unemployment rates that were close to historical lows.These low inflation outcomes could have reflected a correlated reduction in the natural rate of unemployment across countries. However, such explanation appears unlikely given that only a few years ago the predominant puzzle was missing deflation with high unemployment. A more plausible candidate explanation for these outcomes is that the Phillips curve may be quite flat, at least locally.[1]

The object of this paper is to explore the implications and empirical relevance of a relatively flat Phillips curve when a cost channel is present. The paper is divided into two main parts. In a first section, we highlight a set of theoretical implications of having a relatively flat Phillips curve in the presence of a cost channel. In particular, we examine this issue in an environment where the flatness of the Phillips curve is only a local phenomenon. This will allow us to contrast how monetary policy may act in normal times, i.e. when the economy is not too far from its steady state and the Phillips curve is relatively flat, as opposed to special times, i.e. when the economy is in more extreme circumstances and in a region where the Phillips curve may be steep. As we shall show, this type of environment will offer a simple explanation for why inflation can get stuck below target, with low unemployment even if monetary policy appears quite aggressive. Such an outcome depends on parameters of the Phillips curve as well as on the sensitivity of aggregate demand to interest rates. Accordingly, in the second and third sections of the paper we explore the empirical plausibility for this parameter configuration. To this end, we present both *(i)* estimates derived from single-equation estimation of the Phillips curve and *(ii)* estimates based on structural estimation of a full model. In most of our estimations, we adopt a standard linear specification as to examine whether the relevant condition is reflected in average outcomes of the economy and therefore relevant in normal times. We complement these linear estimations with non-linear estimates which provide support to the notion that the effects we find may be local

[1] When referring to the slope of the Phillips curve we are referring to the partial relationship between inflation and a measure of market tightness such as either the output gap or the labor gap.

phenomena and that monetary policy may in fact have more conventional implications when the economy is far from its steady state.

In terms of monetary policy, the findings of this paper have novel implications for how policy should be conducted to keep inflation close to an inflation target. In particular, our results support what may be called a "Go Big or Stay Home" principle. This refers to the idea that stabilizing inflation in response to shocks may require either bold moves or, when that is not feasible, doing nothing rather than making small moves. To be more precise, in our framework, stabilizing inflation in response to either demand shocks or supply shocks may require rather large changes in real interest rates, that is, it may be necessary to "Go Big" to help inflation remain on target. However, and more importantly, if one cannot "Go Big" – for example due to an Effective Lower Bound constraint or due to some institutional constraints– then going only part of the way may be worse than "Staying Home". Our notion of "Staying Home" refers simply to the policy of maintaining real interest rates at their long run steady state value independent of shocks. If instead, a central bank tries to fine tune inflation using limited interest rates movement, this can generate outcomes where inflation is either persistently below (with low unemployment) or persistently above (with high unemployment) its target.

In a similar vein, our framework suggests that, when trying to compensate for past departures from inflation target, inflation targeting central banks should not aim for quick redress by adopting slightly more aggressive non-standard interest rate policy. Within our framework, this is precisely the type of strategy that can cause a persistent deviation of inflation from target. In such a situation, it is likely best to leave bygones be bygones and return quickly to a historical rule that has given good inflation results in the past. Overall, our framework gives conditions under which in the presence of small shocks, a passive policy of maintaining real interest rates at their long run value may keep inflation closer to its target compared to an active policy that continuously reacts to the state of the economy but does so using limited tools.

The remaining sections of the paper are structured as follows. In Section 1, we derive some simple theoretical implications of having a relatively flat Phillips curve when a cost channel may also be operative. We show under which conditions this can change how an inflation targeting monetary authority should conduct policy to stabilize inflation. In this section we also discuss how an economy can get stuck in a low inflation trap. In Section 2, we begin by examining the plausibility of the parameter configuration of interest by presenting estimates of the Phillips curve when we allow for a cost channel. We consider both specifications where we impose rational expectations as well as cases where we allow for departures from rational expectations (by using measures of expectations drawn from survey data). Since the proper identification of Phillips curve parameters is key, we explore several different instrumental variable strategies. In Section 3, we complement this partial equilibrium evidence by presenting estimates derived by the Bayesian estimation of the full model. Finally, in Section 4, we offer some concluding comments.

1 Monetary Policy Implications of a Relatively Flat Phillips Curve

The aim of this section is to highlight how the slope of the Phillips curve – or more precisely the sensitivity of the real marginal cost to market tightness–can affect the link between monetary policy and inflation stabilization. We define a condition – the "Patman condition" – under which restrictive monetary policy can increase inflation. Throughout this section, it is important to emphasize that we will only be examining positive implications of different interest rate stances with a focus on stances aimed at stabilizing inflation. We will not be doing welfare analysis nor looking for optimal rules.

1.1 The Patman Condition

Since most of the elements of the model we use are rather standard, explicit derivation of the main equations presented here and used in the estimation is presented in Appendix C. Our starting point are the two key equations of the canonical New Keynesian setup, where we introduce minor changes. As is standard, all variables are expressed in deviations from the steady state. We are abstracting from capital accumulation, and we are assuming that technological progress follows a deterministic trend. Deviations of economic activity from its steady state therefore correspond to deviations of employment from its steady state. As a result, when talking about market tightness, we can refer interchangeably to the output gap or the labor gap.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \, \mathrm{mc}_t + \mu_t, \tag{1}$$

$$y_t = \alpha_y E_t y_{t+1} - \alpha_r (i_t - E_t \pi_{t+1}) + d_t, \qquad 0 < \alpha_y < 1. \tag{2}$$

Equation (1) is the New Keynesian Phillips curve where inflation depends on expected inflation, the real marginal cost and on a markup shock μ_t. This equation is entirely conventional. Equation (2) is an Euler equation (the forward looking IS curve) which is subject to preference shocks d_t. In this equation, we allow for a discounted Euler equation specification by having $0 < \alpha_y < 1$ (we provide micro-foundations for this in Appendix C). Such a modification is not very substantial as we allow α_y to be arbitrarily close to one. However, it has the advantage of allowing us to consider a wider set of monetary policy rules without needing to worry about a unit root (induced when α_y is exactly equal to one). In particular, we are able to consider environments where a central bank aims to influence real interest rates which, in addition to being plausible, will be very convenient.

The main element we want to focus on is our specification of the real marginal cost as given by Equation (3):

$$\mathrm{mc}_t = \gamma_y y_t + \gamma_r (i_t - E_t \pi_{t+1}). \tag{3}$$

In Equation (3), we are including the real interest rate in the marginal cost. In Appendix C, we show how this formulation can arise in the presence of intermediate goods that are financed at the beginning of the period. It is common to refer to this term γ_r as the cost channel of monetary policy, even though the cost channel of monetary policy is most often associated with nominal interest rates affecting the real marginal cost. As our main results are not substantially modified by allowing for a nominal versus a real interest rate in the cost channel, we choose to maintain a real cost channel specification which is theoretically more appealing, offers clearer results and, most importantly, finds greater support in our later estimation.

The central message we want to convey in this section is that the way monetary policy influences inflation is closely tied to a particular condition involving α_r, γ_y and γ_r. We will call it the Patman condition after US Senator Patman who argued in the late 1970s that the Fed's policy of increasing interest rates could be more of a contributor to inflation than a cure.[2]

In an economy given by equations (1), (2) and (3), a marginal increase in the real interest rate $i_t - E_t\pi_{t+1}$ has two effects on current inflation π_t, holding expectations constant. A direct effect γ_r goes through the impact on the marginal cost. An indirect effect $-\alpha_r\gamma_y$ runs through y_t.

Definition 1. Patman Condition: *Current inflation will increase following a rise in real interest rate, holding expectations constant, if and only if the Patman condition $\gamma_r > \alpha_r\gamma_y$ is satisfied.*

Accordingly, it is not surprising that the effect of monetary policy on inflation will be influenced by this condition. It is important to point out that most models with a cost channel have micro-foundations that rule out the possibility that $\alpha_r\gamma_y \leq \gamma_r$ (see Appendix B). [3] However, this is not the case for the micro-foundations we present in Appendix C.

1.2 The Patman Condition Cannot Hold Globally

The main problem associated with a model in which the Patman Condition holds is that it can at most be thought of as a local possibility, that is, as a condition that is only valid over a restricted domain. The easiest way to see this is to consider the case where $\alpha_r\gamma_y$ is exactly equal to γ_r. Despite being a knife edge case, it helps illustrate the type of untenable implication one could get by focusing on a linear model with $\alpha_r\gamma_y \leq \gamma_r$. In this knife edge case, a monetary authority could in principle engineer a temporary activity expansion of arbitrary size without having any effect on inflation. This is certainly not reasonable as we know that resources are bounded, implying that, at some point, marginal cost has to increase if interest rates have sufficiently been reduced and activity becomes sufficiently high. Therefore, the condition $\alpha_r\gamma_y \leq \gamma_r$ only makes sense, if at all, as a local condition. There are two ways one could address this. On the one hand, we could stay with

[2]The view that tight monetary policy could be inflationary was discussed in Tobin [1980]:

> *"More fundamentally, heretics from the populist Texas Congressman, Wright Patman, to John Kenneth Galbraith have disputed the orthodox view that tight money policies are anti-inflationary, claiming that borrowers mark up interest charges like other cost." (page 35)*

See also Driskill and Sheffrin [1985] who introduced interest costs into Taylor's [1979] model of overlapping wage contracts.

[3]However, this is not true for larger DSGE models. See for instance Rabanal [2007].

the above linear specification (as implied by the first order approximation of our model) and discuss implications only for small shocks. The drawback is that it might be hiding some of the potential constraints on monetary policy and thereby can potentially lead to very misleading policy prescriptions. On the other hand, we could study the full non-linear version of our micro-founded model so that the global constraints are made explicit. Here, the drawback is that it complicates the exposition enormously. We therefore choose to take an intermediate route. To keep the analysis transparent, we will consider an environment where we make the relevant global constraints between marginal cost and market tightness explicit, but we restrict our attention to this one non-linearity. In particular, we will assume that the marginal cost is of the form:

$$\mathrm{mc}_t = \Gamma(y_t) + \gamma_r(i_t - E_t\pi_{t+1}). \tag{4}$$

In this slight generalization, we are now allowing market tightness, represented by y_t, to affect the marginal cost in a non-linear way as captured by the function $\Gamma(\cdot)$. The main assumptions made on $\Gamma(\cdot)$ are that *(i)* $\Gamma'(\cdot) \geq 0$, *(ii)* $\Gamma'(\cdot)$ is convex and has its minimum at 0 [4] and *(iii)* the $\lim_{x->\infty} \Gamma'(x) = \lim_{x->-\infty} \Gamma'(x) = \infty$. These assumptions imply that the Phillips curve is upward sloping but flatness occurs near the steady state. Moreover, these assumptions allow for the possibility of the Phillips being quite flat – and even linear – for a wide range of y_t around zero, while limiting that such a flatness for extreme values of y_t. In this non-linear environment, our Patman condition needs to be restated as $\alpha_r\gamma_y \leq \gamma_r$, now expressed as a local condition related to the steady state with $\gamma_y = \Gamma'(0)$. As we want to consider an environment with only one steady state consistent with $\pi = 0$, we restrict our attention to the case where $\alpha_r\Gamma'(0) \geq (1-\alpha_y)\gamma_r$. We do recognize that our approach of introducing a non-linearity in one mechanism only may appear quite arbitrary even if it allows for simple exposition.[5] For this reason, in our basic estimations, we will always start by reporting results based on the standard linear approximation formulation of our model and examine if the Patman condition holds in that case. Only after that will we examine whether the condition continues to hold when we allow for non-linearities in the Phillips

[4]Note that as all variables are in deviations from the steady state, $\Gamma(0) = 0$.

[5]This approach of only introducing only one non-linearity is shared with a large fraction of the literature on the Effective Lower Bound.

curve.

1.3 Implications for Monetary Policy

We first need to emphasize that, as long as monetary policy is conducted in a way that maintains equilibrium determinacy, the Patman condition does not generally affect how the economy qualitatively responds to either demand or supply (markup) shocks. In particular, for a large class of monetary rules, it is easy to show that demand shocks will always cause both activity and inflation to rise regardless of whether the Patman condition is met or not. Similarly, cost push shocks will lead to higher inflation and lower activity regardless of whether this condition is met. This observation is very important as it implies that the relevance – or irrelevance – of the Patman condition can not be evaluated by simply examining the qualitative properties of how the economy reacts to such shocks. To see this, the easiest is to consider demand shocks and markup shocks sequentially. For demand shocks (d shocks), if the response of monetary authorities is to increase real interest rates but do not over-compensate by causing a fall in activity (which is the case for a large set of policy rule including the form $i_t = E_t[\pi_{t+1}] + \phi_d d_t$ with $\phi_d < \frac{1}{\alpha_r}$), then the demand shock will lead to an increase in both inflation and output regardless of whether the Patman condition is met or not. For markup shocks (μ shocks), if the response of monetary authorities is to increase real interest rates but do not over-compensate by leading to a fall in inflation (which again is the case for a large set of rules), then the shock will lead to both an increase in inflation and a fall in activity regardless of whether the Patman condition is met or not.

We now turn to deriving implications of how different monetary stances affect the properties of the system given by equations (1), (2) and (3). In particular, we will want to emphasize how traditionally prescribed anti-inflationary responses to shocks can have qualitatively different effects on inflation depending on whether the Patman condition is met or not. For example, in response to either a supply shock or a demand shock, the standard prescription for an inflation targeting central bank is to engineer an increase in real interest rates as to slow down the economy and thereby bring inflation closer to its target. In our setup, this insight is maintained as long as the Patman condition is not met, as expressed in Proposition 1. Note that in the following three propositions, we take the economy to be initially at its steady state.

Proposition 1. *Suppose the Patman condition is not met. Then, in response to either a positive (negative) supply shock or demand shock, engineering a rise (drop) in real interest rates – as long as it is not too large or too persistent – will always help bring inflation closer to its target relative to keeping interest rates at their steady state value.*[6]

This proposition highlights that the changes we have made relative to a standard New Keynesian setting – in terms of both the non-linearity $\Gamma(\cdot)$ and the cost channel– do not alter how monetary policy can be used to help stabilize inflation if the Patman condition is not met. The content of this proposition is illustrated on Figure 1, where in this figure we plot inflation at time t as a function of the real interest rate at time t (under the assumption that this real interest rates is held constant N periods, before returning to zero). The resulting relationship, which simply combines equations (1) and (2) (for arbitrary processes for μ_t and d_t), is given explicitly by :

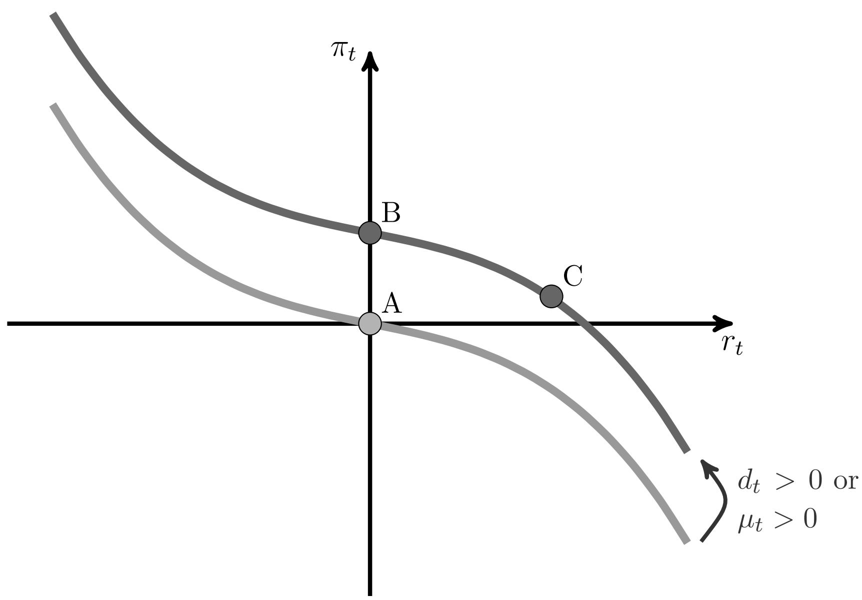
$$\begin{aligned}\pi_t =&\kappa \sum_{j=0}^{N-1} \beta^j E_t \Gamma\Bigg(-\alpha_r \frac{1-\alpha_y^{N-j}}{1-\alpha_y}\, r + \sum_{k=0}^{\infty} \alpha_y^k E_{t+N+j} d_{t+N+j+k} \Bigg) \\ &+\kappa \sum_{j=0}^{\infty} \beta^{N+j} E_t \Gamma\Bigg(\sum_{k=0}^{\infty} \alpha_y^k E_{t+N+j} d_{t+N+j+k} \Bigg) \\ &+\kappa \gamma_r \frac{1-\beta^N}{1-\beta}\, r + \sum_{j=0}^{\infty} \beta^j E_t \mu_{t+j}.\end{aligned} \tag{5}$$

The important property of the resulting relationship between π and r– as illustrated in Figure (1)– is that it is negatively sloped for all values of r, regardless of the value of $N > 0$. Moreover, both demand and supply shocks shifts this curve upwards. Therefore, in the absence of any move in interest rates, positive shocks to either demand or supply will increase inflation. Since the slope of the curve is negative, an increase in real interest rates will act to reduce inflation. As long as this increase is not too large, the resulting increase in interest rates will help bring inflation closer to its target.

The content of this proposition can also be illustrated in a more familiar way when the interest rate is determined by a rule of the form $r_t = \psi_\pi \pi_t$. However, to easily plot this on

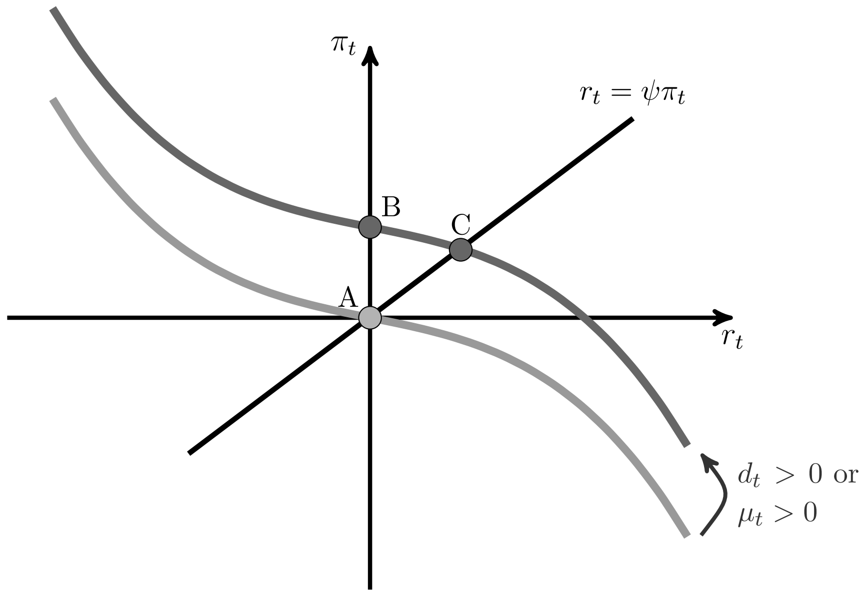
[6]We need to add the qualifier "not too large" in the proposition because a very large increase in r can over-compensate the shock with inflation falling below target. See appendix A for more details on the derivation of Propositions 1 to 3.

Figure 1: π_t as a Function of r_t when the Patman Condition is Not Met.



Notes: equilibrium relationship between π_t and r_t for periods t to $t + N$ as implied by Equation (5) when the Patman condition is not met. Point A is the steady state, where the economy was supposed to be before period t. Consistent with standard New Keynesian theory, the shape of the π_t curve is downward sloping and both positive demand and supply shocks will push the curve upwards. Point B corresponds to a shock $d_t > 0$ or $\mu_t > 0$. At B, inflation is higher than its target (the steady state). A policy that increases r moves the economy to point C, where inflation is brought closer to its target. In this case, conventional monetary policy that increases interest rate will stabilize inflation.

Figure 2: More or Less Reactive Monetary Policy when the Patman Condition Does Not Hold.



Notes: the curved lines represent the equilibrium relationship between π_t and r_t for periods t to $t+N$ as implied by Equation (5) when the Patman condition is not met. The black line represents a monetary policy rule of the type $r_t = \psi\pi_t$. Point A is the steady state where the economy was supposed to be before period t. Point B corresponds to the model equilibrium when the economy is hit by a positive supply shock μ_t or demand shock d_t and when the real interest rate is kept constant ($\psi = 0$). Point C corresponds to the equilibrium after the shock and when $\psi > 0$.

a graph, one needs to focus on the case of i.i.d. shocks. In this case, inflation as a function of interest rates and shocks is simply given by :

$$\pi_t = \Gamma(d_t - \alpha_r r_t) + \gamma_r r_t + \mu_t.$$

We plot both this relationship and the policy rule in the (r, π) space in Figure 2. Note that the policy rule $r_t = \psi\pi_t$ is vertical if $\psi = 0$ and rotates as policy gets more aggressive due to a higher ψ. This implies that inflation will always be more stable in response to shocks if ψ is bigger. This is consistent with Proposition 1 and is well known.

In contrast to Proposition 1, the role of monetary policy in stabilizing inflation is not so straightforward when the Patman condition is met. In this case, the easiest is to start by focusing on an economy that has been hit by a supply shock. We then have the following proposition.

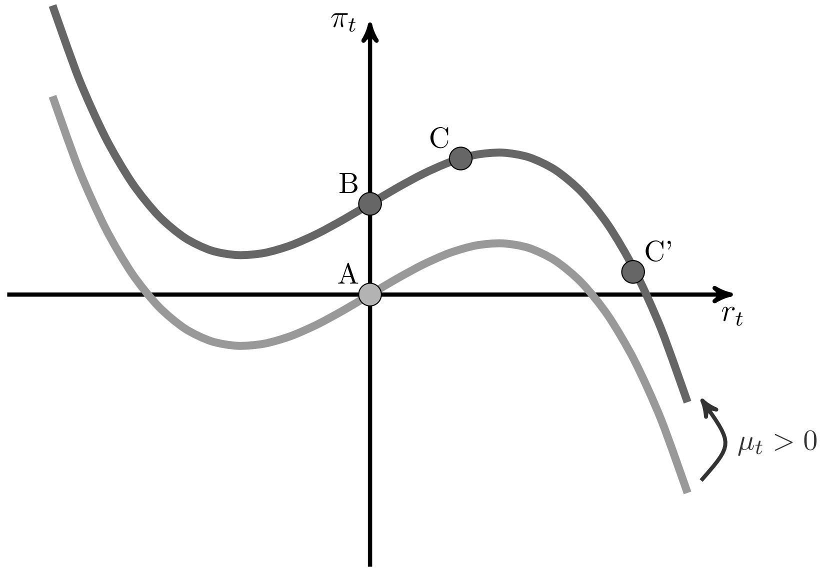
Proposition 2. *Assume the Patman condition is met. Then, in response to a supply shock,* *engineering a rise in real interest rates that is either too small or not sufficiently persistent will push inflation further away from its target relative to keeping real rates at their steady state value.*

The proposition indicates that, if monetary policy does not take a sufficiently aggressive stance in response to a supply shock, then doing nothing may be better at stabilizing inflation than increasing interest rates timidly. This reflects what we propose to call the "Go Big or Stay Home" implication of the Patman condition. [7] To see why this is the case, it is helpful to visualize how the Patman condition changes the relationship between inflation and real interest rates as graphed previously in Figure 1 in the (r, π) space. In particular, Figure 3 plots this relationship for the case where the Patman condition is met. In this figure, all shocks are set to zero and the real interest rate is assumed to be set to r for N periods and then to zero thereafter (as in Figure 1). When the Patman condition is met, the relationship between inflation and real interest rates will generally be non-monotonic as illustrated in Figure 3. Most importantly, it becomes positively sloped near zero if N is not too large (which is the implicit assumption behind the figure). In fact, if the condition $\alpha_r \gamma_y \geq (1 - \alpha_y)\gamma_r$ holds with equality, then this non-monotonicity holds for all finite values of N. In this set-up, a supply shock shifts this curve upwards while always maintaining a positive slope at zero. Therefore, following a supply shock, a marginal increase in interest rates will increase inflation instead to decreasing it. However, if the increase in interest rates is large enough, we enter a region of the curve where higher interest rates have the traditional effect of lowering inflation. In other words, for large enough movements in real rates, we come back to the conventional result that increasing interest rates after a supply shock helps to stabilize inflation. When the Patman condition is met, the increase in real interest rates needs to be sufficiently large to help the inflation targeting objective. It is also the case that, when N becomes sufficiently large, the relationship between inflation and real interest rates can reverse slope if $\alpha_r \gamma_y > (1 - \alpha_y)\gamma_r$.[8] Hence, another way of making

[7]Underlying our "Go Big or Stay Home" principle is the assumption that the only choice following a positive shock is to either increase interest rates or keep them unchanged. However, if this were not the case, an inflation targeting central bank could actually want to reduce interest rates in response to a small supply shock.

[8]As N goes to infinity, the slope of the relationship between inflation and real rates evaluated at $r = 0$ becomes equal to $\frac{-\alpha_r \Gamma'(0)}{1-\alpha_y} + \gamma_r$.

Figure 3: π_t as a Function of r_t when the Patman Condition is (Locally) Met.



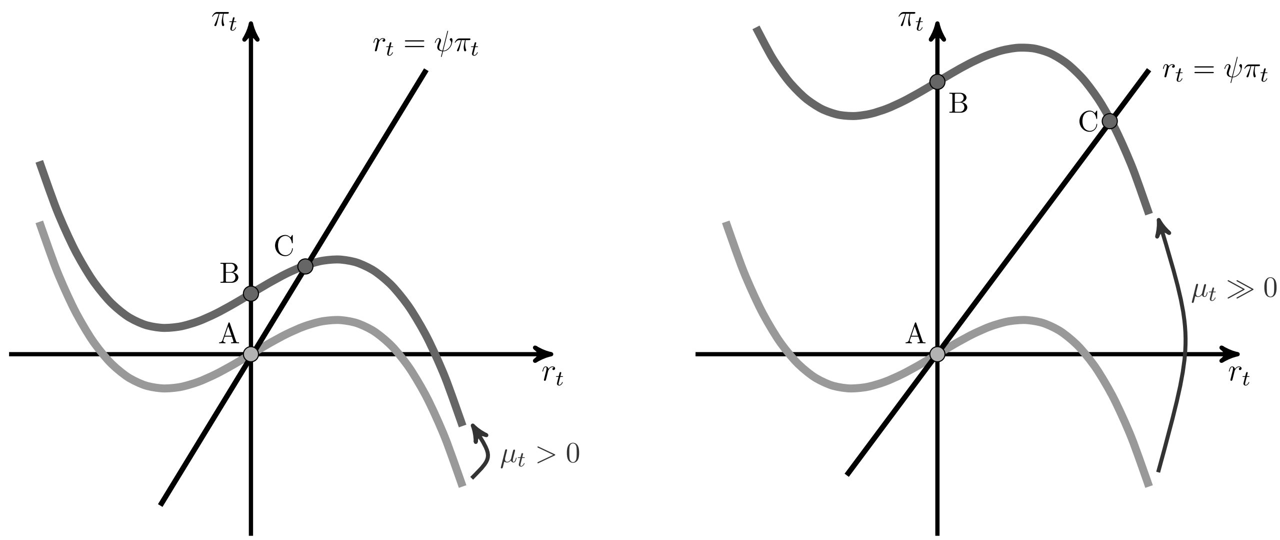
Notes: equilibrium relationship between π_t and r for periods t to $t+N$ as implied by Equation (5) when the Patman condition is (locally) met. In contrast to Figure 1, the π_t curve's slope becomes locally positive around the steady state. As the magnitude of r increases, the slope becomes negative again. Point A is the steady state, in which the economy was supposed to be before period t. As before, a positive supply shock $\mu_t > 0$ shifts the curve upwards. Point B corresponds to a shock $\mu_t > 0$. Considering monetary policy, point B represents keeping real interest rate at its steady state level, point C represents a slight interest rate increase in response to the supply shock, and C' stands for our "Go Big" monetary policy (large change in interest rate). We observe that a small rise in interest rate increases inflation rather than stabilizing it.

sure that an increase in real interest rates leads to a fall in inflation is for the increase to be perceived by agents as being very persistent, which has a flavor of "forward guidance" policy. Therefore, "Going Big" to stabilize inflation should be interpreted as either a large enough or a persistent enough increase in real interest rates. If too small or too short-lived, an increase in real interest rates following a supply shock will push inflation away from its target instead of towards it.

Once again, one can visualize part of the content of the proposition by focusing on the case of an i.i.d. shock and a standard policy rule of the form $r_t = \psi\pi_t$.[9] This is illustrated in Figure 4 for an i.i.d. supply shock. Here, supply shocks always push inflation upwards, moving the economy along the policy curve $r_t = \psi\pi_t$. The size of the inflation response depends on ψ. As we can see in Figure 4, starting from $\psi = 0$, increasing ψ will actually destabilize inflation

[9] We are restricting our attention to the case where $\psi < \frac{1}{\alpha_r\gamma_y+\gamma_r}$ to ensure equilibrium determinacy.

Figure 4: More or Less Reactive Monetary Policy when the Patman Condition Holds (locally) and Shocks are Either Small or Large.



Notes: the curved lines represent the equilibrium relationship between π_t and r_t for periods t to $t + N$ as implied by Equation (5) when the Patman condition is locally met. The black line represents a monetary policy rule of the type $r_t = \psi\pi_t$. The left panel corresponds to a small supply shock μ_t while the right panel plots a large supply shock. Shocks are i.i.d. In both panels, point A is the steady state, where the economy was supposed to be before period t. Point B corresponds to the model equilibrium when the economy is hit by a positive supply shock μ_t and when the real interest rate is kept constant ($\psi = 0$). Point C corresponds to the equilibrium after the shock and when $\psi > 0$.

instead of stabilizing it.[10] The intuition for why a standard anti-inflationary prescription might destabilize inflation instead of helping stabilize it is rather evident. Recall that the Patman condition relates to the property that the direct effect of an increase in interest rates is larger than the indirect effect. Hence, in a zone where the Patman condition is met, increases in interest rates have the opposite effect than what is traditionally predicted. The traditional assumption is that the indirect effect always dominates the direct effect, with the later often assumed to be zero.[11]

In the case of demand shocks, the simple prescription of increasing interest rates in response to a positive shock to help stabilize inflation will again not hold in general when the Patman condition is met. However, expressing this property is slightly more involved

[10]With a policy of the form $r_t = \psi\pi_t$, a higher ψ will tend to destabilize inflation in response to small shocks but tend to help stabilizing inflation in response to big shocks.

[11]In the canonical New Keynesian model there is not direct effect.

than in the case of supply shocks. This is because, even if the condition $-\alpha_r\gamma_y + \gamma_r > 0$ is satisfied, it may be the case that, for a sufficiently big demand shock, we have that $-\alpha_r\Gamma'(d_t) + \gamma_r < 0$. Hence with demand shocks, one has to consider the size of the shock to know whether increasing interest rates is likely to help stabilizing inflation. This is expressed in Proposition 3.

Proposition 3. *Assume the Patman condition is met and that the demand shock is neither too big nor too persistent.*[12] *Responding to this shock with a rise in real interest rates that is either too small or not sufficiently persistent will push inflation away from its target relative to keeping real rates at their steady state value.*

Compared to our previous figures, the added complexity is that demand shocks do not simply induce a vertical translation of the curve without shocks. Instead, demand shocks trigger a shift up and a movement to the right. Accordingly, if a demand shock is big or persistent enough, the rightward translation will cause the slope of the relationship between inflation and real interest rates evaluated at $r = 0$ to be negative even if it was positively sloped for $d = 0$.[13]

Part of Proposition 3 can again be understood in the (r, π) space, assuming the central bank sets the interest rate to r for N periods and returns it to zero afterwards. Starting from the steady state, a shock that is not too large will shift up the inflation schedule while its slope remains positive at $r = 0$. Hence, following this type of demand shock, small increases in interest rates will bring inflation further away from its target than if the central bank had kept interest rates unchanged. It would then be preferable to "Stay Home" by keeping interest rates at steady state value instead of generating small interest rates increases. However, the central bank always has the option to "Go Big" and increase interest rates sufficiently to actually bring inflation closer to its target. In this sense, the idea "Go Big or Stay Home" still applies for demand shocks when they are not too large. However, note that when "Going Big", the central bank needs to increase rates sufficiently

[12] If we assume that demand shocks follow the first order process $d_t = \rho d_{t-1}$, then the relevant condition on demand shocks for Proposition 3 is d_t being smaller than $\bar{d}$ defined by $-\alpha_r\Gamma'(\frac{\bar{d}}{1-\alpha_y\rho}) + \gamma_r > 0$.

[13] Note that if Γ' is close to zero for a wide range of y values, then demand socks would not directly affect inflation much. In such a case, the main relationship between demand shocks and inflation would reflect how monetary policy reacts to these shocks as opposed to the effects of the shocks themselves.

to induce a net contraction in activity ($y_t < 0$), otherwise the rise will not help to stabilize inflation. This may be unrealistic as it is unlikely that a central bank would want to be that aggressive. Therefore, in response to small demand shocks, "Staying Home" may be seen as the desirable strategy for an inflation targeting central bank. In other words, when the Patman condition holds, it may not be desirable to respond to small demand shocks by fine tuning the economy, as this is more likely to destabilize inflation.

In contrast, if the demand shock is large enough, the slope of the $\pi(r)$ will become negative at $r = 0$. In this case, we are back to a more standard result pattern, as a small increase in real interest rates can now help push the inflation rate towards its target. Hence, with large demand shocks, "Go Big or Stay Home" does not exactly hold. Nonetheless, it remains the case that there exists a set of timid interest rate responses that would be worse in terms of inflation than "Staying Home". For this reason, when the Patman condition holds, one strategy to help inflation stay close to its target following demand shocks would be to "Stay Home" in response to small shocks and, to change interest rates substantially in response to large demand shocks (while still avoiding timid middle grounds).

1.4 Missing Deflation and Low Inflation Trap

In this section we want to illustrate how a country can get trapped in a situation where simultaneously interest rates are at the ELB, inflation is below target and unemployment is below its steady state value. In particular, we want to emphasize how this situation can arise when monetary authorities depart from their traditional rules after a period of ELB constraint and low inflation – either to undo past inflation misses or simply to quickly bring inflation closer to its target.

To simplify exposition, we will assume that we are in a case where Γ' is zero over a sufficiently wide interval around the steady state so that we can treat $\Gamma(y_t)$ as zero. This extreme assumption of a perfectly flat Phillips curve in a neighborhood of the steady state is not necessary for the point we want to make but it simplifies our presentation substantially. We also assume that the only shock present is an i.i.d. demand shock, again for clarity of exposition. Note that in this case, all expected terms will be zero.

Under the iid assumptions, inflation is simply given by:

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} + \gamma_r (i_t - E_t \pi_{t+1}),\\ &= \gamma_r i_t.\end{aligned}$$

We call this economy an extreme Patman economy. By contrast, the inflation equation in a standard New Keynesian economy will be:

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} + \gamma_y y_t,\\ &= \gamma_y y_t.\end{aligned}$$

In both economies, the Euler equation can be written as

$$\begin{aligned}y_t &= \alpha_y E_t y_{t+1} - \alpha_r (i_t - E_t \pi_{t+1}) + d_t,\\ &= -\alpha_r i_t + d_t.\end{aligned}$$

Finally, assume that the traditional monetary stance is to decrease real interest rates when demand falls, so that the desired policy rate would be to set:

$$i_t^d = E_t \pi_{t+1} + \psi_d d_t, \quad \psi_d > 0. \tag{6}$$

However, this policy is constrained by the ELB, which we denote by $\underline{i}$, so that the policy nominal rate is :

$$i_t = \max \left\{ i_t^d, \underline{i} \right\}. \tag{7}$$

Since variables are expressed in deviation from their steady state value, $\underline{i} < 0$.

Missing deflation : Suppose the extreme Patman economy faces a temporary demand shock in period t and the monetary authorities follow the policy in (6) and (7). If the shock $d_t = -d < 0$ is not too large in absolute terms, the ELB will not bind, and monetary authorities will decrease the interest rate to the level $i_t = -\psi_d d$, so that $y_t = -(1 - \alpha_r \psi_d) d$ and $\pi_t = -\gamma_r \psi_d d$. The slope of the observed Phillips curve will be $\sigma^N = \frac{\pi_t}{y_t} = \frac{\gamma_r \psi_d}{1 - \alpha_r \psi_d}$. Note that the apparent slope is a function of the monetary policy stance ψ_d.

Assume now that the demand shock is negative enough for the ELB constraint to be binding and $i_t = \underline{i} = -\delta(d) \psi_d d$, where $\delta(d)$ is comprised between zero and one and is inversely related to the bindingness of the ELB constraint ($\delta(d) = 1$ if the constraint is not binding). In this case, it is easy to compute the apparent slope of the Phillips curve, $\sigma = \frac{\delta(d)\gamma_r\psi_d}{1-\delta(d)\alpha_r\psi_d} < \sigma^N$, as long as $\delta(d) < 1$. In contrast, in the New Keynesian economy, the apparent slope of the Phillips curve will be constant and equal to γ_y

In a extreme Patman economy, the period of mild deflation at the ELB could easily be mis-interpreted as an episode of missing deflation. In particular, if the monetary authority uses the past (linear) historical relationship between inflation and activity to predict how inflation should react in this episode, the fall in inflation when hitting the ELB would be smaller than predicted. This reflects the fact that, when the ELB is not constraining, inflation falls less in proportion to the demand shock than in normal times. This is the case because interest rates cannot fall as much. Therefore, when there is a cost channel to monetary policy, a period of perceived missing deflation at the ELB is readily explained.

Low Inflation Trap : Let us now consider a slightly different policy stance which can lead to poorer inflation outcomes despite looking more aggressive by design. Such a policy will be very much in line with the one suggested by Ben Bernanke in a blog post on the Brookings website:

> "To be more concrete on how the temporary price-level target would be communicated, suppose that, at some moment when the economy is away from the ZLB, the Fed were to make an announcement something like the following:
>
> - The Federal Open Market Committee (FOMC) has determined that it will retain its symmetric inflation target of 2 percent. The FOMC will also continue to pursue its balanced approach to price stability and maximum employment. In particular, the speed at which the FOMC aims to return inflation to target will depend on the state of the labor market and the outlook for the economy.
>
> - The FOMC recognizes that, at times, the zero lower bound on the federal funds rate may prevent it from reaching its inflation and employment goals, even with the use of unconventional monetary tools. The Committee therefore agrees that, in future situations in which the funds rate is at or near zero, a necessary condition for raising the funds rate will be that average inflation *since the date at which the federal funds rate*

first hit zero be at least 2 percent. Beyond this necessary condition, in deciding whether to raise the funds rate from zero, the Committee will consider the outlook for the labor market and whether the return of inflation to target appears sustainable." (Bernanke [2017], italics added by Bernanke)

We model such an idea in the following way. Consider the extreme Patman economy described in the previous paragraph. The desired policy stance is assumed to remain $i_t^d = E_t\pi_{t+1} + \psi_d d_t$ in normal times. However, normal times are here defined in a slightly stricter way than previously. They correspond to either *(i)* when the interest rate was not at the ELB last period or *(ii)* when the interest rate was at the ELB last period but $\pi_{t-1} \geq 0$. In abnormal times, when both $i_{t-1} = \underline{i}$ and $\pi_{t-1} < 0$, the rule is to set interest rates at the ELB, $i_t = \underline{i}$. Policy is then given by:

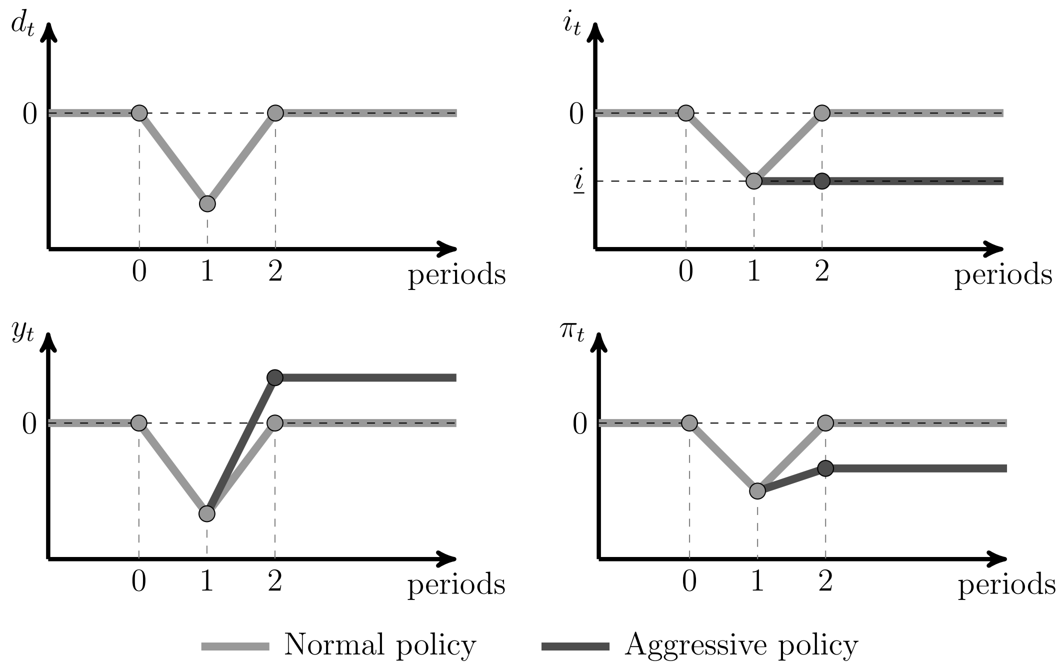
$$i_t = \begin{cases} \max\left\{\psi d_t, \underline{i}\right\} & \text{in normal times, i.e. when } [i_{t-1} > \underline{i}] \text{ or } [i_{t-1} = \underline{i} \text{ and } \pi_{t-1} \geq 0], \\ \underline{i} & \text{if } [i_{t-1} = \underline{i} \text{ and } \pi_{t-1} < 0]. \end{cases}$$

This policy corresponds to keeping interest rates lower than standard policy when the economy has recently been at the ELB and inflation has been below target. From a standard perspective, this approach may seem aggressive as it is potentially correcting for low inflation episodes by keeping interest rates at the ELB even if the state of the economy would push the standard policy stance to increase interest rates. This type of policy can lead to situations where, in the absence of any new shocks, the policy rate gets stuck at the ELB even after the negative demand shock that initiated the ELB episode has dissipated. Such a situation is plotted in Figure 5, where we display responses of the nominal policy rate, the output gap and inflation to a negative demand shock that occurs in period 1 and puts the economy at the ELB. When the above described aggressive policy is followed, inflation is stuck below target and unemployment is above its steady state value.[14] The economy could potentially remain stuck in such a low inflation trap until a sufficiently big supply shock pushes inflation up and leads to a re-normalization of policy.[15]

[14] One of the reasons monetary authorities may be tempted by this policy is the fear that inflation becomes unanchored after a period of low inflation at the ELB. However, if the Patman condition is met, it is precisely following this policy that might trigger a de-anchoring of inflation expectations.

[15] Note that even if in this example we have a policy prescription somewhat similar to those associated with Neo-Fisherian view, the mechanism is very different. In particular, in the current framework, the inflation trap can arise even if inflation expectations remain well anchored. The main mechanism is not through expectations but thought the cost channel.

Figure 5: Inflation Trap with an Aggressive Monetary Policy.



Notes: this figure plots responses of the nominal policy rate, the output gap and inflation to a negative demand shock that occurs in period 1 and puts the economy at the ELB. In each panel, the light line corresponds to a normal policy while the dark one represents the aggressive policy stance. See main text for the definition of those two policies. With the aggressive policy, equilibrium values of inflation and activity/unemployment after the demand shock has dissipated are, for $t \geq 2$, $\pi_t = \frac{\gamma_r}{1-\beta+\gamma_r}\underline{i} < 0$, $i_t - \pi_t = \frac{1-\beta}{1-\beta+\gamma_r}\underline{i} < 0$ *and* $y_t = \frac{-\alpha_r}{1-\alpha_y}\frac{1-\beta}{1-\beta+\gamma_r}\underline{i} > 0$.

1.5 Monetary Shocks

Up to now, we have focused on the effects of the systematic part of monetary policy rules and we have not considered the effects of pure monetary shocks. To look at this issue, it is preferable to extend the model slightly to allow for some internal dynamics in order not to focus on knife edge cases. The easiest way to do this is to allow for external habit in consumption. In this case, the Euler equation for consumption takes the form[16]

$$y_t = \alpha_{y,f} E_t y_{t+1} + \alpha_{y,b} y_{t-1} - \alpha_r (i_t - E_t \pi_{t+1}) + d_t.$$

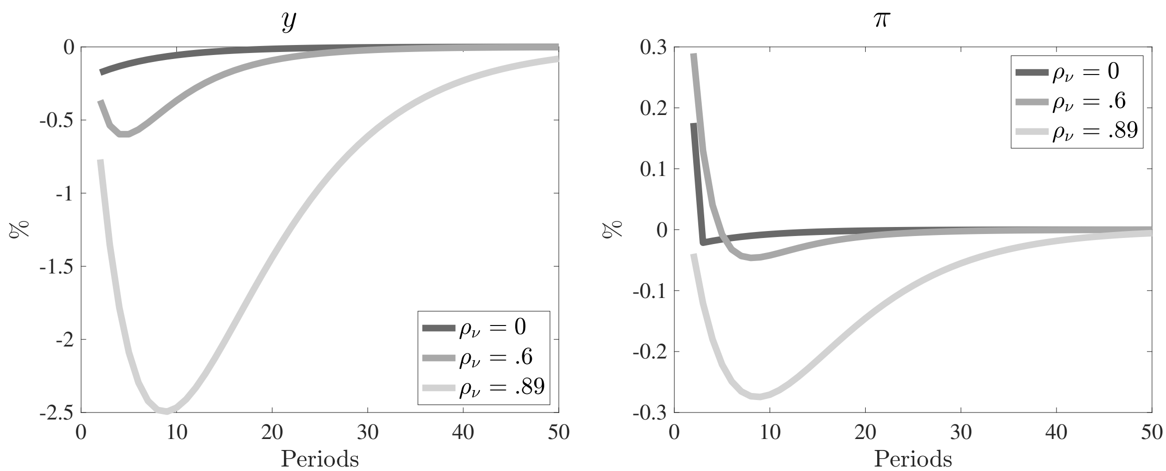
Now consider a monetary shock that aims to increase real rates for a while. For instance, this would be the case of an interest rate rule of the form $i_t = E_t \pi_{t+1} + \nu_t$, $\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_{\nu t}$, where $\varepsilon_{\nu t}$ is the monetary shock. In this case, it is clear that a tightening of monetary policy will lead to a persistent decline in economic activity as long as either ρ_ν or $\alpha_{y,b}$ are not equal to zero.

Inflation response to such a monetary shocks is slightly more complicated because of the non-linearity in the Phillips curve and the possibility that the Patman condition holds. The presence of the cost channel will potentially cause the emergence of a price puzzle in response to a monetary shock, i.e. inflation can rise on impact after a monetary contraction before declining below zero at later dates. The occurrence of a price puzzle following a monetary shock is not surprising in environments where the Patman condition is met. However, the more interesting observation is that the length of the price puzzle will vary depending on both the persistence of the shock (ρ_ν), the size of the shock and the extent of internal dynamics (i.e. the size of $\alpha_{y,b}$). In order to get a better sense of these forces, it is helpful to consider the effects of a temporary change in interest rates of size r occurring at time 0. In this case, inflation for $t \geq 0$ is given by:

$$\pi_0 = \sum_{i=0}^{\infty} \left(\beta^i \Gamma(-\alpha_r \lambda^i r) \right) + \gamma_r r,$$
$$\pi_t = \sum_{i=0}^{\infty} \beta^i \Gamma(-\alpha_r \lambda^{t+i} r),$$

[16] To avoid a unit root associated with real interest rate rules, we are again assuming that $1 - \alpha_{y,f} - \alpha_{y,b}$ is greater than zero but can be arbitrarily close to zero.

Figure 6: Impulse Responses to a Monetary Shock when the Patman Condition Holds (for Various Persistence of the Shock, Linearized Model).



Notes: This shows the response of y and π to a 1% shock to the real interest rate. The model is linearized. Solutions are $y_t = \lambda_1 y_{t-1} - \frac{\alpha_r}{\alpha_{y,f}} \frac{\rho_\nu^t}{1-\rho_\nu \lambda_2^{-1}}$ *and* $\pi_t = \gamma_y \sum_{j=0}^{\infty} \beta^j y_{t+j} + \gamma_r \frac{\rho_\nu^t}{1-\rho_\nu \lambda_2^{-1}}$*. The parameters values for these responses are* $\beta = .99$, $\alpha_{y,f} = \alpha_{y,b} = .99/2$, $\alpha_r = .1$, $\gamma_y = .02$, $\gamma_r = .2$ *and* $\rho_\nu \in \{0, .6, .89\}$.

where λ is the stable root of the polynomial $\alpha_{y,f}X^2 - X + \alpha_{yb}$. Here there would be a price puzzle in period 0 if $\sum_{i=0}^{\infty} (\beta^i \Gamma(-\alpha_r \lambda^i r)) + \gamma_r r$ is greater than zero. The condition $\sum_{i=0}^{\infty} (\beta^i \Gamma(-\alpha_r \lambda^i r)) + \gamma_r > 0$ is the natural extension of the Patman condition for the case when there is external habit. Note that the basic Patman condition $-\alpha_r \gamma_y + \gamma_r$ is a necessary condition for the price puzzle but is not sufficient. With a purely temporary increase in r, the price puzzle lasts one period in this case. After one period, inflation drops below steady state inflation and then converges back to its steady state value from below. In Figure 6, this response is displayed in dark grey. We also plot responses to a mildly persistent and very persistent shock. When persistence is mild, one observes several periods of "price puzzle" while there are none in the case of a more persistent shock. Note that output gap decreases in all scenarios.

1.6 Summary

In this section we explored some theoretical implications of an environment where there is a cost channel of monetary policy and where the Phillips curve is relatively flat. We showed that, when the Patman condition is met, one can understand situations like missing deflation at the ELB or the possibility of a low inflation trap. In the next two sections, we explore the empirical relevance of the Patman condition.

2 Estimating the Phillips Curve with Unrestricted Cost Channel

In this section, we explore properties of the New Keynesian Phillips curve when interest rates are allowed to directly affect real marginal costs. We do so by using the limited information-single equation approach initiated in the New Keynesian literature by Roberts [1995] and Galí and Gertler [1999][17]. While there is a substantial body of literature that allows for a monetary policy cost channel, most papers impose parameter restrictions which rule out by assumption the Patman configuration that is of interest to us. Therefore, our objectives in this section are threefold. First, we want to examine, within the confines of the New Keynesian Phillips Curve, whether interest rates have significant direct effects on inflation. Second, we want to examine whether the omission of interest rates in the estimation of the New Keynesian Phillips curve may have biased other parameters, especially the importance of market tightness. Third, we want to focus on a non-linear version of the Phillips curve where the effects of market tightness on inflation are allowed to vary depending on how far the economy is from its steady state. Finally, and most importantly, we want to look at whether the direct channel of monetary policy on inflation (ie. the direct effect of interest rates) is large in comparison to the more standard indirect channel (i.e. working through market tightness) when the economy is near its steady state. To achieve this, we will focus on two specifications: a linear and a non-linear version.

2.1 Linear Specification

We start by focusing on the linear specification as to both emphasize the first order forces we estimate to be at play in the determination of inflation and to allow for an easy comparison to the literature. The linear specification corresponds to the the first order approximation

[17] See the surveys of Nason and Smith [2008] and Mavroeidis, Plagborg-Møller, and Stock [2014].

of the model derived in Section 1, that is:[18]

$$\pi_t = \beta\pi^e_{t+1} + \gamma_y x_t + \gamma_r(i_t - \pi^e_{t+1}) + \theta z_t + \mu_t, \tag{8}$$

where as before π^e_{t+1} is expected inflation, x_t is a measure of market tightness, i_t represents the nominal interest rate, μ_t is a markup shock and z_t are other factors which may influence inflation. Note that all the variables are demeaned, so that there is no constant term in the equation.

It is worth immediately noting that, from an estimation point of view, the distinction between whether one should allow real interest rates or nominal interest rates in this equation is irrelevant. Both lead essentially to the same regression up to a recombination of terms. We will return to this point later when discussing the interpretation of coefficients.

There are many data choices associated with estimating Equation (8). In our baseline results, we use quarter-to-quarter headline CPI as our measure of inflation (in appendix E, we report results using Personal Consumption Expenditures inflation and the Gross Domestic Product deflator). As our measure of market tightness, we use either the unemployment gap or the output gap as measured by the U.S. Congressional Budget Office. For our measure of the interest rate we use the Federal Funds Rate. For expected inflation we follow two strands of the literature. We either use survey information or we impose rational expectations. When using survey data, we exploit the Michigan Survey of Consumer expectations.[19] In z_t we include the real price of oil and its lag.[20] Further details on the data used can be found in Appendix G. The sample we focus on covers the period from 1969 to 2017.[21].

The biggest challenge in estimating (8) relates to endogeneity of the regressors. The literature has addressed this in different ways, and it remains a difficult issue. In our case, the endogeneity problem is compounded by the fact that we allow for interest rates to have a direct effect on inflation, knowing very well that the setting of interest rates is likely responding

[18]Notice that we normalize the coefficient attached to the marginal cost, $\kappa = 1$, as it is not separately identifiable from γ_r and γ_y. However this is not restrictive for our case as the value of κ is irrelevant when considering Patman condition, which is about the ratio $\frac{\gamma_y}{\gamma_r}$

[19]See Coibion, Gorodnichenko, and Kamdar [2018] for a recent overview of the literature that uses survey data in the estimation of Phillips curves.

[20]The real price of oil is calculated as the spot price of crude oil deflated by core CPI. We generally include 2 lags of the real price of oil as regressors unless only one is significant.

[21]We start in 1969 as this is the start of the identified monetary shocks isolated in Romer and Romer [2004], which are key to our identification strategy.

to inflation. For this reason, in all our estimations we will treat interest rates as endogenous and follow Barnichon and Mesters [2019] in using identified monetary policy shocks as instruments. In particular, we will use 6 lags of of the monetary policy shocks isolated in Romer and Romer [2004] (hereafter R&R shocks) and their squares as instruments.[22] To present results in the clearest fashion, we will proceed by steps where we sequentially treat more variables as endogenous and correspondingly add more instruments.

Table 1: New Keynesian Phillips Curve with Headline CPI, 1969-2017.

$E\pi_{t+1}$	Michigan Survey					
Gap	*minus* Unemployment gap $(-u)$				Output gap (y)	
	(1)	(2)	(3)	(4)	(5)	(6)
β	0.99***	0.98***	0.96***	0.95***	0.95***	0.95***
	(0.043)	(0.043)	(0.020)	(0.020)	(0.021)	(0.020)
γ_y	0.17***	0.15**	-0.01	0.02	0.01	0.02
	(0.062)	(0.067)	(0.057)	(0.054)	(0.038)	(0.036)
γ_r			0.20***	0.20***	0.20***	0.20***
			(0.029)	(0.028)	(0.028)	(0.027)
Observations	196	196	196	196	196	196
J Test		0.368	7.661	7.887	7.963	8.248
(jp)		(0.544)	(0.865)	(0.895)	(0.846)	(0.876)
Weak ID Test		1964.109	72.238	69.417	59.038	51.323

Notes: column (1) corresponds to OLS; in column (2) we use two lags of corresponding gaps as IV for the gap; in columns (3) and (5) we use six lags of the monetary shocks with their squares as IV for the real rate, gap is not instrumented; in columns (4) and (6) we use six lags of monetary shocks with their squares and two lags of gap as IV for both gap and real rate.

In Table 1, we provide a first set of estimates of (8) where we measure expected inflation using survey responses obtained from the Michigan Survey of Consumers.[23]. In this table, we follow Coibion, Gorodnichenko, and Kamdar [2018] and treat expected inflation as an

[22] The original Romer and Romer's [2004] shocks series ends in 1996. We instead use the shocks series extended to 2007 by Wieland and Yang [2020]. For post 2007 quarters in our baseline sample, we patch the series with zeros as in most of these periods nominal interest rate was at zero lower bound. Our results are not affected if we use a subsample from 1969 to 2007, as shown in Table E.6.

[23] In the Michigan Survey of Consumers, every month a representative sample of consumers are asked the following question: "*By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?*" The answer to this question is then the one-year-ahead inflation expectation denoted as $E_t\pi_{t+4,t}$. To keep consistency with the quarter-to-quarter inflation we use in the estimation, we rescaled the one-year-ahead expected inflation assuming survey respondents believe that quarter-to-quarter inflation

exogenous regressor. In column (1) we report the OLS regression of inflation on consumer's expected inflation and on a measure of market tightness. In the first four columns of this table, we are using the negative of the unemployment gap as our measure of market tightness, while in columns (4) and (5) we use the GDP gap as the measure of market tightness. In line with results in Coibion, Gorodnichenko, and Kamdar [2018], we see in Column 1 that market tightness is a significant determinant of inflation with a coefficient of .17. In the second column, we repeat the same exercise but now treat the market tightness measure as endogenous and follow Galí and Gertler [1999] in using lags as instruments.[24] In both these first two columns of the table we are not including interest rates in the regression.[25] In Column (3), we now include the real interest rate as an additional regressor and use identified monetary shocks as instruments. Here, the real interest rate is calculated as the nominal rate minus expected inflation obtained from the Michigan survey.[26] In this column, we are only instrumenting the real rate of interest. As we can see, the real rate is entering the equation with a positive and highly significant coefficient. Moreover, the inclusion of the real rate drastically changes the coefficient on market tightness. This later coefficient becomes essentially zero. In column (4), we now instrument both the real interest rate and the market tightness measure. For this, we are using both the identified monetary shocks and the lags of the unemployment rate as instruments (results with alternative instrument sets are available in Appendix E). Columns (5) and (6) replicate the IV regressions of columns (3) and (4) but now using the output gap as our measure of labour market tightness. This table suggests a clear pattern whereby real interest rates affect inflation much more than labor market tightness. In particular, when real interest rates are included, the effect of market tightness on inflation is estimated to be close to zero.

Table 2 is a continuation of Table 1 where the main difference is that we now treat

follows an AR(1) process with persistence ρ, so that:

$$E_t\pi_{t+1,t} = \frac{E_t\pi_{t+4,t}}{1/4(1+\rho+\rho^2+\rho^3)}.$$

We obtain an estimate of ρ by an AR(1) regression using headline CPI.

[24] We are using two lags of the of unemployment gap as instruments.

[25] Recall that in all regressions, we are including the real price of oil and its lags as regressors.

[26] When we assume rational expectations and use π_{t+1} as our measure of expected inflation, we also use π_{t+1} in the construction of the real rate.

expected inflation as being potentially endogenous and therefore in need of instrumentation. In this table, all three key variables are being instrumented. As before, the instrument set

Table 2: New Keynesian Phillips Curve with Headline CPI, Instrumented for All Regressors.

$E\pi_{t+1}$	Michigan Survey		Rational Expectations	
Gap	$-u$	y	$-u$	y
	(1)	(2)	(3)	(4)
β	0.96***	0.96***	0.86***	0.87***
	(0.023)	(0.024)	(0.034)	(0.034)
γ_y	0.04	0.02	0.04	-0.01
	(0.051)	(0.037)	(0.082)	(0.054)
γ_r	0.18***	0.18***	0.22***	0.21***
	(0.028)	(0.027)	(0.037)	(0.037)
Observations	195	195	196	196
J Test	9.220	9.423	9.773	10.036
(jp)	(0.866)	(0.854)	(0.834)	(0.817)
Weak ID Test	50.007	42.885	33.500	34.746

Notes: in columns (1) and (2) we use the Michigan Survey of Consumers as measure of expected inflation; in columns (3) and (4) we use realized headline CPI under the assumption of Full Information Rational Expectations. Columns (1) and (3) use negative unemployment gap as measure of market tightness; columns (2) and (4) use output gap. All regressors are instrumented.

consists of identified monetary shocks and lags of the market tightness measure. To this set, we add two lags of inflation to help identify the effect of expected inflation. In the first two columns of the table, expected inflation is taken from the Michigan survey, while in the third and fourth column we assume full information rational expectations and use realized inflation at $t+1$ as our measure (with error) of expectations. The columns (1) and (3) use the negative of the unemployment gap as our measure of labor market tightness, while columns (2) and (4) use the output gap.[27] The results in this table are very similar to those in Table 1. In all cases, we see that with the inclusion of the real interest rate, market tightness tends to have a coefficient very close to zero while real rates have a coefficient close to .2. Recall that the Patman condition is satisfied if the real interest rate effect on inflation is greater than the effect of market tightness multiplied by the elasticity of aggregate demand

[27] In the appendix we re-estimate this table allowing for the first lag of inflation to enter directly in the equation. This specification is often referred to as a hybrid Phillips Curve.

to interest rates. Since the latter is almost always estimated to be smaller than 1 (and in the next section we estimate it to be significantly below 1), the coefficients in Table 1 and 2 suggest that the Patman condition is easily met when looking at the linear approximation of the model.

Before turning to the estimation of a non-linear specification, it is interesting to note that, in Table 1 and 2, our finding of a very weak effect of market tightness on inflation is not driven by recent data only. It has been widely noticed that the Phillips curve may have become quite flat over the last three decades. The results of Table 1 and 2 could therefore be thought as mainly reflecting this change. However, this inference would be inappropriate. For instance, even if we constrain our estimation to the period 1969-1992 – which is thought to be a period with a steeper Phillips curve – we get very similar results to that observed over our longer sample. In this shorter sample, we find that, in the absence of real interest rates, the estimated effect of market tightness on inflation is almost twice as larger as in the longer sample. However, once we include interest rates (and instrument them using monetary shocks), these coefficients drop very close to zero. These results are reported in Table 3.[28]

2.2 Non-linear Specification

In this subsection, we extend the results of Table 1 and 2 by allowing the effect of market tightness to enter in a non-linear fashion in order to be in line with the model presented in Section 1. To that effect, we estimate the following equation:

$$\pi_t = \beta\pi^e_{t+1} + \gamma_{y,1}x_t + \gamma_{y,2}x_t^2 + \gamma_{y,3}x_t^3 + \gamma_r(i_t - \pi^e_{t+1}) + \theta z_t + \mu_t, \tag{9}$$

Once again, the main challenge in estimating this type of equation relates to the potential endogeneity of regressors. To address this issue, we proceed with a stepped approach similar to that taken in the linear case. All results in Table 4 use the unemployment gap as the measure of market tightness. In Table 5 we repeat Table 4 but we use the output gap as the measure of market tightness. In these two tables we use the data from the Michigan survey to

[28] In Table 3 we use the (negative of the) unemployment gap as our measure of labor market. We get very similar results if we use the output gap.

Table 3: New Keynesian Phillips Curve with Headline CPI, 1969-1992.

$E\pi_{t+1}$	Michigan Survey					
Gap	$-u$				y	
	(1)	(2)	(3)	(4)	(5)	(6)
β	0.83***	0.77***	1.13***	1.10***	1.08***	1.12***
	(0.081)	(0.060)	(0.051)	(0.053)	(0.045)	(0.041)
γ_y	0.41**	0.49***	-0.16	-0.08	-0.03	-0.07
	(0.167)	(0.121)	(0.109)	(0.105)	(0.051)	(0.048)
γ_r			0.30***	0.29***	0.28***	0.30***
			(0.037)	(0.038)	(0.033)	(0.027)
Observations	96	96	96	96	96	96
J Test		0.330	6.318	6.735	6.570	7.427
(jp)		(0.566)	(0.934)	(0.944)	(0.923)	(0.917)
Weak ID Test		664.800	60.771	46.328	45.963	25.810

Notes: in column (1) we use OLS; in column (2) we use two lags of corresponding gaps as IV for the gap; in columns (3) and (5) we use six lags of monetary shocks with their squares as IV for the real rate, gap is not instrumented; in columns (4) and (6) we use six lags of monetary shocks with their squares and two lags of gap as IV for both gap and real rate.

measure inflation expectations. In Appendix E.3, we include corresponding tables where we instead impose rational expectations and use realized inflation as a proxy for expectations.

Column (1) of Table 4 reports the OLS estimation of our non-linear specification when we omit the real interest rate as an additional regressor. Column (2) adds the real interest rate while still estimating by OLS. Column (3) treats the real interest rate as endogenous and uses identified monetary shocks as instruments. Column (4) treats both the real rate and the market tightness measure as endogenous, while Column (5) instruments all the variables in the table. Since we are allowing market tightness to enter in a non-linear fashion, the only change we make relative to previous instrument sets is to include the square and the cube of the two lags of market tightness when treating market tightness as endogenous. There are two main observations to take from Table 4. First, when including real interest rates in this non-linear specification, we find that the linear coefficient on market tightness is now slightly negative although not generally significantly different from zero. These estimates imply that the Patman condition, when evaluated near the steady state, is met. Second, when we include real interest rates, there is evidence of non-linearity and in particular there

Table 4: New Keynesian Phillips Curve with Headline CPI, Using u, 1969-2017.

Gap: $-u$	OLS		IV		
$E\pi_{t+1}$: Michigan Survey	(1)	(2)	(3)	(4)	(5)
β	0.99***	0.97***	0.95***	0.95***	0.95***
	(0.043)	(0.030)	(0.021)	(0.021)	(0.024)
$\gamma_{y,1}$	0.05	-0.12	-0.13	-0.15*	-0.12
	(0.138)	(0.114)	(0.096)	(0.087)	(0.087)
$\gamma_{y,2}$	0.06**	0.05**	0.04**	0.04*	0.05*
	(0.026)	(0.024)	(0.021)	(0.022)	(0.024)
$\gamma_{y,3}$	0.02	0.03**	0.02**	0.03***	0.03**
	(0.016)	(0.012)	(0.009)	(0.010)	(0.010)
γ_r		0.15***	0.20***	0.20***	0.18***
		(0.030)	(0.029)	(0.029)	(0.028)
Observations	196	196	196	196	195
J Test			8.032	9.469	10.361
(jp)			(0.841)	(0.893)	(0.888)
Weak ID Test			68.599	62.382	43.087

Notes: in column (3) we only instrument for the real rate with six lags of monetary shocks and their squares; in column (4) we instrument for both real rate and non-linear terms of gaps, with two lags of the gap and six lags of monetary shocks and their squares; in column (5) we instrument for all regressors by adding two lags of the Michigan survey to the IV set.

Table 5: New Keynesian Phillips Curve with Headline CPI, Using y, 1969-2017.

Gap: y	OLS		IV		
$E\pi_{t+1}$: Michigan Survey	(1)	(2)	(3)	(4)	(5)
β	1.00***	0.96***	0.94***	0.93***	0.94***
	(0.045)	(0.028)	(0.022)	(0.022)	(0.025)
$\gamma_{y,1}$	-0.03	-0.14**	-0.14**	-0.12**	-0.10*
	(0.070)	(0.065)	(0.053)	(0.053)	(0.052)
$\gamma_{y,2}$	0.01	0.01	0.01	0.02**	0.02**
	(0.011)	(0.009)	(0.007)	(0.008)	(0.008)
$\gamma_{y,3}$	0.01*	0.01***	0.01***	0.01***	0.01***
	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)
γ_r		0.18***	0.22***	0.21***	0.19***
		(0.028)	(0.029)	(0.026)	(0.027)
Observations	196	196	196	196	195
J Test			8.223	9.375	10.403
(jp)			(0.829)	(0.897)	(0.886)
Weak ID Test			57.009	32.569	26.103

Notes: in column (3) we only instrument for the real rate with six lags of monetary shocks and their squares; in column (4) we instrument for both real rate and non-linear terms of gaps, with two lags of the gap and six lags of monetary shocks and their squares; in column (5) we instrument for all regressors by adding two lags of the Michigan survey to the IV set.

is evidence that the cube of labor market tightness enters significantly.[29] Finding a significant effect of the cubic term is relevant because it suggests that, if the economy is far enough from its steady state, monetary policy's effects on inflation return to being more standard – even if the Patman condition is valid near the steady state.[30]

To gain more insights on the non-linearity implied by Table 5 estimates, Figure 7 plots $\pi_t - \beta E_t \pi_{t+1}$ as a function of real interest rates. To do this, we need to know how labor market tightness is responding to interest rates. To this end, we assume that market tightness responds to auto-correlated changes in interest rates with elasticity α_r and autocorrelation coefficient ρ_r. In that case, we have $y = -\frac{\alpha_r}{1-\rho_r} r$. In figure 7, we set $\alpha_r = .1$ (as estimated in the next section) and $\rho_r = .9$ (to capture the common persistence of real rates). With these parameter estimates, we see on the figure that the total effect (both direct and indirect effects) of the real interest rate on inflation is positive when the real interest rate is within a range of roughly -300 basis points to 400 basis points around its steady state value. This suggests that changes in interest rates within this range do not have the standard effect on inflation. Instead, they work in the opposite direction. Hence, if an inflation targeting monetary authority is constrained to move only within such a limited range (possibly due to the ELB), responding to shocks by keeping interest rates at their steady state value ("Staying Home") instead of moving them would be a better strategy to keep inflation close to target.

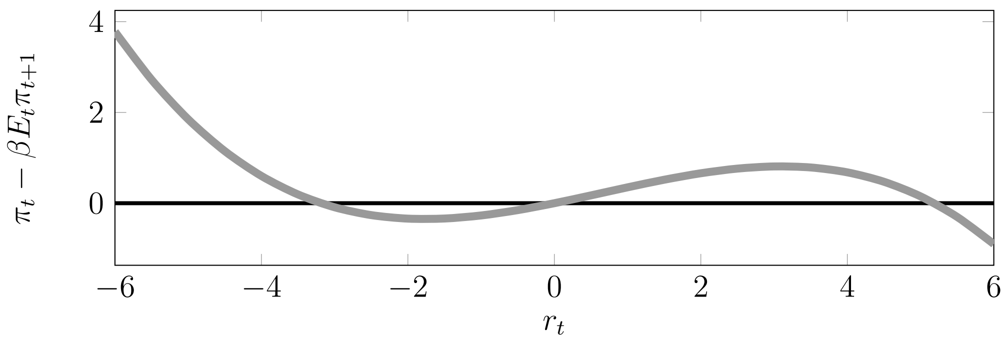
2.3 Nominal versus Real Interest Rates?

So far, we have presented theory and data that emphasize the impact of the real interest rate on the marginal cost and thereby on inflation. However, in most of the literature on the cost channel of monetary policy, it is the nominal interest rate that is highlighted to affect inflation. As noted previously, in terms of Phillips curve estimation, this distinction does not matter as both approaches lead to the same estimating equation. However, by focusing on

[29]We also explored for potentially non-linear effects of interest rates. However, we did not find any evidence to support them.

[30]Finding significance for higher order terms of labor market tightness also sheds light on why estimations of the Phillips curve with typical linear specifications usually find a smaller slope in post 1990s samples. Omitting higher order terms of the gap will induce a bias in the estimates on the linear part of the gap. Therefore, the larger (and positive) the non-linear gap magnitude is in the sample, the higher the estimated slope will be for linear Phillips curve. For this reason one should expect to obtain a steep Phillips curve with pre-1990s samples (where gaps are typically large in absolute values), and to find that the Phillips curve becomes "flatter" in later samples (when economic activity gets less volatile).

Figure 7: The Estimated Non-Linearity.



Notes: this figure shows the shape of the inflation-real rate curve implied by the non-linear New Keynesian Phillips curve. We use estimates from column (3) in Table 4 to plot the left-hand-side variable $\pi_t - \beta\pi^e_{t+1}$ as a function of real interest rate $i_t - \pi^e_{t+1}$. We obtain very similar graph when using estimates in columns (4)-(5) from either Table 4 or 5.

estimated coefficients, especially the coefficient on expected inflation, one can get a sense of whether a real interest rate or a nominal interest rate interpretation is preferable. One of the implications of the New Keynesian Philips curve literature is that the coefficient of expected inflation should be close to agents' discount factor. Given the fact that we use quarterly data, this would suggest a coefficient of expected inflation close to .99. In our estimations of the Philips curve that include real rate – looking through Tables 1, 2, 4 and 5 – we most often observe coefficients on expected inflation that are around .95 and that not significantly very different from .99. However, if we were to adopt a nominal rate specification, then we would need to subtract the coefficient we found for real rates (which is around .2) from the coefficient we estimated for expected inflation. This would imply that the coefficient on expected inflation in a nominal cost channel specification would be around .75 instead of around .95. This would be quite far from theoretical predictions. For this reason, we believe that the real rate specification is better supported by the data.

2.4 Summary

Estimations of New Keynesian Phillips curves with a real interest rate cost channel have shown consistently that *(i)* the coefficient on the gap γ_y (the "slope" of the Phillips curve) is very small, sometimes negative and often non-significant, *(ii)* the coefficient on the real interest rate γ_r enters positively and significantly in the equation, *(iii)* γ_r is always greater than γ_y, so that the Patman condition is satisfied for any value of α_r smaller than one. We now move to the Full Information estimation of the model presented in Section 1.

3 Structural Estimation

The goal of this section is to estimate our simple extended three-equation New Keynesian model, where we do not *a priori* take any stance on whether parameters satisfy the Patman condition. Our objective is to see whether the Patman parameterization may offer a better fit to the data than more standard parameterizations implicit in most New Keynesian models.

We immediately want to emphasize how we choose to address estimation, and especially evaluating whether $\alpha_r \gamma_y \lesseqgtr \gamma_r$, given that in the previous sections we sometimes allowed for $\Gamma(\cdot)$ to be non-linear. To remain close to much of the literature, we focus on estimating the linear approximation of the model and consider the resulting parameter estimates as informing on whether the Patman condition is operative near the steady state. This is in line with standard inference.

3.1 The Estimated Equations

The initial model we want to estimate includes the following two equations

$$y_t = \alpha_y E_t[y_{t+1}] - \alpha_r (i_t - E_t[\pi_{t+1}]) + d_t, \quad \text{(EE)}$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa\big(\gamma_y y_t + \gamma_r (i_t - E_t[\pi_{t+1}])\big) + \mu_t, \quad \text{(PC)}$$

where d_t and μ_t are assumed to be independent AR(1) processes. Here we are expressing market tightness by y, which should be interpreted as labor gap. Since in our simple framework the labor gap and the output gap are interchangeable, we chose to express market tightness by y to remind the reader of this property.

We choose to close the model with the following class of policy rules:

$$i_t = E_t[\pi_{t+1}] + \phi_d d_t + \phi_\mu \mu_t + \nu_t. \quad \text{(Policy)}$$

This class of policy rules is attractive as it minimizes difficulties associated with indeterminacy while simultaneously being very flexible as it allows monetary policy to react to the state space of the system. With such a real rate rule, the equilibrium is determinate as long as $|\alpha_y| < 1$. In the baseline estimation, we assume quasi-no Euler discounting by setting $\alpha_y = .99$. Note that in this policy rule, ν_t will represent monetary shocks that we also assume to be AR(1). In Appendix D, we prove that for any monetary rule that reacts to current endogenous variables and that guarantees determinacy of equilibrium – which includes the typical Taylor rule estimated in the literature – equilibrium allocations can be replicated with our class of policy rules.[31] Estimating a model with our policy is therefore not restrictive, and nests a Taylor rule specification.

The model is a simple linear system of three equations in three unknowns: y, π and i. As such, this system has low dimension, is entirely forward looking and is unlikely to fully capture the rich dynamics of the economy. An attractive feature of using such a simple model is that all the mechanisms at play can be understood easily. The drawback is that it may be an over-simplification. We believe that it is a useful starting point as it allows us to ask whether the simple narrative of a striped down New Keynesian model offers a better interpretation of the data than what could be offered by a Patman parametrization– a parameterization that is generally not considered in the literature.

[31]It is important to note that our estimation results do depend on the specification of the policy rule. There are several reasons why we believe that a policy rule $i_t = E_t[\pi_{t+1}] + \phi_d d_t + \phi_\mu \mu_t + \nu_t$ (a real rate rule that reacts only to shocks, or for short "Real rule" rule) is a better option than a more traditional rule of type $i_t = +\phi_y y_t + \phi_\pi \pi_t + \nu_t$ (Taylor Rule). First, as it has been illustrated in Section 1, a Real rule makes the New Keynesian model nicely recursive as the Euler Equation and the Policy Rule pin down y and r while the Phillips curve residually pins down π. This makes the model mechanisms – following a shock or for different stances of policy – much more transparent. Second, it does not constraint the model estimation and, if the policy rule actually implemented is a Taylor rule, then the estimation will not be biased. This is what we prove in Appendix D. In a set of Monte-Carlo experiments, we have used a Taylor rule model as the Data Generating Process and estimated on simulated data a model with either a Taylor rule or a Real rule. All the model parameters are well estimated with either of the two rules, which is not surprising given Appendix D theoretical result. More interesting, when the DGP features a Real rule, the model estimated with a Taylor rule might be biased. This is the case, as the DGP allocations might correspond to a Taylor rule that does not guarantee determination, and would therefore have been ruled out int the estimation procedure – e.g. when using DYNARE. Furthermore, for the estimated Real model, it is indeed the case that the corresponding Taylor rule produces indeterminacy, so that all parameters are biased when estimated under a Taylor rule (see Beaudry and Portier [2020] for more details on this). Third, it is not obvious that a Taylor rule is always and everywhere an accurate description of how monetary policy is actually implemented. For instance, Debortoli, Galí, and Gambetti [2019] argue that in periods where non conventional monetary policy is used, a short interest rate Taylor rule might not be the best description of Monetary policy implementation. The advantage of the Real rule is that it can accomodate many different implementation of monetary policy. Finally, it is reassuring to find that in our extended model with a Real rule, the Phillips curve estimates on γ_r and γ_y are in line with those obtained in Section 2.

3.2 Estimation, Identification and Sample Period

We begin by estimating the above model on post-war U.S. data. We follow a classical maximum likelihood method. As commonly done in the empirical macroeconomic literature, we calibrate some parameters. First, one cannot separately identify κ, γ_y and γ_r. Instead we can only get estimates of $\kappa\gamma_y$ and $\kappa\gamma_r$. Without loss of generality, we therefore normalize $\kappa = 1$ when estimating over one sample period. We set β to .99, which is in line with large parts of the literature. Our results are not sensitive to changing β around this level. We set α_y to .99, so that although there is almost no discounting in the Euler equation, the model is always determinate.

Our initial data sample is for quarterly U.S. data over the period `1957Q3-2008Q4` and our measure of interest rates is the Federal Funds rate. For inflation, we use Core CPI growth rate. For the gap variable y, we use the opposite of the unemployment gap, as computed by the Congressional Budget Office. This estimation is referred to as the baseline one. In Appendix F, we present results from estimating the model with different samples (including the ZLB period), different measures of inflation (GDP deflator or Headline CPI) and using the output gap. All estimations are performed using Dynare.[32]

3.3 Results

Table 6 presents the baseline estimation of our forward-looking sticky prices model. In this estimation, parameters ρ_d, ρ_μ and ρ_ν are restricted to be in the unit interval. The first thing to note from the table is that the signs of the estimates are the expected ones. Monetary policy is observed to increase interest rates in response to demand shocks ($\phi_d > 0$) and to decrease it in response to cost-push shocks ($\phi_\mu < 0$). The estimated value of α_r is low, which is in line with recent evidence from micro data obtained by Best, Cloyne, Ilzetzki, and Kleven [2020]. Finally, the Phillips curve slope (γ_y) is not significantly different from zero and smaller than the real interest rate channel (γ_r), which is positive and significant. Henceforth, the Patman condition is significantly satisfied. This simple model estimation confirms what we have found in the previous section: New Keynesian Phillips curves are flat, but have a strong, positive and significant cost channel.

[32] See Adjemian et al. [2020]

Table 6: Estimated Parameters, Simple Model, Baseline.

α_r	0.01	σ_d	0.03
	(0.01)		(0.01)
γ_y	0.02	σ_μ	0.36
	(0.03)		(0.06)
γ_r	0.05	σ_ν	0.24
	(0.02)		(0.09)
ϕ_d	0.38	ρ_d	0.94
	(0.14)		(0.03)
ϕ_μ	-0.73	ρ_μ	0.74
	(0.08)		(0.05)
		ρ_ν	0.98
			(0.01)
Patman			0.05
			(0.02)

Notes: this table shows the estimated coefficients of equations (EE), (PC) and (Policy) with unemployment gap, Core CPI and using the sample 1957Q3-2008Q4. Parameters β and α_y are not estimated and set to .99 and .99. Parameter κ is normalized to one. Standard errors are between parenthesis.

The estimated value of γ_r (0.05) is significant but lower than in the Phillips curve estimates of Section 2 (about 0.2). This might be the consequence of estimating a purely forward model that is extremely constrained. In order to loosen those constraints, we extend the model to add some internal propagation mechanisms.

3.4 Extending the Model

We now consider an extended version of our baseline model where we now allow for internal propagation mechanisms in the three equations. To that effect, we follow the literature and introduce habit persistence, hybrid Phillips curve and persistence in the policy rule. The derivation of the Euler equation and Phillips curve are presented in Appendix C.

The model now writes:

$$y_t = \alpha_y\big(\alpha_{y,f}E_t[y_{t+1}] + (1-\alpha_{y,f})y_{t-1}\big) - \alpha_r(i_t - E_t[\pi_{t+1}]) + d_t, \tag{EE'}$$

$$\pi_t = \beta\big((1-\beta_b)E_t[\pi_{t+1}] + \beta_b\pi_{t-1}\big) + \kappa\big(\gamma_y y_t + \gamma_{y,b}y_{t-1} + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t, \tag{PC'}$$

$$i_t = E_t[\pi_{t+1}] + \phi_{r,b}\big(i_{t-1} - E_{t-1}[\pi_t]\big) + \phi_{\pi,b}\pi_{t-1} + \phi_{y,b}y_{t-1} + \phi_d d_t + \phi_\mu\mu_t + \nu_t. \tag{Policy'}$$

With habit persistence, past output also enters in the marginal cost. In order to facilitate comparison with the Phillips curve estimations of Section 2, we present present estimates of the Phillips curve where we do not include lagged market tightness in the marginal cost – i.e. we use the Phillips Curve equation of the form:

$$\pi_t = \beta\big((1-\beta_b)E_t[\pi_{t+1}] + \beta_b\pi_{t-1}\big) + \kappa\big(\gamma_y y_t + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t, \tag{PC”}$$

In Appendix F, we show that results are unaffected when we estimate the model with (PC’) rather than (PC”). The policy rule includes past real interest rate on top of all the states of the economy – i.e. $\{d_t, \mu_t, \nu_t, \pi_{t-1}, y_{t-1}\}$.

We estimate the extended model using Bayesian estimation. As we have five more parameters than in the simple model, a classical maximum likelihood method would become a nonlinear optimization problem that is quite unstable. We therefore perform a Bayesian estimation, as in this case the use of prior distributions over the structural parameters makes this optimization more stable in that case. In Appendix F.1, we present the choice of priors and show detailed results such as parameters priors and posterior distributions. Table 7 presents the parameters estimates.

Parameters are well identified and have the expected sign. Interestingly, at the median of the posterior distribution, the slope of the Phillips curve γ_y is essentially zero while the cost channel γ_r is significant and equal to .15 – which is closer to the values we found in the previous section when we estimated the Phillips curve alone. In Appendix F, we show that the results we found here are robust to choices of samples and variables. Once again, the parameters configuration is such that the Patman condition is met.

4 Conclusion

During the last two decades prior to the Covid-19 pandemic, the behavior of inflation has been puzzling in several countries. First, during 2008-09 recession, inflation fell by less than anticipated given the depth of the recession. This became known as the missing deflation puzzle. After that, the puzzle reversed with inflation generally remaining below target in many countries despite the experience of historically low rates of unemployment. This in turn became known as the missing inflation puzzle. Both these puzzles could reflect a relatively flat Phillips curve. This paper builds on this observation and goes a step further by exploring the

Table 7: Estimated Parameters, Baseline.

α_r	0.01	ρ_d	0.01
	[0.00, 0.03]		[0.81, 0.90]
γ_y	-0.04	ρ_μ	-0.04
	[-0.21, 0.03]		[0.62, 0.78]
γ_r	0.14	ρ_ν	0.14
	[0.05, 0.47]		[0.90, 0.96]
ϕ_d	0.25	β_b	0.25
	[-0.08, 0.50]		[0.01, 0.09]
ϕ_μ	-0.73	$\phi_{\pi,b}$	-0.73
	[-0.85, -0.63]		[-0.07, 0.09]
σ_d	0.04	$\phi_{r,b}$	0.04
	[0.03, 0.05]		[0.16, 0.46]
σ_μ	0.50	$\alpha_{y,f}$	0.50
	[0.36, 0.82]		[0.65, 0.73]
σ_ν	0.21	$\phi_{y,b}$	0.21
	[0.07, 0.31]		[-0.01, 0.42]
Patman			0.11
			[0.05, 0.43]

Notes: This table shows the posterior median estimates of the coefficients in equations (EE'), (PC") and (Policy') using unemployment gap, Core CPI and the sample 1957Q3-2008Q4. Parameters β and α_y are not estimated and set to .99 and .99. Parameter κ is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis Algorithm with two chains of 1,000,000 draws each and discarding the first 500,000 draws of each chains. The numbers between brackets represent the 95% confidence band using the posterior distribution.

monetary policy implications of a locally flat Phillips curve when a cost channel of monetary policy may also be present. We show how standard prescriptions for monetary policy may need to be modified in such an environment. In particular, to keep inflation close to its target, we argue that a central bank may not want to fine tune monetary policy by reacting to both small and large shocks in the same way. In response to small shocks, keeping real interest rates unchanged may be the best way to keep inflation close to its target, whereas in response to large shocks, the best reaction my be to follow more standard prescription by increasing (decreasing) real interest rates in response to positive (negative) demand or supply shocks. One of the interesting features of this framework is that it offers a simple explanation to why and when a country may find itself trapped for a considerable amount of time at the ELB with inflation below target and employment above its steady state value. A large part of the paper has been devoted to show that the condition under which these features arise are supported in US data.

References

Adjemian, S., H. Bastani, M. Juillard, F. Kankanamge, Sumudu andKaramé, D. Kocsis, J. Maih, F. Mihoubi, W. Mutschler, J. Pfeifer, M. Ratto, and S. Villemot (2020): "Dynare: Reference Manual Release 4.6.1," .

Barnichon, R., and G. Mesters (2019): "Identifying Modern Macro Equations with Old Shocks," working paper.

Beaudry, P., and F. Portier (2018): "Real Keynesian Models and Sticky Prices," NBER Working Papers 24223, National Bureau of Economic Research, Inc.

——— (2020): "The Perils of Estimated Taylor Rules," mimeo.

Bernanke, B. (2017): "Temporary price-level targeting: An alternative framework for monetary policy," Blog post on Brookings.edu.

Best, M. C., J. S. Cloyne, E. Ilzetzki, and H. J. Kleven (2020): "Estimating the Elasticity of Intertemporal Substitution Using Mortgage Notches," *Review of Economic Studies*, 87(2), 656–690.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Coibion, O., Y. Gorodnichenko, and R. Kamdar (2018): "The Formation of Expectations, Inflation, and the Phillips Curve," *Journal of Economic Literature*, 56(4), 1447–1491.

Debortoli, D., J. Galí, and L. Gambetti (2019): "On the Empirical (Ir)relevance of the Zero Lower Bound Constraint," in *NBER Macroeconomics Annual 2019, volume 34*, NBER Chapters, pp. 141–170. National Bureau of Economic Research, Inc.

Driskill, R. A., and S. M. Sheffrin (1985): "The "Patman Effect" and Stabilization Policy," *The Quarterly Journal of Economics*, 100(1), 149–163.

Galí, J., and M. Gertler (1999): "Inflation dynamics: A structural econometric analysis," *Journal of Monetary Economics*, 44(2), 195–222.

MAVROEIDIS, S., M. PLAGBORG-MØLLER, AND J. STOCK (2014): "Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve," *Journal of Economic Literature*, 52(1), 124–188.

NASON, J. M., AND G. W. SMITH (2008): "The New Keynesian Phillips curve : lessons from single-equation econometric estimation," *Economic Quarterly*, Fall, 361–395.

RABANAL, P. (2007): "Does inflation increase after a monetary policy tightening? Answers based on an estimated DSGE model," *Journal of Economic Dynamics and Control*, 31(3), 906–937.

RAVENNA, F., AND C. E. WALSH (2006): "Optimal monetary policy with the cost channel," *Journal of Monetary Economics*, 53(2), 199–216.

ROBERTS, J. M. (1995): "New Keynesian Economics and the Phillips Curve," *Journal of Money, Credit and Banking*, 27(4), 975–984.

ROMER, C. D., AND D. H. ROMER (2004): "A New Measure of Monetary Shocks: Derivation and Implications," *American Economic Review*, 94(4), 1055–1084.

ROSSI, B., AND S. ZUBAIRY (2011): "What Is the Importance of Monetary and Fiscal Shocks in Explaining U.S. Macroeconomic Fluctuations?," *Journal of Money, Credit and Banking*, 43(6), 1247–1270.

SURICO, P. (2008): "The Cost Channel Of Monetary Policy And Indeterminacy," *Macroeconomic Dynamics*, 12(5), 724–735.

TAYLOR, J. B. (1979): "Staggered Wage Setting in a Macro Model," *The American Economic Review*, 69(2), 108–113.

TOBIN, J. (1980): "Stabilization Policy Ten Years After," *Brookings Papers on Economic Activity*, 1980(1), 19–89.

WIELAND, J. F., AND M.-J. YANG (2020): "Financial Dampening," *Journal of Money, Credit and Banking*, 52(1), 79–113.

Appendix

A Steps in the Derivation of Propositions 1 to 3

In Propositions 1 to 3, we discuss the consequences on $\{y_{t+j}, \pi_{t+j}\}_{j=0}^{\infty}$ of the following interest rate policy: $r_{t+j} = r \ \forall j \in [0, N-1]$, $r_{t+j} = 0 \ \forall t \geq N$ and for an arbitrary sequence of shocks $\{d_{t+j}, \mu_{t+j}\}_{j=0}^{\infty}$.

Using (2)-(3), the real rate policy and substituting forward, we obtain, for $t \leq j \leq N-1$:

$$y_{t+j} = -\alpha_r \underbrace{\left(\sum_{k=0}^{N-j} \alpha_y^j\right)}_{\frac{1-\alpha_y^{N-j}}{1-\alpha_y}} r + \sum_{k=0}^{\infty} \alpha_y^k E_t d_{t+j+k}. \tag{A.1}$$

For $j \geq N$, when r reverts to zero, we have:

$$y_{t+j} = \sum_{k=0}^{\infty} \alpha_y^{j+k} E_{t+j} d_{t+j+k}. \tag{A.2}$$

Plugging these expressions in the Phillips curve (1) and solving forward, one obtains equation (5) in the main text:

$$\begin{aligned}
\pi_t =& \kappa \sum_{j=0}^{N-1} \beta^j E_t \Gamma\Big(-\alpha_r \frac{1-\alpha_y^{N-j}}{1-\alpha_y} r + \sum_{k=0}^{\infty} \alpha_y^k E_{t+N+j} d_{t+N+j+k}\Big) \\
&+ \kappa \sum_{j=0}^{\infty} \beta^{N+j} E_t \Gamma\Big(\sum_{k=0}^{\infty} \alpha_y^k E_{t+N+j} d_{t+N+j+k}\Big) \\
&+ \kappa \gamma_r \frac{1-\beta^N}{1-\beta} r + \sum_{j=0}^{\infty} \beta^j E_t \mu_{t+j}.
\end{aligned} \tag{5}$$

In order to understand the logic of Propositions 1 to 3, it is useful to write the local approximation of (5) when $\Gamma(y) \approx \gamma_y y$:

$$\begin{aligned}
\pi_t =& \kappa \underbrace{\left(-\gamma_y \alpha_r \sum_{j=0}^{N-1} \frac{1-\alpha_y^{N-j}}{1-\alpha_y} + \gamma_r \frac{1-\beta^N}{1-\beta}\right)}_{\mathcal{A}(N)} r \\
&+ \kappa \gamma_y \left(\sum_{j=0}^{\infty} \beta^j E_t \left(\sum_{k=0}^{\infty} \alpha_y^k E_{t+j} d_{t+j+k}\right)\right) + \sum_{j=0}^{\infty} \beta^j E_t \mu_{t+j}.
\end{aligned} \tag{A.3}$$

$\mathcal{A}(N)$ is the slope of the $\pi - r$ locus for given shocks. If the increase in interest rate lasts for one period ($N = 1$), then $\mathcal{A}(1) = \gamma_r - \alpha_r \gamma_y$, which is exactly the expression in the Patman condition. Therefore, the slope will be positive if the Patman condition holds and negative if it does not. For $N > 1$, we have:

$$\mathcal{A}(N) = -\gamma_y \alpha_r \underbrace{\sum_{j=0}^{N-1} \frac{1-\alpha_y^{N-j}}{1-\alpha_y}}_{\mathcal{B}(N)} + \gamma_r \underbrace{\frac{1-\beta^N}{1-\beta}}_{\mathcal{C}(N)}. \tag{A.4}$$

$\mathcal{B}(N)$ and $\mathcal{C}(N)$ are two terms that increase with N at different speeds. Depending on the relative speed, $\mathcal{A}(N)$ can change sign when N increases. Hence the condition on the persistence of the r increase in Propositions 1 and 2.

In Proposition 1 the Patman condition is assumed not to holds. Therefore, the non-linearity cannot change the sign of the $r - \pi$ locus. From (5), one can easily check that an increase in d or μ pushes inflation up while a simultaneous increase in r pushes it down.

Proposition 2 and 3 assumes that the Patman condition holds and are local results. When shocks and real interest surges are small and not too persistent, one can think of the local approximation Equation (A.3) as correct, and the results are immediate. When shocks are shocks are large , one needs to notice that the $r - \pi$ locus changes slope sign.

B The Patman Condition in Standard Cost Channel Models

Here we explore two simple models that are typical references for New Keynesian models with a cost channel, namely Ravenna and Walsh [2006] and the no-capital version of Rabanal [2007] proposed by Surico [2008]. Note that in those models, it is the nominal interest rate that enters the marginal cost and not the real interest rate. However, the Patman condition is computed holding expectations fixed, so that real and nominal rates move as one. Also note that the Patman condition is a necessary condition for inflation to increase following an rise in the interest rate when expectations are not held constant. If the Patman condition does not hold, then inflation will never respond positively to monetary tightening when the monetary shock is persistent.

B.1 Ravenna and Walsh [2006]

Firms must borrow the wage bill at the nominal interest rate. Preferences are $\frac{c^{1-\sigma}}{1-\sigma} - \chi\frac{N^{1+\eta}}{1+\eta}$. Euler equation and Phillips curve are given by:

$$\begin{aligned} y_t &= E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}), \\ \pi_t &= \beta E_t\pi_{t+1} + \kappa(\sigma+\eta)y_t + \kappa i_t. \end{aligned}$$

The Patman condition writes $\frac{\gamma_r}{\gamma_y} > \alpha_r$, with $\gamma_r = \kappa$, $\gamma_y = \kappa(\sigma+\eta)$ and $\alpha_r = \frac{1}{\sigma}$. Patman condition implies $\frac{1}{\sigma+\eta} > \frac{1}{\sigma}$. It cannot hold as $\eta \geq 0$. Therefore, the Patman condition is never satisfied.

B.2 Surico [2008]

Here, only a fraction θ of firms need to borrow the wage bill in advance. Euler equation and Phillips curve are given by:

$$\begin{aligned} y_t &= E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}), \\ \pi_t &= \beta E_t\pi_{t+1} + \kappa(\sigma+\eta)y_t + \kappa\theta i_t. \end{aligned}$$

The Patman condition writes $\frac{\gamma_r}{\gamma_y} > \alpha_r$, with $\gamma_r = \theta\kappa$, $\gamma_y = \kappa(\sigma+\eta)$ and $\alpha_r = \frac{1}{\sigma}$. The Patman condition implies $\frac{1}{\sigma+\eta} > \frac{1}{\theta\sigma}$. A lower bound of the right-hand side is attained at $\theta = 1$. In that case, the Patman condition cannot hold as $\eta \geq 0$. This implies that the Patman condition cannot hold for values of θ lower than one. Therefore, the Patman condition is never satisfied.

C Model Microfoundations

C.1 Discounted Euler Equation Specification

The derivation of the discounted Euler equation relies on two sets of assumptions. First, because of asymmetry of information and lack of commitment, individual households will face an upward sloping supply of funds when borrowing. To maintain tractability, we will consider an equilibrium in which agents never default, so that the income and wealth distributions will have a unique mass point. For exposition simplicity, we will derive the main features of the equilibrium in a two-period model and explain why the extension to an infinite horizon is trivial. Second, we will assume a particular timing of income and expenditure flows. Those two assumptions will allow us to derive a discounted Euler equation.

C.1.1 A simple two-period model with asymmetric information and lack of commitment

We consider a deterministic mode with two periods. There are two types of households and a zero-profit risk neutral representative bank that has access to an unlimited supply of funds at cost $\overline{R}$. Households receive no endowment in the first period, and ω in the second period. The consumption good is the numéraire.

Some households (superscript c) have access to commitment and always repay their debt while other households (superscript nc) cannot commit to repay. Type is not observable. Because of this, the risk neutral bank will want to charge a risk premium on its loans. More specifically, the bank proposes to the households a schedule $R(d)$ that is increasing in the level of debt d.

Preferences over consumption are given by $u(c_1) + \beta u(c_2)$. Households also bear an additively separable utility cost of defaulting $\psi(d)$ which is an increasing and convex function of the amount of defaulted debt.

When households borrow (as they will always do under regularity conditions on preferences u), they will consume (c_1, c_2) and their debt is $d = c_1$. Committed type households maximize their utility under the budget constraint $c_2 = \omega - R(c_1)c_1$. Their optimal choice for c_1 satisfies

$$u'(c_1^c) = \beta\Big(R(c_1^c) + R'(c_1^c)c_1^c\Big)u'(\omega - R(c_1^c)). \tag{C.5}$$

The non-committed type households optimally decide whether they will default (superscript d) or not (superscript nd) in period 2, and this choice can be made in period 1 because there is no uncertainty in this example. If they repay (no default), non-commited households behave as the committed type, so that

$$c_1^{nc,nd} = c_1^c.$$

If they default, then they will borrow (in period 1) as much as they need to equalise marginal utility of consumption with marginal psychological cost of default. The optimal choice will then satisfy:

$$u'(c_1^{nc,d}) = \psi'(c_1^{nc,d}), \tag{C.6}$$

while $c_2^{nc,d} = \omega$.

The optimal decision to default or not depends on the direction of the following inequality:

$$\underbrace{u(c_1^c) + \beta u\Big(\omega - R(c_1^c)c_1^c\Big)}_{\text{if no default}} \gtrless \underbrace{u(c_1^{nc-d}) + \beta u(\omega) - \psi(c_1^{nc,d})}_{\text{if default}}.$$

For given $u(\cdot)$, β and ω, there is always a psychological cost function $\psi(\cdot)$ such that household of the non-committed type choose to behave as committed households. In this case, we have a pooling equilibrium in which all households behave the same and in which there are no defaults. From the bank's zero-profit condition, we should have $R(c_1^c) = \overline{R}$ (as there is no default). This condition is the only restriction put on the $R(\cdot)$ schedule, so that any off-equilibrium belief $R'(\cdot) > 0$ is consistent with a no default pooling equilibrium.

Extension to an infinite horizon model : If we assume that past actions (default or not) are not observable, the logic of the two-period model still holds in a standard infinite horizon model. With asymmetric information on the household types (access or not to commitment), one can sustain an equilibrium with no default with the following properties: (*i*) households always make the same consumption and saving choices (no observed heterogeneity), (*ii*) there is no risk premium on the interest rate in equilibrium and (*iii*) households consistently face an upward sloping interest schedule $R(b)$. The interest of this modelling is the absence of observed heterogeneity that allows for a simple solving of the model.

C.1.2 Household's problem with upward sloping interest schedule.

There is a measure one of identical households indexed by i. Each household chooses a consumption stream and labor supply to maximizes discounted utility $E_0 \sum_{t=0}^{\infty} \beta^t \zeta_{t-1}(U(C_{it}) - \nu(L_{it}))$, where ζ is a discount shifter.

We split each period into a morning and an afternoon. There is no difference in information between morning and afternoon. In the morning, household i must order and pay consumption expenditures $P_t C_{it}$ and cannot use previous savings to do so. Household i must therefore borrow $D_{it+1}^M = P_t C_{it}$ units of money (say dollars) at a nominal interest rate i_{it}^H that, for the reasons mentioned above, will depend on her total borrowing in period t (hence the subscript i). In the afternoon, household i can borrow D_{it+1}^A for intertemporal smoothing motives, receives labor income $W_t L_{it}$ and profits from intermediate firms Ω_{it} and must repay principal and interest on the total debt inherited from the previous period $(1 + i_{it-1}^H)(D_{it}^M + D_{it}^A)$. The morning budget constraint is therefore given by:

$$D_{it+1}^M = P_t C_{it},$$

and the afternoon budget constraint writes:

$$D_{it+1}^A + W_t L_{it} + \Omega_{it} = (1 + i_{it-1}^H)(D_{it}^M + D_{it}^A).$$

Putting these together, we obtain the following budget constraint for period t:

$$D_{it+1}^A + W_t L_{it} + \Omega_{it} = (1 + i_{it-1}^H) D_{it}^A + (1 + i_{it-1}^H) P_{t-1} C_{it-1}.$$

As there is no new information between morning and afternoon, the interest rate i_{it}^H faced by household i is a function of the total real net debt subscribed in period t. We write it as a premium over the risk-free nominal rate:

$$1 + i_{it}^H = (1 + i_t)\left(1 + \rho\left(\frac{D_{it+1}^M + D_{it+1}^A}{P_t}\right)\right) = (1 + i_t)\left(1 + \rho\left(C_{it} + \frac{D_{it+1}^A}{P_t}\right)\right),$$

with $\rho > 0$, $\rho' > 0$ and $\rho'' > 0$.

The decision problem of household i is therefore given by:

$$\max \sum_{t=0}^{\infty} \beta^t \zeta_{t-1} E_0\big[U(C_{it}) - \nu(L_{it})\big],$$

$$\begin{aligned}
\text{s.t.} \quad D^A_{it+1} + W_t L_{it} + \Omega_{it} &= (1+i^H_{it-1})D^A_{it} + (1+i^H_{it-1})P_t C_{it}, \\
1 + i^H_{it} &= (1+i_t)\left(1+\rho\left(C_{it} + \frac{D^A_{it+1}}{P_t}\right)\right).
\end{aligned}$$

The first order conditions (evaluated at the symmetric equilibrium in which $D^A_{it+1} = 0 \ \forall i$) associated with this problem are:

$$\begin{aligned}
U'(C_t) &= \beta \frac{\zeta_t}{\zeta_{t-1}} E_t\left[U'(C_{t+1})(1+i_t)\big(1+\rho(C_t) + C_t\rho'(C_t)\big)\frac{P_t}{P_{t+1}}\right], \\
\frac{\nu'(L_t)}{U'(C_t)} &= \frac{W_t}{P_t}.
\end{aligned}$$

Assuming that consumption utility is CRRA ($U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$), the Euler equation can be log-linearized to obtain (omitting constant terms and using $C_t = Y_t$) :

$$y_t = \alpha_y E_t[y_{t+1}] - \alpha_r(i_t - E_t[\pi_{t+1}]) + d_t,$$

where y is the log of Y and with $\alpha_y = \frac{\sigma}{\sigma+\varepsilon_\rho} \in]0,1[$, $\alpha_r = \frac{1}{\sigma+\varepsilon_\rho} > 0$, $\varepsilon_\rho = \frac{C(2\rho'+C\rho'')}{\rho+C\rho'} > 0$ and $d_t = -\frac{1}{\sigma+\varepsilon_\rho}(\log\zeta_t - \log\zeta_{t-1})$. This give us equation (EE) in the main text.

C.1.3 Adding habit persistence

Assume that utility is $\frac{(C_{it}-\gamma C_{t-1})^{1-\sigma}}{1-\sigma} - \nu(L_{it})$. Note that we assume external habit. The first order conditions (evaluated at the symmetric equilibrium in which $D^A_{it+1} = 0 \ \forall i$) become:

$$(C_t - \gamma C_{t-1})^{-\sigma} = \beta \frac{\zeta_t}{\zeta_{t-1}} E_t\left[(C_{t+1} - \gamma C_t)^{-\sigma}(1+i_t)\big(1+\rho(C_t) + C_t\rho'(C_t)\big)\frac{P_t}{P_{t+1}}\right], \quad \text{(C.7)}$$

$$\frac{\nu'(L_t)}{(C_t - \gamma C_{t-1})^{-\sigma}} = \frac{W_t}{P_t}, \quad \text{(C.8)}$$

and the log-linearized Euler equation writes:

$$y_t = \alpha_{y,f} E_t[y_{t+1}] + \alpha_{y,b} y_{t-1} - \alpha_r(i_t - E_t[\pi_{t+1}]) + d_t.$$

C.2 Derivation of the Augmented New Keynesian Phillips Curve

The introduction of the real interest rate in the marginal cost of firms is not new (Christiano, Eichenbaum, and Evans [2005], Ravenna and Walsh [2006]). However, the twists we introduce here allow for arbitrary elasticities of the marginal cost with respect to respectively the real wage and the real interest rate. In what follows, we present the derivation of the marginal cost, that can be done considering the static optimal choice of inputs.

C.2.1 Production

Each monopolist produces a differentiated good using a basic input as the only factor of production, and according to a one to one technology. The marginal cost of production will therefore be the price of that basic input. It is assumed that the basic input is produced by a representative competitive firm. The representative firm produces basic input Q_t with labor L_t and the final good M_t according to the following Leontief technology:

$$Q_t = \min(a\Theta_t L_t, bM_t).$$

As in the main text, we assume that Θ_t is constant and normalized to one. The optimal production plan implies $Q_t = aL_t = bM_t$, so that the optimal input demands are $L_t = \frac{Q_t}{a}$ and $M_t = \frac{Q_t}{b}$. Denote by $\mathcal{C}(Q_t) = W_t L_t + \Phi_t M_t$ the total cost of production, where the exact expression of Φ_t will be derived later. Using the optimal input demands, we obtain:

$$\mathcal{C}(Q_t) = \left(\frac{W_t}{a} + \frac{\Phi_t}{b}\right) Q_t,$$

so that marginal cost is

$$\mathcal{C}'(Q_t) = \frac{W_t}{a} + \frac{\Phi_t}{b}.$$

Log-linearizing the above gives the following expression of the real marginal cost, where the variables are now in logs and where constant terms have been omitted:

$$mc_t = \left(\frac{\frac{W}{a}}{\frac{W}{a} + \frac{\Phi}{b}}\right)(w_t - p_t) + \left(\frac{\frac{\Phi}{b}}{\frac{W}{a} + \frac{\Phi}{b}}\right)(\phi_t - p_t).$$

C.2.2 Derivation of the cost Φ_t

The unit price of the final good that enters the production of basic input is P_t. We assume that, in the morning of each period, the basic input representative firm must borrow D^B_{t+1} at the risk-free nominal interest rate i_t to pay for the input M_t. In the afternoon, it produces, sells its production, pays wages, repays the debt contracted the previous period D^B_t and distributes all the profits Ω^B_t as dividends. Those profits will be zero in equilibrium. The period t budget constraint of the firm is therefore:

$$D^B_{t+1} + \widetilde{P}_t Q_t = W_t L_t + (1 + i_{t-1}) D^B_t + P_t M_t,$$

with $D^B_{t+1} = P_t M_t$. Period t profit writes:

$$\Omega^B_t = \widetilde{P}_t Q_t - W_t L_t - (1 + i_{t-1}) P_{t-1} M_{t-1},$$

where $\widetilde{P}_t$ is the price of the basic input. Assuming that the firm maximizes the expected discounted sum of profits real profits Ω^B_t / P_t with discount factor β, and using $Q_t = aL_t = bM_t$, we obtain the first order condition:

$$\widetilde{P}_t = \left(\frac{W_t}{P_t} + \frac{\beta}{b} E_t\left[\frac{1 + i_t}{1 + \pi_{t+1}}\right]\right) P_t.$$

Therefore, the real marginal cost of the basic input firm will be given by:

$$MC_t = \frac{W_t}{P_t} + \frac{\beta}{b} E_t\left[\frac{1 + i_t}{1 + \pi_{t+1}}\right].$$

The price of the basic input $\widetilde{P}_t$ is equal to the nominal marginal cost of the basic input firm and is also equal to the marginal cost of the intermediate input firm (which is the relevant one for pricing decisions). In logs, the real marginal cost will write (omitting constants):

$$mc_t = \underbrace{\left(\frac{\frac{1}{a}\frac{W}{P}}{\frac{1}{a}\frac{W}{P} + \frac{\beta}{b}\frac{1+i}{1+\pi}}\right)}_{\widehat{\gamma}_y}(w_t - p_t) + \underbrace{\left(\frac{\frac{\beta}{b}\frac{1+i}{1+\pi}}{\frac{1}{a}\frac{W}{P} + \frac{\beta}{b}\frac{1+i}{1+\pi}}\right)}_{\gamma_r}(i_t - E_t[\pi_{t+1}])\,.$$

C.2.3 Pricing

As in the standard New Keynesian model, intermediate firms play a Calvo lottery to draw price setting opportunities. Except for the use of the basic input, the modelling is very standard. The optimal household labor supply, that we will derive later, will give us:

$$\frac{\nu'(L_t)}{U'(C_t)} = \frac{W_t}{P_t},$$

which writes in logs, using $C_t = aL_t$ and omitting constant terms:

$$w_t - p_t = \left(\frac{L\nu''(L)}{\nu'(L)} - \frac{CU''(C)}{U'(C)}\right) y_t.$$

As $C_t = Y_t = aL_t$, the marginal cost does not depend on the scale of production and is the same for all the intermediate input firms. It is written as

$$mc_t = \underbrace{\widetilde{\gamma}_y \left(\frac{L\nu''(L)}{\nu'(L)} - \frac{CU''(C)}{U'(C)}\right)}_{\gamma_y} y_t + \gamma_r\,(i_t - E_t[\pi_{t+1}])\,.$$

The rest of the model is standard, and we obtain the New Keynesian Phillips curve:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa\; mc_t + \mu_t.$$

Plugging in the expression for the real marginal cost, we have:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa\Big(\gamma_y y_t + \gamma_r\,(i_t - E_t[\pi_{t+1}])\Big) + \mu_t.$$

This give us equation (PC) in the main text.

C.2.4 Adding habit persistence

When habit persistence is added, labor supply depends on current and last period consumption (see section C.1.3). The Phillips curve writes :

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa\Big(\gamma_y y_t + \gamma_{y,b} y_{t-1} + \gamma_r\,(i_t - E_t[\pi_{t+1}])\Big) + \mu_t.$$

D Equivalence of Different Forms of Policy Rules

For generality, the proof is written in a model with three shocks (demand, markup and money). We show below that two classes of policy rules can replicate the same allocations. Those two classes are a standard Taylor rule:

$$i_t = \phi_y y_t + \phi_\pi \pi_t + \nu_t, \tag{D.9}$$

and a real interest rate rule:

$$i_t = E_t[\pi_{t+1}] + \psi_d d_t + \psi_\mu \mu_t + \psi_\nu \nu_t. \tag{D.10}$$

Below are the elements of the proof. It is without loss of generality as long as the Taylor rule is restricted to produce a determinate equilibrium.

The Euler equation and Phillips curve of our extended sticky prices model can be written as:[33]

$$X_t = AE_t[X_{t+1}] + B\Big(i_t - E_t[X_{t+1}]\Big) + CS_t, \tag{D.11}$$

where $X_t = (y_t, \pi_t)'$, $S_t = (d_t, \mu_t, \nu_t)'$ and each shock $x \in \{d, \mu, \nu\}$ follows $x_t = \rho_x x_{t-1} + \varepsilon_{xt}$. Denote R the diagonal matrix with the persistence parameters ρ_x on the diagonal, with $|\rho_x| < 1$. Let's also define $K = [0\ 1]$ so that $E_t[\pi_{t+1}] = KE_t[X_{t+1}]$.

Solution under a Taylor rule (D.9): Note that policy rule (D.9) can be written:

$$i_t = \Phi X_t + JS_t \tag{D.12}$$

with $\Phi = (\phi_y, \phi_\pi)$ and $J = [0\ 0\ 1]$. Plugging (D.12) in (D.11), we obtain:

$$X_t = \underbrace{(I - B\Phi)^{-1}(A - BK)}_{\mathcal{A}} E_t[X_{t+1}] + \underbrace{(I - B\Phi)^{-1}(BJ + C)}_{\mathcal{B}} S_t \tag{D.13}$$

We assume that the standard Taylor rule is restricted to give equilibrium determinacy, so that the eigenvalues of $\mathcal{A}$ are inside the unit disk.

Solving forward, we obtain :

$$X_t = \underbrace{\left(\sum_{i=0}^{\infty} \mathcal{A}^i \mathcal{B} R^i\right)}_{F(\Phi)} S_t.$$

Under the assumption that the equilibrium is determinate, $\sum_{i=0}^{\infty} \mathcal{A}^i \mathcal{B} R^i$ converges and $F(\Phi)$ is well defined.

Solution under the real interest rule (D.10): The policy rule (D.10) can be written:

$$i_t - E_t[\pi_{t+1}] = \underbrace{[\psi_d\ \psi_\mu \psi_\nu]}_{\Psi} S_t. \tag{D.14}$$

Plugging (D.14) in (D.11), we obtain:

$$X_t = AE_t[X_{t+1}] + \underbrace{(B\Psi + C)}_{\widehat{\mathcal{B}}} S_t. \tag{D.15}$$

[33]This does not cover the case where α_y is exactly 1. We can easily generalize the following analysis for this case.

Solving forward, we obtain:

$$X_t = \underbrace{\left(\sum_{i=0}^{\infty} A^i \widehat{\mathcal{B}} R^i\right)}_{\widehat{F}(\Psi)} S_t,$$

with $\Psi = (\psi_d, \psi_\mu, \psi_\nu)$. Ψ is uniquely defined given that A has its eigenvalues inside the unit disk as long as $|\alpha_y| < 1$.

Equivalence: Policy rules (D.9) and (D.10), which are respectively characterized by the parameters Φ and Ψ, will give similar allocations if:

$$F(\Phi) = \widehat{F}(\Psi).$$

Given a standard Taylor rule with parameters Φ that guarantees determinacy, the mapping F is typically invertible. One can recover the equivalent real interest rule with parameters Ψ, that will be given by $\Psi = \widehat{F}^{-1}(F(\Phi))$.

E Phillips Curve Estimation

In this appendix we include a set of additional estimation results for our single equation estimation of the Phillips Curve.

E.1 Hybrid Version of the Phillips Curve

In Table E.1 we extend the entries of Table 2 in the main text to include one lag of inflation. This type of specification is generally referred to in the literature as a hybrid specification of the Phillips curve, which is our case takes the form:

$$\pi_t = \beta_f \pi^e_{t+1} + \beta_b \pi_{t-1} + \gamma_y x_t + \gamma_r (i_t - \pi^e_{t+1}) + \mu_t. \tag{E.16}$$

The lag term of inflation has been justified in the hybrid New Keynesian Phillips curve by either assuming some form of indexation (e.g. Christiano, Eichenbaum, and Evans [2005]) or assuming the presence of rule-of-thumb behavior in price-setting (e.g. Galí and Gertler [1999]).

E.2 Estimating the Phillips Curve with Alternative Measures of Inflation

In this subsection, we consider three measures of inflation other than headline CPI which we used in our baseline specification: (1) Headline PCE; (2) GDP Deflator and (3) Core CPI.

As these alternative measures of inflation are not usually addressed in surveys of expectations, we focus here only on estimation the Phillips curve under rational expectations. In other words, we use one quarter forward inflation in place of expectation, in both computation of real interest rate and New Keynesian Phillips curve itself. Table E.2 below summarizes the results with these three alternative measures of inflation. We report results with the negative of the unemployment gap as our measure of labor market tightness in odd columns and we use the output gap in even columns.

Table E.1: Hybrid New Keynesian Phillips Curve with Headline CPI, Instrumenting for all Regressors.

$E\pi_{t+1}$	Michigan Survey		Rational Expectations	
Gap	$-u$	y	$-u$	y
	(1)	(2)	(3)	(4)
β_f	0.98***	0.98***	0.59***	0.58***
	(0.061)	(0.061)	(0.058)	(0.055)
β_b	-0.02	-0.02	0.26***	0.29***
	(0.059)	(0.058)	(0.061)	(0.061)
γ_y	0.05	0.02	0.09	0.03
	(0.050)	(0.036)	(0.064)	(0.045)
γ_r	0.18***	0.18***	0.18***	0.17***
	(0.032)	(0.032)	(0.048)	(0.047)
Observations	195	195	196	196
J Test	9.601	9.887	11.509	10.524
(jp)	(0.844)	(0.827)	(0.716)	(0.786)
Weak ID Test	42.584	40.804	20.513	16.983

Notes: in columns (1) and (2) we use the Michigan Survey of Consumers as measure of expected inflation; in columns (3) and (4) we use the realized headline CPI under the assumption of Full Information Rational Expectations. In columns (1) and (3), we use negative unemployment gap as measure of labor market tightness; in columns (2) and (4) we use the output gap. All regressors are instrumented.

Table E.2: New Keynesian Phillips Curve with Different Measures of Inflation, 1969-2017.

$E\pi_{t+1}$ = FIRE	HL PCE		GDP Deflator		Core CPI	
	$-u$	y	$-u$	y	$-u$	y
	(1)	(2)	(3)	(4)	(5)	(6)
β	0.91***	0.91***	0.97***	0.98***	0.90***	0.90***
	(0.025)	(0.026)	(0.023)	(0.024)	(0.035)	(0.034)
γ_y	-0.01	-0.04	-0.01	-0.03	-0.08	-0.07*
	(0.051)	(0.034)	(0.042)	(0.029)	(0.058)	(0.040)
γ_r	0.11***	0.11***	0.07***	0.07***	0.25***	0.24***
	(0.017)	(0.016)	(0.017)	(0.016)	(0.029)	(0.029)
Observations	196	196	196	196	196	196
J Test	9.311	9.369	8.139	7.905	12.205	12.499
(jp)	(0.861)	(0.857)	(0.918)	(0.928)	(0.663)	(0.641)
Weak ID Test	63.756	61.337	59.938	52.241	87.591	57.467

Notes: in columns (1) and (2) we use headline CPI; in columns (3) and (4) we use GDP Deflator; in columns (5) and (6) we use Core CPI. We use negative unemployment gap in odd columns and output gap in even ones. All regressions include real oil price and its lags. Oil price is not significant when Core CPI is used. Identification is under the assumption of Full Information Rational Expectations (FIRE). We use our baseline set of instruments: two lags of the gap and corresponding inflation measure, six lags of monetary shocks and their squares.

E.3 Non-linear Specification of the Phillips Curve under Rational Expectations

In this subsection, we report estimates of our non-linear specification, equation (9), under the assumption of rational expectations. In Table E.3 and E.4, columns (1) and (2) present results for the baseline specification (Equation (9)) while columns (3) and (4) report results for a hybrid extension where we include one lag of inflation. We treat the lag of inflation as an exogenous regressor in the two tables. In Table E.3, we use negative unemployment gap as measure of labor market tightness, whereas in Table E.4 we use output gap.

Table E.3: New Keynesian Phillips Curve with Headline CPI, 1969-2017.

using $-u$	non-hybrid		Hybrid	
$E\pi_{t+1}$ = FIRE	(1)	(2)	(3)	(4)
β or β_f	0.86***	0.83***	0.59***	0.58***
	(0.034)	(0.033)	(0.059)	(0.056)
β_b	–	–	0.28***	0.26***
			(0.066)	(0.061)
$\gamma_{y,1}$	-0.24**	-0.23**	-0.08	-0.09
	(0.123)	(0.114)	(0.115)	(0.117)
$\gamma_{y,2}$	-0.01	0.00	-0.01	0.00
	(0.026)	(0.023)	(0.022)	(0.020)
$\gamma_{y,3}$	0.02*	0.03***	0.01	0.02*
	(0.012)	(0.012)	(0.010)	(0.011)
γ_r	0.22***	0.25***	0.17***	0.21***
	(0.036)	(0.034)	(0.049)	(0.047)
Observations	196	196	196	196
J Test	9.821	11.520	10.441	12.565
(jp)	(0.775)	(0.828)	(0.729)	(0.765)
Weak ID Test	36.259	22.350	22.109	18.238

Notes: in columns (1) and (3) we instrument both the real interest rate and future inflation, with two lags of inflation and six lags of the monetary shocks and their squares; in columns (2) and (4) we instrument for all regressors by adding two lags of the gap and higher orders of the gap in the IV set.

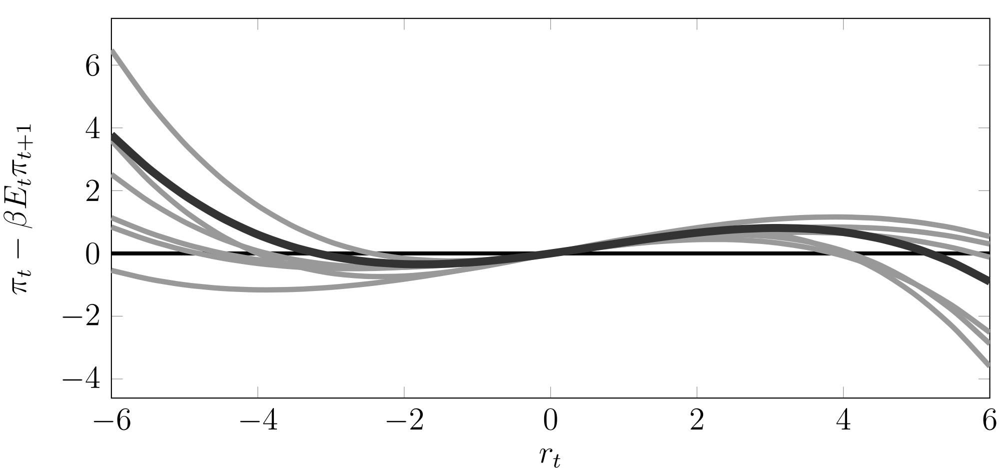
The results from Table E.3 and E.4 are in line with our baseline results presented in Section 2. To illustrate how the implied real interest rate curve is qualitatively consistent across different specifications, we plot (Figure E.1) the $\pi_t - \beta_1 \pi_{t+1}$ as a function of real interest rates, using different estimates from Table 4, 5, E.3 and E.4. Specifically, we choose the results where we instrument for all regressors. These include column (5) in Table 4 and 5 and columns (2) and (4) from Table E.3 and E.4. The light grey lines are implied real interest rate curves for all these estimations and the thick black line is the same curve for our baseline result (column (3) in Table 4).

Table E.4: New Keynesian Phillips Curve with Headline CPI, 1969-2017.

using y	non-hybrid		Hybrid	
$E\pi_{t+1}$ = FIRE	(1)	(2)	(3)	(4)
β or β_f	0.84***	0.83***	0.58***	0.57***
	(0.032)	(0.031)	(0.056)	(0.051)
β_b	–	–	0.29***	0.28***
			(0.064)	(0.055)
$\gamma_{y,1}$	-0.17**	-0.20***	-0.08	-0.08
	(0.070)	(0.074)	(0.068)	(0.075)
$\gamma_{y,2}$	-0.01	-0.00	-0.01	-0.01
	(0.008)	(0.009)	(0.007)	(0.009)
$\gamma_{y,3}$	0.01***	0.01***	0.01***	0.01***
	(0.002)	(0.003)	(0.003)	(0.003)
γ_r	0.27***	0.25***	0.20***	0.20***
	(0.033)	(0.028)	(0.052)	(0.039)
Observations	196	196	196	196
J Test	10.492	10.637	11.202	11.755
(jp)	(0.725)	(0.875)	(0.670)	(0.815)
Weak ID Test	28.858	29.813	25.933	24.454

Notes: in columns (1) and (3) we instrument both the real interest rate and future inflation, with two lags of inflation and six lags of the monetary shocks and their square; in columns (2) and (4) we instrument for all regressors by adding two lags of the gap and higher orders of the gap in the IV set.

Figure E.1: The estimated nonlinearity, robustness.



Notes: this figure plots the shape of the inflation-real rate relation implied by the non-linear New Keynesian Phillips curve. For the light gray lines we use the estimates of column (5) in Table 4 and 5 and columns (2) and (4) in Table E.3 and E.4. The black line is from our baseline result, column (3) in Table 4. In each case, we plot the left-hand-side variable $\pi_t - \beta\pi^e_{t+1}$ as a function of real interest rate $i_t - \pi^e_{t+1}$.

E.4 Alternative Instruments

In this subsection, we report estimation results using alternative sets of instruments. As mentioned in the main text, proper identification of New Keynesian Phillips curve is a topic that is unsettled.

Table E.5 mimics our baseline Table 1 but considers two alternative sets of instruments for real interest rate. We present the table in a progressive fashion like in Table E.1. For column (1) we use monetary shocks identified in Rossi and Zubairy [2011] (henceforth R&Z shocks) in place of those identified by Romer and Romer [2004]. As in Table 1 column (3), here we are only instrumenting the real interest rate. In column (2), we add two lags of the associated labor gap measure to instrument for both real interest rate and the gap. In column (3), we replace Rossi and Zubairy's [2011] shocks with two lags of Federal Funds Rate to instrument the real interest rate. In column (4), we again add two lags of labor gap to instrument for both real interest rate and the gap. Finally, in columns (5)-(8), we reproduce (1)-(4) but use output gap to measure labor market tightness. Notice that as in Table E.1, expected inflation is from Michigan Survey of Consumers and is not instrumented. The endogeneity of the survey response is instead addressed in Table E.6.

Table E.5: Different IV Sets to Estimate the New Keynesian Phillips Curve with Headline CPI, 1969-2017.

	$-u$				y			
$E\pi_{t+1} = MSC$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β	0.96***	0.95***	0.97***	0.95***	0.96***	0.95***	0.98***	0.96***
	(0.023)	(0.023)	(0.030)	(0.028)	(0.025)	(0.025)	(0.031)	(0.029)
γ_y	0.01	0.04	-0.00	0.06	0.01	0.03	-0.02	0.02
	(0.054)	(0.054)	(0.063)	(0.058)	(0.036)	(0.036)	(0.041)	(0.043)
γ_r	0.20***	0.20***	0.15***	0.14***	0.20***	0.20***	0.15***	0.14***
	(0.022)	(0.022)	(0.028)	(0.026)	(0.023)	(0.022)	(0.027)	(0.026)
Observations	196	196	196	196	196	196	196	196
J Test	7.229	7.345	1.862	4.046	7.487	7.845	2.251	4.244
(jp)	(0.890)	(0.921)	(0.172)	(0.132)	(0.875)	(0.897)	(0.134)	(0.120)
Weak ID Test	28.644	28.987	818.824	616.663	24.931	27.584	1010.862	261.147

Notes: columns (1) and (5) are instrumenting for only the real interest rate only, with R&Z shocks; (2) and (6) are instrumenting the real interest rate and the gap, with both R&Z shocks and lags of the gap; (3) and (7) again only instrument for the real interest rate, but with two lags of FFR; (4) and (8) are instrumenting for both the real interest rate and the gap, adding two lags of the gap in the IV set.

From Table E.5 we see that the direct effect of monetary policy (through real interest rates) appears to be strong and significant again, with an estimate ranging from 0.14 to 0.2. Conversely, the labor gap is once more close to zero. Not surprisingly, in columns (3)-(4) and (7)-(8) our first stage test statistics are much higher because of the lags of Federal Funds Rate as instruments.

In Table E.6, we use only monetary shocks as instruments, following Barnichon and Mesters [2019]. One caveat of using only monetary shocks as instruments is that we have to restrict our sample to 1969-2006 as this is the period where the Romer and Romer's [2004] shocks are available. We again present our results progressively: in columns (1) and (4) we instrument the real interest rate only; in columns (2) and (5) we instrument both the labor market tightness measure and the real interest rate; finally in columns (3) and (6) we instrument for all variables. We use negative unemployment gap for market tightness in (1)-(3) and the output gap in (4)-(6).

Table E.6: New Keynesian Phillips Curve with Headline CPI, with Only the Monetary Shocks as Instruments, 1969-2006.

Gap		$-u$			y	
$E\pi_{t+1} = MSC$	(1)	(2)	(3)	(4)	(5)	(6)
β	0.99***	0.96***	1.23***	0.99***	0.93***	1.16***
	(0.037)	(0.042)	(0.122)	(0.038)	(0.047)	(0.096)
γ_y	-0.02	0.15	-0.05	-0.01	0.16*	0.10
	(0.088)	(0.180)	(0.258)	(0.049)	(0.094)	(0.108)
γ_r	0.20***	0.17***	0.21***	0.20***	0.18***	0.20***
	(0.045)	(0.045)	(0.053)	(0.045)	(0.045)	(0.053)
Observations	152	152	152	152	152	152
J Test	9.554	10.081	8.178	9.770	9.479	8.767
(jp)	(0.730)	(0.609)	(0.697)	(0.713)	(0.662)	(0.643)
Weak ID Test	104.105	3.211	4.213	63.278	6.156	6.743

Notes: in columns (1) and (4) we instrument the real interest rate only; in columns (2) and (5) we instrument both the gap and the real interest rate; in columns (3) and (6) we instrument all variables.

Table E.6 shows that the same pattern holds for the direct effect (real interest rate) and the indirect effect (labor tightness) of monetary policy. The one caveat is that, when we instrument for all these variables using only monetary shocks, a weak instrument problem may arise. In effect, our weak identification test statistics drop from as large as 104 to around 4. It is also worth noting that in this last table, we find slightly stronger effects of labor gap.

E.5 Other Robustness and Sensitivity

As the literature has already noted, results obtained from single equation estimation of New Keynesian Phillips curve are very sensitive to the choice of specification made. Following the spirit of Mavroeidis, Plagborg-Møller, and Stock [2014], we also consider a wide range of different settings to assess the robustness of the baseline results we present in Section 2. Table E.7 summarizes all the different configurations that we used to estimate the New Keynesian Phillips curve with unrestricted cost channel through real interest rate.

We sort our estimations into two groups: the ones using Survey Expectations on inflation and the ones identified under the assumption of Full Information Rational Expectations. When using Survey Expectations, we follow Coibion, Gorodnichenko, and Kamdar [2018] and restrict our attention to the use of two CPI indices as measures of inflation. Conversely, under Full Information Rational Expectations, we can explore the possibility of using all five different inflation measures available to us. We estimate a total of 984 equations, each standing for a different configuration picked from Table E.7 (600 under Full Information Rational Expectations and 384 using survey expectation). We present a scatter plot of these results in Figure E.2, where x-axis shows the point estimate of γ_r and y-axis the point estimate of γ_y. A black dot means that both γ_ℓ and γ_r are each significant at 5%. A dark dot means that γ_r is significant at 5% and that γ_ℓ is not. A light gray dots means that γ_ℓ is significant at 5% and that γ_r is not. When the dot is white, none of the coefficient is significant. Note that none of the tests are joint tests.

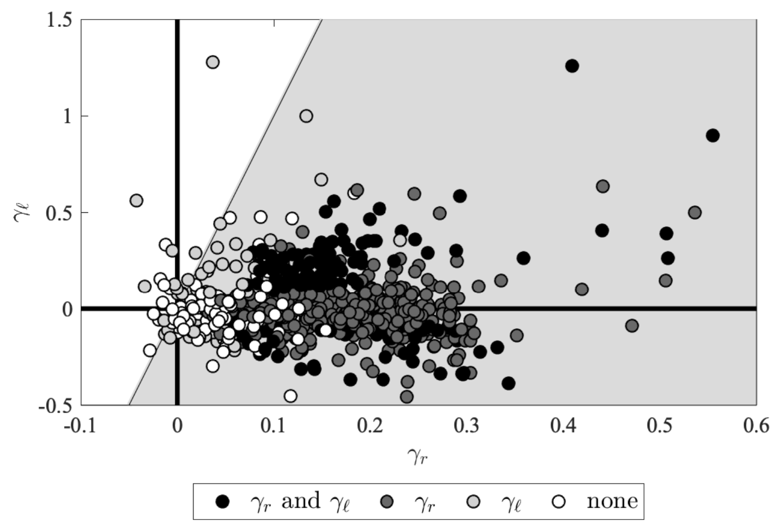
Table E.7: Various Choices of Variables, Samples and Specifications to Estimate γ_y and γ_r.

	Survey Expectation	FIRE
Inflation Measure (π_t) :	Core CPI, HL CPI	Core CPI, HL CPI, Core PCE, HL PCE, GDP Deflator
Gap Measure (x_t) :	unemployment gap, output gap	unemployment gap, output gap
Specification:	hybrid, non-hybrid	hybrid, non-hybrid
IV Set:	(1)6 lags of R&R shocks and squares only (2) R&Z shocks and squares only (3) 2 lags of endogenous regressor (4) combination of (1) and (3) or (2) and (3)	Same combinations as in "Survey Expectations" column
Endogenous Variables:	Only real interest rate, all regressors (except for lag inflation)	All regressors (except for lag inflation)
Sample:	1969-1992,1969-2006, 1969-2017	1969-1992,1969-2006, 1969-2017
Others :	control for real oil price or not	control for real oil prices or not

There are several patterns in Figure E.2. First, the direct cost channel parameter γ_r is significant at 5% level for most of the cases (751 out of 984, around 76%) and is always positive when significant. Conversely, the parameter on the labor gap, γ_y, is only significant for 256 cases, and is negative around half of the time when significant (120 out of the 256 cases). Comparing the magnitude of γ_r and γ_y estimates among the 984 regressions ran, more than 80% of the cases (795 out of 984 cases) feature $\gamma_r > \max\{\gamma_y, 0\}$, suggesting direct cost channel is stronger. Moreover, among the results where $\gamma_r < \gamma_y$, there are 70 cases where γ_y is insignificant. Finally, the mean of point estimates on γ_r is 0.13 and that of γ_y is 0.009.

We then check how many of these results satisfy the Patman condition $\gamma_r > \alpha_r\gamma_y$ where we use $\alpha_r = 0.1$. If we directly use the point estimates obtained from our 984 configurations, 964 cases satisfy the Patman condition, this is 98% of all the specifications we considered. These cases are the points within the shaded area in Figure E.2. We then perform a Wald test on $H_0 : \gamma_r - \alpha_r\gamma_y < 0$ for each specification we run, and we can reject the null at 5% significance level for 724 cases (74% of all the specifications considered). Conversely, if we test $H_0 : \gamma_r - \alpha_r\gamma_y > 0$, we can reject the null at 5% level for only 3 out of 984 cases. Moreover, for most of the cases where we fail to reject non-Patman environment, this corresponds to a specification in which we used headline inflation rates and did not control for oil prices.

Figure E.2: Estimates of γ_ℓ and γ_r with Significance at 5%.



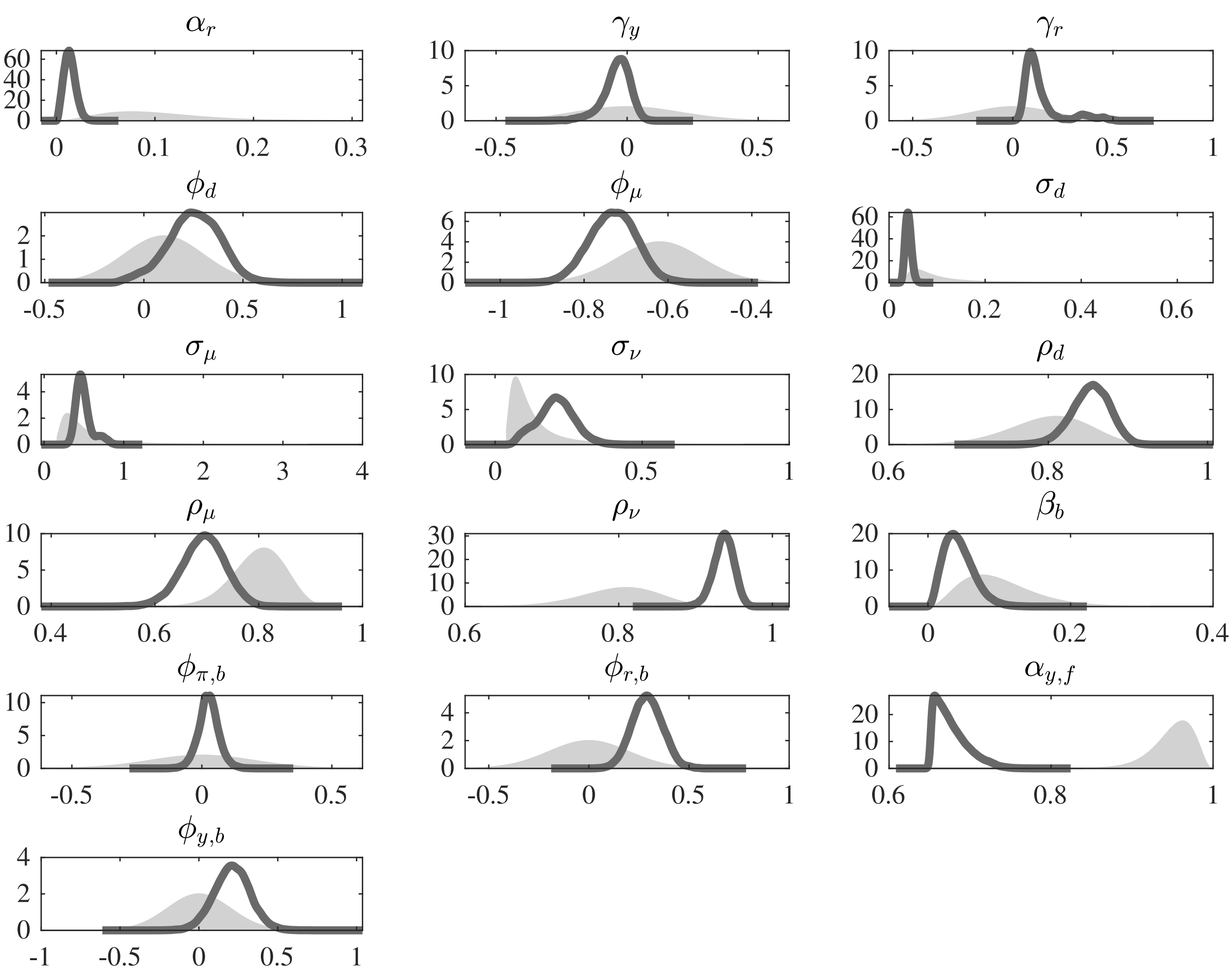
Notes: there are 984 dots on the graph. Each dot corresponds to one of the different specifications of the Phillips Curve listed in Table E.7. A black dot means that both γ_ℓ and γ_r are each significant at 5%. A dark grey dot means that γ_r is significant at 5% and that γ_ℓ is not. A light gray dots means that γ_ℓ is significant at 5% and that γ_r is not. A white dot means that none of the coefficients are significant. None of the tests are joint tests. The shaded area corresponds to the configurations in which the Patman condition holds, ie. $\gamma_r > \alpha_r\gamma_y$ with $\alpha_r = .1$. These points account for 98% of all the specifications we considered.

F Robustness of the Full Information Estimates

F.1 More Details on the Bayesian Estimation of the Extended Model in the Baseline Case

We assume relatively dispersed priors. For the parameters that were estimated in the simple model, we center the prior distributions on the previously estimated value. For the new parameters, we center the priors around zero. Figure F.3 displays prior and posterior distributions for all the estimated parameters. One can check that all the parameters are indeed well identified. Table F.8 presents mode details about the prior and posterior distributions.

Figure F.3: Prior and Estimated Posterior Distributions for Parameters, Extended Model, Baseline.



Notes: this figure plots the prior (the light gray area) and posterior (the dark gray line) distributions for the extended model parameters. The posterior distribution is obtained using the Random Walk Metropolis Algorithm, with two chains of 1,000,000 draws each and discarding the first 500,000 draws of each chain.

Table F.8: Detailed Results on Parameters Estimation, Extended Model, Baseline.

	Prior distribution			Max. posterior		Posterior distribution MH			
Parameter	Type	a	b	Mode	s.d. (Hessian)	Mean	Med.	2.5%	97.5%
α_r: Euler coef. on real rate	Beta([a,b])	0.10	0.05	0.01	0.00	0.01	0.01	0.00	0.03
γ_y: Marginal cost loading to labour market	Normal([a,b])	0.00	0.20	-0.04	0.01	-0.04	-0.03	-0.21	0.03
γ_r: Marginal cost loading to the real interest rate	Normal([a,b])	0.00	0.20	0.10	0.01	0.14	0.11	0.05	0.47
ϕ_d: Policy rule reaction to demand shock	Normal(a,b)	0.10	0.20	0.27	0.06	0.25	0.25	-0.08	0.50
ϕ_μ: Policy rule reaction to markup shock	Normal(a,b)	-0.62	0.10	-0.74	0.02	-0.73	-0.73	-0.85	-0.63
σ_d: Demand shock s.d.	InvGamma(a,b)	0.12	2.00	0.04	0.00	0.04	0.04	0.03	0.05
σ_μ: Markup shock s.d	InvGamma(a,b)	0.61	2.00	0.44	0.03	0.50	0.47	0.36	0.82
σ_ν: Monetary shock s.d.	InvGamma(a,b)	0.15	2.00	0.20	0.02	0.21	0.21	0.07	0.31
ρ_d: Demand shock persistence	Beta([a,b])	0.80	0.05	0.85	0.01	0.85	0.85	0.81	0.90
ρ_μ: Markup shock persistence	Beta([a,b])	0.80	0.05	0.70	0.02	0.69	0.69	0.62	0.78
ρ_ν: Monetary shock persistence	Beta([a,b])	0.80	0.05	0.94	0.01	0.94	0.94	0.90	0.96
β_b: Phillips curve intertia	Beta(a,b)	0.10	0.05	0.04	0.02	0.04	0.04	0.01	0.09
$\phi_{\pi,b}$: Past inflation in policy rule	Normal(a,b)	0.00	0.20	0.07	0.02	0.02	0.02	-0.07	0.09
$\phi_{r,b}$: Persistence in policy rule	Normal(a,b)	0.00	0.20	0.31	0.04	0.29	0.29	0.16	0.46
$\alpha_{y,f}$: Habit persistence	Beta(a,b)	0.95	0.03	0.65	0.01	0.68	0.67	0.65	0.73
$\phi_{y,b}$: Past gap in policy rule	Normal(a,b)	0.00	0.20	0.20	0.03	0.21	0.21	-0.01	0.42

Notes: this table shows the estimated coefficients of equations (EE'), (PC") and (Policy') using unemployment gap, Core CPI and the sample `1957Q3-2008Q4`. *Parameters β and α_y are not estimated and set to .99 and .99. Parameter κ is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis Algorithm, with two chains of 1,000,000 draws each and discarding the first 500,000 draws of each chains. "Med." is the median of the posterior distribution.*

F.2 Estimating with Phillips Curve (PC') Instead of (PC")

Here we repeat the benchmark estimation but we use Phillips curve (PC')

$$\pi_t = \beta\big((1-\beta_b)E_t[\pi_{t+1}] + \beta_b\pi_{t-1}\big) + \kappa\big(\gamma_y y_t + \gamma_{y,b}y_{t-1} + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t, \tag{PC'}$$

instead of (PC").

$$\pi_t = \beta\big((1-\beta_b)E_t[\pi_{t+1}] + \beta_b\pi_{t-1}\big) + \kappa\big(\gamma_y y_t + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t. \tag{PC"}$$

Table F.9 shows that all the parameters are close to what was estimated in the benchmark case and the Patman condition is again satisfied. Table F.10 gives more details about the prior and posterior distributions.

Table F.9: Estimated Parameters, Extended Model with Phillips Curve (PC')

α_r	0.01	ρ_d	0.01
	[0.00, 0.03]		[0.81, 0.89]
γ_y	-0.13	ρ_μ	-0.13
	[-0.33, 0.04]		[0.62, 0.77]
γ_r	0.12	ρ_ν	0.12
	[0.05, 0.34]		[0.91, 0.96]
ϕ_d	0.26	β_b	0.26
	[0.02, 0.49]		[0.01, 0.09]
ϕ_μ	-0.74	$\phi_{\pi,b}$	-0.74
	[-0.85, -0.64]		[-0.05, 0.08]
σ_d	0.04	$\phi_{r,b}$	0.04
	[0.03, 0.05]		[0.14, 0.44]
σ_μ	0.48	$\alpha_{y,f}$	0.48
	[0.35, 0.71]		[0.65, 0.72]
σ_ν	0.21	$\phi_{y,b}$	0.21
	[0.10, 0.30]		[-0.01, 0.45]
		$\gamma_{y,b}$	0.09
			[-0.07, 0.25]
Patman			0.11
			[0.05, 0.31]

Notes: this table shows the posterior median estimates of the coefficients in equations (EE'), (PC') and (Policy') using unemployment gap, Core CPI and the sample `1957Q3-2008Q4`*. Parameters β and α_y are not estimated and set to .99 and .99. Parameter κ is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis Algorithm, with two chains of 1,000,000 draws each and discarding the first 500,000 draws of each chain. The numbers between brackets represent the 90% confidence band using the posterior distribution.*

Table F.10: Detailed Results on Parameters Estimation, Extended Model with Phillips Curve (PC').

Parameter	Prior distribution			Max. posterior		Posterior distribution MH			
	Type	a	b	Mode	s.d. (Hessian)	Mean	Med.	2.5%	97.5%
α_r: Euler coef. on real rate	Beta([a,b])	0.10	0.05	0.01	0.01	0.01	0.01	0.00	0.03
γ_y: Marginal cost loading to labour market	Normal([a,b])	0.00	0.20	-0.19	0.16	-0.13	-0.13	-0.33	0.04
γ_r: Marginal cost loading to the real interest rate	Normal([a,b])	0.00	0.20	0.35	0.08	0.12	0.10	0.05	0.34
ϕ_d: Policy rule reaction to demand shock	Normal(a,b)	0.10	0.20	0.06	0.21	0.26	0.25	0.02	0.49
ϕ_μ: Policy rule reaction to markup shock	Normal(a,b)	-0.62	0.10	-0.73	0.05	-0.74	-0.74	-0.85	-0.64
σ_d: Demand shock s.d.	InvGamma(a,b)	0.12	2.00	0.03	0.01	0.04	0.04	0.03	0.05
σ_μ: Markup shock s.d	InvGamma(a,b)	0.61	2.00	0.69	0.09	0.48	0.47	0.35	0.71
σ_ν: Monetary shock s.d.	InvGamma(a,b)	0.15	2.00	0.08	0.02	0.21	0.21	0.10	0.30
ρ_d: Demand shock persistence	Beta([a,b])	0.80	0.05	0.86	0.02	0.85	0.85	0.81	0.89
ρ_μ: Markup shock persistence	Beta([a,b])	0.80	0.05	0.73	0.04	0.69	0.69	0.62	0.77
ρ_ν: Monetary shock persistence	Beta([a,b])	0.80	0.05	0.93	0.01	0.94	0.94	0.91	0.96
β_b: Phillips curve intertia	Beta(a,b)	0.10	0.05	0.03	0.02	0.04	0.04	0.01	0.09
$\phi_{\pi,b}$: Past inflation in policy rule	Normal(a,b)	0.00	0.20	0.02	0.03	0.03	0.02	-0.05	0.08
$\phi_{r,b}$: Persistence in policy rule	Normal(a,b)	0.00	0.20	0.38	0.07	0.28	0.28	0.14	0.44
$\alpha_{y,f}$: Habit persistence	Beta(a,b)	0.95	0.03	0.65	0.07	0.67	0.67	0.65	0.72
$\phi_{y,b}$: Past gap in policy rule	Normal(a,b)	0.00	0.20	0.20	0.11	0.21	0.20	-0.01	0.45
$\gamma_{y,b}$: Marginal cost loading to past labour market	Normal(a,b)	0.00	0.10	0.08	0.09	0.09	0.09	-0.07	0.25

Notes: This table shows the estimated coefficients of equations (EE'), (PC') and (Policy') using unemployment gap, Core CPI and the sample 1957Q3-2008Q4. Parameters β and α_y are not estimated and set to .99 and .99. Parameter κ is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis Algorithm, with two chains of 1,000,000 draws each and discarding the first 500,000 draws of each chain. "Med." is the median of the posterior distribution.

F.3 Robustness to Samples and Variables

Table F.11 presents the simple model estimates for different samples and choices of variables. Parameters estimates are pretty similar across estimations. Furthermore, the Patman condition is always satisfied and significant. When the GDP deflator is chosen as a measure of inflation, estimates of α_r are close to zero but negative. The estimations we present in that case are those when α_r is not estimated but set to 0.1. In the extended model, we do not have such a problem when GDP deflator is chosen.

Tables F.12 and F.13 show estimates of the extended model. We use the same priors than in the benchmark estimation. Again, parameters estimates are pretty similar across estimations. Furthermore, the Patman condition is always satisfied and significant.

Table F.11: Estimated Parameters, Simple Model, Robustness.

	Unemployment gap						Output gap					
	Core CPI		HL CPI		GDP defl.		Core CPI		HL CPI		GDP defl.	
Parameter	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
α_r	0.01	0.01	0.01	0.00	0.10	0.10	0.02	0.02	0.01	0.01	0.10	0.10
	(0.01)	(0.01)	(0.01)	(0.01)	(NaN)	(NaN)	(0.03)	(0.03)	(0.03)	(0.02)	(NaN)	(NaN)
γ_y	0.02	0.00	0.06	0.01	-0.01	-0.01	0.00	0.00	0.04	0.01	-0.02	-0.01
	(0.03)	(0.02)	(0.05)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)
γ_r	0.05	0.04	0.07	0.04	0.02	0.02	0.07	0.05	0.09	0.05	0.04	0.04
	(0.02)	(0.01)	(0.03)	(0.02)	(0.01)	(0.01)	(0.03)	(0.02)	(0.04)	(0.02)	(0.03)	(0.02)
ϕ_d	0.38	0.36	0.20	0.21	0.57	0.45	0.30	0.27	0.05	0.15	0.41	0.36
	(0.14)	(0.11)	(0.18)	(0.19)	(0.12)	(0.08)	(0.16)	(0.16)	(0.21)	(0.15)	(0.10)	(0.08)
ϕ_μ	-0.73	-0.70	-1.10	-1.03	-0.92	-0.89	-0.75	-0.71	-1.12	-1.03	-0.98	-0.93
	(0.08)	(0.07)	(0.13)	(0.12)	(0.07)	(0.06)	(0.08)	(0.07)	(0.13)	(0.11)	(0.08)	(0.07)
σ_d	0.03	0.02	0.02	0.01	0.03	0.02	0.09	0.07	0.08	0.06	0.08	0.07
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)
σ_μ	0.36	0.33	0.81	0.81	0.25	0.28	0.36	0.33	0.81	0.82	0.25	0.27
	(0.06)	(0.05)	(0.12)	(0.10)	(0.05)	(0.05)	(0.06)	(0.05)	(0.11)	(0.10)	(0.05)	(0.05)
σ_ν	0.24	0.25	0.26	0.33	0.31	0.30	0.32	0.34	0.31	0.36	0.40	0.38
	(0.09)	(0.09)	(0.07)	(0.09)	(0.07)	(0.06)	(0.09)	(0.10)	(0.09)	(0.07)	(0.10)	(0.08)
ρ_d	0.94	0.97	0.95	0.97	0.94	0.97	0.91	0.93	0.92	0.93	0.92	0.93
	(0.03)	(0.02)	(0.03)	(0.02)	(0.02)	(0.01)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)
ρ_μ	0.74	0.74	0.60	0.58	0.79	0.76	0.74	0.75	0.61	0.58	0.80	0.77
	(0.05)	(0.04)	(0.06)	(0.05)	(0.05)	(0.04)	(0.05)	(0.04)	(0.06)	(0.05)	(0.04)	(0.04)
ρ_ν	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.97	0.97	0.97	0.97	0.97
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)
Patman	0.05	0.04	0.07	0.04	0.02	0.02	0.07	0.05	0.09	0.05	0.04	0.04
	(0.02)	(0.01)	(0.03)	(0.02)	(0.01)	(0.01)	(0.03)	(0.02)	(0.04)	(0.02)	(0.03)	(0.02)

Notes: this table shows the estimated coefficients of equations (EE), (PC) and (Policy) for various measures of y, π and for various samples. Columns (1) use the sample `1957Q3-2008Q4`*; columns (2) use the sample* `1957Q3-2017Q4`*. Parameters β and α_y are not estimated and set to .99 and .99. When the GDP deflator is used, α_r is not estimated and set to 0.1. Parameter κ is normalized to one. Standard errors are between parenthesis.*

Table F.12: Estimated Parameters, Extended Model, Robustness, Unemployment Gap.

	Core CPI		HL CPI		GDP defl.	
Parameter	(1)	(2)	(1)	(2)	(1)	(2)
α_r	0.01	0.01	0.01	0.01	0.01	0.01
	[0.00,0.02]	[0.00,0.02]	[0.01,0.02]	[0.00,0.02]	[0.00,0.02]	[0.00,0.01]
γ_y	-0.03	-0.04	0.04	-0.00	-0.04	-0.03
	[-0.12,0.04]	[-0.09,0.01]	[-0.02,0.11]	[-0.04,0.04]	[-0.09,0.01]	[-0.08,0.01]
γ_r	0.11	0.09	0.11	0.07	0.04	0.05
	[0.04,0.32]	[0.04,0.15]	[0.05,0.17]	[0.04,0.11]	[0.01,0.07]	[0.02,0.09]
ϕ_d	0.25	0.24	0.22	0.25	0.53	0.37
	[0.04,0.47]	[0.09,0.38]	[0.01,0.42]	[0.10,0.40]	[0.30,0.76]	[0.21,0.55]
ϕ_μ	-0.73	-0.72	-0.95	-0.94	-0.83	-0.86
	[-0.82,-0.64]	[-0.81,-0.63]	[-1.06,-0.84]	[-1.04,-0.83]	[-0.94,-0.71]	[-0.95,-0.77]
σ_d	0.04	0.03	0.04	0.03	0.04	0.03
	[0.03,0.05]	[0.03,0.04]	[0.03,0.05]	[0.03,0.04]	[0.03,0.05]	[0.03,0.04]
σ_μ	0.47	0.42	0.89	0.86	0.29	0.33
	[0.34,0.68]	[0.33,0.52]	[0.73,1.05]	[0.73,0.99]	[0.21,0.38]	[0.25,0.41]
σ_ν	0.21	0.22	0.26	0.28	0.36	0.31
	[0.10,0.30]	[0.13,0.30]	[0.15,0.36]	[0.20,0.38]	[0.25,0.49]	[0.20,0.42]
ρ_d	0.85	0.88	0.85	0.88	0.85	0.88
	[0.81,0.89]	[0.86,0.91]	[0.81,0.89]	[0.85,0.91]	[0.81,0.88]	[0.85,0.91]
ρ_μ	0.69	0.70	0.55	0.55	0.76	0.73
	[0.62,0.76]	[0.63,0.76]	[0.49,0.62]	[0.48,0.61]	[0.69,0.82]	[0.66,0.79]
ρ_ν	0.94	0.94	0.94	0.94	0.94	0.94
	[0.92,0.96]	[0.92,0.96]	[0.91,0.96]	[0.92,0.96]	[0.92,0.96]	[0.92,0.96]
β_{-1}	0.04	0.04	0.03	0.03	0.03	0.03
	[0.01,0.07]	[0.01,0.07]	[0.01,0.06]	[0.01,0.06]	[0.01,0.06]	[0.01,0.06]
$\phi_{\pi,-1}$	0.02	0.04	0.08	0.08	0.08	0.08
	[-0.04,0.09]	[-0.02,0.09]	[0.00,0.16]	[0.02,0.14]	[-0.01,0.18]	[-0.00,0.16]
$\phi_{r,-1}$	0.29	0.29	0.22	0.21	0.15	0.18
	[0.17,0.41]	[0.17,0.40]	[0.09,0.34]	[0.10,0.32]	[0.04,0.25]	[0.09,0.28]
α_{y+1}	0.67	0.66	0.68	0.66	0.67	0.66
	[0.65,0.71]	[0.65,0.68]	[0.65,0.72]	[0.65,0.68]	[0.65,0.70]	[0.65,0.68]
$\phi_{y,-1}$	0.21	0.20	0.15	0.19	0.27	0.23
	[0.02,0.40]	[0.04,0.36]	[-0.06,0.36]	[0.01,0.37]	[0.02,0.48]	[0.04,0.42]
Patman	0.11	0.09	0.11	0.07	0.04	0.05
	[0.05,0.43]	[0.04,0.19]	[0.05,0.21]	[0.04,0.13]	[0.01,0.09]	[0.02,0.10]

Notes: this table shows the posterior mean for coefficients of equations (EE'), (PC') and (Policy') for various measures of π and for various samples, when the gap measure is unemployment gap. Columns (1) use the sample `1957Q3-2008Q4`; columns (2) use the sample `1957Q3-2017Q4`. Parameters β and α_y are not estimated and set to .99 and .99. Parameter κ is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis Algorithm, with two chains of 1,000,000 draws each and discarding the first 500,000 draws of each chain. The numbers between brackets represent the 95% confidence band using the posterior distribution.

Table F.13: Estimated Parameters, Extended Model, Robustness, Output Gap.

	Core CPI		HL CPI		GDP defl.	
Parameter	(1)	(2)	(1)	(2)	(1)	(2)
α_r	0.03	0.03	0.03	0.03	0.03	0.02
	[0.01,0.06]	[0.01,0.05]	[0.01,0.06]	[0.01,0.05]	[0.01,0.05]	[0.01,0.04]
γ_y	-0.10	-0.09	0.01	0.00	-0.04	-0.03
	[-0.23,-0.00]	[-0.25,-0.00]	[-0.06,0.07]	[-0.03,0.03]	[-0.12,0.00]	[-0.09,0.00]
γ_r	0.27	0.22	0.17	0.09	0.05	0.06
	[0.08,0.51]	[0.06,0.46]	[0.06,0.41]	[0.05,0.13]	[-0.01,0.16]	[0.02,0.14]
ϕ_d	0.13	0.15	0.01	0.10	0.40	0.29
	[-0.03,0.28]	[0.02,0.29]	[-0.18,0.21]	[-0.05,0.26]	[0.18,0.63]	[0.12,0.46]
ϕ_μ	-0.73	-0.72	-0.97	-0.96	-0.87	-0.89
	[-0.82,-0.63]	[-0.81,-0.63]	[-1.08,-0.86]	[-1.07,-0.86]	[-0.99,-0.75]	[-0.98,-0.80]
σ_d	0.12	0.11	0.12	0.10	0.13	0.10
	[0.09,0.16]	[0.08,0.13]	[0.09,0.16]	[0.08,0.13]	[0.09,0.18]	[0.08,0.13]
σ_μ	0.64	0.55	0.98	0.87	0.30	0.33
	[0.43,0.87]	[0.36,0.78]	[0.76,1.28]	[0.74,1.00]	[0.18,0.45]	[0.23,0.44]
σ_ν	0.13	0.13	0.21	0.29	0.40	0.33
	[0.06,0.22]	[0.05,0.24]	[0.08,0.34]	[0.20,0.39]	[0.11,0.61]	[0.16,0.51]
ρ_d	0.86	0.88	0.86	0.88	0.85	0.88
	[0.82,0.90]	[0.84,0.91]	[0.82,0.90]	[0.84,0.91]	[0.81,0.90]	[0.84,0.91]
ρ_μ	0.72	0.73	0.55	0.54	0.77	0.74
	[0.65,0.79]	[0.65,0.81]	[0.48,0.62]	[0.48,0.60]	[0.70,0.83]	[0.67,0.80]
ρ_ν	0.93	0.93	0.93	0.93	0.93	0.94
	[0.90,0.95]	[0.91,0.95]	[0.90,0.95]	[0.91,0.95]	[0.90,0.95]	[0.92,0.96]
β_{-1}	0.04	0.03	0.03	0.03	0.03	0.03
	[0.01,0.07]	[0.01,0.06]	[0.01,0.06]	[0.01,0.06]	[0.01,0.06]	[0.01,0.06]
$\phi_{\pi,-1}$	-0.01	0.00	0.04	0.07	0.05	0.06
	[-0.06,0.03]	[-0.05,0.05]	[-0.03,0.11]	[0.01,0.14]	[-0.04,0.16]	[-0.02,0.14]
$\phi_{r,-1}$	0.37	0.35	0.27	0.23	0.14	0.17
	[0.25,0.48]	[0.25,0.46]	[0.13,0.39]	[0.12,0.35]	[0.03,0.25]	[0.07,0.27]
α_{y+1}	0.89	0.86	0.90	0.88	0.88	0.87
	[0.81,0.96]	[0.79,0.93]	[0.83,0.96]	[0.81,0.95]	[0.81,0.95]	[0.79,0.94]
$\phi_{y,-1}$	0.17	0.18	0.18	0.20	0.21	0.20
	[0.07,0.27]	[0.09,0.27]	[0.07,0.28]	[0.10,0.30]	[0.09,0.33]	[0.09,0.30]
Patman	0.27	0.22	0.17	0.09	0.05	0.06
	[0.09,0.62]	[0.07,0.72]	[0.08,0.66]	[0.05,0.15]	[0.00,0.33]	[0.02,0.27]

Notes: this table shows the posterior mean for coefficients of equations (EE'), (PC') and (Policy') for various measures of π and for various samples, when the gap measure is output gap. Columns (1) use the sample `1957Q3-2008Q4`; columns (2) use the sample `1957Q3-2017Q4`. Parameters β and α_y are not estimated and set to .99 and .99. Parameter κ is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis Algorithm, with two chains of 1,000,000 draws each and discarding the first 500,000 draws of each chain. The numbers between brackets represent the 95% confidence band using the posterior distribution.

G Data Definition and Sources

- Inflation, GDP deflator: Gross Domestic Product: Implicit Price Deflator, Percent Change from Preceding Period, Quarterly, Seasonally Adjusted Annual Rate, obtained from the FRED database, (`A191RI1Q225SBEA`). Sample is `1947Q1-2017Q4`.
- Inflation, Headline CPI: Consumer Price Index for All Urban Consumers: All Items in U.S. City Average, Percent Change, Quarterly, Seasonally Adjusted, obtained from the FRED database, (`CPIAUCSL_PCH`). Sample is `1947Q1-2017Q3`.
- Inflation, Core CPI: Consumer Price Index for All Urban Consumers: All Items Less Food and Energy in U.S. City Average, Percent Change, Quarterly, Seasonally Adjusted, obtained from the FRED database, (`CPILFESL_PCH`). Sample is `1947Q1-2017Q3`.
- Inflation, Headline PCE: Personal Consumption Expenditures: Chain-type Price Index, Percent Change, Quarterly, Seasonally Adjusted, obtained from the FRED database, (`PCEPI_PCH`). Sample is `1947Q1-2017Q3`
- Nominal interest rate: Effective Federal Funds Rate, Percent, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (`FEDFUNDS`). Sample is `1954Q3-2017Q3`.
- Unemployment: Civilian Unemployment Rate, Percent, Quarterly, Seasonally Adjusted, obtained from the FRED database, (`UNRATE`). Sample is `1948Q1-2017Q3`.
- Natural rate of unemployment: Natural Rate of Unemployment (Long-Term), Percent, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (`NROU`). Sample is `1949Q1-2017Q3`.
- Unemployment gap: constructed as `UNRATE - NROU`.
- Oil price: Spot Crude Oil Price: West Texas Intermediate (WTI), Dollars per Barrel, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (`WTISPLC`). Sample is `1946Q1-2017Q3`. The real oil price is then obtained by deflating by GDP deflator `A191RI1Q225SBEA`.
- GDP: Real Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate, obtained from the FRED database, (`GDPC1`). Sample is `1949Q1-2017Q3`.
- Potential output: Real Potential Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (`GDPPOT`). Sample is `1949Q1-2017Q3`.
- Output gap: constructed as `GDPC1 - GDPPOT`.
- Expected Inflation: Expected Change in Price During the Next Year, obtained from Surveys of Consumers, University of Michigan. Transformed into annualized quarterly expected inflation. Sample is `1960Q1-2017Q4`.
- Romer and Romer shocks: Monetary Shocks isolated following the method in Romer and Romer [2004]. Obtained from Wieland and Yang [2020]. Sample is `1969Q1-2007Q4`.
- Rossi and Zubairy shocks: Monetary Shocks from Rossi and Zubairy [2011]. Sample is `1957Q1-2006Q4`.