Uncovering Subjective Models from Survey Expectations

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Abstract

Expectations about different macroeconomic aspects correlate with each other. Using Michigan Survey of Consumers (MSC), I found consumers' inflation expectation is positively correlated with expectations on unemployment status. Such a correlation is inconsistent with realized data, professionals' belief, and the standard New Keynesian Model. I then perform a structural test in the framework of noisy information model and show that consumers form their expectations on multiple macroeconomic variables jointly rather than independently, thus causing these expectations to be correlated with each other. These results imply the consumers have a subjective model about how macroeconomics variables are correlated that is different from the professionals and the reality. In particular, consumers believe economic conditions will be worse during episode with extensive inflation news, even if there's only mild inflation, causing their average expectation on inflation to co-move with that of unemployment and business condition. These patterns call for explanations on how agents form beliefs on interactions between macroeconomic variables that are different from the actual structure of data. They also suggest Central Bank should use inflation-related expectation management policy with cautious, as such policy may induce pessimistic responses among households.

Keywords: Expectation Formation, Noisy Information Model, Survey Data

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1 Introduction

The last decade has seen a growing literature documenting behaviors that deviate from Full Information Rational Expectation (FIRE) theories. Most of these studies focus on inflation expectation (e.g. (Coibion and Gorodnichenko, 2015), (Malmendier and Nagel, 2015) etc), or expectation on a single economic variable.[1] Few papers examine the link between expectations on different macroeconomics variables,[2] despite the fact that in classical macroeconomics models, cross-correlations between these variables are particularly important.

In the context of expectation formation models, these correlations can offer us new insights into why the households' expectations deviate from FIRE. To be specific: (1) if agents form their expectations on various macroeconomic aspects jointly, the cross-correlations between expectational variables will have implications on their subjective beliefs about the interactions between these variables. Such a subjective belief may or may not be consistent with the complicated modern macroeconomics models. In other words, the agents may have a different model in mind. Then even if they have full information, they will form expectations that are different from the FIRE benchmark. (2) If they do have the same model in mind, a noisy information environment will generate correlations between expectations, which are absent in standard models under the FIRE assumption.

Both these two possibilities are important to policymakers as the current policy will serve as signals to economic agents. For example, if an agent believes that inflation is a signal of a possible economic downturn, moving inflation expectations up in the Zero Lower Bound (ZLB) episode may have an additional contractionary effect than suggested in (Eggertsson and Woodford, 2003). In fact, there is evidence suggesting that inflation expectations have a negative impact on household consumption, especially in the ZLB episode.[3]

In this paper, I first examine the correlation across expectations on two key macroeconomic variables: unemployment rate and inflation. I find that consumers believe higher unemployment rate and worse economic conditions are more likely to happen together with high inflation, a feature that is neither seen in realized data nor in the Survey of Professional Forecast (SPF) and at the same time inconsistent with standard New Keynesian Model.

This data pattern is hard to be explained by learning models where agents make inferences about a single variable of interest. I then modify the noisy information framework

[1]For example (Barsky and Sims, 2012), (Doms and Morin, 2004) look at consumers' expectation on economic condition only.

[2]Exceptions include recent studies using survey experiment such as (Candia *et al.*, 2020) and (Andre *et al.*, 2019).

[3]For example, (Bachmann *et al.*, 2015) and (Burke and Ozdagli, 2013) found expected inflation has a negative impact, if any, on durable good consumption attitude for US consumers, and this impact is even more negative during the Zero Lower Bound (ZLB) episode. (Candia *et al.*, 2020) found similar results from a field experiment in Netherland.

to allow agents to jointly form their expectations across different macroeconomic variables. Following the same veins as (Coibion and Gorodnichenko, 2012), I describe the different testable implications on observed expectational data that can distinguish between different models of expectation formation. On top of testing for deviations from FIRE as proposed by (Coibion and Gorodnichenko, 2012), my empirical test can also tell whether the expectations on different variables are formed jointly or independently. Moreover, the test results can shed lights on the subjective beliefs about the correlation between different macroeconomic variables.

I perform the joint learning test using survey data on consumer's expectation (the Michigan Survey of Consumers, thereafter MSC) and professional's belief (SPF). I find that the consumers form expectations jointly, taking into the consideration that inflation and unemployment are correlated. In particular, they believe past high inflation will lead to higher unemployment rate in the future. Such a belief is different from the professionals. I then show the consumers' subjective belief is consistent with the positive correlation between expected unemployment rate and inflation that I documented before. It also explains why the same correlation is not observed in SPF.

Two possibility arises when expectations are formed jointly. The agents may have a subjective belief about the correlations between these variables (the transition matrix in the noisy information model). Or they may believe the two variables are not correlated but receive signals that contain information about both variables. The latter is the friction through which (Kamdar, 2019) explains the same positive correlation documented in this paper. I show that the test results from survey data are at odds with such friction being the sole explanation for the observed correlation.

Finally, I also provide independent and new evidence to show that the correlation is likely due to consumers having a specific subjective model. I use the perceived news measure documented by MSC to show news heard or believed by consumers has a significant yet different impact on their expectations. For example, consumers who heard of the news about inflation are likely to believe in not only higher inflation but also worse economic conditions in the future; whereas bad news about the labor market mostly affects consumers' beliefs on unemployment conditions and has at best negative impacts on inflation expectations. I also show these self-reported news are co-moving with realized inflation and unemployment rate correspondingly, suggesting the consumers can distinguish between signals on inflation and real economic activities.

This paper is related to the empirical literature on information rigidity in expectation formation process. This literature considers structure from noisy-information model (Woodford, 2001) and (Sims, 2003) or information rigidity model ((Mankiw *et al.*, 2004)) and perform tests using the model implications on forecasting error and forecast revisions.[4] The joint learning test developed in this paper is in the same spirit of the tests as in (Coibion and Gorodnichenko, 2012) and (Andrade and Le Bihan, 2013). It allows me to test for new forms of information friction: the agents' subjective beliefs on the transition matrix of macroeconomic variables as well as whether the signals contain mixed information about different macro variables. My test also nests the original tests on deviation from FIRE as a special sub-case.

The positive correlation between expected inflation and unemployment rate is also documented in (Bhandari *et al.*, 2019) and (Kamdar, 2019). The former explains these stylized facts with robustness where the consumers are concerned about unknown unfavorable events including unemployment rate increase or inflation hikes. The latter proposed a rational inattention model in which consumers optimally choose a signal as linear combination of inflation and unemployment rate. The test proposed in this paper serves as an empirical test for the model in (Kamdar, 2019). The results are hard to be reconciled with the mixed signal friction suggested in that paper. Furthermore, both these two explanations have a prediction that the consumers cannot distinguish between news about inflation and unemployment status. In this paper I offer empirical evidence that the consumers subjectively label the news they heard about inflation and labor market. These self-reported news are co-moving with realized inflation and unemployment rate correspondingly. Furthermore, only news labelled as inflation will lead to positive adjustment on both inflation and unemployment expectations.

This paper is organized as follows: Section 2 organizes the empirical findings on cross-correlation between expectations on different variables. Section 3 derives the testable implications and performs the test of joint expectation formation under the noisy information model. Section 4 documents independent evidence on the connection between the cross-correlation and joint learning using perceived news data in MSC. Section 5 concludes.

2 Cross-correlation between Expectational Variables

In this section, I show the cross-correlation structure of expectational variables is quite different between households/consumers and professional. I focus on the correlation between inflation and the change of unemployment rate because the correlations between beliefs on other economic variables are consistent with those in realized data. I include these results in **Appendix B.1**.

I first confirm that the cross-correlation between consensus expectations on inflation and

[4]The seminal work (Coibion and Gorodnichenko, 2012) and (Andrade and Le Bihan, 2013) consider tests using current and lag forecast errors. (Coibion and Gorodnichenko, 2015) and (Bordalo *et al.*, 2018) use forecast error and forecast revisions obtained from survey data.

unemployment is positive for consumers. These results are also documented by (Bhandari *et al.*, 2019) and (Kamdar, 2019). I then show such correlations in realized data and professional forecasts are both close to zero. Finally, I use micro-data from surveys of expectations to show this pattern holds true at individual level, and is not likely induced by time or individual specific factors.

In my baseline analysis, I use the Reuters/Michigan Survey of Consumers (MSC) as proxy for households' expectations and the Survey of Professional Forecasts (SPF) from Federal Reserve Bank of Philadelphia for professional's expectations.[5] For realized macroeconomic variables I obtain data from website of Federal Reserve Bank of St. Louis (FRED). Detailed data description is included in **Appendix A.1**.

2.1 Aggregate Time Series

I first report the simultaneous correlation between consensus expectations on inflation and unemployment from MSC, SPF and realized data. All the expectational variables are average of individual expectations within the quarter.[6]

Table 1: Correlations: 1981q3-2018q4

	MSC	SPF	FRED
$corr(E\pi, Eun)$	0.16^{**}	0.03	0.00
$corr(E\pi, Ey)$	-0.25^{***}	-0.01	0.08

* **** means significant at 1%,** means 5 % and * means 10%, indicating significance level of Pearson Correlation.*

The first column in Table 1 summarizes the Pearson correlation between (expected) inflation and unemployment. As unemployment and real GDP growth is negatively correlated in all the three datasets, I also report in the second column the correlation between inflation and real GDP growth in the second column as a robustness check. For households there is a significant positive correlation between expected inflation and expected unemployment increase. It suggests that the agents believe future inflation will occur together with the unemployment rate increase or real GDP growth fall.

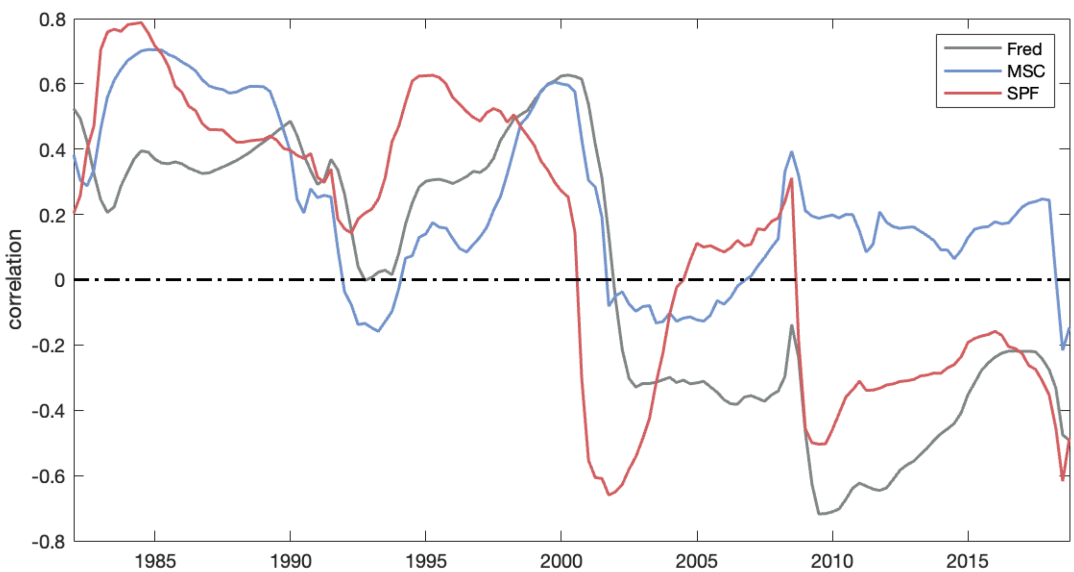
[5] Another dataset on households' expectations is Survey of Consumer Expectation (SCE) from the Federal Reserve Bank of New York. As MSC is available for a longer period and has a wider range of questions on households' expectations, I use it in the baseline results and show that it is robust to the use of SCE over the same period when the SCE is available.

[6] In MSC, expectation data is available at monthly frequency. I use quarterly data to keep MSC at the same frequency as SPF. The use of monthly data doesn't change my results qualitatively.

There can be various reasons for such a correlation to exist. If agents are making predictions using adaptive learning or rational expectation models, we may also see a cross-correlation structure of their expectations similar to that of realized variables. However, from Table 1, I found such a correlation doesn't exist in either SPF or realized data over the same period. This pattern suggests the cross-correlation structure of expectational variables is hard to be reconciled with the rational expectation or adaptive learning models, because both these models suggest expectations should be closely linked with realized data so that expectational variables should have a similar correlation structure as the realized ones.

Another interesting fact one may notice is the correlation between realized inflation and unemployment change is around 0 instead of being negative. This seems to contradict the Phillips Curve relationship between inflation and the unemployment rate. One explanation is that the correlation between inflation and unemployment is time-varying. When the dynamics of these variables are mainly driven by supply shocks, the correlation is likely to be positive. Whereas demand shocks lead to negative correlations between inflation and unemployment rate. [7]

Figure 1: Time-varying correlation between inflation and unemployment change



Correlation using 10-year rolling window, 1982-2018. Grey line: realized data from FRED. Blue line: expectations from MSC. Red line: expectations from SPF.

Figure 1 depicts the time-varying correlation between (expected) inflation and unemployment change using 10-year rolling windows. The grey line shows this correlation is mostly

[7] However, I want to point out this correlation I document is not directly comparable to a Phillips Curve relation because I'm using year-to-year unemployment rate change rather than a gap that measures economic slackness which is typically used in modern Phillips Curve analysis.

positive prior to 1990s and turns negative after 2000 for realized variables, similar to that of SPF. Whereas the correlation from MSC depicted by the blue line is always positive throughout 1982-2018. This again suggests the cross-correlation structure in households's beliefs is different from the professionals and actual realization. [8]

2.2 Individual-level Cross-correlation

There are potentially many possible explanations for the observed positive correlation between consensus expectations. One possibility is there are waves of pessimism and optimism that move the average of unemployment and inflation beliefs in the same direction. Furthermore, as one can see from Figure 1, the time-series correlation will heavily depend on the presence of aggregate shocks.

To rule out these possibilities, I examine whether individual respondents in household surveys make a similar association. This will help me to understand whether the patterns in aggregate level data have a micro-level foundation or they are mainly coming from the aggregation process. Various former researches suggest that the properties of consensus expectation may differ from those of individual expectations.[9] Figure 2 shows the estimated correlation from the cross-section regression in each year.

The top panel of Figure 2 uses data from MSC. In this survey the respondents are asked whether they think unemployment a year from now will go up, stay the same or go down. The two lines are the differences in inflation expectations relative to consumers that believe unemployment will stay the same, for each year. The figure suggests that households' beliefs on inflation is again positively associated with their beliefs on unemployment change. Such a positive relation is significant and relatively stable across time. This finding is the same as in (Kamdar, 2019).

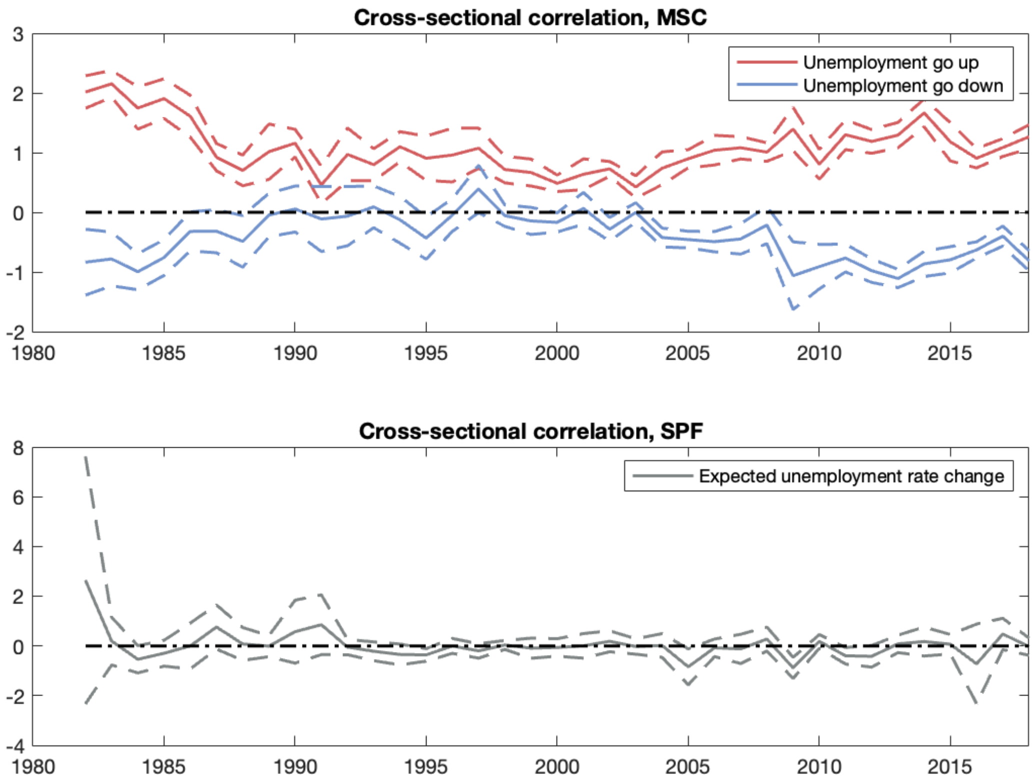
The bottom panel of Figure 2 is the cross-sectional correlation between expected inflation and unemployment rate change in SPF. In contrary to consumers, the professionals do not associate inflation with unemployment rate when forming their beliefs.

Could this correlation be driven by specific group of individuals? For example, if there are groups of pessimistic individuals, they will always form worse than average unemployment expectations together with higher than average inflation expectations. This will create a positive association in the cross-sectional analysis above. I then utilize the panel dataset in

[8] One other interesting aspect is that the time-varying correlation from SPF leads those from both realized variables and MSC. This suggests that the professionals have a good knowledge about the comovment between macroeconomic variables, or they are exposed to news about future economic activities.

[9] For instance, (Coibion and Gorodnichenko, 2015) suggests the predictability of forecasting error from forecast revision is an emergence property of aggregation across individuals and may not be seen at the individual level; (Bordalo *et al.*, 2018) documents over-reaction of inflation expectation to new information on the individual level, in contrary to under-reaction typically found with consensus expectations.

Figure 2: Time-varying correlation between inflation and unemployment change



The top panel reports estimates β_1 from: $E_{i,t}\pi_{t+12,t} = \beta_0 + \beta_1 U_{t+12,t} + \theta\mu_i + D_t + \epsilon_{i,t}$. Where $U_{t+12,t}$ stands for two dummy variables indicating the MSC consumer believes unemployment rate will go up or go down in the next 12 months. The bottom panel reports estimates β_1 from: $E_{i,t}\pi_{t+4,t} = \beta_0 + \beta_1 E_{i,t}un_{t+4,t} + \theta\mu_i + D_t + \epsilon_{i,t}$. Where $E_{i,t}un_{t+4,t}$ stands for expected change of unemployment rate from SPF. The data from MSC is monthly and from SPF is quarterly. 10% confidence interval is reported in dash lines.

MSC and SPF to control for individual fixed effect as well as time fixed effect.

$$E_{i,t}\pi_{t+12,t} = \beta_0 + \beta_1 E_{i,t}un_{t+12,t} + \beta_2 E_{i,t}i_{t+12,t} + \theta X_{i,t} + D_t + \mu_i + \epsilon_{i,t} \tag{1}$$

Again because in MSC the expected unemployment change is categorical variable, β_1 in (1) contains coefficients when expected unemployment go up or down. $X_{i,t}$ includes controls such as expectations on other subjects and social-economic status, μ_i and D_t stand for individual and time fixed effect respectively. Because the panel dataset from MSC contains fewer observations and only keeps the participants for 2 waves of surveys that are 6 months apart. I also report the results from the same regression using panel data from SCE. [10]

[10] When using MSC, the expected unemployment and interest rate change are categorical variables, and I construct dummies that stand for increase or decrease for each of these variables. In SCE those variables are reported percentage points for the likelihood of the corresponding variable to increase.

Table 2: FE Panel Regression

	MSC		SCE		SPF
Unemployment up	0.30^{***}	$\hat{\beta}_1$	0.012^{***}	$\hat{\beta}_1$	-0.17^{***}
	(0.05)		(0.002)		(0.06)
Unemployment down	-0.22^{***}				
	(0.05)				
FE	Y		Y		Y
Time dummy	Y		Y		Y

* ***,**,*: Significance at 1%,5% and 10% level. Estimation results for specification (1) controlling for individual and time varying characteristics, individual fixed effect and time fixed effect. Standard errors are adjusted for heteroskesticity and autocorrelation.

Table 2 column 1 shows that for MSC, an agent that expects the unemployment to go up will predict inflation to be 0.3% higher on average than one that believes unemployment be stable; and 0.52% higher than one that believes the unemployment rate will fall. Meanwhile, the standard deviation of expected inflation across this episode is 1.17%, and the standard deviation of CPI is around 2.19%. These results are comparable to those from (Kamdar, 2019), where the author estimates a similar fixed-effect model but only on correlation between expected inflation and unemployment change, without controlling for other expectational variables. The estimates shown in column 2 from SCE are consistent with those from MSC: if the agent expects there is a 22% higher chance (which is the standard deviation of the variable) unemployment rate will increase in 12 months, he will also expect inflation to be 0.22% higher. It's worth noting the controls of fixed individual and time effect means the positive correlation between unemployment and inflation is not due to a common time-varying bias, which should have been captured by the time fixed effect; and is not due to the effect of "pessimistic individuals" which is taken out by individual fixed effects. Finally, in contrast to the consumers' expectations, the column three shows that there is a negative correlation between expected inflation and change of unemployment rate. On average a 1% increase in expected unemployment rate is associated with 0.17% fall of expected inflation for professionals. This again coincides with the message from aggregate correlation that professionals believe in a different relationship between future inflation and unemployment movements than consumers.

3 Test of Joint Expectation Formation

From the last section, we see significant reduced-form cross-correlations between households' expectations on inflation and unemployment status. They constantly believe economic performance will be worse when there is concern about future inflation. This stylized fact is specific to household expectations and is not present in professionals' beliefs. It is also inconsistent with the realized data in the same period and obviously at odds with predictions from New Keynesian models. Furthermore, such a correlation exists on both individual and aggregate levels, and it's not due to time-specific or individual-specific factors. This distinction between expectations and the reality gives rise to the possibility of a joint learning model, in which the household may believe in a different model from professionals as well as the reality.

In this section, I develop a test on joint expectation formation to formally test whether households are forming expectations on different variables jointly or independently, and to shed lights on the different models the households and professionals believe in. The test is within the framework of a noisy information model that is most commonly used in the empirical literature with survey data on expectations.

The noisy information model has long history dated back to (Lucas, 1976) and the recent version was proposed by (Woodford, 2001) and (Sims, 2003). It is then widely adopted for tests on information friction and deviation from FIRE assumptions. The seminal research of (Coibion and Gorodnichenko, 2012) shows the existence of imperfect(noisy) information implies predictability of forecasting errors and provides evidence of imperfect information using consensus expectations of consumers, professionals, and policymakers; similarly (Andrade and Le Bihan, 2013) provides evidence in support of information friction in ECB professional forecasts. More recently, researchers have focused on estimating the implied structure of noisy information model with individual level data.[11] The joint expectation formation test I propose in this section is in the same spirit as (Coibion and Gorodnichenko, 2012), but allows for more general forms of imperfect information.

However all of these empirical tests are assuming for each variable the agent tries to predict, the filtering and updating process is done independently from other variables that the same agent wants to predict. This serves as an extra assumption when agents try to predict more than one outcome of the future at the same time, which is usually the case in daily life and in a survey environment. I call such a model "joint expectation formation" model, in which agents form expectations on multiple variables using the same set of information. One important benefit of allowing joint expectation formation is that the survey data on different expectational variables will help to uncover the agents' beliefs on the underlying State Space

[11]For inflation expectation of US consumers, see (Ryngaert, 2017); for expectations on various subjects of SPF, see (Bordalo *et al.*, 2018).

Representation. This then can help explain the cross-correlation structures I documented before and bring new insights on different agents' expectation formation process. In this section, I follow the baseline noisy information model in the literature and allow for joint expectation formation and test whether household surveys indicate agents form expectation jointly rather than independently.

Consider that the Actual Law of Motion(ALM) takes the form of state-space representation of multiple macroeconomic variables $\boldsymbol{L}_{t+1,t}$ as in (2). and agents observe noisy signals on these variables, the observational equation is given by (3).

$$\boldsymbol{L}_{t+1,t} = A\boldsymbol{L}_{t,t-1} + w_{t+1,t} \tag{2}$$

$$\boldsymbol{s}_t^i = G\boldsymbol{L}_{t,t-1} + v_t^i + \eta_t \tag{3}$$

Agents face four different channels of imperfect information: (1) the functional form of ALM (linear in this case); (2) the correct structural parameters in (2);(3) a mixed signal generating process (3);(4) observability of $\boldsymbol{L}_{t,t-1}$. Most of the noisy information models assume that the only source of imperfect information comes from not observing $\boldsymbol{L}_{t,t-1}$ perfectly. Recently researchers also consider the possibility of misspecified parameters or models. For example, in the context of forecasting inflation, (Ryngaert, 2017) found households use a different persistence parameter than ALM in predicting inflation. In (Hou, 2021), the author found the U.S households form expectations on unemployment and economic conditions in a non-linear and asymmetric way. In (Kamdar, 2019), the author argues that a mixed signal generating process (3) creates the positive correlation between expected inflation and unemployment. In the joint expectation formation framework introduced in this section, I allow for the last three forms of imperfect information and assume the agents always have the correct (linear) functional form of ALM.

I follow the existing literature on noisy information model but allow agents to have a subjective model, which is possibly mis-specified, where they use $\hat{A}$ in place of $\boldsymbol{A}$. Their Perceived Law of Motion (PLM) then can be expressed as:

$$\boldsymbol{L}_{t+1,t} = \hat{A}\boldsymbol{L}_{t,t-1} + w_{t+1,t} \tag{4}$$

It is obvious that $\hat{A}$ represents households' subjective model about the economy. In the single-variable expectation formation context, this usually means agents misperceived the persistence of state variables, as in (Ryngaert, 2017). In a joint expectation formation model, a $\hat{A}$ that is different from A also suggests that agents believe in cross-correlation between macroeconomic variables that is different from actual data or models that the professionals use. Intuitively, $\hat{A}$ can then help to explain the differences of cross-correlation structure between survey expectations and actual data. Another form of joint expectation formation would be that the agents observe signals that mix information of multiple variables in $\boldsymbol{L}_{t,t-1}$. This possibility nests (Kamdar, 2019) as a special case, where the author assumes G to be a vector and $\boldsymbol{s}_t^i$ contains both information on inflation and unemployment status.

These two forms of joint learning stand for different reasons why we see discrepancies between expectational and realized variables: a misspecified model of the economy from $\hat{A}$, or a mixture of information represented by G. The test I propose in this section will shed lights on these two mechanisms, I leave the discussion to Section 3.1.

In the joint learning model, I also allow for an individual-specific noise v_t^i as well as a time-specific one η_t, both of which follow a normal distribution with mean zero. The individual noise is independent across agent and time and the time-specific noise is not autocorrelated and independent with the structural shock $w_{t+1,t}$. Each element in v_t^i, η_t and $w_{t+1,t}$ are also assumed to be independent with each other for simplicity. Adding a time-specific noise doesn't change the nature of the individual's signal extraction problem, the only difference it makes is to allow for an imprecise signal after aggregation at each time point. To ease notations I define $\epsilon_{i,t} := v_t^i + \eta_t$. The distribution of shocks and noises:

$$w_{t+1,t} \sim N(0, Q) \quad \epsilon_{i,t} := v_t^i + \eta_t \sim N(0, R)$$

Where Q and R are diagonal variance-covariance matrices.

The agents then update their beliefs upon observing s_t^i and form expectations according to a linear Kalman Filter as described in (5), where K is the Kalman Gain.[12]

$$\begin{aligned}
\boldsymbol{L}^i_{t+1,t|t} &= \hat{A}\boldsymbol{L}^i_{t,t-1|t} \\
&= \hat{A}\Big(\boldsymbol{L}^i_{t,t-1|t-1} + K(\boldsymbol{s}^i_t - G\boldsymbol{L}^i_{t,t-1|t-1})\Big)
\end{aligned} \tag{5}$$

The forecasting error for one period ahead is given by:

$$\begin{aligned}
FE^i_{t+1,t|t} &\equiv \boldsymbol{L}_{t+1,t} - \boldsymbol{L}^i_{t+1,t|t} \\
&= A\boldsymbol{L}_{t,t-1} - [\hat{A}(I-KG)\boldsymbol{L}^i_{t,t-1|t-1} + \hat{A}KG\boldsymbol{L}_{t,t-1} + \hat{A}K(v^i_t + \eta_t)] + w_{t+1,t} \\
&= \hat{A}(I-KG)(\boldsymbol{L}_{t,t-1} - \boldsymbol{L}^i_{t,t-1|t-1}) + \underbrace{(A - \hat{A}KG - \hat{A}(I-KG))}_{M}\boldsymbol{L}_{t,t-1} + w_{t+1,t} - \hat{A}K(v^i_t + \eta_t) \\
&= \hat{A}(I-KG)FE^i_{t,t-1|t-1} + M\boldsymbol{L}_{t,t-1} + w_{t+1,t} - \hat{A}K(v^i_t + \eta_t)
\end{aligned} \tag{6}$$

The above equation is the testable implications on the dynamics of forecasting errors.

[12] For derivation of standard Kalman Filter please see Appendix C.1.

Averaging across agents i at each time t we get an aggregate test on forecasting errors:

$$FE_{t+1,t|t} = \hat{A}(I - KG)FE_{t,t-1|t-1} + M\boldsymbol{L}_{t,t-1} + w_{t+1,t} - \hat{A}K\eta_t \tag{7}$$

The equation (6) is the individual level forecasting error test and (7) is the aggregate test. Both equations can be tested against survey data using OLS as $w_{t+1,t}$ and η_t are independent with $FE_{t,t-1|t-1}$ and $\boldsymbol{L}_{t,t-1}$. Now consider the state vector $\boldsymbol{L}$ contains unemployment rate change and inflation. I will discuss the formal properties in the context of two-dimensional $\boldsymbol{L}$.

3.1 Properties of Joint Learning Test

First to define relevant notations and declare the formal assumptions under which the test results will hold. I assume that the structural shocks w and noise on signals ϵ are not correlated:

Assumption 1. *The shocks and noises on signals are not correlated, so that:*

$$R := \begin{pmatrix} \sigma^2_{1,s} & 0 \\ 0 & \sigma^2_{2,s} \end{pmatrix} \quad Q := \begin{pmatrix} \sigma^2_{1,t} & 0 \\ 0 & \sigma^2_{2,t} \end{pmatrix}$$

Furthermore, I assume the priors on the two state variables are also uncorrelated and common for each individual:[13]

Assumption 2. *The var-cov matrix of prior $\boldsymbol{L}^i_{t,t-1|t-1}$ is a diagonal matrix and common to each individual:*

$$\Sigma := \begin{pmatrix} \sigma^2_1 & 0 \\ 0 & \sigma^2_2 \end{pmatrix}$$

Under these assumptions, different types of imperfect information will lead to different values in the coefficient matrix $\hat{A}(I - KG)$. Following the convention from the literature, I first consider the case of FIRE. Notice FIRE under joint learning requires more than noise going to zero (or variance of noise going to zero). Rather we need $\hat{A} = A$ and $G = I$.[14] These two extra conditions suggest that the agent has the correct belief about the parameters in the State-space model and she observes two separate signals on the two states perfectly. Then the following proposition holds:

[13] The prior variance-covariance matrix is useful in deriving Kalman Gain K.

[14] The condition $\hat{A} = A$ is not necessary for the test results of FIRE to hold. See the proof of Proposition 1 below.

Proposition 1. *Under Full Information Rational Expectation, that is when $A = \hat{A}$, $G = I$ and the variances of signals $\sigma_{1,s}, \sigma_{2,s} \to 0$, the coefficient matrix attached to $FE^i_{t,t-1|t-1}$ have all-zero elements.*

Proof. First we derive Kalman Gain K in FIRE case:

$$\begin{aligned} K &= \Sigma(\Sigma + R)^{-1} \\ &= \begin{pmatrix} \frac{\sigma_1^2}{\sigma_1^2+\sigma_{1,s}^2} & 0 \\ 0 & \frac{\sigma_2^2}{\sigma_2^2+\sigma_{2,s}^2} \end{pmatrix} \end{aligned} \tag{8}$$

Take the Limit:

$$\lim_{\sigma_{1,s},\sigma_{2,s}\to 0} K = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

We have then the coefficient matrix: $\hat{A}(I - KG) = \mathbf{0}$. □

The above proposition makes clear that even under joint expectation formation, lag forecast errors will not predict current forecast errors under FIRE. This is consistent with the standard results from single variable noisy information model.

Then we turn to the case where the agent learns two state variables independently. This means she believes that the two states are not related in the state-space representation ($\hat{A}$ is diagonal) and observes two separate noisy signals on the two states (without loss of generality $G = I$).[15] The following proposition holds:

Proposition 2. *If $G = I$ and $\hat{A}$ is diagonal: then $\hat{A}(I - KG)$ is a diagonal matrix.*

Proof. When $G = I$ and $\hat{A} = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}$. From (8) we get:

$$\begin{aligned} \hat{A}(I - KG) &= \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \times \begin{pmatrix} \frac{\sigma_{1,s}^2}{\sigma_1^2+\sigma_{1,s}^2} & 0 \\ 0 & \frac{\sigma_{2,s}^2}{\sigma_2^2+\sigma_{2,s}^2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{\sigma_{1,s}^2\rho_1}{\sigma_1^2+\sigma_{1,s}^2} & 0 \\ 0 & \frac{\sigma_{2,s}^2\rho_2}{\sigma_2^2+\sigma_{2,s}^2} \end{pmatrix} \end{aligned} \tag{9}$$

□

[15]Note that separate signals actually imply G being diagonal. It is straight-forward to extend the next proposition to the case where G has elements other than one on its diagonal. For simplicity of the exposition, I always stay with $G = I$ in these cases.

The above two propositions then made clear that when $\hat{A}$ and G are both diagonal, this formulation collapses to the single-variable noisy information model on each variable in $\boldsymbol{L}$ and one can perform the forecasting error tests separately for each variable, which is done in (Coibion and Gorodnichenko, 2012) and (Andrade and Le Bihan, 2013). This is a special case for the joint-learning specification. I call it independent-learning model. Under these restrictions, forecast error tests accroding to (6) and (7) have implications on whether information rigidity exists as shown in (Coibion and Gorodnichenko, 2012) and (Andrade and Le Bihan, 2013). The resulted coefficient matrix under independent-learning is different from that under FIRE, because the lag forecast errors can predict current forecast errors of the same state variable. This suggests there is information friction in belief formation process so that the agents's mistakes become persistent.

One special case that falls into the scope of the proposition 2 is when actual states are correlated but the agents believe they are not. In this situation the coefficient matrix will still be diagonal, the difference between $\hat{A}$ and A will appear in matrix M defined in (6).

Finally, I consider the case when agents form expectations jointly. The joint expectation formation takes two different forms. The agents could believe in a non-diagonal $\hat{A}$, or they could receive a mixed signal on the two state variables. I discuss the implications on the coefficient matrix $\hat{A}(I - KG)$ separately for these two cases.

Proposition 3. *If $G = I$ and $\hat{A}$ is non-diagonal, $\hat{A}(I - KG)$ have non-zero off-diagonal elements. The signs of these off-diagonal elements are the same as their counter-parts in $\hat{A}$.*

Proof. Suppose $\hat{A}$ has non-zero elements off-diagonal:

$$\hat{A} = \begin{pmatrix} \rho_1 & m_1 \\ m_2 & \rho_2 \end{pmatrix}$$

The coefficient matrix then becomes:

$$\begin{aligned} \hat{A}(I - KG) &= \begin{pmatrix} \rho_1 & m_1 \\ m_2 & \rho_2 \end{pmatrix} \times \begin{pmatrix} \frac{\sigma_{1,s}^2}{\sigma_1^2 + \sigma_{1,s}^2} & 0 \\ 0 & \frac{\sigma_{2,s}^2}{\sigma_2^2 + \sigma_{2,s}^2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{\sigma_{1,s}^2 \rho_1}{\sigma_1^2 + \sigma_{1,s}^2} & \frac{\sigma_{2,s}^2 m_1}{\sigma_2^2 + \sigma_{2,s}^2} \\ \frac{\sigma_{1,s}^2 m_2}{\sigma_1^2 + \sigma_{1,s}^2} & \frac{\sigma_{2,s}^2 \rho_2}{\sigma_2^2 + \sigma_{2,s}^2} \end{pmatrix} \end{aligned} \tag{10}$$

As long as $m_1, m_2 \neq 0$ the coefficients on cross-terms of past forecasting errors will be non-zero. Furthermore, the sign of these coefficients are the same as those in their subjective belief matrix $\hat{A}$. ∎

The above proposition shows that when the two signals are not mixed,[16] the coefficient matrix will have non-zero off-diagonal elements if and only if the agent believes in a non-diagonal $\hat{A}$. Moreover, the signs on the off-diagonal elements in $\hat{A}(I - KG)$ is directly linked to off-diagonal elements in $\hat{A}$.

The intuition behind this proposition is also straight-forward. Suppose that the first element in $\boldsymbol{L}_{t,t-1}$ is change of unemployment rate and the second element is inflation. If one under-predicted inflation yesterday, this means that she will also under-predict current inflation due to information rigidity. Such an under-prediction will create an under-prediction of unemployment tomorrow if the agent believes that higher inflation leads to higher unemployment rate in the future. Or it will create over-prediction of unemployment in the future if she believes that current inflation lowers future unemployment. Such a pattern holds no matter what the actual transition matrix A is.

Proposition 4. *If $G = (g_1 \quad g_2)$ and $\hat{A} = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}$, the off-diagonal elements of $\hat{A}(I-KG)$ are non-zero and of the same signs.*

Proof. Notice in this case signal is one dimentional so $R = \sigma_s^2$. We have:

$$K = \Sigma G'(G\Sigma G' + R)^{-1} = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} \times \frac{1}{g_1^2\sigma_1^2 + g_2^2\sigma_2^2 + \sigma_s^2}$$
$$= \begin{pmatrix} \frac{g_1\sigma_1^2}{g_1^2\sigma_1^2+g_2^2\sigma_2^2+\sigma_s^2} \\ \frac{g_2\sigma_2^2}{g_1^2\sigma_1^2+g_2^2\sigma_2^2+\sigma_s^2} \end{pmatrix} \tag{11}$$

Denote $m = g_1^2\sigma_1^2 + g_2^2\sigma_2^2 + \sigma_s^2$. Then the coefficient matrix is then:

$$\hat{A}(I - KG) = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \begin{pmatrix} \frac{g_2^2\sigma_2^2+\sigma_s^2}{m} & -\frac{g_1 g_2\sigma_1^2}{m} \\ -\frac{g_1 g_2\sigma_2^2}{m} & \frac{g_1^2\sigma_1^2+\sigma_s^2}{m} \end{pmatrix}$$
$$= \begin{pmatrix} \rho_1\frac{g_2^2\sigma_2^2+\sigma_s^2}{m} & -\rho_1\frac{g_1 g_2\sigma_1^2}{m} \\ -\rho_2\frac{g_1 g_2\sigma_2^2}{m} & \rho_2\frac{g_1^2\sigma_1^2+\sigma_s^2}{m} \end{pmatrix} \tag{12}$$

Now because $\rho_1, \rho_2 > 0$, and $m > 0$, it is obvious that the diagonal elements of the coefficients are positive and off-diagonal elements are non-zero and of the same signs. $\square$

Proposition 4 is about the second type of joint learning: when the agent believes in a diagonal transition matrix $\hat{A}$ but observes a mixed signal containing information on both state variables. For simple exposition, I consider the special case as in (Kamdar, 2019),

[16] Here I show it with $G = I$ for simple exposition, but it is straight-forward to extend the results to the case when G is merely diagonal.

where the author suggests the optimal signal an inattentive consumer will choose is created by $G = (g_1 \quad g_2)$. Furthermore to explain the positive correlation between expected inflation and unemployment, one needs $g_1 g_2 > 0$. In this case the coefficient matrix will also have non-zero off-diagonal elements, but the signs of these elements will be the same. The intuition behind is also simple. Consider the same example as before and an extreme case that in the last period inflation is 0 and unemployment is positive. When $g_1 g_2 > 0$, because the agent cannot tell whether a positive signal means positive inflation or unemployment change, she will adjust beliefs on both upwards thus creating positive forecasting error on unemployment and negative one on inflation. The past mistake is persistent due to information rigidity. For any new signal realized, the agent will start with a prior on unemployment lower than reality and a prior on inflation higher than reality. In other words, the positive FE on unemployment in the past creates lower FE on inflation in the future. The similar logic follows for the case of $g_1 g_2 < 0$.

It is worth point out that the information friction for mixed signal is not restricted to the case where G is a vector as in Proposition 4. I chose the specific form to simplify the analysis and relate to existing explanations for the correlated expectations in the literature. In **Appendix D** I include an extended version of this proposition allowing for G being a non-diagonal matrix. The results hold true: such a friction alone will imply non-zero off-diagonal elements of $\hat{A}(I - KG)$ with the same signs.

3.2 Correlation between Beliefs

In section 3.2, I show that the coefficient matrix $\hat{A}(I - KG)$ in the joint learning test I proposed has different properties when beliefs are formed under FIRE, single-variable learning, or joint learning. It is now useful to link the results from such test with implied correlations between belief variables under these different scenarios. Recall the individual mean forecast is given by (5). Define $Y_t = \begin{pmatrix} L^i_{t,t-1|t-1} \\ L_t \end{pmatrix}$ and we can write (5) and ALM (2) as the following vectorial auto regression (VAR) model:

$$Y_{t+1} = \underbrace{\begin{pmatrix} \hat{A}(I - KG) & \hat{A}KG \\ \mathbf{0}_{2\times 2} & A \end{pmatrix}}_{:=\Phi} \cdot Y_t + \underbrace{\begin{pmatrix} \hat{A}K & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 2} & I_{2\times 2} \end{pmatrix}}_{F} \cdot \begin{pmatrix} \epsilon_{i,t} \\ w_{t+1,t} \end{pmatrix} \tag{13}$$

Then we know the stationary Variance-covariance matrix is given by:

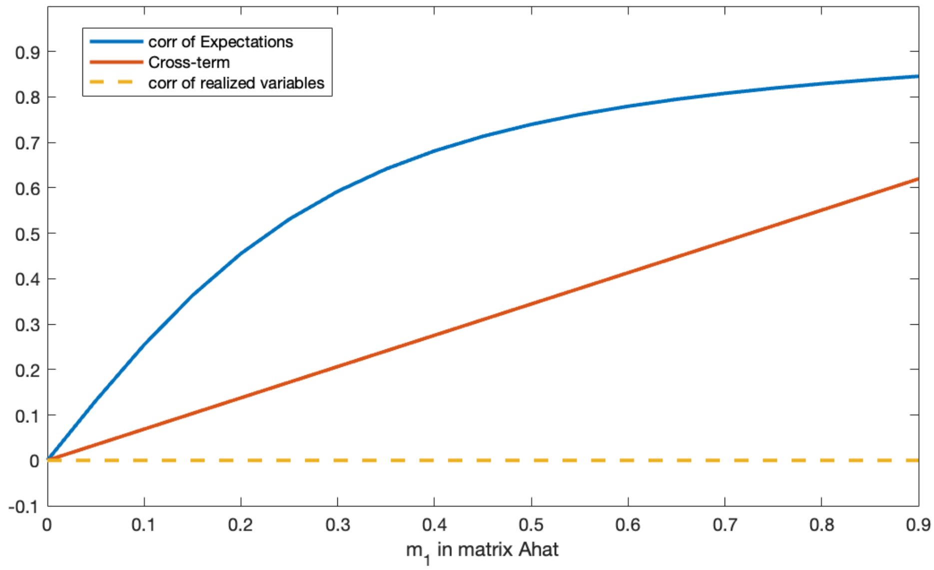
$$vec(\Sigma_L) = (I_{16} - \Phi \otimes \Phi)^{-1} vec\left(F(R + Q)F'\right) \tag{14}$$

It is obvious that the correlation between belief variables implied by the above covariance matrix will be different depending on whether expectations are formed independently, jointly or under FIRE. However the closed form expression for the above matrix is complicated even in 2-d case. I will illustrate the property of the correlations under different parametrizations of $\hat{A}$ and G. First because I saw no correlation between unemployment and inflation in the realized data, I always consider the case where A is diagonal.

FIRE and Independent Expectation Formation: It is then straight-forward that under FIRE or independent expectation formation both correlations between realized and expected unemployment and inflation should be 0. This follows directly from the fact that in (5), expectational variables $\boldsymbol{L}_{i,t+1,t|t}$ are linear combinations between their own past values, the corresponding current state variable and noise. Because $\hat{A} = A$ and G is diagonal, the elements in the first expectational variable are not correlated with those in the second expectation variable. From Proposition 1 and 2, in these two scenarios the joint learning test implies the off-diagonal elements in the coefficient matrix are zeros.

Joint Learning and $\hat{A}$ is non-diagonal: If the agent forms expectation jointly, there are two cases. First I consider the case where $\hat{A}$ is non-diagonal but the true A is diagonal matrix. The following figure plots the correlation as derived in (14) for expectational variables $L^i_{t+1,t|t}$, the off-diagonal term of $\hat{A}(I - KG)$ as derived in (12) as well as the correlation of the realized time series. I plot these objects as function of m_1, the off-diagonal term in matrix $\hat{A}$.

Figure 3



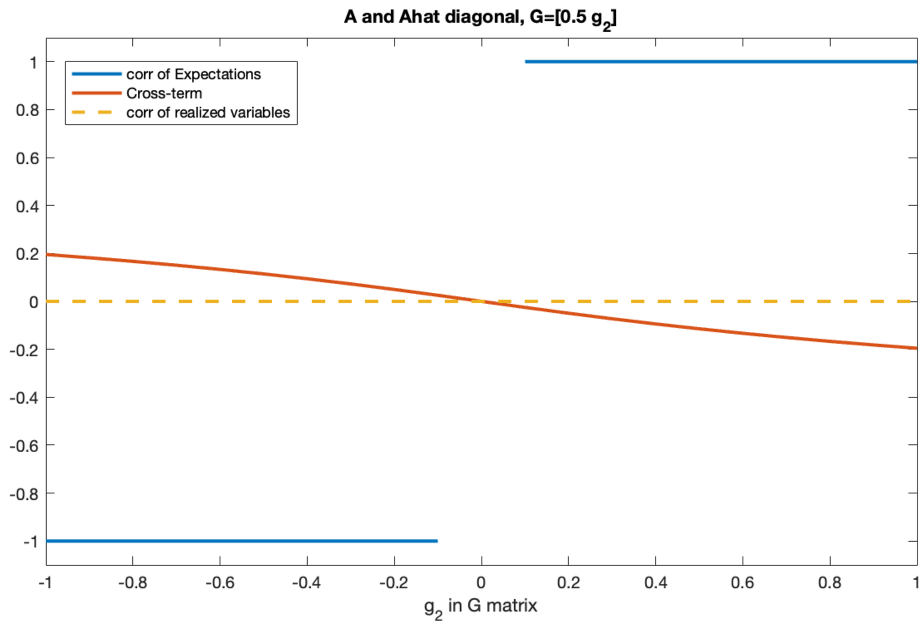
Blue line is correlation between expectations; red line is off-diagonal cross-term as derived in (12) and yellow line is the correlation of realized variables. The figure uses $A = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.9 \end{pmatrix}$, $\hat{A} = \begin{pmatrix} 0.9 & m_1 \\ 0 & 0.9 \end{pmatrix}$, $G = I_2$.

This figure shows that when $\hat{A}$ has off-diagonal terms $m_1 > 0$, the expectation variables will be positively correlated, despite the corresponding realized macro variables are not correlated. Meanwhile, as suggested by Proposition 3, one of the off-diagonal cross-term in $\hat{A}(I-KG)$ will be positive. Moreover, this cross-term and correlation between expectational variables are positively correlated.

Joint Learning and $A = \hat{A}$ diagonal matrix, $G = [g_1 \quad g_2]$: Then I look at the case where the agents have the same model $\hat{A}$ as the truth A, but their signals are correlated. Figure 2 then plots the correlation, cross-term and correlation of realized variables as function of g_2:[17]

[17]Without loss of generality, I fix g_1 and allow g_2 to vary and plot the corresponding correlation and cross-term. According to proposition 4, different values of g_1 will only affect the cross-term quantitatively in the figure and the correlation will not be affected.

Figure 4



Blue line is correlation between expectations; red line is off-diagonal cross-term as derived in (12) and yellow line is the correlation of realized variables. The figure uses $\hat{A} = A = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.9 \end{pmatrix}$, $G = \begin{pmatrix} 0.5 & g_2 \end{pmatrix}$.

The above figure shows that the correlation between expectational variables will be positive (and always be 1) only when the cross-term in $\hat{A}(I - KG)$ is negative. Moreover, from Proposition 4 we know the two off-diagonal cross-terms should be of the same sign. This implies that if expectational variables are positively correlated due to mix of signals, the joint learning regression should give negative coefficients on both cross-terms of forecast errors.

3.3 Empirical Tests on Joint Learning

Guided by the results from previous section, I can now summarize the testable implications under joint learning and test these using survey data. The expectational and realized macroeconomic variables are inflation and change of unemployment rate. Define the h-period ahead forecasting errors of variable x as $fe^{x}_{t+h,t|t}$. According to the aggregate testable equation (7) I can estimate the following regressions using survey data:

$$\begin{pmatrix} fe^{\pi}_{t+1,t|t} \\ fe^{un}_{t+1,t|t} \end{pmatrix} = \boldsymbol{\beta}_0 + \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} fe^{\pi}_{t,t-1|t-1} \\ fe^{un}_{t,t-1|t-1} \end{pmatrix} + \boldsymbol{\Theta} X_{t,t-1} + e_t \tag{15}$$

However with MSC we do not observe $fe^{x}_{t+1,t|t}$ directly, rather we have data on year-ahead forecast errors $fe^x_{t+4,t|t}$. We can then use the 4 period ahead version of equation (7):

$$FE_{t+4,t|t} = \hat{W}\hat{A}(I-KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (I-\hat{W}\hat{A}(I-KG)\hat{W}^{-1})\boldsymbol{L}_{t+3,t-1} \\ -(\hat{W}\hat{A}KG+I)\boldsymbol{L}_{t,t-1} + A\boldsymbol{L}_{t+3,t+2} + w_{t+4,t+3} - \hat{W}\hat{A}K\eta_t \tag{16}$$

where $\hat{W} = I+\hat{A}+\hat{A}^2+\hat{A}^3$, the fact that $\hat{A}$ is stationary guarantee that $\hat{W}$ is invertible. The derivation that extends (7) to (16) is in **Appendix C.2**. More importantly, the properties of β's derived in the last section hold true for the year-ahead specification as well. To illustrate the similar performance of the proposed quarter-ahead test (7) and year-ahead test (16), I perform the proposed tests with simulated data and include these results in **Appendix E**. We can then estimate:

$$\begin{pmatrix} fe^{\pi}_{t+4,t|t} \\ fe^{un}_{t+4,t|t} \end{pmatrix} = \boldsymbol{\beta}_0 + \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} fe^{\pi}_{t+3,t-1|t-1} \\ fe^{un}_{t+3,t-1|t-1} \end{pmatrix} + \boldsymbol{\Theta} X_{t+3,t-1} + e_t \tag{17}$$

The parameters of interest are $\beta_{11}, \beta_{12}, \beta_{21}$ and β_{22}. They can be estimated using OLS because in equation (16), the two components of the error term are uncorrelated with all the regressors. The $w_{t+4,t+3}$ is unpredictable error happening after $t+3$, thus uncorrelated with forecasting errors up to $t+3$ as well as any variable realized before $t+4$. The noise attached to public signal η_t is realized at time t thus not correlate with forecast error with the information set at time $t-1$, and here I have to assume there is not feedback effect of η_t on realized macroeconomic variables after time t through general equilibrium so that η_t is uncorrelated with any variable(except for expectational ones) realized beyond time t.[18]

In sections 3.1 and 3.2, I show that different learning structures imply different β's and correlations between expected unemployment and inflation. Table 3 summarizes these implications. One key take-away of this table is that when realized variables are uncorrelated, as documented in section 2, the fact that expectational variables are correlated can not be reconciled with either models under FIRE or standard noisy information models with independent learning. Joint learning models can create the positive correlation between expected inflation and unemployment, through either a subjective model $\hat{A}$ or mixed signal generated by G. Different patterns on β's will then help us to distinguish between these two forms of joint-learning. To illustrate how the proposed test scheme is able to distinguish between different models that generate expectational data, I include test results with simulated data in **Appendix E**.

[18] Notice v^i_t disappeared as we derive the consensus expectation, this is because the idiosyncratic noise has mean zero at each time point.

Table 3: **Summary of Models and Testable Implications**

Model:	Implied Estimate Results
FIRE	$\beta_{11} = \beta_{12} = \beta_{21} = \beta_{22} = 0$, $corr(E\pi, Edun)$ same as realized $corr(\pi, dun)$
Independent Learning: $m_1 = m_2 = 0$, G diagonal	$\beta_{12} = \beta_{21} = 0$, $\beta_{11}, \beta_{22} \neq 0$, $corr(E\pi, Edun) = 0$
Joint Learning: $m_i \lesseqgtr 0$, $m_j = 0$, G diagonal	$\beta_{ij} \lesseqgtr 0$, $\beta_{ji} = 0$, $corr(E\pi, Edun) \lesseqgtr 0$
Joint Learning: $m_1 = m_2 = 0$, $G = \begin{pmatrix} g_1 & g_2 \end{pmatrix}$, $g_1 g_2 \lesseqgtr 0$	$\beta_{12} \gtreqless 0$, $\beta_{21} \gtreqless 0$, $corr(E\pi, Edun) \lesseqgtr 0$

3.4 Test Results with Survey Data

I then perform the test for joint expectation formation as described above, using MSC and SPF. I focus on two variables to be forecasted: inflation and unemployment rate change. Four coefficients in (17) are estimated. Among these, β_{11} and β_{22} are the typical indicators for presence of information friction as in (Coibion and Gorodnichenko, 2012) and (Andrade and Le Bihan, 2013). I call them *the own-terms* of coefficients on forecast errors. A higher value of the own-terms imply a higher degree of information rigidity (noisier signals). The key coefficients related to joint learning are β_{12} and β_{21}. I call them *the cross-terms* of coefficients on forecast errors. Their property was summarized in Table 3. The goal of this section is to assess which model of expectation formation can be reconciled with the estimates of these four coefficients from survey data.

One complication to perform the test is that it requires unemployment rate change to be in comparable levels as the realized data in order to create forecast errors, whereas the data in MSC on unemployment expectation is categorical. I follow (Bhandari *et al.*, 2019) and (Mankiw *et al.*, 2004) to impute the expectational series. I confirmed that the same cross-correlation structure remains for imputed series. [19]

It is worth noting here the assumption essential to recover unemployment expectation is the predicted unemployment change follows a normal distribution with a constant variance across time. This assumption is particularly plausible in the framework of a noisy information model with stationary Kalman Filter, as the posterior distribution of forecasted variables

[19] The imputation approach is discussed in Appendix A.3. The cross-correlation using recovered expectational variables is in Appendix B.2.

are normally distributed and stationarity guarantees a time-invariant posterior variance.

I then estimate (17) with year-ahead forecast errors on expected inflation and expected unemployment rate change with OLS, controlling for corresponding realized variables according to (16).[20] The following Table summarize the results with MSC and SPF.

Table 4: Aggregate Test on Joint Learning, MSC v.s. SPF

	MSC		SPF	
	1981-2018	1990-2018	1981-2018	1990-2018
	(1)	(2)	(3)	(4)
β_{11}	0.61^{***}	0.65^{***}	0.63^{***}	0.61^{***}
	(0.066)	(0.085)	(0.056)	(0.086)
β_{12}	-0.15	-0.02	-0.17	0.00
	(0.094)	(0.102)	(0.181)	(0.221)
β_{21}	0.10^{***}	0.20^{***}	0.03	0.06
	(0.036)	(0.059)	(0.032)	(0.053)
β_{22}	0.59^{***}	0.50^{***}	0.41^{***}	0.40^{***}
	(0.080)	(0.092)	(0.101)	(0.143)
Observations	150	116	150	116

* ***,**,*: Significance at 1%,5% and 10% level. Estimation results for joint-learning test (17). The first and third columns are using full sample 1981-2018; the second and fourth columns are results for sub-sample 1990-2018. Newey-West standard errors are reported in brackets.

The first column of Table 4 contains estimation results using the full sample. The estimates on *own-terms* being significantly positive means that the consumers form expectations with limited information. More importantly, the significant estimates on *cross-term* β_{21} suggests the consumers form expectations on unemployment and inflation jointly rather than independently. The fact that β_{12} and β_{21} are having opposite signs suggest that such a joint learning friction is likely due to their subjective beliefs about structure relationship between inflation and unemployment, $\hat{A}$, rather than the signal generating process G. Furthermore, the estimation results are consistent with a positive correlation between expected inflation and unemployment change. According to Table 3, β_{21} significantly positive means that the agents believe past inflation will lead to unemployment rate increase. Such a belief structure

[20]The imputation method involves the use of SPF and uses the consensus expectation on unemployment status. Such an approach is not applicable to panel data. For this reason, in the baseline analysis for SPF and MSC I consider the aggregate version of joint-learning test (16). For SPF, I also include results with panel data as a robustness check because I do not need to impute the expectation variable on unemployment in SPF.

$\hat{A}$ induces positive correlation between the two expectations.

On the other hand, the results from column (3) shows that the professionals seem to have a different $\hat{A}$ from consumers. In particular, the small and insignificant β_{21} implies that they do not believe lagged inflation will raise future unemployment rate. From the discussions before, this may likely create the fact there is a positive correlation between unemployment and inflation in MSC whereas such a correlation doesn't appear in beliefs of SPF. The estimates on β_{11} and β_{22} are comparable to previous studies imposing independent learning. All in all the estimates from SPF suggest that professionals are closer to independent expectation formation or at least use a different structure $\hat{A}$ from consumers when forming expectations.

Recall in Figure 1 the correlations between realized inflation and unemployment fall below zero after 1990s.[21] Meanwhile, the correlation between expected variables in MSC stays positive. It is in this episode the two correlations have the most stark disconnection. I then include the results using a subsample 1990-2018 for both MSC and SPF. The results are qualitatively in line with those using the full sample. Moreover, the estimated β_{21} is twice as large, suggesting the consumers believe in a stronger response of future unemployment rate to current inflation.

Unlike MSC, the SPF is a panel dataset that contains unemployment expectations in units comparable to realized data. This allows me to perform the individual version of joint learning test (17). The reduced form regression becomes:

$$\begin{pmatrix} fe^{\pi}_{i,t+4,t|t} \\ fe^{un}_{i,t+4,t|t} \end{pmatrix} = \boldsymbol{\beta}_0 + \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} fe^{\pi}_{i,t+3,t-1|t-1} \\ fe^{un}_{i,t+3,t-1|t-1} \end{pmatrix} + \boldsymbol{\Theta} D_t + \mu_i + e_{i,t} \tag{18}$$

Where the $fe_{i,t+h,t|t}$ is the individual level forecasting error, D_t stands for time dummy and μ_i is the individual fixed effect. Integrating this expression will lead to the aggregate specification (17) I used before. The benefit of this approach is it improves the efficiency of the corresponding estimates as pointed out in (Ryngaert, 2017).[22] Furthermore, controlling for time dummy makes the estimation results robust to other possible aggregate confounders besides the realized macro variables in (17).

[21] In Figure 1 I used 10-year rolling window and plot the correlation against ending date of that window. The figure then suggests using realized data after 1990s inflation and unemployment become negatively correlated.

[22] In fact, under independent learning ($\beta_{12} = \beta_{21} = 0$), the one-period ahead expression of (18) collapse to the same form as in (Ryngaert, 2017).

Table 5: Panel Test on Joint Learning, SPF

	1981-2018		1990-2018	
	(1)	(2)	(3)	(4)
β_{11}	0.59^{***}	0.36^{***}	0.61^{***}	0.36^{***}
	(0.121)	(0.030)	(0.014)	(0.037)
β_{12}	-0.16^{***}	-0.13^{***}	-0.13^{***}	-0.11^{***}
	(0.026)	(0.035)	(0.027)	(0.037)
β_{21}	0.04^{***}	-0.004	0.04^{***}	-0.01
	(0.007)	(0.014)	(0.006)	(0.013)
β_{22}	0.81^{***}	0.48^{***}	0.83^{***}	0.50^{***}
	(0.014)	(0.028)	(0.016)	(0.031)
Observations	3388/3449	3388/3449	2976/3010	2976/3010
Individual Fixed Effect	Yes	Yes	Yes	Yes
Time Fixed effect	No	Yes	No	Yes

* ***,**,*: Significance at 1%,5% and 10% level. Estimation results for joint-learning test (18). The first and third columns are using full sample 1981-2018; the second and fourth columns are results for sub-sample 1990-2018. Newey-West standard errors are reported in brackets.

Table 5 shows the results of estimating (18) using panel data from SPF. In column (2) and (4) I control for both time and individual fixed effect, using full sample or sub-sample 1990-2018. The results are consistent with those from Table 4: the past forecast errors on inflation has no prediction power for current forecast error of unemployment, suggesting professionals do not believe inflation leads to unemployment rate increase. Meanwhile, interestingly professionals seem to believe unemployment leads to lower inflation. This is a typical "Phillips Curve" correlation. Such a result is then consistent with the fact that we see a negative correlation between individual expectations on unemployment and inflation for professionals as shown in Table 2.

I also include results without time fixed effect in columns (1) and (3) to illustrate the importance of aggregate confounders. Without controlling for time fixed effect the realized variables are omitted and will create bias on all the four regressors. Because according to the forecast error relationship (16), the omitted realized inflation and unemployment rate are correlated with lag forecast errors. Finally, the lower standard errors in the panel test results illustrate the efficiency gain of using panel data.

All in all the test results show strong evidence in support of joint expectation formation instead of independent learning for consumers. The straightforward implication of joint expectation formation is that consumers take into account the link between inflation and unemployment when they are learning to predict the future from signals. The estimates on forecast errors suggest consumers believe past inflation lead to future unemployment rate hikes, which can create the counterfactual positive correlation between expected inflation and unemployment observed in survey data. Meanwhile the results from SPF suggest the professionals have a different belief from the consumers, which is consistent with negative or no correlation between their expectations on inflation and unemployment rate change. Finally, the test results also suggest the correlation between expectational variables is not explainable solely by the mix of information in the observed signal, as proposed by (Kamdar, 2019). Because such an explanation implies that both estimates on *the cross-terms* will be negative, which is inconsistent with the test results using MSC. Whereas the explanation through subjective belief $\hat{A}$ is consistent with both the positive correlation and the estimated *cross-terms*.

4 Independent Evidence: Perceived News and Expectation

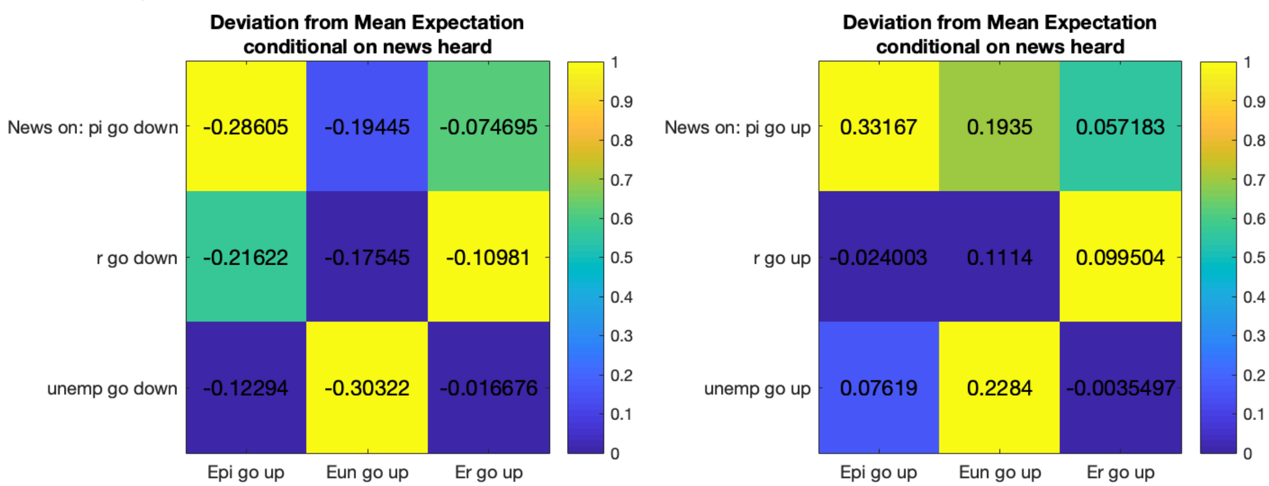
So far I have shown that the consumers form expectations on inflation and unemployment jointly rather than independently. They believe in a specific transition matrix $\hat{A}$ where past inflation will lead to higher unemployment rate. I then argue this is the major reason for them to make positive association between inflation and unemployment expectations. In particular, I distinguish the information friction in my explanation from the one through a mixed signal. However, the joint learning test results are not sufficient to make such a conclusion. For example, it is possible that both frictions are at play to create the positive association. In this section I aim to provide some independent evidence using additional information from MSC to make the connection between the positive correlation and the subjective model $\hat{A}$.

One key distinction between the two frictions is the response of the expectational variable to news. With the mixed-signal friction agents typically can't distinguish between news about inflation or unemployment and unlabelled bad news will affect both inflation and unemployment expectation positively. On the other hand, if there are signals specific to one subject, it will only affect the expectation on this subject. Whereas friction on subjective model $\hat{A}$ suggests agents can distinguish between different signals, and according to the estimates in Table 4, those signals on inflation will move both inflation and unemployment forecasts up whereas news about unemployment will only increase unemployment forecast, with negative or no impact on inflation forecast.

To examine these implications I use the perceived news measures from MSC as in (Doms and Morin, 2004), (Pfajfar and Santoro, 2013) and (Lamla and Maag, 2012). This variable includes a label on what kind of news that the agent has heard of in the recent 3 months. The description of these variables is included in Appendix F.1.

In presence of only mixed-signal friction, suppose these labels on news heard truthfully reflect agents' understanding of the content, we may expect different news have impacts only on expectational variable of the same subject. If we believe that agents still cannot distinguish the content of this news and they randomly pick a label in reporting, we should expect both expectations to adjust in the same direction in response to receiving such news, as long as it's unfavorable. Both these are different from the implication of subjective model friction: under this friction, we should observe news on inflation has a positive impact on both unemployment expectation as well as inflation expectation itself, whereas unemployment news will mainly affect unemployment forecasts positively. The response of inflation expectation should either be negative or close to zero. We can test these implications using micro-level data from MSC.

Figure 5: Heatmap for Expectation Responses to News: Cross-sectional



On y-axis is the news heard for each subgroup, on x-axis is expectation under examination. The number reported in each box is the percentage deviation of expectations reported by the agents who received corresponding news, from the mean expectations of all the survey participants at each point of time. The left panel are results upon receiving good/favorable news, the right are those of bad/unfavorable news.

I first split the samples into subgroups conditional on news the agents heard of. I focus for now on only news about inflation, employment, and interest rate, favorable or unfavorable. For every group, I compute the percentage deviation of expectations on inflation, unemployment change and interest rate change[23] from their means of all the survey participants at

[23] In Appendix F.2 the same experiment with more expectational variables are available, here for ease to read I only report the three key expectations.

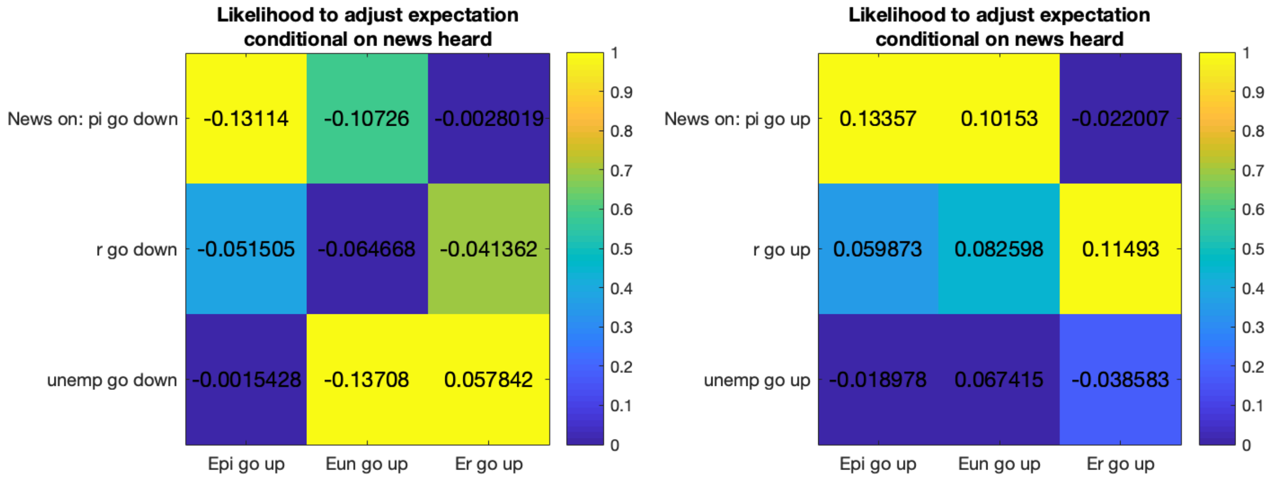
each time point, to eliminate the time-specific effect in each expectational variable. I then take the average of this deviation across time, conditional on the news they have heard. Figure 5 shows two matrices for the deviations of these conditional expectations.

To interpret Figure 5, consider the top left corner in the left panel(matrix), -0.286 means for a person who has heard of inflation being lower, he/she also reports expected inflation 28.6% lower than the average at the time(the unconditional time mean in that cross-section). The color of boxes inside each panel is normalized vertically: the most yellow box means agents with that type of news have the highest deviation in absolute value, whereas the darkest blue one has the lowest. For example, the first column in the left panel means agents heard of inflation being low have inflation expectation further lower than those with interest rate and employment news.

Figure 5 shows that news has the biggest impact on the variable it is labeled with. Furthermore, inflation news has big impacts on all three expectational variables when comparing to other news, especially when it is news on high inflation (in the right panel). For agents with news on high inflation, they report 33% higher in expected inflation, 19% higher in unemployment change, and 5.7% higher in interest rate expectation. However, we also see a similar response to unfavorable employment news, though with a smaller impact. This is due to the fact I haven't controlled for individual fixed effects. As news is self-reported, it is possible pessimistic agents pay attention to all kinds of bad news and also more likely to form worse expectations than average. Then when I condition on agents with bad employment news, they have higher inflation expectations, not because of the news, but the fact they almost always expect higher inflation than average.

To control for this fixed effect, I consider the likelihood each agent increases his/her expectation upon receiving different news, similar to (Pfajfar and Santoro, 2013). I use the two-wave panel available for MSC and compute the fraction of agents who adjust their expectations upwards or downwards, conditional on receiving news in the second period. The likelihood of adjusting expectation is reflected in two ways: (1) agents with specific news are more likely to adjust expectation upwards comparing to others; (2) agents with specific news are less likely to adjust expectation downwards. To capture both these two ways I sum up these two types of likelihood difference between agents with specific news and others. Figure 6 shows the results.

Figure 6: Heatmap for Expectation Responses to News: Panel



On y-axis is the news heard for each subgroup, on x-axis is expectation under examination. The number reported in each box is the likelihood each agent increases her expectation upon receiving different news. The left panel are results upon receiving good/favorable news, the right are those of bad/unfavorable news.

The two panels are organized in the same way as Figure 5, except the the interpretations of values inside boxes differ. Now it stands for the difference of likelihood in adjusting expectations between agents with specific news and those who don't hear of such news. For example in the first row of the right panel, for an agent that has heard of news about high inflation, he/she has 13% higher chance to adjust his/her inflation expectation upwards and 10% higher chance to believe in a higher unemployment rate in the future, the opposite is true for those who have hear of news on low inflation (the first row of left panel). However employment news barely has any impact on inflation expectation now and in presence of unfavorable employment news, agents are more likely to adjust inflation forecasts downwards.

Finally, I perform a panel regression controlling for the individual as well as time fixed effect, the parameters of interest are dummy variables on what kind of news the agent receives. This can be seen as a compliment result for the previous ones. The contents of the self-report news are grouped by the topic of the news as well as the tone of the news as indicated by the survey respondent. Table 6 suggests hearing news on high (low) inflation increase reported expected inflation by about 0.5% (0.31%) and increase the probability to believe unemployment rate will rise (fall) by 6.5%. However, news about employment only has a significant impact on unemployment expectation but not on inflation expectation. Moreover, such a pattern exists not only for news about employment itself. In general, information about other economic topics such as news on specific industries or sectors in the economy and news on the financial market also may affect expectations. In Table 6 I also examine how expectations change conditional on receiving this news. The results on industry and financial market related news are qualitatively similar to those with news on employment status: none of them have a significant impact on inflation expectation and the point estimate on favorable news are usually negative, which is in line with findings from (Candia *et al.*, 2020) and (Andre *et al.*, 2019).

Table 6: FE Panel Regression with Self-reported News

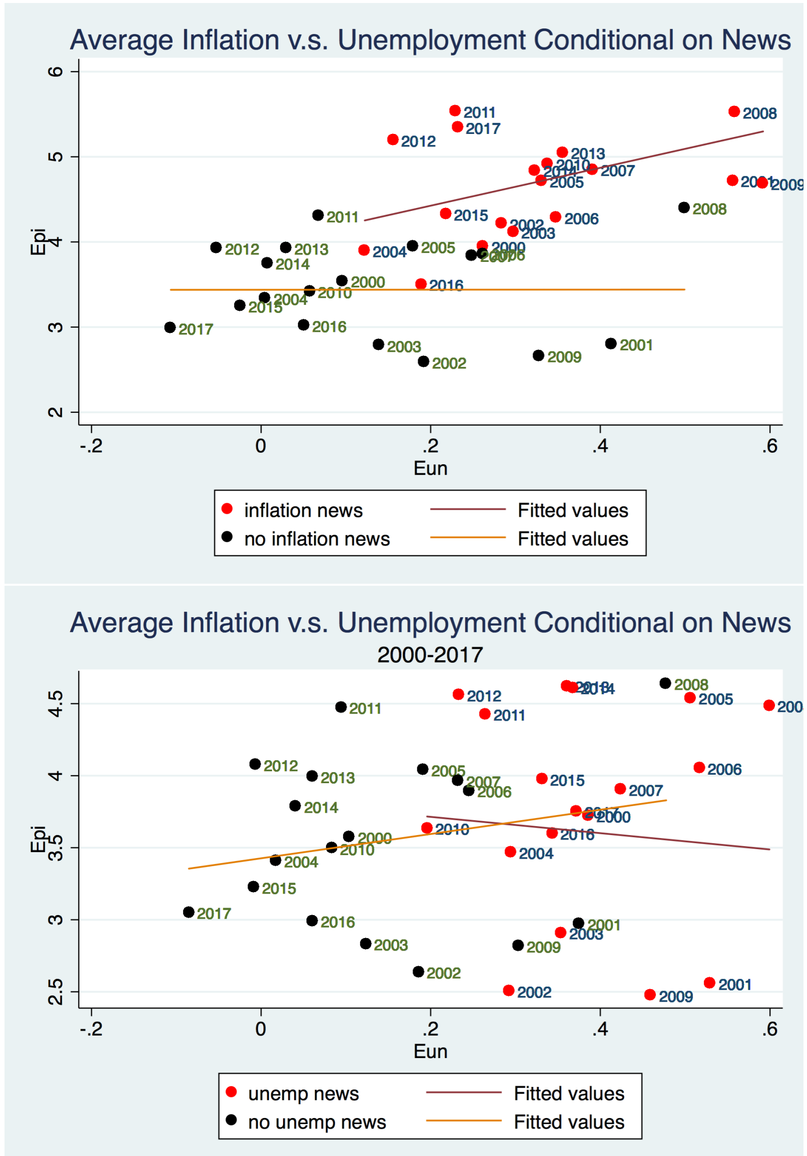
Expectation on:	Inflation	Likelihood Unemployment Increase
news on:	(1)	(2)
high inflation	0.50***	0.060***
	(0.09)	(0.011)
low inflation	−0.31***	−0.059***
	(0.10)	(0.016)
employment unfavourable	−0.001	0.10***
	(0.052)	(0.007)
employment favourable	−0.08	−0.14***
	(0.057)	(0.009)
industry unfavourable	0.08	0.08***
	(0.050)	(0.006)
industry favourable	−0.08	−0.10***
	(0.053)	(0.008)
high interest rate	0.18**	0.09***
	(0.085)	(0.012)
low interest rate	−0.17**	−0.05***
	(0.071)	(0.011)
financial market unfavourable	0.03	0.07***
	(0.074)	(0.011)
financial market favourable	−0.08	−0.08***
	(0.061)	(0.012)
Observations	163233	162369
R^2	0.68	0.69

[*] ***,**,*: Significance at 1%,5% and 10% level. Results come from fixed-effect panel regressions of different dummies of self-reported news on expectations. The first column is result using expected inflation as dependent variable; the second column is result using expected probability of unemployment rate increase as dependent variable. The results controlled for individual and time varying characteristics, individual fixed effect and time fixed effect. Standard errors are adjusted for heteroskesticity and autocorrelation.

Furthermore, the individual-level impact of inflation and employment news seem to transmit into consensus expectation perfectly through aggregation. In Figure 7, I plot the mean of each year for consensus expectations on inflation and unemployment, conditional on hearing inflation news or unemployment news. In Figure 7, the red dots are consensus expectation in each year, conditional on hearing inflation (top panel) or unemployment (bottom panel) news and black dots are that of people without that news. It is clear that agents with inflation news have both higher inflation and higher unemployment expectation comparing to those who didn't hear such news. Whereas unemployment news only shifts unemployment expectation to the right.[24] Moreover, in the top panel we clearly see the positive correlation between the two expectations across time for those agents with inflation news. Once we take these people out, the black dots present no correlation at all. On the other hand, such a correlation doesn't exist among the consumers who heard about unemployment news. In the bottom panel, there is a positive correlation for consumers without unemployment news. This is because these consumers may have received news about inflation.

[24]For data from 1984-1999 the same pattern persists, I'll include them in Appendix F.2.

Figure 7: Consensus Expectation on Inflation and Unemployment, Conditional on News



Scatter plot for consensus expected inflation and unemployment each year from 2000-2017. Top panel: conditional on having heard inflation news or not, red dots are expectations conditional on hearing inflation news, black dots are those without inflation news. Bottom panel: conditional on having heard unfavourable unemployment news.

These findings together strongly support the subjective model friction. With the mixed-signal friction alone we cannot observe the pessimistic correlation between unemployment and inflation expectation as documented in Section 2 together with the test results in Section 3.4 and responses of expectations to perceived news measures presented in this section.

5 Conclusion

In this paper I document that the expectational variables from US household surveys correlate with each other. In particular, the US consumers predict high inflation together with worse economic performances including higher unemployment rate and weaker growth. This correlation is different from what is observed in realized macroeconomic variables and the professional forecasts. It is also inconsistent with the predictions from the standard New Keynesian Model.

These patterns are hard to be explained by standard single-variable noisy information model. I propose a joint expectation formation model and a simple test to distinguish it from standard single variable models. I then show survey data strongly support the idea that consumers form expectations on various subjects jointly rather than independently. The joint learning model then can help to understand the cross-correlation I documented in household survey expectations. The cross-correlation can arise from either agents holding subjective beliefs in the structure of the economy that are different from realized data or economic theories, or the agents observing mixed signals generated by multiple state variables. I then examine the testable implications from survey data to show that the cross-correlation is majorly driven by agents' subjective belief in the structure of the economy. The test results suggest U.S. consumers hold beliefs that past inflation will lead to deterioration of future real economic conditions. Meanwhile, the professionals do not hold such beliefs. This explains why I did not find the same positive correlation in SPF.

To further support this argument, I supplement the above results with evidence from self-reported news measures in MSC. I show that information related to inflation moves expectations on unemployment and inflation in the same direction, whereas information about real economic variables typically fails to create the co-movement of these expectational variables. These results are consistent with the notion that agents' subjective beliefs about the economic model are the main reasons for the cross-correlation documented before.

These findings have important implications on households' behaviors in response to their expectations and Central Bank Communication. Multiple researchers have found negative responses of household's consumption attitudes to their inflation expectations. This paper shows inflation-specific news makes agents believe economic conditions in general will be worse. The precautionary motive and anticipated income decrease can generate a negative response of consumption. For Central Bank Communication, signals on current or future inflation are likely to create pessimistic beliefs on economic performance among households. The findings suggest Central Bank should use inflation-related expectation management policy with cautious, and clear messages that distinguish inflation from real economic conditions will be beneficial.

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A Data Appendix

A.1 Data Description

SCE: SCE run by New York Fed started in June 2013. It is a nationally representative, internet-based rotating panel of about 1300 household heads, each stay in the panel for 12 months. The survey is month by month and in each month new respondents are drawn to match various demographic targets[25]. The survey contains a richer set of questions comparing to existing surveys about consumer expectations, including individual's employment status and different characteristics of the household. The panel feature of SCE allows me to control for individual fixed effect that could induce spurious correlation between different perceptions and to follow individual along time which is important to capture learning behaviour.

MSC: The monthly component Michigan Survey of Consumers started from 1978[26]. I will use the aggregate component of MSC as well as the cross-sectional archive as a complement part to the SCE. So far most of the literature using aggregate or micro-level data are utilizing this dataset.

A.2 Aggregate Survey Forecast and Real-time Data

To first illustrate the difference between the survey expectation and realized data, Figure 8 plots raw data on average expectation from MSC with realized data for inflation, unemployment rate change and real GDP growth. All real time series are change from a year ago, as the corresponding expectation series are one-year-forward forecasts.

A.3 Recover Survey Mean from Categorical Data

From the cross-sectional dataset of MSC, I can acquire information on the fraction of respondents with different answers. Denote f_t^u as fraction of responses that are "increase" and f_t^d as "decrease". Assume for each period of t, there is a cross-section of answers formed by individuals about the change of the asked subject (unemployment rate or business condition and price). And assume this measure follows a normal distribution with mean μ_t and variance σ_t^2.

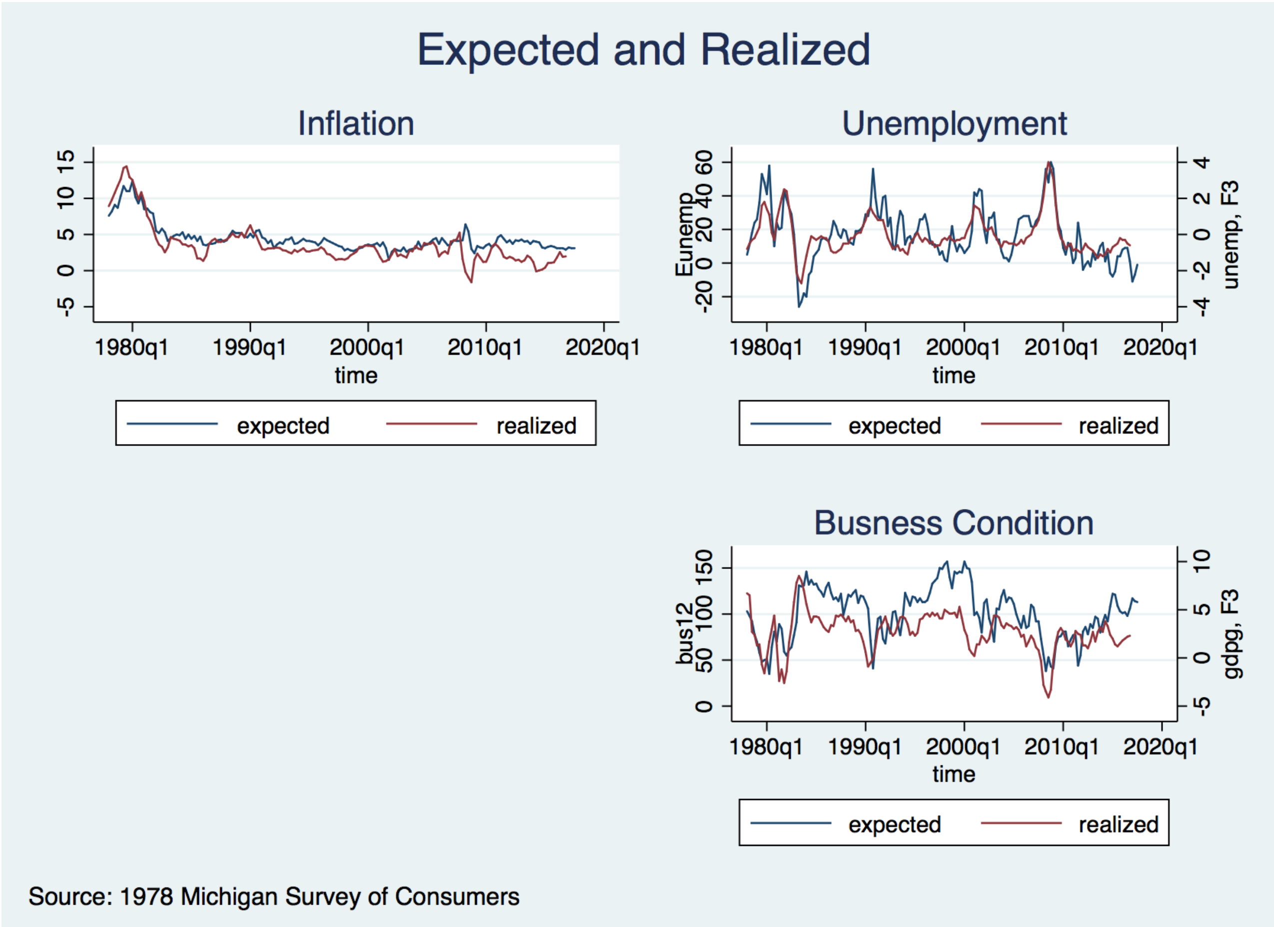
Assumption 3. *At each period t, survey respondent i forms a belief $x_{i,t}$ that indicates the change of asked variable x, this belief follows a normal distribution:*

$$x_{i,t} \sim N(\mu_t, \sigma_t^2)$$

[25]For details of SCE see (Armantier *et al.*, 2016)

[26]Quarterly data starts earlier from 1960 but with a lot of dimensions missing.

Figure 8: Data on Consensus Expectations from MSC



Survey expectation from MSC against realized data. All macro data are changes from a year ago, survey expectations are one-year-forward forecasts. Unemployment and business condition expectations are aggregated from categorical data. Positive number (over a hundred) means more people believes unemployment (business condition) will increase (be better) in the future.

Then suppose the agents have a common scale in answering the categorical question: If $x_{i,t}$ is close to some level b, then he will consider the subject will barely change; if $x_{i,t}$ is much bigger than b, he will answer increase, otherwise answer decrease.

$$category_{i,t} = \begin{cases} increase & x_{it} > b + a \\ decrease & x_{it} < b - a \\ same & x_{it} \in [-a + b, b + a] \end{cases}$$

Then the fraction of answer "increase", denoted as f_t^u, and "decrease" , denoted f_t^d, will directly follow from normality:

$$f_t^d = \Phi\left(\frac{b-a-\mu_t}{\sigma_t}\right) \tag{19}$$

$$f_t^u = 1-\Phi\left(\frac{a+b-\mu_t}{\sigma_t}\right) \tag{20}$$

The items I want to recover is μ_t, which is the corresponding average change of the asked subject a year from now. This can be computed using:

$$\sigma_t = \frac{2a}{\Phi^{-1}(1-f_t^u)-\Phi^{-1}(f_t^d)} \tag{21}$$

$$\mu_t = a+b-\sigma_t\Phi^{-1}(1-f_t^u) \tag{22}$$

From (21) and (22), compute the average across time we have:

$$\hat{\sigma} = 1/T\sum_t^T \sigma_t = 1/T\sum_t^T \frac{2a}{\Phi^{-1}(1-f_t^u)-\Phi^{-1}(f_t^d)} \tag{23}$$

$$\hat{\mu} = 1/T\sum_t^T \mu_t = 1/T(a+b-\sigma_t\Phi^{-1}(1-f_t^u)) \tag{24}$$

As in MSC there is no information on $\hat{\sigma}$ and $\hat{\mu}$, I use the time-series mean of the data from Survey of Professional Forecast (SPF) on comparable questions to approximate those from MSC[27]. Following (Bhandari *et al.*, 2019) I assume the ratio of time-series average between inflation expectation and other expectation in MSC equals to its counterpart in SPF:

Assumption 4. *For the variable x asked in the survey:*

$$\hat{\sigma}_x^{MCS} = \frac{1/T\sum_t^T \sigma_{E\pi,t}^{MCS}}{1/T\sum_t^T \sigma_{E\pi,t}^{SPF}} \times 1/T\sum_t^T \sigma_{x,t}^{MCS}$$

And

$$\hat{\mu}_x^{MCS} = \frac{1/T\sum_t^T \mu_{E\pi,t}^{MCS}}{1/T\sum_t^T \mu_{E\pi,t}^{SPF}} \times 1/T\sum_t^T \mu_{x,t}^{MCS}$$

Then from (23) and (24) and Assumption 4 I can back out a and b, and with (22) I can recover $\mu_{x,t}$ for the expectational variable x.

[27]For unemployment rate change, I use the average difference between projected unemployment rate at $t+3$ and the historical data at $t-1$ which is the last information available to the economist. For real GDP growth I use the real GDP growth projection for the next four quarters after $t-1$.

Recovered series: To test whether the above method is plausible, I use the cross-sectional data of MSC for inflation expectation to construct categorical variable using different ranges $1\% - 2\%$, $3\% - 4\%$ and $4\% - 5\%$ for answers to be "stay the same". Then I use the proposed method to recover the $\mu_{\pi,t}$ and compare it with the actual average of expected inflation. Figure 9 plots the recovered mean and the actual mean.

Figure 9: Recovered Expected Inflation v.s. Actual

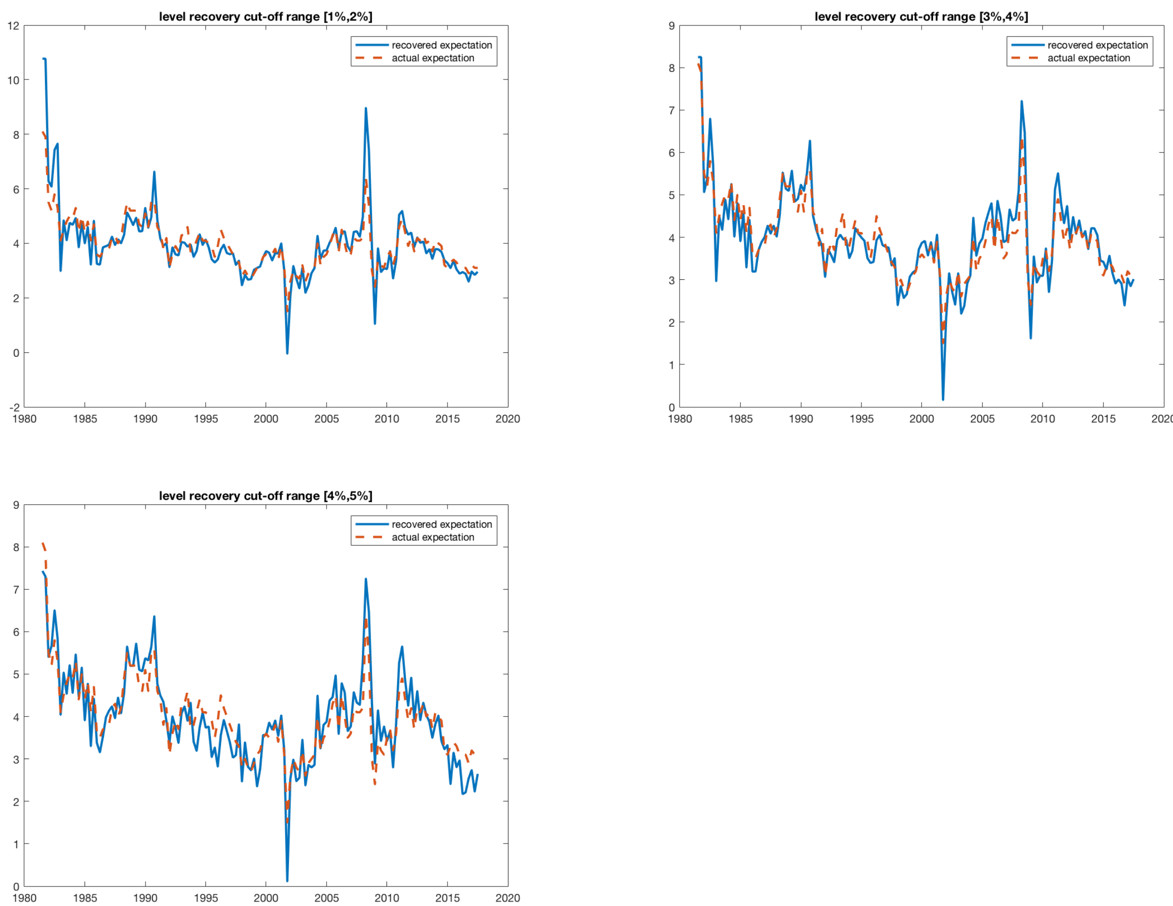


Figure 9 shows that the recovered data is actually quite close to the actual mean expectation, with correlation of 0.93, 0.95 and 0.91 respectively. Figure 10 shows the recovered data on unemployment change and real GDP growth (economy condition change) comparing to actual data.

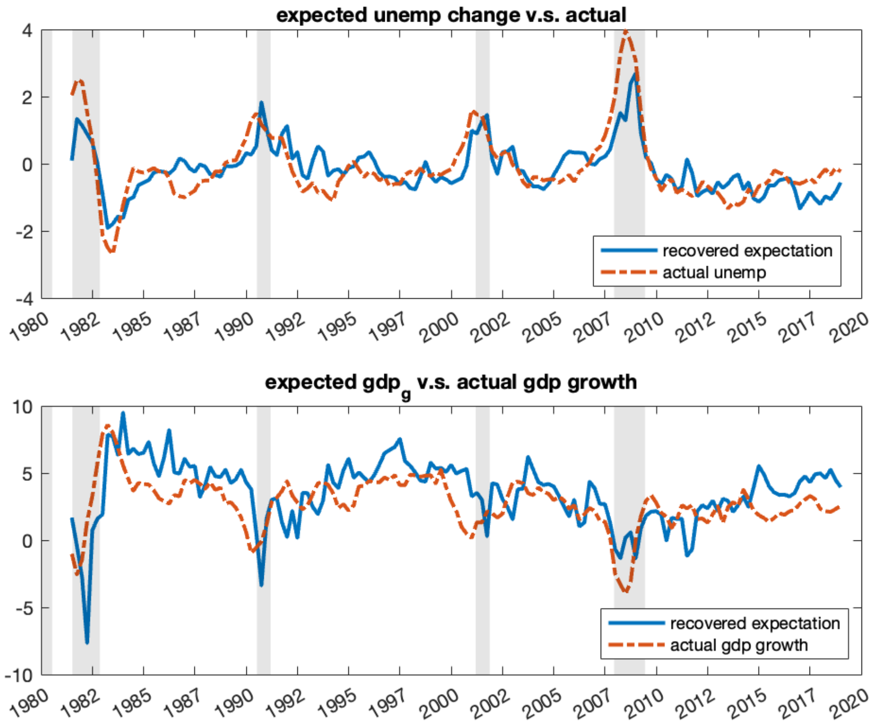


Figure 10: Recovered Expected Series v.s. Realized Data.

Data from 1981q3 to 2018q4 due to availability of quarterly SPF on CPI inflation.

B Cross-correlations

B.1 More variables than inflation and unemployment

I offer cross-correlations on a larger set of variables beyond inflation and unemployment rate change. I include expectations on interest rate change, business condition, nominal income change and real income change from MSC. For the counter-parts of these expectational variables in real data, I use change of federal funds rate, real GDP growth, change of wage and salary disbursements as well as wage net of CPI inflation. The earliest data available for MSC and FRED is 1978 quarter 1, so I use sample from 1978q1 to 2018q4. The cross-correlations from MSC and FRED are reported in Table 7 and Table 8 respectively.

Table 7: Correlation MCS: more variables

	(1)	(2)	(3)	(4)	(5)	(6)
(1) inflation ($E\pi_{t+4,t}$)	1.00	0.31^{***}	-0.13	-0.43^{***}	0.20^{***}	-0.51^{***}
(2) unemp change ($E\Delta un_{t+4,t}$)		1.00	-0.41^{***}	-0.64^{***}	0.02	-0.28^{***}
(3) interest rate change ($E\Delta i_{t+4,t}$)			1.00	0.40^{***}	0.18^{**}	0.07
(4) Busi Condition change ($E\Delta y_{t+4,t}$)				1.00	0.5^{***}	0.77^{***}
(5) nominal income change ($E\Delta W_{t+4,t}$)					1.00	0.62^{***}
(6) real income change ($E\Delta w_{t+4,t}$)						1.00

* **** means significant at 1%,** means 5 % and * means 10%, data in use are quarterly 1978q1-2018q4 from MSC.*

Table 8: Correlation FRED: more variables

	(1)	(2)	(3)	(4)	(5)	(6)
(1) CPI	1.00	0.11	0.38^{***}	-0.03	0.63^{***}	-0.32^{***}
(2) Δun		1.00	-0.52^{***}	-0.79^{***}	-0.53^{***}	-0.77^{***}
(3) ΔFFR			1.00	0.43^{***}	0.56^{**}	0.26^{***}
(4) $\Delta RGDP$				1.00	0.61^{***}	0.79^{***}
(5) ΔW					1.00	0.53^{**}
(6) Δw						1.00

* **** means significant at 1%,** means 5 % and * means 10%, data in use are quarterly 1978q1-2018q4 from FRED.*

As one can clearly see from the above tables. The major difference between expectational variables and realized data lies among the correlations between inflation and unemployment change. Such a difference also shows up in the correlation between inflation and real GDP growth. Meanwhile, the consumers understand unemployment and GDP growth are negatively correlated. They also understand inflation is positively related with nominal income but negatively related with real income.

B.2 Cross-Correlation with recovered data and SPF

In Table 9, I report the same cross-correlation exercise using the imputed data as mentioned in Appendix A.3. I also include the cross correlation structure for the same set of expectational variable from SPF and FRED for comparison. To illustrate that the consumers understand the negative relationship between economic condition and unemployment change, I also include the correlations using recovered RGDP growth expectation. Finally in Panel B of Table 9 I include the results from monthly data when SCE is available and compare it to the correlations using monthly MSC.

Table 9: Correlation: Recovered MSC, SPF, Realized Data and SCE

	Panel A: quarterly 1981q3-2018q4			Panel B: monthly 2013m6-2018m12	
Correlation of:	MSC	SPF	Real time	MSC	SCE
$E\pi, E\Delta un$	0.16^{*}	0.03	0.00	0.36^{***}	0.32^{***}
$E\pi, E\Delta y$	-0.25^{***}	-0.01	0.08	-	-
$E\Delta un, E\Delta y$	-0.64^{***}	-0.79^{***}	-0.78^{***}	-	-

*** means significant at 1%,** means 5 % and * means 10%, data in use are quarterly from MSC.

Panel A of Table 9 shows using recovered data from MSC starting from 1981, we still see the stark positive association between expected inflation and worse economic performance(both unemployment increase and business condition worsen). Whereas in SPF we cannot find such a correlation. The cross correlation structure of SPF is very similar to that of the realized data, suggesting the correlation between inflation expectation and the projection of future economic condition is not an artifact of expectation formation in general, but rather a unique feature of household expectation. Panel B illustrates the cross correlation structure of household is robust to use of monthly data, a more recent time period, and other data source(SCE).

C Derivation of Noisy Information Model

C.1 Basic stationary Kalman Filter

Consider the ALM and observational equation as in (2) and (3), where $w_{t+1,t}$, v_t^i and η_t are independent normally distributed:

$$w_{t+1,t} \sim N(\mathbf{0}, Q) \quad v_t^i \sim N(\mathbf{0}, R_1) \quad \eta_t \sim N(\mathbf{0}, R_1)$$

Consistent with the main-text, I denote $R = R_1 + R_2$, and the perceived value of $\boldsymbol{L}_{t,t-1}$ for individual i at time t as $\boldsymbol{L}^i_{t,t-1|t}$. The Filtering process is:

$$\boldsymbol{L}^i_{t,t-1|t} = \hat{A}\boldsymbol{L}^i_{t,t-1|t} = \boldsymbol{L}^i_{t,t-1|t-1} + K(\boldsymbol{s}^i_t - G\boldsymbol{L}^i_{t,t-1|t-1}) \tag{25}$$

The Kalman Filter is given by:

$$K = \Sigma G'(G\Sigma G' + R)^{-1}$$

$$\Sigma_p = \hat{A}\Sigma\hat{A}' - \hat{A}K_t G\Sigma\hat{A}' + Q$$

Where Σ is the covariance matrix of priors as defined in assumption 2, Σ_p is the covariance matrix of posteriors.[28] Then the expectation is given by:

$$\boldsymbol{L}^i_{t+1,t|t} = \hat{A}\Big(\boldsymbol{L}^i_{t,t-1|t-1} + K(\boldsymbol{s}^i_t - G\boldsymbol{L}^i_{t,t-1|t-1})\Big)$$

C.2 Derivation of Year-ahead Forecasting Error Rule

Consider the year-ahead consensus forecast $\boldsymbol{L}^c_{t+4,t|t}$ and year-ahead realization $\boldsymbol{L}_{t+4,t}$, using ALM (2) we have:

$$\boldsymbol{L}_{t+4,t} \equiv \sum_{j=1}^{4} \boldsymbol{L}_{t+j,t+j-1} = A\boldsymbol{L}_{t+3,t-1} + \sum_{j=1}^{4} w_{t+j,t+j-1} \tag{26}$$

Similar to equation (5), the year-ahead consensus expectation is:

$$\boldsymbol{L}^c_{t+4,t|t} = (\hat{A}^3 + \hat{A}^2 + \hat{A} + I)[\hat{A}(I - KG)\boldsymbol{L}^c_{t,t-1|t-1} + \hat{A}KG\boldsymbol{L}_{t,t-1} + \hat{A}K\eta_t] \tag{27}$$

[28]Given common beliefs on $\hat{A}$ and G, it can be shown prior and posterior covariance matrices converge.

Meanwhile from (25) and ALM we know:

$$\boldsymbol{L}^c_{t+3,t-1|t-1} = \sum_{j=0}^{3} \boldsymbol{L}^c_{t+j,t+j-1|t-1} = (\hat{A}^3 + \hat{A}^2 + \hat{A} + I)\boldsymbol{L}^c_{t,t-1|t-1}$$

Denote $\hat{W} = (\hat{A}^3 + \hat{A}^2 + \hat{A} + I)$ and stationarity of $\hat{A}$ guarantees $\hat{W}$ is invertible. Plug above equation into (27) we have:

$$\boldsymbol{L}^c_{t+4,t|t} = \hat{W}[\hat{A}(I - KG)\hat{W}^{-1}\boldsymbol{L}^c_{t+3,t-1|t-1} + \hat{A}KG\boldsymbol{L}_{t,t-1} + \hat{A}K\eta_t]$$

Now write the forecasting error $FE_{t+4,t|t}$ as defined:

$$\begin{aligned}
FE_{t+4,t|t} &\equiv \boldsymbol{L}_{t+4,t} - \boldsymbol{L}^c_{t+4,t|t} = A\boldsymbol{L}_{t+3,t-1} + \sum_{j=1}^{4} w_{t+j,t+j-1} - \boldsymbol{L}^c_{t+4,t|t} \\
&= \hat{W}\hat{A}(I - KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (A - \hat{W}\hat{A}(I - KG)\hat{W}^{-1})\boldsymbol{L}_{t+3,t-1} \\
&- \hat{W}\hat{A}KG\boldsymbol{L}_{t,t-1} - \hat{W}\hat{A}K\eta_t + \sum_{j=1}^{4} w_{t+j,t+j-1} \\
&= \hat{W}\hat{A}(I - KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (A - \hat{W}\hat{A}(I - KG)\hat{W}^{-1})\boldsymbol{L}_{t+3,t-1} \\
&- \hat{W}\hat{A}KG\boldsymbol{L}_{t,t-1} + \boldsymbol{L}_{t+3,t} - A\boldsymbol{L}_{t+2,t-1} - \hat{W}\hat{A}K\eta_t + w_{t+4,t+3} \\
&= \hat{W}\hat{A}(I - KG)\hat{W}^{-1}FE_{t+3,t-1|t-1} + (I - \hat{W}\hat{A}(I - KG)\hat{W}^{-1})\boldsymbol{L}_{t+3,t-1} \\
&- (I + \hat{W}\hat{A}KG)\boldsymbol{L}_{t,t-1} + A\boldsymbol{L}_{t+3,t+2} - \hat{W}\hat{A}K\eta_t + w_{t+4,t+3}
\end{aligned} \tag{28}$$

The last equation follows from the fact:

$$\boldsymbol{L}_{t+3,t-1} = \boldsymbol{L}_{t+3,t+2} + \boldsymbol{L}_{t+2,t+1} + \boldsymbol{L}_{t+1,t} + \boldsymbol{L}_{t,t-1} = \boldsymbol{L}_{t+2,t-1} + \boldsymbol{L}_{t+3,t+2}$$

D Extended Proposition

Here I extend Proposition 4 to the case where $G = \begin{pmatrix} g_1 & g_2 \\ g_3 & g_4 \end{pmatrix}$.

Proposition 5. *If $G = \begin{pmatrix} g_1 & g_2 \\ g_3 & g_4 \end{pmatrix}$ and $\hat{A} = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}$, the off-diagonal elements of $\hat{A}(I - KG)$ are non-zero and of the same signs.*

Proof.

$$\hat{A}(I - KG) = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \begin{pmatrix} m_1 & m_0 \\ m_0 & m_2 \end{pmatrix} = \begin{pmatrix} \rho_1 m_1 & \rho_1 m_0 \\ \rho_2 m_0 & \rho_2 m_2 \end{pmatrix}$$

Given that $0 < \rho_1, \rho_2 < 1$, the off-diagonal elements have the same sign.

To link the signs of these off-diagonal elements to elements in G, I can derive them analytically:

$$\begin{aligned} K = \Sigma G'(G\Sigma G' + R)^{-1} &= \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \begin{pmatrix} g_1 & g_3 \\ g_2 & g_4 \end{pmatrix} \times \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} \\ &= \begin{pmatrix} g_1\sigma_1^2 & g_3\sigma_1^2 \\ g_2\sigma_2^2 & g_4\sigma_2^2 \end{pmatrix} \times \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \end{aligned}$$

Where

$$\begin{cases} a = g_1^2\sigma_1^2 + g_2^2\sigma_2^2 + \sigma_{1,s}^2 \\ b = g_1 g_3 \sigma_1^2 + g_2 g_4 \sigma_2^2 \\ c = g_1 g_3 \sigma_1^2 + g_2 g_4 \sigma_2^2 \\ d = g_3^2\sigma_1^2 + g_4^2\sigma_2^2 + \sigma_{2,s}^2 \end{cases}$$

Denote the matrix $KG := \frac{1}{ad-bc}\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}$. Then the coefficient matrix is given by:

$$\hat{A}(I - KG) = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \begin{pmatrix} 1 - \frac{x_1}{ad-bc} & -\frac{x_2}{ad-bc} \\ -\frac{x_3}{ad-bc} & 1 - \frac{x_4}{ad-bc} \end{pmatrix}$$

Given $0 < \rho_1, \rho_2 < 1$, the off-diagonal elements have the same sign if and only if x_2 and x_3 have the same sign. With some algebra it is easy to show:

$$\begin{cases} x_2 = \sigma_1^2(g_1g_2d - g_2g_3c - g_1g_4b + g_3g_4a) = \sigma_1^2(g_1g_2\sigma_{2,s}^2 + g_3g_4\sigma_{1,s}^2) \\ x_3 = \sigma_2^2(g_1g_2d - g_1g_4c - g_3g_2b + g_3g_4a) = \sigma_2^2(g_1g_2\sigma_{2,s}^2 + g_3g_4\sigma_{1,s}^2) \end{cases}$$

The off-diagonal elements have the same signs. They will be zeros if $g_2 = g_3 = 0$, which is the case for separate signals. □

The above proposition conveys the same message as Proposition 4: if the correlation is created by mixed signals the off-diagonal elements for the coefficient matrix will have the same sign. Furthermore, the sign is related to how the information on inflation and unemployment mixed. For example, if $g_3 = 0$ and $g_2 \neq 0$, the only mixed signal is the first one and the signs for off-diagonal elements in $\hat{A}(I - KG)$ will be negative (positive) iff $g_1g_2 > 0$ ($g_1g_2 < 0$). The intuition here is the same as the case in Proposition 4. Meanwhile recall that $g_1g_2 > 0$ will lead to positive correlation between expected variables. This suggests that if mixed signal is the friction that generate such correlation, we should expected $\beta_{12}, \beta_{21} < 0$

when we run the test (17). These also suggest that considering the case where G is a vector is without loss of generality.

E Monte Carlo Simulation

I consider the different learning structures discussed in Table 3 and simulate expectation data according to the noisy information model with sample sizes similar to the survey data being used in Section 3.4. To assess the performance of the tests, I consider several different empirical specifications discussed throughout the paper: (1) both (7) and (16); (2) with time-series of consensus expectation and panel of individual expectation. I also provide the correlations of the simulated data and their analytical values. To fix the idea, consider the following parametrization:

Table 10: Parameters for simulation

Fixed Parameters		
Variable	Value	Description
$Q := \begin{pmatrix} \sigma^2_{1,t} & 0 \\ 0 & \sigma^2_{2,t} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Cov matrix of shocks
$R := \begin{pmatrix} \sigma^2_{1,s} & 0 \\ 0 & \sigma^2_{2,s} \end{pmatrix}$	$\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$	Cov matrix of noises
$\Sigma_{t\|t-1} := \begin{pmatrix} \sigma^2_1 & 0 \\ 0 & \sigma^2_2 \end{pmatrix}$	$\begin{pmatrix} 2.65 & 0 \\ 0 & 2.26 \end{pmatrix}$	Cov matrix of prior
$A := \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}$	$\begin{pmatrix} 0.9 & 0 \\ 0 & 0.9 \end{pmatrix}$	Structural parameters from ALM
N	40	cross-section sample size
T	150	time-series sample size
Model-specific Parameters		
$\hat{A} := \begin{pmatrix} \rho_1 & m_1 \\ m_2 & \rho_2 \end{pmatrix}$		Structural parameters from PLM
G		Signal Generating Matrix

First we look at the other two cases when the agent is using FIRE or forming expectation jointly. In both cases, $\hat{A} = A$ and $G = I$. The difference is that under FIRE, $\sigma_{1,s} = \sigma_{2,s} = 0$.

Table 11: Simulation Results: $\hat{A} = A$, $G = I$

	FIRE or Single Variable Learning: $\hat{A} = A$, $G = I$							
	FIRE				Single Variable Learning			
	Spec (16)		Spec (7)		Spec (16)		Spec (7)	
	Time Series	Panel	Time Series	Panel	Time Series	Panel	Time Series	Panel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_{11}	-0.002	-0.03^{*}	-0.19^{**}	0.01	0.73^{***}	0.59^{***}	0.59^{***}	0.59^{***}
	(0.028)	(0.015)	(0.091)	(0.017)	(0.048)	(0.008)	(0.097)	(0.008)
β_{12}	-0.026	-0.03^{*}	0.06	-0.01	-0.027	0.004	-0.10	0.004
	(0.030)	(0.017)	(0.116)	(0.016)	(0.034)	(0.010)	(0.13)	(0.010)
β_{21}	-0.005	0.02	0.09	-0.02	-0.049	0.01	-0.22^{***}	0.008
	(0.029)	(0.014)	(0.104)	(0.013)	(0.058)	(0.009)	(0.084)	(0.010)
β_{22}	0.035	0.002	0.020	-0.01	0.68^{***}	0.60^{***}	0.68^{***}	0.60^{***}
	(0.032)	(0.014)	(0.094)	(0.015)	(0.036)	(0.014)	(0.080)	(0.014)
Time FE?	N/A	Yes	N/A	Yes	N/A	Yes	N/A	Yes
Indiv FE?	N/A	Yes	N/A	Yes	N/A	Yes	N/A	Yes

* ***,**,*: Significance at 1%,5% and 10% level. The odd columns are estimation results for year-ahead joint-learning test (16), the even columns are for quarter-ahead specification (7). Newey-West standard errors are reported in brackets.

The results in Table 11 shows the clear differences of test results under FIRE or Single-variable learning. For all specifications considered, if expectation is formed under FIRE all the β's will be insignificantly different from zero. Meanwhile if expectations are formed independently but with information friction, only the own-terms (β_{11} and β_{22}) are significantly positive. The cross-terms will be insignificant.

Table 12: Simulation Results: $m_1 = 0.4$, $m_2 = 0$, $G = I_{2\times 2}$

	Joint Learning: $m_1 = 0.4$, $m_2 = 0$, $G = I_{2\times 2}$					
		Year-ahead spec (16)			Quarter-ahead spec (7)	
	Truth	Time Series	Panel	Truth	Time Series	Panel
	(1)	(2)	(3)	(4)	(5)	(6)
β_{11}	0.59	0.66***	0.56***	0.59	0.54***	0.56***
	–	(0.057)	(0.013)	–	(0.086)	(0.009)
β_{12}	0.29	0.20***	0.30***	0.28	0.26***	0.28***
	–	(0.10)	(0.020)	–	(0.145)	(0.012)
β_{21}	0	−0.05	0.001	0	−0.19***	0.001
	–	(0.05)	(0.010)	–	(0.078)	(0.010)
β_{22}	0.62	0.77***	0.59***	0.62	0.90***	0.59***
	–	(0.094)	(0.017)	–	(0.126)	(0.015)
Time FE?	N/A	N/A	Yes	N/A	N/A	Yes
Individual FE?	N/A	N/A	Yes	N/A	N/A	Yes
$corr(E\pi, Edun)$	0.8	0.87	0.87	0.68	0.81	0.81

* ***,**,*: Significance at 1%,5% and 10% level. Columns (2) and (5) are estimation results for year-ahead joint-learning test (16). columns (3) and (6) are for quarter-ahead specification (7). Columns (1) and (4) are the ground truth for these coefficients. Newey-West standard errors are reported in brackets.

The above table shows the results using data from a joint learning model with wrong belief $\hat{A}$. It's clear that in presence of noisy information, if the agent believes $m_1 > 0$, so that $\hat{A} \neq A$. Both regressions with (7) and (16) correctly uncover such joint learning behaviors with the estimate on $\beta_{12} > 0$, implying that in agent's subjective model, $m_1 > 0$. In this case, either time series or panel data is sufficient. However, comparing columns (2) and (3) we see the efficiency gain of using panel data. The last column of the table documents the correlation between expectations theoretically and in the data.

I then consider the other form of joint learning: when $\hat{A} = A$ but the signals are generated using $G = \begin{pmatrix} g_1 & g_2 \end{pmatrix}$. I report the same results as in columns (2), (4), (6) and (8) as in Table 12:

Table 13: Simulation Results: $\hat{A} = A$, $G = \begin{pmatrix} g_1 & g_2 \end{pmatrix}$

	Joint Learning: $\hat{A} = A$, $G = \begin{pmatrix} g_1 & g_2 \end{pmatrix}$					
	Year-ahead spec (16)			Quarter-ahead spec (7)		
	Truth	Time Series	Panel	Truth	Time Series	Panel
	(1)	(2)	(3)	(4)	(5)	(6)
β_{11}	0.79	0.89^{***}	0.79^{***}	0.79	0.72^{***}	0.80^{***}
	-	(0.059)	(0.064)	-	(0.047)	(0.064)
β_{12}	-0.12	-0.12^{**}	-0.15^{**}	-0.12	-0.14^{***}	-0.15^{***}
	-	(0.047)	(0.065)	-	(0.067)	(0.065)
β_{21}	-0.12	-0.16^{**}	-0.11^{*}	-0.12	-0.13^{**}	-0.10
	-	(0.061)	(0.064)	-	(0.069)	(0.064)
β_{22}	0.79	0.86^{***}	0.75^{***}	0.79	0.74^{***}	0.75^{***}
	-	(0.051)	(0.065)	-	(0.050)	(0.064)
Time FE?	N/A	N/A	Yes	N/A	N/A	Yes
Individual FE?	N/A	N/A	Yes	N/A	N/A	Yes
$corr(E\pi, Edun)$	1.00	1.00	1.00	1.00	1.00	1.00

* ***,**,*: Significance at 1%,5% and 10% level. The odd columns are estimation results for year-ahead joint-learning test (16), the even columns are for quarter-ahead specification (7). Newey-West standard errors are reported in brackets.

The comparison from Table 11 to Table 13 shows clearly that the joint learning test yields informative results on how the agents form expectation that are in line with the summary from Table 3. When all β's are zeros, the expectation is formed under FIRE. When noisy information friction is present, $\beta_{12} = \beta_{21} = 0$ suggests expectations are likely formed independently; whereas either these two estimates being non-zero means that expectations are formed jointly. Furthermore, if the correlation between expectations comes from signal generating process, β_{12} and β_{21} should have the same signs; whereas a non-diagonal $\hat{A}$ would impose less restrictions on the signs and magnitudes of β_{12} and β_{21}. Moreover, the test using either consensus expectation (aggregate time-series) or panel data gives qualitatively same results.

F News Measure from MSC

F.1 Description

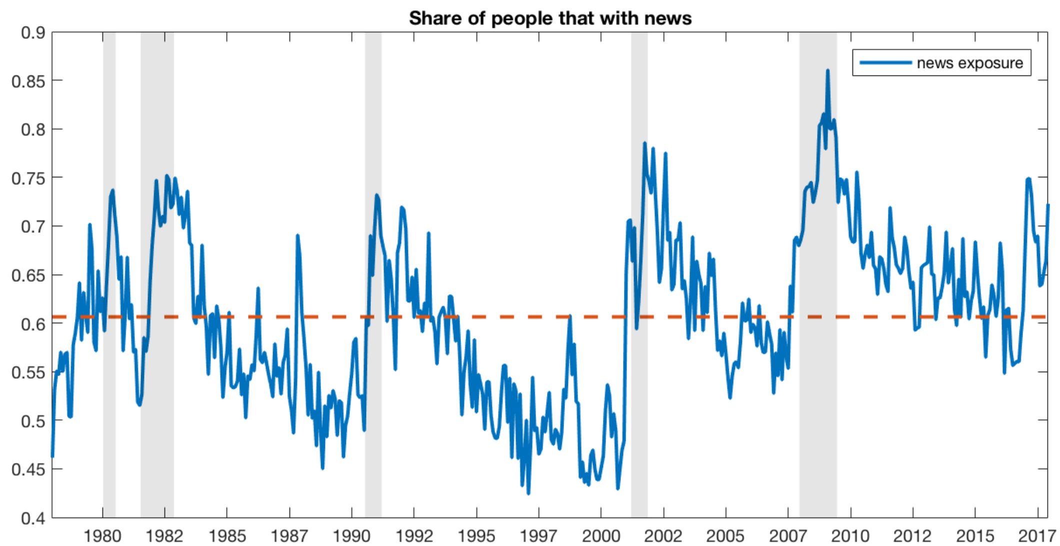
The news measures from MSC are usually referred as "perceived news" as the question asked in the survey is:

A6. During the last few months, have you heard of any favorable or unfavorable changes in business conditions?

A6a. What did you hear?

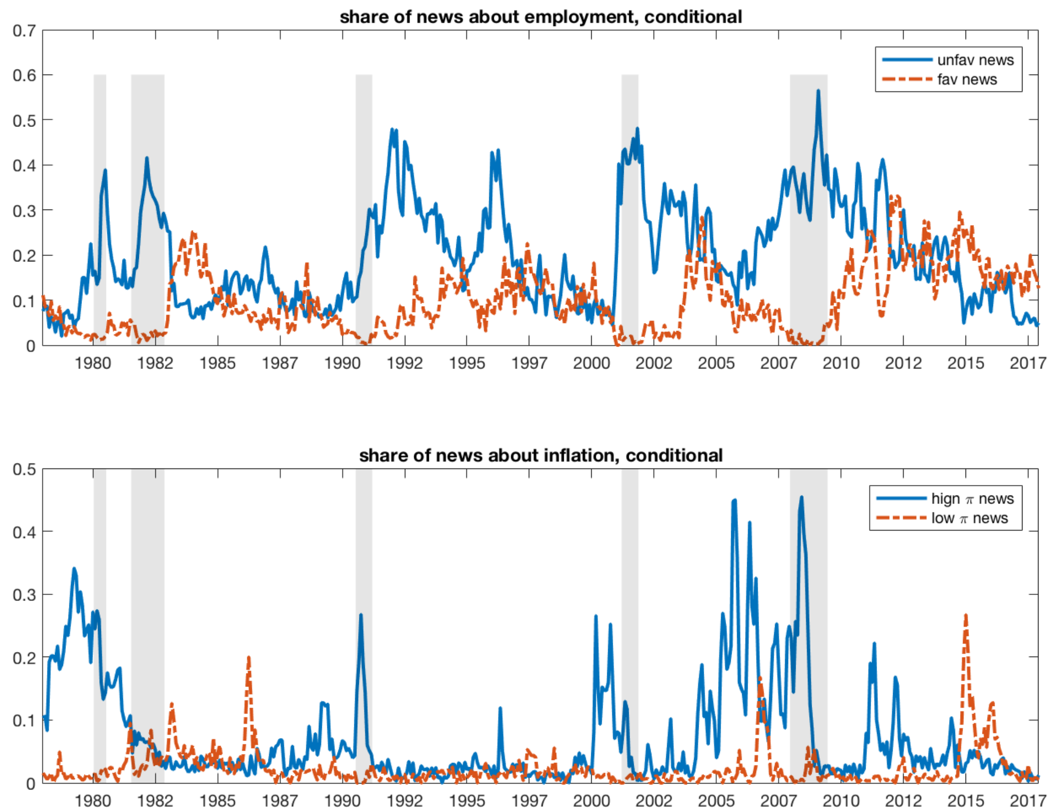
The news reported in this question should be considered as self-reported information, it may contain both public and private information heard by the surveyee. The content of news is described by the surveyee and then categorized into 80 different categories. In Figure 11 I plot the share of surveyees that report hearing any news. And Figure 12 depicts the fraction of agents hearing news about unemployment and inflation conditional on hearing any news.

Figure 11: Share of People that Report Hearing of News



Share of people that report hearing any news across time. The dashed line represents on average 60% survey participants reported hearing about some news in the past 3 months.

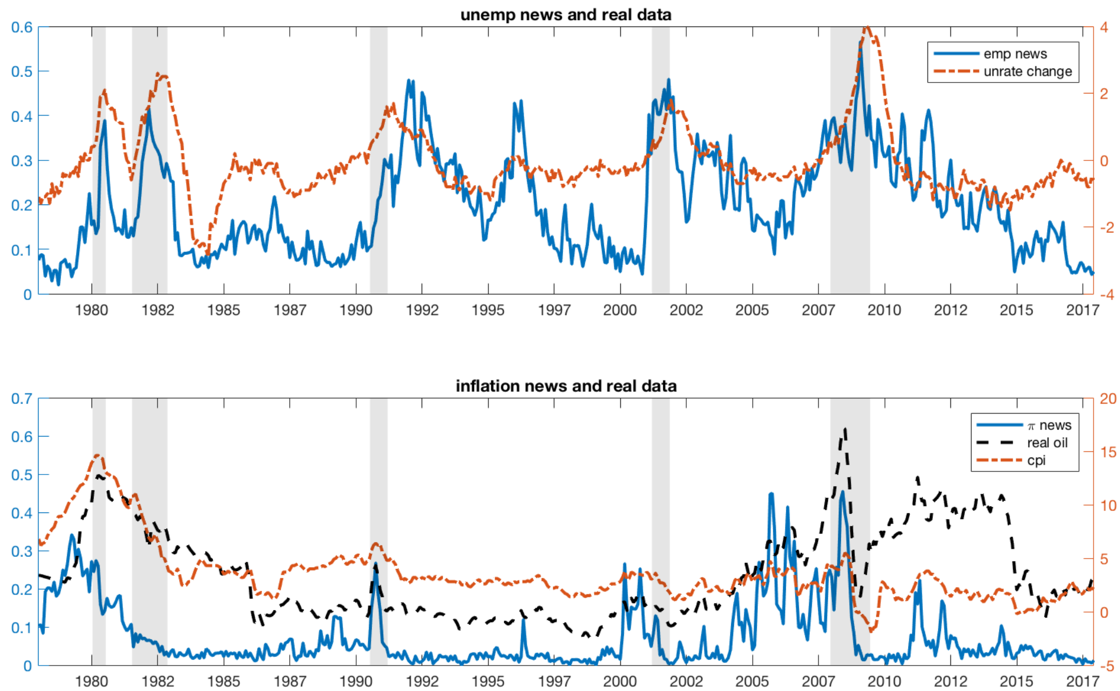
Figure 12: Share of People that Report Hearing of News on Inflation and Employment



Share of people that report hearing news on employment or inflation, conditional on hearing news. In top panel, the blue line is fraction with unfavourable news on employment and red dash line is fraction with favourable news. In bottom panel, blue line is fraction with news on higher inflation.

On average there are more than 60% agents report they have heard some news about the economy, and the fraction is comoving with business cycle, peaking in each recessions. Among this news about unemployment and inflation accounts for more than 40% on average, peaking at about 80% in the recent recession. And there is an asymmetry in tones of news: the blue curve is almost always above red ones, which suggests agents report to hear of bad news more often than good ones. At first pass it seems agents are making distinctions in labelling news about inflation and employment. Figure 13 plots the specific news against realized data, the news heard is highly comoving with corresponding macroeconomic variable. And the news on inflation is also highly correlated with real oil price (0.51) which indicates households' inflation expectations are sensitive to gas prices, as various researchers have suggested.

Figure 13: News Heard with Actual Data

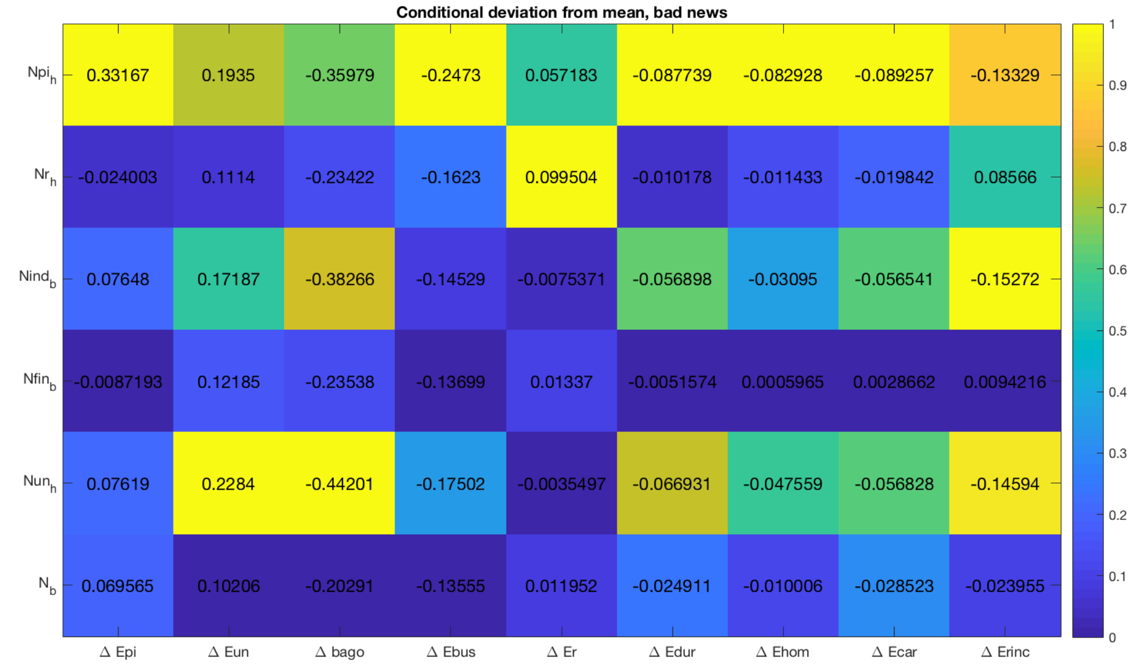


In both panels blue lines are fraction of news on employment or inflation, red dash lines are corresponding actual data. In the bottom panel the black dotted line is real oil price obtained from FRED.

F.2 Extra Figures

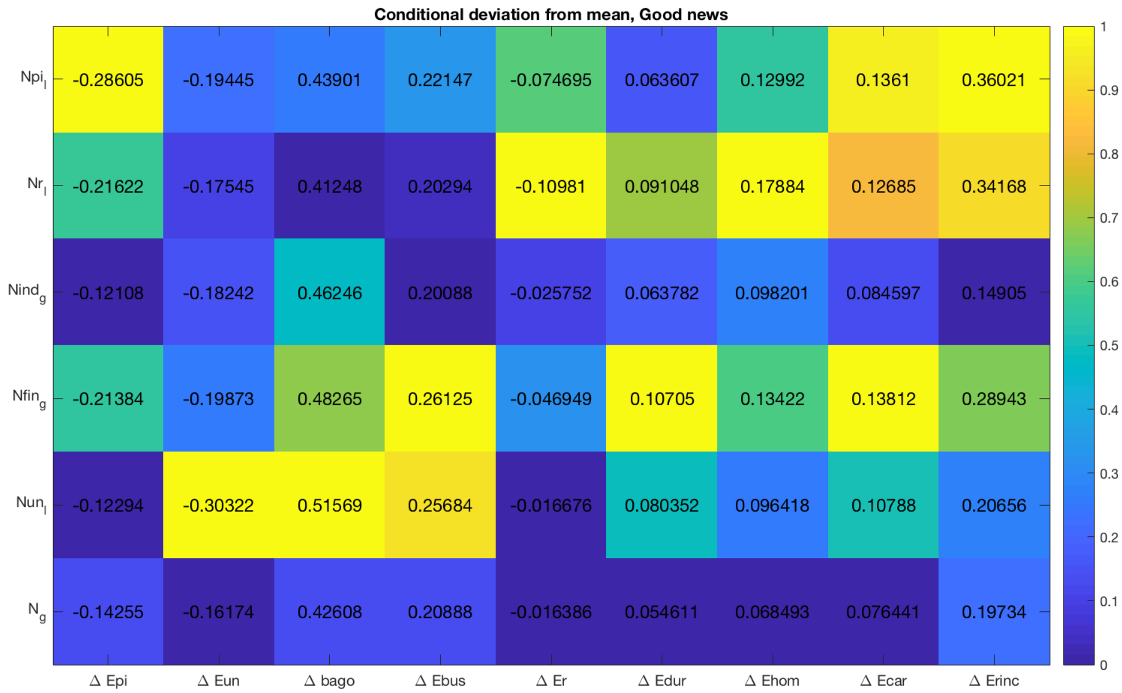
Figure 14 and 15 are similar matrices to Figure 6 from **Section 4**, with more news categories and expectational variables as response variables. Figure 14 are deviation of expectational variables from their unconditional mean, conditional on hearing unfavorable news. Figure 15 are the same exercise conditional on hearing good news.

Figure 14: Heatmap for Expectation Responses to Unfavourable News:Cross-sectional



On y-axis is the news heard for each subgroup, on x-axis is expectation under examination. The number reported in each box is the percentage deviation of expectations reported by the agents who received corresponding news, from the mean expectations of all the survey participants at each point of time. The figure is responses conditional on hearing unfavourable news.

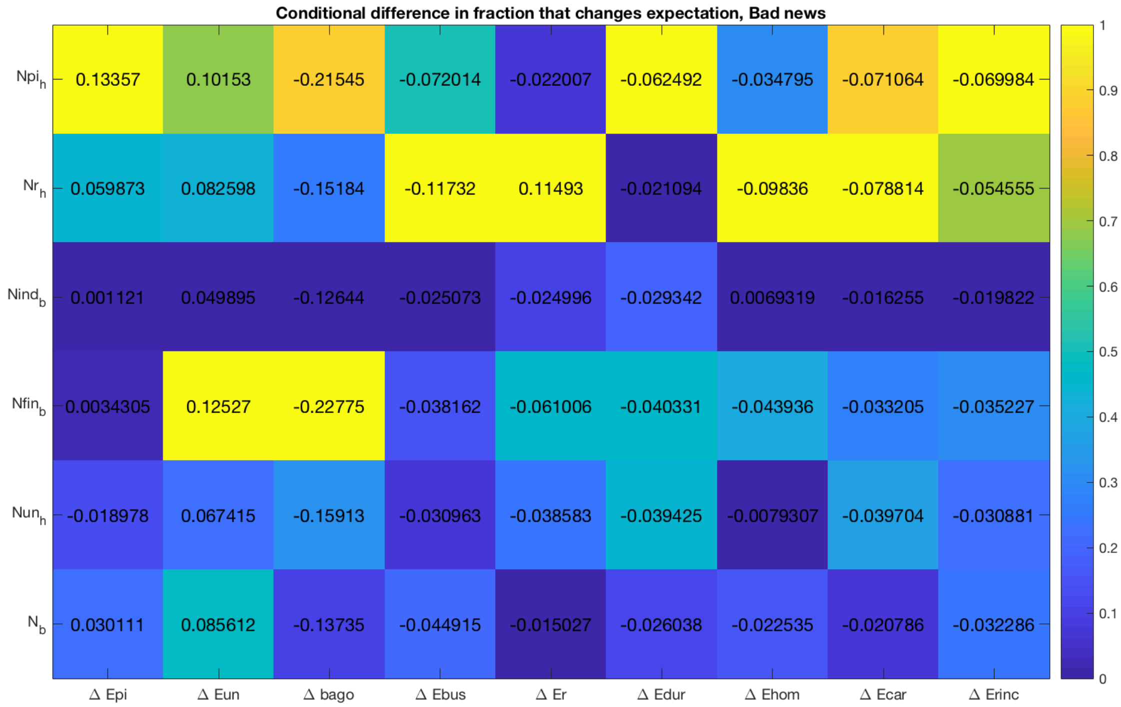
Figure 15: Heatmap for Expectation Responses to Favorable News:Cross-sectional



On y-axis is the news heard for each subgroup, on x-axis is expectation under examination. The number reported in each box is the percentage deviation of expectations reported by the agents who received corresponding news, from the mean expectations of all the survey participants at each point of time.The figure is responses conditional on hearing favourable news.

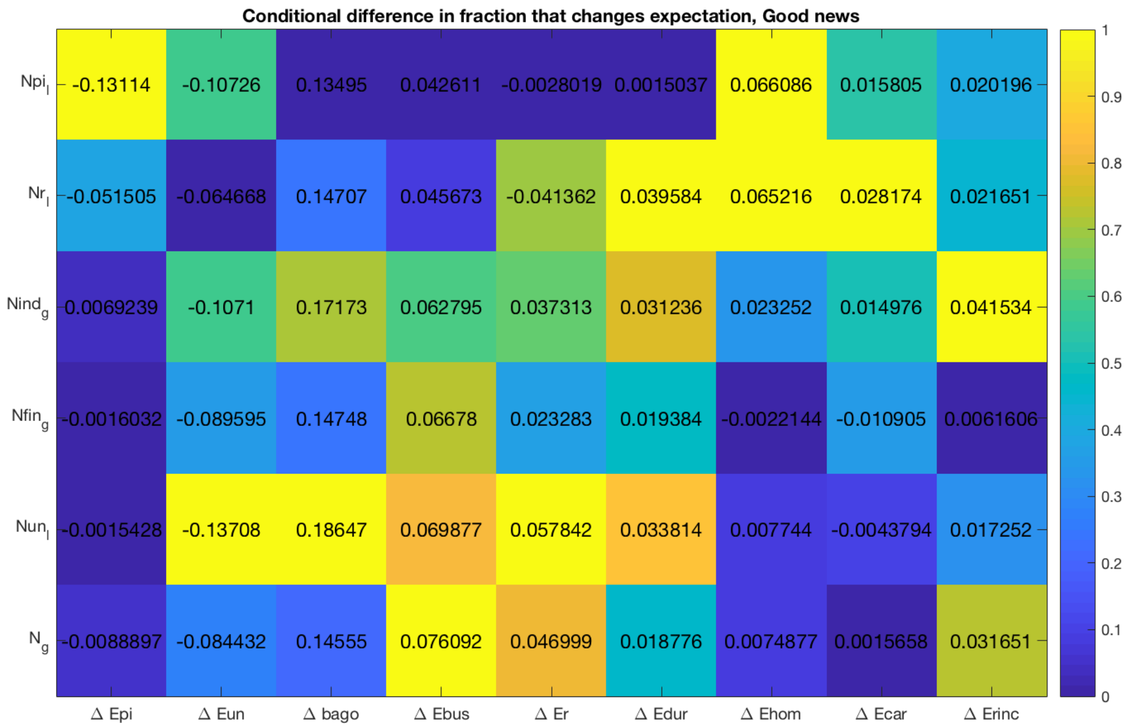
Figure 16 and 17 are similar matrices to Figure 6 from **Section 4**, with more news categories and expectational variables as response variables. Figure 16 is for unfavorable news, Figure 17 is for favorable news.

Figure 16: Heatmap for Expectation Responses to Unfavourable News:Panel



On y-axis is the news heard for each subgroup, on x-axis is expectation under examination. The number reported in each box is the likelihood each agent increases her expectation upon receiving different news. The figure is responses conditional on hearing unfavourable news.

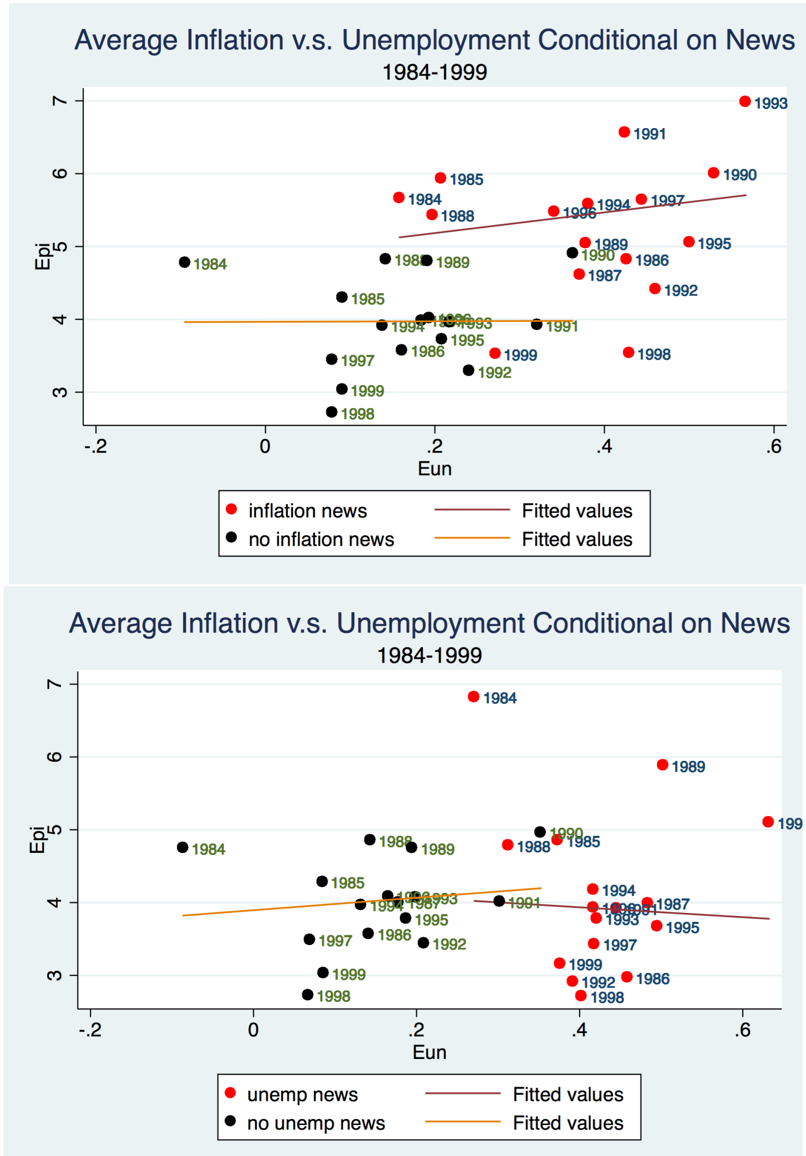
Figure 17: Heatmap for Expectation Responses to Favourable News:Panel



On y-axis is the news heard for each subgroup, on x-axis is expectation under examination. The number reported in each box is the likelihood each agent increases her expectation upon receiving different news. The figure is responses conditional on hearing favourable news.

Finally, I include the scatter plots for consensus expectation on inflation and unemployment, conditional on getting news about inflation and unemployment status but for sample period 1984-1999. Figure 18 shows similar pattern as in Figure 7

Figure 18: Consensus Expectation on Inflation and Unemployment, 1984-1999



Scatter plot for consensus expected inflation and unemployment each year from 1984-1999. Top panel: conditional on having heard inflation news or not, red dots are expectations conditional on hearing inflation news, black dots are those without inflation news. Bottom panel: conditional on having heard unfavourable unemployment news.