The Dominant Role of Expectations and Broad-Based Supply Shocks in Driving Inflation.

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Abstract

In light of the experience of the last few years, the object of this paper is to re-examine the role of supply shocks, labour market tightness and expectation formation in explaining bouts of inflation. We begin by showing that a quasi-flat Phillips curve, which was popular prior to the pandemic, still fits the post-2020 US data well with the implication that (1) labour market tightness likely played a limited role in generating recent inflation and (2) changes in short term inflation expectations induced by supply shocks likely played a major role. We then explore how best to capture the join dynamics of inflation and inflation expectations in response to supply shocks. Given the difficulty of capturing these dynamics under rational expectations, we propose and evaluate a model with bounded rationality. In our model, supply shocks that affect many goods affect inflation expectations and this drive persistent inflation dynamics, while supply shocks that are concentrated in a sector lead to much more temporary changes in both inflation and inflation expectations. Although our departure from full rationality is minor, it allows perceived common shocks to dis-aggregated inflation series to be amplified and propagated over time in a manner consistent with observation.

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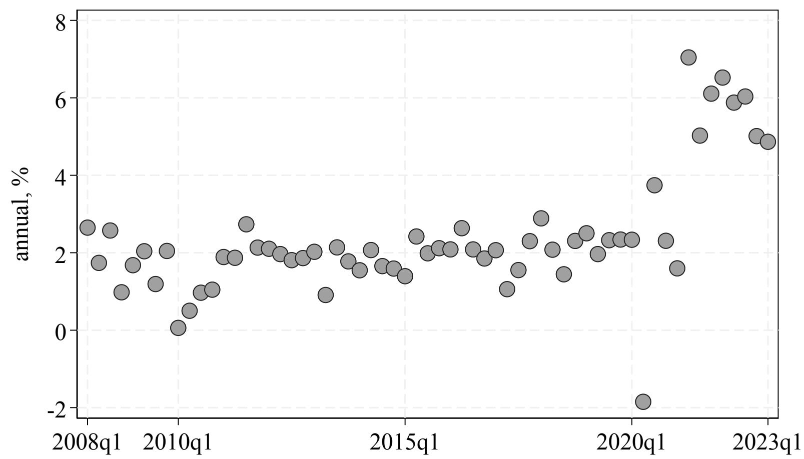
Introduction

While inflation in many countries has been coming down quickly from the heights of 2022, core inflation has been coming down at a slower pace and this can be challenging for central banks who aim for a timely return to a pre-pandemic level of near 2%. For the US, this pattern for core inflation can be seen in Figure 1.

Figure 1: US Core CPI Inflation



Notes: Each dot represents the annualised inflation rate of the corresponding quarter.

One of the main tools used by central bankers to understand inflation is the expectations-augmented Phillips curve which links inflation to expectations of future inflation and overall market tightness, either measured by an output gap, an unemployment gap or vacancy-to-unemployment ratio.[1] When viewed through the lens of the Phillips curve, the persistent period of high inflation that is not explained by supply shocks can be attributed to either an overly-hot labour market or by elevated expectations of inflation. A third possibility is that the Phillips curve is a rather unstable object and should be considered of limited value for thinking about current inflation. The later more unorthodox view is sometimes motivated

[1] The later has been advocated as a better proxy for labour market tightness by, among others, Ball, Leigh, and Mishra [2022], Michaillat and Saez [2022] and Benigno and Eggertsson [2023].

by perceived repeated failures of the Phillips Curve. For example, in the decade between the "Great Financial Crisis" (GFC) and the pandemic –instead of under-predicting inflation as has recently been common– the Phillips curve often lead to a "missing deflation puzzle".

This paper begins by examining what a pre-2020 view of the Phillips curve tells us about the recent determinants of inflation and whether such a Phillips curves actually remains a good tool for understanding inflation. We complement our Phillips Curve analysis with a VAR analysis of the joint dynamics of inflation, inflation expectations and labour market tightness, where inflation expectations used in both case are taken from the Michigan Household survey . The VAR analysis is especially relevant as it will become the target for our later structural analysis of how inflation and inflation expectations interact to create rich joint dynamics. Both our Phillips curve exploration and our VAR analysis come to similar conclusions. First, the recent episode does not suggest any important break from past behaviour. Second, both analyses suggest that inflation is mainly driven by expectations and quasi-iid supply shocks, with labour market tightness playing a very secondary role. In other words, the data maintain support for a close-to-flat Phillips curve view with short term inflation expectations being an important driver of inflation. In particular, the VAR analysis highlights how inflation expectation formed at t almost perfectly match realized inflation at $t + 1$, while inflation and inflation expectations are almost unrelated to labour market outcomes. We finish this section by showing how a Philips curve with rational expectation does not match the VAR observations.

We then turn to examining some of the empirical determinants of household inflation expectations. In particular, we show that these inflation expectations react to sectorial inflation measures in a way that deviate markedly from their importance in the overall CPI basket. Instead, inflation expectations seem to be influence by the common component of disaggregated data, where the main common component place very similar weights across items/sectors. So agents appear to update their expectations very differently if inflation is driven by a broad-based increase in prices than if the same level of inflation is driven by only one or two items. This observation will be a key element in our model of inflation expectations.

In the third section of the paper we propose a model of the joint determination of inflation and inflation expectations that builds on the previously mentioned empirical patters with the aim of explaining the VAR patterns. For the determinants of inflation, our model simply imports the Philips Curve specification we discuss and evaluate in Section 1. For the determinant of inflation expectations, we make two departures relative to a full information rational expectation benchmark. First, we endow agents with two inflation realizations; a realization of headline inflation and a realization of an inflation signal that can be interpreted as reflecting broad based shocks. Second, we model the agents as quite sophisticated but not fully informed of the data generating process. In particular, they are assumed to not take into account that their shared expectations is an important force driving inflation. Instead, they extract a common component from their inflation signals using a Kalman filter and use this common component to predict future inflation. In this filtering process, agents treat the realization of the common component in inflation as uncorrelated with the noise in their signal even though the two may not be independent in equilibrium due to the role of expectations themselves driving inflation. This is the key departure from rational expectations. We show that such a model of inflation expectations can capture very well the joint dynamics of inflation and inflation expectations as captured by the VAR presented in Section 2, while simultaneously being consistent with easily observable moments of the inflation process.

While the narrative behind our model is most easily expressed in reference to the role of broad based inflation shocks in driving inflation expectations, we do not actually use the disaggregated data in the estimation of our model. Instead, the “broad based” inflation signal is treated as a latent variable and we show how our estimate of this latent variable correlates with the common component directly estimated from the disaggregated data.

The narrative that arise from our model of inflation and inflation expectations can be expressed as follows. Inflation can be seen as driven by two different types of shocks: first by narrow supply shocks that creates very little dynamics even when substantial since such shocks have not readily transmitted to expectations as they do not create a perceived generalized increase in prices; second by a broad-based supply shocks that transmits to inflation expectations and thereby creates something close to a self-fulling inflation episode. Such a model is shown to explain why inflation can remain stable during long periods even in the presence of many narrow but unsynchronized supply shocks. But, in contrast, inflation tends to broaden and persist when the economy is hit simultaneously by several supply shocks in different sectors.

1 Should We Maintain or Throw out the Flat Phillips Curve View?

Prior to the Covid pandemic, many studies of the Phillips Curve suggested that its slope was quite flat. Although the identification of the slope of the Phillips curve can be difficult, the work by Hazell, Herreño, Nakamura, and Steinsson [2022] advanced on this point by exploiting cross-state variation. Their finding was that the slope of the Phillips curve was significantly positive, but nevertheless quite flat. Furthermore, they found that this flatness property did not arise post 90’s, but appears to be a feature of the data going back into the 1960s. They concluded that inflation expectations likely played a dominant role in the inflation episode on the 1970s and early 1980s. In related work (Beaudry, Hou, and Portier [2023]), we found similar results with the inflation expectations drawn from the Michigan survey of consumer helping to explain much of the variation in inflation since the late 1960.

In this section, we aim to examine whether the recent mis-inflation episode that followed the 2008 recession, followed by the inflation episode post-Covid lockdowns, should lead one to revise/reject/update the flat Phillips curve view. To this end, we start with the following very parsimonious view of the (quarterly) Phillips curve:

$$\pi_t = \beta \pi^e_{t+1} + \gamma_g \, \mathrm{gap}_t + \epsilon_t, \tag{1}$$

π_t is quarter-to-quarter Headline CPI inflation (in annual terms), π^e_{t+1} is based the mean of the one-year-ahead expectation of CPI inflation drawn from the Michigan Survey of Consumers.[2] We choose this measure of expectations as it goes back all the way to 1960s. As prices are set by firms, using a measure of business expectation could be preferred. However, such measures are not readily available over such a long time span.[3] [4]The gap represents labour market tightness and is measured by minus unemployment gap.[5]

We first do not estimate this Phillips curve, but choose commonly accepted parameters

[2]Our preference would be to use quarter-to-quarter expected inflation. However, the Michigan Survey of Consumers reports expectations for CPI inflation for the next year. To extract a quarter estimate from this data we rescale the one-year-ahead expected inflation assuming survey respondents believe that quarter-to-quarter inflation follows an AR(1) process with persistence $\tilde{\rho}$, that needs not to be equal to the actual persistence of inflation. The estimated $\tilde{\rho} = 0.89$. For more details, we refer to Beaudry, Hou, and Portier [2023].

[3]The Cleveland Fed Survey of Firms' Inflation Expectations (SoFIE) is available since 2018Q2. Over the sample 2018Q2-2023Q1, the correlation between the Michigan Survey of Consumers' inflation expectations and the Cleveland Fed Survey of Firms' ones is 0.9330

[4]A related question is which measure of expectation is most relevant to the Phillips Curve. This problem is well discussed in Coibion, Gorodnichenko, and Kamdar [2018], suggesting the expectations of households and firms are the most relevant. We check for robustness of our analysis with other measures of expected inflation in Appendix B. We also make an important remark that the expected inflations of households and firms differ significantly from those of the professionals and central banks after 2020.

[5]The unemployment gap is computed as the unemployment rate minus the noncyclical rate of unemployment from FRED (series name *NROU*).

values. γ_g is set to 0.0138 according to what estimated in Hazell, Herreño, Nakamura, and Steinsson [2022] [6] and β is set to 0.99, which is a common value in the literature. We refer to this as the baseline Phillips curve.

From this baseline Phillips curve, we construct residuals $\epsilon_t = \pi_t - 0.99\pi^e_{t+1} - 0.0138 \text{ gap}_t$ over the period 2008q1-2023q1 and plot them against our measure of the gap. This is the plot in Panel (a) of Figure 2.[7] The dark dots are Phillips curve residuals for the post-2020 period, while the grey dots cover the period from 2008q1 to 2019q4. We overlayed on these figures two estimated relations between the Phillips curve residuals and labour market tightness over 2008q1 to 2019q4; one in linear form and one in cubic form. The coefficient for this regression are presented in Columns 1 and 2 of Table 1. We also calculate the standard deviation σ_ϵ of the Phillips curve residuals for this sample as well as that from the prior sample running from 1968q1 to 2007q4. The first thing to note is that the standard deviation of the Phillips curve residuals are only slightly higher over the period 2008q1-2023q1 than over the period 1968q1-2007q4, although we are considering the Headline inflation and not controlling for supply shocks. More interestingly, we detect no significant link between these residuals and the unemployment gap.[8] If the residuals were strongly and positively associated with the labour market tightness measure, including possibly in a non-linear fashion, this would put

[6] In Hazell, Herreño, Nakamura, and Steinsson [2022], the authors provide an implied aggregate slope of the Phillips curve, with R-CPI as the measure of inflation and negative unemployment gap as the measure of market slackness. From footnote 24 in Hazell, Herreño, Nakamura, and Steinsson [2022], this aggregate slope of the Phillips curve is $= 0.58 \times 0.0062 + 0.42 \times 0.0243 = 0.0138$.

[7] In our analysis for post-2008 data, we exclude the 2020q2 observation given that measuring labour market tightness at a time of massive lock downs is very controversial.

[8] Endogeneity issues can complicate inference from this figure. Under the null hypothesis that our Phillips curve is well specified, forecast errors should reflect supply or markup shocks. In contrast, if the slope is mis-specified, then the forecast errors would also include a term directly related to labour market tightness. In this later case, such mis-specification should show up as a systematic relationship between forecast errors and labour market tightness. However, a strong endogenous relation between supply shocks and labour market tightness could hide this effect. Although we see this as a possibility, we believe it is likely of second order importance over this sample. We have explored the potential relevance of this issue by using high frequency identified monetary shocks to instrument labour market tightness when regressing forecast errors on labour market tightness. We have not found evidence of endogeneity bias.

in question the validity of a flat Phillips curve view in the more recent data. However, these forecast errors do not suggest that a steeper Phillips curve would better explain the more recent data episode. The same exercise is repeated using Core CPI inflation as a measure of inflation. As it can be seen in Columns 3 and 4 of Table 1 and on Panel (b) of Figure 2, there is again no sign of a steepening of the Phillips curve. Furthermore, the standard deviation of the Phillips curve residuals are is now smaller post 2008 as compared to previously. If anything, this flat Phillips curve fits better post 2008 data than over the period 1968-2007.

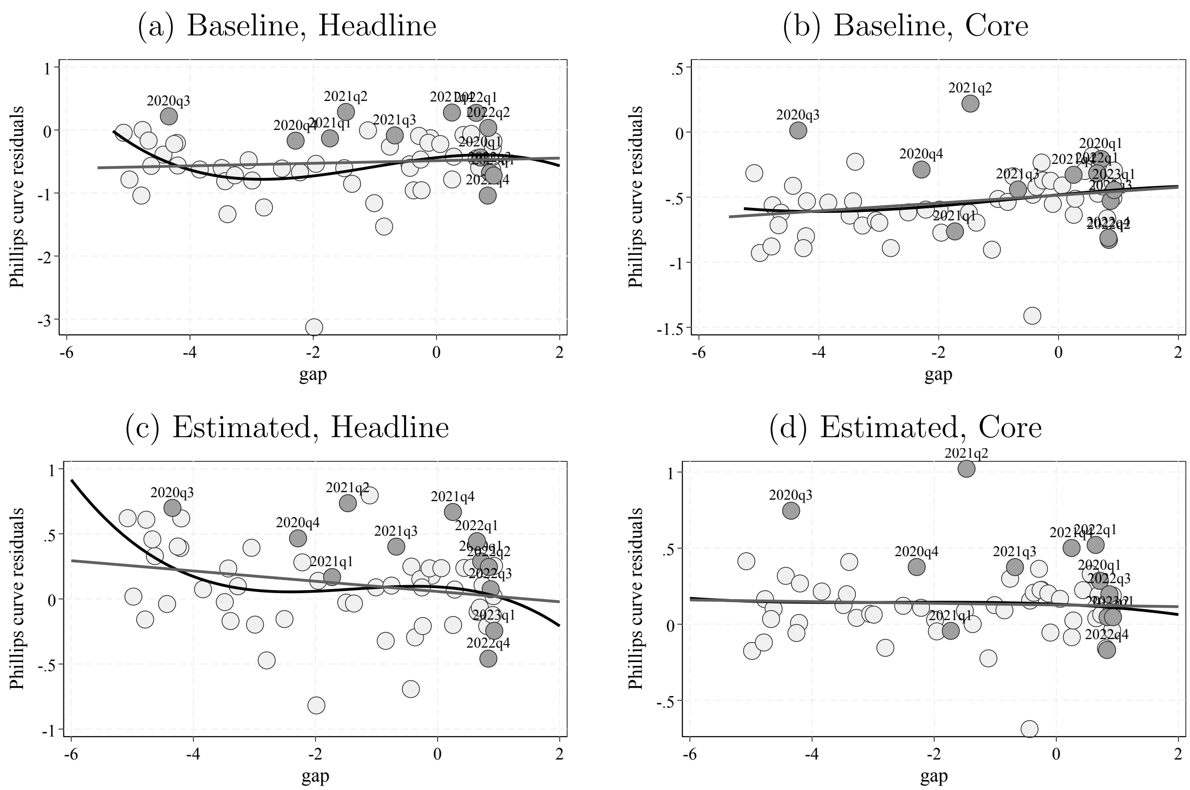
Table 1: Projection of the Philips Curve Residuals ϵ on the Gap, 2008-2023

	Headline Baseline		Core Baseline		Headline Estimated		Core Estimated	
	linear	nonlinear	linear	nonlinear	linear	nonlinear	linear	nonlinear
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
gap	0.02	0.11	0.03	0.04	-0.04	-0.03	-0.01	-0.02
	(0.035)	(0.113)	(0.016)	(0.056)	(0.022)	(0.072)	(0.016)	(0.055)
gap^2		-0.05		-0.00		-0.04		-0.01
		(0.087)		(0.043)		(0.056)		(0.042)
gap^3		-0.02		-0.00		-0.01		-0.00
		(0.015)		(0.007)		(0.009)		(0.007)
N	60	60	60	60	60	60	60	60
σ_ϵ 60-07	.324		.315		.276		.291	
σ_ϵ 08-23	.528		.256		.338		.245	

Notes: in columns (1) to (4), the Phillips curve residuals are obtained from the baselien Phillips curve (1). In columns (5) to (8), they are obtained from the augmented Phillips curve (2) estimated over the sample 1969Q1-2007Q4.

In addition to this baseline Phillips curve, we also directly estimate a Phillips curve over the sample 1969q1 to 2007q4, and look at the implied out of sample forecast errors for the period 2008q1-2023q1. Details of this estimation in provided in the appendix and follow Beaudry, Hou, and Portier [2023]. Since the determinants of inflation in Phillips Curve are endogenous, we follow Barnichon and Mesters [2020] and estimate our Phillips curve by

Figure 2: Out-of-Sample Residuals from Phillips Curves



Notes: Panels (a) and (b) of this figure plots the out-of-sample residuals of the baseline Phillips curve (1) against a measure of labor market tightness ((minus) the U.S. Congressional Budget Office unemployment gap), for two measures of inflation (Headline and Core CPI). The gray lines show the estimated linear or cubic relation between residuals and labor market tightness (see Table 1 for the estimated coefficients). Light dots correspond to pre-2020 observations and dark ones to post-2020. We exclude 2020q2 from this graph. Panels (c) and (d) of this figure repeat the analysis with the augmented estimated Phillips curve (2).

Instrumental Variables using estimated monetary shocks as instruments.[9] To reduce the risk of biases, we estimated the Phillips curve using Core inflation, and as argued in Beaudry, Hou, and Portier [2023], we include the real interest rate as an additional explanatory variable to capture a potential cost channel of monetary policy.[10] The estimated Phillips curve is therefore

$$\pi_t = \beta\pi^e_{t+1} + \gamma_g \, \mathrm{gap}_t + \gamma_r(i_t - \pi^e_{t+1}) + \epsilon_t, \tag{2}$$

The coefficients we obtain for γ_g is quite close to that our of baseline Phillips curve, with the exception that we obtain a slightly steeper Phillips curve than that reported in Hazell, Herreño, Nakamura, and Steinsson [2022]. Our estimate our γ_g is .04 and significant at standard levels, which is more than twice that of .0138 from Hazell, Herreño, Nakamura, and Steinsson [2022], but not significantly different from it. Our estimate for β is also .99.

In Panel (d) of Figure 2, we report the out of sample Phillips curve residuals from our estimated model for the period 2008q1-2023q1 plotted against our measure of labour market tightness, with again a linear and quadratic fit superimposed. We see a very similar pattern to that observed in Panels (a) and (b), with very little link between the forecast errors and labour market tightness, even though these are out of sample forecast. The estimated coefficients for these linear and cubic fits are displayed in Columns (7) and (8) of Table 1 So again, these observation do not suggest that a steeper Phillips curve is helpful/needed to explain the 2008q1-2023q1 period. Note that the fit of this estimated Phillips Curve, as measured by the standard deviation of the residuals σ_ϵ is better out-of-sample than in-sample. In appendix A, we confirm similar finding using the vacancy to unemployment rate as an alternative measure of labour market tightness, as well as exploring the effect of the use

[9] To be specific, we use twelve lags of the Romer and Romer's [2004] shocks (extended by Wieland and Yang [2020]) and their square as instruments.

[10] The presence of direct cost channel matters in the context of asking which expectations are more relevant when estimating Phillips curve. See a comprehensive discussion in Reis [2023].

market base measures of expectations. For completeness, panel (c) of Figure 2 and Columns (5) and (6) of Table 1 repeat the analysis when using the Headline CPI to estimate the Phillips curve. Results are similar.[11]

The overall out-of-sample fit of the baseline Phillips curve can also be seen in Panel (a) and (b) (for Headline) and (c) and (d) (for Core) of Figure 3. As can be seen, the forecasted inflation tracks actual inflation reasonably well, with exceptions such as 2021q2, where actual inflation was well above predicted, as is consistent with a strong supply shock in that quarter associated with transport bottlenecks. In Panels (a) and (c), we also we plot a counterfactual path of inflation over the period assuming that the unemployment gap had been equal to its sample mean over the forecast horizon. Here we see that this counterfactual inflation series is very similar to the predicted inflation path using the full model. This reflects the very weak direct role that labour market tightness is playing in our estimated Phillips curve given that the slope is quite small. The second counterfactual we examine imposes that inflation expectations are constant over the forecast period and set at their average. This counterfactual path is reported together with our forecasted rates of inflation in Panel (b). and (d). Here we see clearly that it is expected inflation that mainly drives the fit of the predicted inflation, as when we omit the role of inflation expectations, the fit greatly deteriorates.

In summary, Figures 2 and 3 illustrate how a very simple three-variable Phillips curve, either estimated prior to the recent periods of missing deflation and high inflation, or based on the work of Hazell, Herreño, Nakamura, and Steinsson [2022], fits the post-2007 data and post-2020 quite well. The lack of a relationship between forecast errors and labour market tightness indicates that the Phillips curve is likely still quite flat. This ongoing flat-

[11]We also confirm the findings from Benigno and Eggertsson [2023] that the Phillips Curve appear to be steeper after 2020. However, such a result is only present when using SPF as

ness of the Phillips curve implies that persistently high levels of core inflation in 2022 and early 2023 are unlikely explained by labour market tightness. Instead, it appears primarily driven by elevated levels of inflation expectations. Such observations can be both reassuring and uncomfortable. On the one hand, they support the view that the Phillips curve framework likely remains a relevant framework for thinking about inflation developments. However, they also suggest that the main driver of inflation is expected inflation itself. The latter observations can be uncomfortable as it questions/downplays the traditional role of pure demand management as key to controlling inflation and instead pushes forward the central importance of expectation management– and the more amorphous role of inflation psychology– in controlling inflation.

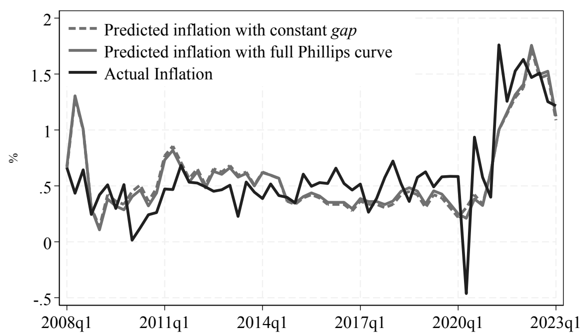
Table 2: Estimated Phillips Curves, 1969-2007

	Headline	Core
β	$1.15^{\star}$	$0.99^{\star}$
	(0.031)	(0.048)
γ_g	$0.07^{\star}$	$0.04^{\star}$
	(0.020)	(0.015)
γ_r	$0.13^{\star}$	$0.25^{\star}$
	(0.031)	(0.042)
Observations	144	144
J Test	15.201	10.684
(jp)	(0.887)	(0.986)
Weak ID Test	7.825	12.245

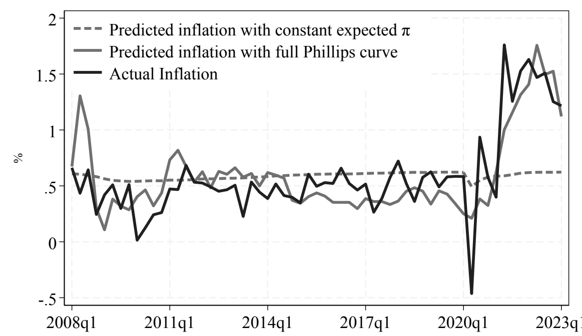
Notes: All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. Real oil price and its lag are alos omitted for the regression with Headline CPI. All regressors are instrumented using six lags of Romer and Romer's [2004] shocks (as extended by Wieland and Yang [2020]) and their squares as instruments. A $\star$ indicates significance at 5%. Sample is 1969Q1-2007Q4.

Figure 3: Counterfactual Simulations from the Baseline Phillips Curve

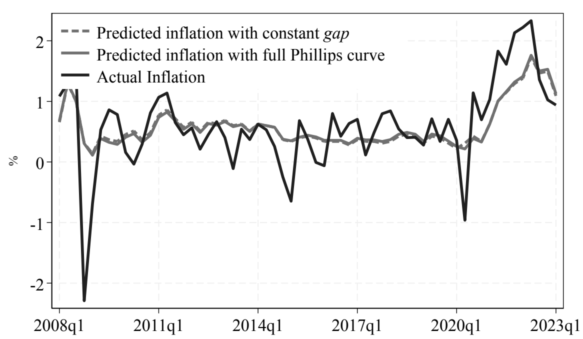
(a) With gap at its mean, Core



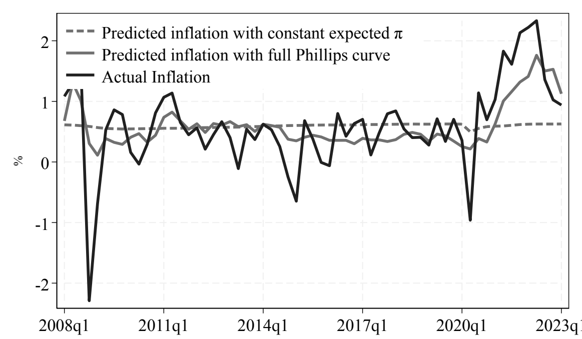
(b) With inflation expectations at their mean, Core



(c) With gap at its mean, Headline



(d) With inflation expectations at their mean, Headline



Notes: These counterfactual simulations are done using the baseline Phillips curve (1), using either Headline or Core CPI.

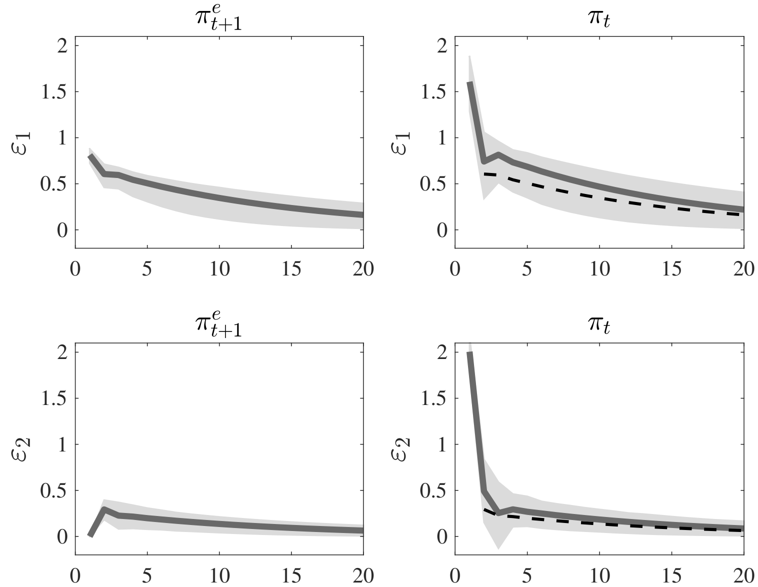
2 The Joint Dynamics of Inflation, Inflation Expectations and Labour Market Tightness

In this section we provide first some VAR based evidence to give further support to the view that inflation dynamics appears largely be explained by inflation expectations, with labour market tightness playing a very secondary role. We then show that our baseline Phillips curve model with rational expectations and full information is unlikely to account for that joint dynamics.

2.1 VAR Evidence

To this end, Figure 4 plots impulse response generated from a bi-variate VAR using inflation and the inflation expectations estimated over the period 1969q1-2023q2. The data for inflation remains headline inflation and that for expectations is again drawn from the Michigan Survey of Expectations. The figure corresponds to impulse response associated with a Choleski orthogonalization of the VAR residuals, where the shock ε_2 does not have impact effect on inflation expectations while ε_1 is unrestricted but to be orthogonal to ε_2. We are not interested that these impulses responses as reflecting the effects of structural shocks. Instead, we simply view these impulse responses as a means of summarizing properties of these data. In particular, these two shocks will be non trivial combinations of the structural shock in our modelling of section 4. In Figure 4, together with the response of inflation to the two shocks $\{\pi_t\}_{t=1}^{20}$, we also represent (dashed line) the expected inflation response $\{\pi_{t+1}\}_{t=1}^{19}$ shifted by one period. Two properties of this joint dynamics can be observed. First, inflation expectations formed at time t match very closely realized inflation at $t+1$. This is true for both shocks, and therefore for any linear combination of these shocks. Accordingly, it suggests that either agents are very good at predicting inflation or instead

Figure 4: Impulse Responses in the 2-VAR (π_t, π^e_{t+1})



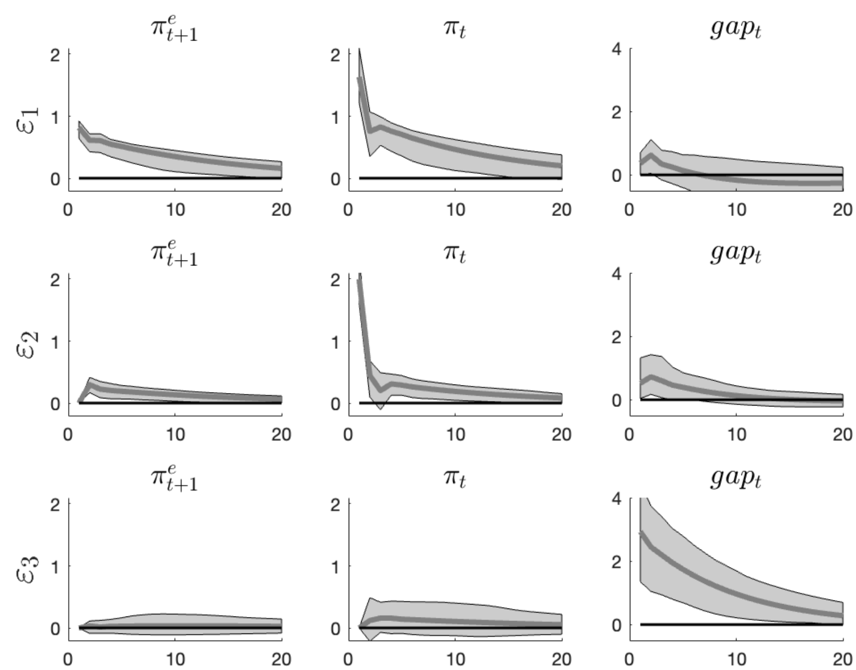
Notes: this Figure plots the impulse responses to a one standard deviation shocks ε_1 and ε_2. These shocks are obtained from a Choleski orthogonalization. The estimated VAR features two lags of Headline CPI inflations and the MSC inflation expectations. Sample is 1969Q1-2023Q1. Shaded area is the 95% confidence band.

that expected inflation may be driving inflation. Second, these impulse responses suggest that there is some combination of shocks (that forms ε_2) that don't transmit to expected inflation and this leads to very temporary rise in inflation. While on the other had, there is some combination of shocks (that forms ε_2) that have a large effect of inflation, transmit to expected inflation, and this is associated with persistent inflation

In Appendix C, we show that these impulse response are essentially unchanged if we change the number of lags or if drop post-2008 data from the estimation. This suggests that the recent inflation dynamics continue to obey dynamics that were observed earlier, suggesting no substantial break in the process.

In Figure 5, we present the impulse responses from a trivariate VAR that build on our bi-variate VAR by including (minus) the unemployment gap as the third variable, again using a Choleski indentification; ε_3 affects only the gap on impact, ε_2 affects only the gap

Figure 5: Impulse Responses in the 3-VAR ($\pi_t, \pi^e_{t+1}, \mathrm{gap}_t$)



Notes: this Figure plots the impulse responses to a one standard deviation shocks ε_1, ε_2 and ε_3. These shocks are obtained from a Choleski orthogonalization. The estimated VAR features two lags of Headline CPI inflations, the MSC inflation expectations and (minus) the employment gap. Sample is 1969Q1-2023Q1. Shaded area is the 95% confidence band.

and inflation on impact and ε_1 is unrestricted but to be orthogonal to the two other shocks.

The main feature we want to highlight from this figure is the quasi complete separation between inflation and inflation expectations on one hand and labour market tightness on the other hand. This can seen examining how the sub-block of impulse response to ε_1 and ε_2 for inflation and inflation expectations generated by this three variable VAR is almost identical to that generated by our two-variable VAR. Furthermore, we can see that the shock to unemployment ε_3 has essentially no impact on inflation or inflation expectations, while the two other shocks that drive almost all the variance of inflation have very effect on unemployment. In summary, these two (non-structural) VARs appear consistent with the view that the Phillips curve is likely very flat and that the persistent component of inflation may well be driven by expected inflation.

2.2 Could the Observed Joint Dynamics of Inflation and Inflation Expectations Reflect Rational Expectations?

As we have argued above, inflation expectations appear to potentially be an important driver of inflation. However, this may only be a proximate cause. It may well be the case that both inflation and inflation expectations are driven by a third variable, and expectations are simply good as capturing this force. Accordingly, we ask in this subsection whether the joint dynamics of inflation and inflation expectation reflected in our bi-variate VAR could be produced by our baseline Phillips curve model under rational expectations, where the dynamics of the labour market tightness and supply shocks would be the more fundamental factor. To that end, we assume that inflation, inflation expectations and the gap are generated by a model that comprise the baseline Phillips curve and exogenous processes for the gap and the supply shock:

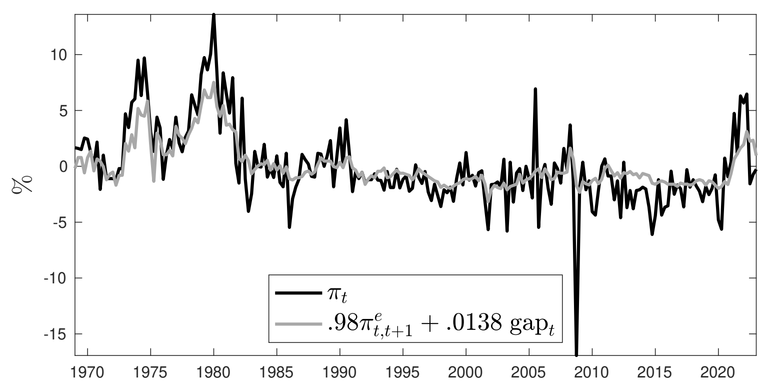
$$\pi_t = 0.99 E_t[\pi_{t+1}] + 0.0138\,\text{gap}_t + e_t \tag{3}$$

$$\text{gap}_t = \rho\,\text{gap}_{t-1} + v_t \tag{4}$$

$$e_t \sim N(0, \sigma_e^2) \quad v_t \sim N(0, \sigma_v^2) \tag{5}$$

We use Headline inflation and the Michigan Survey of Expectations as a measure of expectations. Figure 6 shows the fit of the baseline Phillips curve. As discussed in the previous section, the fit of the baseline Phillips curve hasn't worsened in the recent period, as illustrated on Figure 6.

Figure 6: The Fit of the Baseline Phillips Curve, 69-23



Notes: this Figure plots Headline CPI inflation together with inlfation predicted by the baseline Phillips curve (1)

Full Information Rational Expectation (FIRE) When the agent forms rational expectation, she uses the correct model (3)-(5) and forms expectation $E_t^{FIRE}[\pi_{t+1}]$. When the agent has full information, v_t and e_t are fully observable. The solution to the model is given by:

$$\pi_t = \frac{\alpha}{1-\beta\rho}\text{gap}_t + e_t \tag{6}$$

$$E_t^{FIRE}[\pi_{t+1}] = \frac{\alpha\rho}{1-\beta\rho}\text{gap}_t \tag{7}$$

Incomplete Information Rational Expectation (IIRE) To explore whether limited information alone can help to explain the bi-variate VAR pattern, we also consider a model when the expectation is formed with the correct model (3)-(5) but gap_t is not fully observable. We refer to this case as "Incomplete Information Rational Expectation" (IIRE). In this model, the agent knows that $E_t[\pi_{t+1}]$ will affect inflation, and she will form $E_t[\pi_{t+1}]$ using Bayes Rule. As a result, she understands that:

$$\pi_t - \beta E_t[\pi_{t+1}] = \alpha\text{gap}_t + e_t \tag{8}$$

But she cannot observe gap_t and w_t separately. In particular, we can assume she observes a disaggregate series as a signal:[12]

$$s_t = \alpha\text{gap}_t + \epsilon_t \tag{9}$$

$$\epsilon_t \sim N(0, \sigma_\epsilon^2) \tag{10}$$

The aggregate inflation is given by:

$$\pi_t = \beta E_t[\pi_{t+1}] + \alpha\text{gap}_t + \underbrace{\gamma\epsilon_t + w_t}_{\equiv e_t} \tag{11}$$

$$w_t \sim N(0, \sigma_e^2 - \gamma^2\sigma_\epsilon^2) \tag{12}$$

[12] This is similar to our baseline model that will be present later.

The agent then forms expectations using the Bayes Rule. Denoting the Kalman gain as k_i, the solution to this model is:[13]

$$E_t^{IIRE}[\pi_{t+1}] = (1 - k_i\alpha)\rho E_{t-1}^{IIRE}[\pi_t] + \frac{\alpha\rho k_i}{1-\beta\rho}(\alpha \text{gap}_t + \epsilon_t) \tag{13}$$

$$\pi_t = \beta(1 - k_i\alpha)\rho E_{t-1}^{IIRE}[\pi_t] + \frac{\alpha^2 k_i\rho\beta + (1-\beta\rho)\alpha}{1-\beta\rho}\text{gap}_t + \frac{k_i\rho\beta\alpha + (1-\beta\rho)\gamma}{1-\beta\rho}\epsilon_t + w_t \tag{14}$$

We examine under parameters consistent with our baseline Phillips curve, how well these rational expectation models can explain the bi-variate VAR we obtained from the last subsection. We estimate an AR(1) process for gap_t.[14] The estimated $\rho = 0.89$ (s.e. 0.03) and $\sigma_v^2 = 9.05$ (s.e. 0.33). We obtain an estimate of $\sigma_e^2 = 5.00$ from the residual of (3). In the IIRE model (13)-(14), there are two more free parameters: σ_ϵ and γ. We estimate these two parameters to match the IRFs from our empirical bivariate VAR, the resulting estimates are $\sigma_\epsilon = 1.08$ and $\gamma = 1$. The implied estimate of $\sigma_w = 1.96$.

Full Information Rational Expectation (FIRE) with persistent supply shock: When we estimate an AR(1) process for the implied supply shock e_t, we find low autocorrelation, with an estimated $\rho_e = 0.165$ (standard deviation $= 0.05$). In light of this, we also consider an extension of the FIRE model where we allow e_t to be persistent.[15]

For each of the three models considered, using the estimated parameters, we simulate 200 random samples with the same length as the data. With these generated data, we estimate the bi-variate VAR and see how well it can match the impulse responses derived

[13]The detailed derivations are left to Appendix F

[14]When we estimate an AR process with more lags, only the estimate on the first lag is significant.

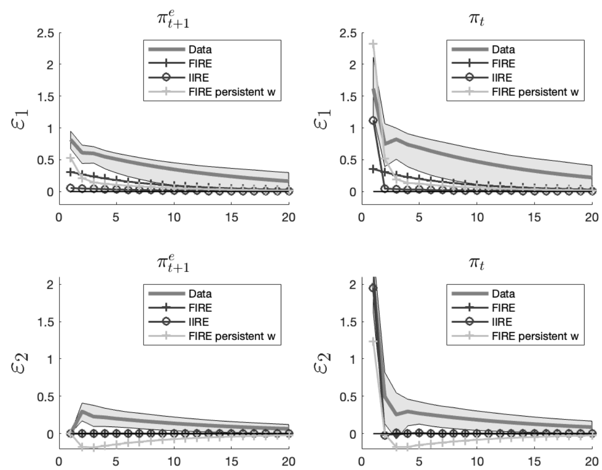
[15]The solution to such a FIRE model with persistent w_t is given by:

$$\pi_t = \frac{\alpha}{1-\beta\rho}\text{gap}_t + \frac{1}{1-\beta\rho_e}e_t \tag{15}$$

$$E_t^{FIREp}[\pi_{t+1}] = \frac{\alpha\rho}{1-\beta\rho}\text{gap}_t + \frac{\rho_e}{1-\beta\rho_e}e_t \tag{16}$$

from our empirical bi-variate VAR. The result can be seen in Figure 7. In this Figure, the solid lines are impulse responses associated with the empirical data, and the marked lines are the average impulse responses from generated data of rational expectation models. As can be seen in the Figure, the impulse responses from the generated data do not match that derived from actual data neither in shape nor level, even if in the IIRE model we estimate σ_ϵ and γ to deliberately match these IRFs.[16]

Figure 7: The Joint Process of π and π^e_{t+1} in the Data and Under models with Rational Expectations



Notes: on this Figure, the solid grey line plots the impulse responses to a one standard deviation shocks ε_1 and ε_2 estimated with data from MSC and Headline CPI. Sample is 1969Q1-2023Q1. Shaded area is the 95% confidence band. The marked black lines plot the average impulse responses (over 200 simulations of length 217) obtained from the same VAR estimated on simulated data, when the Data Generating Process is either the FIRE model (6)-(7) or the IIRE model (13)-(14), assuming e_t is i.i.d. The light grey line plots the average impulse responses estimated on simulated data from FIRE model assuming e_t is persistent.

The failure of rational expectation models not only lies in explaining the impulse responses. Table 3 reports the variance and covariance structure of inflation and expected inflation from the actual data and the generated data of rational expectation models. Not

[16]We also explore models with the hybrid Phillips Curve or adaptive expectations. We show that these models cannot match the empirical VAR with expected and headline inflation. These results are included in Appendix F.2

surprisingly, none of the rational expectation models can create inflation as volatile as the data. The simulated data have much lower persistence and variances in both realized and expected inflation, compared to the actual data.

Table 3: Variance-Covariance Data v.s. Rational Expectation Models

	Moments				
	$var(\pi_t)$	$cov(\pi_t, \pi_{t-1})$	$cov(\pi_t, \pi_{t-2})$	$var(E_t\pi_{t+1})$	$corr(E_t\pi_{t+1}, E_{t-1}\pi_t)$
Data	12.52	7.70	6.99	3.92	0.90
FIRE	5.49	0.45	0.39	0.42	0.87
IIRE	5.10	0.05	0.04	0.00	0.87
FIRE, w_t persistent	7.59	1.61	0.57	0.61	0.64

Notes: The "Model" moment is the average moment across 200 random samples, we could think about the "Data" moment as the moment from one particular random sample.

In part, this should not be too surprising. Given the rather small slope of the Phillips curve implied by the work of Hazell, Herreño, Nakamura, and Steinsson [2022], it is hard to see how shocks to labor market tightness could generate important volatility in inflation and inflation expectations. In particular, the difficulty for the rational expectation models to match reality is to generate persistent responses of inflation expectations. The rational agent understands that the underlying persistent factor moving inflation plays only a limited role. As a result, they shouldn't expect future inflation to have persistent movement as well. This is still true even if we consider the very low auto-correlation of supply shocks implied by the residuals of our baseline Phillips curve. Our results above show that these shocks are also unlikely to be able to generate sufficient volatility and persistence in inflation and inflation expectations. Moreover, limited information won't help with the situation. In fact, the IIRE case has the most muted response among all the three models and generates the least volatile as well as the least persistent inflation. This is not only because the rational agent understands the persistent labor market condition plays little role. They also realize that their signals are not informative because of the large noise-to-signal ratio due to low α. As a result, their expectation doesn't respond much to the movements in labor market conditions.

All in all, we conclude from this section that there exists a persistent dynamics of inflation expectations that is highly correlated with actual inflation and that is unlikely to be explained by a Rational Expectation New Keynesian Phillips curve. In the next section, we explore what may inflation expectations contain.

3 The Information Encoded in Inflation Expectations

A simple Phillips curve under full information rational expectation does not appear to be able to explain the joint dynamics of inflation and inflation expectations. This raises the quation of what could be driving inflation expectation? In this section we want to provide preliminary evidence for our conjecture that inflation expectations may be driven by a perceived common component across disaggregated categories of inflation. Our narrative is quite simple. Inflation is defined as a generalized increase in prices. So when agents (both firms and households) see prices increase in many sectors (although not necessarily in *all* sectors), they take this to be sign that inflation is taking hold. This affects their expectations and this incite firms to increase prices, either for reasons in standard sticky price model or because they expect to have to pay higher wages to keep their employees.

To support this conjecture, we take the set of disaggregated inflation data for the 25 expenditure categories used by the BLS to construct the CPI (see Table D.1 in Appendix D for a description) and extract the common component of these categorical inflations. To do so, we estimate a Dynamic Factor Model[17] in which categorical inflations are driven by one common persistent factor $\mathcal{C}_t$ which follows an AR(1) process. The model takes the following form:

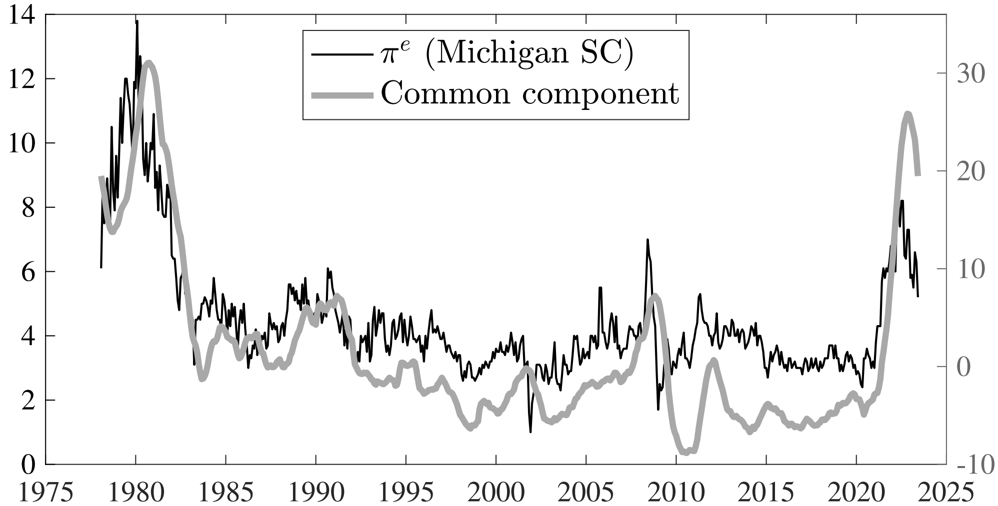
$$\pi_{i,t} = \alpha_i \, \mathcal{C}_t + e_{i,t} \tag{17}$$

$$\mathcal{C}_t = \rho_{\mathcal{C}} \, \mathcal{C}_t + v_t \tag{18}$$

where i stands for different categories and $\pi_{i,t}$ is the montly y-to-y inflation of each category i. How the common component $\mathcal{C}_t$ affects different $\pi_{i,t}$ may be different, and is captured by α_i. Data are montlhy over the sample 1978m1-2023-m5 for seventeen categorical inflations and shorter for the remaining seven (see Table D.1). We estimate that model with Kalman Filter Smoothing using a Maximum Likelihood Estimator that accounts for the missing observations. Estimated parameters are $\rho_{\mathcal{C}}$ and $\{\alpha_i, \sigma_i^2\}_{i=1}^{24}$. Note that σ_v^2 is not separately identifiable from α_i's, so that we normalize it to 1. The parameters estimates we obtained are presented in get are in Table D.2 in Appendix D. The important element to note is that all the α_is are positive, so that $\mathcal{C}_t$ is indeed capturing a common component in inflation. In Figure 8, we plot expected inflation and this common component (normalized to have the same mean and variance than expected inflation) which reflects broad-based supply shocks. As can be seen, this measure of broad-based inflation tracts the expected inflation quite well. It is interesting to compare the (normalized) weights of each category in the common components $\left(\frac{\alpha_i}{\sum_{j=1}^{25} \alpha_j}\right)$ to the expenditure share of that category, as used by the BLS to compute CPI. Those two sets of weights are represented one against the other in Figure 9. If the grey circles were aligned along the 45 degree line, that would mean that the common

[17] see Stock and Watson [2011] for a thorough review.

Figure 8: Michigan Survey of Consumers Inflation Expectations and the Common Component



Notes: this Figure plots inflation expectations (as measured by the Michigan Survey of Consumers) together with the common component $\mathcal{C}_t$. $\mathcal{C}_t$ is obtained from the estimation of (17) and (18) on the 25 expenditure categories used by the BLS to construct the CPI.

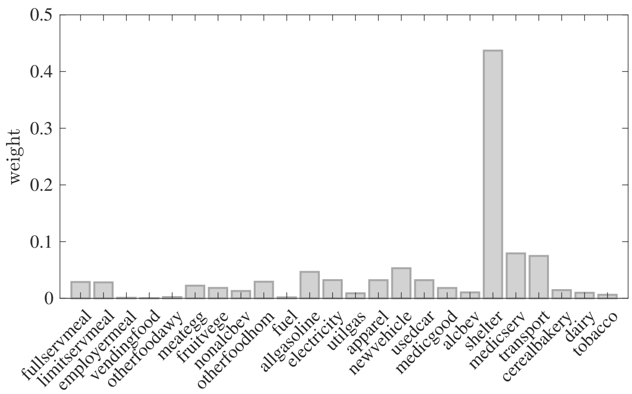
component encodes the categorical inflations in the same way than way CPI does. As we can see, it is far from being the case. The correlation between these two set of weight is 0.12, with a 95% confidence set [-0.29, 0.5]. This clear shows that the common component encodes some information about broad-based shock, as the weights differ from the CPI ones. We then regress our measure of expected inflation (MSC) on the common component of the disaggregated inflation data and CPI (headline) inflation. The resulting regression is:

$$\pi^e_{t+1} = \underset{(0.10)}{1.82} + \underset{(0.04)}{0.17}\ \mathcal{C}_t + \underset{(0.02)}{0.56}\ \pi^{HL}_t, \tag{19}$$

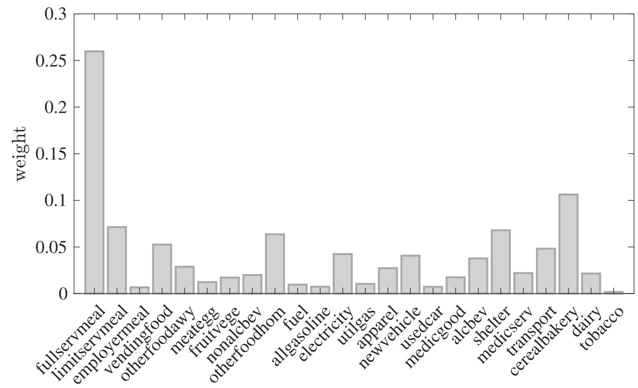
where the data are at a monthly frequency with 545 observations and an adjusted R-Squared of 0.837 (standard errors between parenthesis). The common component enters significantly in the regression on top of Headline CPI, which shows that economic agents (here the households) use it to forecast inflation. Regressing the common component on Headline CPI shows that 30% of its variations are not explained by the CPI inflation. These 30% encode

Figure 9: Comparing the Weights in the Common Component and in the CPI Basket

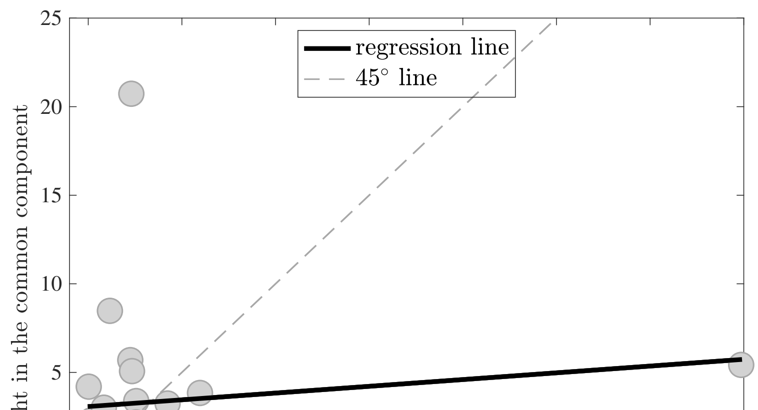
(a) BLS weights



(b) Common Component weights



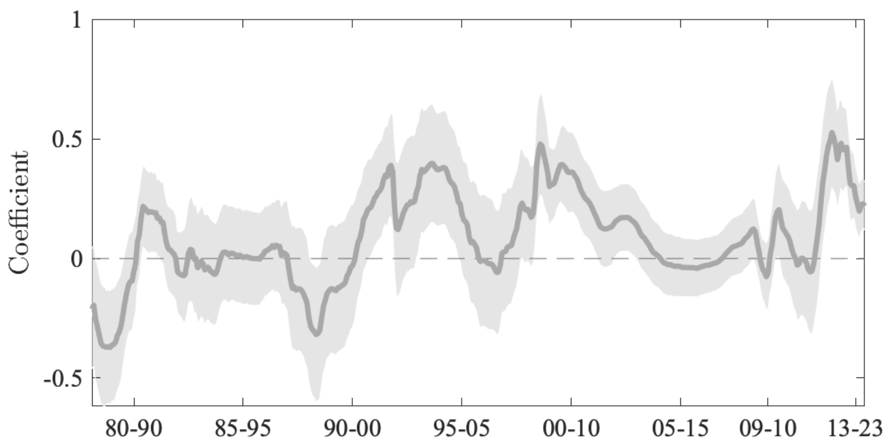
(c) BLS weights against Common Component weights



Notes: Panel (a) plots the weight of each category in the CPI basket, while panel (b) plots the (normalized) weights in the common component $\mathcal{C}_t$*. Panel (c) plots these two sets of weights one against the other, together with the regression line.*

joint movements of the categorical inflations that are not strictly speaking inflation, and will capture broad-based supply shocks. Finally, we estimate Equation (19) on a rolling sample of 10 years, and report the sequence of coefficients on the common component in Figure 10. As one can see, there are some periods in which the information in the common component is particularly relevant for inflation expectations. This is the case in the recent period.

Figure 10: Rolling Estimation of the Coefficient on the Common Component $\mathcal{C}_t$ in Equation (19)



Notes: this Figure plots the coefficient on the common component in Equation (19), when this equation is estimated on a rolling sample of 10 years.

4 Model

In this section our aim is to take up the following challenge, building on the idea that agents may form expectations by extracting a common component from disaggregated data. We want to explore whether this view of expectations formation, combined with our baseline

Phillips curve, can generate data that matches the features we documented in the bi-variate VAR for inflation and expected inflation. To this end, it is helpful to describe the setting in terms of the actual law of motions of the economy, and a perceived law of motion as seen by agents.

We start with the aggregate Phillips Curve as in our previous sections:

$$\pi_t = \beta \mathbb{E}[\pi_{t+1}] + \alpha \text{gap}_t + e_t \tag{20}$$

The headline aggregate inflation π_t can be thought of as a weighted average of disaggregate inflations of goods and services from different categories. Consider each of these series follows a disaggregate Phillips Curve similar to the aggregate one:[18]

$$\pi_{j,t} = \beta \mathbb{E}[\pi_{t+1}] + \alpha \text{gap}_t + e_{j,t} \tag{21}$$

where j stands for a specific category, $e_{j,t}$ is the i.i.d category specific shock with distribution $N(0, \sigma_j^2)$, and $\mathbb{E}[\pi_{t+1}]$ is the private sector's expected inflation. We postpone the complete description of the expectation formation process to a later point. As in section 2.2, gap_t follows an AR(1) process:[19]

$$\text{gap}_t = \rho \text{gap}_{t-1} + v_t \tag{22}$$

The aggregate headline inflation is the weighted average of the disaggregate series, assuming

[18] Note that in general, we can allow for different responses of disaggregate inflations to expectation (β_j) and labor market conditions (α_j). This will give extra degrees of freedom of the model in our quantitative analysis later. For ease of exposition, we stay with the simple case where $\beta_j = \beta$ and $\alpha_j = \alpha$ for all j.

[19] We estimate an AR(4) process on the negative unemployment gap using sample 1969-2023, only the first lag enters significantly with an estimate $\rho = 0.89$.

there are N categories:

$$\pi_t = \sum_{j=1}^{N} \gamma_j \beta \mathbb{E}[\pi_{t+1}] + \sum_{j=1}^{N} \gamma_j \alpha \text{gap}_t + \sum_{j=1}^{N} \gamma_j e_{j,t} \tag{23}$$

where γ_j is the corresponding weight, and $\sum_j \gamma_j = 1$. Among all the categorical inflations, the agent may only observe a subset of them, denote the set of these indexes as $\mathcal{S} \subseteq \{1, 2, ..., N\}$.[20] They use these categorical inflations as signals to form expectations. Now we can explicitly write the actual law of motion that generates HL inflation:

ALM:

$$\pi_{j,t} = \beta \mathbb{E}[\pi_{t+1}] + \alpha \text{gap}_t + e_{j,t}, \quad \forall j \in \mathcal{S} \tag{24}$$

$$\pi_t = \beta \mathbb{E}[\pi_{t+1}] + \alpha \text{gap}_t + \underbrace{\sum_{j \in \mathcal{S}} \gamma_j e_{j,t} + w_t}_{\equiv e_t} \tag{25}$$

$$\text{gap}_t = \rho \text{gap}_{t-1} + v_t \tag{26}$$

$$e_{j,t} \sim N(0, \sigma_j^2), \quad v_t \sim N(0, \sigma_v^2), \quad w_t \sim N(0, \sigma_w^2) \tag{27}$$

where $w_t \equiv \sum_{j \notin \mathcal{S}} \gamma_j e_{j,t}$. The set of $\pi_{j,t}$ for $j \in \mathcal{S}$ can be thought of as observed signals used by the agent to form expectations. We denote the cardinality of $\mathcal{S}$ as $m \leq N$.

Expectation formation: We now describe how expectations are formed. As shown in section 2, neither the FIRE nor the IIRE model can be reconciled with the empirical VAR and variance-covariance structures from inflation data. As a result, simple imperfect information cannot explain the joint dynamics of inflation and expectation. In our expectation formation process, we allow the agent to have a perceived law of motion (PLM) for inflation

[20] We do not take a stand on what are the causes for the agents not being fully attentive to all the categorical inflations. Many frameworks such as rational inattention (Sims [2003]) or sparsity (Gabaix [2014]) can create such a partial information environment.

slightly different from the actual one described above. The agent will use the PLM to form expectations.

In particular, the agent observes $\{\pi_{j,t}, \pi_t\}$ and forms expectations using this information. They perceive the prices they observe as generated by a common persistent component $\tilde{z}_t$ and price-specific transitory components $\tilde{e}_{j,t}$. The persistent part is useful to form expectations about future aggregate inflation. The agents then face a signal-extraction problem with many signals. This can be summarized into the following perceived law of motion (PLM):[21]

PLM:

$$\pi_{j,t} = \tilde{z}_t + \tilde{e}_{j,t} \tag{28}$$

$$\pi_t = \tilde{z}_t + \sum_{j \in \mathcal{S}} \gamma_j \tilde{e}_{j,t} + \tilde{w}_t \tag{29}$$

$$\tilde{z}_t = \tilde{\rho} \tilde{z}_{t-1} + \tilde{v}_t \tag{30}$$

$$\tilde{e}_{j,t} \sim N\left(\mathbf{0}, \tilde{\sigma}_j^2\right), \quad \tilde{v}_t \sim N(0, \tilde{\sigma}_v^2), \quad \tilde{w}_t \sim N(0, \tilde{\sigma}_w^2) \tag{31}$$

The tilde variables denote the perceived variances and persistence for the law of motion that are possibly different from the actual ones. The agents endowed with this PLM are still very sophisticated. They will use the observed inflation series to infer state $\tilde{z}_t$ - the common component from their perspective - and use these beliefs about $\tilde{z}_t$ to form expectation of aggregate inflation according to (29). However, they are bounded rational because they treat the state-space system (28)-(31) as a standard signal-extraction problem where the noise of a signal is uncorrelated with the state. However, because the disaggregate prices are common to all the agents, when the aggregate expectation responds to the signals, these noises will affect the state variable and the signal. When the agent fails to recognise the

[21] Such a signal-extraction formulation is widely used in the literature about inflation expectations. Examples are like Coibion and Gorodnichenko [2015] and Bordalo, Gennaioli, Ma, and Shleifer [2020] etc.

hidden state is correlated with noise in such a way, it creates a self-confirming loop. In other words, the agents don't realize the signals they observe are endogenous. This bounded rationality is the key to create the amplification mechanism in our expectation formation model.

Because we do not observe what (subset of) disaggregate inflation series the agents use to form expectations, mathematically, it is equivalent to treat the signals in (28) as an average signal.[22] Write it together with the aggregate inflation (29) we get the observational equations from the agent's perspective:

$$S_t \equiv \begin{pmatrix} s_t \\ \pi_t \end{pmatrix} = \iota \tilde{z}_t + \begin{pmatrix} \tilde{\epsilon}_t \\ \gamma \tilde{\epsilon}_t + \tilde{w}_t \end{pmatrix} \tag{32}$$

whereas the actual signals are generated by:

$$S_t \equiv \begin{pmatrix} s_t \\ \pi_t \end{pmatrix} = \iota \beta \mathbb{E} \pi_{t+1} + \iota \alpha z_t + \begin{pmatrix} \epsilon_t \\ \gamma \epsilon_t + w_t \end{pmatrix} \tag{33}$$

where s_t is an average signal from m independent categorical signals, with precision $1/\tilde{\sigma}_\epsilon^2$. This gives equivalent results as we write explicitly the m signals out. However, the interpretation of the average noise ϵ_t is subtle. The probability of a positive realization of ϵ_t is equivalent to that of the same positive realizations of $e_{j,t}$ in all m categories jointly, which is much more unlikely than just one category having a positive realization. As a result, inflation in one particular category j is unlikely to induce a sizable movement in inflation expectation. However, when there are *broad-based positive inflation shocks*, this will be a large positive realization in ϵ_t. The coefficient γ can be interpreted as the fraction of categories the agent

[22]For independent signals $j = 1, ..., m$ with different precisions $\frac{1}{\tilde{\sigma}_j^2}$. Observing m different signals is equivalent to observing a signal with precision: $\frac{1}{\tilde{\sigma}_\epsilon^2} = \sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2}$. See Appendix E for the derivation. For a similar reason, the ALM for disaggregate price series (24) can also be written as an average price signal with weight $\frac{\frac{1}{\sigma_i^2}}{\sum_{j=1}^m \frac{1}{\sigma_j^2}}$. This gives the representation (33) with possibly $\sigma_\epsilon \neq \tilde{\sigma}_\epsilon$.

paid attention to.

Another feature of the above observational equation is that the signals are weighted by their precisions that do not depend on their actual weights γ_j in the aggregate price index. In other words, a category that has a very low weight in the aggregate price index or expenditure share is as informative as a category that has significant weight but with a similar signal-to-noise ratio. This feature coincides with our finding that the weights of disaggregate price indices in the CPI basket are not highly correlated with the estimated weights from Kalman Filter Smoothing in section 3.

The agent then faces the state-space model described by (30)-(32). She solves a signal-extraction problem to form belief $z_{t|t}$ and uses it to form expectations about inflation. Because expectations formed in the current period will also affect current inflation and signals the agent sees. We want to be very specific about the timing of expectation formation and how expectations affect inflation. To be specific, consider at each period t, there are two sub-periods. The expectations are formed in the following steps:

1. At the first sub-period of t, gap_t realizes. The agent forms expectation using the disaggregate signals s_t, which are affected simultaneously by the consensus expectation formed in this sub-period. We denote the expected inflation formed in this sub-period as $\pi_{t+1|t,0}$. This is the expectation $\mathbb{E}\pi_{t+1}$ affecting in (24) and (25). Consistent with the treatment in section 1 and section 2, we take this expectation as those observed in the MSC.

2. In the second sub-period of t, the agent also observes aggregate signal π_t. She uses this extra information to update her belief about future inflation, denoting as $\pi_{t+1|t,1}$. This will be the prior mean she carried on entering period $t+1$.

3. Both expectations $\pi_{t+1|t,0}$ and $\pi_{t+1|t,1}$ are formed from beliefs on $\tilde{z}_t$. Similarly, we denote these beliefs as $z_{t|t,0}$ and $z_{t|t,1}$. From PLM (30)-(32) the links between expected inflation and common component are:

$$\pi_{t+1|t,k} = z_{t+1|t,k} = \tilde{\rho} z_{t|t,k} \quad \forall k = 1, 2 \tag{34}$$

4. The beliefs on $\tilde{z}_t$ are formed using Kalman Filter according to the state-space representation (the PLM) (30)-(32). Denote the noise in agent's perceived signal-generating process (32) as:

$$\begin{pmatrix} \tilde{\epsilon}_t \\ \gamma\tilde{\epsilon}_t + \tilde{w}_t \end{pmatrix} \sim N(\mathbf{0}, \tilde{R}), \quad \tilde{R} \equiv \begin{pmatrix} \tilde{\sigma}_\epsilon^2 & \gamma\tilde{\sigma}_\epsilon^2 \\ \gamma\tilde{\sigma}_\epsilon^2 & \gamma^2\tilde{\sigma}_\epsilon^2 + \tilde{\sigma}_w^2 \end{pmatrix} \tag{35}$$

The agent forms weights on signals, according to Kalman Filter, taking into consideration that s_t and π_t are correlated. Denote ι_n as a $n \times 1$ vector of ones. The stationary Kalman Filter is given by:

$$\hat{K} = \sigma^2 \iota_2' (\iota_2 \sigma^2 \iota_2' + \tilde{R})^{-1} \tag{36}$$

$$\sigma^2 = \tilde{\rho}^2 (\sigma^2 - \hat{K} \iota_2 \sigma^2) + \tilde{\sigma}_v^2 \tag{37}$$

where σ^2 is the stationary posterior variance of belief on $\tilde{z}_t$. Now note the $\hat{K}$ is an 1×2 vector, with first element being the weight on the categorical signal, and the second weight on aggregate inflation π_t: $\hat{K} \equiv (K \quad k)$.

5. First sub-period of t, agent observes s_t, forms nowcast of $\tilde{z}_t$:

$$z_{t|t,0} = (1 - K) z_{t|t-1,1} + K s_t \tag{38}$$

6. Second sub-period of t, the agent sees π_t and updates belief on z_t:

$$z_{t|t,1} = ((1 - \hat{K}\iota_2)z_{t|t-1,1} + \hat{K}S_t) \tag{39}$$

7. Note that signals s_t and π_t are realized according to ALM (33), so expectation $\pi_{t+1|t,0}$ and signal s_t will affect each other simaltaneously. The above system boils down to the following representation:

$$\pi_{t+1|t,0} = \frac{\tilde{\rho}(1-K)}{1-K\beta\tilde{\rho}} z_{t|t-1,1} + \frac{\tilde{\rho}K\alpha}{1-K\beta\tilde{\rho}}\text{gap}_t + \frac{\tilde{\rho}K}{1-K\beta\tilde{\rho}}\epsilon_t \tag{40}$$

$$\pi_t = \beta\pi_{t+1|t,0} + \alpha\text{gap}_t + \gamma\epsilon_t + w_t \tag{41}$$

$$\begin{aligned} z_{t+1|t,1} = {} & \tilde{\rho}\left(1 - (K+k)\frac{1-\beta\tilde{\rho}}{1-K\beta\tilde{\rho}}\right) z_{t|t-1,1} + \tilde{\rho}\frac{(K+k)\alpha}{1-K\beta\tilde{\rho}}\text{gap}_t \\ & + \tilde{\rho}\left(\frac{K+\tilde{\rho}Kk\beta}{1-K\beta\tilde{\rho}} + \gamma k\right)\epsilon_t + k\tilde{\rho}w_t \end{aligned} \tag{42}$$

$$\text{gap}_t = \rho\text{gap}_{t-1} + v_t \tag{43}$$

The above expectation formation process creates persistent inflation in a quasi self-confirming way through expectation even if the persistent component gap_t itself can only induce small volatility in inflation, consistent with the estimates of the NKPC in section 1. When a positive broad-base price shock ϵ_t hits the economy, the agent observes price hikes in many disaggregate price series. The agent easily confuses it with a positive shock to the common component $\tilde{z}_t$. When the agent believes in large volatility of $\tilde{z}_t$, she will increase her expected inflation. Because the disaggregated price indices are public signals, the average expectation will increase which drives up both the disaggregate signals as well as the actual inflation they observe. This feedback channel largely amplify the response of expected inflation to broad-base shocks on impact.[23]

[23] Note that this amplification comes from the fact that the agents fail to understand expectation $\pi_{t+1|t,0}$ is part of the perceived common component $\tilde{z}_t$. More precisely, they fail to understand the hidden state is

Moreover, in the second sub-period, the agent observes π_t which is also pushed up by expectation. This justifies her belief of positive realization in the common component. As a result, the agent enters next period with a prior of high inflation. In the proceeding periods, both disaggregate and aggregate inflations will remain high even if there are no more shocks, because the expectations are weighted averages of new signals and the priors. This then creates very persistent responses of both actual and expected inflation to broad-base price shocks. Moreover, after the response on impact, the actual inflation will react almost one-to-one to expectation, as what we have seen in the empirical VAR. This mechanism gives a way to reconcile with the limited role of labor market tightness implied by a flat Phillips Curve, the highly persistent inflation, as well as our findings with the empirical VAR. In the next subsection, we perform a quantitative analysis bringing the model to closely match our empirical VAR.

4.1 Quantitative Analysis

Our quantitative exercise aims to explore whether our proposed model above can match the findings in our empirical VAR while maintaining a flat Phillips Curve as in our section 1 or estimated by Hazell, Herreño, Nakamura, and Steinsson [2022]. We first fix the values of β, ρ, α, and σ_v at the values of point estimates in section 1, then we estimate the other parameters by minimum distance as in Gourieroux, Monfort, and Renault [1993] and Mertens and Ravn [2011]. Denote the vector of parameters to be estimated as Θ, the estimates are given by:

$$\hat{\Theta} = \underset{\Theta}{\operatorname{argmin}} \left(\hat{\mathcal{M}} - \frac{1}{N} \sum_{n}^{N} \hat{\mathcal{M}}_T^n(\Theta) \right)' W \left(\hat{\mathcal{M}} - \frac{1}{N} \sum_{n}^{N} \hat{\mathcal{M}}_T^n(\Theta) \right) \tag{44}$$

where $\hat{\mathcal{M}}$ is the empirical moments from the data, including the 20 periods impulse response functions (IRF) estimated from the bivariate VAR in section 2.1. $I\hat{R}F_T^n(\Theta)$ is the average

correlated with the noise of their signal. This arises because the signals the agents observe are endogenous.

estimated IRF from N simulated random sample of the model with parameter Θ. T is the length for the simulated random sample, which is set to be the same as our actual sample 1969-2023.

Furthermore, because we allow the agents to have subjective parameters of their PLM, and they also observe the aggregate inflation. When the variance-covariance structure of actual inflation is far from those implied by their subjective parameters, they may realize that they have incorrect beliefs and want to adjust. This makes the misperception of the PLM held by the agents to be non-sustainable. To guarantee a sustainable misperception, we also restrict the variance-covariance structure of actual inflation created by the model to match with the implied moments from the agent's PLM.[24] In our baseline estimation, we use $N = 200$ and the identity matrix as the weighting matrix. The estimated parameters from NKPC and minimum distance are reported in Table 4:

From Table 4, the subjective standard deviation of the innovations to the "common shock" is 0.7, whereas the actual volatility of the persistent shock (the labor market tightness) from the NKPC estimates is $\alpha\sigma_v = 0.04$. As a result, the perceived standard deviation of "common shock" $\tilde{z}_t$ is 2.5 and the actual variance of z_t is only 0.09.[25] Most of the perceived volatility comes from the consensus expectation that is driving the observed inflation series. This pattern coincides with our empirical exercise with disaggregate inflation series, where we show the extracted "common component" from these series follows the inflation expectations from MSC very well. Another observation is that the actual standard deviation of the broad-base shock, σ_ϵ, is greater than the perceived $\tilde{\sigma}_\epsilon$. Note that in the signal-extraction problem, $1/\tilde{\sigma}_\epsilon$ is the perceived accuracy of the disaggregate signal. $\sigma_\epsilon > \tilde{\sigma}_\epsilon$ then reflects the possibility

[24]To be specific, in our minimum distance estimation we also minimize the distance between the variance, the 1st and 2nd order auto-covariances of the actual inflation and the corresponding implied moments from the agent's PLM.

[25]The difference between subjective and actual volatility of z_t is even larger because the perceived persistence is greater than actual persistence estimated from the NKPC, $\tilde{\rho} > \rho$.

Table 4: Estimated parameters

From NKPC			
Parameter:	Estimate	Parameter:	Estimate
β	0.99	ρ	0.89
σ_v	3.02	α	0.0138
From IRF Matching			
Parameter:	Estimate	Parameter:	Estimate
$\tilde{\sigma}_v$	0.70	σ_ϵ	3.82
$\tilde{\rho}$	0.96	$\tilde{\sigma}_w$	2.13
γ	0.26	$\tilde{\sigma}_\epsilon$	2.34
σ_w	2.01		

Notes: The estimates from NKPC are from our baseline estimation with Hazel et. al. estimates of the gap. The σ_w is implied by the variance of the residual from NKPC estimation, σ_ϵ, and γ: $\sigma_w = \sqrt{var(res) - \gamma^2 \sigma_\epsilon^2}$.

that the agent perceives the disaggregate signals are more accurate than they actually are.

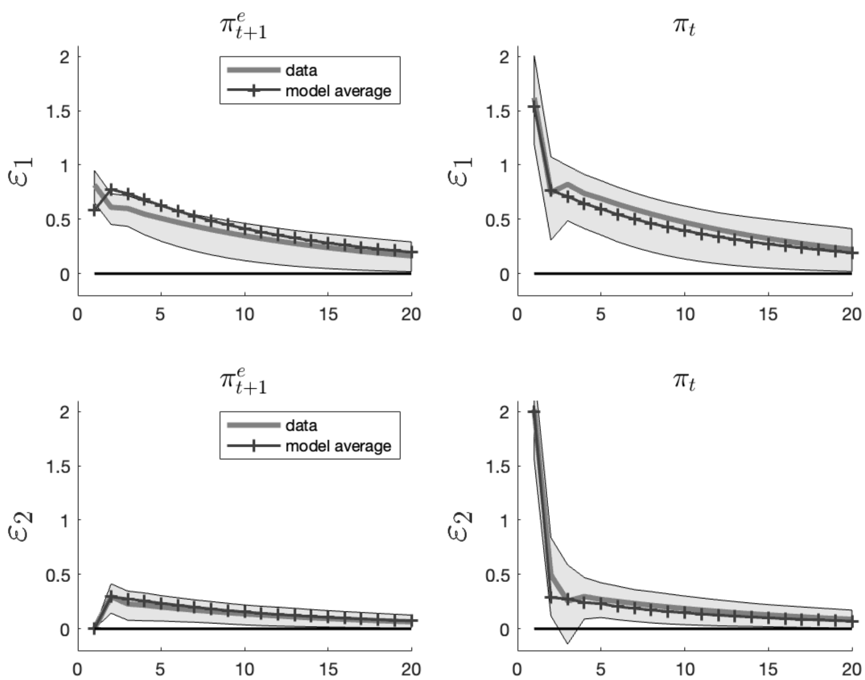
In the signal-extraction problem, the agents form expectations using the PLM and their subjective parameters. The large $\tilde{\sigma}_v$ and relatively smaller $\tilde{\sigma}_\epsilon$ make the agents believe the disaggregate inflation series is much more informative about the common shock than they really are. This is the reason why in the current model the broad-base shocks can trigger larger movements of expectations than those under FIRE or IIRE. Moreover, the adjustments in expectations are persistent because the agents fail to realize the expectations will affect inflation they see. Finally, we want to point out that under the semi self-fulfilling force described above, the joint system of inflation and expected inflation is still stationary under the estimated parameters. Figure 11 shows how the estimated model matches the empirical IRF we obtained in section 2.

From Figure 11, the marked line is the average IRF across 200 random samples simulated from the estimated model. From the model's perspective, the IRF of the first shock reflects responses to the combination of common and broad-base shock. Because the actual variation of the common shock is quite small, the shape of the IRF mainly reflects the impact of the broad-base shock. The second shock corresponds to the average inflation shocks that are not used in forming expectations reported in the survey, w_t.[26]

Figure 12 depicts the IRFs of the structural shocks from the estimated model. We see clearly that the ϵ_t shock plays a dominant role in shaping the IRF of the first shock in Figure 11. This is because the variation in inflation created by v_t is minimal due to the flatness of Phillips Curve. Meanwhile, the IRF of the second shock follows from the IRF of w_t. Moreover, the broad-base shock ϵ_t itself only increase inflation on impact by a factor of γ.

[26] These are the shocks to the disaggregate price series that are not observed by the agents when forming the expectations reported in the survey. The agents observe the aggregate inflation after forming the expectations that they report in the survey. As a result, this shock affects the actual inflation on impact but affects the reported inflation expectation only with a lag.

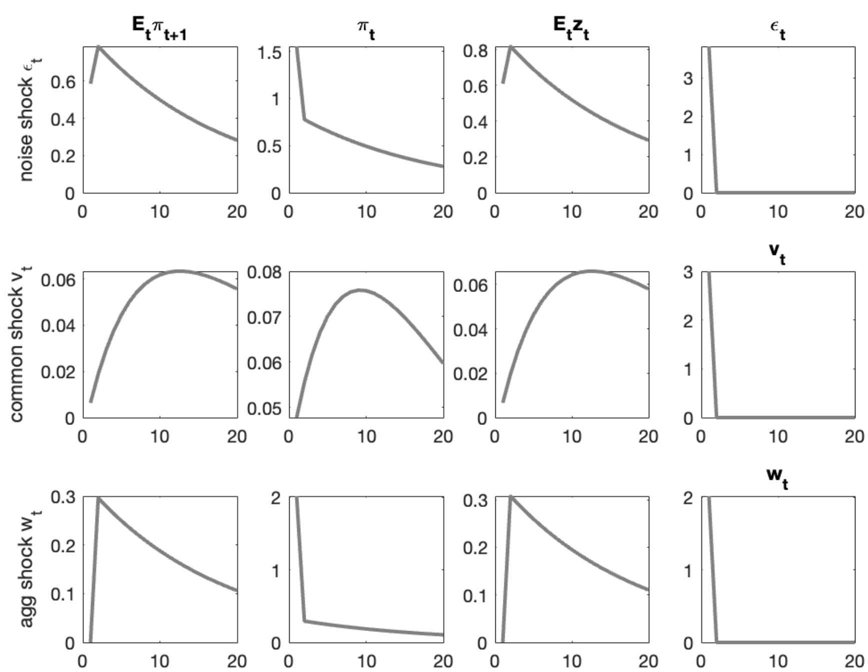
Figure 11: IRF from data and model simulation



Notes: thick gray lines are IRFs from the VAR with actual data; marked black lines are average IRFs from simulated data across 200 random samples. The estimated VAR is VAR(2) with Cholesky Decomposition ordering π_{t+1}^e - π_t. The shaded area is 95% CI.

However, the agent confuses them with the "common component" and triggers large increase of inflation expectation, which pushes up the aggregate inflation. On the other hand, the aggregate inflation shock w_t increases aggregate inflation by a lot on impact, however it doesn't affect expectation right away and doesn't cause the self-confirming amplification. As a result, the shock appears to affect aggregate inflation in a more transitory way.

Figure 12: IRF of structural shocks from estimated model



Notes: The structural impulse response functions of structural shocks from the estimated model. ϵ_t is the broad-base supply shock that has direct impact on expectations. v_t is the innovation to gap_t. w_t is the aggregate supply shock that does not directly affect expectations on impact.

As mentioned before, when the variance-covariance structure of the actual inflation is far from that of the agent's subjective model, the agent will realize the discrepancy between their subjective model and reality. When estimating the model we deliberately match the variance-covariance structures of inflation from objective and subjective models. Table 5 reports these moments. Importantly, the fact that the subjective incorrect belief is supported by the realized inflation is a model property: the volatile and persistent common component perceived by the agents is mostly coming from the consensus expectation. However, the agents believe this component to be exogenous, whereas in reality such a component is endogenously created by expectation formation.

Table 5: Moments

	Untargeted	Targeted	
	Data	Subjective Model	Model
		HL CPI	
$var(\pi_{t,h})$	12.52	11.33	11.33
$cov(\pi_{t,h}, \pi_{t-1,h})$	7.70	6.17	6.28
$cov(\pi_{t,h}, \pi_{t-2,h})$	6.99	5.92	5.82

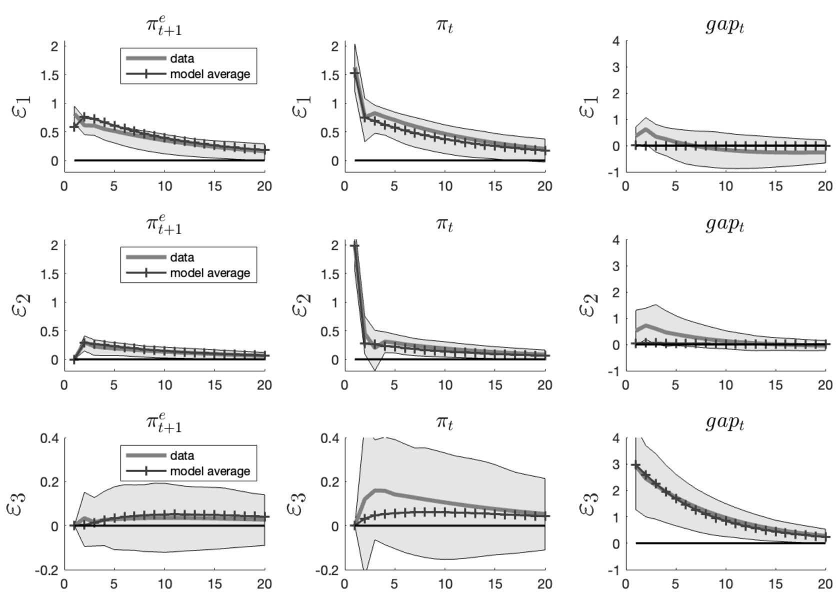
Notes: The "Model" moment is the average moment across 200 random samples.

Finally, in our estimation procedure we didn't directly use the negative unemployment gap as one observable to match the empirical tri-variate VAR that we obtained with expectation, inflation and negative unemployment gap in section 2. In figure 13 we show the IRFs from the same tri-variate VAR we get from 200 random samples simulated using our baseline model and compare them with the IRF from the empirical VAR in section 2. We see that the IRFs of the third shock ε_e match very well even if we are not targeting them in our estimation.[27] [28]

[27]Note that the IRFs of gap_t to the first two shocks ε_1 and ε_2 from the model are by construction always zero because gap_t is modeled as exogenous shocks.

[28]In Appendix XXX, we also include the IRFs using actual and simulated data from our baseline model with alternative orders of variables for Choleski Decomposition. Because the Choleski VAR are used as a method to summarize joint dynamics, we would expect the data generated by a correct model to give similar results as the actual data when summarized with a different order in doing Choleski VAR. These are confirmed in Appendix XXX.

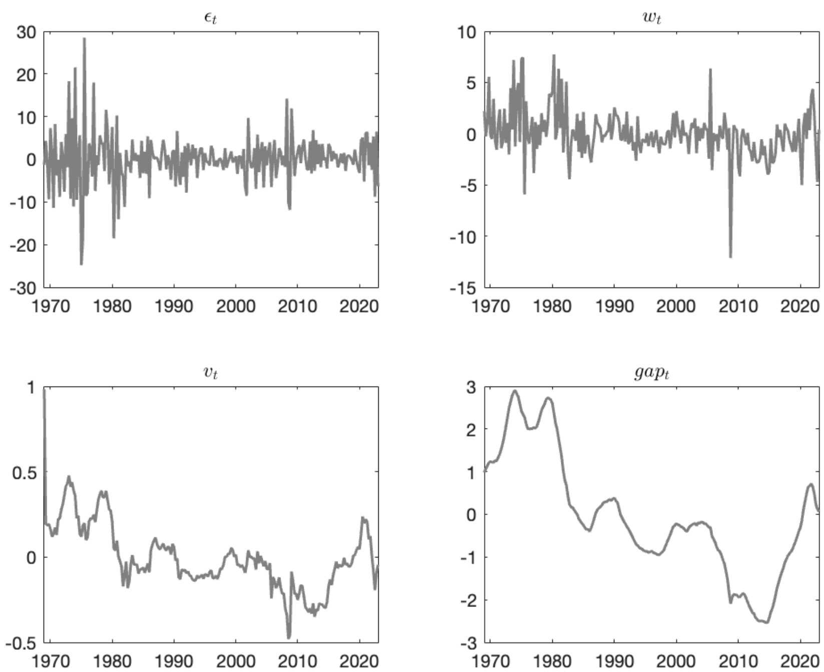
Figure 13: Trivariate VAR IRF from data and model simulation



Notes: thick gray lines are IRFs from trivariate-VAR with actual data; marked black line is average IRF from simulated data across 200 random samples. The estimated VAR is VAR(2) with ordering π^e_{t+1}, π_t, and gap_t. The shaded area is 95% CI.

4.2 Recovered Shocks and Counterfactuals

Figure 14: Filtered shocks from the estimated model



Notes: Model implied structural shocks backed out using the estimated parameters and Kalman Filter Smoothing. ϵ_t is the broad-base supply shock. w_t is the aggregate supply shock to headline inflation. v_t is the innovation to gap_t.

With our model, there are three different types of shocks whereas only two observables $\mathbb{E}_t\pi_{t+1}$ and π_t. With the estimated model we can filter out the three shocks by formalizing our model into a state-space representation and use Kalman Filter Smoothing to form projections of the latent states and shocks implied by our model. We include the details in Appendix H. Figure 14 depicts the shocks recovered from our estimated model. Because the Phillips Curve is quite flat (α being really low), the data on inflation and expected inflation have low signal-to-noise ratio. As a result, the recovered series on v_t and gap_t is very imprecise. This can be partly seen from the fact that the variance of recovered v_t is merely 0.04 whereas the true variance of v_t should be 3.01, meaning that the algorithm doesn't really forecast

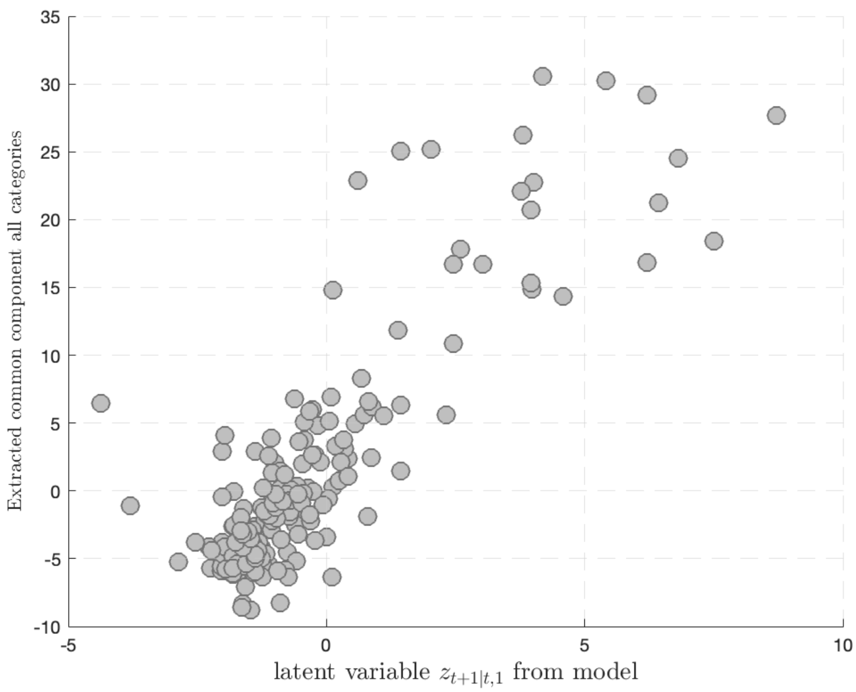
v_t using the observables. However, the recovered series for ϵ_t and w_t are quite trustworthy because the observables contain lots of information on these two shocks. We illustrate these points using simulated data in Appendix H.

In our model, the system of inflation and expected inflation is driven by a latent variable $z_{t+1|t,1}$, which is the "common component" the agents perceived using the disaggregate signals. This variable can be obtained using the Kalman Filter Smoothing approach mentioned above. In figure 15, we scatter the backed-out $z_{t+1|t,1}$ against the "common component" we extracted using the actual disaggregate price series in section 3. The correlation between these two series is 0.85, suggesting that the perceived "common component" in our model has high empirical relevance to one that can be extracted from the actual data.[29]

We then perform a counter-factual simulation for the sample after 2020, where we start with only one shock: the aggregate supply shock w_t, and mute both the broad-base shock ϵ_t and the innovations to the gap v_t. Then we add in the broad-base shock ϵ_t, and finally we add the shock to gap v_t. Figure 16 depicts the counter-factual paths of expected and actual headline inflation in this exercise. First note that by construction the dashed lines with all three shocks coincide with the actual data. The solid black lines correspond to the counter-factual paths when shocks to the gaps are muted. It is clear that the impact of the variations from the labor market tightness is negligible in causing the recent inflation. This echoes our finding in the previous sections that Phillips Curve is very flat and labor market conditions play a limited role in creating inflation. The solid gray lines are the counter-factual paths when the broad based shocks are missing. The expected inflation would have dropped from around 7% as in the data to around 4.5% if there are no broad-base shocks happening in the

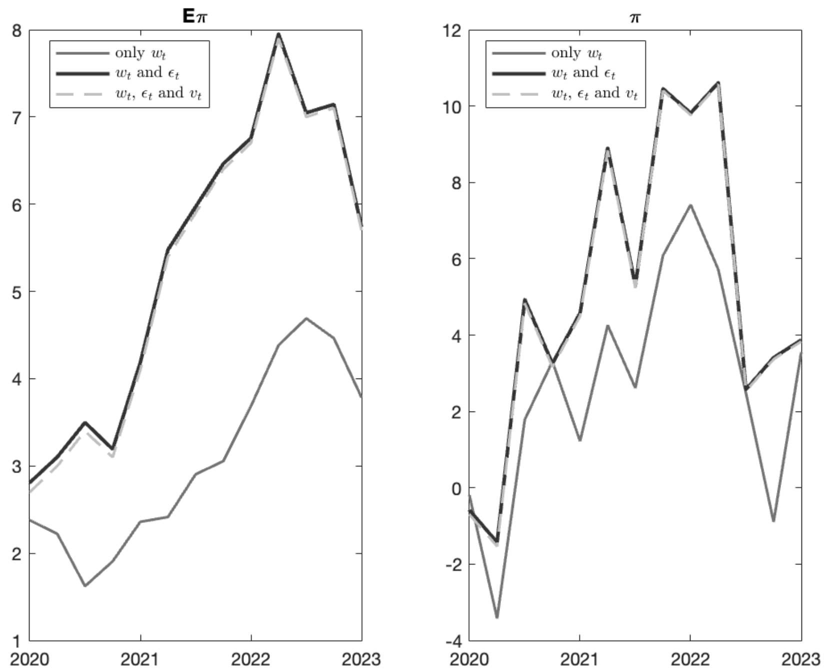
[29]However, one should not expect the two series to be perfectly correlated because the disaggregate series the agents use may be different from what we used in section 3. But one should expect these two series to be highly correlated if our model is empirically very relevant because both series contain disaggregate signals that the agents actually use. The high correlation comes from these common price series included in these two very different ways for extracting common component.

Figure 15: Correlation between extracted common component using disaggregate prices and latent variable $z_{t+1|t,1}$ backed out from the model



economy after 2020. This will translate into a drop of actual inflation from 10% to around 7% when it is the highest. On average, after 2020 the expected inflation would have dropped by 2.13% and the realized inflation would have dropped by 2.25% if the broad-based shocks are missing.

Figure 16: Filtered shocks from the estimated model



Notes: the thick gray lines are counterfactual paths of expected and realized inflation when aggregate supply shock w_t is the only shock at present. The solid black lines are counterfactual paths with w_t and the broad-base supply shock ϵ_t. The dash lines are with all three shocks, which correspond to the actual data.

5 Conclusion

In this paper, we examine the causes for the high and persistent inflation after the COVID-19 pandemic through the lens of the Phillips Curve. We show that for the U.S. economy, the Phillips Curve has been very flat and can still predict the recent inflation dynamics very well. The supply shocks appear to be transitory but inflation expectations from households and firms appear to be very persistent. Empirical evidence from our Philips Curve estimation and empirical VAR analysis show that inflation follows inflation expectations closely and the joint dynamics between the two cannot be explained by models with rational expectations or simple information frictions. The major challenge for understanding inflation dynamics is then to explain why expected inflations are volatile and persistent when labor market tightness has limited impact and supply shocks are transitory.

We propose a model where the agents in the economy are sophisticated but bounded rational in understanding the inflation dynamics. They use many disaggregate price series to infer a common shock that drives aggregate inflation, but don't realize their common expectations push up the disaggregate inflations they see. As a result, when they see price increase in many items due to broad-based supply shocks, they confuse them with the common shock and increase their expected inflation. Expectations push up actual inflation that confirms their beliefs of the common shock. This starts a self-confirming loop that amplifies the impact of transitory supply shock and creates persistent response of both expected and actual inflation. We show that our model is consistent with the estimated flat Phillips Curve and matches the impulse responses from the empirical VAR very well. Through the lens of our model, the immediate surge of inflation after the pandemic is driven by supply chain disruptions, but the persistent increase in the expected inflation from households and firms plays a dominant role in creating the stagnant high inflation after 2021. From our estimated model, in absence of the broad-base supply shock, the headline inflation would have dropped by 2.25% on average after 2020.

References

Ball, L. M., D. Leigh, and P. Mishra (2022): "Understanding U.S. Inflation During the COVID Era," Working Paper 30613, National Bureau of Economic Research.

Barnichon, R., and G. Mesters (2020): "Identifying Modern Macro Equations with Old Shocks," *The Quarterly Journal of Economics*, 135(4), 2255–2298.

Beaudry, P., C. Hou, and F. Portier (2023): "Monetary Policy when the Phillips Curve is Quite Flat," *American Economic Journal: Macroeconomics (forthcoming).*

Benigno, P., and G. B. Eggertsson (2023): "It's Baaack: The Surge in Inflation in the 2020s and the Return of the Non-Linear Phillips Curve," Working Paper 31197, National Bureau of Economic Research.

Bordalo, P., N. Gennaioli, Y. Ma, and A. Shleifer (2020): "Overreaction in Macroeconomic Expectations," *American Economic Review*, 110(9), 2748–82.

Coibion, O., and Y. Gorodnichenko (2015): "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review*, 105(8), 2644–78.

Coibion, O., Y. Gorodnichenko, and R. Kamdar (2018): "The Formation of Expectations, Inflation, and the Phillips Curve," *Journal of Economic Literature*, 56(4), 1447–91.

Gabaix, X. (2014): " A Sparsity-Based Model of Bounded Rationality *," *The Quarterly Journal of Economics*, 129(4), 1661–1710.

Gourieroux, C., A. Monfort, and E. Renault (1993): “Indirect Inference,” *Journal of Applied Econometrics*, 8, S85–S118.

Hazell, J., J. Herreño, E. Nakamura, and J. Steinsson (2022): “The Slope of the Phillips Curve: Evidence from U.S. States,” *The Quarterly Journal of Economics*, 137(3), 1299–1344.

Mertens, K., and M. O. Ravn (2011): “Understanding the aggregate effects of anticipated and unanticipated tax policy shocks,” *Review of Economic Dynamics*, 14(1), 27–54, Special issue: Sources of Business Cycles.

Michaillat, P., and E. Saez (2022): “$u^\star = \sqrt{uv}$,” NBER Working Papers 30211, National Bureau of Economic Research, Inc.

Reis, R. (2023): “Four Mistakes in the Use of Measures of Expected Inflation,” *AEA Papers and Proceedings*, 113, 47–51.

Romer, C. D., and D. H. Romer (2004): “A New Measure of Monetary Shocks: Derivation and Implications,” *American Economic Review*, 94(4), 1055–1084.

Sims, C. A. (2003): “Implications of rational inattention,” *Journal of Monetary Economics*, 50(3), 665 – 690, Swiss National Bank/Study Center Gerzensee Conference on Monetary Policy under Incomplete Information.

Stock, J. H., and M. W. Watson (2011): “35 Dynamic Factor Models,” in *The Oxford Handbook of Economic Forecasting*. Oxford University Press.

Wieland, J., and M.-J. Yang (2020): “Financial Dampening,” *Journal of Money, Credit and Banking*, 52(1), 79–113.

A Benigno and Eggertsson [2023]

In this appendix, we repeat the analysis of section 1 when we use $\log(V/U)$ as a measure of labour market tightness and either the SPF or the MSC as a measure of inflation expectations.

Table A.1: Estimated Phillips Curves using $\log(V/U)$ as a Measure of the Gap, 1969-2007

	Using SPF	Using MSC
β	1.26*	0.99*
	(0.047)	(0.048)
γ_g	0.28*	0.10
	(0.062)	(0.069)
γ_r	-0.42*	0.23*
	(0.098)	(0.040)
Observations	144	144
J Test	10.538	10.700
(jp)	(0.987)	(0.986)
Weak ID Test	29.469	10.201

Notes: this table reports estimates of the augmented Phillips curve (2). All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. The measure of inflation is Core CPI and the gap is measured with $\log(V/U)$ *. All regressors are instrumented using six lags of Romer and Romer's [2004] shocks (as extended by Wieland and Yang [2020]) and their squares as instruments. A* $\star$ *indicates significance at 5%. Sample is 1969Q1-2007Q4.*

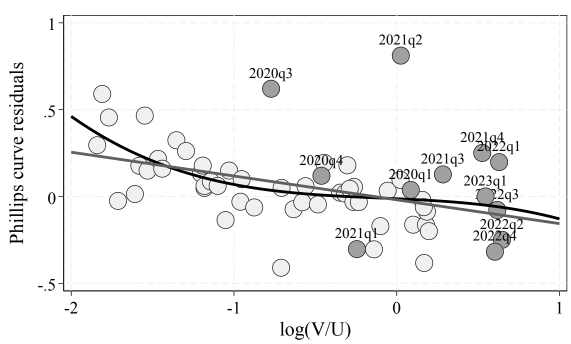
Table A.2: Projection of the Philips Curve Residuals ϵ on the Gap $\log(V/U)$, 2008-2023

	Using SPF		Using MSC	
	linear	nonlinear	linear	nonlinear
$\log(V/U)$	-0.14*	-0.04	0.05	0.07
	(0.038)	(0.076)	(0.042)	(0.083)
$\log(V/U)^2$		-0.02		-0.25
		(0.149)		(0.162)
$\log(V/U)^3$		-0.06		-0.15*
		(0.082)		(0.089)
N	60	60	60	60
σ_ϵ 60-07	.302		.291	
σ_ϵ 08-23	.232		.232	

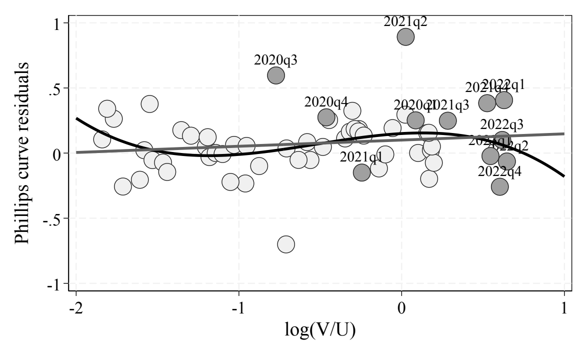
Notes: the Phillips curve residuals are obtained from the augmented Phillips curve (2) estimated over the sample 1969Q1-2007Q4, using $\log(V/U)$ *as a measure of the gap and either SPF or MSC as a measure of expectations.*

Figure A.1: Out-of-Sample Residuals from Phillips Curve, using $\log(V/U)$ as a Measure of the Gap

(a) Using SPF

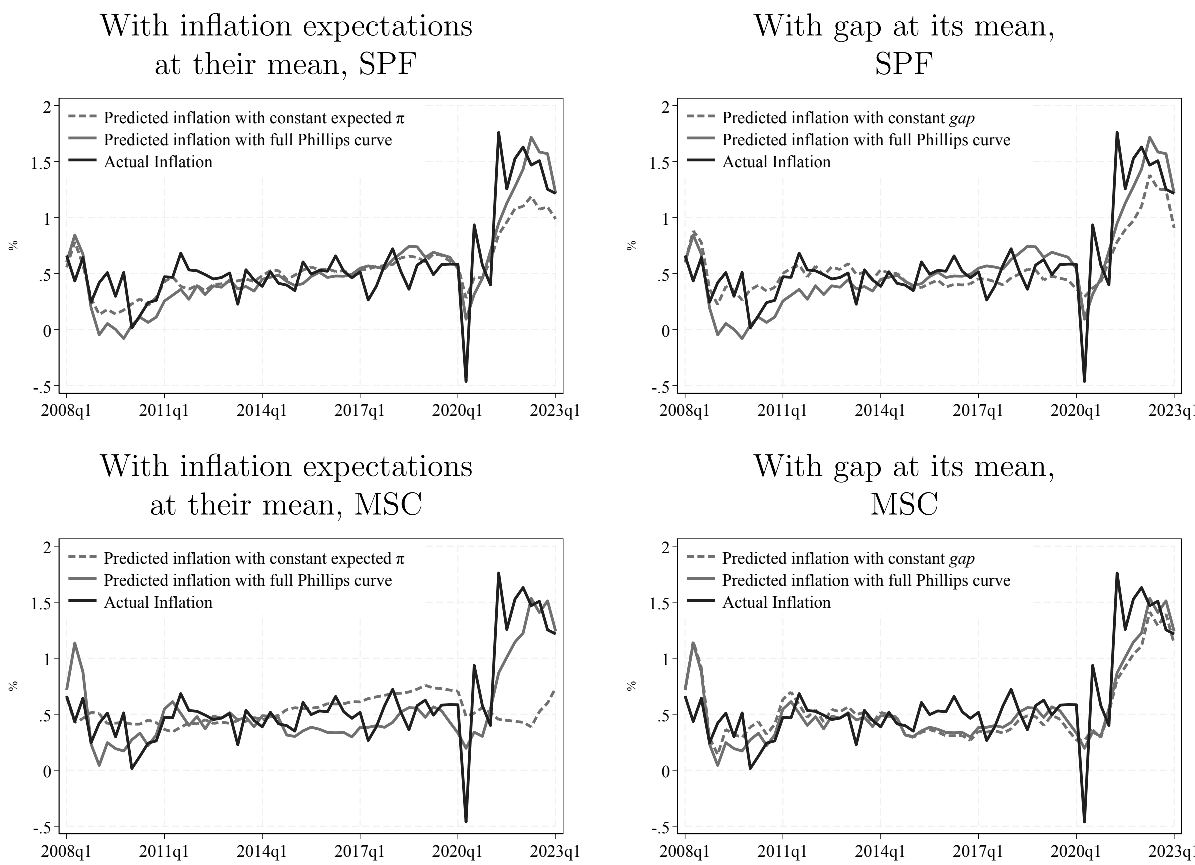


(b) Using MSC



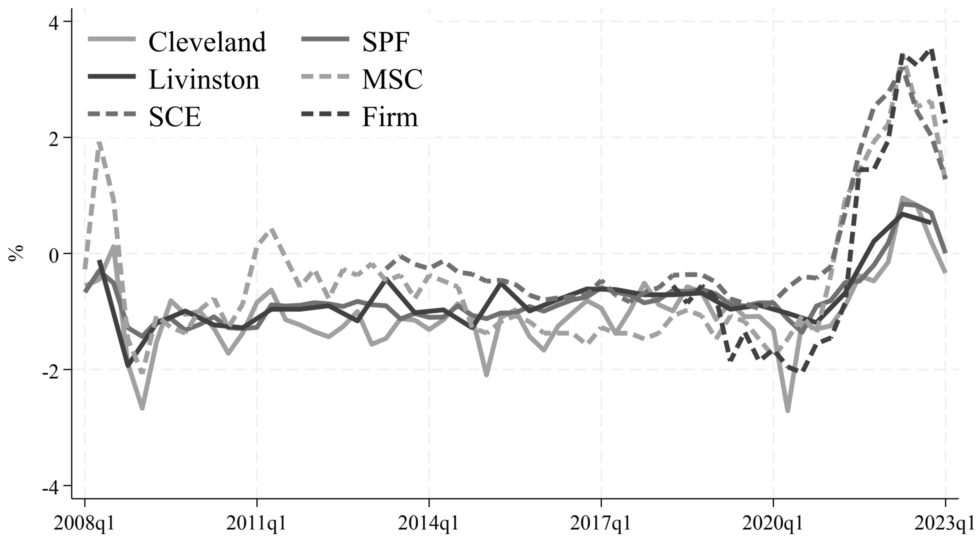
Notes: Panels (a) and (b) of this figure plots the out-of-sample residuals of the estimated Phillips curve (2) against $\log(V/U)$ *as the measure of labor market tightness, for two measures of inflation expectations. The gray lines show the estimated linear or cubic relation between residuals and labor market tightness (see Table A.2 for the estimated coefficients). Light dots correspond to pre-2020 observations and dark ones to post-2020. We exclude 2020q2 from this graph.*

Figure A.2: Counterfactual Simulations from the Estimated Phillips Curve



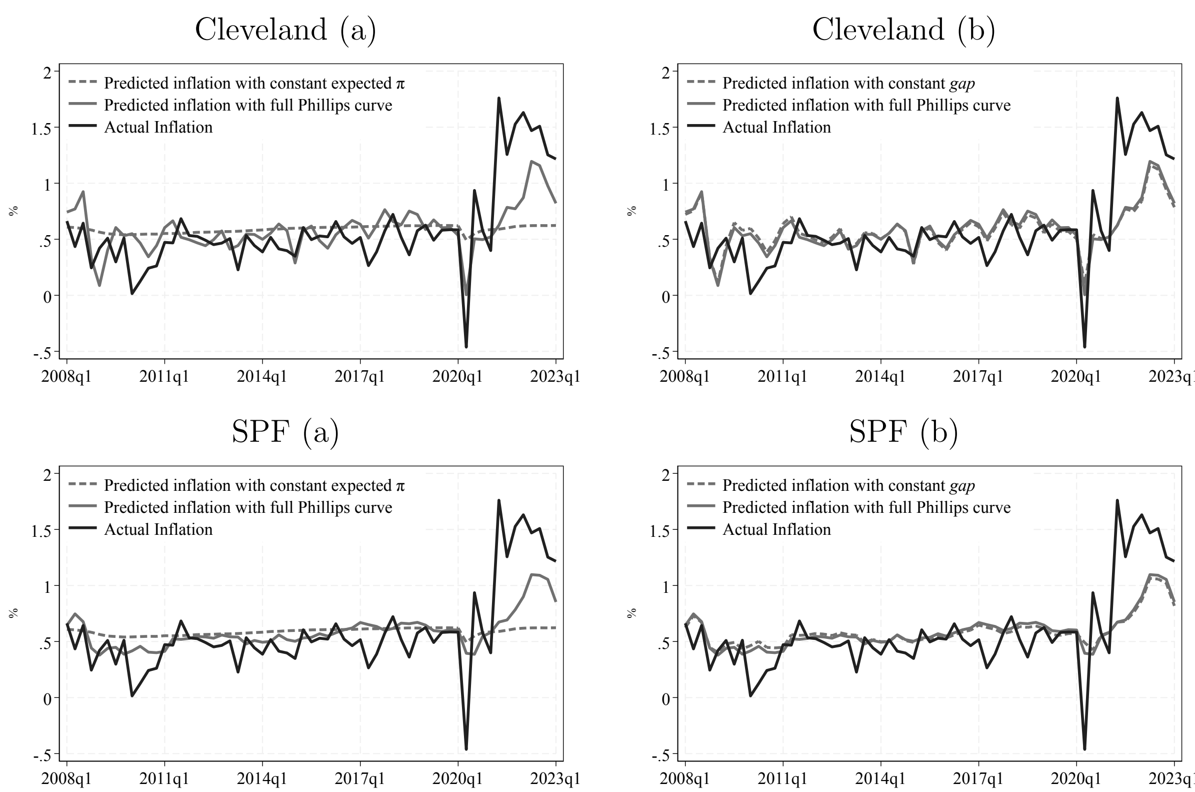
B Baseline Phillips Curve with Various Measures of Inflation Expectations

Figure B.1: Different Inflation Expectation Series



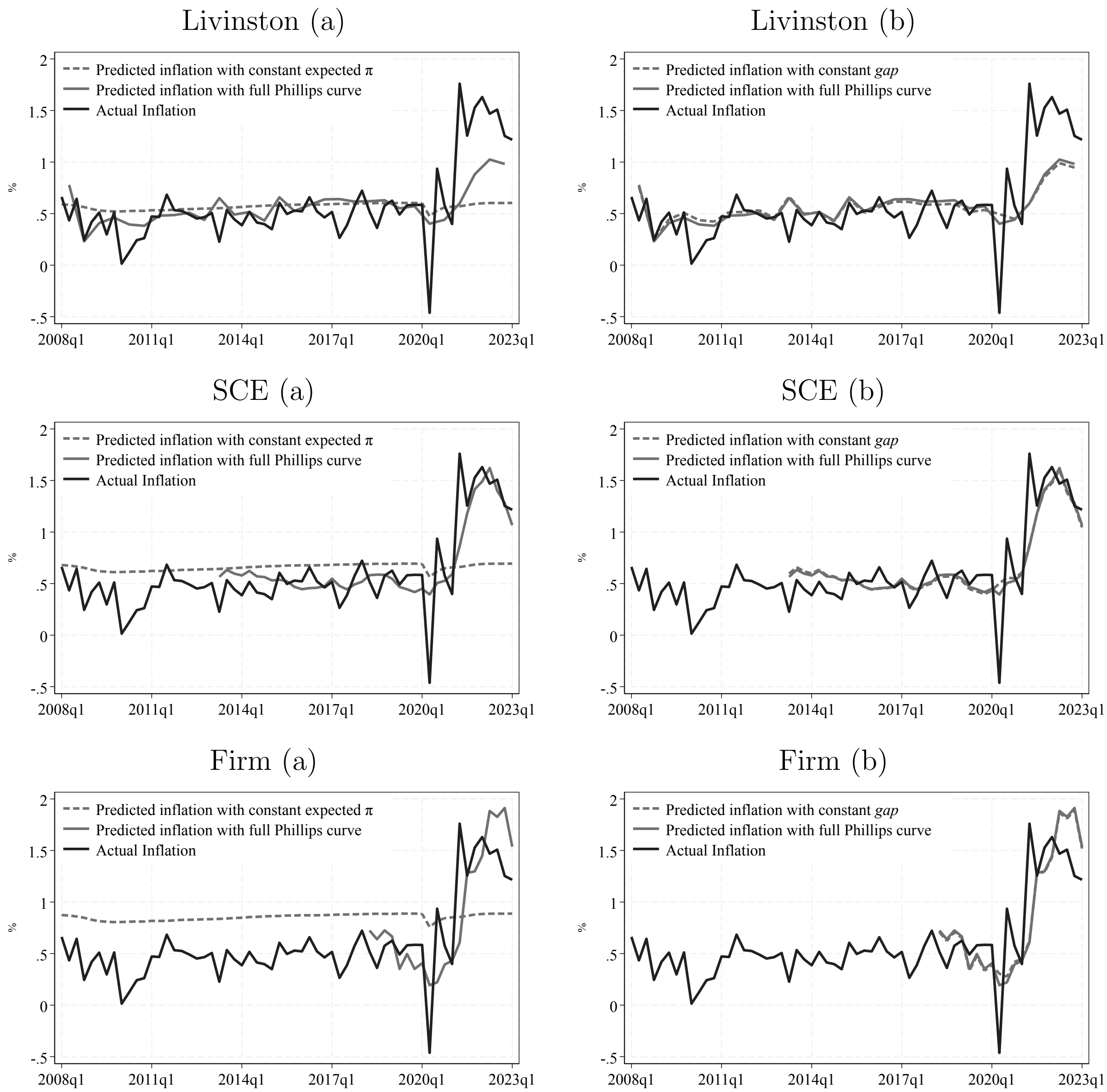
Notes: Demeaned series of inflation expectations. The dash lines are expectations from households and firms. The solid lines are expectations from professionals and central banks. "Cleveland" is the inflation expectation from the Cleveland Fed; "SPF" is the expected CPI of professional forecasts from the Philadelphia Fed; "Livingston" is from the Livingston Survey; "MSC" is the household inflation expectation from Michigan Survey of Consumers; "SCE" is the expected inflation from the NY Fed Survey of Consumer Expectations; "Firm" is the Cleveland Fed Survey of Firms' Inflation Expectations (SoFIE).

Figure B.2: Counterfactual Simulations from the Estimated Phillips Curve, Using Various Measure of Inflation Expectations (a) With inflation expectations at their mean, (b) With gap at its mean



Notes: These counterfactual simulations are done using the estimated Phillips curve (2), using either Cleveland or SPF as a measure of expectations.

Figure B.3: Counterfactual Simulations from the Estimated Phillips Curve, Using Various Measure of Inflation Expectations (a) With inflation expectations at their mean, (b) With gap at its mean



Notes: These counterfactual simulations are done using the estimated Phillips curve (2), using either Livinston, SCE or Firms Survey as a measure of expectations.

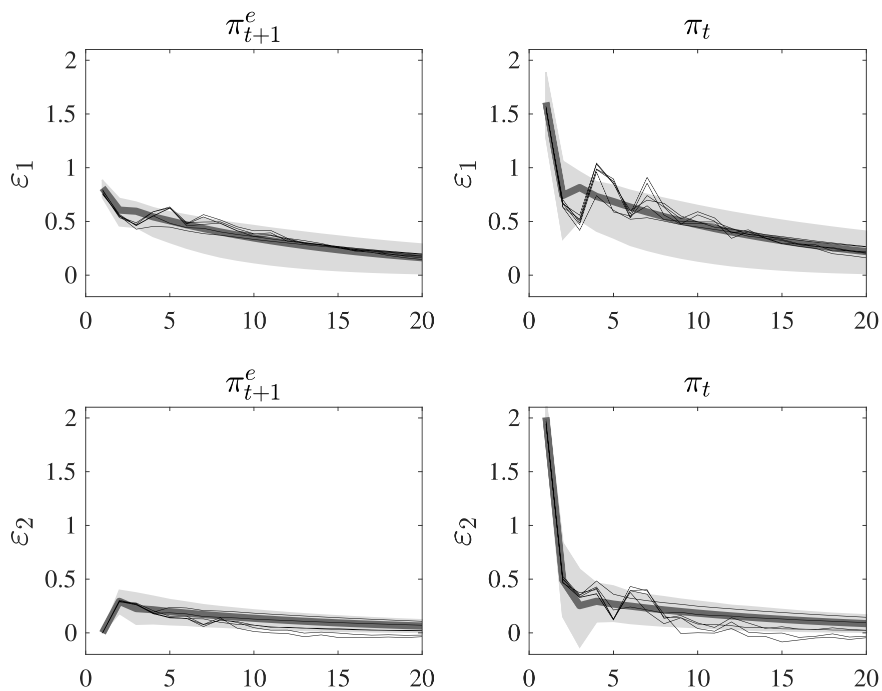
We estimate our augmented Phillips Curve with direct cost channel (2) including both expected CPI from the Professional Forecast and the Michigan Survey of Consumers. We consider three different specifications: (1) use *minus* unemployment gap as a measure of labor market tightness and estimate with monetary shocks as IV; (2) fixing the estimate γ_g at the value estimated by Hazell, Herreño, Nakamura, and Steinsson [2022]; (3) use *log* vacancy to unemployment ratio as a measure of gap and estimate with monetary shocks as IV. The results suggest that the Michigan Survey is the more relevant measure for expectation.

Table B.1: Horse-race regression with SPF and MSC

π	Sample 1981q3-2007q4		
	(1) -ugap free slope	(2) -ugap fixed slope	(3) $\ln V/U$
MSC	0.40**	0.62***	0.47***
	(0.177)	(0.161)	(0.173)
SPF	0.29	-0.02	0.25
	(0.272)	(0.233)	(0.267)
γ_g	0.08***	0.0138	0.19***
	(0.019)	(-)	(0.071)
γ_r	0.30***	0.28***	0.24***
	(0.082)	(0.054)	(0.080)
Observations	106	106	106
(jp)	(0.979)	(0.971)	(0.980)
Weak ID Test	135.594	121.041	39.712

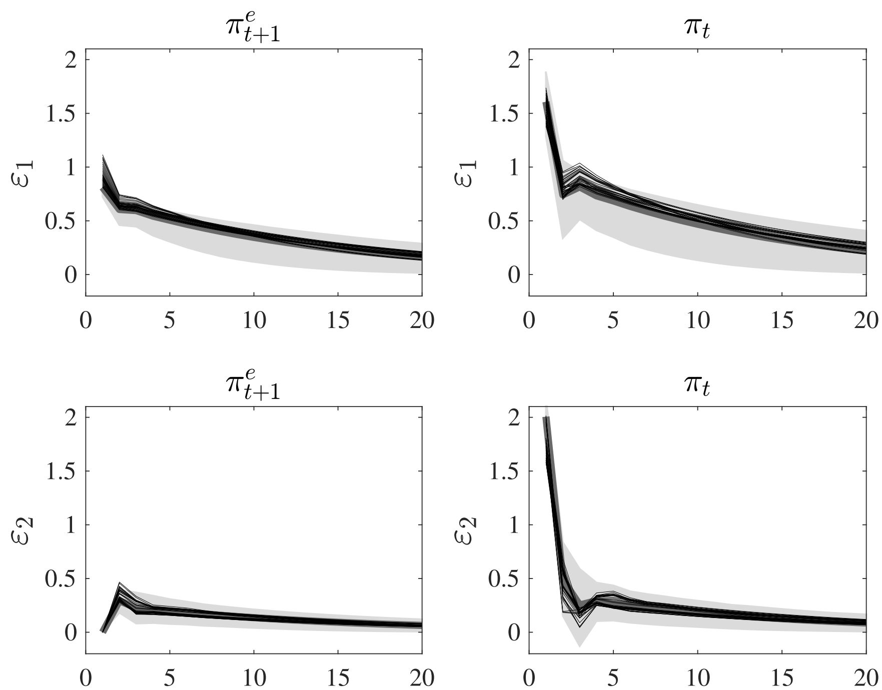
C Robustness of the 2-VAR to Lags and Sample

Figure C.4: Impulse Responses in the 2-VAR (π_t, π^e_{t+1}), with 2 to 8 Lags



Notes: this Figure plots the impulse responses to a one standard deviation shocks ε_1 and ε_2. These shocks are obtained from a Choleski orthogonalization. The estimated VAR with two lags of Headline CPI inflations and the MSC inflation expectations are in gray. Sample is 1969Q1-2023Q1. Shaded area is the 95% confidence band. The black lines corresponds to an estimation with 3 to 8 lags.

Figure C.5: Impulse Responses in the 2-VAR (π_t, π^e_{t+1}), Starting with First 20 Years and Adding Years One by One



Notes: this Figure plots the impulse responses to a one standard deviation shocks ε_1 and ε_2. These shocks are obtained from a Choleski orthogonalization. The estimated VAR with two lags of Headline CPI inflations and the MSC inflation expectations are in gray. Sample is 1969Q1-2023Q1. Shaded area is the 95% confidence band. The black lines corresponds to an estimation samples 1969Q1-1989Q1, 1969Q1-1990Q1, 1969Q1-1991Q1,... etc.

D Using Disaggregated Prices

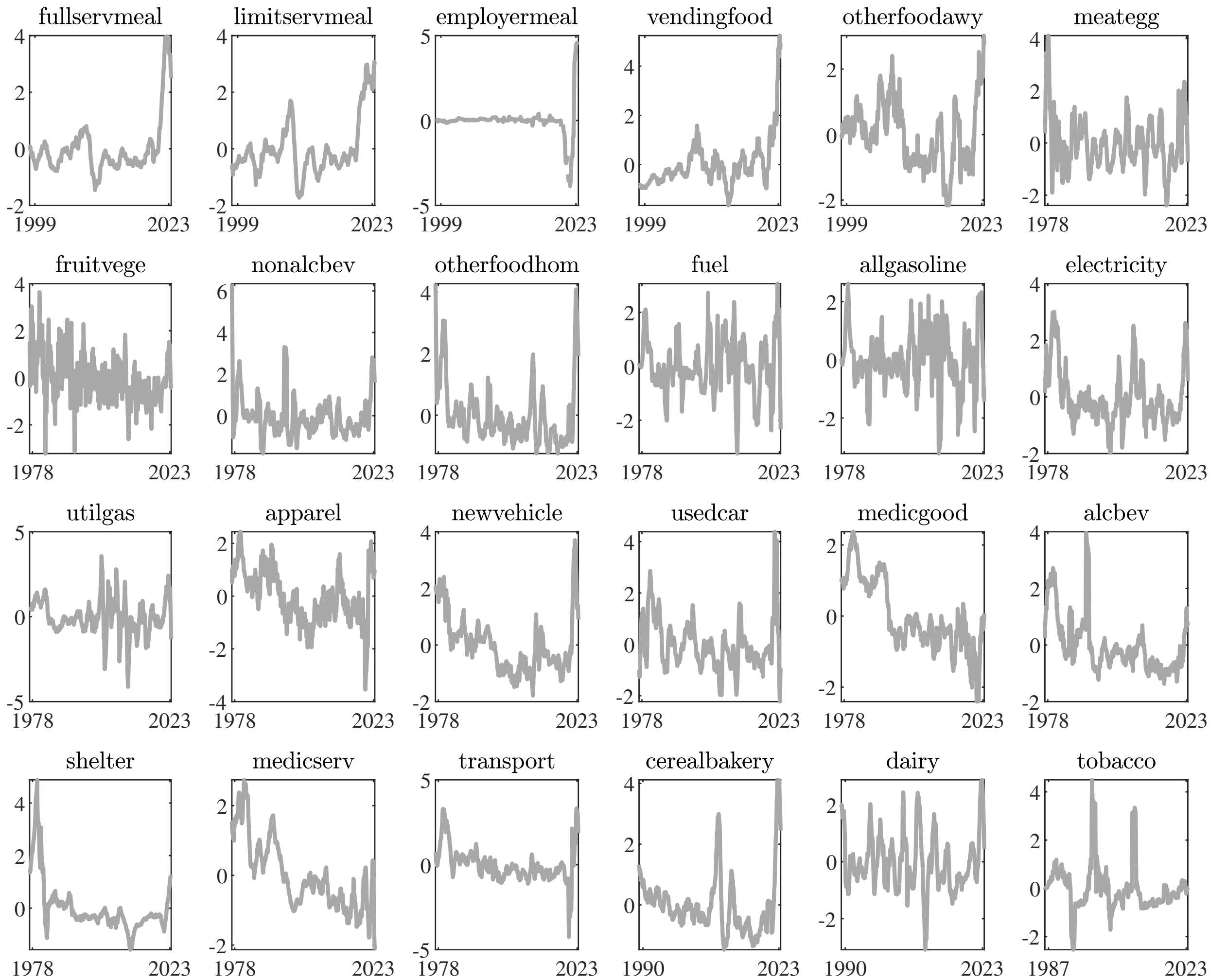
Table D.1 display the 25 expenditures categories we use, as obtained from BLS and used to compute the CPI.

Table D.1: Summary of Categories

Categories and items	Available sample	Categories and items	Available sample
Food at Home		**Food away from Home**	
Cereals and bakery products	1990m1-2023m5	Full service meals and snacks	1999m1-2023m5
Meats, poultry, fish, and eggs	1978m1-2023m5	Limited service meals and snacks	1998m12-2023m5
Dairy and related products	1990m1-2023m5	Food at employee sites and schools	1999m1-2023m5
Fruits and vegetables	1978m1-2023m5	Food from vending machines and mobile vendors	1998m12-2023m5
Nonalcoholic beverages and beverage materials	1978m1-2023m5	Other food away from home	1999m1-2023m5
Other food at home	1978m1-2023m5		
Energy commodities		**Energy services**	
Fuel oil	1978m1-2023m5	Electricity	1978m1-2023m5
Motor fuel (gasoline)	1978m1-2023m5	Utility gas	1978m1-2023m5
less food and energy commodity		**less energy services**	
Apparel	1978m1-2023m5	Shelter	1978m1-2023m5
New vehicles	1978m1-2023m5	Medical care service	1978m1-2023m5
Used cars and trucks	1978m1-2023m5	Transportation service	1978m1-2023m5
Medical care commodities	1978m1-2023m5		
Alcoholic beverages	1978m1-2023m5		
Tobacco and smoking products	1987m1-2023m5		

Notes: the main sample starts from 1978m1 due to the availability of the monthly Michigan Survey of Consumers. The disaggregated data are level 3 categories from BLS.

Figure D.1: Inflation for the Components in the CPI Basket



Notes: this Figure displays the time series of the 25 expenditures categories we use, as obtained from BLS and used to compute the CPI.

Table D.2: Estimated parameters

Parameter	Estimate		Parameter	Estimate	
ρ	0.997				
	α_i	σ_i		α_i	σ_i
fullservmeal	0.13	0.32	limitservmeal	0.12	0.57
employermeal	0.03	0.97	vending	0.11	0.65
otherfoodawy	0.09	0.79	meategg	0.05	0.90
fruitvege	0.06	0.84	nonalcohol	0.07	0.82
otherfoodhome	0.09	0.55	fuel	0.04	0.93
gasoline	0.03	0.96	electricity	0.09	0.64
utility gas	0.04	0.92	apparel	0.08	0.74
new vehicle	0.09	0.65	used car	0.03	0.96
medical good	0.06	0.84	alcoholbev	0.08	0.67
shelter	0.10	0.53	medical service	0.07	0.80
transportation	0.09	0.61	cereal bakery	0.14	0.51
dairy product	0.08	0.86	tobacco	0.01	1.00

Notes: we normalize $\sigma_v = 1$, and we input series that are normalized (demeaned and standard deviations normalized to 1) because the price series have different volatilities. But the normalization doesn't qualitatively change the results.

E Average Signal from Disaggregate Price Indices

We show that the signal-extraction problem with multiple disaggregate price indices is equivalent to one with an average signal across these disaggregate indices. The disaggregate signals the agent faces are given by (28). Without loss of generality, consider the m disaggregate signals the agent uses to form expectations:

$$X_t \equiv \begin{pmatrix} \pi_{1,t} \\ \pi_{2,t} \\ \cdots \\ \pi_{m,t} \end{pmatrix} = \ell_m \tilde{z}_t + \begin{pmatrix} \tilde{e}_{1,t} \\ \tilde{e}_{2,t} \\ \cdots \\ \tilde{e}_{m,t} \end{pmatrix} \tag{E.1}$$

$$\tilde{e}_{j,t} \sim N(0, \tilde{\sigma}_j^2) \tag{E.2}$$

where we denote the vector of signals as X_t and ℓ_m is an $m \times 1$ vector of ones. Denote the prior of z_t at $t-1$ as:

$$z_t^{prior} \sim N(z_{t|t-1}, \sigma^2) \tag{E.3}$$

where $z_{t|t-1}$ denotes the prior mean and σ^2 the stationary prior variances. The posterior mean of the nowcast is:

$$z_{t|t} = z_{t|t-1} + \kappa(X_t - \ell_m z_{t|t-1}) \tag{E.4}$$

where κ is the Kalman Gain:

$$\kappa = \sigma^2 \ell_m' (\sigma^2 \ell_m \ell_m + V)^{-1} \tag{E.5}$$

where V is a diagonal matrix with entries $\tilde{\sigma}_j^2$ on the main diagonal. The Kalman Gain can then be explicitly written as a function of σ^2 and $\tilde{\sigma}_j^2$'s:

$$\begin{aligned}
\kappa &= \sigma^2 \ell_m' \left(V^{-1} - \frac{V^{-1}\sigma^2 \ell_m \ell_m' V^{-1}}{1 + \sigma^2 \ell_m' V \ell_m} \right) \\
&= \sigma^2 \left[\begin{pmatrix} \frac{1}{\tilde{\sigma}_1^2} & \frac{1}{\tilde{\sigma}_2^2} & \cdots & \frac{1}{\tilde{\sigma}_m^2} \end{pmatrix} - \begin{pmatrix} \frac{1}{\tilde{\sigma}_1^2} \sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2} & \frac{1}{\tilde{\sigma}_2^2} \sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2} & \cdots & \frac{1}{\tilde{\sigma}_m^2} \sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2} \end{pmatrix} \frac{1}{\frac{1}{\sigma^2} + \sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2}} \right] \\
&= \begin{pmatrix} \frac{1}{\tilde{\sigma}_1^2} & \frac{1}{\tilde{\sigma}_2^2} & \cdots & \frac{1}{\tilde{\sigma}_m^2} \end{pmatrix} \frac{1}{\frac{1}{\sigma^2} + \sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2}}
\end{aligned} \tag{E.6}$$

Now consider an average signal:

$$x_t = \tilde{z}_t + \underbrace{\frac{\sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2} \tilde{e}_{j,t}}{\sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2}}}_{\equiv \epsilon_t} \tag{E.7}$$

$$\epsilon_t \sim N(0, \underbrace{\frac{1}{\sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2}}}_{\equiv \sigma_\epsilon^2}) \tag{E.8}$$

With the same prior, the Kalman Gain is given by:

$$\hat{\kappa} = \frac{\frac{1}{\sigma_\epsilon^2}}{\frac{1}{\sigma^2} + \frac{1}{\sigma_\epsilon^2}} \tag{E.9}$$

Using (E.6) and (E.9), it is then straightforward to show that the posterior mean $z_{t|t}$ is the same when the agent uses disaggregate signals (E.1) with the aggregate signal (E.7). Moreover, the stationary posterior variances are the same as well. As a result, the stationary posterior belief formed by observing multiple signals (E.1) is equivalent to one formed using an average signal (E.7)

F Models with Rational Expectations

F.1 Incomplete Information Rational Expectation

Take the IIRE model described by (9)-(12). The agent forms expectations about gap_t. Denoting the nowcasts of which as $\text{gap}_{t|t}$, the expectation is given by:

$$\begin{aligned} E_t[\pi_{t+1}] &= E_t[\beta E_{t+1}\pi_{t+2} + \alpha \text{gap}_{t+1} + e_{t+1}] \\ &= \frac{\alpha\rho}{1-\beta\rho}\text{gap}_{t|t} \end{aligned} \tag{F.1}$$

The nowcast $z_{t|t}$ is given by:[30]

$$z_{t|t} = z_{t|t-1} + k_i(s_t - \alpha z_{t|t-1}) \tag{F.2}$$

$$k_i = \sigma_i^2\alpha(\alpha^2\sigma_i^2 + \sigma_\epsilon^2)^{-1} \tag{F.3}$$

$$\sigma_i^2 = \rho^2(\sigma_i^2 - k\alpha\sigma_i^2) + \sigma_v^2 \tag{F.4}$$

[30]Note in (F.3) the agent uses correct σ_ϵ because when the agent is rational, she can easily back-out the correct ρ, $\alpha\sigma_v$ and σ_ϵ with the variance-covariance structure of s_t. As a result, the agent's information will not support any subjective $\tilde{\sigma}_\epsilon \neq \sigma_\epsilon$.

where k_i is the stationary Kalman Gain and σ_i^2 is the stationary posterior variance. This leads to the following system:

$$E_t^{IIRE}[\pi_{t+1}] = (1 - k_i\alpha)\rho E_{t-1}^{IIRE}[\pi_t] + \frac{\alpha\rho k_i}{1-\beta\rho}(\alpha \text{gap}_t + \epsilon_t) \tag{F.5}$$

$$\pi_t = \beta(1 - k_i\alpha)\rho E_{t-1}^{IIRE}[\pi_t] + \frac{\alpha^2 k_i\rho\beta + (1-\beta\rho)\alpha}{1-\beta\rho}\text{gap}_t + \frac{k_i\rho\beta\alpha + (1-\beta\rho)\gamma}{1-\beta\rho}\epsilon_t + w_t \tag{F.6}$$

Note that as $\sigma_\epsilon \to 0$, $k_i \to 1/\alpha$ and $w_t \to e_t$. The IIRE case converges to the FIRE case.

F.2 Hybrid Phillips Curve or Adaptive Expectation

One related question is whether a hybrid Phillips Curve or adaptive expectations can help to explain the joint dynamics between expected and actual inflation. We consider a Phillips Curve taking the following hybrid form:

$$\pi_t = \beta \underbrace{\left(\tau\pi_{t-1} + (1-\tau)E_t^{FIRE}\pi_{t+1}\right)}_{\text{observed in MSC}} + \alpha \text{gap}_t + e_t \tag{F.7}$$

$$\text{gap}_t = \rho \text{gap}_{t-1} + v_t \tag{F.8}$$

where τ can represents the indexation, the fraction of people using adaptive expectation, or motivated by k-level thinking as in *BCL-2022*. The expectation formed under FIRE takes into account that there are backward looking component in inflation. It is easy to show that inflation takes the following form using undetermined coefficients:

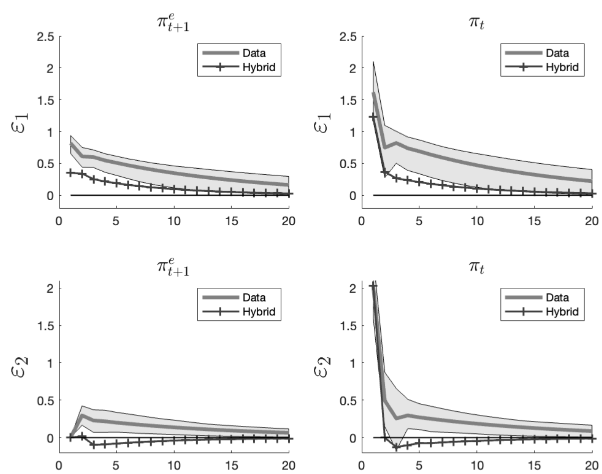
$$\pi_t = a\pi_{t-1} + b\text{gap}_t + ce_t \tag{F.9}$$

with

$$\begin{cases} a &= \frac{\beta\tau}{1-a\beta(1-\tau)} \\ b &= \frac{\beta(1-\tau)b\rho+\alpha}{1-a\beta(1-\tau)} \\ c &= \frac{1}{1-a\beta(1-\tau)} \end{cases}$$

Following our approach in section 2.2, we fix β, ρ, σ_v and α at values consistent with our Phillips Curve, and we estimate the free parameter τ to match the IRFs from the bivariate VAR(2). Our estimate is $\hat{\tau} = 0.05$, and the above model cannot match the empirical IRFs well:

Figure F.1: The Joint Process of π and π^e_{t+1} in the Data and Under models with Rational Expectations

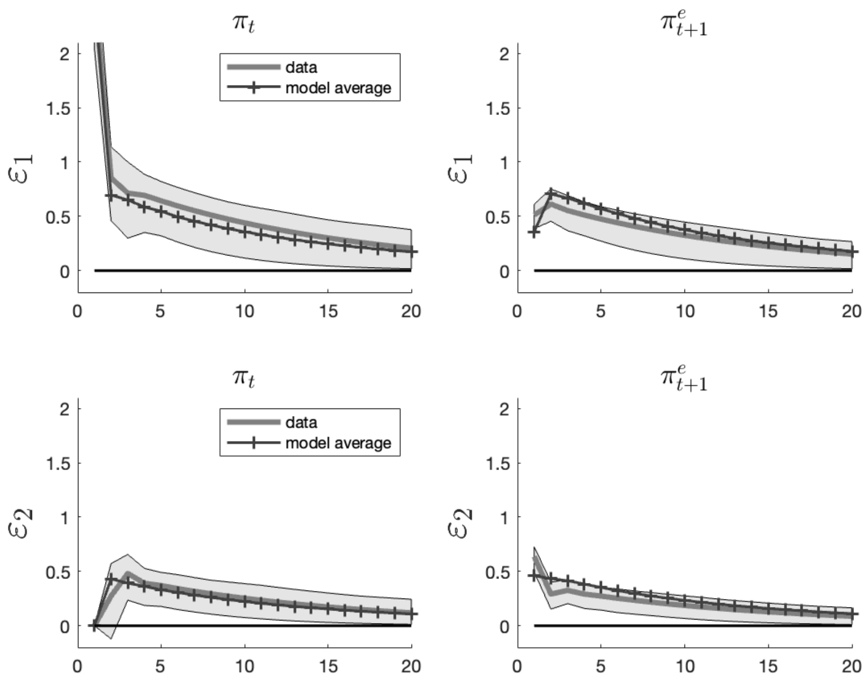


Notes: on this Figure, the solid grey line plots the impulse responses to a one standard deviation shocks ε_1 and ε_2 estimated with data from MSC and Headline CPI. Sample is 1969Q1-2023Q1. Shaded area is the 95% confidence band. The marked black lines plot the average impulse responses (over 200 simulations of length 217) obtained from the same VAR estimated on simulated data, when the Data Generating Process is the estimated hybrid model (F.7).

G Alternative Order of Cholesky VAR

We present the IRFs from Cholesky VAR(2) where we order headline inflation first and expected inflation the second. This change of ordering reflects an alternative identification assumption that the first underlying shock affects π_t and π^e_{t+1} simultaneously and the second shock affects π^e_{t+1} on impact then π_t with a lag. Note our Cholesky VAR is just method to summarize joint dynamics of inflation and expectation. If our model is close to the true data generating process for inflation and expected inflation, we would expect IRF from this VAR with alternative (but possibly incorrect) ordering to have similar result. Figure G.2 shows the results from this exercise. Our model matches very well with the VAR using alternative ordering.

Figure G.2: IRF from data and model simulation



Notes: thick gray line is IRF from bi-VAR with actual data; marked black line is average IRF from simulated data across 200 random samples. The estimated VAR is VAR(2) with Cholesky Decomposition ordering π_t *-* π^e_{t+1}*. The shaded area is 95% CI.*

H State-space Representation of Model

The two observables $\pi_{t+1|t,0}$ and π_t can be written as system of equations of latent states $z_{t|t-1,1}$ and gap_t.

$$\begin{aligned}\pi_{t+1|t,0} &= \tilde{\rho} z_{t|t,0} = \tilde{\rho}((1-K)z_{t|t-1,1} + K s_t) \\ &= \tilde{\rho}((1-K)z_{t|t-1,1} + K(\beta\pi_{t+1|t,0} + \alpha\text{gap}_t + \epsilon_t)) \\ &= \frac{\tilde{\rho}(1-K)}{1-K\beta\tilde{\rho}} z_{t|t-1,1} + \frac{\tilde{\rho}K\alpha}{1-K\beta\tilde{\rho}}\text{gap}_t + \frac{\tilde{\rho}K}{1-K\beta\tilde{\rho}}\epsilon_t \end{aligned} \tag{H.1}$$

From (33):

$$\begin{aligned} s_t &= \pi_{t+1|t,0} + \alpha\text{gap}_t + \epsilon_t \\ &= \frac{\beta\tilde{\rho}(1-K)}{1-K\beta\tilde{\rho}} z_{t|t-1,1} + \frac{\alpha}{1-K\beta\tilde{\rho}}\text{gap}_t + \frac{1}{1-K\beta\tilde{\rho}}\epsilon_t \end{aligned} \tag{H.2}$$

Inflation is given by:

$$\begin{aligned} \pi_t &= \beta\pi_{t+1|t,0} + \alpha\text{gap}_t + \gamma\epsilon_t + w_t \\ &= \frac{\beta\tilde{\rho}(1-K)}{1-K\beta\tilde{\rho}} z_{t|t-1,1} + \frac{\alpha}{1-K\beta\tilde{\rho}}\text{gap}_t + \left(\frac{K\beta\tilde{\rho}}{1-K\beta\tilde{\rho}} + \gamma\right)\epsilon_t + w_t \end{aligned} \tag{H.3}$$

From (39) and expression of s_t, we get recursion of $z_{t|t-1,1}$:

$$\begin{aligned} z_{t+1|t,1} &= \tilde{\rho} z_{t|t,1} = \tilde{\rho}(1-K-k)z_{t|t-1,1} + \tilde{\rho}K s_t + \tilde{\rho}k\pi_t \\ &= \tilde{\rho}\left(1-(K+k)\frac{1-\beta\tilde{\rho}}{1-K\beta\tilde{\rho}}\right) z_{t|t-1,1} + \tilde{\rho}\frac{(K+k)\alpha}{1-K\beta\tilde{\rho}}\text{gap}_t \\ &+ \tilde{\rho}\left(\frac{K+\tilde{\rho}Kk\beta}{1-K\beta\tilde{\rho}} + \gamma k\right)\epsilon_t + \tilde{\rho}k w_t \end{aligned} \tag{H.4}$$

We could write the state-space representation as:

$$X_t \equiv \begin{pmatrix} \pi_{t+1|t,0} \\ \pi_t \\ z_{t+1|t,1} \\ \text{gap}_t \end{pmatrix} = FX_{t-1} + B \begin{pmatrix} \epsilon_t \\ w_t \\ v_t \end{pmatrix} \tag{H.5}$$

where

$$F = \begin{pmatrix} 0 & 0 & \frac{\beta\tilde{\rho}(1-K)}{1-K\beta\tilde{\rho}} & \rho\frac{\tilde{\rho}K\alpha}{1-K\beta\tilde{\rho}} \\ 0 & 0 & \frac{\beta\tilde{\rho}(1-K)}{1-K\beta\tilde{\rho}} & \rho\frac{\alpha}{1-K\beta\tilde{\rho}} \\ 0 & 0 & \tilde{\rho}\left(1-(K+k)\frac{1-\beta\tilde{\rho}}{1-K\beta\tilde{\rho}}\right) & \tilde{\rho}\frac{(K+k)\alpha}{1-K\beta\tilde{\rho}}\rho \\ 0 & 0 & 0 & \rho \end{pmatrix} \tag{H.6}$$

$$B = \begin{pmatrix} \frac{\tilde{\rho}K}{1-K\beta\tilde{\rho}} & 0 & \frac{\tilde{\rho}K\alpha}{1-K\beta\tilde{\rho}} \\ \frac{K\beta\tilde{\rho}}{1-K\beta\tilde{\rho}} + \gamma & 1 & \frac{\alpha}{1-K\beta\tilde{\rho}} \\ \left(\frac{K+\tilde{\rho}Kk\beta}{1-K\beta\tilde{\rho}} + \gamma k\right)\tilde{\rho} & \tilde{\rho}k & \tilde{\rho}\frac{(K+k)\alpha}{1-K\beta\tilde{\rho}} \\ 0 & 0 & 1 \end{pmatrix} \tag{H.7}$$

The observational equation is given by:

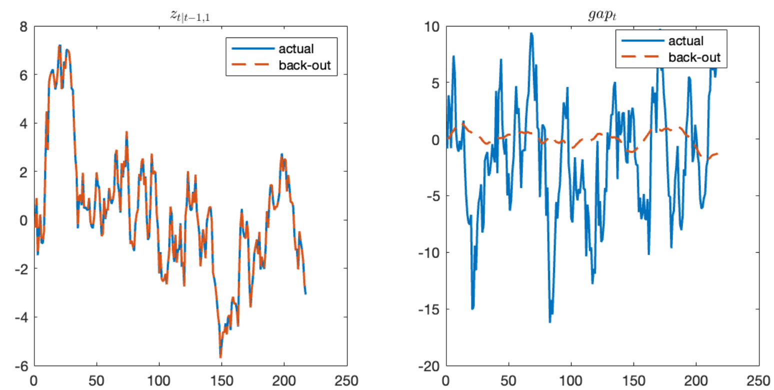
$$O_t = \begin{pmatrix} I_{2\times 2} & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 2} & \mathbf{0}_{2\times 2} \end{pmatrix} X_t \tag{H.8}$$

We can then estimate the hidden states $z_{t|t-1,1}$ and gap_t using Kalman Filter Smoothing, then get the corresponding shocks implied by the state-space representation.

The precision of the estimated states and shocks depend on the parameters in F and B. To illustrate the performance of this approach in the context of our estimated model, we simulate data using parameters from Table 4. We then plot the estimated states $\{E_t\pi_{t+1}, \pi_t, z_{t|t-1,1}, \text{gap}_t\}$ in Figure H.1. The blue solid lines are the actual hidden states from simulated data and the red dash lines are the estimated ones from the above approach. Not surprisingly, the hidden state $z_{t|t-1,1}$ is very well recovered where the estimated values are almost identical to the actual ones. Whereas the gap_t is very illy recovered. This is because the observables $\{E_t\pi_{t+1}, \pi_t\}$ contain a lot more information for $z_{t|t-1,1}$ and almost no information about gap_t

due to the fact α is very small in the actual DGP. As a result, the Kalman Smoothing algorithm puts low weights on the observables when making predictions about this hidden state. Figure H.2 depicts the backed-out shocks and compares them with actual shock series

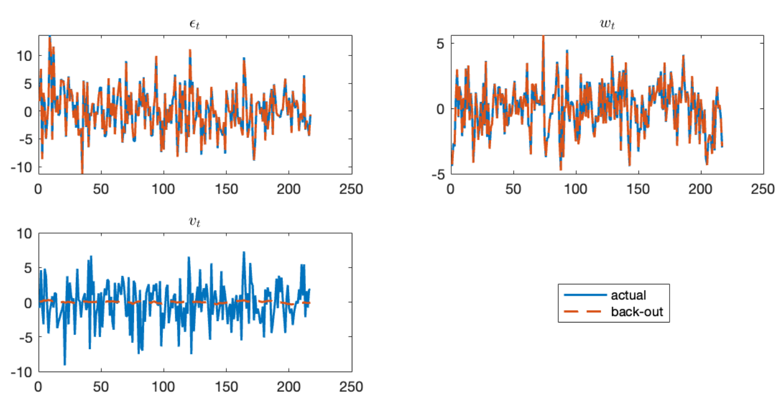
Figure H.1: Estimated v.s. actual latent states $z_{t|t-1,1}$ and gap_t from simulated data



Notes: the blue solid lines are actual latent states from simulated data. The red dash lines are the recovered latent states using the Kalman Smoothing.

simulated. For the same reason as the latent states, the observations are quite informative about the broad-base shocks ϵ_t and the aggregate shock w_t, but they are not informative about the common shock v_t. As a result, the recovered v_t series are much less volatile and not really comparable to the actual shock series.

Figure H.2: Estimated v.s. actual latent states $z_{t|t-1,1}$ and gap_t from simulated data

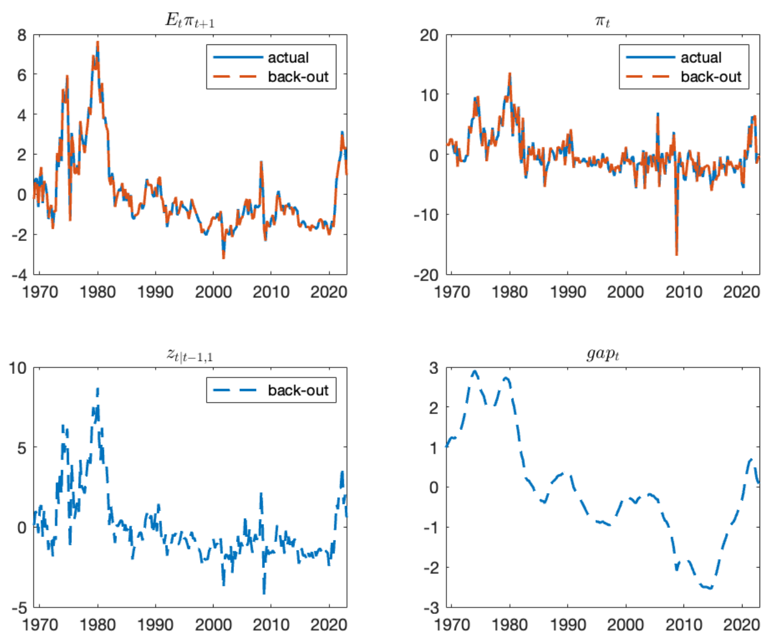


Notes: the blue solid lines are actual latent states from simulated data. The red dash lines are the recovered latent states using the Kalman Smoothing.

H.1 Backed-out Series from Real Data

The figure H.3 depicts the recovered states $\{E_t\pi_{t+1}, \pi_t, z_{t|t-1,1}, \text{gap}_t\}$ applying Kalman Smoothing. By construction, the expected inflation and aggregate inflation are observable so they coincide with each other. Because the actual values for $z_{t|t-1,1}$ and gap_t are treated as unobserved, there are no corresponding actual values plotted for these two variables.

Figure H.3: Estimated v.s. actual states from actual data



Notes: the states π_t and $E_t\pi_{t+1}$ are observables. The blue solid lines are actual states. The red dash lines are the recovered latent states using the Kalman Smoothing. Because the actual values for $z_{t|t-1,1}$ and gap_t are treated as unobserved, there are no corresponding actual values plotted for these two variables.

I Extended Model with Core CPI

We also present an extended version of our model where we separately model HL CPI and Core CPI. The model exposition is very similar to our baseline model:

ALM:

$$
\begin{aligned}
s_{t,h} &= \beta \mathbb{E}\pi_{t+1} + \text{gap}_t + \epsilon_{t,h} && \text{(HL signal)} \\
s_{t,c} &= \beta \mathbb{E}\pi_{t+1} + \text{gap}_t + \epsilon_{t,c} && \text{(Core signal)} \\
\pi_{t,c} &= \beta \pi_{t+1|t-1} + \text{gap}_t + \gamma_c \epsilon_{t,c} + w_{t,c} && \text{(Core CPI)} \\
\hat{\pi}_{t,h} &= \beta \pi_{t+1|t-1} + \text{gap}_t + \gamma_h \epsilon_{t,h} + w_{t,h} && \text{(Only food and energy)} \\
\pi_{t,h} &= \theta \pi_{t,c} + (1-\theta)\hat{\pi}_{t,h} \\
&= \beta \pi_{t+1|t-1} + z_t + \theta(\gamma_c \epsilon_{t,c} + w_{t,c}) + (1-\theta)(\gamma_h \epsilon_{t,h} + w_{t,h}) && \text{(HL CPI)} \\
\text{gap}_t &= \rho \text{gap}_{t-1} + v_t && \text{(Common component)}
\end{aligned}
$$

Where θ is the weights on the core components in computing HL CPI, γ_h is the fraction of HL items the agent is attentive to as signals, and γ_c is the fraction of Core items the agent attentive to. The $w_{t,c}$ and $w_{t,h}$ are the transitory shocks to the items in core and HL that the agent wasn't attentive to in the first place. As in our baseline model, $\epsilon_{c,t}$ and $\epsilon_{h,t}$ are the broad-base supply shocks that enter agents' expectations through the sectors in core or headline categories.

All shocks are normal with mean zero:

$$
e_{t,i} \sim N(0, \sigma^2_{e,i}) \quad w_{t,i} \sim N(0, \sigma^2_{w,i}) \quad v_t \sim N(0, \sigma^2_v) \quad \forall i = c, h
$$

PLM:

$$
\tilde{S}_t = \begin{pmatrix} \tilde{s}_{t,h} \\ \tilde{s}_{t,c} \\ \tilde{\pi}_{t,h} \end{pmatrix} = \begin{pmatrix} \tilde{z}_t \\ \tilde{z}_t \\ \tilde{z}_t \end{pmatrix} + \begin{pmatrix} \tilde{\epsilon}_{t,h} \\ \tilde{\epsilon}_{t,c} \\ \theta\gamma_c \tilde{\epsilon}_{t,c} + (1-\theta)\gamma_h \tilde{\epsilon}_{t,h} + \underbrace{\theta \tilde{w}_{t,c} + (1-\theta)\tilde{w}_{t,h}}_{\equiv \tilde{w}_t} \end{pmatrix} \tag{I.9}
$$

The agent sees three types of signals: the subset of items in HL, Core, and the HL CPI.

They have no clue what is "core CPI". The PLM above describes both how they think of HL CPI is created, as well as how the signals they see are generated. Similar to our baseline model, we still maintain the timing restriction of seeing $s_{t,h}$ and $s_{t,c}$ first, then $\pi_{t,h}$ later. The expectation $\mathbb{E}\pi_{t+1}$ as observed in MSC are formed after observing $s_{t,h}$ and $s_{t,c}$.

The agent's perceived shock processes:

$$\tilde{\epsilon}_{t,i} \sim N(0, \tilde{\sigma}_{e,i}^2) \quad \tilde{w}_t \sim N(0, \tilde{\sigma}_w^2) \quad \tilde{v}_t \sim N(0, \tilde{\sigma}_v^2) \quad \forall i = c, h$$

It is useful to then write (I.9) in matrix representation, denoting $\tilde{G}_n$ as $n \times 1$ vector of ones:

$$\tilde{S}_t = \tilde{G}_3 \tilde{z}_t + \xi_t \tag{I.10}$$

$$\tilde{z}_t = \tilde{\rho}\tilde{z}_{t-1} + \tilde{v}_t \tag{I.11}$$

$$\xi_t \sim N\left(0, \tilde{R}\right) \qquad \tilde{v}_t \sim N(0, \tilde{\sigma}_v^2) \tag{I.12}$$

where:

$$\tilde{R} = \begin{pmatrix} \tilde{\sigma}_{e,h}^2 & 0 & (1-\theta)\gamma_h \tilde{\sigma}_{e,h}^2 \\ 0 & \tilde{\sigma}_{e,c}^2 & \theta\gamma_c \tilde{\sigma}_{e,c}^2 \\ (1-\theta)\gamma_h \tilde{\sigma}_{e,h}^2 & \theta\gamma_c \tilde{\sigma}_{e,c}^2 & \theta^2\gamma_c^2 \tilde{\sigma}_{e,c}^2 + (1-\theta)^2\gamma_h \tilde{\sigma}_{e,h}^2 + \tilde{\sigma}_w^2 \end{pmatrix}$$

I.1 Quantitative Analysis

We can now take our model and estimate the parameters using Minimum Distance to match the IRFs from the model simulated data and actual data. In particular, because the extended model incorporate core CPI, we target the IRFs from the empirical Cholesky VAR(2) with ordering expected inflation, HL inflation and Core inflation. Again, we fix the values of β, α, σ_v and ρ as in our baseline Phillips Curve and estimate:

$$\Theta = \{\beta, \rho, \sigma_v, \sigma_{e,c}, \sigma_{e,h}, \sigma_{w,c}, \sigma_{w,h}, \gamma_c, \gamma_h, \tilde{\rho}, \tilde{\sigma}_w, \tilde{\sigma}_v, \theta\} \tag{I.13}$$

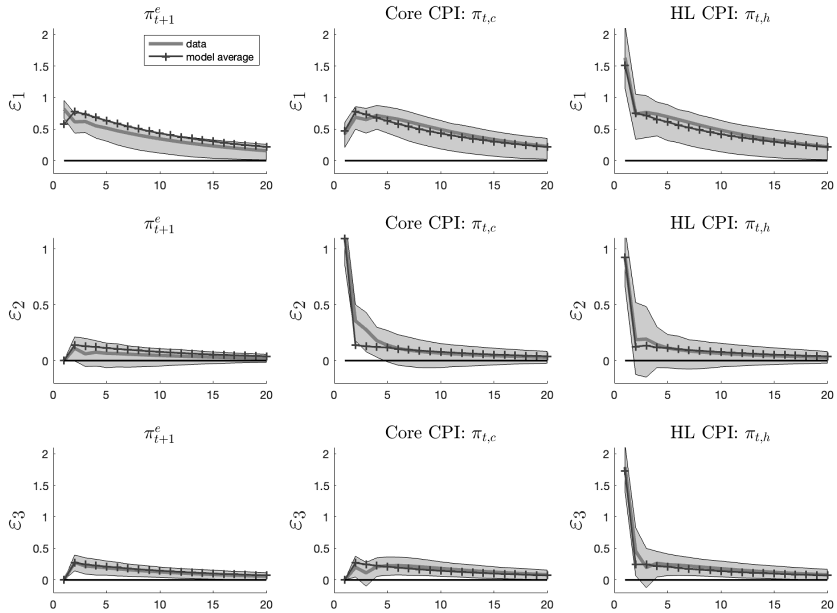
The estimated results are included in Table I.1. The estimated parameters are pretty much in line with our baseline results. More interestingly, the estimate of θ, the fraction of items counted as core in the hl inflation, is estimated to be 0.65, which is very close to the actual data. Moreover, the estimate γ_h is much bigger than γ_c. This suggests that most of the disaggregate prices the household used to form expectations are coming from HL categories, and they use fewer price series in core categories. This implies that the supply shocks that affect headline inflations, such as energy and food prices, will have bigger impact on the aggregate inflations through their effects on expected inflation.

Table I.1: Estimated parameters

From NKPC			
Parameter:	Estimate	Parameter:	Estimate
β	0.99	ρ	0.89
σ_v	3.01	α	0.0138
From IRF Matching			
Parameter:	Estimate	Parameter:	Estimate
$\tilde{\sigma}_v$	0.73	$\sigma_{w,h}$	4.48
$\tilde{\rho}$	0.96	$\tilde{\sigma}_w$	2.14
γ_h	0.39	γ_c	0.05
$\sigma_{e,h}$	22.4	$\sigma_{e,c}$	2.90
$\tilde{\sigma}_{e,h}$	4.31	$\tilde{\sigma}_{e,c}$	2.51
θ	0.65	$\sigma_{w,c}$	1.05

Finally, figure I.4 shows the match of the IRFs from our model and the actual data. Table I.2 shows the variance and covariance structures of headline and core CPIs generated by our extended model. Our extended model does very well in both targeted and untargeted moments.

Figure I.4: IRF from data and model simulation



Notes: thick gray line is IRF from bi-VAR with actual data; marked black line is average IRF from simulated data across 200 random samples. The estimated VAR is VAR(2) with Cholesky Decomposition ordering expected inflation, core inflation, hl inflation. The shaded area is 95% CI.

Table I.2: Moments

	Targeted		Untargeted	
	Model	Subjective Model	Data	Model
HL CPI				
$var(\pi_{t,h})$	11.59	11.59	12.52	11.59
$cov(\pi_{t,h}, \pi_{t-1,h})$	6.73	6.67	7.70	6.73
$cov(\pi_{t,h}, \pi_{t-2,h})$	6.35	6.41	6.99	6.35
Core CPI				
$var(\pi_{t,c})$	-	-	7.16	6.96
$cov(\pi_{t,c}, \pi_{t-1,c})$	-	-	6.11	5.69
$cov(\pi_{t,c}, \pi_{t-2,c})$	-	-	5.72	5.31