Learning and Subjective Expectation Formation: A Recurrent Neural Network Approach *

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Abstract

I propose a flexible non-parametric method using Recurrent Neural Networks (RNN) to estimate a generalized model of expectation formation. This approach does not rely on restrictive assumptions of functional forms and parametric methods yet nests the standard approaches of empirical studies on expectation formation. Applying this approach to data on macroeconomic expectations from the Michigan Survey of Consumers (MSC) and a rich set of signals available to U.S. households, I document three novel findings: (1) agents' expectations about the future economic condition have asymmetric and non-linear responses to signals; (2) agents' attentions shift from signals about the current state to signals about the future: they behave as adaptive learners in ordinary periods and become forward-looking as the state of economy gets worse; (3) the content of signals on economic conditions, rather than the amount of news coverage on these signals, plays the most important role in creating the attention-shift. Double Machine Learning approach is then used to obtain statistical inferences of these empirical findings. Finally, I show these stylized facts can be generated by a model with rational inattention, in which information endogenously becomes more valuable when economic status worsens.

Keywords: Expectation Formation, Bounded Rationality, Information Acquisition, Non-parametric Method, Recurrent Neural Network, Survey Data

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1 Introduction

Models on expectation formation have played an important role in modern macroeconomic theories. The past decade has seen a surge of empirical studies using survey data to examine how information about aggregate economic status, such as unemployment and inflation rate, affects households' macroeconomic expectations. For example, in their seminal work, Coibion and Gorodnichenko (2012) document pervasive evidence that expectations from the Michigan Survey of Consumers (MSC) deviate from Full Information Rational Expectation (FIRE) and conclude that households have limited information. However, these empirical frameworks usually use restrictive assumptions on functional forms to apply parametric methods. Empirical findings with these approaches are then subject to these parametric assumptions and might miss important features of the relationship between households' macroeconomic expectations and signals. For example, when facing information about different macroeconomic aspects or from various sources, agents may be selective about the information they use to form expectations. Positive and negative news about economic status may have different impacts in terms of magnitudes on their expectations. Furthermore, the way they utilize various information may differ when the state of the economy changes.[1] This paper aims to explore whether these patterns exist in the data.

To achieve this goal, I first make a methodological contribution by proposing an empirical framework that allows for a flexible relationship between macroeconomic signals and households' expectations.[2] Specifically, it nests the dynamic structure adopted by most expectation formation models in macroeconomics, where households form expectations about the future by perceiving some latent variables according to a rich set of signals. Such a structure is common in many learning models and empirical studies, but the latent variables may take different forms depending on the parametric assumptions made in the model. For

[1]For example, Coibion and Gorodnichenko (2015) documents that the level of information rigidity falls in recessions and is particularly high during the Great Moderation. This indicates that the way economic agents process information may change as economic status changes.

[2]To be specific, I consider a rich set of macroeconomic signals, including current and past unemployment rates, inflation rates, real GDP growth, and interest rates. I also include signals that contain information about the future, such as professional forecasts on the unemployment rate and inflation, as well as some signals at the individual level. For households' expectations, I consider expectations on unemployment rates, inflation, interest rates, and economic condition. A detailed description of the data used in my empirical analysis can be found in Section 4.1

example, in the standard noisy information model (e.g. Woodford (2001)), where the state variable is unobserved, the latent variable is the posterior mean of that state. In Markov Switching Models (e.g. Hamilton (2016)), these latent variables become their posterior beliefs on the Markovian state.

The novelty of my empirical method compared to standard approaches is that I impose no restrictions on what the latent variables are, how the signals affect the latent variables, and how the latent variables affect households' expectations. Instead, the relation between signals and expectational variables through the dynamic structure is estimated using a non-parametric method, Recurrent Neural Networks (henceforth RNN). RNN can be used in this specific context because it can universally approximate the dynamic system that represents the general structure proposed above. Such a property follows from the Universal Approximation Theorem in the context of dynamic system as proved in Schäfer and Zimmermann (2006). The strength of this approach is that it can capture the flexible relationship between signal and expectational variables without further parametric assumptions on functional forms while maintaining the dynamic structure described above. In particular, suppose the macroeconomic signals affect the expectations non-linearly, or through interacting with other signals or the latent variables. In these cases, the relations will be captured by RNN but are usually missed by models that are linear or with pre-assumed structures. On the other hand, if the underlying mapping between signals and expectations is linear, this approach will uncover a linear relationship.[3]

The estimated functional form then offers important insight on plausible structures for households' expectation formation process. It also provides a way to evaluate how macroeconomic signals affect households' expectations. Following the functional estimation with RNN, I apply the Double Machine Learning (DML) method proposed by Chernozhukov *et al.* (2018) to estimate the average marginal effect of the macroeconomic signals on households' expectations. This approach is usually used to correct the bias induced by the plug-in estimators following machine learning methods. It is also known to deliver valid inference on these estimators under high-level assumptions on the corresponding moment condition model and machine learning estimators, thus allowing for tests on the statistical significance

[3]In **Appendix B.4** I illustrate these properties with examples using simulated data.

of my empirical findings. In Section 3, I describe the details on how to apply DML, and in Appendix B, I verify the high-level conditions needed to obtain valid inference.

Applying my empirical methods to the Michigan Survey of Consumers, I document three major findings new to the literature. I first show that households' expectations on the economic conditions, namely the unemployment rate and the real GDP growth,[4] are non-linear functions of signals about the change of unemployment rate and real GDP - the effect of an incremental change in such a signal depends on the level of the signal itself. The relationship is also asymmetric - positive and negative signals with the same magnitude have an asymmetric impact on expectations. In particular, households respond more aggressively to signals that suggest the economic status worsens.

Furthermore, using the approximated functional form of the expectation formation model, I find the marginal effects of these signals change over time. Specifically, the absolute values for the marginal effects of signals on the economic conditions fall as the GDP growth slows down or the unemployment rate hikes up. However, the opposite is true for the signals that contain information about the future. When interpreting marginal effects as weights that households put on signals, this finding suggests that households shift their attention from signals about current and past states to those about the future. In other words, the households behave as "adaptive learners" when economic conditions are stable and become more "forward-looking" when the situation gets worse.

Lastly, the estimated functional form of the expectation formation model suggests such an attention-shift is mainly driven by the signals on economic conditions rather than information related to the interest rate or inflation. Furthermore, they contribute to the attention-shift through both the contemporaneous signals newly observed in each period and the latent variables that capture the past signals' impacts. This is consistent with the empirical evidence on the presence of info rigidity largely documented in the literature on households' expectations. Moreover, in my empirical framework, I also include measures on the amount of news coverage about various macroeconomic aspects from both local and national news

[4] On a side note, the "economic conditions" I defined here is different from that in MSC. In MSC there is a question asking households' expectations on economic condition. I consider this as proxy for expectation on the real GDP growth.

media.[5] I refer to such a measure as "volume of news". I then find that a higher volume of news about the economic condition from media leads to a higher weight on signals about the future, as suggested by Carroll (2003). However, it does not explain the drop of weights on signals about current and past states. Instead, it is the content of signals on economic conditions, rather than the volume of news on these signals, that plays the most important role in creating the attention-shift.

These new stylized facts are consistent with rational inattention models but hard to be reconciled with many other commonly used frameworks for modelling beliefs. For example, for a model with Full Information Rational Expectation to explain the attention-shift between signals on current and future states, one has to believe that the economic conditions, such as the unemployment rate, follow a more persistent or volatile process during recession episodes. Standard noisy information and sticky information models are also insufficient. To create weight changes on signals in these models, one needs state-dependency in either precision of signals or the underlying state-space model that agents believe in, both of which are exogenous in those models. One possible explanation for the attention-shift is through the volume of news reported by media as first proposed in Carroll (2003). Lamla and Lein (2014) formalized the idea by showing that greater media coverage increases the precision of signals about the future in agents' signal-extraction problem, leading to a higher weights on these signals. For this explanation to work, one should observe that the weights on the current signals fall as the volume of news on economic conditions increases. Moreover, the volume of this news alone should account for most of the variations in the change of marginal effects. However, neither of these is true according to my empirical findings.

I then develop a model featuring rational inattention to explain these stylized facts. When agents have limited ability to acquire information, they will choose to allocate their limited resources optimally on a subset of signals available to them. These choices can change as economic status changes, thus creating the attention-shift and the non-linear responses to different signals. Moreover, the state-dependency created by this type of model is not ad hoc: it comes from agents' optimal behaviour in the face of information constraints. In the

[5] I scraped the number of news stories on related macroeconomic topics (i.e. inflation, interest rate and unemployment rate) from TV news scripts and local newspaper articles in LexisNexis Database. Then I construct a measure of news coverage on these topics following PFAJFAR and SANTORO (2013).

rational inattention model I propose, information about the future becomes more valuable endogenously when the state of the economy gets worse. For this reason, households start to seek for more information about the future actively and end up placing higher weights on these signals when forming their expectations.

Literature Review This paper contributes to several different strands of literature. It first relates to the growing empirical literature using survey data to investigate how expectations are formed. These studies have documented substantial evidence on information rigidity in the agents' expectation formation process and shown they utilize information from different sources.[6] For example, D'Acunto *et al.* (2019) shows that the U.S. consumers use personal shopping experience, information from family members and friends, as well as news from media as the top three information sources to form their inflation expectations. More recent papers have linked information rigidity with agents' limited attention to various sources of information. Coibion and Gorodnichenko (2015) shows evidence that the degree of information rigidity is highest during Great Moderation - when macroeconomic conditions are less volatile, and agents have less incentive to pay attention to information - and that rigidity falls during recessions. Roth *et al.* (2020) finds that U.S. households demand an expert forecast about the likelihood of recession when perceiving higher unemployment risk in a random experiment setting. My paper adds to this literature using observational data by showing that various sources of information compete for households' attention, and they acquire more information about the future from experts when the state of the economy gets worse. This paper also presents another brand new aspect of households' expectation: their expectation formation model is non-linear and asymmetric. These findings come from the flexible empirical framework to model the relationship between survey expectation and a large set of information available to households, which are not available in empirical analysis motivated by the class of linear models.

The empirical framework proposed in this paper is built on the literature about learning and information acquisition. This literature has a long history in macroeconomics. The mod-

[6] For example, Coibion and Gorodnichenko (2012) and Andrade and Le Bihan (2013) show that expectations are formed under a limited information structure, and public information is rigid. Carroll (2003) and Lamla and Lein (2014) show that households obtain information about future inflation and unemployment from professional forecasters through media exposure to this news.

els developed in this literature include Constant Gain Learning (e.g. Evans and Honkapohja (2001), Milani (2007), Eusepi and Preston (2011)),[7] Noisy Information (e.g. Woodford (2001)), Markov Regime Switching (e.g. Hamilton (2016)) and Rational Inattention (e.g. Sims (2003), Mackowiak and Wiederholt (2009), Maćkowiak *et al.* (2018)). All these models adopt the same dynamic structure as in my empirical framework but differ in parametric functional form assumptions made when brought to data. Empirical findings with this approach may miss important features of the relationship between households' macroeconomic expectations and signals. For example, in a standard noisy information model,[8] agents are assumed to believe in a linear state-space model with known structural parameters and fixed precision of signals. Following these assumptions, least-square methods can be used. One will find a signal with the same magnitude has the same impact on the expectation as long as agents have the same prior variance, even if the signal's actual impact is time-varying. The method proposed in this paper is more flexible on these fronts. It avoids making these restrictive assumptions and directly estimates the function form from data while maintaining the same dynamic structure adopted by these theoretical models. The estimated function can then offer insights into agents' expectation formation process. Indeed I find that the new stylized facts are more consistent with models that feature rational inattention.

The two-period rational inattention model developed in this paper is similar to the partial-equilibrium consumer problem setup in Kamdar (2019), but with only a stochastic return on capital rather than labor income. In the literature a standard approach to solve rational inattention models is by taking a second-order approximation (e.g. Mackowiak and Wiederholt (2009), Maćkowiak *et al.* (2018), Afrouzi (2020) etc).[9] In this paper I solve the model numerically and restrict my setup to Gaussian signals.[10] In this setup, I show that

[7]The Constant Gain Learning Framework is later extended to models in which experiences affect expectations (Malmendier and Nagel (2015)), and models to explain heterogeneity across agents (Cole and Milani (2020)).

[8]In these models, the agent forms expectation on a single unobservable state using stationary Kalman Filter. See examples as Coibion and Gorodnichenko (2015),Coibion and Gorodnichenko (2012) and Andrade and Le Bihan (2013). For noisy information model allowing for joint expectation formation, see Hou (2020) and Kamdar (2019).

[9]Exceptions include Sims (2006).

[10]This method of taking second-order or log-quadratic approximation leads to the well-known result that the optimal distribution of signals is Gaussian. It also results in that the optimal precision of the signal is independent of the perceived current state, which is the key to create state-dependency in my model. For this reason, I avoid taking second-order approximation and restrict the distribution of signal as Gaussian.

information about the future return on capital endogenously becomes more valuable in bad states. This is because the utility loss induced by the difference between optimal saving choice under full information and that under limited information is larger in those states. This mechanism is enough to capture both the non-linearity and state-dependency in agents' expectation formation process.

Finally, the methodology part of this paper contributes to the recent literature using machine learning techniques to solve economic problems. There is a surge in applications of modern machine learning tools in economics for the past several years, including prediction problems as discussed in Kleinberg *et al.* (2015) as well as more recent work on causal inference such as in Athey and Imbens (2016) and Chernozhukov *et al.* (2017).[11] Among these tools, the use of Deep Neural Networks in macroeconomics can date back to the early 2000s. Back then, different types of Neural Networks (both fully-connected ones and recurrent ones as used in this paper) were used to solve pure prediction problems. Examples like Nakamura (2005) and Kuan and Liu (1995) have shown RNN out-performs standard linear models in forecasting inflation and exchange rate respectively.[12] However, this paper uses RNN to solve both prediction and estimation problems. RNN is first used to approximate the average structural function (henceforth ASF) as described in Blundell and Powell (2003) derived from my empirical framework, which is essentially a prediction problem. Then with the standard identification restrictions the same as empirical literature on learning and expectation formation, I follow Chernozhukov *et al.* (2018) to obtain the DML estimator and its inference for the average marginal effect of signals on households' expectations. The estimation procedure of this paper is closely related to those in Chernozhukov *et al.* (2018) and Farrell *et al.* (2018). The latter offers convergence-speed conditions for deep Neural Networks to acquire valid inference. To my best of knowledge, this is the first time RNN is applied to learning and expectation formation problems in an estimation context.

The rest of this paper is organized as follows: in **Section 2** I describe the empirical framework I propose and the Average Structural Function implied by such framework. In **Section 3** I introduce the method to approximate Average Structural Function using RNN

[11] For a complete review on recent applications of Machine Learning tools in economics, see Athey (2018).

[12] See also Almosova and Andresen (2018), R. and Hall (2017) for recent application to macroeconomic forecasting with state-of-art architecture of RNN – Long Short Term Memory (henceforth LSTM) layers.

and how to estimate average marginal effect of signals using the DML method. **Section 4** presents the results from applying the method to survey expectation and macroeconomic signal data. Then I propose the rational inattention model that can explain these news stylized facts in **Section 5**. And **Section 6** concludes.

2 Generic Learning Framework

In this section, I describe the empirical framework about how expectation is formed by households, which I refer to as the Generic Learning Framework. It is worth describing the similarity and key differences between this model to the standard learning models such as stationary Kalman Filter or Constant Gain Learning. In the standard models, several types of assumptions are made: (1) assumptions about information structure faced by agents that are forming expectations; (2) assumptions on identification, which involves the restrictions on unobservable error terms in the model; and (3) parametric assumptions on learning behavior. These parametric assumptions include both the underlying structure agents learn about and how learning is carried out. For example, in standard noisy information models, the perceived law of motion that the agents learn is assumed to be linear in the hidden states, and the prior and posterior beliefs on these states are structured as Gaussian. These assumptions lead to specific parametric regression methods used in different learning models. The Generic Learning Framework maintains standard assumptions on information structure and identification but imposes only minimal restrictions on the functional forms of learning. It then naturally requires the use of non-parametric or semi-parametric methods such as RNN. Such a feature also implies the Generic Learning Framework can represent a large class of learning models existing in the literature despite these models may differ in their functional forms. In **Appendix B.4**, I include an example that illustrates how this framework can represent a stationary Kalman Filter.

I introduce the Generic Learning Framework in two parts. First, I show how the agents form their expectations after observing a set of signals. This part is typically referred to as the "agent's problem". Then I describe the econometrician's information set as an observer and what she can do to learn about the agent's expectation formation process. This part is usually referred to as the "econometrician's problem".

2.1 Agent's Problem

Consider the agents observe a set of signals. These signals include both public signals that are common to each individual and private signals that are individual specific. Denote the public signal as $X_t \in \mathbb{R}^{d_1}$ with dimension d_1 and private signal as $S_{i,t} \in \mathbb{R}^{d_2}$ with dimension d_2. An example of the public signal will be official statistics such as CPI inflation or professional forecast on CPI inflation a year from now. An example of the private signal will be state-level inflation matched to the location agent lives at or the fraction of news stories about inflation published in local newspapers.

Other than public and private signals, there is an individual level noise term denoted as $\epsilon_{i,t}$ in the agent's information set. This term represents the observational noise attached to signals in the standard noisy information model as in Woodford (2001) and Sims (2003). It can also stand for any unobserved individual-level information that is not captured by public and private signals but is used by the agent when forming expectations. If it takes the form of observational noise, $\epsilon_{i,t}$ is typically separable additive to the public and private signals. Here for generality, I do not restrict the form of how it enters the expectation formation process.

After observing the set of signals $\{X_t, s_{i,t}, \epsilon_{i,t}\}$, agent forms expectation of variables Y_{t+1} and denote the corresponding subjective expectation as $Y_{i,t+1|t}$ [13].The agents' expectation formation model then can be written as:

$$Y_{i,t+1|t} = \hat{\mathbb{E}}(Y_{t+1}|X_t, S_{i,t}, \epsilon_{i,t}, X_{t-1}, S_{i,t-1}, \epsilon_{i,t-1}...) = G(X_t, S_{i,t}, \epsilon_{i,t}, ...) \tag{1}$$

The formulation in (1) is the most general form of an expectation formation model. The expectation operator $\hat{\mathbb{E}}$ stands for subjective expectations formed by agents, which could be different from a statistical expectation operator. Without further assumptions the expectations formed through this model can be non-stationary and non-tractable. To avoid these properties I make the following assumption for the Generic Learning Framework:

[13] To save notations I drop the step t, however generally speaking this could be h step expectations agents form, and it can be over any object Y.

Assumption 1. *Agents form expectation through two steps: updating and forecasting. In the updating step, agents form a finite dimensional latent variable* $\Theta_{i,t}$*, which follows a Stationary Markov Process:*

$$\Theta_{i,t} = H(\Theta_{i,t-1}, X_t, S_{i,t}, \epsilon_{i,t}) \tag{2}$$

In the forecasting step, they use $\Theta_{i,t}$ *to form expectation:*

$$Y_{i,t+1|t} = F(\Theta_{i,t}) \tag{3}$$

Where both $H(.)$ *and* $F(.)$ *are measurable functions.*

The updating step suggests that agent holds some beliefs about the economy which can be summarized with $\Theta_{i,t}$. In each period he updates this belief from its previous level $\Theta_{i,t-1}$ with the new signals observed $\{X_t, S_{i,t}, \epsilon_{i,t}\}$. The Markov property helps to simplify the time-dependency and guarantees tractability of the model. Stationarity makes sure the signals from history further back in time can affect expectational variables today but in a diminishing way. Furthermore, in this set up I allow expectation to be affected by signals in the past without explicitly specifying a fixed length of memory.[14].

These two steps are commonly seen in standard learning models. For example, in stationary Kalman Filter, this is usually referred to as "Filtering Step", where the agent uses the new signals to form a "Now-cast" variable about the current state of the economy. They will then use this "Now-cast" to form the expectation about the future using their perceived law of motion.[15] This step is the same as the "forecasting step" in the Generic Learning Framework.

It is then worth noting that the structure of my framework described in assumption 1 covers a large class of learning models existing in the literature, other than the stationary Kalman Filter. Obviously, this formulation includes adaptive learning models where agents use only past information to form expectations[16]. It also covers models where agents get information about the future from professional forecasts through reading news stories, as

[14]For example, one may want consider a case where expectation $Y_{i,t+1|t}$ is a function of signals from a fixed window of time $\{X_t, S_{i,t}, X_{t-1}, S_{i,t-1}, ..., X_{t-h}, S_{i,t-h}\}$ Such a function is also covered by the system described by (2) and (3)

[15]Refer to Appendix B.4 for a detailed example in the context of the standard noisy information model.

[16]See Evans and Honkapohja (2001) for example.

in Carroll (2003). Recall the $\hat{\mathbb{E}}$ in equation (1) means agents may form expectation using subjective beliefs, instead of assuming the full structure of agents' knowledge and the statistical property agent believes in as usually done in the learning literature. This allows for behavioral models such as Bordalo *et al.* (2018). To further illustrate the flexibility of this generic framework, in Appendix B.4 I will take the stationary Kalman Filter that is typically used in noisy information models and a Constant Gain Learning model as two examples, and represent them in the form of the Generic Learning Framework.

In addition to Assumption 1, I also need independence assumptions on the observational noise term $\epsilon_{i,t}$. This assumption states that the noise unobservable by economists is independent with observed public and individual specific signals as well as across individuals and time. While such an assumption is commonly made in noisy information and other learning models with unobserved noise, the economic intuition behind it is simple as well. Consider an agent wants to predict inflation, and they observe a signal on price change when they went grocery shopping. Such a signal is an imperfect measure of current inflation as it is price change only for one or several products. Mathematically this signal can be thought of as drawn from a distribution, with the official measure of inflation being the mean of this distribution. An individual may draw the signal from the left tail or right tail of the distribution, depending on the specific product she picked up. The public signal X_t (or private signal $S_{i,t}$) is then the mean of this distribution, and $\epsilon_{i,t}$ measures the deviation of the actual signal agent observes from this mean. The assumption suggests this deviation is independent of its mean as well as across individual and time.

Assumption 2. *The idiosyncratic noise on public signal, $\epsilon_{i,t}$ is i.i.d across individual and time. It is orthogonal to past and future public and private signals:*

$$\epsilon_{i,t} \perp X_\tau \quad \epsilon_{i,t} \perp S_{i,\tau} \quad \forall t \leq \tau$$

$$\epsilon_{i,t} \perp \epsilon_{j,t} \quad \forall j \neq i, \quad \epsilon_{i,t} \perp \epsilon_{i,s} \quad \forall t \neq s$$

The flexible form of expectation formation in (1) together with the two assumptions summarize the Generic Learning Framework. One can fully recover agents' expectations if $F(.)$ and $H(.)$ are known and $\{X_\tau, S_{i,\tau}, \epsilon_{i,\tau}\}_{\tau=0}^{t}$ and $\Theta_{i,0}$ are observable[17].

[17] One do not need to observe $\{\Theta_{i,\tau}\}_{\tau=1}^{t}$ as they can be derived from function $H(.)$, $F(.)$ and history of

2.2 Econometrician's Problem

Econometricians don't have all the information endowed by agents. In econometrician's problem, $\epsilon_{i,t}$ and $\Theta_{i,t}$ are typically unobservable. Furthermore, econometricians also don't have information on the functional form of $H(.)$ and $F(.)$. Denote the observable signals as $Z_{i,t} = \{X_t, S_{i,t}\}$, the econometrician only observes signals $\{Z_{i,\tau}\}_{\tau=0}^{t}$ and households' expectations $Y_{i,t+1|t}$.

The goal of an econometrician is to evaluate the impact of observable signals on the household's expectations. In standard learning literature, this is achieved by making structural assumptions on the expectation formation process, for example the functional forms of $F(.)$ and $H(.)$, and estimate the average marginal effect of signals or structural parameters through parametric methods. The findings from this approach are model-specific and prone to model misspecification. An alternative way to estimate the average marginal effect is through estimating the Average Structural Function (ASF) without imposing assumptions on the form of $F(.)$ and $H(.)$. Then one can use the ASF as a nuisance parameter to estimate the average marginal effect.

Average Structural Function The ASF follows from Blundell and Powell (2003). In my case the dependent variable is household expectation $Y_{i,t+1|t}$, independent variables are observed signals $\{Z_{i,\tau}\}_{\tau=0}^{t}$ and unobserved error term is $\epsilon_{i,t}$. With strict exogeneity between independent variables and unobserved errors, ASF is the counterfactual conditional expectation of dependent variable $Y_{i,t+1|t}$ given the signals $\{Z_{i,\tau}\}_{\tau=0}^{t}$. It is obtained by integrating out the unobserved i.i.d noise $\epsilon_{i,t}$:

$$
\begin{aligned}
y_{i,t+1|t} &\equiv \mathbb{E}_{\{\epsilon_{i,\tau}\}_{\tau=0}^{t}}[Y_{i,t+1|t}] \\
&= \int G(Z_{i,t}, \epsilon_{i,t}...) d\mathcal{F}_{\epsilon}(\{\epsilon_{i,\tau}\}_{\tau=0}^{t}) \\
&= \int F(H(\Theta_{i,t-1}, Z_{i,t}, \epsilon_{i,t})) d\mathcal{F}_{\epsilon}(\{\epsilon_{i,\tau}\}_{\tau=0}^{t}) \qquad (4)
\end{aligned}
$$

In (4), function $\mathcal{F}_{\epsilon}(.)$ is the joint CDF of all the past noise $\{\epsilon_{i,\tau}\}_{\tau=0}^{t}$. With the independence assumption 2, the ASF is equivalent to counterfactual conditional expectation function

signals. In this sense $\Theta_{i,t}$ can be treated as part of the functional form of $H(.)$ and $F(.)$.

$\mathbb{E}[Y_{i,t+1|t}|\{Z_{i,\tau}\}_{\tau=0}^{t}]$.

It is immediately worth noting that the ASF can offer insight into the underlying model $G(.)$, $F(.)$ and $H(.)$ (the expectation formation process employed by agents in this case). For example, if both updating and forecasting steps follow a linear rule so that $F(.)$ and $H(.)$ are linear functions. The ASF will be linear in $Z_{i,t}$ as well. On the contrary, if the estimated ASF is highly non-linear, it suggests non-linearity in the expectation formation process.

As economists, we want to first learn features of agents' expectation formation model under the generic formulation, in this case, the structural function $G(.)$, with information we have. We then want to assess how signals affect households' expectations. In nonparametric methods, the ASF can be seen as a summarization of the structural functions $G(.)$, and a finite-dimensional measure of the ASF is useful to understand the properties of these structural functions. In particular, the "average derivative" of ASF can be an important measure for the marginal effects of input variables. In this paper, I define such a derivative as the average marginal effect of signals on expectations. The goal now is to estimate the ASF and the average marginal effect of the Generic Learning Framework.

3 Methodology

The estimation of Average Structural Function in forms of (4) is difficult. Under no further assumptions on updating and forecasting steps, $F(.)$ and $H(.)$ are unknown and possibly non-linear. Furthermore, the latent variable $\Theta_{i,t}$ is not directly observable, so its dimensionality is unknown.

In standard learning literature, these problems can be solved by parametric assumptions on structural function. For example, in models with an explicit form on forecasting and updating steps, such as stationary Kalman Filters, $F(.)$ and $H(.)$ are parametric functions. Parametric methods can be applied to the reduced form relation between expectational variables and signals. This method will lead to a "best estimate" of ASF within the models that satisfy the parametric assumptions. In this paper, I take an alternative approach to directly estimate the ASF with a nonparametric method – Recurrent Neural Network. Then using the estimated ASF as a first-stage nuisance parameter, I construct a second-stage DML estimator of the average marginal effect following Chernozhukov *et al.* (2018). I start by introducing the RNN approach to estimate the Average Structural Function directly.

3.1 Estimate Average Structural Function with RNN

To estimate the ASF (4), I need a method that can capture the mapping from observed signals $\{Z_{i,t}\}$ to expectational variables flexibly. Artificial Neural Networks are known for their ability to approximate any functional forms between input and output variables. Such a property is implied by the Universal Approximation Theorem addressed in Hornik *et al.* (1989), which suggests a single layer neural network with sigmoid activation function can approximate any continuous function. However, the most popular Feedforward Neural Networks do not fit the problem well because of its inability to model time dependency between output variables and past input variables induced by the dynamic structure described before. To better fit this empirical framework, Recurrent Neural Networks are used.

RNN are neural networks designed to model time-dependency between input and output variables. When a dynamic system describes the mapping between input and output variables, it is shown by Schäfer and Zimmermann (2006) that RNN can approximate the dynamic system of any functional form arbitrarily well. This is usually referred to as the Universal Approximation Theorem for RNN. To justify that RNN can approximate the ASF of the Generic Learning Framework arbitrarily well, I need to show that the ASF (4) takes the form of a dynamic system considered by this Universal Approximation Theorem. For the ASF to be represented in form of such a dynamic system, I need the assumptions 1 and 2 that $\Theta_{i,t}$ is a Markov Process and $\epsilon_{i,t}$ is i.i.d across individual and time. Theorem 1 shows that the ASF (4) can be well-approximated by a dynamic system of equations with a finite dimensional $\theta_{i,t}$. This justifies why Recurrent Neural Networks can be used to estimate the ASF (4).

Theorem 1. *For any dynamic system described in* (2) *and* (3)*, with assumptions 1 and 2 hold, input vector* $Z_{i,t} \in \mathbb{R}^s$*, where* $s = d_1 + d_2$*, and output vector* $Y_{i,t+1|t} \in \mathbb{R}^l$*. Denote the average structural function* (4) *as:*

$$y_{i,t+1|t} \equiv g(\{Z_{i,\tau}\}_{\tau=0}^{t}, \theta_{i,-1}) \tag{5}$$

There exists a finite dimensional $\theta_{i,t} \in \mathbb{R}^d$*, a continuous function* $f : \mathbb{R}^d \to \mathbb{R}^l$ *and a* *measurable function $h : \mathbb{R}^s \times \mathbb{R}^d \to \mathbb{R}^d$ s.t. the average structural function described in* (4) *can be written as a dynamic system:*

$$\begin{aligned} y_{i,t+1|t} &= f(\theta_{i,t}) \\ \theta_{i,t} &= h(\theta_{i,t-1}, Z_{i,t}) \end{aligned} \tag{6}$$

Notice equation (5) is an alternative representation of ASF (4). In (5) the inputs of function $g(.)$ are the history of observed signals $\{Z_{i,\tau}\}_{\tau=0}^{t}$ and the initial levels of θ at time $t = 0$, $\theta_{i,-1}$. The unobserved noise $\epsilon_{i,t}$ are integrated out and the information contained in hidden states $\Theta_{i,t}$ is captured by the construction of $\theta_{i,t}$. The proof of Theorem 1 can be found in **Appendix A**.

Following theorem 2 in Schäfer and Zimmermann (2006), Recurrent Neural Network (RNN) is the universal approximator of the dynamic system in forms of (6).[18] Theorem 1 then implies I can use a state-of-art RNN with Rectifier Linear (ReLu) activation function to approximate the ASF (4) derived from the Generic Learning Framework.[19] Now denote the class of functions in RNN $\mathcal{G}_{foh}^{RNN}$, the estimator is computed by minimizing the sample mean squared errors:

$$\hat{g}_{rnn} := \underset{g_w \in \mathcal{G}_{foh}^{RNN}}{\arg\min} \sum_{i,t} \frac{1}{2} \Big(Y_{i,t+1|t} - g_w(\{Z_{i,\tau}\}_{\tau=0}^{t}, \theta_{i,-1}) \Big)^2$$

In Theorem 1 the alternative representation (5) also shows with the same realization of $Z_{i,t}$, $y_{i,t+1|t}$ may differ at different point of time. Moreover, such a difference comes from the accumulation of signals they see, $\{Z_{i,\tau}\}_{\tau=0}^{t}$ rather than the underlying structural functional forms $f(.)$ and $h(.)$. In other words, such a flexible formulation allow for endogenous time-varying marginal effect of signals $Z_{i,t}$. This point will become more clear when I introduce average marginal effect.

[18] According to the Universal Functional Approximation Theorem (See Hornik *et al.* (1989) for the results for Feed Forward Networks and Schäfer and Zimmermann (2006) for Recurrent Networks), a single layer neural network with sigmoid activation function can approximate any continuous function. The result is extended to nerual networks with Rectifier Linear (ReLu) activation function by Sonoda and Murata (2015).

[19] The RNN approximate dynamic systems (6) by constructing representations of $\theta_{i,t}$ as well as $f(.)$ and $h(.)$.

3.2 Estimate Average Marginal Effect with DML

Now I turn to the other object of interest: the average marginal effect of a particular signal. This is the mean of gradient for Average Structural Function $g(\{Z_{i,\tau}\}_{\tau=0}^{t}, \theta_{i,-1})$:

$$\beta = \mathbb{E}[\nabla g(\{Z_{i,\tau}\}_{\tau=0}^{t}, \theta_{i,-1})] \tag{7}$$

Or for a single signal $z_{j,i,t}$ which is the j-th element in vector $Z_{i,t}$, this can be written as:

$$\beta^{j} = \mathbb{E}[\frac{\partial g(\{Z_{i,\tau}\}_{\tau=0}^{t}, \theta_{i,-1})}{\partial z_{j,i,t}}] \tag{8}$$

The equation (7) can be thought of as a moment condition used to estimate β. With the functional estimator obtained from RNN, a plug-in estimator of β is available by computing the sample mean of the partial derivative using estimator of conditional expectation function: $\mathbb{E}_n[\nabla \hat{g}_{rnn}(\{Z_{i,\tau}\}_{\tau=0}^{t}, \theta_{i,-1})]$. However, such an estimator typically has two problems: (1) when regularization is used in RNN, which is the case here, the estimate using moment condition (7) is usually biased; (2) the functional estimates obtained by Machine Learning (RNN in this case) methods typically have slower than $\sqrt{n}$ convergence speed. This makes the estimate not well-behaved asymptotically, thus making inference hard.[20]

One way to solve these problems is to use the DML method as proposed by Chernozhukov *et al.* (2018) and Chernozhukov *et al.* (2017). I can form the estimation problem as a semi-parametric moment condition model with a finite-dimensional parameter of interest, β; infinite-dimensional nuisance parameter η (including functional estimator from Machine Learning methods, $\hat{g}_{rnn}$ in this case), and a known moment condition $\mathbb{E}[\psi(W; \beta, \eta)]$. The benefits of this approach are two folds, it first corrects for biases in the estimator, and it also offers a way to obtain valid inference on the estimator. The plug-in estimator is usually biased and not asymptotically normal because the construction of the estimator of β involves the regularized nuisance parameters obtained by Machine Learning methods (in this case RNN). This Machine Learning estimator usually has a convergence speed slower than $\sqrt{n}$ and makes the estimator on β exploding as sample size goes to infinity. Using orthogonalized moment conditions solves this problem because the moment conditions used

[20] These issues are well discussed in Chernozhukov *et al.* (2018), they also propose ways to solve these problems. One way they proposed is the DML approach, which is what I follow to estimate the average marginal effect in this paper.

to identify β are locally insensitive to the value of the nuisance parameter. This allows me to plug in noisy estimates of these parameters obtained from RNN.

The estimator $\hat{\beta}$ is then $\sqrt{n}$ asymptotic normal under appropriate assumptions on estimate of nuisance parameter $\hat{\eta}$ and the moment condition. These conditions typically require the moment condition to be (Near) Neyman Orthogonal; function $\psi(.)$ to be linearizable and a fast enough convergence speed of nuisance parameter.[21]

The convergence speed requirement for Neural Networks with ReLu activation functions is verified in Farrell *et al.* (2018). Then following the concentrating-out approach in Chernozhukov *et al.* (2018), I can derive the Neyman Orthogonal Moment Condition for β^j:

$$\mathbb{E}[\beta^j - \frac{\partial g(\{Z_{i,\tau}\}_{\tau=0}^{t}, \theta_{i,-1})}{\partial z_{j,i,t}} + \frac{\partial ln(f_z(\{Z_{i,\tau}\}_{\tau=0}^{t}, \theta_{i,-1}))}{\partial z_{j,i,t}}(Y_{i,t+1|t} - g(\{Z_{i,\tau}\}_{\tau=0}^{t}, \theta_{i,-1}))] = 0 \tag{9}$$

The nuisance parameters associated with moment condition (9) then include both the average structural function $g(.)$ as well as the joint density function $f_z(\{Z_{i,\tau}\}_{\tau=0}^{t}, \theta_{i,-1})$. One complication here is the joint density function could be high-dimension, and it includes both current and past signals. Here I make an extra assumption that the signal Z follows a VAR(1) so that to get the estimate of the partial derivative of log density, I only need to estimate the joint density of $f_z(Z_{i,t}, Z_{i,t-1})$. The joint density is then obtained using higher-order multivariate Gaussian Kernel Density Estimation with bandwidth chosen according to Silverman (1986) to guarantee the appropriate convergence speed of the density estimator.

The estimator of β^j is obtained by the following steps:

1. Estimate nuisance parameter $\eta = \{g, f_z\}$. g is estimated by RNN and f_z is estimated by Gaussian Kernel Density Estimation. Denote the estimates as $\hat{g}_{rnn}$ and $\hat{f}_z$ respectively.

2. Obtain estimate of average structural function from computing derivative numerically:

$$\frac{\partial \hat{g}_{rnn}}{\partial z_{j,i,t}} = \lim_{\delta \to 0} \frac{\hat{g}_{rnn}(Z_{i,t} + \Delta_j/2, \{Z_{i,\tau}\}_{\tau=0}^{t-1}, \theta_{i,-1}) - \hat{g}_{rnn}(Z_{i,t} - \Delta_j/2, \{Z_{i,\tau}\}_{\tau=0}^{t-1}, \theta_{i,-1})}{\delta}$$

   Where $\Delta_j \in \mathbb{R}^s$ is a vector of zeros, with jth element being δ.

[21] For the formal formulation of semi-parametric moment condition model, derivation of Neyman Orthogonality condtion and convergence speed requirements of nuisance parameter, refer to **Appendix B**

3. The estimate of $\frac{\partial ln(\hat{f}_z(Z_{i,t},Z_{i,t-1}))}{\partial z_{j,i,t}}$ is obtained similarly using numerical derivatives.

$$\frac{\partial ln(\hat{f}_z(\{Z_{i,\tau}\}_{\tau=0}^{t},\theta_{i,-1}))}{\partial z_{j,i,t}} = \lim_{\delta \to 0} \frac{\hat{f}_z(Z_{i,t}+\Delta_j/2, Z_{i,t-1}) - \hat{f}_z(Z_{i,t}-\Delta_j/2, Z_{i,t-1})}{\delta \hat{f}_z(Z_{i,t}, Z_{i,t-1})}$$

4. Then the DML estimate is given by:

$$\hat{\beta}^j = \frac{1}{N}\sum_i \frac{1}{T}\sum_t \underbrace{[\frac{\partial \hat{g}_{rnn}(\{Z_{i,\tau}\}_{\tau=0}^{t},\theta_{i,-1})}{\partial z_{j,i,t}} - \frac{\partial ln(\hat{f}_z(\{Z_{i,\tau}\}_{\tau=0}^{t},\theta_{i,-1}))}{\partial z_{j,i,t}}(Y_{i,t+1|t} - \hat{g}_{rnn}(\{Z_{i,\tau}\}_{\tau=0}^{t},\theta_{i,-1}))]}_{\equiv \hat{\beta}^j_{i,t}}$$

4 Application to Survey Data

In this section, I use survey data of expectation and a rich set of macroeconomic signals to estimate the Average Structural Function of the Generic Learning Framework. There is a growing literature using survey data to estimate learning models. For example, Coibion and Gorodnichenko (2015) and Andrade and Le Bihan (2013) use survey data in the framework of the noisy information model and information rigidity models; and Malmendier and Nagel (2015) use survey data to estimate a Least Square Learning model with a time-varying decay of macroeconomic signals observed.

The respondents in the surveys that researchers use are different as well. The most widely explored expectations are those from households and professionals. In this paper, I focus on households' expectations from the US, and I use professional forecasts as a signal that households can utilize to form their expectations.

4.1 Data Description

Table 1 summarizes the data on expectation and signals used to estimate generic learning model as well as the notations being used.

For outcome variable $Y_{i,t+1|t}$ I use Reuters/Michigan Survey of Consumers (MSC). It is a monthly survey for a representative sample of US households with a preliminary interview usually conducted at the beginning of the month. The survey asks about the respondent's

Table 1: Data Description: some key notations

Input variable ($X_t, S_{i,t}$)	Variable and Notation	Source
Macro variable	CPI: π_t, unemployment: Δu_t, Federal Funds Rate: r_t, real GDP growth: $\Delta rgdp_t$, Real Oil price: o_t Stock price index: $stock_t$	FRED
Professional Forecasts	CPI: $F_t \pi_{t+1}$, unemployment change: $F_t \Delta u_{t+1}$, short term Tbill: $F_t \Delta r_{t+1}$, real GDP growth: $F_t \Delta rgdp_{t+1}$ anxious index: $F_t rec_{t+j}$	Survey of Professional Forecasters (Philadelphia FED)
Individual Signals	regional CPI: $\pi_{i,t}$, regional unemployment: $\Delta u_{i,t}$ news on recession: $Nrec_{i,t}$ news on inflation: $N\pi_{i,t}$ news on boom: $Nboom_{i,t}$ news on interest rate: $Nr_{i,t}$	Bureau of Labor Statistics, LexisNexis Uni
Individual Lag Expectation	inflation rate: $\hat{\pi}_{i,t\|t-1}$ change of economic condition: $\Delta \hat{y}_{i,t\|t-1}$ unemployment change: $\Delta \hat{u}_{i,t\|t-1}$ interest rate change: $\Delta \hat{r}_{i,t\|t-1}$	Michigan Survey of Consumers
Output variable ($Y_{i,t+1\|t}$)	**Variable and Notation**	**Source**
Expectational Variable	inflation rate: $\hat{\pi}_{i,t+1\|t}$ change of economic condition: $\Delta \hat{y}_{i,t+1\|t}$ unemployment change: $\Delta \hat{u}_{i,t+1\|t}$ interest rate change: $\Delta \hat{r}_{t+1\|t}$	Michigan Survey of Consumers

one-year-ahead expectation on various macroeconomic aspects. In this paper I include four expectational variables of interest: (1) expected inflation rate, denoted as $\hat{\pi}_{i,t+1|t}$; (2) whether economic condition will be better, denoted as $\Delta\hat{y}_{i,t+1|t}$; (3) whether unemployment rate will increase, denoted as $\Delta\hat{u}_{t+1|t}$; (4) whether interest rate will increase $\Delta\hat{r}_{t+1|t}$.

I include two sets of public signals X_t. One is the realized economic statistics from the Federal Reserve of St. Louis. These signals contain information about the current state of the economy. In the adaptive learning literature, agents rely only on (the history of) this information to form forecasts. Another set of public signals I consider are the professional forecasts from the Federal Reserve of Philadelphia. These signals are considered as containing information about the future because they usually lead and Granger-Cause the predicted macroeconomic variables.[22]

Then in individual-level signals $S_{i,t}$, I include local unemployment rate and CPI inflation matched with the individual in MSC according to their location information. I also include the intensity of news story reports on recessions, inflation and interest rates at both local and national level.[23] The idea that information about future flows from professional forecasts to households through media reports can be dated back to Carroll (2003) and has lots of follow-up researches.[24] I include the news measure as RNN allows for interaction between input variables, so the transmission of information can also be captured. I also include the lagged expectations of households as extra inputs. The assumption that observational noise is uncorrelated across time guarantees the lagged expectation won't be correlated with the unobserved error term $\epsilon_{i,t}$.

Because the panel component of MSC only has two waves for each individual, whereas capturing the latent state accumulated by observing the history of signals requires a longer time dimension. For this reason, the data set is compiled as a synthetic panel. Each synthetic agent is grouped by its social-economic status, including income quantile, region of living, age and education level, which are the four characteristics found significantly affecting

[22]See Carroll (2003) for details

[23]I scraped volume of reports on related macroeconomic topics from TV news scripts and local newspaper articles. Following PFAJFAR and SANTORO (2013) I construct a measure of news coverage on these topics by computing the number of news stories on each topic (for example, news about inflation) in each quarter as a fraction of total news stories in the same quarter, and I include only news with more than 120 words to exclude short reviews or notice. The data is available from LexisNexis Database.

[24]See PFAJFAR and SANTORO (2013) and LAMLA and MAAG (2012) for examples.

expectation by Das *et al.* (2019). The baseline sample I am using is quarterly from 1988 quarter 1 to 2019 quarter 1. The length of the sample is due to the availability of data on news stories.[25] The frequency of data is quarterly because professional forecasts are quarterly data.

4.2 Results

Estimation of functions with RNN usually requires selection of network architecture. Because of the superior performance in applications of modern neural networks, I choose Rectified Linear (ReLu) Activation functions for all the layers in RNN and use Long-Short Term Memory (LSTM) recurrent layer. It is worth noting the requirements for convergence speed offered by Farrell *et al.* (2018) are also for neural networks with ReLu activation functions, and the width (number of neurons) and depth in my baseline architecture of RNN satisfy these requirements. The rest configurations of hyper parameters are chosen using a standard K-Fold Cross Validation, in my case $K = 6$.[26] Table 2 summarizes the architecture of RNN I use.

[25] Prior to 1988, there are too few local published newspapers included in LexisNexis Database.

[26] I also tried RNN with smaller width and no regularization (dropout) as well as more complex architectures, the results don't change qualitatively. To assess the stability of the neural networks I also tried with multiple random initial weights and the results are stable across different initial weights used.

Table 2: Architecture RNN

Tuned Hyper Parameter	Configuration
Num. of Recurrent Neurons	32
Feed-forward Neurons	20
Dropout on recurrent layer	0.5
Epochs	200
Learning Rate	$1e^{-6}$
Depth	2(4)
Un-tuned Hyper Parameter	Configuration
Type of Recurrent Layer	Long-Short Term Memory (LSTM)
Activation Function:	ReLu

* Tuned hyper parameters are picked using 6-Fold cross-validation across individuals. There is 1 layer of recurrent neurons that are connected to 1 layer of feed-forward neurons. Because each one LSTM layer contains 3 layers of neurons, this makes the actual depth of network being 4. It is worth noting such depth satisfies the requirement for fast enough convergence of estimated Average Structural Function so that functional estimators from this Neural Network can be used to obtain inference on DML estimators.

It is important to note the estimated ASF has a 4-dimensional output, and more than 20 inputs are considered. The ASF and marginal effects can be presented in each signal-expectation pair. In this paper, I will only focus on the impact of signals on expectations regarding the same subjects, which I refer to as "self-response". For example, I will look at the impact of the realized unemployment rate on unemployment expectations for the future. Another interesting direction is to examine "cross-response", for example, how signals on inflation affect unemployment expectation. This will help us to understand how households believe different economic aspects interact with each other. Results on these topics are documented in detail in my other work Hou (2020) thus are not included in this paper.

Because the estimation procedure described in **Section 3** involves several steps, in this subsection, I present results progressively following those steps. I first show the estimated Average Structural Function from the baseline RNN described in Table 2. Then I present the time-varying marginal effects of macroeconomic signals implied by the estimated ASF to illustrate the key finding that households are adaptive learners in ordinary periods and become more "forward-looking" when economic conditions worsen. I interpret this finding as an "attention-shift" of households from signals about the past and current state of the economy to signals that contain information about the future. Then I obtain the DML estimator of marginal effects with inference and perform tests to show that such an "attention shift" is statistically significant. Finally, I explore reasons for the "attention shift" by doing a decomposition of the time-varying marginal effects of interest. The identified key driving forces are then used in the rational inattention model I proposed to rationalize findings from RNN.

4.2.1 Estimated Average Structural Function

First, it is worth examining the estimated Average Structural Function. As the ASF in (4) is a complex object with high-dimensional input and 4-dimension output, it is hard to visualize such a function in all possible dimensions. I decide to focus on presenting expectations as a function along one dimension of the input signal as a starting point because it serves as a foundation to understand the results presented in this section. Before I plot the function in a two-dimension space, it's useful to define the estimated function in that space. Denote the signal considered in the input dimension as x_t, and the one dimensional output is the expectational variable on the same subject, denoted as $E_t x_{i,t+1}$. Then use $Z_{i,t}^{-x}$ to represent contemporaneous signals other than x_t. Following from (6), the estimated functional estimator can be expressed as the following function:

$$E_t x_{i,t+1} = \hat{g}_x(\theta_{i,t-1}, Z_{i,t}^{-x}, x_t) \tag{10}$$

Now take unemployment as an example subject. Figure 1 plots the average structural function of expected probability for unemployment rate increase, along the signal on change of actual unemployment rate. Following (10), this function can be written as:

$$E_t \Delta u_{t+1} = \hat{g}_u(\theta_{i,t-1}, Z_{i,t}^{-u}, \Delta u_t) \tag{11}$$

In Figure 1, the function (11) is plotted at three different points of time: quarter 2,3 and 4 in 2016, and different from each other. However, such a difference is not due to estimated functional form $\hat{g}_u(.)$ is different across time, but because of different inputs of $\theta_{i,t-1}$ and $Z_{i,t}$[27]. This means at a different point of time, households may form different expectations

[27] Given that they are close to each other in time (should have similar hidden state accumulated) and

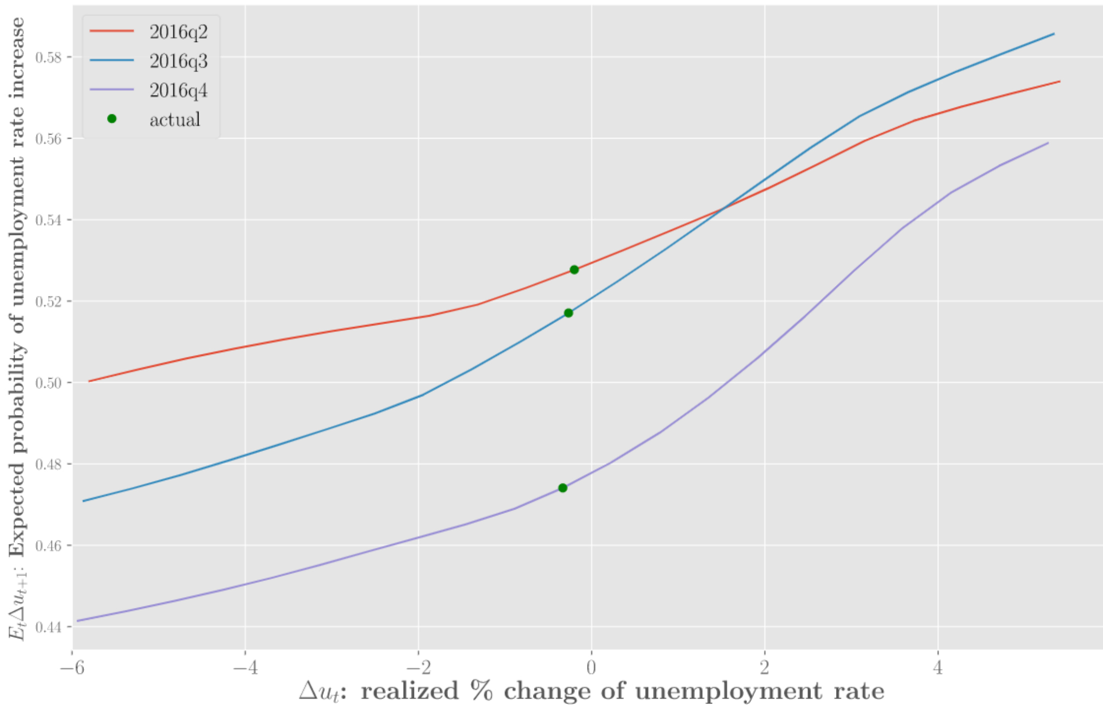


Figure 1: Average of expected probability for unemployment rate increase $E_t\Delta u_{t+1}$ as function of realized unemployment rate change Δu_t, at different point of time. Purple curve: 2016q4, blue curve: 2016q3, red curve: 2016q2. The dot on each curve represents the prediction from estimated function when actual data in that period is input.

in response to the same signal on the realized unemployment rate change. However, such a difference comes from either the hidden states ($\theta_{i,t-1}$) they accumulated from observing a different path of signals or the interactions between newly observed signals $Z_{i,t}$. In other words, any state-dependency I find with the estimated ASF is endogenously resulted from the signals households observed. This is a crucial implication of the model that comes from the flexibility of the Generic Learning Framework and RNN method.

Then I turn to the properties of estimated ASF. The first thing to notice is that it is highly non-linear. This is common to all three points of time despite the potential difference between hidden states and covariates. In particular, the slope of ASF changes in three regions. First, when actual changes in the unemployment rate are negative and big in absolute value, for example, less than 0% or lower in Figure 1, the slope of ASF is relatively

current Δu_t is roughly at the same level. The primary reason for the level difference here is that the lag expectation $E_{t-1}\Delta u_t$ was higher in 2016q2 and q3. The fact that expected unemployment is gradually falling illustrates how expectation is slowly adjusting downwards when the actual unemployment rate keeps falling($\Delta u_t < 0$) throughout the three quarters plotted.

flat. This means households are less sensitive to this information, which can be considered as "good" news. The slope gets steeper and steeper in the region $0\% < \Delta u_t < 2\%$, which means agents' expectations become more responsive to the actual unemployment status when the unemployment rate increases. Finally, when the unemployment rate increased too much that Δu_t becomes higher than 3% or more, the slope of ASF becomes flat again.

A second important observation of Figure 1 is that the ASF is asymmetric. Take 2016 quarter four as an example, which is the purple curve in Figure 1. In that quarter, the unemployment rate decreased by around 0.4%, and on average householdes expect the unemployment rate to increase a year from now by probability 0.45, keeping other signals (and the history of them) fixed. The curve implies that if unemployment had increased by 1.6% instead of falling by 0.4%, the model predicts households will be 5% more likely to believe unemployment will increase in the future. However, if unemployment decreased further by 2.4%, households will only be 3% less likely to expect the unemployment rate to go up. This implies households may be more sensitive to unfavorable news, which is the unemployment rate increase in this case. Such a pattern will not be seen in a linear model[28]if the underlying expectation formation model is linear in signals, the ASF will be linear as well.

A final point to notice is because of the time-variation of latent state $\theta_{i,t-1}$ and covariates $Z_{i,t}$, the slope of ASF becomes time-dependent. This gives rise to the time-varying marginal effects of signals. I will discuss the details on this in **Section 4.2.2**.

Now I have showed you the estimated ASF is non-linear and asymmetric with fixed input $\{Z_{i,\tau}\}_{\tau=0}^{t}$, as it is still an estimated object it's useful to get a sense of how significant these patterns are. To achieve that I turn to estimate average deviations of expectation and obtain valid inference using DML as described in **Appendix B.1**.

$$\gamma_\delta = \mathbb{E}[g(Z_{i,t} + \delta, \{Z_{i,\tau}\}_{\tau=0}^{t-1}, \theta_{i,-1}) - g(Z_{i,t}, \{Z_{i,\tau}\}_{\tau=0}^{t-1}, \theta_{i,-1})] \tag{12}$$

The average deviation is defined in equation (12), it describes the average (across $\{Z_{i,\tau}\}_{\tau=0}^{t}$) change of expectational variable when signal $Z_{i,t}$ increase by δ, relative to its original level. As this needs to be done for each output-input pair, I focus on the pairs in which the output expectational variable and input signal variable are on the same subject.

[28]See discussion in Appendix B.4

In Figure 2 I plot the average deviation for unemployment expectation along with the change of unemployment signal as a leading example again. I consider 20 different values of δ symmetrically centered around 0. For each point estimate at δ, I present the 95% confidence interval. It shows the average deviation exhibits the same curvature as the shape of the estimated ASF presented in Figure 1. It indicates the responsiveness of expectation on unemployment status to realized unemployment signal is relatively weak when the unemployment rate falls or increases by a large magnitude. Meanwhile, the expectation is most sensitive to the unemployment signal when it increases but by smaller magnitudes. The confidence interval shows the asymmetry is significant. With a positive change to Δu_t, expected unemployment will increase more in absolute value comparing to how much it decreases in response to a negative change of Δu_t with same magnitude, and such a difference is statistically significant.

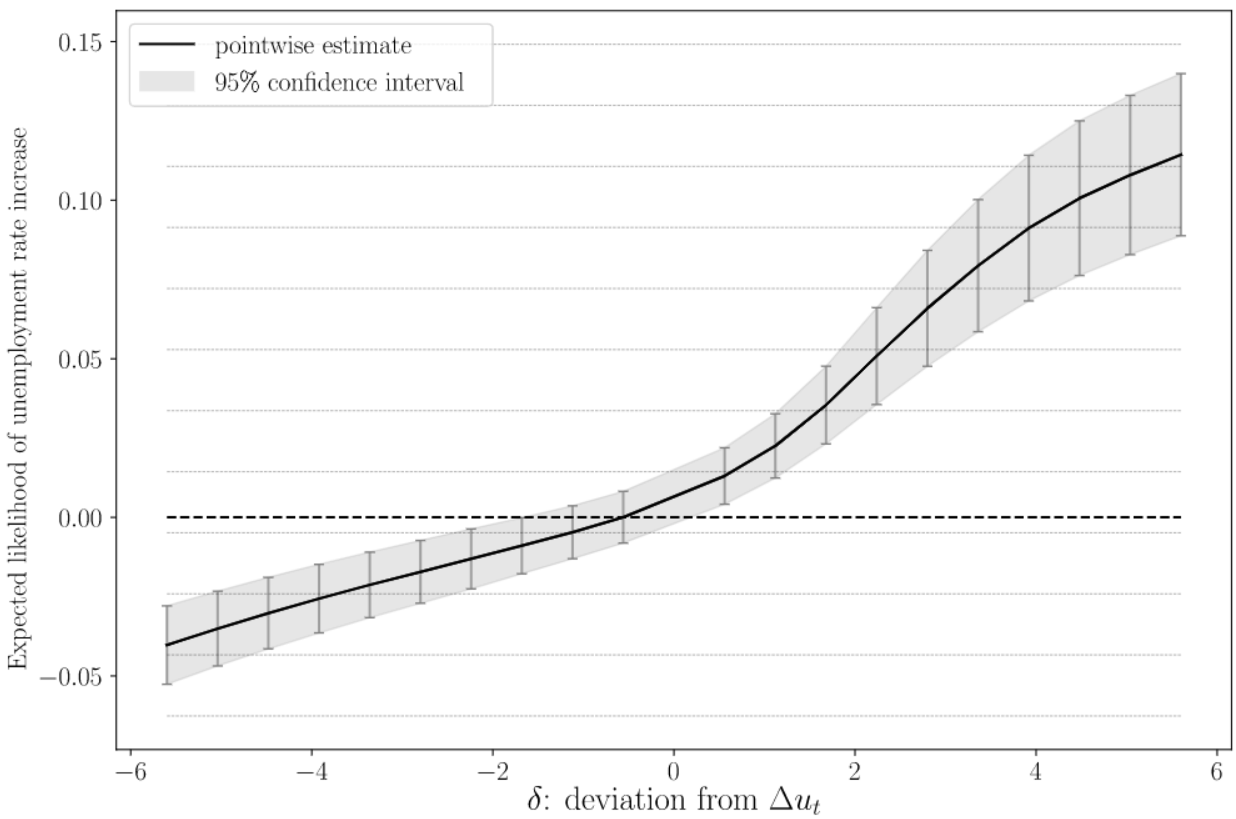


Figure 2: Average change of unemployment expectation $E_t \Delta u_{t+1}$, when unemployment signal Δu_t change by δ. This is obtained by point-wise estimation of (12) at 20 different points of δ, following the Double-Debiased Machine Learning approach from Chernozhukov *et al.* (2018), 95% confidence band is reported at each point-wise estimate. The estimate is depicted as solid black line, shaded area is implied 95% region.

If I look at the average deviation for all four expectational variables and again focus on the "self-response". I find such a non-linearity shows up consistently in cases of unemployment expectation and economic condition expectation. This can be seen from the comparison between panel (a) and (b) in Figure 3. In panel (b) when Δy falls drastically, the slope of ASF becomes flat, the same as the case when the unemployment signal is high in panel (a). Then it gets steeper as δ becomes closer to 0, and gets flat again when δ keeps increasing and becomes positive. On the other hand, in panel (c) and (d) of Figure 3, which correspond to inflation and interest rate expectation as functions of inflation and interest rate signal, the relationships are closer to linear.

These observations lead to two major patterns among all the findings in my application of RNN to survey data: (1) findings are most stark in cases with expectations on economic condition (e.g. unemployment change $E_t \Delta u_{i,t+1}$ and economic condition change $E_t \Delta y_{i,t+1}$), and these results are consistent between these two measures. One can think of unemployment (expectation or signal) as a negative counterpart of economic condition/RGDP. (2) findings on expected inflation and interest rate are more consistent with those from existing literature. These patterns also hold for my later findings on time-varying and average marginal effects. For these reasons, I will focus on presenting results with the expected economic condition, $E_t \Delta y_{i,t+1}$, from now on. For results on other three expectational variables I include the results in **Appendix C.1**.

4.2.2 State-dependent Marginal Effect

Following from the estimated ASF in (10), I can define the average (across individual) time-specific marginal effect of signal x on expectational variable Ex as:

$$\beta_{x,t}^{Ex} = \mathbb{E}_n [\frac{\partial \hat{g}_x(\theta_{i,t-1}, Z_{i,t}^{-x}, x_t)}{\partial x_t}] \tag{13}$$

This marginal effect is different at each point of time t for the same reason as discussed in **Section 4.2.1**: different internal state $\theta_{i,t-1}$ and contemporaneous signal $Z_{i,t}$. It describes on average how responsive the expectation $E_t x_{t+1}$ is to change of signal x_t at time t after observing all signals up to that time. It can then be interpreted as weights applied to signals following the standard learning literature. In the rest of this paper, I will use weights and

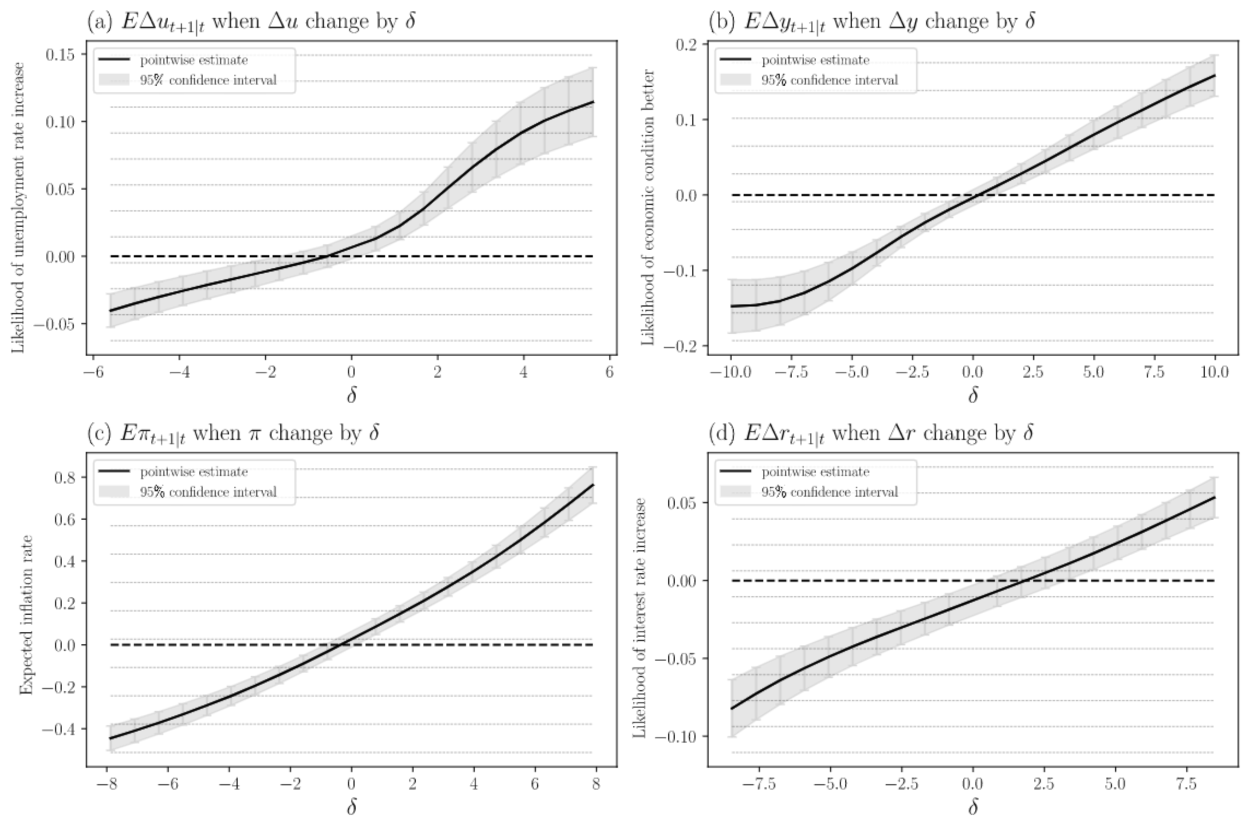


Figure 3: Average deviation of four expectational variables in response to signals on themselves. Panel (a): expected likelihood of unemployment increase as unemployment signal change by δ. Panel (b): expected likelihood of economic condition be better as real GDP signal change by δ. Panel (c): expected inflation rate as inflation signal change by δ. Panel (d): expected likelihood of interest rate increase as interest rate signal change by δ.

marginal effects interchangeably. If the underlying learning model doesn't feature endogenous states or interactions between signals and states, for example, stationary Kalman Filter, this marginal effect will not have a time-varying slope.[29] In this section, I show profound time-variation in the average marginal effect of signals on expectations about the economic condition. Specifically, such a time-variation implies households' attention to signals is cyclical: they put lower weights on signals about current and past states and, at the same time, more weights on signals about future during periods with bad economic conditions. In other words, when agents form expectations about economic conditions, they change from adaptive learners to forward-looking during bad times.

[29] It is closely related to the curvature of estimated ASF presented in the previous section but not related to the level difference. For example, in stationary Kalman Filter, its ASF recovered by RNN may still be different in levels at each point of time.

Before I proceed to these results, it is useful to define two related notions: (1) signal about past and signal about the future; (2) bad times and ordinary times.

Signals about past v.s. future: Following the adaptive learning literature, agents can acquire information about the current state of the economy from macroeconomic statistics. They get this information either directly as it is publicly available or partially through daily activities. I will use realized key macroeconomic variables as a proxy for the signal about the past. Expectations formed majorly relying on this information are then treated as adaptive. For signals about the future, I follow Carroll (2003) and use consensus (average) expectation from the Survey of Professional Forecasters as a proxy. Information about the future can take the form of news or anticipated shocks as in Beaudry and Portier (2006) and Barsky and Sims (2012), and it flows into the household's information set through news media as suggested in Carroll (2003).

Bad time v.s. ordinary time: For periods characterized as "bad time", I take the ones that have at least 2 consecutive quarters with unemployment rate increasing[30]:1990q3-1992q3,2001q1-2002q4 and 2007q3-2010q3. This is because I use a year-to-year change of unemployment rate as the measure of unemployment rate signal, and this measure appears to return to zero 2 to 4 quarters after the day that marks the end of NBER recessions. Using such a characterization shows weights on signal change is related to the signal itself rather than an external definition of "bad period" as it is reasonable to think that households won't have the information on the end date of NBER recessions when they form expectations around the same time[31]. The results will not change qualitatively if I use the NBER recession dates to measure "bad time". These results are included in **Appendix C.2**.

I then present the time-specific marginal effect from (13) of signals on real GDP growth. I consider both signals about the past and future. In Figure 4, the color bars in top panel are the marginal effects of real GDP growth signal, $x_t = \Delta y_t$, on expected economic condition next year; those in bottom panel are the marginal effects of professionals' forecasts about

[30]Notice the unemployment rate change I use, Δu_t is year-to-year unemployment rate change. I pick the quarters that have $\Delta u_t > 0$ with 2 consecutive quarters around it also have $\Delta u_t > 0$

[31]The announcement typically comes out at least 2 quarters after the official end day of NBER recession.

real GDP growth next year, $x_t = F_t \Delta y_{t+1}$, on expected economic condition. Both marginal effects are normalized by standard deviations for ease of comparison.

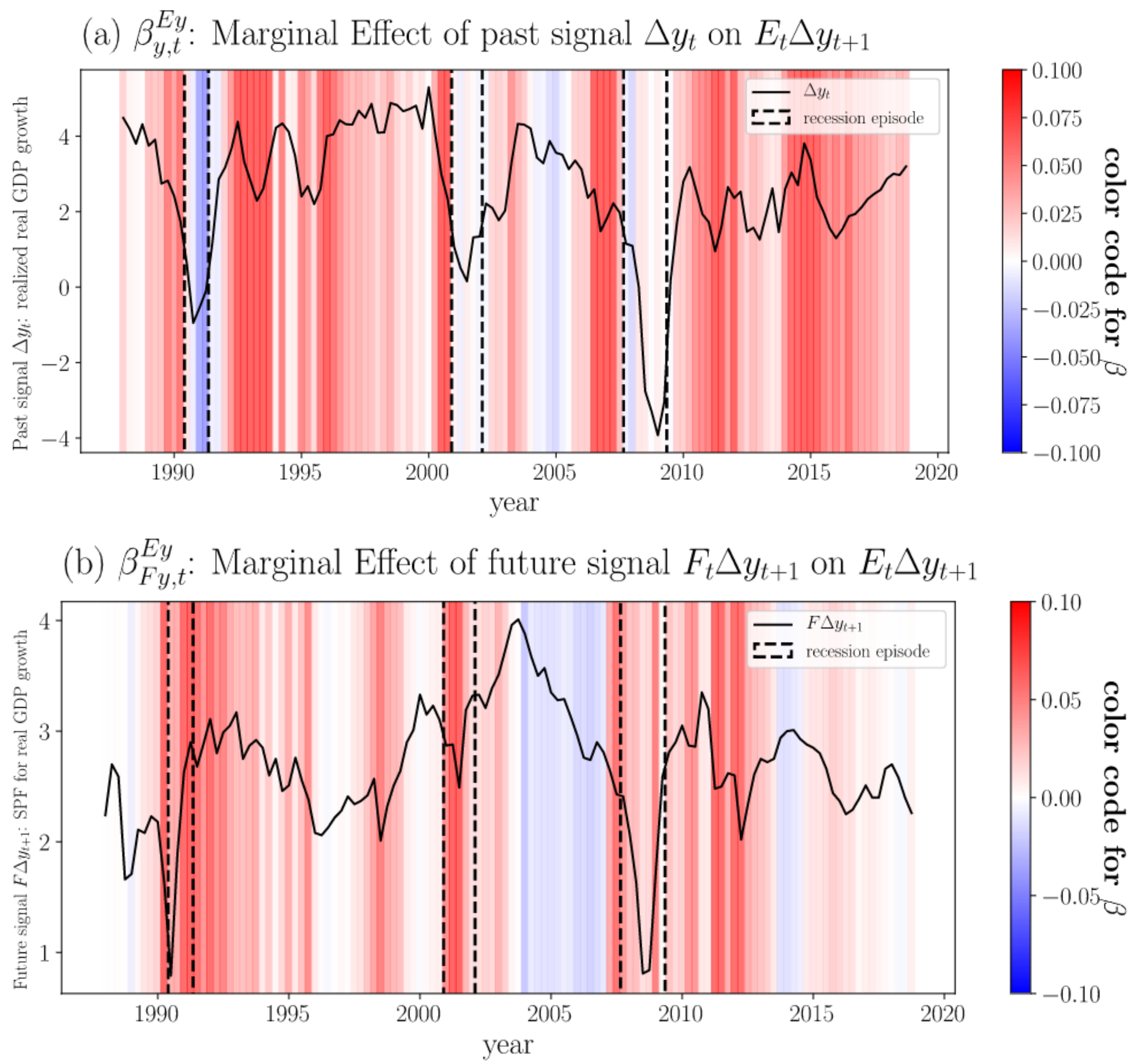


Figure 4: Color bars in panel (a): the marginal effects of real GDP growth signal Δy_t on expected economic condition next year $E\Delta y_{t+1|t}$. Panel (b): the marginal effects of professionals' forecasts about real GDP growth next year $F\Delta y_{t+1|t}$ on expected economic condition. Red color: positive marginal effect; blue color: negative marginal effect. Black solid line: data on frequency of news about recession.

The color bars in each panel stand for the corresponding marginal effect at that point of time. A red color means a positive marginal effect; the blue color means a negative marginal effect, and white means the marginal effect is zero. The color map is on the right side of each panel, and the scale stands for normalized marginal effect. For example, 0.1 on the color map means when signal x_t change by 1 standard deviation, corresponding expectation changes by 0.1 standard deviation. This is then represented by a dark red color bar in the graph. The darker the color, the bigger the magnitude for marginal effect. The solid black line is the series of signal x_t at which I evaluate the marginal effect. The dotted area is the NBER recession episode.

In general, both higher real GDP growth and higher forecasted growth by the professionals make households predict better economic conditions. The maximum of the marginal effect of real GDP growth is 0.24 in 1996 quarter 1, which indicates 1 standard deviation increase of real GDP growth (approximately 1.66%) leads to 0.24 standard deviation increase in expected business condition (on average 0.125 more likely to believe economic condition to be better).

One key observation comes from comparing the top panel to the bottom. In panel (a), the pale color during recession periods in panel (a) suggests that the marginal effect of the past signal is close to zero or negative. In contrast, the red color bars indicate the marginal effects are usually sizeable during non-recession episodes. On the other hand, in panel (b), the patterns for marginal effects on the future signals are the opposite: higher during the recession period than ordinary periods. Such an observation indicates that households are more sensitive to signals about the past during ordinary periods and put more weights on signals about the future when the economic condition gets worse. It is also important to note that it does not necessarily mean they are more pessimistic during bad times because negative or close-to-zero marginal effects do not mean worse expectation on economic conditions, rather it means the expectation is less responsive to the signal considered.

Such a finding is obviously at odds with constant gain learning or noisy information models. In a standard noisy information model with stationary Kalman Filters, the marginal effect across time is fixed and depends only on variances of noise and prior. In constant gain learning models, the marginal effect of signals may be time-varying when the data is limited but will be stabilized as more data is available to the learner. However, the finding here suggests a strong cyclicality of weights that households put on specific signals. It is more consistent with the case that agents shift their attention to signals about the future thus becoming more "forward-looking" during bad times of the economy.

Moreover, such a finding does not only exist in expectation and signals on economic condition Δy, but it also qualitatively holds for expectation and signals on unemployment status Δu. However, there is still a caveat to the evidence I presented in this section: are the differences between marginal effects during ordinary and bad times statistically significant? As I discussed in **Section 3.2**, the naive estimates for marginal effect derived from functional estimations may be biased. To correct the potential biases and obtain estimates on average marginal effects with valid inference, I follow Chernozhukov *et al.* (2018) and obtain the DML Estimator. Then I can perform statistical tests on the difference between marginal effects in bad and ordinary times.

4.2.3 DML Estimator of Average Marginal Effects

To test whether the difference of marginal effects between ordinary and bad periods is statistically significant. I compute the DML Estimator following the procedures described in Section 3.2. Table 3 reports the estimated average marginal effect of past and future signals on expected economic condition and expected unemployment rate change. I separate the time-varying marginal effects into two groups, β_{rec} denotes the average marginal effect during "bad periods" defined in **Section 4.2.2**.[32] And β_{ord} denotes the average marginal effect in periods other than the recession episodes. I then perform a Wald-test on $\beta_{rec} = \beta_{ord}$, the p-value is also reported in the table.

The first thing to notice in Table 3 is that the estimates are consistent with findings in Figure 4, where I use the naive estimate (as in equation (7)) of marginal effect computed at each point time. The signals about the unemployment rate and inflation have a negative impact on households' expectations about the economic condition, whereas signals about real GDP growth and interest rate usually have a positive impact. The opposite is true for expectation on unemployment rate change. Both signals on past/current and future economic indicators are significantly affecting expectations. This suggests households are not complete adaptive learners, and they have access to some information about the future.[33] Moreover, signals on unemployment and real GDP growth have a bigger impact when comparing to signals about the interest rate and inflation.

[32] For the same table using NBER recession dates for "bad periods", refer to **Appendix C.2**.

[33] This is consistent with findings from Barsky and Sims (2012).

Table 3: Average Marginal Effect of Past and Future Signals on Expectation

Expectation:		$E\Delta y_{t+1\|t}$			$E\Delta u_{t+1\|t}$		
	Signal	β_{bad} (std)	β_{ord} (std)	$\beta_{bad}=\beta_{ord}$ (p-val)	β_{bad} (std)	β_{ord} (std)	$\beta_{rec}=\beta_{ord}$ (p-val)
Future Signal	$F_t\Delta u_{t+1}$	**−0.037*****	0.009**	<0.01	**0.029*****	0.007***	<0.01
		(0.004)	(0.002)		(0.003)	(0.002)	
	$F_t\Delta y_{t+1}$	**0.049*****	0.016***	<0.01	**−0.022*****	−0.009***	<0.01
		(0.005)	(0.003)		(0.002)	(0.001)	
	$F_t\Delta r_{t+1}$	0.026***	0.025***	0.92	−0.022***	−0.021***	0.79
		(0.007)	(0.004)		(0.004)	(0.002)	
	$F_t\pi_{t+1}$	0.014***	0.003**	<0.01	−0.008***	0.000	<0.01
		(0.002)	(0.001)		(0.002)	(0.001)	
Past Signal	Δu_t	−0.006	**−0.021*****	0.04	0.005	**0.012*****	0.08
		(0.006)	(0.004)		(0.004)	(0.002)	
	Δy_t	0.004*	**0.017*****	<0.01	−0.006***	**−0.01*****	0.04
		(0.003)	(0.001)		(0.001)	(0.002)	
	Δr_t	0.002	0.003***	0.80	0.004*	0.004**	0.99
		(0.002)	(0.001)		(0.002)	(0.001)	
	π_t	−0.007***	−0.008***	0.67	−0.000	0.001	0.40
		(0.003)	(0.002)		(0.001)	(0.001)	

* ***,**,*: Significance at 1%,5% and 10% level. β_{bad} is average marginal effect in bad periods defined before, β_{ord} is average marginal effect in ordinary period. $\beta_{bad}=\beta_{ord}$ is test on equality between average marginal effects, its p-value is reported for each expectation-signal pair. Bold estimates denote the marginal effect with significantly bigger magnitude. Standard errors are adjusted for heteroskesticity and clustered within time.

The key message from Table 3 can be seen by comparing the marginal effects of the same signal between bad and ordinary periods. For future signals on unemployment and real GDP growth, their marginal effects always have a bigger magnitude during bad episodes, whereas the effects of past signals are always bigger in ordinary episodes. The p-values on the Wald test with the null hypothesis: $H_0:\beta_{bad}=\beta_{ord}$ range from 0.08 to less than 0.01 for these signals, which suggest the difference of marginal effects is statistically significant at least at 10% level. However, the same pattern does not hold true for signals on inflation and interest rate, with the exception of the future signal on inflation. In fact, average marginal effects on these signals are either insignificant or with small magnitudes. The marginal effect of the signal can be interpreted as weights put on these signals when forming expectations. This pattern then indicates households shift their attention from signals about past and current economic conditions to information about future economic conditions. Moreover, such an attention-shift is statistically significant. In other words, they are more adaptive learners when economic conditions are stable and become more forward-looking when the situation gets worse.[34]

4.2.4 Decomposing Time-varying Marginal Effect

Now I have shown that households put more weights on signals from professional forecasters in bad times; meanwhile, they rely less on realized macroeconomic statistics. However, the explanation for such a weight shift remains unclear. As the time-variation is only created by inputs to the RNN, I can use the trained ASF to decompose the contributions coming from different sets of input signals. I separate input signals for RNN into four categories: signals about economic conditions, signals about inflation, signals about the interest rate, and measure of news exposure about economic conditions.

As estimated ASF is non-linear, a proper way for variance decomposition is to use the Law of Total Variance following Isakin and Ngo (2020). I compute the direct contribution to the time-varying marginal effects of past and future signals on expectations related to economic conditions (those regarding Δu and Δy) for each of the four sets of signals described before. It's important to note that this variance decomposition does not represent the relative importance of specific signals in forming expectations. Rather it can be interpreted as relative importance of these signals to explain the time-variation of marginal effects that I presented in **Section 4.2.2** and **Section 4.2.3**.

Table 4 shows the variance decomposition for time-varying marginal effects of two signals on expected economic conditions as presented in **Section 4.2.2**.[35] The top panel is for past/current signal on real GDP growth, denoted as $\beta^{Ey}_{y,t}$ and the bottom panel is for future signal on real GDP growth (from SPF), denoted as $\beta^{Ey}_{Fy,t}$. In both marginal effects, signals on economic conditions contribute the most for time-variation observed. They explain up

[34]Notice the time-varying marginal effect is created endogenously by the estimated model. One may wonder whether it is a repetition of the non-linear conditional expectation function. Intuitively the input signals such as realized unemployment rate are, on average, higher during recessions. This means the average slope evaluated at those points is with higher absolute values. These concerns will be addressed in **Section 4.2.4**

[35]For same decomposition exercise of unemployment expectations refer to **Appendix C.3**

to 57% of the variation for the marginal effect of past signal and 52% for that of the future signal. News exposure on economic conditions also plays an important role, especially for the marginal effect of future signals. It explains 28% for the future signals and 15% for the past signals. With signals and news exposure on economic condition alone, I can explain as much as 72% and 80% of the total time-variation for the marginal effects of past and future signals.

Table 4: Variance Decomposition of Time-varying Marginal Effects: $E\Delta y$

Marginal Effect of Past Signal:		$\beta_{y,t}^{Ey}$				
Signal Type:		Economic Condition	Inflation	Interest rate	News	Total
	State $\theta_{i,t-1}$	17%	8%	3%	12%	40%
Channel:	Covariate $Z_{i,t}$	**40%**	12%	5%	3%	60%
	Total	57%	20%	8%	15%	
Marginal Effect of Future Signal:		$\beta_{Fy,t}^{Ey}$				
Signal Type:		Economic Condition	Inflation	Interest rate	News	Total
	State $\theta_{i,t-1}$	13%	2%	5%	9%	29%
Channel:	Covariate $Z_{i,t}$	**39%**	7%	6%	19%	71%
	Total	52%	9%	11%	28%	

On the other hand, inflation and interest rate signals account for only little of the time-variation, except for inflation signals in explaining marginal effects of past signal $\beta_{y,t}^{Ey}$. This is due to the signal on real oil price included as signals on inflation. Researchers document that oil price affects consumer expectations not only on inflation but also general economic conditions,[36] it is possible that oil prices either interact with or competing the attention put on signals about economic conditions and thus affecting the sensitivity of the household's expectation to these signals. Excluding oil price cuts down the marginal effect of Δy explained by inflation signals from 20% to 12%.

Another important question is for the same set of signals considered whether the time-variation of marginal effect is coming from contemporaneous signals $Z_{i,t}$ or through the accumulation of past signals which is represented by state $\theta_{i,t-1}$. I then separately evaluate

[36] See Edelstein and Kilian (2009), for example.

the variation explained by these two channels. In Table 4 for each set of signals, I also document the variance explained by each channel separately. For economic condition signals, new information at each period plays the most important role, which is around 70% of the total variation explained by these signals. Meanwhile, the state also accounts for a significant share of the time-variation. It explains 17% and 13% respectively for the marginal effects of past and future signals. This means the weight households put on economic condition signals depend on not only its current level but also the state they accumulated from observing these signals in the past.

From the variance decomposition, I conclude types of signals that explain most of the time-variation of marginal effects are those about economic conditions and news exposure on these topics. However, variance decomposition alone does not offer information about how these signals change marginal effects along time. It is possible that despite these two types of signals explain the most variation, they do not create the weight increase for future signals and decrease for past signals during bad times. To complete the picture, I present the time-varying marginal effects with only signals on economic conditions in Figure 5 and compare it with the actual marginal effects.

In Figure 5, the red curves are the time-varying marginal effects from estimated ASF with all signals as input. They are identical to those presented in **Section 4.2.2**. The blue curves are marginal effects computed from ASF using only actual economic condition signals as input,[37] which are the same series I use to perform variance decomposition in Table 4. This figure shows strong evidence that economic condition signals generate the weight increase on future signals as well as drop of weight on past signals during bad times. They are indeed key driving forces for agents to change from adaptive learners to forward-looking when economic condition gets worse.

Applying the RNN approach to survey data on expectation for US consumers, I find the average expectation formation model for these households are non-linear and asymmetric, and they rely on signals about the past during ordinary times but put more weights on signals about the future economic conditions when the current economic status gets worse. With

[37]For signals other than economic conditions I use random draw from the empirical distribution of these signals.

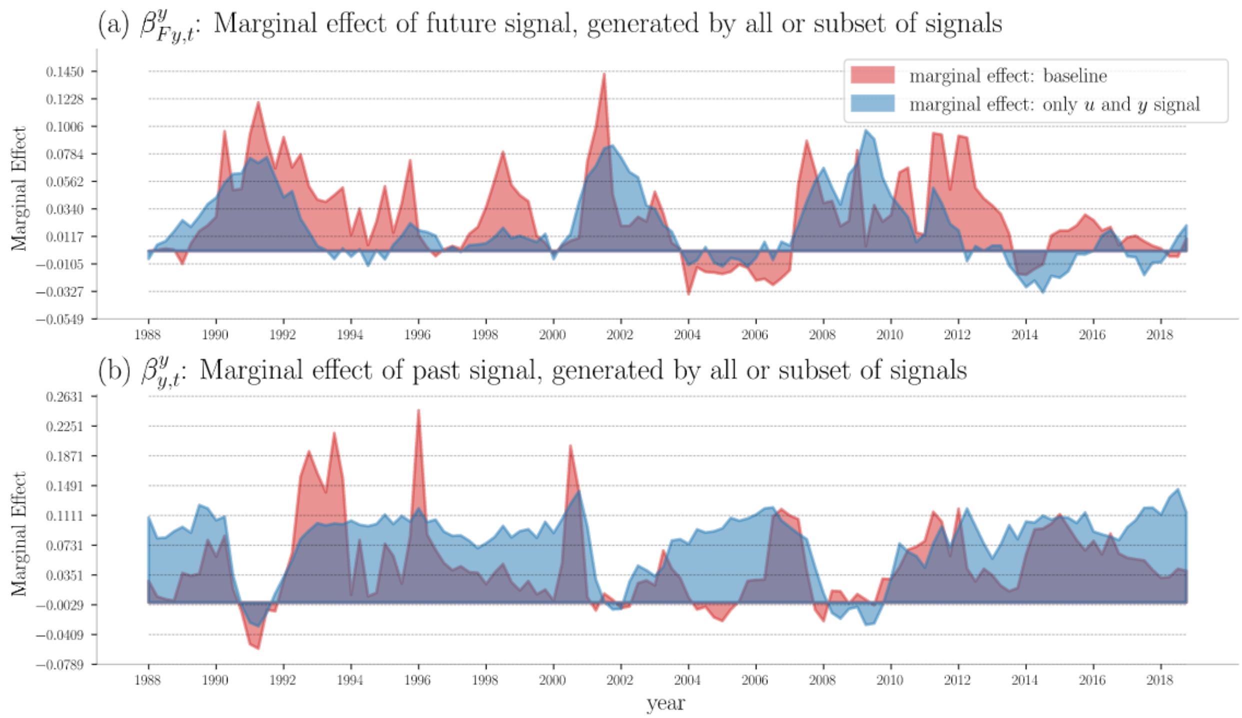


Figure 5: Time-varying marginal effect of past and future signal on real GDP growth. Top panel: marginal effect of future signal, $\beta^{Ey}_{Fy,t}$; bottom panel: marginal effect of future signal, $\beta^{Ey}_{y,t}$. The red curve: marginal effect created by estimated ASF with all signals. The blue curve: marginal effect created by ASF with only economic condition signals.

further exploration using the estimated ASF, I find the main reasons for such attention-shift are economic condition signals themselves.

For these findings to be consistent with FIRE, it has to be the actual fundamental law of motion that the agent tries to learn is highly non-linear and state-dependent. For example, the contribution to unemployment from its past level has to be countercyclical so that a rational agent with full information will appear to put lower weight on it during recessions. Such a law of motion is inconsistent with most of the macroeconomic models.

These findings are clearly at odds with the standard noisy information model, which typically assumes linear rules for expectation formation with the implication of a fixed marginal effect of signals across time. They are also hard to be reconciled with constant gain learning models as usually the time-variation of marginal effect appears when there are few data points to learn about the underlying structure, and it fades away when more information available to the learner.

One possible explanation for the time-variation of marginal effect was addressed by Car-

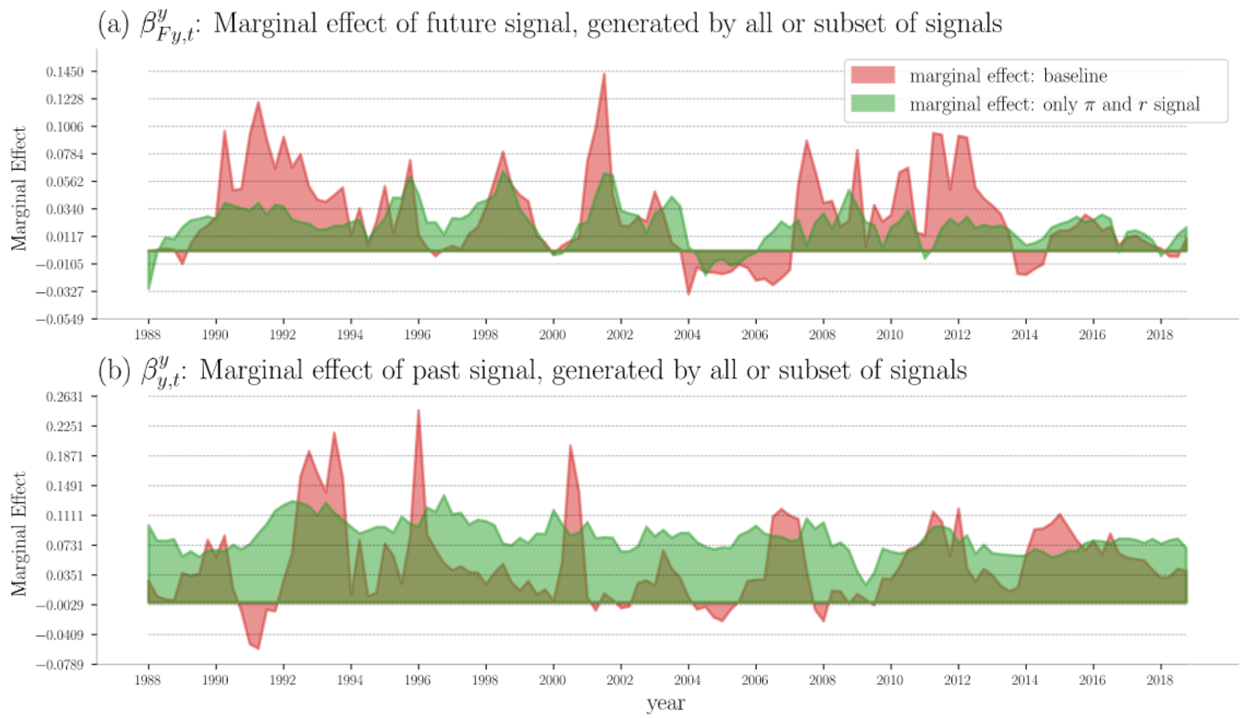


Figure 6: Time-varying marginal effect of past and future signal on real GDP growth. Top panel: marginal effect of future signal, $\beta^{Ey}_{Fy,t}$; bottom panel: marginal effect of future signal, $\beta^{Ey}_{y,t}$. The red curve: marginal effect created by estimated ASF with all signals. The blue curve: marginal effect created by ASF with only interest rate and inflation signals.

roll (2003), in which the author shows how information on inflation transmits from professional forecasts to households through news media. Intuitively, when there are more news stories on economic conditions, it is easier for households to acquire information about the future, thus putting higher weights on those signals[38]. Figure 7 presents how news exposure affects the marginal effects on future and past signals. It shows that news exposure only creates higher weights on future signals (from SPF) exactly when there is more news on economic status but not the weight change of past signals. According to Table 4, news exposure only accounts for 28% and 15% time-variation of weights on future and past signal, whereas economic conditions alone explain more than 50%. Furthermore, economic condition signals without news exposure successfully recreate the key attention-shift pattern. This indicates economic condition signals explain a much bigger fraction of the time-variation.

[38]See LAMLA and MAAG (2012) for example.

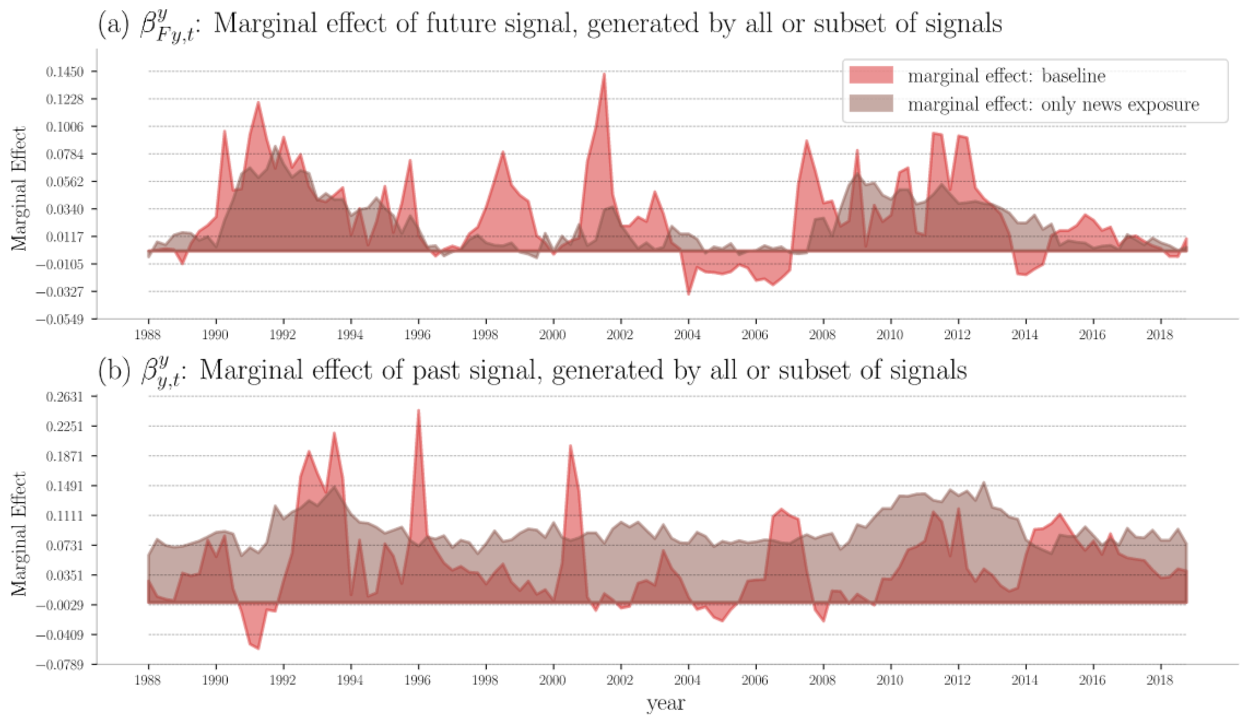


Figure 7: Time-varying marginal effect of past and future signal on real GDP growth. Top panel: marginal effect of future signal, $\beta^{Ey}_{Fy,t}$; bottom panel: marginal effect of future signal, $\beta^{Ey}_{y,t}$. The red curve: marginal effect created by estimated ASF with all signals. The blue curve: marginal effect created by ASF with only exposure of economic condition news.

5 Model with Rational Inattention

So far, I have documented novel stylized facts about the households' expectation formation model, using an innovative and more flexible semi-parametric approach with RNN. These new facts can be summarized with three key patterns: (1) households' expected economic condition is a non-linear and asymmetric function of signals about current economic status. In particular, the shape of the function is described in Figure 2. (2) When forming expectations on economic conditions, agents are adaptive learners in ordinary times and become forward-looking when the state of the economy gets worse. This is referred to as attention-shift. (3) The two major driving forces for the attention-shift are signals on economic condition.

In this section, I develop a simple two-period rational inattention model to explain the three stylized facts. Comparing to a standard rational inattention model, as presented in Sims (2003) and Maćkowiak *et al.* (2018), several modifications are made to the model.

First, I allow agents to acquire information about both the current and future state of the economy, and there are two separate signals associated with this information. This is done by assuming the future state of the economy contains a predictable part. Such a modification is needed to address the attention-shift towards future signal that I documented in the data. Secondly, rather than taking a linear-quadratic approximation of the agent's problem and looking for an analytical solution, I solve the problem numerically to keep the non-linear nature of the agent's optimal choices. Such an approach is more appropriate as the model aims to examine weights on signals when the state of the economy deviates largely (bigger than two standard deviations) from its mean rather than around its steady-state level. This modification then is crucial to make information more valuable to the agent when the state of the economy gets worse, which is the key mechanism to explain stylized facts I documented in data.

As the full model involves several different parts, I present the model progressively. In **Section 5.1** I describe agent' optimal saving-consumption problem given an information set. **Section 5.2** presents the uncertainty agents face and the corresponding information structure. Then in **Section 5.3**, I illustrate the value of information by showing different expected utilities under various information sets. The key mechanism for state-dependent marginal effect and non-linearity is presented in this section. In **Section 5.4** I formalize the full model by allowing agents to choose optimal information set (signals). **Section 5.5** presents the model-generated non-linear ASF, attention-shift between current and future signals, and illustrates key model mechanisms.

5.1 Household's Problem

There is a representative household that faces an individual consumption-saving problem. The household lives for two periods and get endowments $\{e_t, e_{t+1}\}$. The household can only save with a risky asset that pays a random return d_{t+1} at time $t+1$. At time t household will choose both consumption and saving with this risky asset, without knowing the value of d_{t+1} that is going to realize in $t+1$. At time $t+1$, the risky asset pays off, and the household consumes his total income in that period.

As the primary goal of this model is to assess the agent's optimal choice of information structure to forecast variable at $t+1$, for simplification, I assume the endowments $\{e_t, e_{t+1}\}$ are deterministic and the only uncertainty that the agent faces comes from d_{t+1}. I then interpret d_{t+1} as the fundamental about economic condition in the future, as it accounts for all the uncertainty about the agent's future income. If one considers saving as capital investment, with full depreciation d_{t+1} can be thought of as productivity shocks in the standard AK model.

Before the agent chooses consumption and saving in the first period, he can obtain signals that help him to forecast d_{t+1}. After observing these signals, the agent forms a belief on the return of the risky asset and chooses consumption and saving according to this belief. Such a procedure in making optimal decisions is common in models with limited information[39]. In rational inattention models, agents can choose the accuracy (variance) of signals he observes with an information cost. Signals with high accuracy and low variance will have high costs. I will discuss details about information structure and cost in Section 5.2. For now, I will denote the information structure chosen optimally by the agent as $\mathcal{I}_t$.

The household's utility maximization problem then can be written as:

$$\begin{aligned} \max_{c_t, s_{t+1}} \quad & \mathbb{E}[u(c_t) + \beta u(c_{t+1}) | \mathcal{I}_t] \\ s.t. \quad & c_t + s_{t+1} = e_t \\ & c_{t+1} = (1 + d_{t+1}) s_{t+1} + e_{t+1} \end{aligned} \tag{14}$$

5.2 Information Structure

For agents to make forecast on d_{t+1}, I need to specify a law of motion for the stochastic return. Consider the return evolves according to an AR(1) process described in (15).

$$d_{t+1} = \rho d_t + \psi_{t+1} \tag{15}$$

To reflect the fact that there are information available to agents about the future of the fundamental. I assume the shock on return tomorrow has a predictable part η_t and an

[39] For example Sims (2003), Maćkowiak *et al.* (2018) and Kamdar (2019)

unpredictable part $\epsilon_{1,t+1}$. The predictable part itself follows a stationary AR(1) process.

$$\psi_{t+1} = \eta_t + \epsilon_{1,t+1} \tag{16}$$

$$\eta_t = \rho_\eta \eta_{t-1} + \epsilon_{2,t} \tag{17}$$

Both $\epsilon_{1,t+1}$ and $\epsilon_{2,t}$ are i.i.d and mean-zero shocks that follow normal distribution.

$$\boldsymbol{\epsilon}_t \equiv \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix} \sim N(\mathbf{0}, \boldsymbol{Q}) \quad \boldsymbol{Q} \equiv \begin{bmatrix} \sigma^2_{1,\epsilon} & 0 \\ 0 & \sigma^2_{2,\epsilon} \end{bmatrix}$$

Such a formulation is similar to Barsky and Sims (2012), and it can be interpreted as "news shocks" described in Beaudry and Portier (2014). In general, this information may come from the stock market, news, or professionals. In this model, for simplicity I consider that this information is contained in the professional forecast. Throughout the model, I will assume the agent knows the correct law of motion of the stochastic return. In other words, the household is rational, and the only reason for not forming a perfect prediction on d_{t+1} is the limited information he can get.

5.2.1 Signals and Beliefs

At the beginning of time t, agent is endowed with some prior beliefs on states d_t and η_t, this reflects the latent states in empirical part. I denote the fundamental states by $\boldsymbol{X}_t \equiv \begin{bmatrix} d_t \\ \eta_t \end{bmatrix}$ and its prior as:

$$\boldsymbol{X}_0 \equiv \begin{bmatrix} d_0 \\ \eta_0 \end{bmatrix} \sim N(\hat{\boldsymbol{X}}_0, \Sigma_0)$$

Where $\hat{\boldsymbol{X}}_0$ stands for prior mean of the states $\boldsymbol{X}_t$.[40] Now the fundamental states can be written in a state-space representation that is commonly used in noisy information models:

$$\boldsymbol{X}_{t+1} = A\boldsymbol{X}_t + \boldsymbol{\epsilon}_{t+1} \tag{18}$$

Where A is the matrix given by parameters in (15)-(17): $A = \begin{bmatrix} \rho & 1 \\ 0 & \rho_\eta \end{bmatrix}$. Then they face different information sets that contain various signals about the fundamentals. Some of these information may be passive, which means that agents will be exposed to them without cost. And some of these may be costly to acquire.

[40]In steady state one can think of the prior mean being at the long-run mean of each state, which is 0. When an agent observes a history of signals before time t, she may have prior mean different from 0. This then can be thought of as a form of the "internal states" described in Section 4.2.4.

The signals that agents are passively exposed to are on current state d_t. This is summarized as a Gaussian noisy signal $z_0 = d_t + \xi_0$, where $\xi_0 \sim N(0, \sigma_z^2)$. Such a signal can be thought of as an information agent picks up passively during daily life. For example, agents may get a rough idea about the current economic condition when seeing friends or themselves getting unemployed or wage raise. These are information they are exposed to without putting in an effort to collect, thus incurring no cost. This type of information may also be quite inaccurate, with a relatively high variance on noise.

Upon observing passive signals, agents also deliberately choose signals costly to be better informed. To be consistent with my empirical setup, I restrict the choices of signals to one about current state d_t and one about future that comes from SPF:

$$F_t d_{t+1} = \rho d_t + \eta_t \tag{19}$$

Agents observe unbiased signals on these two objects, with additive normal noise $\boldsymbol{\xi}_t$, where:

$$\boldsymbol{\xi}_t \equiv \begin{bmatrix} \xi_{1,t} \\ \xi_{2,t} \end{bmatrix}, \quad \boldsymbol{\xi}_t \sim N(\mathbf{0}, R), \quad R \equiv \begin{bmatrix} \sigma_{1,\xi}^2 & 0 \\ 0 & \sigma_{2,\xi}^2 \end{bmatrix}$$

Denote the vector of signals as $\boldsymbol{Z}_t$, the signal structure is given by:

$$\begin{bmatrix} z_t^{spf} \\ z_t \end{bmatrix} \equiv \boldsymbol{Z}_t = G\boldsymbol{X}_t + \boldsymbol{\xi}_t \tag{20}$$

Where G is given by:

$$G = \begin{bmatrix} \rho & 1 \\ 0 & 1 \end{bmatrix}$$

.

Agent is Bayesian Learner and form posterior beliefs using Kalman Filter. Agent updates his belief twice: first he is exposed to a normal noisy signal z_0 about current state d_t. The variance of noise is σ_z^2. Agent then updates his belief on $\boldsymbol{X}_t$. Because both prior and noise are normally distributed, the updated prior is also normal.

$$\boldsymbol{X}_{t|0} \equiv \begin{bmatrix} d_{t|0} \\ \eta_{t|0} \end{bmatrix} \sim N(\hat{\boldsymbol{X}}_{t|0}, \Sigma_{t|0})$$

I define $\boldsymbol{X}_{t|0}$ as conditional prior as it contains information about d_t. Specifically its mean $\hat{\boldsymbol{X}}_{t|0}$ is a function of d_t, unconditional prior mean and random noise in z_0. However, the agent has no control of variance of this noise σ_z^2. It will not be in the agent's choice set and will be treated as given when the agent solves the rational inattention problem later.

The second time agent updates belief is after observing signal $\boldsymbol{Z}_t$. He forms posterior belief about the fundamentals next period. As this is a two period model, only belief on d_{t+1} is relevant. Again agent forms belief using Bayes Rule:

$$\boldsymbol{X}_{t+1|t} \equiv \begin{bmatrix} d_{t+1|t} \\ \eta_{t+1|t} \end{bmatrix} \sim N(\hat{\boldsymbol{X}}_{t+1|t}, \Sigma_{t+1|t})$$

Where posterior mean $\hat{\boldsymbol{X}}_{t+1|t}$ and variance $\Sigma_{t+1|t}$ is defined as:

$$\hat{\boldsymbol{X}}_{t+1|t} \equiv \mathbb{E}[\boldsymbol{X}_{t+1}|\boldsymbol{Z}_t] = A\bigg((I - KG)\hat{\boldsymbol{X}}_{t|0} + K\boldsymbol{Z_t}\bigg) \tag{21}$$

$$\Sigma_{t+1|t} = A\Sigma_{t|0}A' - AKG\Sigma_{t|0}A' + \boldsymbol{Q} \tag{22}$$

And Kalman Gain is given by (23), where matrices A and G are given by exogenous parameters $\{\rho, \rho_\eta\}$ about the fundamentals.

$$K = \Sigma_{t|0}G'(G\Sigma_{t|0}G' + R)^{-1} \tag{23}$$

From (21)-(23), the choices of signal precision will affect both mean and variance of his posterior through the variance-covariance matrix on noise, R. Signals with lower variance are more accurate, and the agent will put higher weights on these signals. Each different choice of signal accuracy (represented by the variance-covariance matrix on noise, R) gives the agent a different information set. Given different information sets, the agent will form different posterior beliefs even if the signals realized are the same.

5.2.2 Two Special Information Set

At this point, it is worth describing several special information sets:

d_t **Fully Observable:** At time t, an agent has only perfect information about d_t and no information on η_t. This happens when $\sigma_{2,\xi} = 0$ and $\sigma_{1,\xi} \to \infty$. In this case agent will form adaptive expectation about return in the future: $E_t^A d_{t+1} = \rho d_t$.

Both fundamentals X_t Fully Observable: At time t, an agent has all the information about fundamentals at time t. Given the distribution of $\epsilon_{1,t+1}$, an agent with this information set can form a posterior belief on the distribution of d_{t+1} with mean being expressed as (19). This can be thought of as the Full Information Rational Expectation benchmark in this model as the forecasting error in this case will only be the unpredictable shock $\epsilon_{1,t+1}$.

An information set with arbitrary variance-covariance matrix on noise, R, can be thought of as in the middle of two information sets described above. For each information set $\mathcal{I}_t$ given, the agent will solve his optimization problem (14) accordingly. Different information sets will then result in different choices, thus giving the agent different expected utility. In this sense, information has a value that can be evaluated with their expected utility. The rational inattention models suggest the agent realizes the value of information and he chooses an optimal information set with a cost. Before I proceed to the full rational inattention set up, I will first illustrate the value of information in this model using different information sets described here.

5.3 Value of Information

In this section I explicitly compute agent's expected utility conditional on different information sets given. I will illustrate that more information is valuable to agents as it increases their expected utility. Furthermore, the improvement of expected utility obtained by possessing more information depends on the current state of economy, d_t.

The agent solves his optimal consumption and saving choice given the information set $\mathcal{I}_t$. Defining for ease of notation $r_{t+1} = 1 + d_{t+1}$ and following (14), the maximization can be rewritten as:

$$\max_{s_{t+1}} \quad \mathbb{E}[u(e_t - s_{t+1}) + \beta u(r_{t+1}s_{t+1} + e_{t+1})|\mathcal{I}_t] \tag{24}$$

One key distinction I want to make between this model and standard rational inattention model is that I won't take the second-order approximation of the problem (24). Instead I directly solve it numerically using the distribution of signals and fundamentals. Although this problem is well approximated in small deviations from steady state and the approach is commonly used in the rational inattention literature, it also leads to the result that only variance of posterior belief on state variable matters when agent chooses optimal signals. Such a result is intuitive when considering choices around the steady state, however it is not appropriate in my paper as I'm focusing explicitly on difference of attention choices during large deviations from the steady state. An important result my model will deliver is that mean of belief on economic state matters for agents' attention choices, this is the key mechanism to generate attention-shift that is found in empirical part.

To illustrate the value of information, I solve problem (24) under the three different information sets and evaluate agent's expected utility. For simplicity, I use a linear-quadratic utility function.

$$u(c_t) = c_t - bc_t^2$$

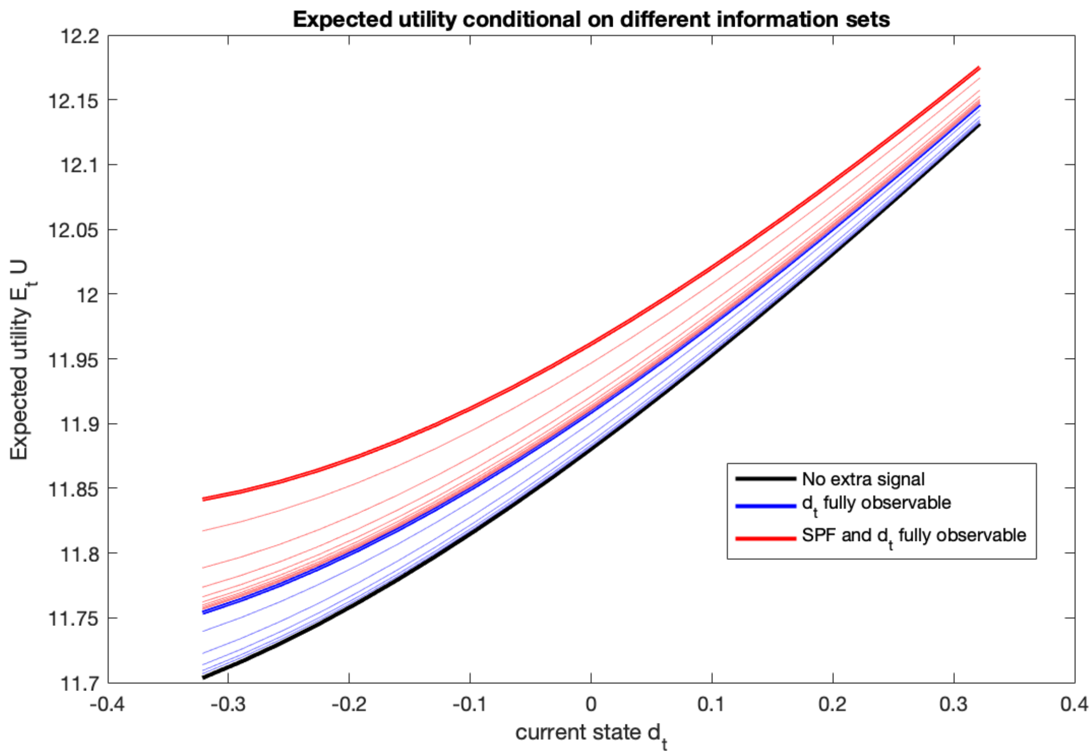
Such a function form also makes the point that the common mean-independent result of rational inattention model is not due to linear quadratic preference per se, rather it's because of the quadratic approximation for the entire problem. However the results are not restricted to such utility function form. Given different information set $\mathcal{I}_t$, the first order condition for problem (24) then takes the form:

$$s^*_{t+1}(\mathcal{I}_t) = \frac{-1 + 2be_t + (\beta - 2b\beta e_{t+1})\mathbb{E}[r_{t+1}|\mathcal{I}_t]}{2b(1 + \beta\mathbb{E}[r^2_{t+1}|\mathcal{I}_t])} \tag{25}$$

From (25) it's clear that agent's optimal saving plan depends on his posterior belief on the return of risky asset, conditional on information set $\mathcal{I}_t$. Furthermore, it relies on both the conditional mean and the conditional variance of the return and is a non-linear function of them. It immediately follows that different information set will lead to differences in utility. When agent considers the value of information, they will evaluate the expected utility for each given information set $\mathcal{I}_t$.

For illustration purpose, I solve the model numerically using the following parametrization: $b = 1/40$, $\beta = 0.95$, $e_t = 10$ and $e_{t+1} = 5$. For the fundamentals I consider $\rho = 0.2$, $\rho_d = 0.9$, $\sigma_{1,\epsilon} = \sigma_{2,\epsilon} = 0.15$. The prior beliefs on states d_t and η_t are assumed to be mean zero with the stationary variance-covariance matrix obtained from recursive Kalman Filter. The standard deviation of noise on passive signal is $\sigma_z = 0.22$. In Figure 8 I plot the expected utility conditional on various information sets, as functions of realized d_t. The thick black curve is expected utility when there's no more information other than the initial passive signal on d_t is available to agent. The thick blue curve is expected utility when d_t fully observable and the thick red curve is when both SPF and d_t are fully observable (the FIRE benchmark)[41]. The curves between the thick lines depicts the increase in expected utilities as precision of signal increases (or variance of noise decreases).

Figure 8: Expected Utility under Different Information Set



Black thick line: Expected utility when no more information other than initial passive signal on d_t; blue thick line: expected utility when d_t becomes fully observable; red thick line: when both SPF and d_t fully observable. Blue thin lines are expected utilities when there are noise attached to extra signal on d_t, the more accurate the signal, the closer it gets to d_t fully observable case. Red thin lines are expected utilities when noise attached to signal on SPF, and d_t is fully observable. The more accurate the signal, the closer it gets to full information case.

There are two key messages from Figure 8. First more information improves agent's expected utility progressively: with more accurate signal on d_t, agent resolves the uncertainty about current state and his utility increases at any given d_t from black line to blue line; and it continues to increase as signal on SPF becomes more accurate, from blue curve to red

[41]With the specific law of motion assumed in (15) - (17) together with definition of SPF (19), the case with only SPF fully observable will coincide with the FIRE case.

curve. This is typical result from informational models.

Secondly and more importantly, the value of information is decreasing in realized state d_t. This can be seen from the differences between expected utilities with different information sets. When realized state d_t is low and negative, getting same amount of information will increase agent's expected utility by more than the case when d_t is high. In other words, information is more valuable when economic status is bad. This is a result different from that of standard rational inattention literature. The reason for such a difference is the non-linearity in the optimal saving/investment function.

The difference between expected utility comes from differences of optimal investment (25). The fact that optimal saving is a non-linear function of both posterior mean and posterior variance of state d_{t+1} makes the expected utility mean-dependent. To see this, we can utilize the assumption of quadratic utility function, and re-write the expected utility as the following form:

$$\mathbb{E}[U(s^*_{t+1}(\mathcal{I}_t))] = -\mathbb{E}[\chi(s^*_{t+1}(\mathcal{I}_t) - \bar{s}_{t+1})^2] + M \tag{26}$$

Where $\chi = b(1+\beta r^2_{t+1})$ and $M = \mathbb{E}[\frac{(-1+2be_t+\beta r_{t+1}-2\beta b r_{t+1}e_{t+1})^2}{4b(1+\beta r^2_{t+1})}] + e_t - be_t^2 + \beta e_{t+1} - \beta b e^2_{t+1}$.[42] The variable $\bar{s}_{t+1}$ is given by (27). It stands for the optimal investment under perfect foresight, when agent observes d_{t+1} perfectly.

$$\bar{s}_{t+1} = \frac{-1 + 2be_t + \beta r_{t+1} - 2b\beta r_{t+1}e_{t+1}}{2b(1+\beta r^2_{t+1})} \tag{27}$$

The transformed utility function (26) is usually referred as a quadratic loss function in rational inattention models, intuitively agent will seek to minimize the expected loss between optimal choice under limited information set $\mathcal{I}_t$ and optimal choice under Full Information Rational Expectation[43]. From (26) it is obvious that if optimal choice of s is linear in state r_{t+1}, the expected utility only depends on posterior variance of r_{t+1} given information set $\mathcal{I}_t$. It is not related to posterior mean of states or realized state at time t.

Using the transformed expected utility (26), I can explore reasons for value of information decreasing in d_t. To see this, consider the cases with or without full information from

[42] For derivation please refer to Appendix D.1

[43] It is worth noting that M is not involved in choosing the optimal information structure $\mathcal{I}_t$ as it is only related to the actual distribution of r_{t+1}.

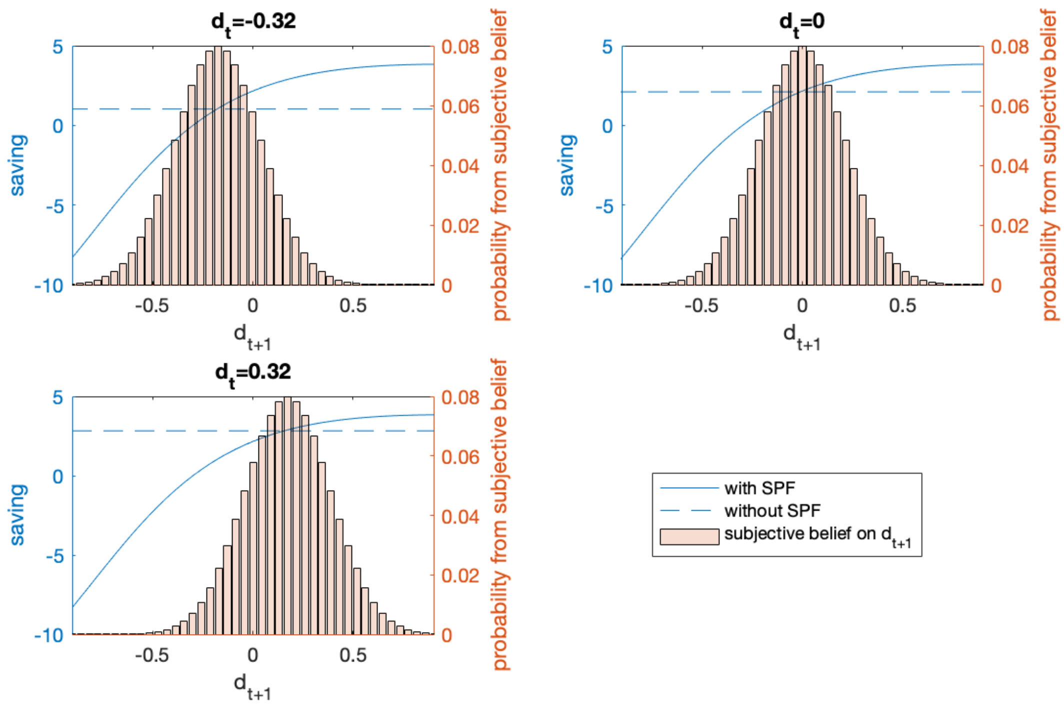
SPF. Conditional on realization of a specific d_t, without information from SPF agent faces uncertainty from both η_t and $\epsilon_{1,t+1}$ being unobservable. With information from SPF uncertainty from η_t is resolved. Because in both cases agents have no information on $\epsilon_{1,t+1}$, the utility improvement comes solely from knowledge on η_t. For simplicity I consider an extreme case when $\epsilon_{1,t+1} = 0$. Then $\bar{s}_{t+1}$ can be seen as the optimal saving choice when SPF is available. The utility loss of agent not having information from SPF can then be evaluated by differences between optimal savings with or without SPF observable, weighted by agent's subjective belief.

In Figure 9 I depict the optimal saving choices at three realized values of d_t: when current state is bad ($d_t = -0.32$), neutral ($d_t = 0$) and good ($d_t = 0.32$). In each case I plot optimal saving choice as a function of future state d_{t+1}. The dotted line is the optimal saving that agent chooses when he only observes initial signal on d_t. It is a flat line because agent's choice does not depend on d_{t+1} (or realization of η_t) when SPF is not observable. The solid line is optimal saving choice when SPF is observable to agent. This line is a function of d_{t+1} because under the assumption $\epsilon_{1,t+1} = 0$, when SPF is observable then η_t and d_{t+1} are fully observed. An important feature is then this function is increasing and concave in d_{t+1}. This is because the higher the return d_{t+1} is, the more agent wants to save. The concavity comes from the fact that substitution effect becomes weaker as return on asset increases and finally dominated by income effect[44].

Now for agent without information from SPF, the solid line is not feasible. For a given realized d_t, the agent will evaluate his utility loss of not having information on η_t following (26). This is done by measuring the distance between optimal saving choices with and without information from SPF and compute the expected value of (the square of) this distance using their posterior belief on d_{t+1} (η_t). In Figure 9 this belief is shown with bar plot. When realized d_t is higher, the belief of distribution on d_{t+1} is centered at a higher mean. Because of the non-linearity of the optimal saving choice, the average distance between saving choices with and without information from SPF is higher when d_t is low. This gives rise to the fact that value of information from SPF is decreasing in d_t.

[44] Interestingly, if one would instead assume riskless asset with a risky endowment in $t+1$, the optimal saving curve under full information will be linear and value of information won't depend on current state any more.

Figure 9: Optimal Saving under Full Information and Limited Information



Solid line: optimal saving choice under full information: when both d_t and SPF are fully observable. Dash line: optimal policy when SPF signal is not available. Bar plot: agent's subjective belief on future state d_{t+1}, when SPF is not observable. Top left panel is when current state is very bad ($d_t = -0.32$), top right panel is when $d_t = 0$ and bottom left panel is when current state very good, $d_t = 0.32$.

With the simple structure presented above, I show the key pattern my model generates: value of information decreases in state of economy. Agent is willing to pay higher costs to acquire information as state of economy gets worse. This gives the key mechanism to create the time-varying marginal effect and non-linearity I documented with RNN. Because when agent can choose the precision of signals (thus information set) optimally, they will make different choices during bad and ordinary times and this will result in different weights on these signals. This is achieved with the full rational inattention model in next section.

5.4 Model with Agent Choosing Signal Optimally

In the model with Rational Inattention, given information set, agent will face the same two period consumption-saving problem as in Section 5.1. Agents face the same uncertainty generated by two fundamentals described by (15)-(17). They also know information has value as described in Section 5.3. The major difference here is that agent now can costly choose the optimal signals on fundamentals that he can observe in his information set. At this point it's useful to describe the time-line for agent's problem.

5.4.1 Set-up

Time-line of Agent's Problem: The agent's problem can be described by the following steps:

1. At beginning of time t, agent is exposed to a signal about current state d_t. The signal was denoted as z_0. Agent is endowed with some normal prior on fundamentals:
$$\boldsymbol{X}_0 \equiv \begin{bmatrix} d_0 \\ \eta_0 \end{bmatrix} \sim N(\hat{\boldsymbol{X}}_0, \Sigma_0)$$
2. Agent chooses optimal signals in information set $\mathcal{I}_t$, knowing the consumption-saving choices he make later will depend on the information set he chooses. The form of signals and contents in information sets are same as those in Section 5.2.
3. Given the chosen information set, agent chooses consumption and saving to maximize his expected utility, conditional on signals observed.

Information Set and Cost: Agent's initial information set contains his prior on fundamentals, $\boldsymbol{X}_0$ as well as the initial passive signal on d_t: z_0. This is because the agent has no control over z_0. This information set can then be fully summarized with updated prior about $\boldsymbol{X}_t$. Denote the initial information set as $\mathcal{I}_0 = \{\boldsymbol{X}_{t|0}\}$, for the entire problem agent will take this information set as given. The information set after agent choosing precision of signals can be then defined as:

$$\mathcal{I}_t = \mathcal{I}_0 \cup \{\boldsymbol{Z}_t\} \tag{28}$$

Where $\boldsymbol{Z}_t$ is a vector of signals defined in (20).

Information comes with a cost. Following Sims (2003) I measure the cost of acquiring more information in set $\mathcal{I}_t$ with the difference of the Shannon entropy, denoted as $\mathcal{H}(.)$. As both random states and signals I considered are normally distributed, results from Maćkowiak *et al.* (2018) show that the entropy cost can be represented by posterior variance-covariance matrices. Denoted as κ, equation (29) formally defines the entropy cost.

$$\kappa = \mathcal{H}(\boldsymbol{X}_{t+1}|\mathcal{I}_0) - \mathcal{H}(\boldsymbol{X}_{t+1}|\mathcal{I}_t) = \frac{1}{2}log_2(\frac{det\Sigma_{t+1|0}}{det\Sigma_{t+1|t}}) \tag{29}$$

Where $\Sigma_{t+1|0}$ stands for posterior variance matrix for hidden states $\boldsymbol{X}_{t+1}$ conditional on information in $\mathcal{I}_0$ and $\Sigma_{t+1|t}$ stands for posterior variance matrix conditional on information in $\mathcal{I}_t$.[45]

Agent's Optimization Problem: Agent's problem comes in two steps. First agent chooses information set $\mathcal{I}_t$. He cannot control the realization of signal $\boldsymbol{Z}_t$ but he can choose the precision of noise $\boldsymbol{\xi}_t$ that is attached to this signal. In this sense choosing information set $\mathcal{I}_t$ is equivalent to choosing variances of signal $\{\sigma^2_{1,\xi}, \sigma^2_{2,\xi}\}$. Then agent solves consumption-saving problem given the information set chosen and signals $\boldsymbol{Z}_t$ realized. This problem can be summarized as follows:

$$\max_{\sigma^2_{1,\xi},\sigma^2_{2,\xi}} \quad \mathbb{E}[u(e_t - s^*_{t+1}) + \beta u(r_{t+1}s^*_{t+1} + e_{t+1})|\mathcal{I}_0] - \lambda\kappa \tag{30}$$

$$s.t. \quad s^*_{t+1} = argmax_{s_{t+1}} \quad \mathbb{E}[u(e_t - s_{t+1}) + \beta u(r_{t+1}s_{t+1} + e_{t+1})|\mathcal{I}_t] \tag{31}$$

$$\kappa = \frac{1}{2}log_2(\frac{det\Sigma_{t+1|0}}{det\Sigma_{t+1|t}}) \tag{32}$$

Equation (30) describes the first step of agent's problem. He chooses optimal precision of signals knowing that his choice of saving depends on the realization of these signals. In this expectation, the agent will take into account uncertainty that comes from both underlying true states and noise in realized signals. His choice on precision of signals will affect $\mathcal{I}_t$ in (31) as well as information constraint (32). The information cost in terms of utility loss is assumed to be a marginal cost parameter λ times the Shannon entropy cost κ. The parameter λ describes how costly it is for the agent to acquire information with some level of entropy reduction. When λ is bigger, it means the agent suffers higher utility loss from acquiring more information. In particular, when $\lambda = 0$, the information cost becomes irrelevant and the agent forms expectation according to FIRE.

[45] For derivations of entropy cost in (29) and the posterior variance-covariance matrices $\Sigma_{t+1|0}$ and $\Sigma_{t+1|t}$, please refer to Appendix D.2.

5.4.2 Solve for Optimal Precisions of Signals

The agent's problem described by (30)-(32) can be solved backwards. This involves solving the optimal saving problem (31) first, taking the information set $\mathcal{I}_t$ as given. The result is given by (25) in Section 5.3:

$$s^*_{t+1}(\mathcal{I}_t) = \frac{-1 + 2be_t + (\beta - 2b\beta e_{t+1})\mathbb{E}[r_{t+1}|\mathcal{I}_t]}{2b(1 + \beta\mathbb{E}[r^2_{t+1}|\mathcal{I}_t])}$$

Intuitively, information matters for the agent as it will affect her optimal saving. To see how the optimal saving is related to the agent's choice on signal precisions $\sigma^2_{1,\xi}, \sigma^2_{2,\xi}$, it is useful to write $\mathbb{E}[r_{t+1}|\mathcal{I}_t]$ and $\mathbb{E}[r^2_{t+1}|\mathcal{I}_t]$ as functions of these precisions.[46] Notice $r_{t+1} = 1 + d_{t+1}$, then from (21):

$$\begin{aligned}\begin{pmatrix}\mathbb{E}[d_{t+1}|\mathcal{I}_t] \\ \mathbb{E}[\eta_{t+1}|\mathcal{I}_t]\end{pmatrix} &= A\Big((I - KG)\hat{\boldsymbol{X}}_{t|0} + K\boldsymbol{Z_t}\Big) \\ &= A\bigg((I - KG)\Big((I - K_0G_0)\hat{\boldsymbol{X}}_0 + K_0z_0\Big) + K\boldsymbol{Z_t}\bigg)\end{aligned} \tag{33}$$

In equation (33), matrices K, G and I are given by Kalman Filter from (21)-(23). Now define $\iota = [1 \quad 0]$, and $G_0 = \iota$ as defined in Appendix D.2 and $K_0 = \Sigma_0 G'_0(G_0\Sigma_0 G'_0 + \sigma^2_z)^{-1}$ is the corresponding Kalman Gain used by the agent when she observes the passive signal z_0. The expected second order term in the optimal saving function is then given by:

$$\mathbb{E}_t[r^2_{t+1}|\mathcal{I}_t] = \iota\Sigma_{t+1|t}\iota' + \Big(1 + \mathbb{E}[d_{t+1}|\mathcal{I}_t]\Big)^2 \tag{34}$$

Now notice (33) and (34) imply that both $\mathbb{E}[r_{t+1}|\mathcal{I}_t]$ and $\mathbb{E}[r^2_{t+1}|\mathcal{I}_t]$ are directly functions of $\sigma^2_{1,\xi}$ and $\sigma^2_{2,\xi}$, as the Kalman Gain $K = \Sigma_{t|0}G'(G\Sigma_{t|0}G' + R)^{-1}$ and R is a diagonal matrix of $\sigma^2_{1,\xi}$ and $\sigma^2_{2,\xi}$. Moreover, it is worth nothing that both terms are also functions of normally-distributed random variables $\boldsymbol{Z}_t$, so that the precisions of signals also affect the expected utility in (30) indirectly. As discussed in Section 5.3, higher precision (or lower variance on noise) leads to higher expected utility. This can be considered as the benefit of more information.

In the agent's objective function of problem (30), she also faces a cost of information, κ, described by (32). This information cost is also affected by signal precisions because the

[46] For full derivation of $\mathbb{E}[r_{t+1}|\mathcal{I}_t]$ and $\mathbb{E}[r^2_{t+1}|\mathcal{I}_t]$, please refer to Appendix D.3

posterior variance-covariance matrix $\Sigma_{t+1|t}$ is given by:

$$\begin{aligned}\Sigma_{t+1|t} &= A\Sigma_{t|0}A' - AKG\Sigma_{t|0}A' + \boldsymbol{Q} \\ &= A\Sigma_{t|0}A' - A\Sigma_{t|0}G'(G\Sigma_{t|0}G' + R)^{-1}G\Sigma_{t|0}A' + \boldsymbol{Q}\end{aligned} \tag{35}$$

The first equality comes from (22). From the equation above, we see the diagonal matrix of signal precisions, R, directly affects the posterior variance-covariance matrix, thus affecting the information cost.

The trade-off agent faces in solving this problem are then between the benefit of more information and its cost. As shown in Figure 8, lower $\sigma_{2,\xi}$ and $\sigma_{1,\xi}$ (thus higher precision on both signals of current state and Professional Forecasts) will increase expected utility. Meanwhile, more accurate signals will also increase information cost κ, as accurate signals decrease the posterior variance of the agent's belief. Because the agent observes an initial signal z_0 which contains information about d_t, her optimal choice of signal precision will depend on d_t:[47] when d_t is negative, information becomes more valuable to the agent thus they are willing to choose higher precision (lower variance) for signals. I will illustrate this trade-off in the next section.

Before moving forward, it is also important to note that other exogenous aspects about the information friction affect the agent's objective. Specifically, both the precision of the passive signal and the prior mean about the current state matter for the agent's optimal choices of signal precisions. They affect both the benefit and the cost of acquiring information. Furthermore, the marginal cost of information λ also has an impact on the agent's optimal choices of signal precisions because it affects directly how easy it is for the agent to acquire information. In the next section, I will also illustrate the impacts of changing these exogenous aspects.

[47] If one assumes no passive signal is observed by agents, then the optimal choice of signal precision does not depend on d_t, but it will still depend on the prior belief about fundamentals. If this is the case, one should observe hidden states capturing most of the variation in time-varying marginal effects in Section 4.2.4. However, instead most variation is explained by the current signal, thus the empirical results are more consistent with the case when the agent observes a passive signal on current state d_t.

5.5 Results

I solve the rational inattention problem (30)-(32) by simulation using the parametrization included in Table 5. The main purpose of this section is to show that non-linear functional form and state-dependency weights can be generated with the proposed model with rational inattention. Meanwhile, I will also illustrate the key mechanisms that lead to these results as well as how the households' optimal information set is affected by other aspects of the proposed information friction.

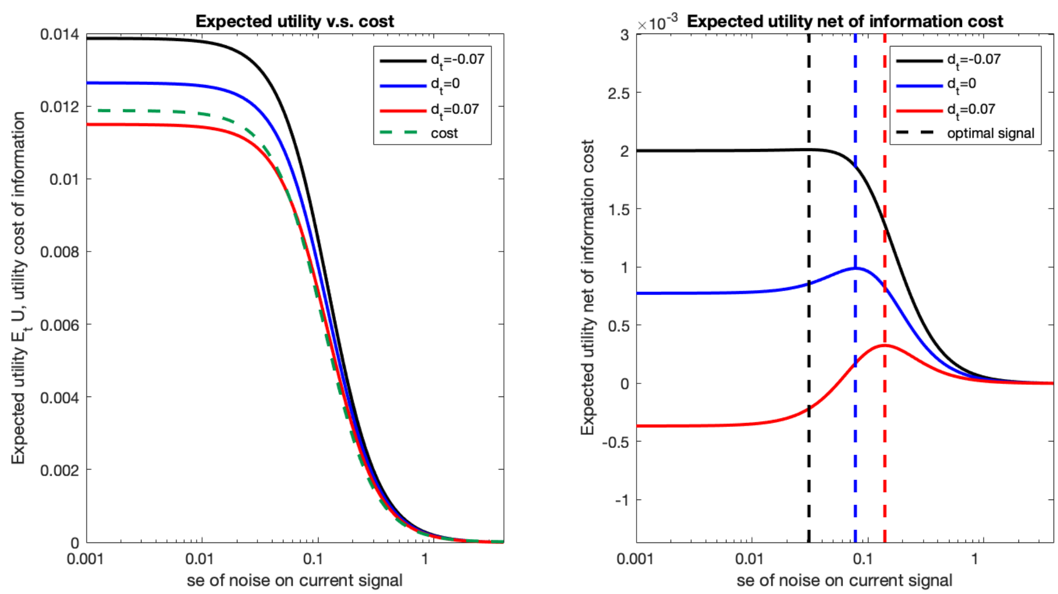
Table 5: Model Parameters

Parameter	Value	Parameter	Value
e_t	10	e_{t+1}	5
b	1/40	β	0.95
ρ	0.2	ρ_η	0.9
$\sigma_{1,\epsilon}$	0.09	$\sigma_{2,\epsilon}$	0.09
σ_z	0.18	λ	0.042
$\hat{\boldsymbol{X}}_0$	$\mathbf{0}$		

5.5.1 State-dependent Optimal Signals

I first show that the trade-off between the benefit and cost of acquiring information changes with the current state d_t. In Figure 10, I present the expected utility, information cost and the objective function in (30) when the current state d_t is negative, at mean zero and positive. The left panel describes how the expected utility changes as the standard error of the current signal ($\sigma_{2,\xi}$) changes for the three cases of d_t. Because at a different level of d_t, the expected utility for the same signal precision will be different, I normalize it by the utility at $\sigma_{2,\xi} = \infty$, which corresponds to the case when the agent acquires no extra signal on the current state. It is obvious in all three cases of d_t, the higher(lower) the precision(standard error) of the signal, the higher the expected utility comparing to the no information case. I then present the information cost for all three cases and show that the information costs are the same across different levels of d_t. This is because, in (33), the passive signal z_0 contains information about the current state d_t thus making the expected utility depending on it. Whereas in the information cost (35), the evaluation of posterior variance is mean-independent, which means only the variance of the passive signal matters in accessing the information cost so that the cost will not change as d_t changes. The key message from the left panel is that both the cost and the benefit of information increase with the precision of the current signal. Meanwhile for the same level of signal precision, the higher the current state d_t, the lower the benefit from that signal.

Figure 10: Information Benefit, Information Cost and Households' Objective: Function of Current Signal

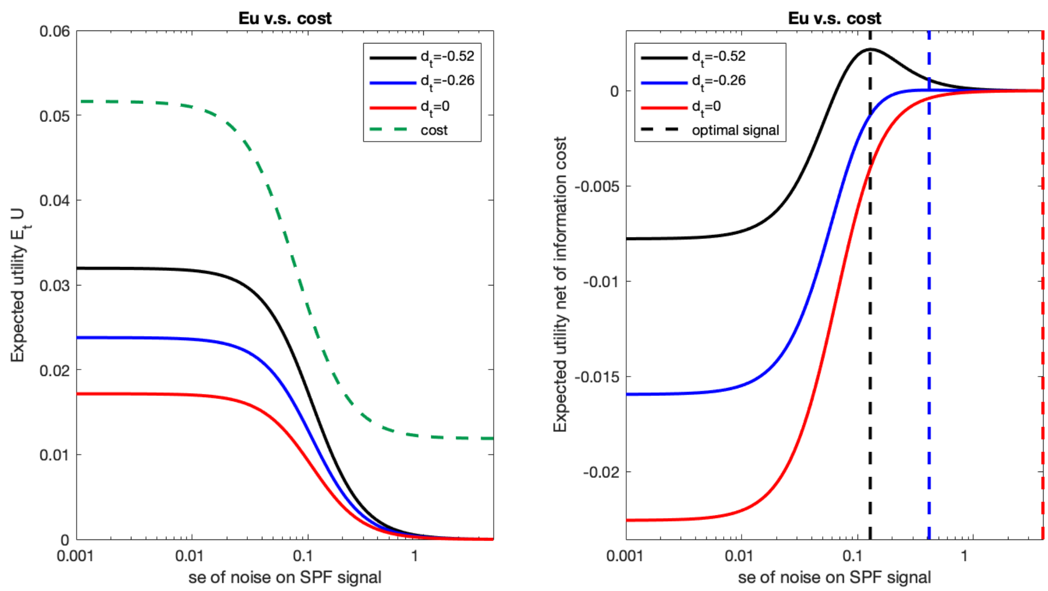


Left panel: the information benefit is evaluated by expected utility and plotted with solid lines, information cost is evaluated by entropy cost and plotted with dashed line. Right panel: objective function is obtained by information benefit minus cost and plotted with solid lines. The figure considers three different cases: current state is high with $d_t = 0.07$, current state is at its mean $d_t = 0$ and current state is low at $d_t = -0.07$. Horizontal axis is standard error of noise on *current signal*, higher s.e. leads to lower weight. Vertical dashed line in the right panel labels optimal s.e. for *current signal* in three scenarios.

The agent's objective function considers both the cost and benefit of acquiring information. The right panel of Figure 10 then presents the objective function under the same three cases of d_t. A utility-maximizing agent will choose the current signal with a standard error that maximizes her objective function. These choices under different states are represented by the dashed line. The right panel then shows that when the current state is worse, the agent will choose a higher precision and a lower standard error for the current signal.

These patterns hold true for precision on signals about SPF as well. In Figure 11, I again show a similar graph as in Figure 10 but for signals on SPF. The major difference between this figure and Figure 10 is that expected utility is computed assuming $\sigma_{2,\xi} = 0.001$, which means the agent chose a very precise current signal. [48] All the objects plotted in Figure 11 are then functions of the precision on SPF signal, $\sigma_{1,\xi}$. Similar to that in Figure 10, we see that both the benefit and the cost increase when the agent acquires more information on SPF. Meanwhile, the optimal precision of the SPF signal decreases with the current state d_t.

Figure 11: Information Benefit, Information Cost and Households' Objective: Function of SPF Signal



Left panel: the information benefit is evaluated by expected utility and plotted with solid lines, information cost is evaluated by entropy cost and plotted with dashed line. Right panel: objective function is obtained by information benefit minus cost and plotted with solid lines. The figure considers three different cases: current state $d_t = 0, -0.26$ and -0.52. For $d_t > 0$ the agent will always choose precision that leads to weight zero because here I plot all the objects under $\sigma_{2,\xi} = 0.001$. Horizontal axis is standard error of noise on *future (SPF) signal*, higher s.e. leads to lower weight. Vertical dashed line in the right panel labels optimal s.e. for *future (SPF) signal* in three scenarios.

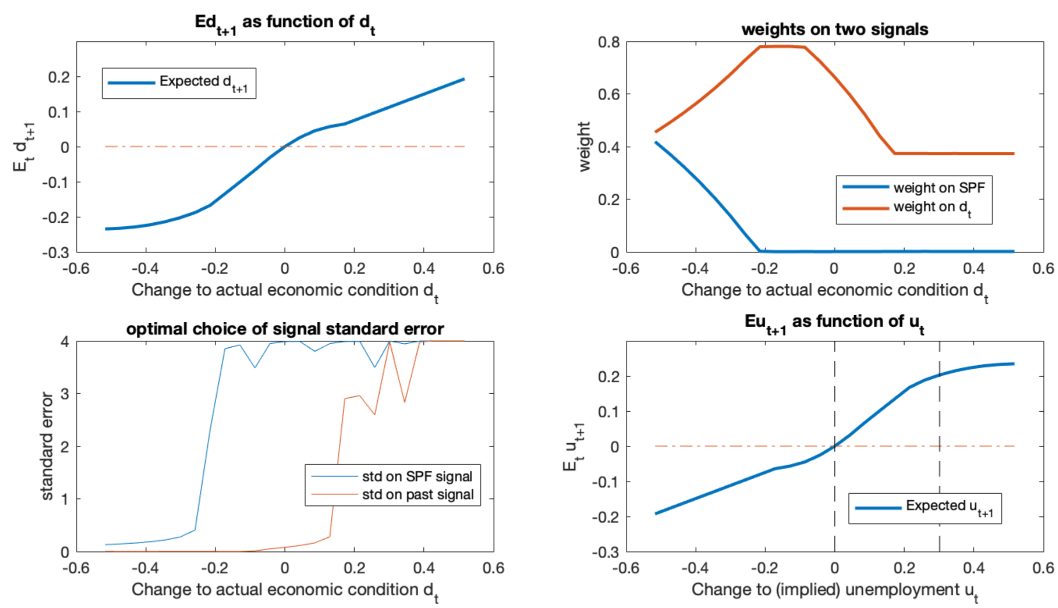
5.5.2 Model Results

For direct comparison with my empirical finding, I first show counterfactual of expectation on d_{t+1} as a function of change to d_t, holding other signals at constant. I present it together with the agent's optimal choices of signal variances as well as the model implied weights on

[48]However, changing the level of $\sigma_{2,\xi}$, in this case, will not change the results qualitatively.

current (d_t) and future (SPF) signals. Recall the weights are computed directly from (33) using Kalman Filter. They are functions of model parameters as well as the endogenously chosen signals precisions. Specifically, the higher the precision on a signal, the higher the weight will be.[49] These results are included in Figure 12.

Figure 12: Results from Rational Inattention Model



Top left panel: expected state of economy Ed_{t+1} as function of current state d_t. Top right panel: red line is weight on past/current signal d_t, blue line is weight on future signal SPF. Bottom right panel: chosen standard deviation of noise attached to corresponding signal. Red line is for signal on d_t, blue line is for signal on SPF. Bottom right panel: Implied expected unemployment as function of current unemployment. This is done by considering unemployment state as the opposite of d_t. It is used to directly compare with Figure 2

Top left panel of Figure 12 can be seen as model implied Average Structural Function of agent's expectation formation process. It describes how expected future state $E_t d_{t+1}$ changes along the change of current state d_t. When realized d_t is high and positive, the slope of this function is quite flat. This is because agent believes it is more likely the state in future will be good, which indicates the return on risky asset is high in expectation. With this prior, more information is not valuable enough for agents thus they are not acquiring accurate signals on either current state d_t or SPF. This can be seen from bottom left panel: under this parametrization, any signal with noise variance higher than 1 implies almost 0 weight

[49] I include the analytical derivation of the weights in Appendix D.3.

on this signal. When current state is good ($d_t > 0.2$) agent chooses variance on both signals to be higher than 10. The weight agent put on signal is depicted in top right panel. The reason why weight on d_t is not 0 is because of the initial signal on d_t that agent gets, before he chooses extra signals in the rational inattention model. This suggests when economic condition is good, agent will be happy to just form fuzzy expectation about future through the initial signal he gets, rather than actively searching for more information.

As the economic condition starts to get worse, in the area where $-0.2 < d_t < 0.2$, the slope of ASF gets most steep. This reflects the increasing weight agent puts on current signal about d_t. As agent realizes economic condition today is getting worse and worse (through observing the initial signal on d_t), information becomes more and more valuable and he is willing to pay higher cost to acquire more precise signals. This can be seen from bottom left graph that standard error on extra signals that agent chooses starts to fall sharply (which means precision of signal increases drastically) when current condition becomes worse. One interesting aspect is that they always get more accurate signal on d_t first before they go for SPF signal. This is because the information cost is increasing as agent's posterior getting more accurate. SPF signal contains more accurate information about future state thus is more costly for agents to get.

Finally when current economic condition is bad enough, when $d_t < -0.2$, agent gets more accurate signals on SPF. And because SPF has higher information content agent will start to put higher weights on signal about future (SPF) and lower weights on signal about current state d_t. Such a structure then created the non-linear ASF as I observed from survey data. Furthermore, it also generates the asymmetric response to good and bad states: as for positive realization of state d_t, agent has less incentive to acquire more information on it and end up attaching lower weights to the signal. This results in a lower mean expectation on d_{t+1}. On the other hand, when realization of d_t is bad, agent actively search for more information and put higher weights on these signals thus his expectation responds to bad states more than good ones.

The right bottom panel is then the ASF for implied unemployment expectation from the model. I consider $-d_t$ as a proxy for unemployment status because d_t can be interpreted as output growth and it is in general negatively correlated with unemployment. By doing this I can create the ASF for unemployment rate, which has the same dynamic as the one I found with RNN.

The time-variation of weights on signals is then reflected in top right panel of Figure 12: the weight on future signal (SPF) starts to increase when economic condition gets worse, meanwhile weight on past signals falls. To better illustrate this property of the model, I simulate the time series of d_t according to equations (15)-(17) for 200 periods[50]. Similar to the empirical part, I define episodes where d_t is 2 standard deviation lower than its mean as "bad periods". I then compute the average weight agent puts on past signal d_t and future signal $F_t d_{t+1}$, together with the optimal standard deviation of noise on each signal. Table 6 summarizes these statistics.

Table 6: Model Implied Weights and Precision during Bad and Ordinary Periods

	Bad Times		Ordinary Times	
Signal on:	Weights	Std. of noise	Weights	Std. of noise
Past/Current signal d_t	0.35	7e-4	0.57	0.94
Future signal $F_t d_{t+1}$	0.55	0.10	0.04	3.13

* Bad time is defined as periods in which d_t is 2 standard deviation lower from its long-run mean, 0. The rest episodes are considered as ordinary time. Weights are average model-implied weight on corresponding signal, during bad or ordinary time. Std. of noise is average model-implied standard deviation of noise on corresponding signal, during bad or ordinary time.

It is obvious in Table 6 that the model implies in ordinary period, agent will on average put higher weight on signals about past and current state when comparing to bad times. The average weight on d_t is 0.57, almost twice as high as that when economic condition is bad. Furthermore, agent puts much higher weight on signals about future during bad times, whereas almost no weight at all during ordinary time. The standard deviation of noise chosen by a rational inattentive agents then suggests such attention shift is induced by them optimally choosing much more accurate signals during bad times, whereas they choose to stay less informed during ordinary periods.

[50]The bad periods account for 12 out of 200 periods of simulate d_t, which is similar to the recession periods as a fraction of post 1980 episode.

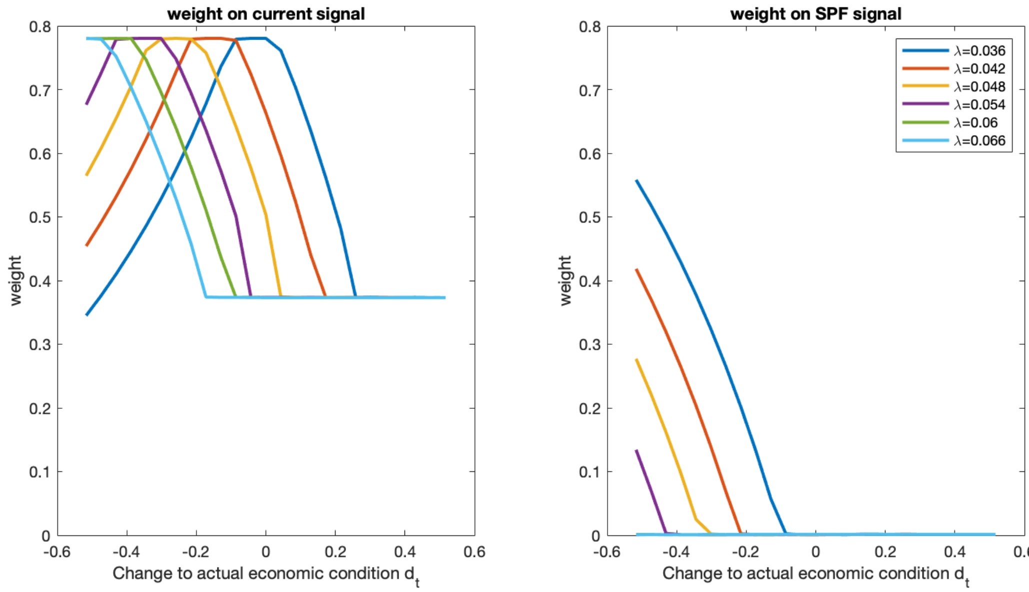
5.5.3 Impact of Marginal Information Cost λ

In Section 5.5.2 above, I illustrate how the information friction imposed in this particular rational inattention model can create both state-dependency and non-linearity in the agent's expectation formation process. I now turn to explore how the marginal information cost parameter λ affects the optimal signal precisions that the agent will choose. As discussed in Section 5.4.2, λ describes how easy it is for the agent to acquire information. The higher λ is, the higher the cost for the same level of signal precision.

Figure 13 shows the optimal weights that the agent puts on current and SPF signals. As the weight is an increasing function of the optimal precision on the corresponding signal, one should associate higher weight with higher precision on the same signal. In the left panel of Figure 13, I present the weight on the current signal as a function of the current state of the economy, d_t, for six different values of λ. It shows that at different values of marginal information cost, the state-dependency of weights on the current signal holds true qualitatively. The agent will still choose higher precision (weight) on the current signal when d_t is getting lower first, then decrease the weight on that same signal when d_t decreases further, but only because they start to put more attention on signals about SPF. When the marginal cost of information is **higher**, the agent will start to pay attention on the current signal at a **worse** state d_t. For example, when $\lambda = 0.036$ (the lowest in Figure 13), the agent will choose to put excess weight on current signal when $d_t < 0.26$.[51] Whereas when $\lambda = 0.066$, the agent will start to pay extra attention to current signal at a worse state, when $d_t < -0.17$. Intuitively, this is because when the marginal cost of information is higher, the originally affordable precision level becomes sub-optimal. She will only choose to acquire that same precision level when the benefit is high enough, which is when the current state d_t gets worse.

[51] Notice in both Figure 13 and Figure 12, there is a constant positive weight on current signal before the agent starts to pay extra attention to current signal. This is because when computing the weight on the current signal here I include the weight that the agent puts on the passive signal, z_0, which contains information about d_t. In this case, even when the agent chooses $\sigma_{2,\xi} = \infty$, we will still observe him responding to the change of state d_t in the data.

Figure 13: Weights on Signals: Changing Marginal Information Cost λ



Left panel: model implied weights on *current signal* as function of actual economic condition d_t. Right panel: model implied weights on *SPF signal*. Each set of weights corresponds to a different value of marginal information cost λ. The baseline results come from $\lambda = 0.042$.

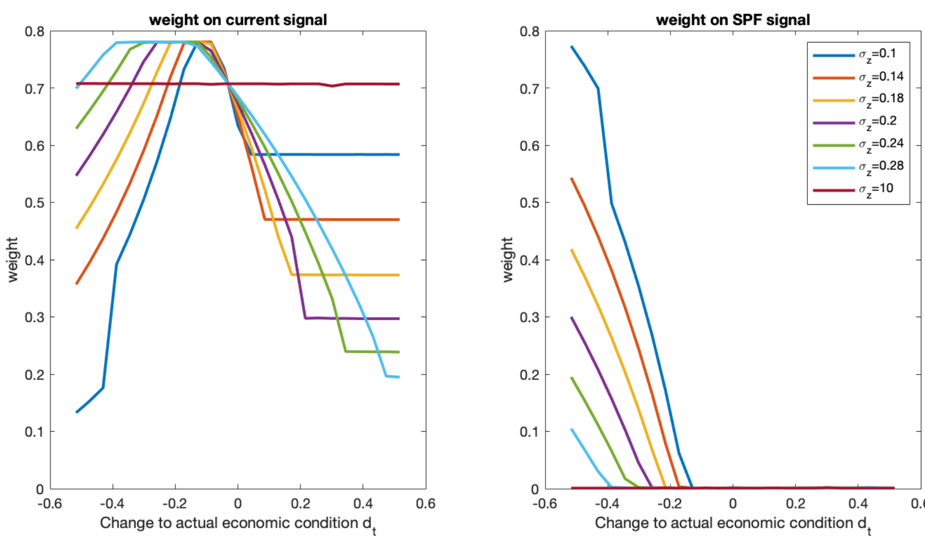
The right panel of Figure 13 is a similar graph but for weights on SPF signal. We see the same pattern as in the case for the current signal that when information cost gets higher, the agent will start to pay attention to the SPF signal at a worse state d_t. Also similar to the results of baseline in Figure 12, for each given λ, the agent will always choose to pay attention to the current signal first as the current state gets worse and shift her attention to SPF when the state is bad enough. This is due to the fact that for the same precision of signal on SPF, it contains more information that helps to predict d_{t+1}. This leads to more entropy reduction thus higher entropy cost κ. Because the benefit of information increases gradually as state d_t gets worse, the agent will only start to choose signals with high entropy costs when the current state gets bad enough.

5.5.4 Impact of Passive Signal Precision σ_z

Another important parameter in the information friction proposed in this chapter is the precision of the passive signal that the agent is exposed to. This passive signal z_0 will contain information about the current state d_t thus making the agent's optimal choice of precision dependent on this state. The precision of this signal then determines the prior variances that the agent considers to evaluate the benefit and cost for more information.[52] Through this channel, it will also affect the agent's optimal choices on signal precisions.

In Figure 14 I show again the optimal weights on current and SPF signals as functions of current state d_t, with different values of σ_z. The left panel shows the results for weights on current signals. First, notice higher precision on the passive signal (lower σ_z) leads to higher weight on the current signal to start with. This shows up in the figure as a higher weight in the flat area before the agent puts excess weight on the current signal. Intuitively, this means the agent already has a better understanding of the current d_t before choosing the extra signals on the current state and SPF. This leads to the fact that in the right panel, the agent with the lowest σ_z will start to pay attention to the SPF signal at a relatively higher state, because the information cost for choosing that precision level is relatively lower to her. An extreme example will be that when $\sigma_z = 0$, which implies that the agent has perfect information on d_t. In this case, we will see her only choosing an extra signal on SPF starting from a relatively high value of d_t.

Figure 14: Weights on Signals: Noise on Passive Signal σ_z



Left panel: model implied weights on *current signal* as function of actual economic condition d_t. Right panel: model implied weights on *SPF signal*. Each set of weights corresponds to a different standard error of noise in the passive signal σ_z. The baseline results come from $\sigma_z = 0.18$.

[52]For analytical illustration please refer to Appendix D.3.

Another interesting aspect in Figure 14 is that when the quality of the passive signal is very low so that σ_z is quite high, the agent's optimal choices of signal precisions will not depend on the current state d_t anymore. This is because the passive signal z_0 contains almost no information about d_t before the agent chooses her information set. As a result, the agent will not be able to choose different precisions according to the realization of d_t. This result is also shown in Figure 14 as the case for $\sigma_z = 0$. Moreover, in this case, the agent will not necessarily choose a very noisy signal about the current state. The optimal precisions will depend on the prior mean of the agent, which is $\hat{\boldsymbol{X}}_0 = \mathbf{0}$ as in the baseline results. Such a pattern then has an important implication: the weights on signals will not only depend on the realized current state of the economy d_t, it will also depend on the prior mean that the agent carried on across time. I will illustrate how the optimal weights change with the prior mean in the next subsection.

5.5.5 Impact of "Internal State": Prior Mean

Intuitively, when an agent chooses the information set she uses to form expectations, her ex-ante belief about the future state should matter. This can be seen directly from (33): when the agent thinks about future state d_{t+1} before she chooses information set that will generate $\boldsymbol{Z}_t$, her effective prior mean should be:

$$\hat{\boldsymbol{X}}_{t|0} = (I - K_0 G_0)\hat{\boldsymbol{X}}_0 + K_0 z_0 \tag{36}$$

From previous sections, I have shown that the current state of economy d_t will affect her choice of optimal precision through the passive signal z_0. For the same reason, the optimal precision on signals should depend on the prior mean $\hat{\boldsymbol{X}}_0$ as well.

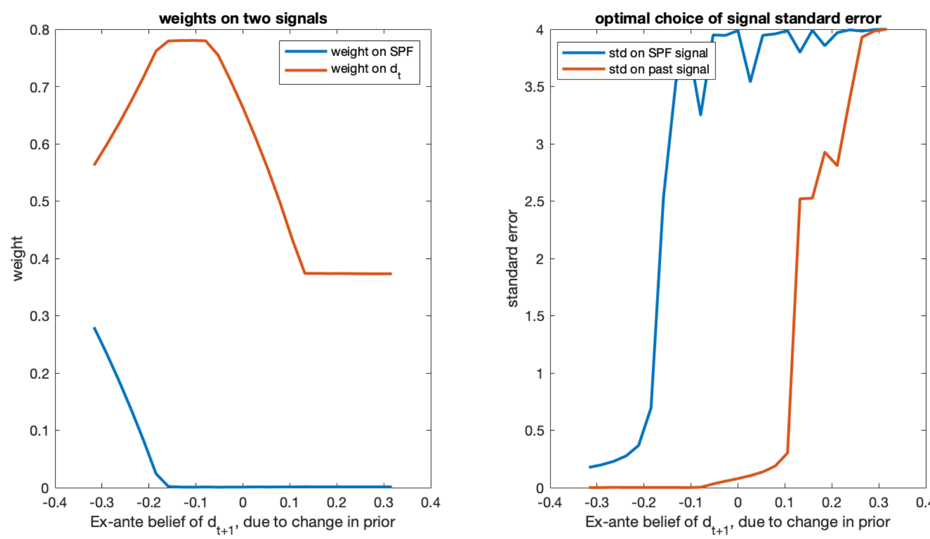
The illustration of the impact of prior mean involves several parts. First I want the change of optimal precision to come solely from the differences of $\hat{\boldsymbol{X}}_0$, so I will keep z_0 at a fixed value. Secondly, the reason why the prior mean will affect the optimal precision choice is that the agent will use the information set $\{\mathcal{I}_0\} = \{\boldsymbol{X}_{t|0}\}$ to evaluate her expected utility. The prior mean vector $\hat{\boldsymbol{X}}_0$ will affect this information set thus affecting the agent's expected benefit for any precision level. As discussed in section 5.3, when the prior makes the agent believe on average the future state will be worse, she will choose a signal will higher precision.

A straightforward way to illustrate this point is to depict the optimal weights and standard errors of the signals as functions of the implied posterior mean on d_{t+1} using the ex-ante information set $\mathcal{I}_0$. For simplicity, I call this "ex-ante belief on d_{t+1}", defined as:

$$\mathbb{E}[d_{t+1}|\mathcal{I}_0] = \iota A\Big((I - K_0G_0)\hat{\boldsymbol{X}}_0 + K_0z_0\Big) \tag{37}$$

In Figure 15, I show the optimal weights and standard errors as function of $\mathbb{E}[d_{t+1}|\mathcal{I}_0]$, while fixing $z_0 = 0$ and $d_t = 0$. This means the variation of the ex-ante belief comes solely from the differences in $\hat{\boldsymbol{X}}_0$.

Figure 15: Weights and Standard Error of Signals: Functions of Prior Beliefs



Left panel: model implied weights on current and future signal as functions of prior mean beliefs on the future state. Right panel: model implied standard error of the noises attached to current and future signal. The blue curves are for future (SPF) signal and red curves are for current signal.

Figure 15 shows that the optimal choices of weights (left panel) and precision (in terms of standard error, right panel) indeed depend on the agent's prior belief. In particular, when the prior belief leads to on average a bad state in the future, the agent will first pay more attention to the current signal, then shift to SPF signal as the implied state getting worse. This piece of evidence is also consistent with my empirical finding. As the prior mean is accumulated from the history of signals and usually not observable in the data, it can then be thought of as a proxy of the "internal state" in my empirical section. As discussed in Section 4.2.4, both the current state of the economy and the internal state accumulated from the past signals play a role in creating the state-dependent marginal effects of signals.

6 Conclusion

How do households form expectations using a rich set of macroeconomic signals? This paper explores the answer to this question by proposing an innovative Generic Learning Framework that is flexible in functional forms and time-dependency that describe the relationship between signals and expectational variables. The unknown function form of agents' expectation formation model is estimated with a state-of-art Recurrent Neural Network. This method can recover any function forms considered by the Generic Learning Framework, including those most commonly used in the learning literature, such as noisy information, constant gain learning, and markov switching models. After the functional estimation, I also obtain estimators on the average marginal effects of signals with valid inferences following the Double Machine Learning approach developed by Chernozhukov *et al.* (2018) .

Applying this method to survey data for US households, I document three stylized facts that are new to the literature: (1) agents' expectations about future economic condition is a non-linear and asymmetric function of signals on well-being of the economy. (2) The attention to past and future signals in the Generic Learning Model are highly state-dependent. The agents behave like adaptive learners in ordinary periods and become forward-looking as the state of the economy gets worse. (3) Among all the signals considered in the empirical setup, signals on economic conditions play the most important role in creating the attention-shift. These findings are at odds with many models widely used in the literature, such as noisy information models and constant gain learning models. These findings then suggest the dynamics of economic variables may be largely different in ordinary and recession episodes as the information contents in households' expectations differ as the state of the economy changes. Another implication is that policy that features forward guidance and expectation management may be less effective as economic conditions become stable as agents pay less attention to information about the future.

Finally, a rational inattention model is developed to match these news stylized facts and help illustrate the impact of attention-shift on agents' expectation formation process. The model highlights that agent's optimal choice of signal precision is a decreasing function of the current state of the economy due to non-linearity in their optimal saving choices. This information friction leads to the agent allocating more efforts to get information about the future when the economic condition is bad today. Such behavior makes them put higher weights on signals about the future and lower weights on information about current and past states. This information friction then is enough to generate both non-linear, asymmetric expectation and state-dependent weights on signals documented in the empirical findings.

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Appendices

A Proof and Derivation

Proof of Theorem 1:

From (4), the average structural function is written as:

$$y_{i,t+1|t} \equiv \mathbb{E}_{\{\epsilon_{i,\tau}\}_{\tau=0}^{t}}[Y_{i,t+1|t}]$$

Under independence assumption 2, this is equivalent to counterfactual conditional expectation functions $\mathbb{E}[Y_{i,t+1|t}|\{Z_{i,\tau}\}_{\tau=0}^{t}]$. This conditional expectation function can then be written as:

$$\begin{aligned}
\mathbb{E}[Y_{i,t+1|t}|\{Z_{i,\tau}\}_{\tau=0}^{t}] &= \int F(\Theta_{i,t}) d\mathcal{F}_{\Theta_{i,t}}(\Theta_{i,t}|\{Z_{i,\tau}\}_{\tau=0}^{t}) \\
&= \int F(\Theta_{i,t}) \mathcal{P}_{\Theta_{i,t}}(\Theta_{i,t}|\{Z_{i,\tau}\}_{\tau=0}^{t}) d\Theta_{i,t} \\
&= \int \left(\int F(\Theta_{i,t}) \mathcal{P}_{\Theta_{i,t}}(\Theta_{i,t}|\{Z_{i,\tau}\}_{\tau=0}^{t}, \Theta_{i,t-1}) d\Theta_{i,t} \right) \mathcal{P}_{\Theta_{i,t-1}}(\Theta_{i,t-1}|\{Z_{i,\tau}\}_{\tau=0}^{t}) d\Theta_{i,t-1}
\end{aligned} \tag{38}$$

Where in equation (38), the first equality holds from Assumption 2, that the expectation formation process admits a latent variable structure, with a finite dimensional latent variable $\Theta_{i,t}$. The conditional CDF of variable X is represented by $\mathcal{F}_X$ and conditional PDF is represented by $\mathcal{P}_X$. The third equality then holds from Bayes Rule.

Now consider the conditional PDF $\mathcal{P}_{\Theta_{i,t}}(\Theta_{i,t}|\{Z_{i,\tau}\}_{\tau=0}^{t}, \Theta_{i,t-1})$, under assumption 2 it can be represented by PDF with respect to the i.i.d random variable $\epsilon_{i,t}$:

$$\begin{aligned}
\mathcal{P}_{\Theta_{i,t}}(\Theta_{i,t} = r'|\{Z_{i,\tau}\}_{\tau=0}^{t}, \Theta_{i,t-1}) &= \mathcal{P}_{\epsilon_{i,t}}(H(\Theta_{i,t-1}, Z_{i,t}, \epsilon_{i,t}) = r'|Z_{i,t}, \Theta_{i,t-1}) \\
&= \mathcal{P}_{\Theta_{i,t}}(\Theta_{i,t} = r'|Z_{i,t}, \Theta_{i,t-1})
\end{aligned} \tag{39}$$

Furthermore, as $\epsilon_{i,t}$ is i.i.d across time, this conditional probability is time-homogenous conditional on same realization of $Z_{i,t}$:

$$\begin{aligned}\mathcal{P}_{\Theta_{i,t}}(\Theta_{i,t}=r'|Z_{i,t}=z,\Theta_{i,t-1}=r) &= \mathcal{P}_{\epsilon_{i,t}}(H(r,z,\epsilon_{i,t})=r') \\ &= \mathcal{P}_{\epsilon_{i,t+s}}(H(r,z,\epsilon_{i,t+s})=r') \\ &= \mathcal{P}_{\Theta_{i,t+s}}(\Theta_{i,t+s}=r'|Z_{i,t+s}=z,\Theta_{i,t+s-1}=r) \quad \forall s>0\end{aligned} \tag{40}$$

In other words, from (39) and (40) the latent variable $\Theta_{i,t}$ is a time-homogenous Markov Process conditional on realization of $Z_{i,t}$. Now one can discretize the continuous-state Markov Process[53]. Denoting the grid points obtained for $\Theta_{i,t}$ as $D_r = \{x_r\}_{r=1}^{N_r}$ and corresponding transition probability from state r to r' as $\{p_{r,r'}(z)\}$. Notice the transition probability will be a function of realized signal in each period z, so that for different realization of Z, we have a different Markov Chain. Now consider a finite dimensional variable:

$$\theta_{i,t}^r = \mathcal{P}_{\Theta_{i,t}}(\Theta_{i,t}=x_r|\{Z_{i,\tau}\}_{\tau=0}^t) \quad \forall r\in\{1,...,N_r\}$$

Then it follows immediately from (38) that:

$$y_{i,t+1|t} = \mathbb{E}[Y_{i,t+1|t}|\{Z_{i,\tau}\}_{\tau=0}^t] = \sum_{r=1}^{N_r} F(x_r)\theta_{i,t}^r = f(\theta_{i,t})$$

Where the last equation is the definition of $f(.)$ function in theorem 1.

And it is obvious that $\theta_{i,t}$ is a function of history of signals $\{Z_{i,\tau}\}_\tau^t$, and it explicitly depends on $\theta_{i,t-1}$ as well as $Z_{i,t}$. This can be easily seen by induction, for $t=0$:

$$\theta_{i,0}^r = \mathcal{P}_{\Theta_{i,0}}(\Theta_{i,0}=x_r|\Theta_{i,-1},Z_{i,0})$$

For $t=1$:

$$\begin{aligned}\theta_{i,1}^{r'} &= \mathcal{P}_{\Theta_{i,1}}(\Theta_{i,1}=x_{r'}|\Theta_{i,-1},Z_{i,0},Z_{i,1}) \\ &= \sum_{r=1}^{N_r}\mathcal{P}_{\Theta_{i,1}}(\Theta_{i,1}=x_{r'}|\Theta_{i,0}=x_r,Z_{i,1}=z)\mathcal{P}_{\Theta_{i,0}}(\Theta_{i,0}=x_r|\Theta_{i,-1},Z_{i,0}) \\ &= \sum_{r=1}^{N_r} p_{r,r'}(z)\theta_{i,0}^r\end{aligned}$$

[53]Following Farmer and Toda (2017), one can discretize non-linear non-Gaussian Markov Process and match exact conditional moments of the process, which is the same as my goal here. The details for the discretization procedure are included in Algorithm 2.2 from their paper.

Where the second equality from above follows from Markov Property (39) then with time-homogeneity (40), one can get time t relation by induction:

$$\theta_{i,t}^{r'} = \sum_{r=1}^{N_r} p_{r,r'}(Z_{i,t})\theta_{i,t-1}^{r} \tag{41}$$

Equation (41) can be summarized as $\theta_{i,t} = h(\theta_{i,t-1}, Z_{i,t})$ from theorem 1. $\square$

B Double De-biased Machine Learning Estimator

In this section I follow the semi-parametric moment condition model of Chernozhukov *et al.* (2018) and Chernozhukov *et al.* (2017). This is a general formulation that can be applied to estimation problems that involve:

- A finite dimensional parameter of interest – the average marginal effect defined in (7) β;
- Nuisance parameters that is usually infinite dimensional, denoted as η;
- Moment Condition that is (near) Neyman Orthogonal, denoted as $\mathbb{E}[\psi(W, \beta, \eta)]$, where $W = \{Y, X\}$ are the data observed;

I first focus to derive the Neyman Orthogonal Moment Condition for the estimation problem of average marginal effect. Throughout this appendix, denote $\ell(.)$ as objective function, g_t as average structural function that can be written as $g(\{X_{i,\tau}\}_{\tau=0}^{t}, \theta_{-1}) = f(h(X_{i,t}, \theta_{i,t-1}))$, $g_{t,x}^{j}$ as partial derivative of g_t with respect to j-th element of X, then $P(.)$ as the joint density function of input variables X. Suppose the true functional form of Average Structural Equation is $\mathbb{E}[Y_{i,t+1|t}|\{X_{i,\tau}\}_{\tau=0}^{t}] = g_{t,0}$ and the parameter of interest for each j-th element of the vector of average marginal effect $\mathbb{E}[\frac{\partial g_{t,0}}{\partial X^{j}}] = \beta_{t,0}^{j}$.

B.1 Neyman Orthogonal Moment Condition

1. Begin by declaring joint objective function, **at each time point** t, denote $X \equiv \{X_{i,\tau}\}_{\tau=0}^{t}$ for short-hand:

$$\min_{\beta_t^j, g_t} \quad \mathbb{E}[\ell(\{Y, X, \theta_{-1}\}; \beta_t, g_t)]$$

$$\ell(\{Y,X,\theta_{-1}\};\beta_t,g_t) = 1/2(y-g_t(X,\theta_{-1}))^2 + \sum_j 1/2(\beta_t^j - g_{t,x}^j(X,\theta_{-1}))^2$$

Following Chernozhukov *et al.* (2018), the only requirement for objective function is the true value $g_{t,0}$ and $\beta_{t,0}^j$ for $\forall j$ minimize the objective function.

2. Concentrated-out non-parametric part:

$$g_{t,\beta_t} = argmin_g \mathbb{E}[\ell(\{Y,X,\theta_{-1}\};\beta_t,g_t)]$$

Need to derive g_{t,β_t} using functional derivative. Notice:

$$\begin{aligned}\mathbb{E}[\ell(\{Y,X,\theta_{-1}\};\beta_t,g_t)] &= \int \mathbb{E}[\ell()|X,\theta_{-1}]P(X,\theta_{-1})d(X,\theta_{-1}) \\ &\equiv \int \mathcal{L}(\{X,\theta_{-1}\};\beta_t,g_t,g_{t,x})d(X,\theta_{-1})\end{aligned} \tag{42}$$

Using Euler-Lagrangian Equation:

$$\begin{aligned}0 &= \frac{\partial \mathcal{L}}{\partial g_t} - \sum_j^J \frac{\partial}{\partial x_t^j}(\frac{\partial \mathcal{L}}{\partial g_{t,x}^j}) \\ &= \underbrace{-(\mathbb{E}[Y|X,\theta_{-1}] - g_t(X,\theta_{-1}))P(X,\theta_{-1})}_{\equiv \frac{\partial \mathcal{L}}{\partial g_t}} \\ &- \sum_j^J \frac{\partial}{\partial x_t^j}\underbrace{\Big(-(\beta_t^j - g_{t,x}^j(X,\theta_{-1}))P(X,\theta_{-1})\Big)}_{\equiv \frac{\partial \mathcal{L}}{\partial g_{t,x}^j}} \\ &= -(\mathbb{E}[Y|X,\theta_{-1}] - g_t(X,\theta_{-1}))P(X,\theta_{-1}) + \\ &\sum_j^J \Big(-g_{t,xx}^j(X,\theta_{-1})P(X,\theta_{-1}) + \frac{\partial P(X,\theta_{-1})}{\partial x_t^j}(\beta_t^j - g_{t,x}^j(X,\theta_{-1}))\Big)\end{aligned} \tag{43}$$

The concentrated-out non-parametric part at time t then is given by:

$$g_{t,\beta_t}(X,\theta_{-1}) = \mathbb{E}[Y|X,\theta_{-1}] + \sum_j^J \Big(g_{t,xx}^j(X,\theta_{-1}) - \frac{\partial ln[P(X,\theta_{-1})]}{\partial x_t^j}(\beta_t^j - g_{t,x}^j(X,\theta_{-1}))\Big)$$

3. Concentrated Objective at each time t:

$$\min_{\beta_t} \mathbb{E}[1/2(Y - g_{t,\beta_t}(X,\theta_{-1}))^2 + \sum_j 1/2(\beta_t^j - g_{t,\beta_t,x}^j(X,\theta_{-1}))^2]$$

Take F.O.C with respect to β_t^j and evaluate at $g_{t,\beta_t} = g_{t,0}$:

$$\mathbb{E}[\beta_t^j - g_{t,0,x}^j(X, \theta_{-1}) + \frac{\partial ln(P(X, \theta_{-1}))}{\partial x^j}(Y - g_{t,0}(X, \theta_{-1}))] = 0$$

Now notice two things here:

- In this set-up basically at each time t the $g_{t,0}()$ function is different, so that β_t is different as well. Without proper regularity the g function could be non-stationary. This is when the markov assumptions come to play. The assumptions with $f()$ and $h()$ functions basically interpret the time-varying β_t is because of different states θ_{t-1}.
- With the previous approach, we get moment condition of β_t^j instead of β^j,they are different because (1) $g_{t,0}(.)$ function is different at each t; (2) $P(X, \theta_0)$ is changing at each t.

The first problem is solved by the Markov property and hidden variable:

$$g_{t,0}(\{X_{i,\tau}\}_{\tau=0}^t, \theta_{i,-1}) \equiv \mathbb{E}[Y_{i,t+1|t}|\{X_{i,\tau}\}_{\tau=0}^t, \theta_{i,-1}] = f(\theta_{i,t}) = f(h(\theta_{i,t-1}, X_{i,t}))$$

Plug this into the moment condition:

$$\mathbb{E}[\beta_t^j - f \circ h_{x^j}(\theta_{i,t-1}, X_{i,t}) + \frac{\partial ln(P(X, \theta_{-1}))}{\partial x^j}(Y - f \circ h(\theta_{i,t-1}, X_{i,t})] = 0 \tag{44}$$

The second problem can be solved by assuming dependency of $X_{i,t}$ and $X_{i,t-s}$. As θ_{-1} are assumed to be zeros in practice, which is deterministic. Here I assume variables $X_{i,t}$ follow a VAR(1) process so that:

$$\begin{aligned} P(X_{i,t}, X_{i,t-1}, ..., X_{i,0}) &= P(X_{i,t}|X_{i,t-1}, ..., X_{i,0})P(X_{i,t-1}|X_{i,t-2}, ..., X_{i,0})...P(X_{i,0}) \\ &= P(X_{i,t}|X_{i,t-1})P(X_{i,t-1}|X_{i,t-2})...P(X_{i,0}) \end{aligned}$$

This leads to the fact that $\frac{\partial ln(P(X_{i,t},X_{i,t-1}))}{\partial X_{i,t}^j} = \frac{\partial ln(P(X_{i,t},X_{i,t-1},...X_{i,0}))}{\partial X_{i,t}^j}$. For this reason, in practice, I just need to estimate the joint density function $P(X_{i,t}, X_{i,t-1})$. Then equation (44) leads to moment condition (9) given the fact that $g(\{X_\tau\}_{\tau=0}^t, \theta_{-1}) \equiv f \circ h(\theta_{i,t-1}, X_{i,t})$.

B.2 Verifying Moment Condition is Orthogonal

This can be done by computing the Frechet Derivative with respect to nuisance parameter g of the moment condition $\mathbb{E}[\psi(W,\beta,\eta)]$, notice that $\eta = \{g, P\}$. The estimate of g will later be obtained from RNN. For sake of simplified notation, I drop the t and consider 1 dimensional case, but the appliction can be easily extended to multidimensional case.

$$\psi(W,\beta,\eta) \equiv \beta - g'(X) + \frac{P'(X)}{P(X)}(\mathbb{E}[Y|X] - g(X)) \tag{45}$$

Define functional $F : C(\mathbb{R}) \to C(\mathbb{R})$:

$$F(g)(\beta, X) = \mathbb{E}[\psi(W,\beta,\eta)]$$

The Frechet Derivative along direction v is given by:

$$\begin{aligned}
F(g+v) - F(g) &= \mathbb{E}[-v'(X) - \frac{P'(X)}{P(X)}v(X)] \qquad (46)\\
&= \lim_{\delta\to 0}\mathbb{E}[-\frac{v(X+\delta)-v(X)}{\delta} - \frac{P(X)-P(X-\delta)}{P(X)\delta}v(X)]\\
&= \lim_{\delta\to 0} 1/\delta[-\int_X v(X+\delta)P(X)dX + \int v(X)P(X)dX\\
&- \int v(X)P(X)dX + \int v(X)P(X-\delta)dX]\\
&= \lim_{\delta\to 0} 1/\delta[\int_y v(y+\delta)P(y)dy - \int_x v(x+\delta)P(x)dx]\\
&= 0
\end{aligned}$$

B.3 High Level Assumptions on Nuisance Parameters

To ensure the asymptotic property of estimate $\hat{\beta}$ obtained from DML approach to hold, I refer to Theorem 3.1 from Chernozhukov *et al.* (2018). First denote the moment condition derived in **Appendix B.1** as $\psi(W,\beta,\eta)$, where β is the parameter of interest, X is data in use and $\eta = \{g, P\}$ are nuisance parameters estimated from functional estimation, where $g(.)$ is ASF and $P(.)$ is joint density function of X. To apply this theorem one needs to verify three condition[54]:

[54] In Chernozhukov *et al.* (2018) these conditions are defined formally by their Assumption 3.1 and 3.2.

1. Moment condition(scores) is linear in parameter of interest, β:

$$\psi(W, \beta, \eta) = \psi^a(W, \eta)\beta + \psi^b(W, \eta)$$

2. (Near) Neyman Orthogonality of score $\psi(W, \beta, \eta)$;

3. Fast enough convergence of nuisance parameters $\eta = \{g, P\}$. Notice such condition is formally described by Assumption 3.2 in Chernozhukov *et al.* (2018). And the authors discussed the sufficient conditions for this assumption to hold: ψ is twice differentiable and $\mathbb{E}[(\hat{\eta}(X) - \eta_0(X))^2]^{1/2} = o(n^{-1/4})$. And the variance of score ψ, $\mathbb{E}[\psi(W, \beta, \eta)\psi(W, \beta, \eta)']$ is non-degenerate.

Condition 1 is obvious given the Neyman Orthogonal score derived in Appendix B.1: equation (45) is linear in β. Condition 2 is verified in Appendix B.2.

The convergence speed requirement in condition 3 needs a bit of work. In practice $g(.)$ function will be estimated by RNN and $P(.)$ function is estimated with gaussian kernel density estimation. For RNN the convergence speed of estimate $\hat{g}$ is offered by Theorem 1 of Farrell *et al.* (2018). To achieve the convergence speed described there, one needs to put restrictions on width and depth of neural network used to approximate $g(.)$. Specifically, for input dimension d, sample size n and smoothness of function $g(.)$, θ, one needs width $H \asymp n^{\frac{d}{2(\theta+d)}}$ and depth $L \asymp \log n$. These conditions will guarantee a convergence speed on a level of $\{n^{-\theta/(\theta+d)} \log^8 n + \frac{\log \log n}{n}\}$ which is faster than $n^{-1/2}$.[55] My baseline architecture satisfies these restrictions.

For convergence speed of joint density $P(.)$, it is estimated by gaussian kernel density estimation with Silverman Rule of Thumb for bandwidth selection. Denote the order of gaussian kernel as ν, and the input dimension of density function $P(.)$ as d' the asymptotic mean integrated squared error (AMISE) is known to be $O(n^{-2\nu/(2\nu+d')})$. The convergence speed requirement in condition 3 needs $2\nu/(2\nu + d') > 1/2$, or $\nu > d'/2$.[56] Notice the density function here is a joint density for $X_{i,t}$ and $X_{i,t-1}$ so its dimensionality is typically twice of the input for RNN. I then need to use a higher order gaussian kernel with at least $\nu = 28$ to ensure the convergence speed requirement for the density estimator.

[55] See Theorem 1 in Farrell *et al.* (2018) for details.

[56] See *Hansen (2009)* for details

Finally, after verifying all three pre-conditions, according to Theorem 3.1 from Chernozhukov *et al.* (2018), denoting the Jacobian matrix from the Neyman Orthogonal score as J_0 and the true value of nuisance parameter as η_0:

$$J_0 = \mathbb{E}[\psi^a(W, \eta_0)]$$

The DML estimator $\hat{\beta}$ is then centered at true values β_0 and are approximately linear and Gaussian:

$$\sqrt{n}\sigma^{-1}(\hat{\beta} - \beta_0) = \frac{1}{\sqrt{n}} \sum_{i}^{n} \bar{\psi}(W_i) \rightarrow N(0, I_d)$$

Where $\bar{\psi}(.)$ is the influence function of the form:

$$\bar{\psi}(W_i) = -\sigma^{-1} J_0^{-1} \psi(W_i, \beta_0, \eta_0)$$

The σ^2 is the variance that is given by:

$$\sigma^2 = J_0^{-1} \mathbb{E}[\psi(W_i, \beta_0, \eta_0)\psi(W_i, \beta_0, \eta_0)'](J_0^{-1})'$$

Notice in my case $\psi^a(W, \eta_0) = 1$ so that $J_0 = 1$. This is the same distribution as if we plug in the true nuisance parameters η_0 and is enough for asymptotic inferences.

B.4 Example: Standard Noisy Information Model in Generic Learning Framework

In this subsection I take the standard noisy information model as an example and show how it can be represented by the Generic Learning Framework. The purpose of this example are three folds. First it gives an example of essential elements in the Generic Learning Model including hidden states $\Theta_{i,t}$, Average Structural Function and the transformed dynamic system (6) in the context of a familiar learning model. Secondly it illustrates how RNN performs in approximating the ASF (in this case linear) and estimating marginal effect without knowledge of the exact functional form of learning model. Lastly as I consider a special case when the expectation formation structure is still linear but OLS is mis-specified and show the performance of RNN in estimating the average marginal effect. This exercise illustrates the possible improvement in using RNN even in a linear case.

Data Generating Process: Consider agents want to predict inflation one period from now denoted as $\pi_{i,t+1|t}$. At time t, they can observe two signals $\{\pi_{i,t}, s_{i,t}\}$. There are two latent variables $\{\pi_t, L_t\}$ that they need to make inference of to form expectation of inflation. Represent the Actual Law of Motion as a Gaussian Linear State Space Model:

$$\begin{bmatrix} \pi_t \\ L_t \end{bmatrix} \equiv X_t = \boldsymbol{A} X_{t-1} + \epsilon_t \tag{47}$$

Where $\boldsymbol{A}$ describes how latent states X_t evolves along time, ϵ_t is i.i.d shock each period. Assume for simplicity the agent's Perceived Law of Motion is the same as (47). Agents do not observe X_t directly, instead they observe a noisy signals about it. Their observational equation is:

$$\begin{bmatrix} \pi_{i,t} \\ s_{i,t} \end{bmatrix} \equiv O_{i,t} = \boldsymbol{G} X_t + \nu_{i,t} \tag{48}$$

Both shock ϵ_t and $\nu_{i,t}$ are i.i.d and follow normal distribution with covariance matrix R and Q:

$$\epsilon_t \sim N(0, R) \quad \nu_{i,t} \sim N(0, Q)$$

This describes the standard noisy information model with two latent states. They use a stationary Kalman Filter to form prediction of the latent variable $X_{i,t+1|t}$, where $\boldsymbol{K}$ is the Kalman Gain.

$$\begin{bmatrix} \pi_{i,t+1|t} \\ L_{i,t+1|t} \end{bmatrix} \equiv X_{i,t+1|t} = \boldsymbol{A}(X_{i,t|t-1} + \boldsymbol{K}(O_{i,t} - \boldsymbol{G} X_{i,t|t-1})) \tag{49}$$

The Generic Learning Formulation The stationary Kalman Filter is a special case of Generic Learning Model. First notice the i.i.d error $\nu_{i,t}$ satisfies assumption 2. The expectation is also formed by filtering step and updating step:

$$X_{i,t|t} = X_{i,t|t-1} + \boldsymbol{K}(O_{i,t} - \boldsymbol{G} X_{i,t|t-1}) \quad \text{(Filtering Step)}$$

$$X_{i,t+1|t} = \boldsymbol{A} X_{i,t|t} \quad \text{(Forecasting Step)}$$

Replace $X_{i,t+1|t}$ with $\hat{Y}_{i,t+1|t}$ and define the "now-cast" variable $X_{i,t|t}$ as latent state variable $\Theta_{i,t}$ in Generic Learning Model, we can re-write Kalman Filter (49) as equation (50) and (51), which reflect the generic formulation of updating step (2) and forecasting step (3). It is obvious that in the stationary Kalman Filter case, both $F(.)$ and $H(.)$ are linear.

$$\hat{Y}_{i,t+1|t} = \boldsymbol{A}\Theta_{i,t} \tag{50}$$

$$\Theta_{i,t} = (\boldsymbol{A} - \boldsymbol{KGA})\Theta_{i,t-1} + \boldsymbol{KG}X_t + \boldsymbol{K}\nu_{i,t} \tag{51}$$

Average Structural Function I then turn to the ASF implied by Kalman Filter (50) and (51). This is simply done by taking expectation of $\hat{Y}_{i,t+1|t}$ conditional on observables X_t. The goal is to integrating out the i.i.d noise term $\nu_{i,t}$ which is not observable by econometrician. Now we can define the sufficient statistics for $\Theta_{i,t}$ as:

$$\theta_{i,t} = \mathbb{E}[\Theta_{i,t}|\{X_\tau\}_{\tau=0}^t] \tag{52}$$

Taking the expectation of (50) and (51) conditional on history of the observable $\{X_\tau\}_{\tau=0}^t$ it immediately follows:

$$y_{i,t+1|t} \equiv \mathbb{E}[\hat{Y}_{i,t+1|t}|\{X_\tau\}_{\tau=0}^t] = \boldsymbol{A}\theta_{i,t}$$

$$\theta_{i,t} = (\boldsymbol{A} - \boldsymbol{KGA})\theta_{i,t-1} + \boldsymbol{KG}X_t$$

This illustrates the link between ASF with the underlying expectation formation model: in the linear case with mean zero error $\nu_{i,t}$, the function form from ASF, $f(.)$ and $h(.)$ are linear and are identical to those from the underlying expectation formation model.

Estimation with Simulated Sample Now suppose as econometricians we want to estimate marginal effect of two signals $\{\pi_t, s_{i,t}\}$ on $\pi_{i,t+1|t}$. The standard approach is to directly estimate the reduced-form equation derived from (49) with OLS. This requires $X_{i,t+1|t}$ observed for each t and the learning model is correctly specified. However in reality it is possible that expectation on latent state $L_{i,t+1|t}$ is not observable or not considered in the model[57]. If this is the case OLS with only lag term $\pi_{i,t|t-1}$ is included in the regression suffers from omitted variable problem.

On contrary, estimation with RNN does not require a correct specification on latent variable $\Theta_{i,t}$, and it doesn't need $L_{i,t|t-1}$ to be observable at all. To show this I simulated 100 random samples according to the Kalman Filter as in (49). In this experiment I consider three different models to estimate marginal effect of the two signals $\{\pi_t, s_{i,t}\}$: (1) the RNN with sequence of $\{\pi_\tau, s_{i,\tau}\}_{\tau=0}^{t}$ and lag expected inflation $\pi_{i,t|t-1}$ as input[58]; (2) mis-specified OLS that uses the same set of variables as dependent variable, the OLS is mis-specified because $L_{i,t+1|t}$ is not available to econometricians; (3) correctly specified OLS with $L_{i,t+1|t}$ observable, which is typically not available. I'll show RNN can still recover the linear relationship between signal and expectational variable as well as obtain comparable estimate on signals as the correctly specified OLS estimator (BLUE in this case), whereas mis-specified OLS is heavily biased.

I first depict the recovered average structural function between inflation expectation $\pi_{i,t+1|t}$ and signals π_t, $s_{i,t}$ in Figure 16. The red solid line is the true Average Structural Function implied by the Kalman Filter (49) and the black solid line is the mean of estimated ASF from 100 random samples using RNN. I also plot estimated ASF for each sample in grey color. The top panel in Figure 16 is the ASF along dimension of realized inflation π_t and the bottom panel is along signal $s_{i,t}$. It is obvious that the estimated ASF all indicate linear relationship between signals and expected inflation. This means RNN will recover a linear function if the underlying expectation formation model is indeed linear. It also shows the stability of the performance of RNN: with 100 random samples it recovers the ASF relative close to the truth.

I then report the (naive) estimates of marginal effects from RNN and compare them to those from the other two models considered. The following table shows the estimation result

[57]For example, when agent form expectation on inflation, if they believe in a three equation New Keynesian Model, they may also want to infer demand and supply shocks as unobserved states. In a Kalman Filter that takes only inflation as unobserved state, OLS will suffer from omitted variable problem.

[58]Interestingly, for estimating ASF and marginal effect, one do not need to include the lag expectation $\pi_{i,t|t-1}$ in RNN, only history of signals are sufficient. The results without lag expectation are similar to these results I include here.

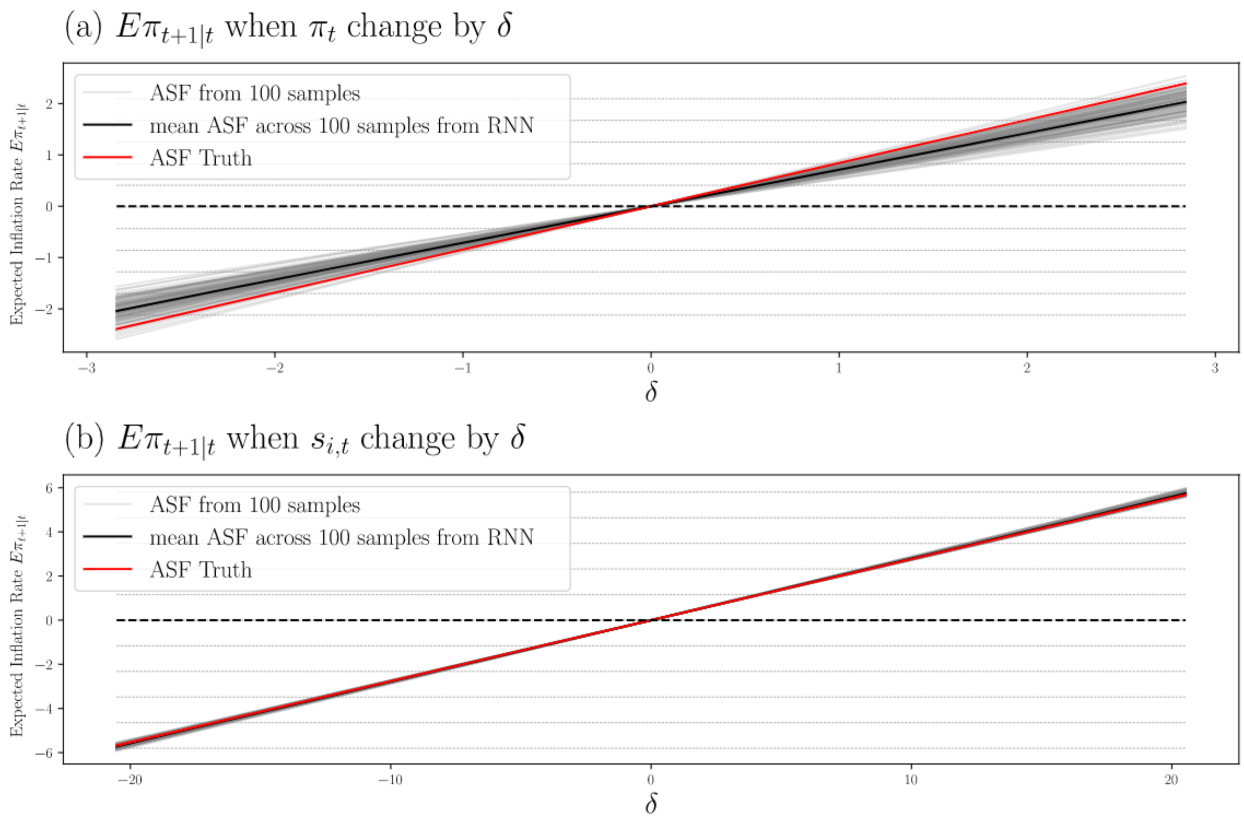

Figure 16: Estimated Average Structural Function from random samples using RNN. Function depicts change of expected variable in response to corresponding signal change by δ. Panel (a): expected inflation as function of inflation signal π_t. Panel (b): expected inflation as function of private signal $s_{i,t}$. Red solid line is the actual ASF implied by linear Kalman Filter. Solid black line is the mean of estimated ASF from 100 random samples. Grey lines are estimated ASFs from each random sample.

from RNN, mis-specified OLS and correctly specified OLS. In this table, the first column is mean squared error on the whole sample, the second column is estimated marginal effect on signal π_t and third column is estimated marginal effect on signal $s_{i,t}$. In brackets I report the standard deviation of the estimate using 100 simulated random samples. Not surprisingly, correctly specified OLS is BLUE in this case with unbiased estimates and small standard deviations. However the key thing to notice here is that mis-specified OLS is biased due to the omitted latent state, whereas RNN has result that is consistent with the true marginal effect, with acceptable standard deviations across 100 samples.

Table 7: Performance of RNN v.s. OLS

	MSE	π_t	$s_{i,t}$
(1) RNN	2.91	0.82	0.276
	(0.054)	(0.037)	(0.003)
(2) OLS mis-specified	3.296	0.720	0.279
	(0.023)	(0.033)	(0.001)
(3) OLS correct	2.835	0.841	0.277
	(0.014)	(0.005)	(0.001)
Truth		0.842	0.277

* The first column is mean squared error on the whole sample, the second column is estimated marginal effect on signal π_t and third column is estimated marginal effect on signal $s_{i,t}$. In brackets I report the standard deviation of the statistics using 100 simulated random samples.

B.5 Example: Constant Gain Learning in Generic Learning Framework

In this subsection I will illustrate how a standard Constant Gain Learning model can be analytically expressed in the form of the Generic Learning Framework. An example of such model is from Evans and Honkapohja (2001). For simplicity I consider the one dimensional case, where an agent observes realized inflation π_t at each time t and try to form forecast about π_{t+1}. I also drop the individual indicator i to same some notations, but the framework can be easily generalized to multi-dimensional multi-agent case. The agent believes in a "Perceived Law of Motion" (PLM) about how inflation is evolving in time and try to estimate the relevant parameters in the PLM using observed data. To do this, she will run OLS at every period and apply a constant weight to the newly available data. With this learning scheme agent perceives different values for parameters in their PLM and form expectation accordingly. The model features a constant gain γ, which represents the weight the agent put on newly observed data. Let's assume the PLM the agent believes in is an AR(1) process:

$$\pi_{t+1} = b_0 + b_1 \pi_t + \eta_{t+1} \quad \text{(PLM)}$$

In this setup, the parameters agent try to learn from realized data are b_0 and b_1. η_{t+1} stands for the mean zero i.i.d random shock realized in each period. The agent uses and OLS method to estimate b_0 and b_1 every period, and this process can be formulated recursively such that in each period the agent forms a different estimate b_t:

$$b_t = b_{t-1} + \gamma R_t^{-1} \boldsymbol{X}_{t-1}(\pi_t - b'_{t-1}\boldsymbol{X}_{t-1})$$

$$R_t = R_{t-1} + \gamma(\boldsymbol{X}_{t-1}\boldsymbol{X}'_{t-1} - R_{t-1})$$

$$\boldsymbol{X}_t = [1 \quad \pi_t]' \quad b = [b_0 \quad b_1]'$$

At time t, the agent then forms expectation about future inflation using the PLM, with some i.i.d noise attached on top of the endogenous component that comes from constant gain learning process, ϵ_t. This exogenous component is sometimes interpreted as "sentiment", for example in Cole and Milani (2020).

$$E_t \pi_{t+1} = b'_t \boldsymbol{X}_t + \epsilon_t \tag{53}$$

Now suppose the agent is learning with the above set-up. As observers we see: $\boldsymbol{X}_t$, $E_t\pi_{t+1}$ up to each time t. We do not see the hidden variables such as b_t and R_t. We also don't know the function form that connects the hidden variables, observables and expectational variables. The goal now is to represent the system described by this constant gain learning model in terms of the Generic Learning Framework. Define the hidden states $\Theta_t = [\boldsymbol{X}_t, b_t, R_t, \epsilon_t]'$. The recursive mapping from observables (and previous hidden states) to hidden states $H(.)$ then can be given by:

$$\Theta_t = H(\boldsymbol{X}_t, \Theta_{t-1}, \epsilon_t)$$

Where

$$\boldsymbol{X}_t \equiv H_1(\boldsymbol{X}_t, \Theta_{t-1}, \epsilon_t) = \boldsymbol{X}_t$$

$$R_t \equiv H_2(\boldsymbol{X}_t, \Theta_{t-1}, \epsilon_t) = R_{t-1} + \gamma(\boldsymbol{X}_{t-1}\boldsymbol{X}'_{t-1} - R_{t-1})$$

$$b_t \equiv H_3(\boldsymbol{X}_t, \Theta_{t-1}, \epsilon_t) = b_{t-1} + \gamma R_t^{-1}\boldsymbol{X}_{t-1}(\pi_t - b'_{t-1}\boldsymbol{X}_{t-1})$$

$$\epsilon_t \equiv H_4(\boldsymbol{X}_t, \Theta_{t-1}, \epsilon_t) = \epsilon_t$$

Notice here, as Θ_t can be any measurable function of $\boldsymbol{X}_t$, Θ_{t-1} and ϵ_t, it can certainly contain elements such as the input $\boldsymbol{X}_t$. Although $\boldsymbol{X}_t$ is actually observable, it remains "hidden" to econometrician as without further knowledge on expectation formation process, one will not know what the exact mapping from observables to elements of Θ_t is. Then the expectation formation model $F(.)$ is given by:

$$E_t\pi_{t+1} \equiv F(\Theta_t) = b_t'\boldsymbol{X}_t + \epsilon_t$$

Now I show that the expectation formed by constant gain learning can be analytically represented by the Generic Learning Framework described by updating step (2) and forecasting step (3). The Average Structural Function implied by this setup is straight forward: one can define $\theta_t = [\boldsymbol{X}_t, b_t, R_t]'$ and obtain $f(.)$ and $h(.)$ by integrating out the i.i.d random variable ϵ_t.

C Appendix on Empirical Findings

C.1 More on Time-varying Marginal Effect

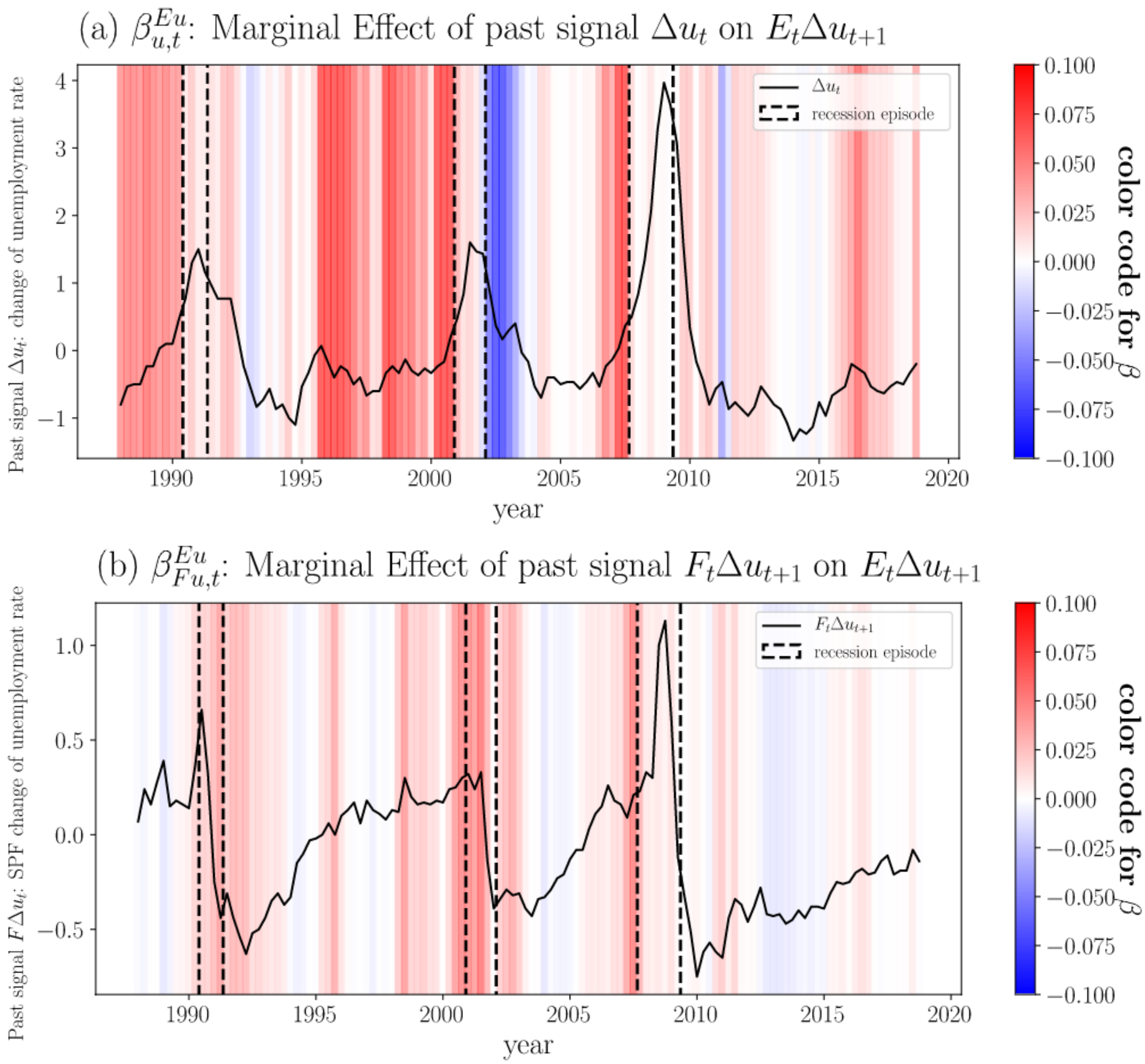

Figure 17: Color bars in panel (a): the marginal effects of unemployment signal Δu_t on expected unemployment rate increase next year $E\Delta u_{t+1|t}$. Panel (b): the marginal effects of professionals' forecasts about unemployment rate change next year $F\Delta u_{t+1|t}$ on expected unemployment rate change. Red color: positive marginal effect; blue color: negative marginal effect. Black solid line: data on frequency of news about recession.

To show the same attention shift pattern holds for all signals and expectations related to economic condition, I first plot the same heatmap for marginal effect of unemployment signals on expectation on unemployment change. This is Figure 17 below. It shows the same pattern holds as in Figure 4: in recession marginal effect of future signal is bigger and the opposite is true for past signal.

For marginal effects of cross-signals, for example, the impact of unemployment signal on economic condition expectation. These results are shown in Figure 18 below. It shows first unemployment signals generally have negative impact on expectation of economic condition. Furthermore, when looking at marginal effects of past signals, such an impact is again weak during recession periods whereas the marginal effects of future signals are again with bigger magnitudes during recessions.

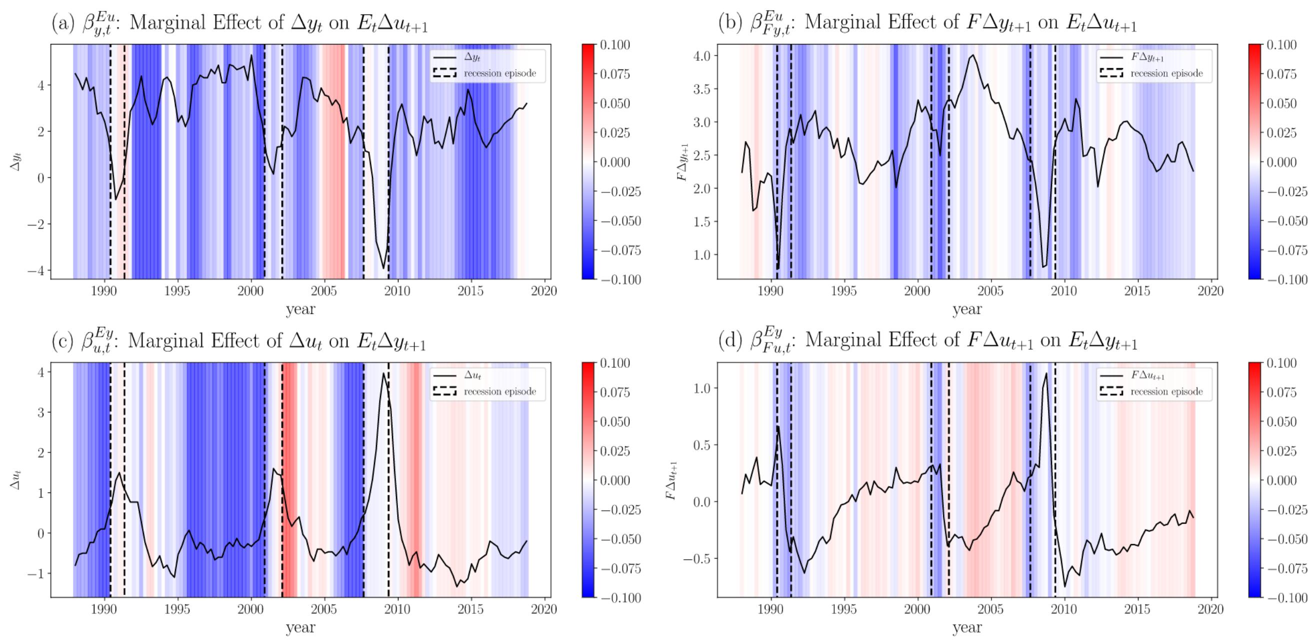

Figure 18:

However these attention shift during recession and ordinary period only holds significantly for expectations and signals related to indicators about economic conditions. Figure 19 plots the time-varying marginal effects for indicators on inflation and interest rate, there is no such attention shift at presence. The DML estimator also suggest the average marginal effects in recession and ordinary periods are not significantly different.

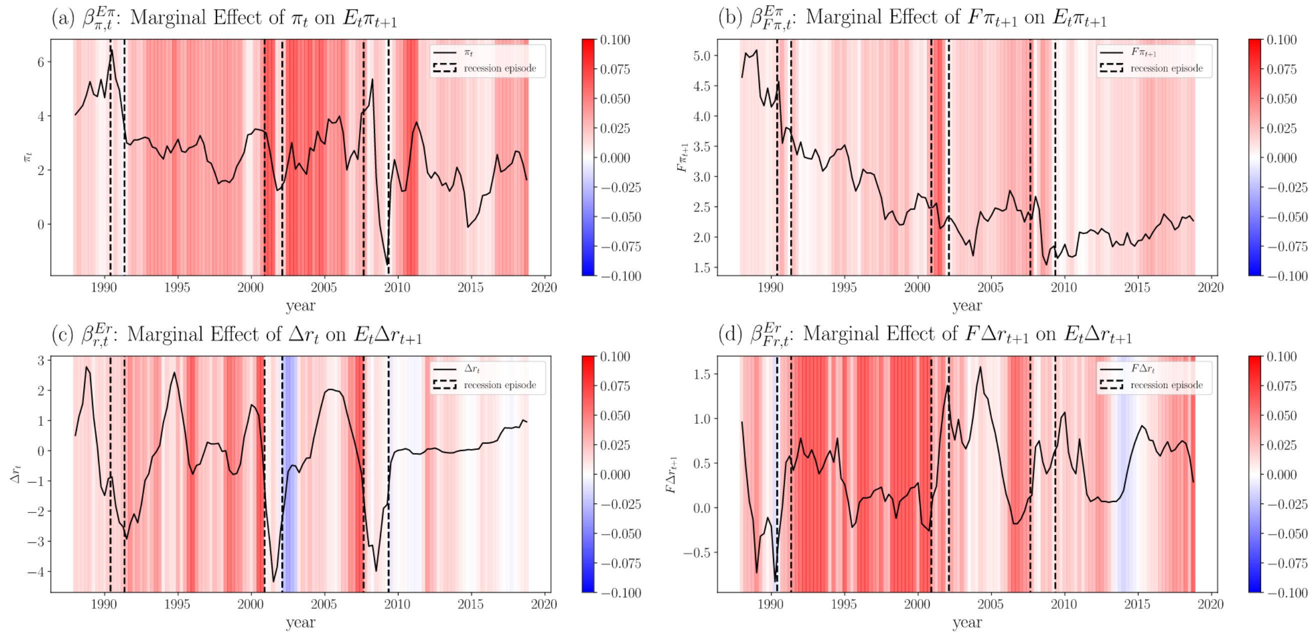

Figure 19:

C.2 Robustness of DML using NBER Recessions

Table 8: Average Marginal Effect of Past and Future Signals: NBER Recession

Expectation:		$E\Delta y_{t+1\|t}$			$E\Delta u_{t+1\|t}$		
	Signal	β_{bad} (std)	β_{ord} (std)	$\beta_{bad}=\beta_{ord}$ (p-val)	β_{bad} (std)	β_{ord} (std)	$\beta_{rec}=\beta_{ord}$ (p-val)
Future Signal	$F_t\Delta u_{t+1}$	**−0.047*** **	0.005	<0.01	**0.033*** **	0.009***	<0.01
		(0.006)	(0.002)		(0.004)	(0.002)	
	$F_t\Delta y_{t+1}$	**0.05*** **	0.02***	<0.01	**−0.024*** **	−0.01***	<0.01
		(0.007)	(0.003)		(0.003)	(0.001)	
Past Signal	Δu_t	−0.016*	−0.018***	0.86	0.012***	0.01***	0.74
		(0.008)	(0.003)		(0.005)	(0.002)	
	Δy_t	0.003	**0.015*** **	0.05	−0.004**	**−0.01*** **	0.04
		(0.004)	(0.002)		(0.002)	(0.001)	

* ***,**,*: Significance at 1%,5% and 10% level. β_{bad} is average marginal effect in bad periods defined by NBER recession dates, β_{ord} is average marginal effect in ordinary period. $\beta_{bad}=\beta_{ord}$ is test on equality between average marginal effects, its p-value is reported for each expectation-signal pair. Bold estimates denote the marginal effect with significantly bigger magnitude. Standard errors are adjusted for heteroskesticity and clustered within time.

Table 8 shows the DML estimates for marginal effects of past and future signals on real GDP growth and unemployment rate, during or out of recession. And the recession dates in use are those from NBER. Although "bad times" defined in **Section 4.2.2** are considered more plausible for reasons discussed before, using NBER recession dates won't qualitatively change the DML estimates much. Future signals still significantly have higher weights during bad periods and the weights on past signals are usually with bigger magnitude in ordinary period.

C.3 Variance Decomposition for Unemployment Expectation

In Table 9 I summarize the variance decomposition of time varying marginal effects of unemployment signal on unemployment expectations. It is consistent with what I find for expectation on economic condition. First the signals that explain most of the time-variation are those related to economic conditions. News exposure also explain a significant part of variation, especially for past signals. Finally these signals affect expectations through both accumulated states and covariates. Current signal usually plays a more important role in explaining the time-variation.

Table 9: Variance Decomposition of Time-varying Marginal Effects: $E\Delta u$

Marginal Effect on Past Signal:		$\beta^{Eu}_{u,t}$				
Signal Type:		Economic Condition	Inflation	Interest rate	News	Total
	State $\theta_{i,t-1}$	28%	3%	6%	20%	57%
Channel:	Covariate $Z_{i,t}$	23%	2%	13%	5%	43%
	Total	52%	5%	18%	25%	
Marginal Effect on Future Signal:		$\beta^{Eu}_{Fu,t}$				
Signal Type:		Economic Condition	Inflation	Interest rate	News	Total
	State $\theta_{i,t-1}$	19%	6%	7%	4%	36%
Channel:	Covariate $Z_{i,t}$	36%	4%	9%	15%	64%
	Total	54%	10%	16%	19%	

D Derivation in Rational Inattention Model

D.1 Quadratic Loss Function

Start from LHS of equation (26), to same notations I denote $s^*_{t+1}(\mathcal{I}_t)$ as s_{t+1}:

$$
\begin{aligned}
\mathbb{E}[U(s^*_{t+1}(\mathcal{I}_t))] &= \mathbb{E}[(e_t - s_{t+1}) - b(e_t - s_{t+1})^2 + \beta(e_{t+1} + r_{t+1}s_{t+1}) - \beta b(e_{t+1} + r_{t+1}s_{t+1})^2] \\
&= \mathbb{E}[-\underbrace{b(1+\beta r^2_{t+1})}_{\equiv\chi} s^2_{t+1} + (2be_t - 1 + \beta r_{t+1} - 2\beta b e_{t+1} r_{t+1})s_{t+1} + (e_t - be^2_t + \beta e_{t+1} - \beta b e^2_{t+1}) \\
&= \mathbb{E}[-\chi(s^2_{t+1} - \underbrace{\frac{2be_t - 1 + \beta r_{t+1} - 2\beta b e_{t+1} r_{t+1}}{\chi}}_{\equiv 2\bar{s}_{t+1}} s_{t+1} + \frac{(2be_t - 1 + \beta r_{t+1} - 2\beta b e_{t+1} r_{t+1})^2}{4\chi^2}) \\
&+ \frac{(2be_t - 1 + \beta r_{t+1} - 2\beta b e_{t+1} r_{t+1})^2}{4\chi} + (e_t - be^2_t + \beta e_{t+1} - \beta b e^2_{t+1})] \\
&= -\mathbb{E}[\chi(s_{t+1} - \bar{s}_{t+1})^2] + \underbrace{\mathbb{E}[\frac{(2be_t - 1 + \beta r_{t+1} - 2\beta b e_{t+1} r_{t+1})^2}{4\chi} + (e_t - be^2_t + \beta e_{t+1} - \beta b e^2_{t+1})]}_{\equiv M}
\end{aligned}
$$

The last line gives the RHS of equation (26). Now notice, M has nothing to do with information set $\mathcal{I}_t$, thus the evaluating the expected utility under choice of $\mathcal{I}_t$ is equivalent to evaluating the quadratic loss term $\mathbb{E}[\chi(s^*_{t+1}(\mathcal{I}_t) - \bar{s}_{t+1})^2]$. This is a standard result from literature of Rational Inattention with linear quadratic preference. However, the key difference here is s^*_{t+1} is non-linear in fundamentals. In standard rational inattention models, the action will be linear in fundamentals thus optimal choice of signal will not depend on prior mean of fundamentals. For example, see Maćkowiak *et al.* (2018) or Kamdar (2019).

D.2 Information Cost

In this subsection I derive the information cost measured by entropy in (29) following Mackowiak and Wiederholt (2009). Recall the state-space representation of fundamentals are:

$$\boldsymbol{X}_{t+1} = A\boldsymbol{X}_t + \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t \sim N(\boldsymbol{0}, \boldsymbol{Q})$$

The initial noisy signal z_0 and chosen signals $\boldsymbol{Z}_t$ are given by:

$$z_0 = d_t + \xi_0 = G_0\boldsymbol{X}_t + \xi_0, \quad G_0 = [1 \quad 0]$$

$$\boldsymbol{Z}_t = G\boldsymbol{X}_t + \boldsymbol{\xi}_t, \quad \boldsymbol{\xi}_t \sim N(\boldsymbol{0}, R)$$

First notice all the random variables $\boldsymbol{X}_t$, z_0, $\boldsymbol{Z}_t$ are normally distributed. The information set $\mathcal{I}_0$ only contains a noisy Gaussian signal z_0, the entropy of $\boldsymbol{X}_{t+1}$ given $\mathcal{I}_0$ is then:

$$\mathcal{H}(\boldsymbol{X}_{t+1}|\mathcal{I}_0) = \mathcal{H}(\boldsymbol{X}_{t+1}|z_0) = \frac{1}{2}log_2[(2\pi e)^2 det\Sigma_{t+1|0}] \tag{54}$$

Where $\Sigma_{t+1|0}$ denotes the variance-covariance matrix of $\boldsymbol{X}_{t+1}$ given z_0. The prior variance covariance matrix of $\boldsymbol{X}_t$ is denoted as Σ_0, then the conditional variance-covariance matrix $\Sigma_{t+1|0}$ is given by:

$$\Sigma_{t+1|0} = A\Sigma_{t|0}A' + \boldsymbol{Q} \tag{55}$$

Where:

$$\Sigma_{t|0} = \Sigma_0 - \Sigma_0 G_0'(G_0\Sigma_0 G_0' + \sigma_z^2)^{-1}G_0\Sigma_0 \tag{56}$$

It is obvious $\Sigma_{t|0}$ by construction the posterior variance-covariance matrix for hidden states $\boldsymbol{X}_t$ after observing z_0 derived from Kalman Filter.

Then recall $\mathcal{I}_t = \mathcal{I}_0 \cup \{\boldsymbol{Z}_t\}$, similar as above we have:

$$\mathcal{H}(\boldsymbol{X}_{t+1}|\mathcal{I}_t) = \mathcal{H}(\boldsymbol{X}_{t+1}|z_0, \boldsymbol{Z}_t) = \frac{1}{2}log_2[(2\pi e)^2 det\Sigma_{t+1|t}] \tag{57}$$

Where:

$$\Sigma_{t+1|t} = A\Sigma_{t|0}A' - A\Sigma_{t|0}G'(G\Sigma_{t|0}G' + R)^{-1}G\Sigma_{t|0}A' + \boldsymbol{Q} \tag{58}$$

Again by construction, the $\Sigma_{t+1|t}$ is the posterior variance-covariance matrix for $\boldsymbol{X}_{t+1}$ after observing $\{z_0, \boldsymbol{Z}_t\}$ derived from Kalman Filter. Moreover, it is obvious the uncertainty after observing $\boldsymbol{Z}_t$ is reduced compared to the uncertainty after only observing z_0.

Now information cost is obtained by measuring uncertainty reduction induced by extra information, using (57) and (54) we have the information cost in (29):

$$\mathcal{H}(\boldsymbol{X}_{t+1}|\mathcal{I}_0) - \mathcal{H}(\boldsymbol{X}_{t+1}|\mathcal{I}_t) = \frac{1}{2}log_2(\frac{det\Sigma_{t+1|0}}{det\Sigma_{t+1|t}})$$

D.3 Derivation of $\mathbb{E}[r_{t+1}|\mathcal{I}_t]$ and $\mathbb{E}[r_{t+1}^2|\mathcal{I}_t]$

From (21):

$$\begin{pmatrix}\mathbb{E}[d_{t+1}|\mathcal{I}_t]\\ \mathbb{E}[\eta_{t+1}|\mathcal{I}_t]\end{pmatrix} \equiv \hat{\boldsymbol{X}}_{t+1|t} = A\Big((I-KG)\hat{\boldsymbol{X}}_{t|0} + K\boldsymbol{Z_t}\Big)$$

Where $\hat{\boldsymbol{X}}_{t|0} = \mathbb{E}[\boldsymbol{X}_t|\mathcal{I}_0]$ is the mean of belief on $\boldsymbol{X}_t$ after observing passive signal z_0. The prior before observing z_0 is denoted as $\boldsymbol{X}_0 \sim N(\hat{\boldsymbol{X}}_0, \Sigma_0)$ from Section 5.2.1. Now denote the Kalman Gain for observing z_0 as K_0, we can write:

$$\hat{\boldsymbol{X}}_{t|0} = (I - K_0G_0)\hat{\boldsymbol{X}}_0 + K_0z_0 \tag{59}$$

Where $G_0 = \iota = [10]$ as defined in Appendix D.2 and K_0 is given by:

$$K_0 = \Sigma_0 G_0'(G_0\Sigma_0G_0' + \sigma_z^2)^{-1} \tag{60}$$

Combine (21), (59) and (60) we have:

$$\begin{aligned}\mathbb{E}[r_{t+1}|\mathcal{I}_t] &= 1 + \iota A\Big((I - KG)\big((I - K_0G_0)\hat{\boldsymbol{X}}_0 + K_0z_0\big) + KZ_t\Big) \\ &= 1 + \iota A\Big((I - KG)\big((I - \Sigma_0G_0'(G_0\Sigma_0G_0' + \sigma_z^2)^{-1}G_0)\hat{\boldsymbol{X}}_0 + \Sigma_0G_0'(G_0\Sigma_0G_0' + \sigma_z^2)^{-1}z_0\big) + KZ_t\Big)\end{aligned} \tag{61}$$

Before I show the derivation of $\mathbb{E}[r_{t+1}^2|\mathcal{I}_t]$, it's useful to consider what is $Var(d_{t+1}|\mathcal{I}_t)$. It is the first element of posterior variance covariance matrix $\Sigma_{t+1|t}$, which is given by (58). So $Var(d_{t+1}|\mathcal{I}_t)$ can be written as:

$$\begin{aligned}Var(d_{t+1}|\mathcal{I}_t) &= \iota\Sigma_{t+1|t}\iota' \\ &= \iota\Big(A\Sigma_{t|0}A' - A\Sigma_{t|0}G'(G\Sigma_{t|0}G' + R)^{-1}G\Sigma_{t|0}A' + \boldsymbol{Q}\Big)\iota' \end{aligned} \tag{62}$$

Now we can derive $\mathbb{E}[r_{t+1}^2|\mathcal{I}_t]$:

$$\begin{aligned}\mathbb{E}[r_{t+1}^2|\mathcal{I}_t] &= Var(r_{t+1}|\mathcal{I}_t) + (\mathbb{E}[r_{t+1}|\mathcal{I}_t])^2 \\ &= Var(d_{t+1}|\mathcal{I}_t) + (\mathbb{E}[r_{t+1}|\mathcal{I}_t])^2 && (63) \\ &= \iota\Big(A\Sigma_{t|0}A' - A\Sigma_{t|0}G'(G\Sigma_{t|0}G' + R)^{-1}G\Sigma_{t|0}A' + \boldsymbol{Q}\Big)\iota' + (\mathbb{E}[r_{t+1}|\mathcal{I}_t])^2 && (64)\end{aligned}$$

In the above equation, $\Sigma_{t|0}$ is given by (56), which contains σ_z^2 and prior variance Σ_0. $\mathbb{E}[r_{t+1}|\mathcal{I}_t]$ is given by (61), which depends on prior mean $\hat{\boldsymbol{X}}_0$, precision (variance) of the signal R and passive signal z_0. From (61) and (64), it is clear that the optimal saving choice is a non-linear function of all these variables related to the information friction.

Now to see how the *ex-post* weights on signal Z_t depend on variances of signals R, denote the weight on SPF signal as w_{spf} and weight on signal about current state as w_t, from (61) we have:

$$\begin{aligned}
\begin{pmatrix} w_{spf} \\ w_t \end{pmatrix} &= (\iota' \quad \iota')AK \\
&= (\iota' \quad \iota')A\Sigma_{t|0}G'(G\Sigma_{t|0}G' + R)^{-1} \qquad (65)
\end{aligned}$$

From we see first for given Σ_{t+0}, G and A, a lower variance of noise on signal (contained in R) leads to higher weights put on corresponding signal. Moreover, as $\Sigma_{t|0}$ is affected by σ_z^2, the weights on signals also change with σ_z^2. This is verified in Section 5.5.4.