

The Dominant Role of Expectations and Broad-Based Supply Shocks in Driving Inflation

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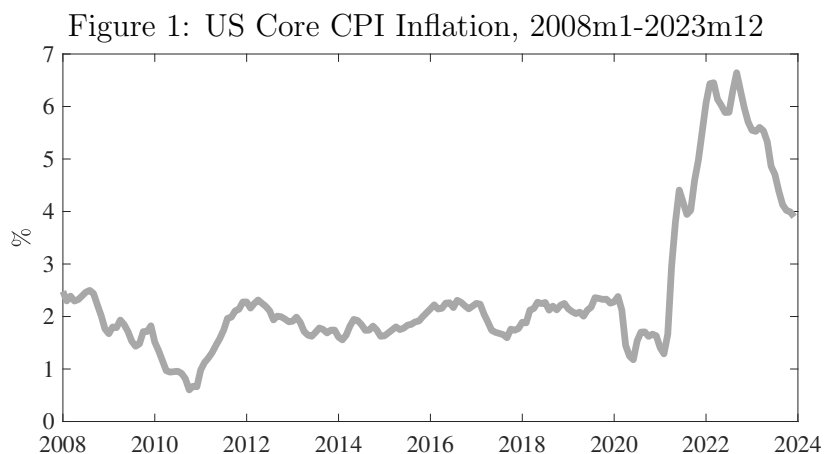
Abstract

In light of the experience of the last few years, the object of this paper is to re-examine the role of supply shocks, labour market tightness and expectation formation in explaining bouts of inflation. We begin by showing that a quasi-flat Phillips curve, which was popular prior to the pandemic, still fits the post-2020 US data well with the implication that *(i)* labour market tightness likely played a limited role in generating recent inflation and *(ii)* changes in short term inflation expectations induced by supply shocks likely played a major role. We then explore how best to capture the joint dynamics of inflation and inflation expectations in response to supply shocks. Given the difficulty of capturing these dynamics under rational expectations, we propose and evaluate a model with bounded rationality. In our model, supply shocks that affect the common component of inflation across many goods drive persistent inflation dynamics through their effect of expectations, while supply shocks that are concentrated in a sector lead to much more temporary changes in both inflation and inflation expectations. Although our departure from full rationality is minor, it allows perceived broad-based supply shocks to be amplified and propagated over time in a manner consistent with observation.

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Introduction

While inflation in many countries has been coming down quickly from the heights of 2022, core inflation has been coming down at a slower pace and this can be challenging for central banks who aim for a timely return to a pre-pandemic level of near 2%. For the US, this pattern for core inflation can be seen in Figure 1.



Notes: The observed series is Consumer Price Index for All Urban Consumers: All Items Less Food and Energy in U.S. City Average. We plot monthly observations of the percent change from one year ago to obtain a slightly smoother picture. In the rest of the paper, we will use quarter-to-quarter changes.

One of the main tools used by central bankers to understand inflation is the expectations-augmented Phillips curve which links inflation to expectations of future inflation and overall market tightness, either measured by an output gap, an unemployment gap or vacancy-to-unemployment ratio.¹ When viewed through the lens of the Phillips curve, the persistent period of high inflation that is not explained by supply shocks can be attributed to either an overly-hot labour market or by elevated expectations of inflation. A third possibility is that the Phillips curve is a rather unstable object and should be considered of limited value for thinking about current inflation. The later more unorthodox view is sometimes motivated

¹The later has been advocated as a better proxy for labour market tightness by, among others, Ball, Leigh, and Mishra [2022], Michaillat and Saez [2022] and Benigno and Eggertsson [2023].

by perceived repeated failures of the Phillips curve. For example, in the decade between the “Great Financial Crisis” and the pandemic –instead of under-predicting inflation as has recently been common– the Phillips curve often leads to a “missing deflation puzzle”.

This paper begins by examining what a pre-2020 view of the Phillips curve tells us about the recent determinants of inflation and whether such a Phillips curves actually remains a good tool for understanding inflation. We complement our Phillips curve analysis with a VAR analysis of the joint dynamics of inflation, inflation expectations and labour market tightness, where inflation expectations used in both cases are taken from the Michigan Survey of Consumers. The VAR analysis is especially relevant as it will become the target for our later structural analysis of how inflation and inflation expectations interact to create a rich joint dynamics. Both our Phillips curve exploration and our VAR analysis come to similar conclusions. First, the recent episode does not suggest any important break from past behaviour. Second, both analyses suggest that inflation is mainly driven by expectations and quasi-iid supply shocks, with labour market tightness playing a very secondary role. In other words, the data maintain support for a close-to-flat Phillips curve view with short term inflation expectations being an important driver of inflation. In particular, the VAR analysis highlights how inflation expectations formed at t almost perfectly match realized inflation at $t + 1$, while inflation and inflation expectations are almost unrelated to labour market outcomes. We finish this section by showing that a Philips curve model with rational expectations cannot account for the joint dynamics of inflation and inflation expectations. .

We then turn to examining some of the empirical determinants of household inflation expectations. In particular, we show that these inflation expectations react to categorical inflation measures in a way that deviates markedly from their importance in the overall CPI basket. Instead, inflation expectations seem to be influenced by the common component of

disaggregated data, where the main common component places very different weights than the CPI across items/sectors. So agents appear to update their expectations very differently if inflation is driven by a broad-based increase in prices than if the same level of inflation is driven by only one or two items. This observation will be a key element in our model of inflation expectations.

In the fourth section of the paper we propose a model of the joint determination of inflation and inflation expectations that builds on the previously mentioned empirical patterns with the aim of explaining the VAR patterns. For the determinants of inflation, our model simply imports the New Keynesian Philips Curve specification we discuss and evaluate in Section 1. For the determinant of inflation expectations, we make two departures relative to a full information rational expectations benchmark. First, we endow agents with two inflation realizations; a realization of headline inflation and a realization of an inflation signal that can be interpreted as reflecting broad-based shocks. Second, we model the agents as quite sophisticated but not fully informed of the data generating process. They extract a common component from their sectoral inflation signals using a Kalman filter and use this common component to predict future inflation. In this filtering process, agents treat the realization of the common component in inflation as uncorrelated with the noise in their signal even though the two may not be independent in equilibrium due to the role of expectations themselves driving inflation. In doing so, agents depart from a fully rational benchmark. We show that such a model of inflation expectations can capture very well the joint dynamics of inflation and inflation expectations, as described by the VAR presented in Section 2, while simultaneously being consistent with easily observable moments of the inflation process, so that agent's beliefs are almost self-confirming.

While the narrative behind our model is most easily expressed in reference to the role of

broad-based inflation shocks in driving inflation expectations, we do not actually use the disaggregated data in the estimation of our model. Instead, the “broad-based” inflation signal is treated as a latent variable and we complete our analysis by showing that our estimate of this latent variable correlates closely with the common component directly estimated from the disaggregated data.

The narrative that arises from our model of inflation and inflation expectations can be expressed as follows. Inflation can be seen as driven by two different types of shocks: first by narrow supply shocks that create very little dynamics even when substantial since such shocks are not readily transmitted to expectations as they do not create a perceived generalized increase in prices; second by a broad-based supply shocks that transmits to inflation expectations and thereby creates something close to a self-fulling inflation episode. Such a model is shown to explain why inflation can remain stable during long periods even in the presence of many narrow but unsynchronized supply shocks. But, in contrast, inflation tends to broaden and persist when the economy is hit simultaneously by several supply shocks in different sectors.

1 Should We Maintain or Throw out the Flat Phillips Curve View?

Prior to the COVID-19 pandemic, many studies of the Phillips curve suggested that its slope was quite flat. Although the identification of the slope of the Phillips curve can be difficult, the work by Hazell, Herreño, Nakamura, and Steinsson [2022] advanced on this point by exploiting cross-state variation. Their finding was that the slope of the Phillips curve was significantly positive, but nevertheless quite small. Furthermore, they found that this flatness property did not arise post 90’s, but appears to be a feature of the data going

back into the 1960s. They concluded that inflation expectations likely played a dominant role in the inflation episode on the 1970s and early 1980s. In related work (Beaudry, Hou, and Portier [2023]), we found similar results with the inflation expectations drawn from the Michigan Survey of Consumers helping to explain much of the variation in inflation since late 1960.

In this section, we aim to examine whether the recent inflation episode should lead one to revise/reject/update the flat Phillips curve view. To this end, we start with the following very parsimonious view of the (quarterly) Phillips curve:

$$\pi_t = \beta \pi_{t+1}^e + \gamma_g \text{gap}_t + e_t, \quad (1)$$

π_t is quarter-to-quarter Headline CPI inflation, π_{t+1}^e is the mean of the one-year-ahead expectation of CPI inflation drawn from the Michigan Survey of Consumers.² We choose this measure of expectations as it goes back all the way to the 1960s. As prices are set by firms, using a measure of business expectations could be preferred. However, such measures are not readily available over such a long time span.³ The gap represents labour market tightness and is measured by the minus unemployment gap.⁴

We initially do not estimate this Phillips curve, but choose commonly accepted parameters values. In particular, γ_g is set to 0.0138 according to what estimated in Hazell, Herreño,

²Our preference would be to use quarter-to-quarter expected inflation. However, the Michigan Survey of Consumers reports expectations for CPI inflation for the next year. To extract a quarter estimate from this data we rescale the one-year-ahead expected inflation assuming survey respondents believe that quarter-to-quarter inflation follows an AR(1) process with persistence $\tilde{\rho}$, that needs not to be equal to the actual persistence of inflation. The estimated $\tilde{\rho} = 0.89$. For more details, we refer to Beaudry, Hou, and Portier [2023].

³The Cleveland Fed Survey of Firms' Inflation Expectations (SoFIE) is available since 2018Q2. Over the sample 2018Q2-2023Q1, the correlation between the Michigan Survey of Consumers' inflation expectations and the Cleveland Fed Survey of Firms' ones is 0.9330

⁴The unemployment gap is computed as the unemployment rate minus the noncyclical rate of unemployment from FRED (series name *NROU*).

Nakamura, and Steinsson [2022]⁵ and β is set to 0.99, which is a common value in the literature. We refer to this as our baseline Phillips curve.

From this baseline Phillips curve, we construct residuals $\hat{e}_t = \pi_t - 0.99\pi_{t+1}^e - 0.0138 \text{ gap}_t$ over the period 2008q1-2023q1 and plot them against our measure of the gap. This is the plot in Panel (a) of Figure 2.⁶ The dark dots are Phillips curve residuals for the post-2020 period, while the grey dots cover the period from 2008q1 to 2019q4. We overlayed on these figures two estimated relations between the Phillips curve residuals and labour market tightness over 2008q1 to 2019q4; one in linear form and one in cubic form. The coefficient for this regression are presented in Columns 1 and 2 of Table 1. We also calculate the standard deviation σ_e of the Phillips curve residuals for this sample as well as that from the prior sample running from 1968q1 to 2007q4. The first thing to note is that the standard deviation of the Phillips curve residuals are only slightly higher over the period 2008q1-2023q1 than over the period 1968q1-2007q4, although we are considering the Headline inflation and not controlling for supply shocks. More interestingly, we detect no significant link between these residuals and the unemployment gap.⁷ If the residuals were strongly and positively associated with the labour market tightness measure, including possibly in a non-linear fashion, this would put in question the validity of a flat Phillips curve view in the more recent data. However, these

⁵In Hazell, Herreño, Nakamura, and Steinsson [2022], the authors provide an implied aggregate slope of the Phillips curve, with R-CPI as the measure of inflation and negative unemployment gap as the measure of market slackness. From footnote 24 in Hazell, Herreño, Nakamura, and Steinsson [2022], this aggregate slope of the Phillips curve is $= 0.58 \times 0.0062 + 0.42 \times 0.0243 = 0.0138$.

⁶In our analysis for post-2008 data, we exclude the 2020q2 observation given that measuring labour market tightness at a time of massive lock downs is very controversial.

⁷Endogeneity issues can complicate inference from this figure. Under the null hypothesis that our Phillips curve is well specified, forecast errors should reflect supply or markup shocks. In contrast, if the slope is mis-specified, then the forecast errors would also include a term directly related to labour market tightness. In this later case, such mis-specification should show up as a systematic relationship between forecast errors and labour market tightness. However, a strong endogenous relation between supply shocks and labour market tightness could hide this effect. Although we see this as a possibility, we believe it is likely of second order importance over this sample. We have explored the potential relevance of this issue by using high frequency identified monetary shocks to instrument labour market tightness when regressing forecast errors on labour market tightness. We have not found evidence of endogeneity bias.

forecast errors do not suggest that a steeper Phillips curve would better explain the more recent data episode. The same exercise is repeated using Core CPI inflation as a measure of inflation. As it can be seen in Columns 3 and 4 of Table 1 and on Panel (b) of Figure 2, there is again no sign of a steepening of the Phillips curve. Furthermore, the standard deviation of the Phillips curve residuals is now smaller post 2008 as compared to previously. If anything, this flat Phillips curve fits better post 2008 data than over the period 1968-2007.

Table 1: Projection of the Philips Curve Residuals \hat{e} on the Gap, 2008-2023

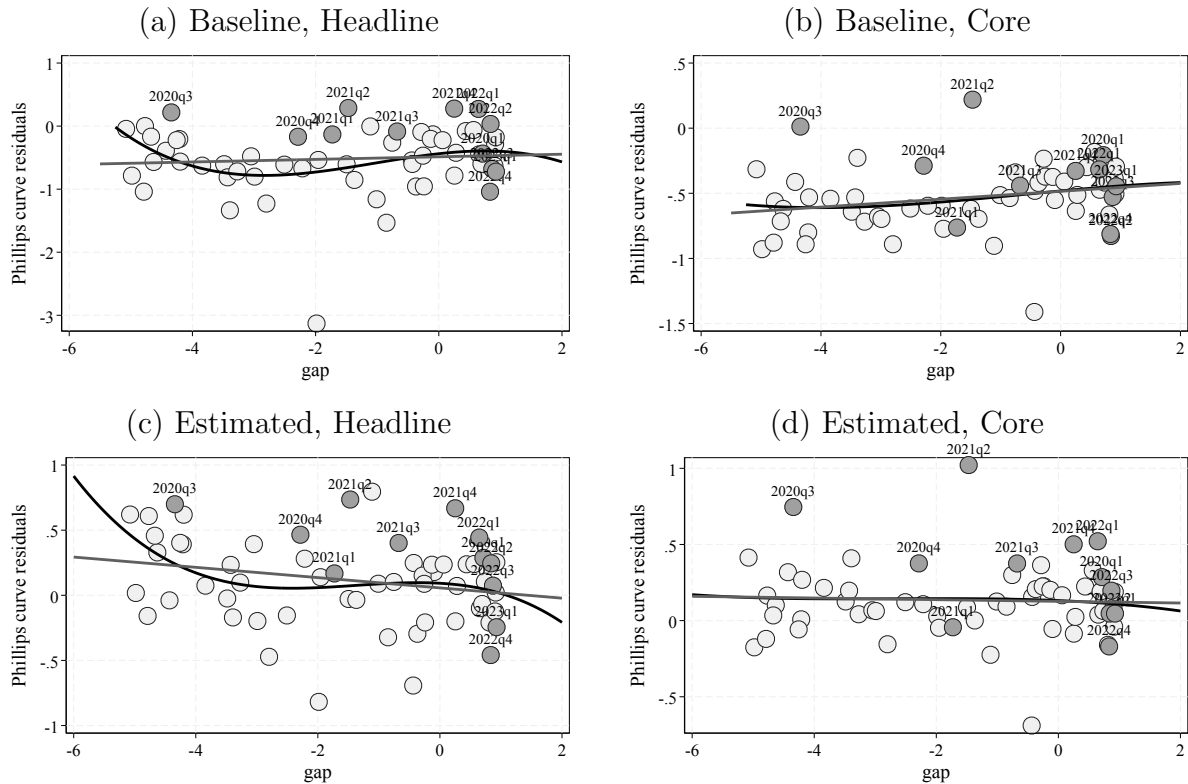
	Headline Baseline		Core Baseline		Headline Estimated		Core Estimated	
	linear (1)	nonlinear (2)	linear (3)	nonlinear (4)	linear (5)	nonlinear (6)	linear (7)	nonlinear (8)
gap	0.02 (0.035)	0.11 (0.113)	0.03 (0.016)	0.04 (0.056)	-0.04 (0.022)	-0.03 (0.072)	-0.01 (0.016)	-0.02 (0.055)
gap ²		-0.05 (0.087)		-0.00 (0.043)		-0.04 (0.056)		-0.01 (0.042)
gap ³		-0.02 (0.015)		-0.00 (0.007)		-0.01 (0.009)		-0.00 (0.007)
N	60	60	60	60	60	60	60	60
σ_{ϵ} 60-07		.324		.315		.276		.291
σ_{ϵ} 08-23		.528		.256		.338		.245

Notes: in columns (1) to (4), the Phillips curve residuals are obtained from the baseline Phillips curve (1). In columns (5) to (8), they are obtained from the augmented Phillips curve (2) estimated over the sample 1969Q1-2007Q4. σ_{ϵ} is the standard deviation of the Phillips curve residuals. The standard errors of estimated coefficients are between parentheses.

In addition to this baseline Phillips curve, we also directly estimate a Phillips curve over the sample 1969q1 to 2007q4, and look at the implied out of sample forecast errors for the period 2008q1-2023q1. Details of this estimation is provided in the appendix and follow Beaudry, Hou, and Portier [2023]. Since the determinants of inflation in Phillips Curve are endogenous, we follow Barnichon and Mesters [2020] and estimate our Phillips curve by Instrumental Variables using estimated monetary shocks as instruments.⁸ To reduce the risk

⁸To be specific, we use twelve lags of the Romer and Romer's [2004] shocks (extended by Wieland and

Figure 2: Out-of-Sample Residuals from Phillips Curves



Notes: Panels (a) and (b) of this figure plots the out-of-sample residuals of the baseline Phillips curve (1) against a measure of labor market tightness (minus the U.S. Congressional Budget Office unemployment gap), for two measures of inflation (Headline and Core CPI). The gray lines show the estimated linear or cubic relation between residuals and labor market tightness (see Table 1 for the estimated coefficients). Light dots correspond to pre-2020 observations and dark ones to post-2020. We exclude 2020q2 from this graph. Panels (c) and (d) of this figure repeat the analysis with the augmented estimated Phillips curve (2).

of biases, we estimated the Phillips curve using Core inflation, and as argued in Beaudry, Hou, and Portier [2023], we include the real interest rate as an additional explanatory variable to capture a potential cost channel of monetary policy.⁹ The estimated Phillips curve is therefore

$$\pi_t = \beta \pi_{t+1}^e + \gamma_g \text{gap}_t + \gamma_r (i_t - \pi_{t+1}^e) + e_t. \quad (2)$$

The coefficients we obtain for γ_g is quite close to that of baseline Phillips curve, with the exception that we obtain a slightly steeper Phillips curve than that reported in Hazell, Herreño, Nakamura, and Steinsson [2022]. Our estimate of γ_g is .04 and significant at standard levels, which is more than twice that of .0138 from Hazell, Herreño, Nakamura, and Steinsson [2022], but not significantly different from it. Our estimate for β is also .99.

In Panel (d) of Figure 2, we report the out of sample Phillips curve residuals from our estimated model for the period 2008q1-2023q1 plotted against our measure of labour market tightness, with again a linear and cubic fit superimposed. We see a very similar pattern to that observed in Panels (a) and (b), with very little link between the forecast errors and labour market tightness, even though these are out of sample forecast. The estimated coefficients for these linear and cubic fits are displayed in Columns (7) and (8) of Table 1. So again, these observations do not suggest that a stepper Phillips curve is helpful/needed to explain the 2008q1-2023q1 period. Note that the fit of this estimated Phillips curve, as measured by the standard deviation of the residuals $\sigma_{\hat{e}}$ is better out-of-sample than in-sample. In appendix A, we confirm similar finding using the vacancy to unemployment rate as an alternative measure of labour market tightness, as well as exploring the effect of the use market base measures of expectations. For completeness, panel (c) of Figure 2 and Columns

Yang [2020]) and their square as instruments.

⁹The presence of direct cost channel matters in the context of asking which expectations are more relevant when estimating Phillips curve. See a comprehensive discussion in Reis [2023].

(5) and (6) of Table 1 repeat the analysis when using the Headline CPI to estimate the Phillips curve. Results are similar.

The overall out-of-sample fit of the baseline Phillips curve can also be seen in Panel (a) and (b) (for Headline) and (c) and (d) (for Core) of Figure 3. As can be seen, the forecasted inflation tracks actual inflation reasonably well, with exceptions such as 2021q2, where actual inflation was well above predicted, as is consistent with a strong supply shock in that quarter associated with transport bottlenecks. In Panels (a) and (c), we also plot a counterfactual path of inflation over the period assuming that the unemployment gap had been equal to its sample mean over the forecast horizon. Here we see that this counterfactual inflation series is very similar to the predicted inflation path using the full model. This reflects the very weak direct role that labour market tightness is playing in our estimated Phillips curve given that the slope is quite small. The second counterfactual we examine imposes that inflation expectations are constant over the forecast period and set at their average. This counterfactual path is reported together with our forecasted rates of inflation in Panel (b) and (d). Here we see clearly that it is expected inflation that mainly drives the fit of the predicted inflation, as when we omit the role of inflation expectations, the fit greatly deteriorates.

In summary, Figures 2 and 3 illustrate how a very simple three-variable Phillips curve, either estimated prior to the recent periods of missing deflation and high inflation, or based on the work of Hazell, Herreño, Nakamura, and Steinsson [2022], fits the post-2007 data and post-2020 quite well. The lack of a relationship between forecast errors and labour market tightness indicates that the Phillips curve is likely still quite flat. The forecast errors are also close to iid, which is consistent with them being interpreted as supply shocks. The ongoing flatness of the Phillips curve implies that persistently high levels of core inflation in 2022 and

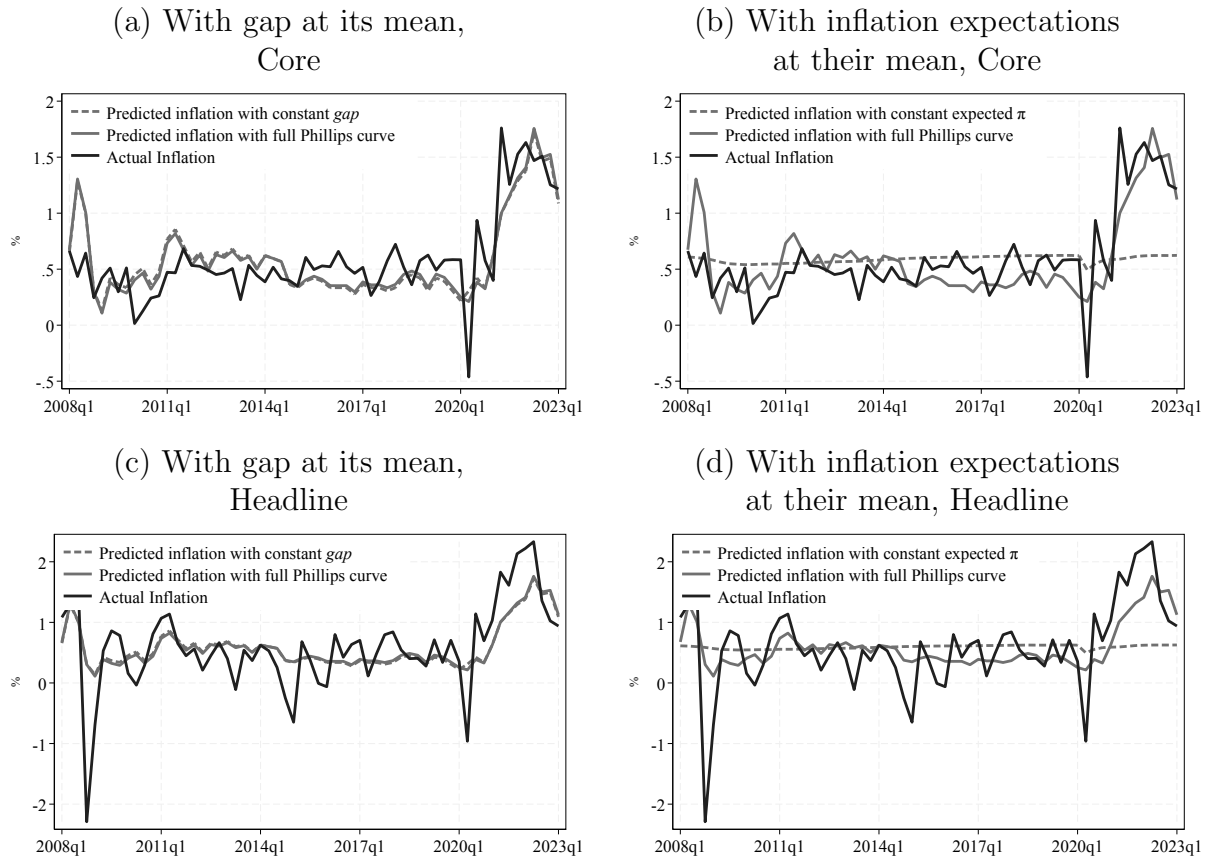
early 2023 are unlikely explained by labour market tightness. Instead, it appears primarily driven by elevated levels of inflation expectations. Such observations can be both reassuring and uncomfortable. On the one hand, they support the view that the Phillips curve framework likely remains a relevant framework for thinking about inflation developments. However, they also suggest that the main driver of inflation is expected inflation itself. The latter observations can be uncomfortable as it questions/downplays the traditional role of pure demand management as key to controlling inflation and instead pushes forward the central importance of expectations management– and the more amorphous role of inflation psychology– in controlling inflation.

Table 2: Estimated Phillips Curves, 1969-2007

	Headline	Core
β	1.15 [*] (0.031)	0.99 [*] (0.048)
γ_g	0.07 [*] (0.020)	0.04 [*] (0.015)
γ_r	0.13 [*] (0.031)	0.25 [*] (0.042)
Observations	144	144
J Test (jp)	15.201 (0.887)	10.684 (0.986)
Weak ID Test	7.825	12.245

Notes: All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. Real oil price and its lag are also omitted for the regression with Headline CPI. All regressors are instrumented using six lags of Romer and Romer's [2004] shocks (as extended by Wieland and Yang [2020]) and their squares as instruments. A ^{} indicates significance at 5%. Sample is 1969Q1-2007Q4.*

Figure 3: Counterfactual Simulations from the Baseline Phillips Curve



Notes: These counterfactual simulations are done using the baseline Phillips curve (1), using either Headline or Core CPI.

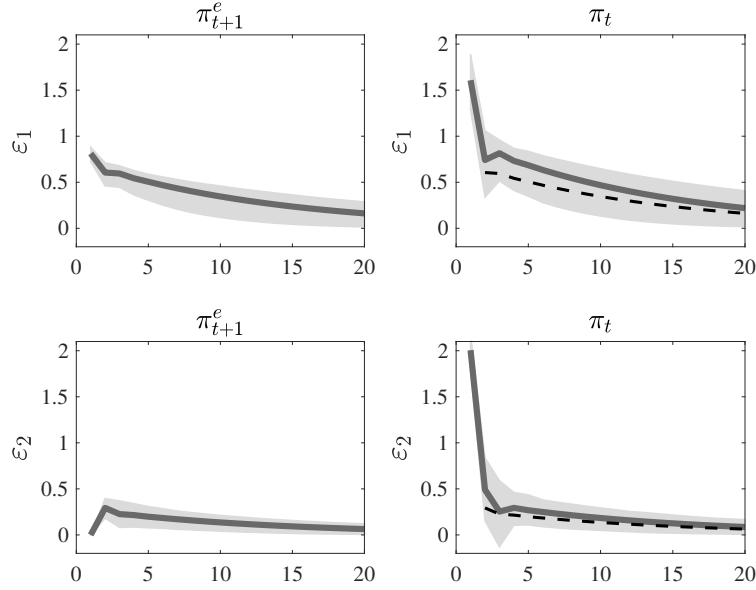
2 The Joint Dynamics of Inflation, Inflation Expectations and Labour Market Tightness

In this section we first provide VAR based evidence to give further support to the view that inflation dynamics appears to be largely explained by inflation expectations, with labour market tightness playing a very secondary role. We then show that our baseline Phillips curve model with rational expectations and full or incomplete information is unlikely to account for the observed joint dynamics.

2.1 VAR Evidence

To this end, Figure 4 plots impulse response generated from a bi-variate VAR using inflation and the inflation expectations estimated over the period 1969q1-2023q2. The data for inflation remains headline inflation and that for expectations is again drawn from the Michigan Survey of Expectations. The figure corresponds to impulse response associated with a Choleski orthogonalization of the VAR residuals, where the shock ε_2 does not have impact effect on inflation expectations while ε_1 is unrestricted but to be orthogonal to ε_2 . We do not claim that these impulses responses capture the effects of structural shocks. Instead, we simply view them as a means of summarizing properties of the data. In particular, these two shocks will eventually be interpreted as combinations of the structural shock in our modelling of section 4. In Figure 4, together with the response of inflation to the two shocks $\{\pi_t\}_{t=1}^{20}$, we also represent (dashed line) the expected inflation response $\{\pi_{t+1}^e\}_{t=1}^{19}$ shifted by one period. Two properties of this joint dynamics can be observed. First, inflation expectations formed at time t match very closely realized inflation at $t + 1$. This is true for both shocks, and therefore for any linear combination of these shocks. Accordingly, it suggests that either agents are very good at predicting inflation or instead that expected inflation may be driving

Figure 4: Impulse Responses in the 2-VAR (π_t, π_{t+1}^e)



Notes: this Figure plots the impulse responses to a one standard deviation shocks ε_1 and ε_2 . These shocks are obtained from a Choleski orthogonalization. The estimated VAR features two lags of Headline CPI inflation and the MSC inflation expectations. The dashed line is the expected inflation response $\{\pi_{t+1}^e\}_{t=1}^{19}$ shifted by one period. Sample is 1969Q1-2023Q1. Shaded area represents the 95% confidence band.

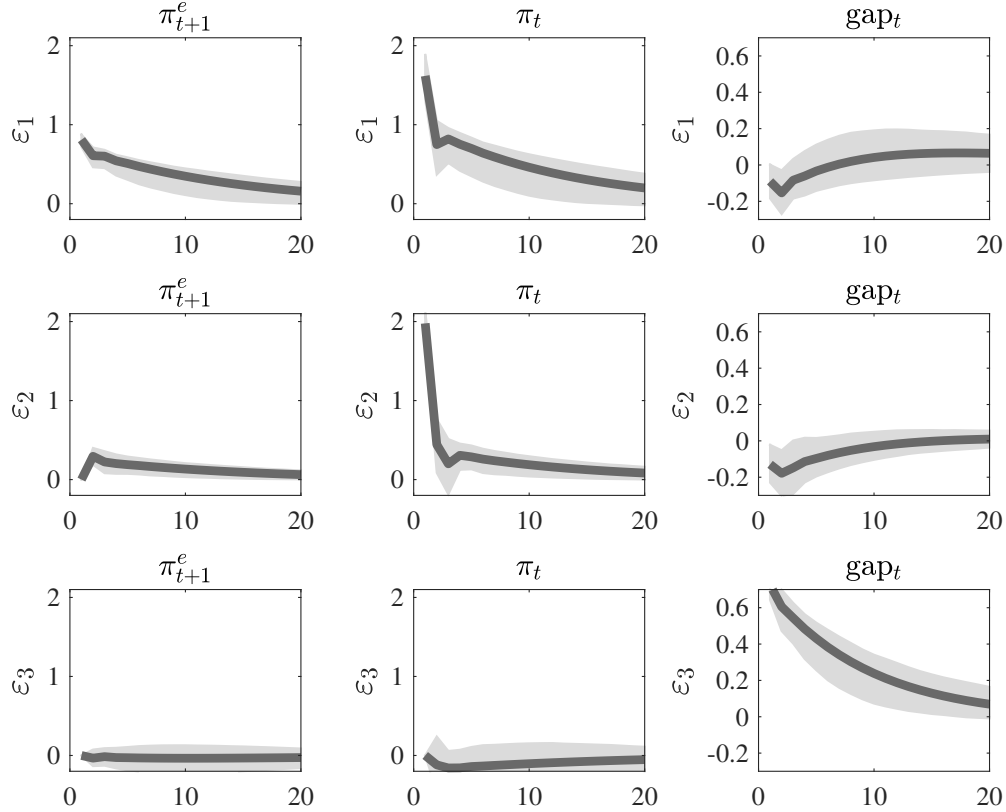
inflation. Second, these impulse responses suggest that there is some combination of shocks (that forms ε_2) that don't transmit to expected inflation and this leads to very temporary rise in inflation. While on the other hand, there is some combination of shocks (that forms ε_1) that have a large effect of inflation, transmit to expected inflation, and this is associated with persistent inflation

In Appendix C, we show that these impulse responses are essentially unchanged if we change the number of lags or if we drop post-2008 data from the estimation. This suggests that the recent inflation dynamics continue to obey the dynamics that was observed earlier, suggesting no substantial break in the process.

In Figure 5, we present the impulse responses from a trivariate VAR that builds on our bi-variate VAR by including (minus) the unemployment gap as the third variable, again using a Choleski identification; ε_3 affects only the gap on impact, ε_2 affects only the gap

and inflation on impact and ε_1 is unrestricted but to be orthogonal to the two other shocks.

Figure 5: Impulse Responses in the 3-VAR $(\pi_t, \pi_{t+1}^e, \text{gap}_t)$



Notes: this Figure plots the impulse responses to a one standard deviation shocks ε_1 , ε_2 and ε_3 . These shocks are obtained from a Choleski orthogonalization. The estimated VAR features two lags of Headline CPI inflation, the MSC inflation expectations and (minus) the employment gap. Sample is 1969Q1-2023Q1. Shaded area represents the 95% confidence band.

The main feature we want to highlight from this figure is the quasi complete separation between inflation and inflation expectations on the one hand and labour market tightness on the other hand. This can be seen by examining how the sub-block of impulse response to ε_1 and ε_2 for inflation and inflation expectations generated by this three variable VAR is almost identical to that generated by our two-variable VAR. Furthermore, we can see that the shock to unemployment ε_3 has essentially no impact on inflation or inflation expectations, while

the two other shocks that drive almost all the variance of inflation and expected inflation have very effect on unemployment. In summary, these two (non-structural) VARs appear consistent with the view that the Phillips curve is likely very flat and that the persistent component of inflation may well be driven by expected inflation.

2.2 The Difficulty in Explaining the Joint Dynamics of Inflation and Inflation Expectations with Rational Expectations

As we have argued above, inflation expectations appear to potentially be an important driver of inflation. However, this may only be a proximate cause. It may well be the case that both inflation and inflation expectations are driven by a third variable, and expectations are simply good as capturing this force. Accordingly, we ask in this subsection whether the joint dynamics of inflation and inflation expectations reflected in our bi-variate VAR could be produced by our baseline Phillips curve model under rational expectations, where the dynamics of the labour market tightness and supply shocks would be the more fundamental factor. To that end, first assume that inflation is generated from the baseline Phillips curve

$$\pi_t = \beta \pi_{t+1}^e + \gamma_g \text{gap}_t + e_t, \quad (3)$$

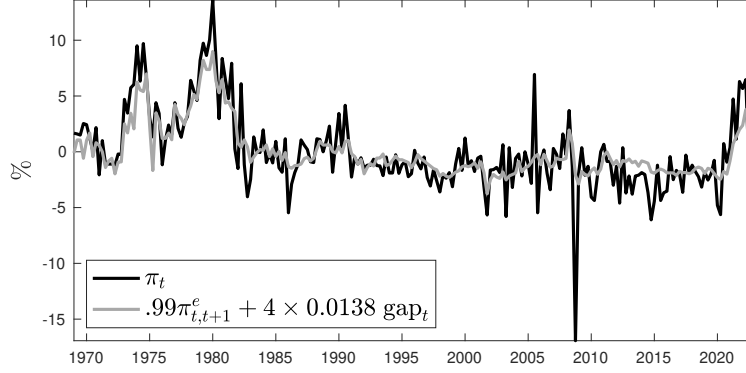
with $e_t \sim N(0, \sigma_e^2)$, $\beta = .99$, $\gamma_g = 4 \times 0.0138^{10}$ and where we use quarterly (annualized) Headline inflation, the Michigan Survey of Expectations and (minus) the unemployment gap. As discussed in the previous section, the fit of the baseline Phillips curve hasn't worsened in the recent period, as illustrated on Figure 6.

We also assume that the gap follows an exogenous AR(1) process:

$$\text{gap}_t = \rho \text{gap}_{t-1} + v_t, \quad (4)$$

¹⁰We multiply the Hazell, Herreño, Nakamura, and Steinsson's [2022] estimate by four because we use here annualized quarterly inflation and expected inflation.

Figure 6: The Fit of the Baseline Phillips Curve, 1969-2023



Notes: this Figure plots Headline CPI inflation together with inflation predicted by the baseline Phillips curve (1)

with $v_t \sim N(0, \sigma_v^2)$. We estimate an AR(1) process for gap_t .¹¹ The estimated parameters are $\rho = 0.89$ (s.e. 0.03) and $\sigma_v^2 = 9.05$ (s.e. 0.33). We obtain an estimate of $\sigma_e^2 = 5.00$ from the residual of (3).

Full Information Rational Expectations (FIRE): When economic agents form rational expectations, they use the correct model (3)-(4) and form expectations $\pi_{t+1}^e = E_t^{FIRE}[\pi_{t+1}]$. Under full information, gap_{t-1} , v_t and e_t are observed. The solution to the model is given by:

$$\pi_t = \frac{\gamma_g}{1 - \beta\rho} \text{gap}_t + e_t, \quad (5)$$

$$E_t^{FIRE}[\pi_{t+1}] = \frac{\gamma_g\rho}{1 - \beta\rho} \text{gap}_t. \quad (6)$$

Full Information Rational Expectations (FIRE) with persistent supply shock:

Looking at the properties of the Phillips curve residual \hat{e} , it is significantly autocorrelated, although the autocorrelation coefficient is small. When we estimate an AR(1) process, we find low autocorrelation, with an estimated $\rho_e = 0.165$ (standard deviation = 0.05). Could

¹¹When we estimate an AR process with more lags, only the estimate on the first lag is significant.

this admittedly low persistence reverse the results we obtain with the iid supply shocks FIRE model? To answer that question, we also consider an extension of the FIRE model where we allow e_t to be persistent. The solution to such a FIRE model with persistent e_t is given by:

$$\pi_t = \frac{\gamma_g}{1 - \beta\rho} \text{gap}_t + \frac{1}{1 - \beta\rho_e} e_t, \quad (7)$$

$$E_t^{FIREp}[\pi_{t+1}] = \frac{\gamma_g\rho}{1 - \beta\rho} \text{gap}_t + \frac{\rho_e}{1 - \beta\rho_e} e_t. \quad (8)$$

Incomplete Information Rational Expectations (IIRE): Here we look at a counterpart of our Section 4 main model but with rational expectations. We assume that agents correctly understand the model (3)-(4), but that they do not observe the gap and the aggregate supply shock e_t . Instead, we assume that the economy is composed of N sectors with sector-specific supply shocks. In sector j , sectoral inflation is generated by:

$$\pi_{j,t} = \beta\pi_{t+1}^e + \gamma_g \text{gap}_t + e_{j,t}, \quad (9)$$

where $e_{j,t} \sim N(0, \sigma_j^2)$. We assume that agents observe a signal that adds demand inflationary pressures $\gamma_g \text{gap}_t$ and a subset \mathcal{S} of the sector-specific supply shocks e_t that we denote ϵ_t and which is given by

$$\epsilon_t = \sum_{j \in \mathcal{S}} e_{j,t}. \quad (10)$$

Therefore, agents observe a signal

$$s_t = \gamma_g \text{gap}_t + \epsilon_t, \quad (11)$$

with $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ and know that inflation is generated by

$$\pi_t = \beta\pi_{t+1}^e + \gamma_g \text{gap}_t + \epsilon_t + w_t, \quad (12)$$

where $w_t = e_t - \epsilon_t$ is the residual supply shock. The agents use Bayes Rule and the knowledge of the model to form beliefs of e_t and gap_t , and to compute $\pi_{t+1}^e = E_t^{IIRE}[\pi_{t+1}]$. Denoting \mathcal{K} the Kalman gain in this computation, the solution to the model is:¹²

$$E_t^{IIRE}[\pi_{t+1}] = (1 - \mathcal{K}\gamma_g)\rho E_{t-1}^{IIRE}[\pi_t] + \frac{\gamma_g\rho\mathcal{K}}{1 - \beta\rho}(\gamma_g \text{gap}_t + \epsilon_t), \quad (13)$$

$$\begin{aligned} \pi_t = & \beta(1 - \mathcal{K}\gamma_g)\rho E_{t-1}^{IIRE}[\pi_t] + \frac{\gamma_g^2\mathcal{K}\rho\beta + (1 - \beta\rho)\gamma_g}{1 - \beta\rho} \text{gap}_t \\ & + \frac{\mathcal{K}\rho\beta\gamma_g + (1 - \beta\rho)\gamma}{1 - \beta\rho} \epsilon_t + w_t. \end{aligned} \quad (14)$$

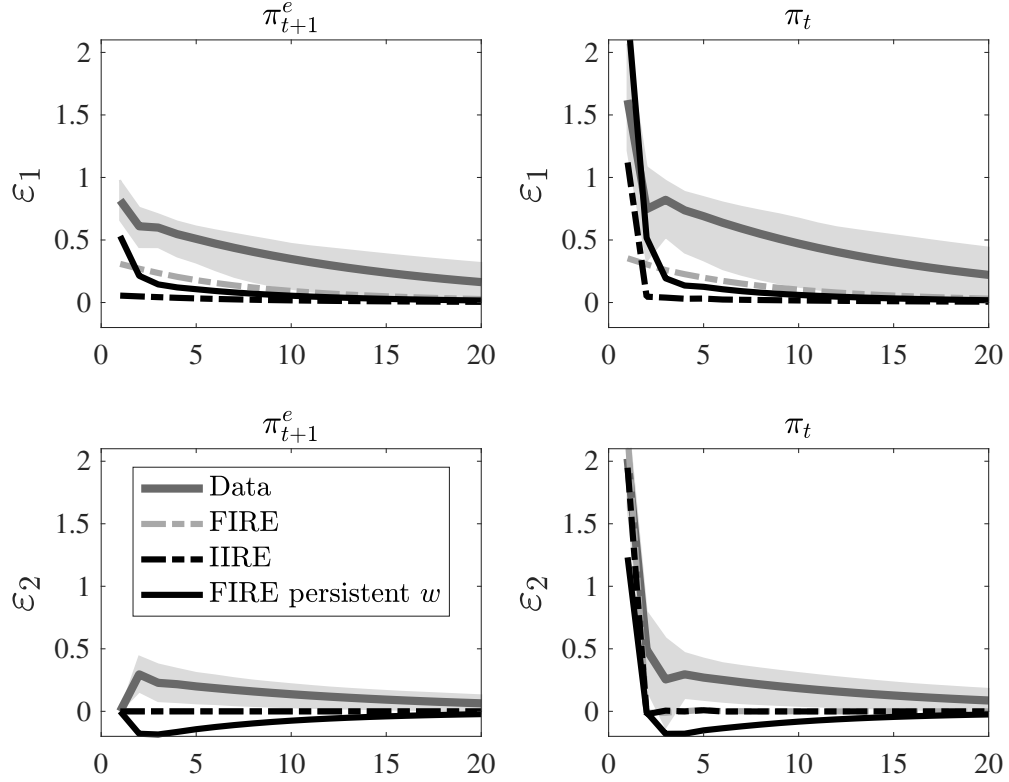
We estimate σ_ϵ by Simulated Method of Moments. The model is simulated 200 times, the (π_t, π_{t+1}^e) VAR is estimated in each simulation and σ_ϵ is set to minimise the distance between the average Impulse Response Functions obtained from the model with their empirical counterpart, when the same Choleski decomposition is used for model and data. The resulting estimate is $\sigma_\epsilon = 1.08$, and this implies $\sigma_w = 1.96$.

Results: For each of the three models considered, using the estimated parameters, we simulate 200 random samples with the same length as the data. With these generated data, we estimate the bi-variate VAR and see how well it can match the impulse responses derived from our empirical bi-variate VAR. The result can be seen in Figure 7. The impulse responses from the generated data do not match that derived from actual data neither in shape nor level, even if in the case of IIRE model we estimate σ_ϵ to deliberately match these IRFs.

We can also illustrate the performances of these models in matching the joint dynamics of inflation and inflation expectations by looking at the set of moments presented in Table 3. We see that none of the rational expectations models can create inflation as volatile as the data. The simulated data have much lower persistence and variances in both realized

¹²The detailed derivations are left to Appendix F.

Figure 7: The Joint Process of π and π_{t+1}^e in the Data and in Models with Rational Expectations



Notes: on this Figure, the solid grey line plots the impulse responses to a one standard deviation shocks ε_1 and ε_2 estimated with data from MSC and Headline CPI. Sample is 1969Q1-2023Q1. Shaded area represents the 95% confidence band. The other lines plot the average impulse responses (over 200 simulations of length 216) obtained from the same VAR estimated on simulated data, when the Data Generating Process is either the FIRE model (5)-(6) assuming w_t is i.i.d, the IIRE model (13)-(14) or the FIRE model assuming w_t is persistent.

and expected inflation, as compared to the actual data.

Table 3: Moments of π and π_{t+1}^e in the Data and in Models with Rational Expectations

	$var(\pi_t)$	$cov(\pi_t, \pi_{t-1})$	$cov(\pi_t, \pi_{t-2})$	$var(\pi_{t+1}^e)$	$corr(\pi_{t+1}^e, \pi_t^e)$
Data	12.52	7.70	6.99	3.92	0.90
FIRE	5.49	0.45	0.39	0.42	0.87
IIRE	5.10	0.05	0.04	0.00	0.87
FIRE, persistent w_t	7.59	1.61	0.57	0.61	0.64

Notes: For the three models, the moments are computed as the average across 200 random samples of same length than the data.

In part, this should not be too surprising. Given the rather small slope of the Phillips curve implied by the work of Hazell, Herreño, Nakamura, and Steinsson [2022] and the low persistence of the supply shocks, it is hard to see how shocks to labor market tightness or supply shocks could generate important volatility in inflation expectations. In particular, the difficulty for the rational expectations models to match the VAR observations reflects the difficulty to generate both large and persistent responses of inflation expectations. Rational agents understand that the underlying persistent factor moving inflation plays only a limited role. As a result, they shouldn't expect future inflation to have persistent movement as well. Moreover, limited information does not help with the situation. In fact, the IIRE case has the most muted response among all the three models and generates the least volatile as well as the least persistent inflation. This is not only because the rational agent understands the persistent labor market condition plays little role. They also realize that their signals are not informative because of the large noise-to-signal ratio due to low γ_g . As a result, their expectations do not respond much to the movements in labor market conditions.

All in all, we conclude from this section that there exists a persistent dynamics of inflation expectations that is highly correlated with actual inflation and that is unlikely to be

explained by a Rational Expectations New Keynesian Phillips curve model, with or without full information. In the next section, we explore what may drive inflation expectations.

3 The Information Encoded in Inflation Expectations

A simple Phillips curve under full information rational expectations does not appear to be able to explain the joint dynamics of inflation and inflation expectations. This raises the question of what could be driving inflation expectations? In this section we want to provide preliminary evidence for our conjecture that inflation expectations may be driven by a perceived common component across disaggregated categories of inflation. Our narrative is quite simple. Inflation is defined as a generalized increase in prices. So when agents (both firms and households) see prices increase in many sectors (although not necessarily in *all* sectors), they take this to be sign that inflation is taking hold. This affects their expectations and this incite firms to increase prices.

To provide support to this conjecture, we take the set of disaggregated inflation data for the 25 expenditure categories used by the BLS to construct the CPI (see Table D.1 in Appendix D for a description) and extract the common component of these categorical inflations. To do so, we estimate a Dynamic Factor Model¹³ in which categorical inflations are driven by one common persistent factor \mathcal{C}_t which follows an AR(1) process. The model takes the following form:

$$\pi_{i,t} = \alpha_i \mathcal{C}_t + e_{i,t}, \quad (15)$$

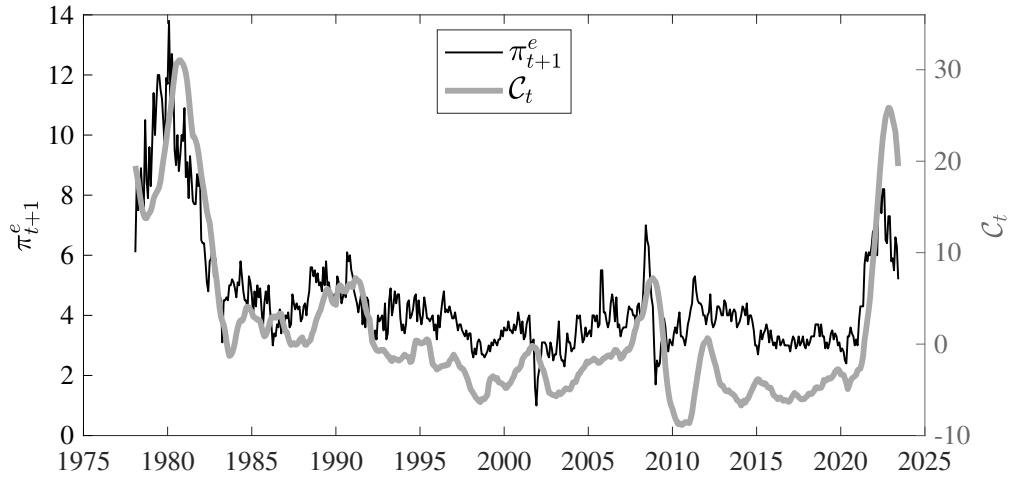
$$\mathcal{C}_t = \rho_C \mathcal{C}_{t-1} + v_t, \quad (16)$$

where i stands for different categories and $\pi_{i,t}$ is the monthly year-to-year inflation of each

¹³See Stock and Watson [2011] for a thorough review.

category i . How the common component \mathcal{C}_t affects different $\pi_{i,t}$ may be different, and is captured by α_i . Data are monthly over the sample 1978m1-2023-m5 for seventeen categorical inflations and shorter for the remaining seven (see Table D.1). We estimate that model with Kalman Filter Smoothing using a Maximum Likelihood Estimator that accounts for the missing observations. Estimated parameters are ρ_C and $\{\alpha_i, \sigma_i^2\}_{i=1}^{24}$. Note that σ_v^2 is not separately identifiable from α_i 's, so that we normalize it to 1. The parameters estimates we obtained are presented in Table D.2 in Appendix D. The important element to note is that all the α_i s are positive, so that \mathcal{C}_t is indeed capturing a common component in inflation. In Figure 8, we plot expected inflation and this common component which reflects broad-based supply shocks. As can be seen, this measure of broad-based inflation tracks the expected inflation quite well. It is interesting to compare the (normalized) weights of each category in

Figure 8: Michigan Survey of Consumers Inflation Expectations π_{t+1}^e and the Common Component \mathcal{C}_t



Notes: this Figure plots inflation expectations (as measured by the Michigan Survey of Consumers) together with the common component \mathcal{C}_t . \mathcal{C}_t is obtained from the estimation of (15) and (16) on the 25 expenditure categories used by the BLS to construct the CPI.

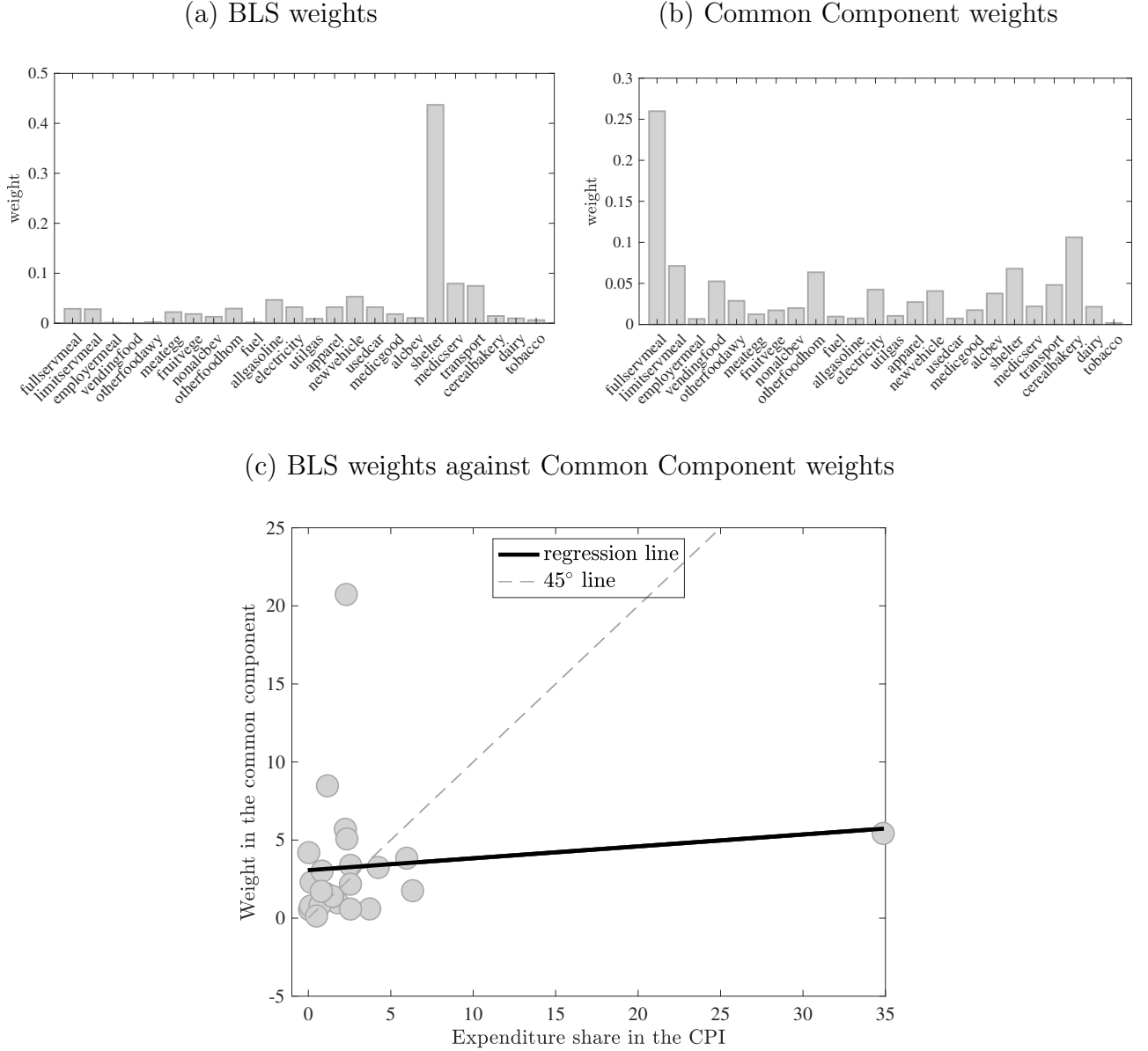
the common components to the expenditure share of that category, as used by the BLS to compute CPI. The weights in the common component are computed as the relative Kalman

gain associated to each sectoral inflation in updating the common component. Those two sets of weights are represented one against the other in Figure 9. If the grey circles were aligned along the 45 degree line, that would mean that the common component encodes the categorical inflations in the same way than way CPI does. As we can see, it is far from being the case. The correlation between these two set of weight is 0.12, with a 95% confidence set $[-0.29, 0.5]$. This shows that the common component encodes some information about broad-based shock, as the weights differ from the CPI ones. We then regress our measure of expected inflation (MSC) on the common component of the disaggregated inflation data and CPI (headline) inflation. The resulting regression is (with standard errors between parenthesis):

$$\pi_{t+1}^e = \underset{(0.10)}{1.82} + \underset{(0.04)}{0.17} \mathcal{C}_t + \underset{(0.02)}{0.56} \pi_t^{HL}, \quad (17)$$

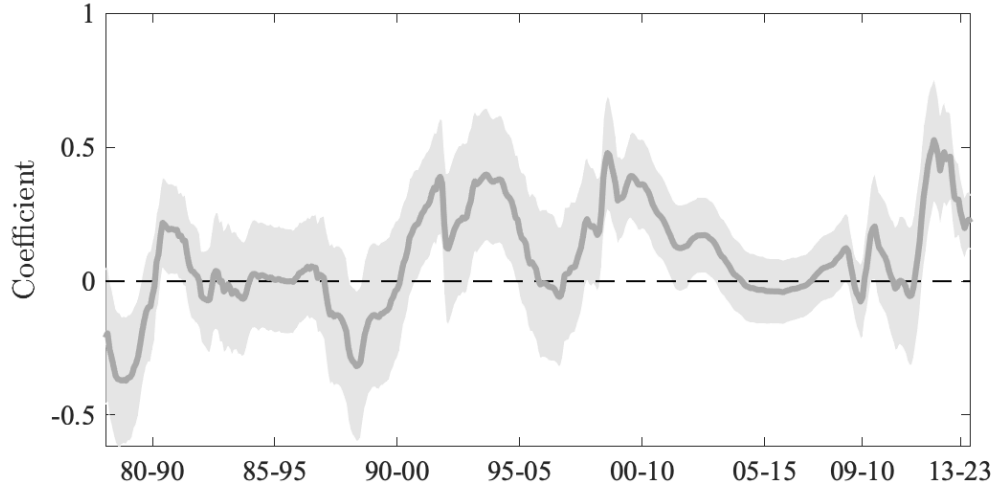
where the data are at a monthly frequency with 545 observations and an adjusted R-Squared of 0.837. The common component enters significantly in the regression on top of Headline CPI, which indicates that economic agents (here the households) may be using it to forecast inflation. Regressing the common component on Headline CPI shows that 30% of its variations are not explained by the CPI inflation. These 30% encode joint movements of the categorical inflations that are not strictly speaking inflation, and will capture broad-based supply shocks. Finally, we estimate Equation (17) on a rolling sample of 10 years, and report the sequence of coefficients on the common component in Figure 10. As one can see, there are some periods in which the information in the common component is particularly relevant for inflation expectations. This is the case in the recent period.

Figure 9: Comparing the Weights in the Common Component and in the CPI Basket



Notes: Panel (a) plots the weight of each category in the CPI basket, while panel (b) plots the (normalized) weights in the common component C_t . Panel (c) plots these two sets of weights one against the other, together with the regression line.

Figure 10: Rolling Estimation of the Coefficient on the Common Component \mathcal{C}_t in (17)



Notes: this Figure plots the coefficient on the common component in Equation (17), when this equation is estimated on a rolling sample of 10 years.

4 A Bounded Rational Model of Inflation and Inflation Expectations Dynamics with Broad-Based Supply Shocks

In this section our aim is to take up the following challenge. We want to combine our baseline Phillips curve with a particular bounded rationality theory of inflation expectations to see whether it can match the bi-variate VAR for inflation and expected inflation. Our theory of expectations will build on the idea that agents may form expectations by extracting a common component from disaggregated data. However, agent's perception of the data generating process will be slightly simpler than the actual data generating process. In particular, we will assume that they don't conceptualize the data generating in its structural form. Instead they view persistent inflation as driven by a common component that they need to infer from the data.

4.1 The Model

The model has two main departures from the Full Information Rational Expectations case. First, agents observe only a subset of the sectoral inflations when forming their inflation expectations. This is the source of imperfect information. Second, they believe that sectoral inflations depend on a common persistent components and a transitory sector-specific component, but fail to fully understand the relations between these two components. The later is the source of bounded rationality.

Imperfect Information: The model directly adopts the baseline aggregate Phillips Curve discussed in the previous sections:

$$\pi_t = \beta \pi_{t+1}^e + \gamma_g \text{gap}_t + e_t. \quad (18)$$

The headline aggregate inflation π_t can be thought of as a weighted average of disaggregate inflations of goods and services from different categories. Consider that each of these series follows a disaggregate Phillips Curve similar to the aggregate one:¹⁴

$$\pi_{j,t} = \beta \pi_{t+1}^e + \gamma_g \text{gap}_t + e_{j,t}, \quad (19)$$

where j stands for a specific category and $e_{j,t}$ is the i.i.d category specific shock with distribution $N(0, \sigma_j^2)$. We postpone the description of the expectations formation process to after we have introduced the agents perceived laws of motion. As in section 2.2, gap_t follows an

¹⁴Note that we could allow for different responses of disaggregate inflations to expectations (β_j) and labor market conditions ($\gamma_{g,j}$). This would give some extra degrees of freedom to the model in our quantitative analysis later. At this stage, we abstract for this extra layer of heterogeneity between sectors and assume $\beta_j = \beta$ and $\gamma_{g,j} = \gamma_g$ for all j .

AR(1) process:

$$\text{gap}_t = \rho \text{gap}_{t-1} + v_t. \quad (20)$$

Assuming there are N categories in the CPI, headline inflation is the weighted average of the disaggregate series:

$$\pi_t = \sum_{j=1}^N \gamma_j \pi_{j,t}, \quad (21)$$

where the γ_j s are the sectoral weights, and $\sum_j \gamma_j = 1$. We can therefore rewrite the Phillips curve as

$$\pi_t = \beta \pi_{t+1}^e + \gamma_g \text{gap}_t + \sum_{j=1}^N \gamma_j e_{j,t}. \quad (22)$$

Among all the categorical inflations, the agent will observe a subset of them. This subset of indexes is denoted $\mathcal{S} \subseteq \{1, 2, \dots, N\}$,¹⁵ and its cardinality is $m \leq N$. Agents will use these categorical inflations as signals to form expectations.

Perceived Law of Motion and Bounded Rationality: We have shown in section 2 that neither the FIRE nor the IIRE models could easily match the empirical VAR. Accordingly, in this section we build on our analysis in Section 3 by assuming that agents form their inflation expectations by extracting a common component from their observations of sectoral inflations. In particular, the agents are assumed to observe $\{\pi_{j,t}\}_{j \in \mathcal{S}}$ and π_t , and forms expectations using this information.¹⁶ The element of bounded rationality is that they perceive the law of motion for sectorial inflations as being composed of a common persistent component \tilde{z}_t and sector-specific transitory components $\tilde{e}_{j,t}$. Their estimate of the persistent

¹⁵We do not take a stand on what are the causes for the agents not being fully attentive to all the categorical inflations. Many frameworks such as rational inattention (Sims [2003]) or sparsity (Gabaix [2014]) can create such a partial information environment.

¹⁶For example, D’Acunto, Malmendier, Ospina, and Weber [2020] documents prevalent evidence that consumers use prices they are exposed to in their daily shopping experience to form inflation expectations.

component will be used to form expectations about future aggregate inflation.¹⁷

Formally, agents have the following perceived law of motion of the economy:

$$\pi_{j,t} = \tilde{z}_t + \tilde{e}_{j,t}, \quad (23)$$

$$\pi_t = \tilde{z}_t + \sum_{j \in \mathcal{S}} \gamma_j \tilde{e}_{j,t} + \tilde{w}_t, \quad (24)$$

$$\tilde{z}_t = \tilde{\rho} \tilde{z}_{t-1} + \tilde{v}_t, \quad (25)$$

with $\tilde{e}_{j,t} \sim N(0, \tilde{\sigma}_j^2) \ \forall j$, $\tilde{v}_t \sim N(0, \tilde{\sigma}_v^2)$ and $\tilde{w}_t \sim N(0, \tilde{\sigma}_w^2)$. The parameters $\tilde{\rho}$, $\tilde{\sigma}_j^2$ and $\tilde{\sigma}_v^2$ denote the perceived persistence and variances for the law of motion. The agents endowed with this PLM will be assumed to be quite sophisticated. They will use the observed inflations series to infer state \tilde{z}_t – the perceived common component affecting inflation – and use these beliefs about \tilde{z}_t to form expectations of aggregate inflation. However, they are bounded rational in the sense that they treat the state-space system (23)-(25) as a standard signal-extraction problem where the noise from the signal is treated as uncorrelated with the state. However, because the disaggregate prices are common to all the agents, when the aggregate expectation responds to the signals, these signals will affect the state variable, which will feedback to the signals via the Phillips curve (22). As agents fail to recognise that the hidden state may be correlated with the noise in their signals, this has the potential to create a positive feedback loop from noise to inflation. This mis-interpretation of the data is the key to creating an amplification mechanism in our expectations formation model.¹⁸

¹⁷Such a signal-extraction formulation is widely used in the literature about inflation expectations. Examples are among others Coibion and Gorodnichenko [2015] and Bordalo, Gennaioli, Ma, and Shleifer [2020].

¹⁸Another feature of the above observational equation is that the signals are weighted by their precisions that do not depend on their actual weights γ_j in the aggregate price index. In other words, a category that has a very low weight in the aggregate price index can be more informative as a category that has significant weight but with a lower signal-to-noise ratio. This feature is in line with our finding in section 3 that the weights of disaggregate price indices in the CPI basket are not highly correlated with the estimated weights from Kalman Filter Smoothing.

Solving the Model: Because we do not observe what (subset of) disaggregate inflation series the agents use to form expectations, it is mathematically equivalent to replace the set of signals in (23) by one aggregate signal.¹⁹ Using equation (23) together with the aggregate inflation (24), we obtain the observational equations from the agent's perspective:

$$S_t \equiv \begin{pmatrix} s_t \\ \pi_t \end{pmatrix} = \iota \tilde{z}_t + \begin{pmatrix} \tilde{\epsilon}_t \\ \gamma \tilde{\epsilon}_t + \tilde{w}_t \end{pmatrix}, \quad (26)$$

whereas the actual signals are generated by:

$$S_t \equiv \begin{pmatrix} s_t \\ \pi_t \end{pmatrix} = \iota \beta \mathbb{E} \pi_{t+1} + \iota \gamma_g z_t + \begin{pmatrix} \epsilon_t \\ \gamma \epsilon_t + w_t \end{pmatrix}, \quad (27)$$

where s_t is an average signal from m independent categorical signals, with precision $1/\tilde{\sigma}_\epsilon^2$ and ι is a column vector of ones with two lines.

The agents face the state-space model described by (25)-(26). They solve a signal-extraction problem to form belief $z_{t|t}$ and use it to form expectations about inflation. Because expectations formed in the current period will also affect current inflation and signals the agents see, we need to be specific about the timing of expectations formation and when expectations affect inflation. In the baseline version of the model, we will allow information extracted from the disaggregated data to be simultaneously determined with inflation, while we will assume that aggregate inflation (CPI) is released to households with a lag. This implies the any residual information in aggregate inflation does not feedback contemporaneously to itself.²⁰ Accordingly, we solve the model adopting the following steps, which is

¹⁹For independent signals $j = 1, \dots, m$ with different precisions $\frac{1}{\tilde{\sigma}_j^2}$. Observing m different signals is equivalent to observing a signal with precision: $\frac{1}{\tilde{\sigma}_\epsilon^2} = \sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2}$. See Appendix E for the derivation. For a similar reason, sectoral inflation equation (19) can also be written as an average price signal with weight $\frac{\frac{1}{\sigma_i^2}}{\sum_{j=1}^m \frac{1}{\sigma_j^2}}$. This gives the representation (27) with possibly $\sigma_\epsilon \neq \tilde{\sigma}_\epsilon$.

²⁰In Appendix I, we reestimate the model under the assumption that inflation expectations are reported *after* observing current inflation. Differences in the results are minor.

easiest to present as having each period t being composed of two sub-periods:²¹

Step 1: In the first sub-period of t , the gap_t realizes. Economic agents form expectations using the disaggregate signals s_t . We denote the expected inflation formed in this sub-period as $\pi_{t+1|t,0}$. This is the expectation π_{t+1}^e that enters in the Phillips curves (18) and (19). Consistent with the treatment in section 1 and section 2, we take these expectations as those observed in the Michigan Survey of Consumers.

Step 2: In the second sub-period of t , agents also observe the aggregate signal π_t . They use this extra information to update their belief about future inflation, denoted as $\pi_{t+1|t,1}$. This will be the prior on inflation that will be carried over to the next period $t + 1$.

Step 3: Both expectations $\pi_{t+1|t,0}$ and $\pi_{t+1|t,1}$ are formed from beliefs on the common component \tilde{z}_t . We denote these beliefs as $z_{t|t,0}$ and $z_{t|t,1}$. From equations (25) and (26), we see that the links between expected inflation and the common component are:

$$\pi_{t+1|t,k} = z_{t+1|t,k} = \tilde{\rho} z_{t|t,k} \quad \forall k = 1, 2. \quad (28)$$

Step 4: The beliefs on \tilde{z}_t are formed using Kalman filter according to the state-space representation of the economy (25) and (26) (the perceived law of motion). Denote the innovations in agents' perceived signal-generating process (26) as:

$$\begin{pmatrix} \tilde{\epsilon}_t \\ \gamma \tilde{\epsilon}_t + \tilde{w}_t \end{pmatrix} \sim N(\mathbf{0}, \tilde{R}), \quad \tilde{R} \equiv \begin{pmatrix} \tilde{\sigma}_\epsilon^2 & \gamma \tilde{\sigma}_\epsilon^2 \\ \gamma \tilde{\sigma}_\epsilon^2 & \gamma^2 \tilde{\sigma}_\epsilon^2 + \tilde{\sigma}_w^2 \end{pmatrix}. \quad (29)$$

The agents form weights on signals, according to the Kalman filter, taking into consideration that s_t and π_t are correlated. Denoting ι_n a $n \times 1$ vector of ones, the stationary Kalman

²¹An alternative formulation would be thinking of the belief on z_t is formed conditional on information set $\mathcal{I}_t = \{\pi_{t-1}, s_t\}$ in each period t . The two will give equivalent representation of the model.

filter is given by:

$$\hat{K} = \sigma^2 \iota_2' (\iota_2 \sigma^2 \iota_2' + \tilde{R})^{-1}, \quad (30)$$

$$\sigma^2 = \tilde{\rho}^2 (\sigma^2 - \hat{K} \iota_2 \sigma^2) + \tilde{\sigma}_v^2, \quad (31)$$

where σ^2 is the stationary posterior variance of belief on \tilde{z}_t . Now note the \hat{K} is an 1×2 vector, with first element being the weight on the categorical signal, and the second weight on aggregate inflation π_t : $\hat{K} \equiv (K \quad k)$.

Step 5: In the first sub-period of t , agents observe s_t and form a nowcast of \tilde{z}_t :

$$z_{t|t,0} = (1 - K)z_{t|t-1,1} + K s_t. \quad (32)$$

Step 6: In the second sub-period of t , agents observe π_t and update belief on z_t :

$$z_{t|t,1} = ((1 - \hat{K} \iota_2)z_{t|t-1,1} + \hat{K} S_t). \quad (33)$$

Following these six steps, the solution of the model will be given by:

$$\pi^{e_{t+1}} = \pi_{t+1|t,0} = \frac{\tilde{\rho}(1 - K)}{1 - K\beta\tilde{\rho}} z_{t|t-1,1} + \frac{\tilde{\rho}K\gamma_g}{1 - K\beta\tilde{\rho}} \text{gap}_t + \frac{\tilde{\rho}K}{1 - K\beta\tilde{\rho}} \epsilon_t, \quad (34)$$

$$\pi_t = \beta\pi_{t+1|t,0} + \gamma_g \text{gap}_t + \gamma\epsilon_t + w_t, \quad (35)$$

$$\begin{aligned} z_{t+1|t,1} &= \tilde{\rho} \left(1 - (K + k) \frac{1 - \beta\tilde{\rho}}{1 - K\beta\tilde{\rho}} \right) z_{t|t-1,1} + \tilde{\rho} \frac{(K + k)\gamma_g}{1 - K\beta\tilde{\rho}} \text{gap}_t \\ &\quad + \tilde{\rho} \left(\frac{K + \tilde{\rho}Kk\beta}{1 - K\beta\tilde{\rho}} + \gamma k \right) \epsilon_t + k\tilde{\rho}w_t. \end{aligned} \quad (36)$$

$$\text{gap}_t = \rho \text{gap}_{t-1} + v_t \quad (37)$$

Note that signals s_t and π_t are realized according to (27), so expectations $\pi_{t+1}^e = \pi_{t+1|t,0}$ and signal s_t will be simultaneously determined.

The above expectations formation process has the potential to create persistent bouts of

inflation in a quasi self-confirming way. When an adverse (positive) broad-base supply shock ϵ_t hits the economy, economic agents observe an inflation hike in many disaggregate inflation series. This causes them to update their view of the common component \tilde{z}_t . If agents believe that their signal is quite informative, this will increase their expected inflation. Because the disaggregated inflations are public signals, the average expectation will increase which drives up the disaggregate prices they observe. This feedback channel can thereby amplify the response of expected inflation to broad-base shocks on impact.²² Moreover, in the second sub-period, agents observe π_t which is also pushed up by expectations. This further supports their belief of positive realization in the common component. As a result, economic agents enter next period with a prior of high inflation. This then creates very persistent responses of both actual and expected inflation to broad-base price shocks. Furthermore, after the initial period of the shock, actual inflation will react almost one-to-one with expectations, which corresponds to the observation we made in the empirical VAR.

4.2 Quantitative Analysis

In this exercise, we estimate the model's parameters in order to match the impulse responses of the (π_{t+1}^e, π_t) VAR while simultaneously imposing that the parameters governing the agents' perceived laws of motion be supported by the data. In particular, we want the parameters of the perceived law of motion to be consistent with that implied by the observed variance, first auto-covariance and second auto-covariance of inflation. In that sense, the model is disciplined in two ways: first it aims at reproducing the VAR impulse responses and second it ensures that there are not large discrepancies between what agents believe and what they observe.

²²Note that this amplification comes from the fact that the agents fail to understand expectations $\pi_{t+1|t,0}$ is part of the perceived common component \tilde{z}_t . More precisely, they fail to understand the hidden state is correlated with the noise of their signal. This arises because the signals the agents observe are endogenous.

We fix the values of β , ρ , γ_g , and σ_v at the values discussed in Section 1 when we use the baseline Phillips curve. We estimate the other parameters $\tilde{\rho}$, $\tilde{\sigma}_v$, γ , σ_w , σ_ϵ , $\tilde{\sigma}_w$ and $\tilde{\sigma}_\epsilon$ by minimum distance as in Gouriéroux, Monfort, and Renault [1993]. Denote the vector of parameters to be estimated as Θ , the estimates are given by:

$$\hat{\Theta} = \underset{\Theta}{\operatorname{argmin}} \left(\begin{array}{c} \widehat{\mathcal{M}} - \frac{1}{N} \sum_n^N \widehat{\mathcal{M}}_T^n(\Theta) \\ \mathcal{M}_{\text{beliefs}}(\Theta) - \mathcal{M}_{\text{model}}(\Theta) \end{array} \right)' W \left(\begin{array}{c} \widehat{\mathcal{M}} - \frac{1}{N} \sum_n^N \widehat{\mathcal{M}}_T^n(\Theta) \\ \mathcal{M}_{\text{beliefs}}(\Theta) - \mathcal{M}_{\text{model}}(\Theta) \end{array} \right). \quad (38)$$

In this minimization problem, we minimize the weighted norm of a vector composed of two sets of elements. The first set corresponds to fitting the data. $\widehat{\mathcal{M}}$ is the vector of moments to match –i.e. the 20 periods impulse response functions estimated from the bivariate VAR in section 2.1. $\widehat{IRF}_T^n(\Theta)$ is computed as the average estimated IRF from N simulated random sample of the model with parameter Θ . T is the length for the simulated random sample, which is set to be the same as our actual sample 1969-2023. The second set corresponds to the requirement for consistency of beliefs. $\mathcal{M}_{\text{beliefs}}(\Theta)$ gathers the variance, first and second order auto-covariances of inflation as computed from the perceived law of motion, while $\mathcal{M}_{\text{model}}(\Theta)$ gathers the same moments as generated by the model. In our baseline estimation, we use the identity matrix as the weighting matrix W . The parameters obtained from the New Keynesian Phillips Curve and the minimum distance estimation are reported in Table 4:

Figure 11 shows how the estimated model matches well the empirical impulse responses we obtained in section 2.²³ In the signal-extraction problem, economic agents form expectations using the perceived law of motion and their subjective parameters. The large $\tilde{\sigma}_v$

²³In Appendix G, we also include the IRFs using actual and simulated data from our baseline model with an alternative orders of variables for the Choleski decomposition. Because the Choleski VAR is here used as a way to summarize the joint dynamics of inflation and inflation expectations, we would expect the data generated by a correct model to give similar results as the actual data when summarized with a different ordering of the variables. Appendix G shows that this is indeed the case.

Table 4: Parameters

Phillips Curve Parameters			
β	0.99	ρ	0.89
σ_v	3.02	γ_g	0.0138
Estimated Parameters (Minimum Distance)			
$\tilde{\sigma}_v$	0.70	σ_ϵ	3.82
$\tilde{\rho}$	0.96	$\tilde{\sigma}_w$	2.13
γ	0.26	$\tilde{\sigma}_\epsilon$	2.34
σ_w	2.01		

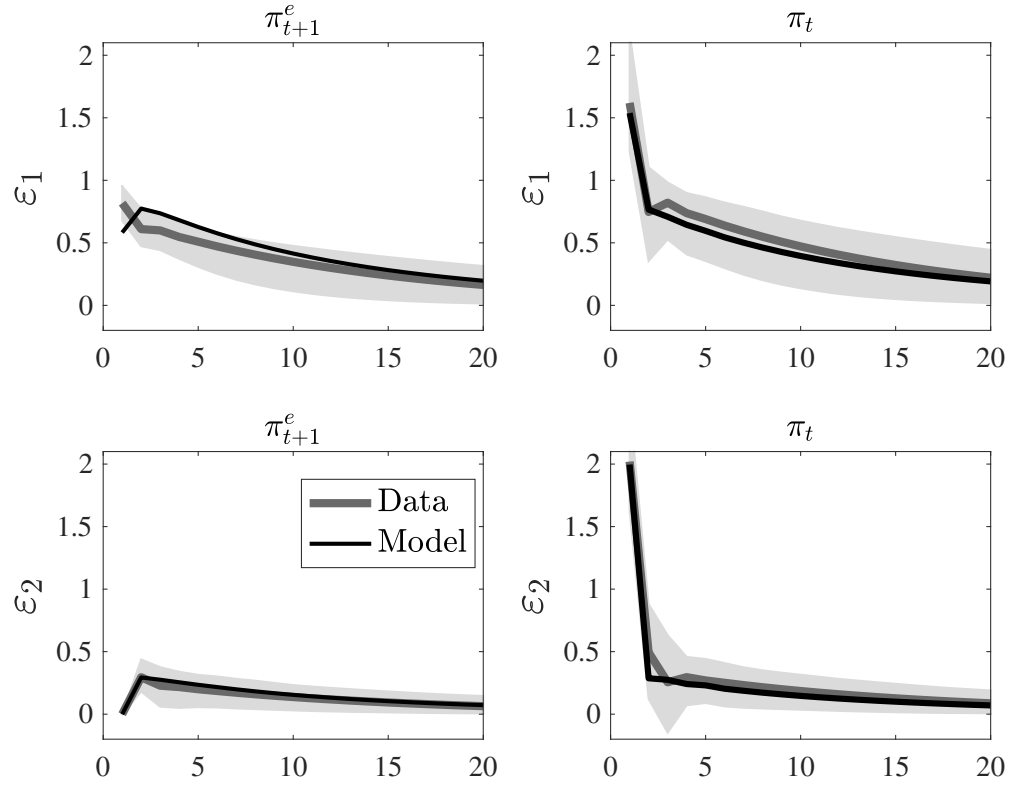
Notes: The Phillips curve estimates use the baseline estimation with Hazell, Herreño, Nakamura, and Steinsson's [2022] estimate of γ_g . σ_w is implied by the variance of the residual e from the Phillips curve, and from σ_ϵ and γ : $\sigma_w = \sqrt{\sigma_\epsilon^2 - \gamma^2 \sigma_\epsilon^2}$.

and relatively smaller $\tilde{\sigma}_\epsilon$ make the agents believe the disaggregated inflation series are very informative about the common shock. This is how the current model with broad-base shocks can trigger larger movements of expectations than those under FIRE or IIRE. Moreover, the adjustments in expectations are persistent because agents fail to realize the expectations may be driving the process.

From the perspective of the model, the VAR shock ε_1 corresponds to combination of demand (v) and broad-base (ϵ) shocks. Because the actual variation in v_t is quite small, the shape of the IRF mainly reflects the impact of the broad-base shock. The other VAR shock, ε_2 , corresponds to the residual supply shocks w that is not used to form the within period expectations. Since agents observe aggregate inflation only after having formed the expectations that they report, this shock affects measured inflation on impact but reported inflation expectations only with a lag.

Figure 12 shows the implied model impulse responses to the three structural shocks. We see why the broad-based supply shock ϵ_t plays a dominant role in shaping the response to ε_1 in Figure 11. This is because the variation in inflation created by v_t is minimal due to the

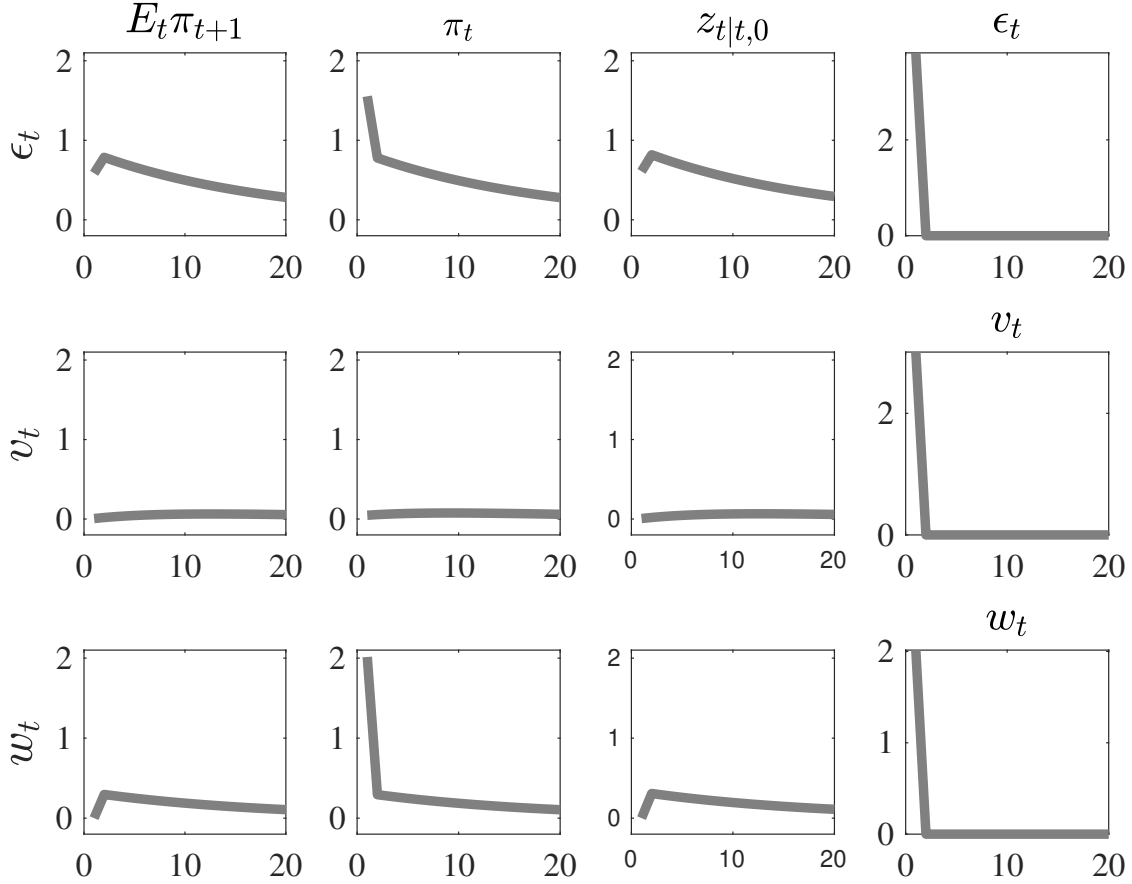
Figure 11: Impulse Responses from Data and Estimated Model



Notes: The thick gray line is the IRF from the 2-VAR with actual data; the black line is the average IRF from simulated data across 200 random samples. The estimated VAR is a VAR(2) with Cholesky decomposition, ordering π_{t+1}^e first. The shaded area represents 95% confidence interval.

flatness of Phillips Curve. Meanwhile, the response to ε_2 corresponds to a residual supply shock w_t . Moreover, the broad-base shock ϵ_t itself only increases inflation on impact by a factor of γ . But the broad-based shock triggers a large increase of inflation expectations, which push up aggregate inflation. In comparison, the residual supply shock w_t increases aggregate inflation substantially on impact, but does not affect expectations right away and therefore does not cause the quasi-self-confirming amplification. As a result, the response of aggregate inflation is more transitory.

Figure 12: Impulse Responses to Structural Shocks in the Estimated Model



Notes: These impulse responses to structural shocks are computed from the model solution (34) to (37) with estimated parameters.

It is worth noting that the three main parameters of the perceived law of motion, $\tilde{\rho}$, $\tilde{\sigma}_v$,

and $\tilde{\sigma}_\epsilon$, will be directly reflected in the process for inflation. For example, given the perceived law of motion, $\frac{cov(\pi_{t,h}, \pi_{t-2,h})}{cov(\pi_{t,h}, \pi_{t-1,h})}$ gives an estimate of $\tilde{\rho}$. Similarly, $\tilde{\sigma}_v$ could then be inferred using $\tilde{\rho}$ and $cov(\pi_{t,h}, \pi_{t-1,h})$. Accordingly, if these moments for actual inflation were far from those implied by the agent’s subjective model, agents should realize the discrepancy between their subjective model and reality, and will adjust their beliefs. Table 5 therefore reports the first three moments of the inflation auto-covariance function generated by the model and those implied by the beliefs of agents. As we can see, they are very close implying that agents would have little reason to change their beliefs.

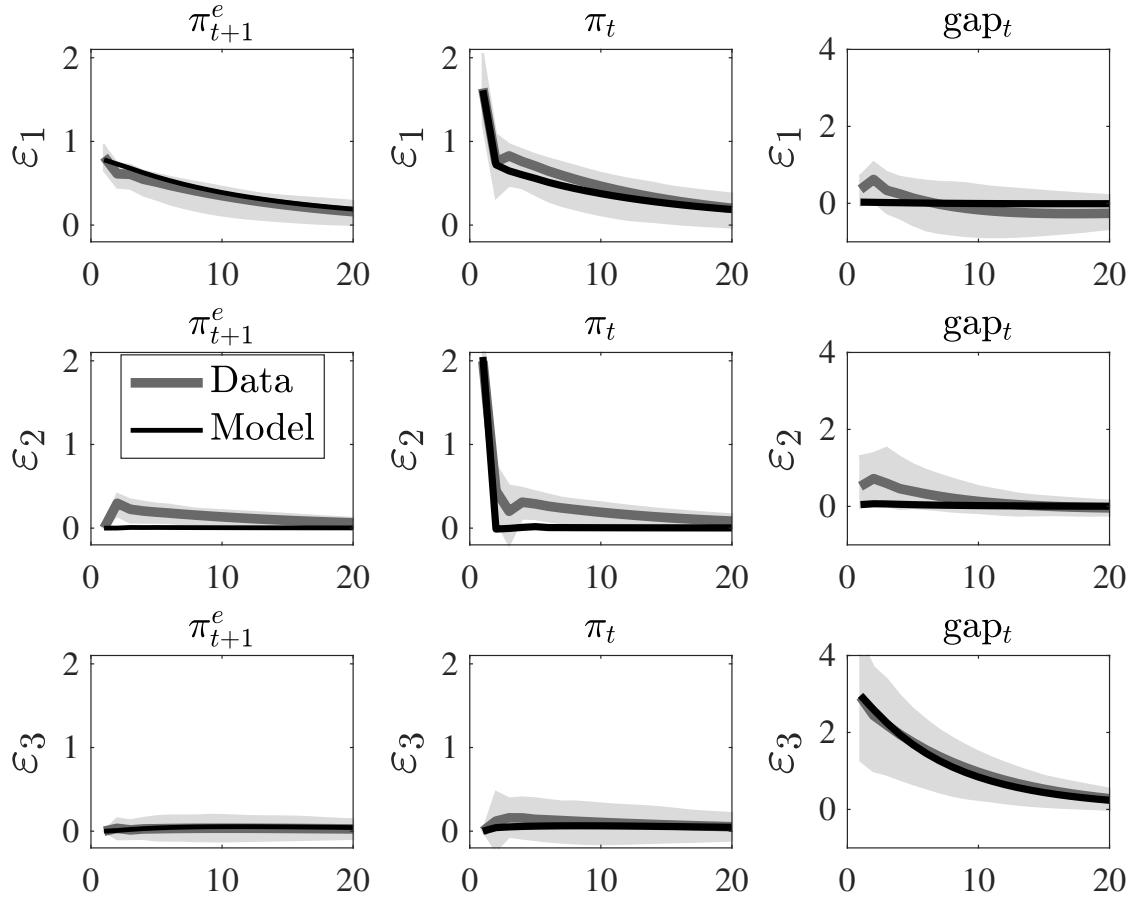
Table 5: Variance and Auto-covariance of Inflation

	Data	PLM	Model
$var(\pi_{t,h})$	12.52	11.33	11.33
$cov(\pi_{t,h}, \pi_{t-1,h})$	7.70	6.17	6.28
$cov(\pi_{t,h}, \pi_{t-2,h})$	6.99	5.92	5.82

Notes: PLM stands for “Perceived Law of Motion”. The PLM and “Model” moments are the average moment across 200 random samples. In the estimations, we penalize distance between data and model 2-VAR responses as well as distance between these three moments in the PLM and in the model.

Finally, in our estimation procedure we did not directly use the gap as one observable and did not try to match the empirical tri-variate VAR $(\pi_t, \pi_{t+1}^e, \text{gap}_t)$ that we estimated in Section 2. In figure 13 we compare these estimated impulse responses with those obtained from the same tri-variate VAR estimated with 200 random samples of our estimated model. The match is indeed very good. Note that the model responses of gap_t to the first two shocks ε_1 and ε_2 are zero by construction as gap_t is modelled as an exogenous shock.

Figure 13: Trivariate VAR IRF from data and model simulation



Notes: thick gray line is IRF from 3-VAR with actual data; black line is average IRF from simulated data across 200 random samples. The shaded area represents the 95% confidence interval.

4.3 Filtered Shocks and Counterfactuals

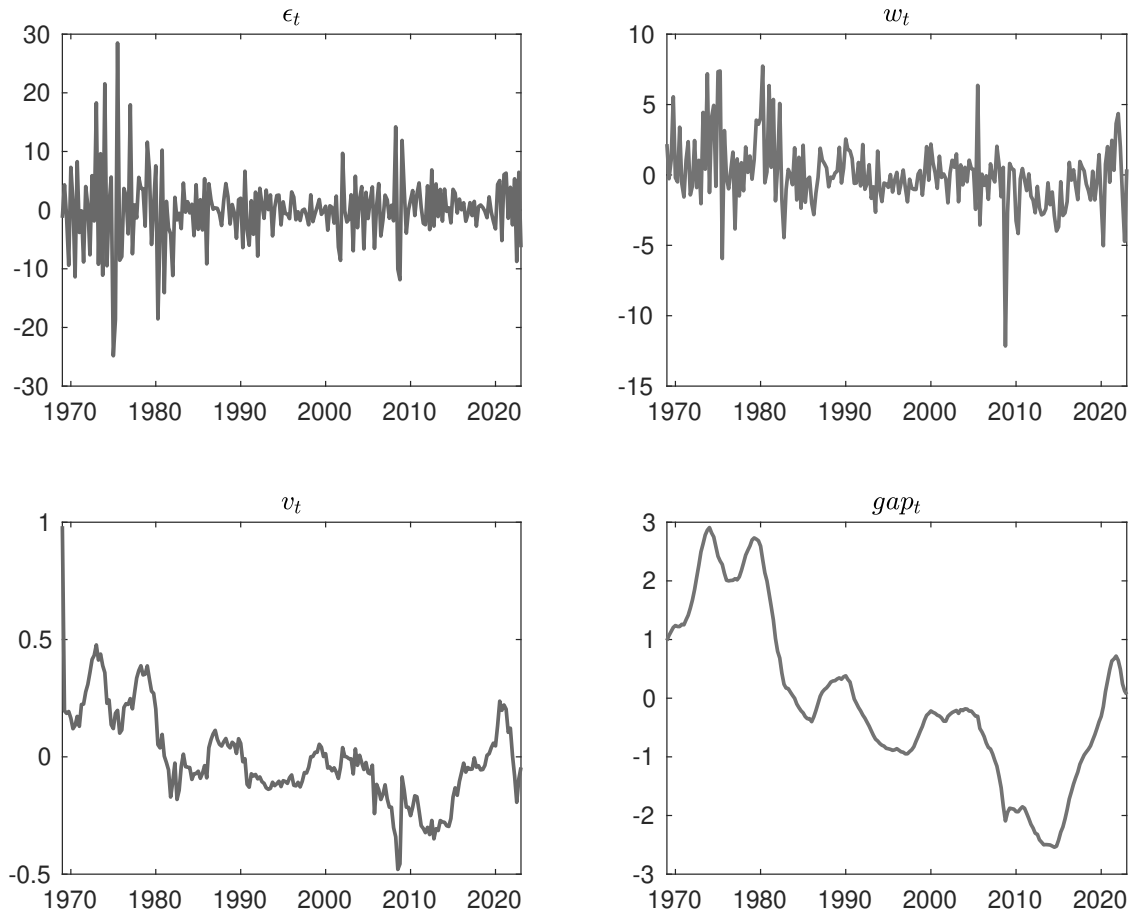
Using the state-space representation of our estimated model, we can use Kalman filter smoothing to form projections of the latent states and the shocks implied by the model. The latent variable $z_{t+1|t,1}$ corresponds to the common component \mathcal{C}_t the agents perceived using the disaggregate signals. The three shocks to the economy are the demand shock v_t , the broad-based supply shock ϵ_t and the residual supply shock w_t . Shocks are displayed in Figure 14. Because the Phillips Curve is quite flat, the data on inflation and expected inflation do not depend much on the gap, and therefore contain little information on it. As a result, the recovered series on v_t is imprecise but mainly irrelevant. This can be partly seen from the fact that the variance of recovered v_t is merely 0.04 whereas the true variance of v_t should be 3.01.

On the contrary, the recovered series for ϵ_t and w_t are quite trustworthy because the observables contain lots of information on these two shocks. We illustrate these points using simulated data in Appendix H.

In our model, the system of inflation and expected inflation is driven by a latent variable $z_{t+1|t,1}$, which is the common component the agents perceived using the disaggregate signals. This variable can be obtained using the Kalman Filter Smoothing approach mentioned above. In figure 15, we scatter the backed-out $z_{t+1|t,1}$ against the common component \mathcal{C}_t we extracted using the actual CPI disaggregated inflation series in Section 3. The correlation between these two series is 0.85, suggesting that the perceived common component in our model contains similar information to the common component directly extracted from disaggregated data, although only aggregate data are used in the model.

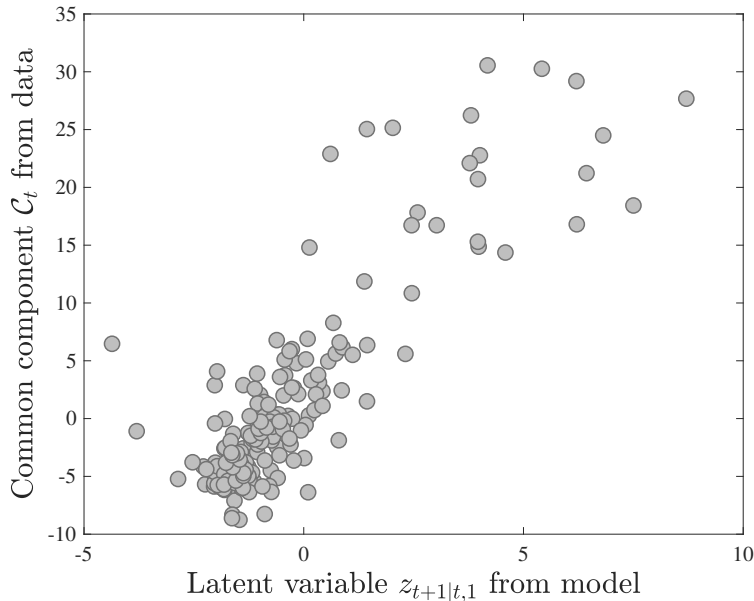
We then perform a counterfactual simulation starting in 2020. We first simulate the economy with the residual supply shock w_t only, and mute both the broad-base shock ϵ_t and

Figure 14: Filtered shocks from the estimated model



Notes: The demand shock v_t , the broad-based supply shock ϵ_t and the residual supply shock w_t are recovered by Kalman filter smoothing using the estimated model.

Figure 15: Filtered shocks from the estimated model

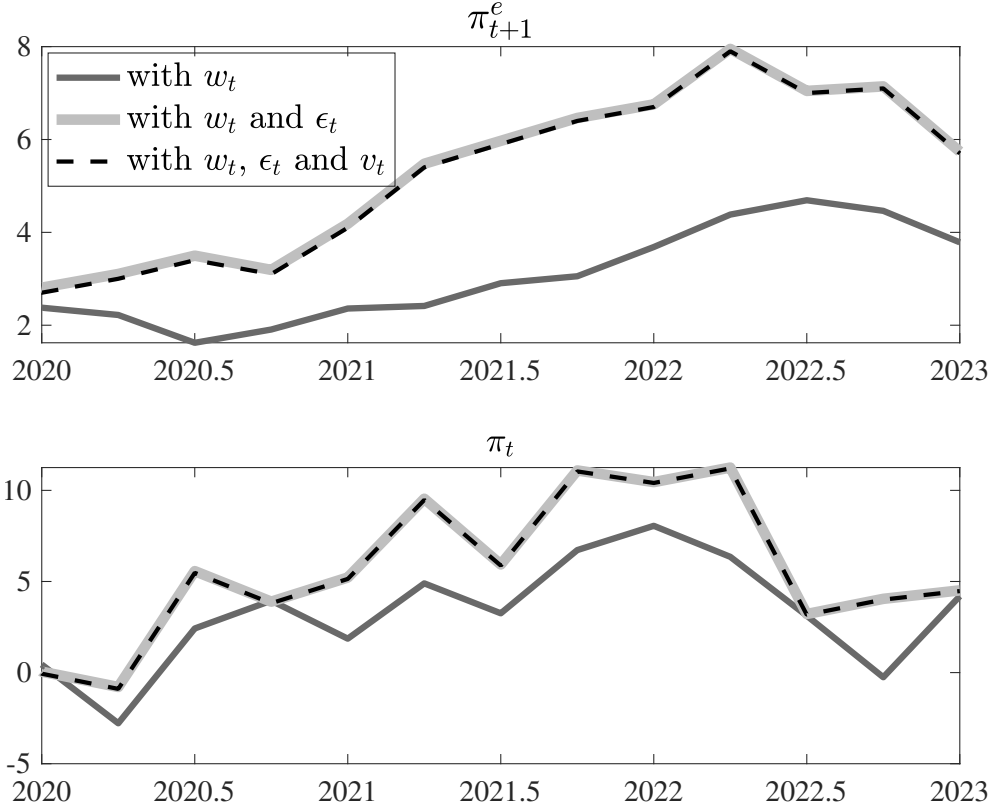


Notes: On this Figure, the latent variable $z_{t+1|t,1}$ is the “common component” the agents perceived using the disaggregate signals, as obtained from Kalman filter smoothing using the estimated model. It is plotted against the common component C_t extracted from the CPI components in Section 3.

the demand shock v_t . Then we add in the broad-base shock ϵ_t , and finally we add the shock to gap v_t . Note that by construction, the model with the three shocks perfectly matches the data.

Figure 16 shows the counterfactual paths for expected inflation and headline inflation. The solid black lines correspond to the counterfactual paths when the shocks to the gap v_t are muted. The difference with the data is negligible. This echoes our findings in the previous sections: because the Phillips curve is flat, labor market tightness plays virtually no role in shaping inflation dynamics. The solid gray lines are the counterfactual paths when the broad-based shocks are missing. The expected inflation run-up after 2020 cannot be accounted for by the absence of this shock. This translates into inflation being also below its observed path during the whole period.

Figure 16: Estimated Model Counterfactual Simulations



Notes: This Figure displays counterfactual simulations of the estimated model, for both expected inflation and actual inflation.

5 Conclusion

In this paper, we examine the causes for the high and persistent inflation after the COVID-19 pandemic through the lens of the Phillips Curve. We show that for the U.S. economy, the Phillips Curve has been very flat and can still predict the recent inflation dynamics very well. The supply shocks appear to be transitory but inflation expectations from households and firms appear to be very persistent. Empirical evidence from our Philips Curve estimation and empirical VAR analysis show that inflation follows inflation expectations closely and the joint dynamics between the two cannot be explained by models with rational expectations

or simple information frictions. The major challenge for understanding inflation dynamics is then to explain why expected inflations are volatile and persistent when labor market tightness has limited impact and supply shocks are transitory.

We propose a model where the agents in the economy are sophisticated but bounded rational in understanding the inflation dynamics. They use many disaggregate price series to infer a common shock that drives aggregate inflation, but don't realize their common expectations push up the disaggregate inflations they see. As a result, when they see price increases in many items due to broad-based supply shocks, they confuse them with the common shock and increase their expected inflation. Expectations push up actual inflation that confirms their beliefs of a common shock. This starts a self-confirming loop that amplifies the impact of transitory supply shock and creates persistent response of both expected and actual inflation. We show that our model is consistent with the estimated flat Phillips Curve and matches the impulse responses from the empirical VAR very well. Through the lens of our model, the immediate surge of inflation after the pandemic is driven by supply chain disruptions, but the persistent increase in the expected inflation from households and firms plays a dominant role in creating the stagnant high inflation after 2021. From our estimated model, in absence of the broad-base supply shock, the headline inflation would have been 2.25% lower on average after 2020.

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Appendix to The Dominant Role of Expectations and Broad-Based Supply Shocks in Driving Inflation

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version 1.8

A Relation to Benigno and Eggertsson [2023]

In this appendix, we repeat the analysis of section 1 when we use $\log(V/U)$ as a measure of labour market tightness and either the SPF or the MSC as a measure of inflation expectations. We first stay with our baseline sample 1969-2007 where we use the expected GDP Deflator from the Professional Forecaster as our measure of expected inflation. We then move to the sample used in Benigno and Eggertsson [2023] (but stopping in 2007 to perform our out-of-sample analysis).

A.1 Sample 1969-2007

Table A.1: Estimated Phillips Curves using $\log(V/U)$ as a Measure of the Gap, 1969-2007

	Using SPF	Using MSC
β	1.26* (0.047)	0.99* (0.048)
γ_g	0.28* (0.062)	0.10 (0.069)
γ_r	-0.42* (0.098)	0.23* (0.040)
Observations	144	144
J Test	10.538	10.700
(jp)	(0.987)	(0.986)
Weak ID Test	29.469	10.201

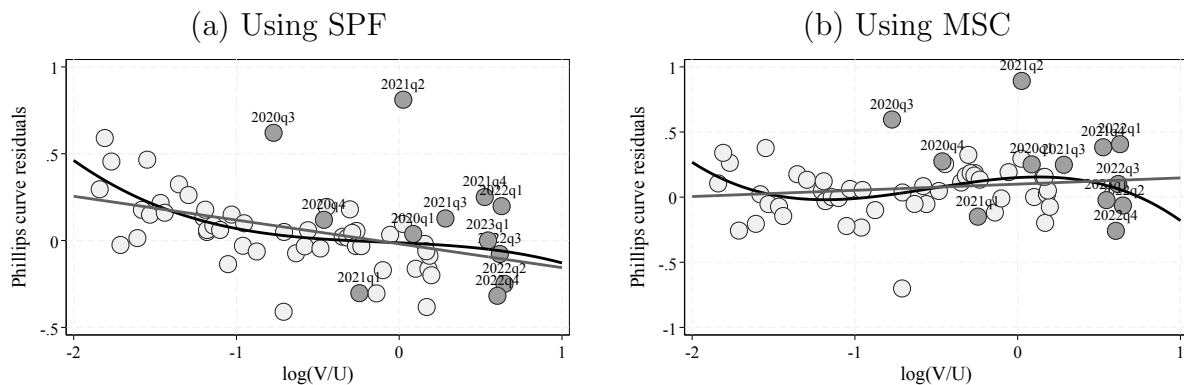
Notes: this table reports estimates of the augmented Phillips curve (2). All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. The measure of inflation is Core CPI and the gap is measured with $\log(V/U)$. All regressors are instrumented using six lags of Romer and Romer's [2004] shocks (as extended by Wieland and Yang [2020]) and their squares as instruments. A * indicates significance at 5%. Sample is 1969Q1-2007Q4.

Table A.2: Projection of the Philips Curve Residuals ϵ on the Gap $\log(V/U)$, 2008-2023

	Using SPF		Using MSC	
	linear	nonlinear	linear	nonlinear
$\log(V/U)$	-0.14* (0.038)	-0.04 (0.076)	0.05 (0.042)	0.07 (0.083)
$\log(V/U)^2$		-0.02 (0.149)		-0.25 (0.162)
$\log(V/U)^3$		-0.06 (0.082)		-0.15* (0.089)
N	60	60	60	60
σ_ϵ 60-07		.302		.291
σ_ϵ 08-23		.232		.232

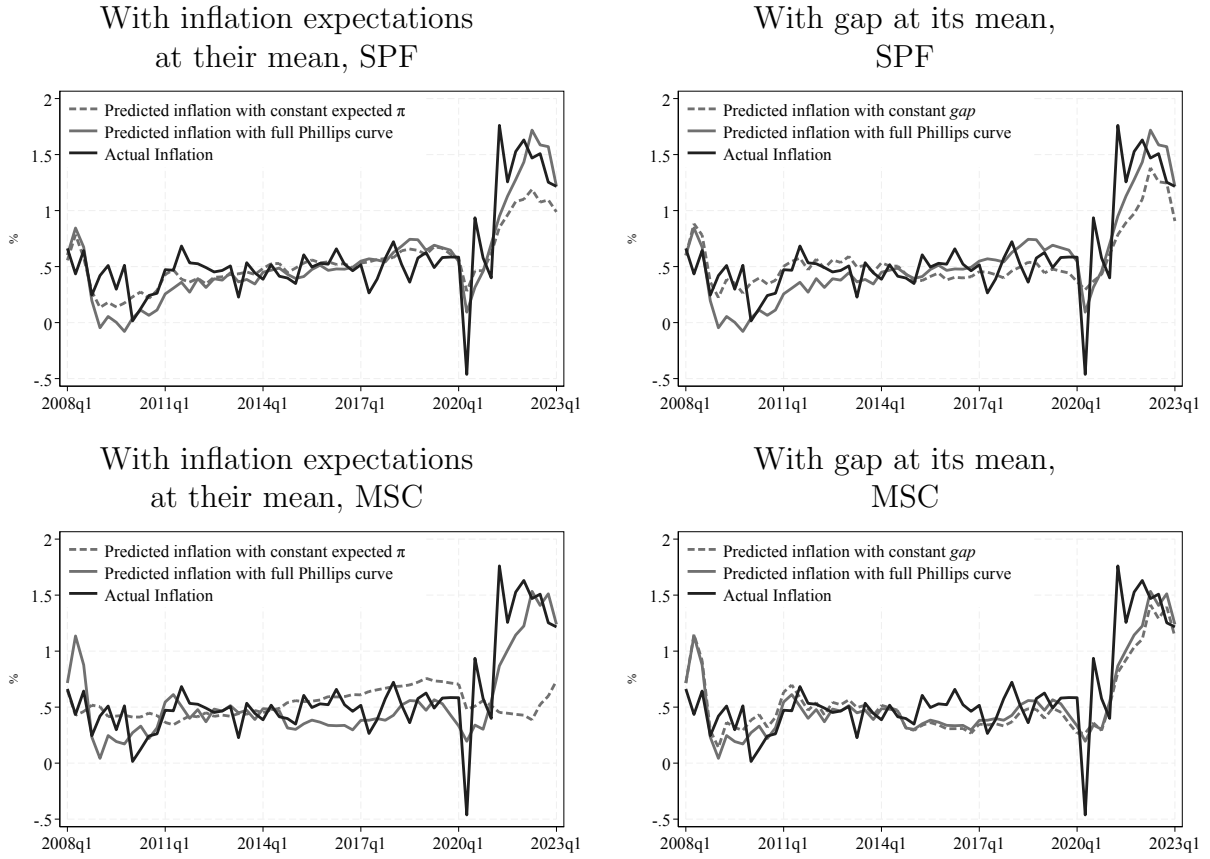
Notes: the Phillips curve residuals are obtained from the augmented Phillips curve (2) estimated over the sample 1969Q1-2007Q4, using $\log(V/U)$ as a measure of the gap and either SPF or MSC as a measure of expectations. The standard errors of estimated coefficients are between parentheses.

Figure A.1: Out-of-Sample Residuals from Phillips Curve, using $\log(V/U)$ as a Measure of the Gap



Notes: Panels (a) and (b) of this figure plots the out-of-sample residuals of the estimated Phillips curve (2) against $\log(V/U)$ as the measure of labor market tightness, for two measures of inflation expectations. The gray lines show the estimated linear or cubic relation between residuals and labor market tightness (see Table A.2 for the estimated coefficients). Light dots correspond to pre-2020 observations and dark ones to post-2020. We exclude 2020q2 from this graph.

Figure A.2: Counterfactual Simulations from the Estimated Phillips Curve



Notes: These counterfactual simulations are done using the estimated Phillips curve (2), using $\log(V/U)$ as the measure of labor market tightness.

A.2 Sample 1981-2007

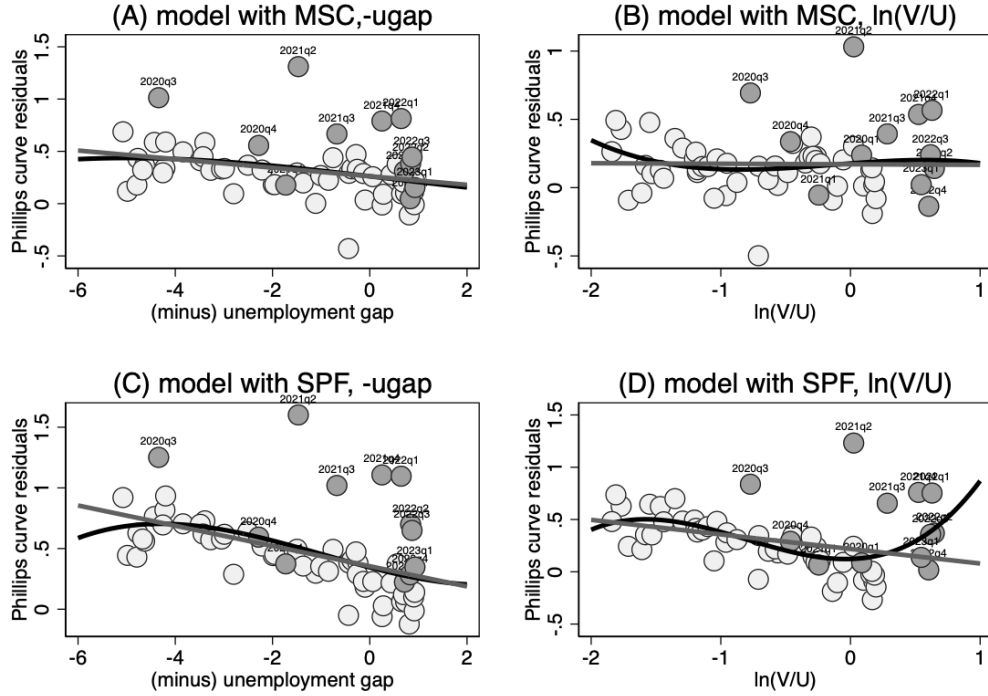
We then move to use the expected CPI series from the Survey of Professional Forecasters. Because that series starts only after 1981 quarter 3, we use the sample 1981q3-2007q4 for our analysis. In Table A.3 we present the estimation results of our Phillips Curve. For comparison, we also include the estimates using *minus* unemployment gap as measure of labor market tightness in this shorter sample.

Table A.3: Estimated Phillips Curves using $\log(V/U)$ as a Measure of the Gap, 1981-2007

Expectations:	Using MSC		Using SPF	
Labor Market Tightness:	(1) -ugap	(2) $\ln V/U$	(3) -ugap	(4) $\ln V/U$
β	0.90*	0.90*	0.97*	0.95*
	(0.033)	(0.033)	(0.048)	(0.045)
γ_g	0.07*	0.17*	0.10*	0.29*
	(0.019)	(0.071)	(0.017)	(0.070)
γ_r	0.35*	0.29*	0.33*	0.26*
	(0.052)	(0.049)	(0.065)	(0.067)
Observations	106	106	106	106
J Test	10.687	10.763	10.952	10.872
(jp)	(0.986)	(0.985)	(0.984)	(0.984)
Weak ID Test	163.460	60.640	121.279	89.040

Notes: this table reports estimates of the augmented Phillips curve (2). All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. The measure of inflation is Core CPI and the gap is measured with $\log(V/U)$. All regressors are instrumented using six lags of Romer and Romer's [2004] shocks (as extended by Wieland and Yang [2020]) and their squares as instruments. A * indicates significance at 5%. Sample is 1981Q3-2007Q4.

Figure A.3: Out-of-Sample Residuals from Phillips Curve, 1981-2007



Notes: Panels (a) and (c) of this figure plots the out-of-sample residuals of the estimated Phillips curve (2) against minus unemployment gap as the measure of labor market tightness, for two measures of inflation expectations. Panels (b) and (d) of this figure plots the out-of-sample residuals against log(V/U) as the measure of labor market tightness, for two measures of inflation expectations. The gray lines show the estimated linear or cubic relation between residuals and labor market tightness (see Table A.4 for the estimated coefficients). Light dots correspond to pre-2020 observations and dark ones to post-2020. We exclude 2020q2 from this graph.

Table A.4: Projection of the Philips Curve Residuals ϵ on measures of gap, 2008-2023

Expectations:	MSC				SPF			
Gap:	-ugap		$\ln V/U$		-ugap		$\ln V/U$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
gap_t	-0.04*	-0.05	-0.00	0.07	-0.08*	-0.10	-0.14*	0.04
	(0.017)	(0.057)	(0.042)	(0.084)	(0.019)	(0.065)	(0.048)	(0.091)
gap_t^2		-0.00		-0.02		0.01		0.50*
		(0.044)		(0.165)		(0.050)		(0.179)
gap_t^3		0.00		-0.05		0.00		0.21*
		(0.007)		(0.090)		(0.008)		(0.098)
Observations	60	60	60	60	60	60	60	60

Notes: the Phillips curve residuals are obtained from the augmented Phillips curve (2) estimated over the sample 1981Q3-2007Q4, using $-\text{ugap}$ or $\log(V/U)$ as a measure of the gap and either SPF or MSC as a measure of expectations. The standard errors of estimated coefficients are between parentheses.

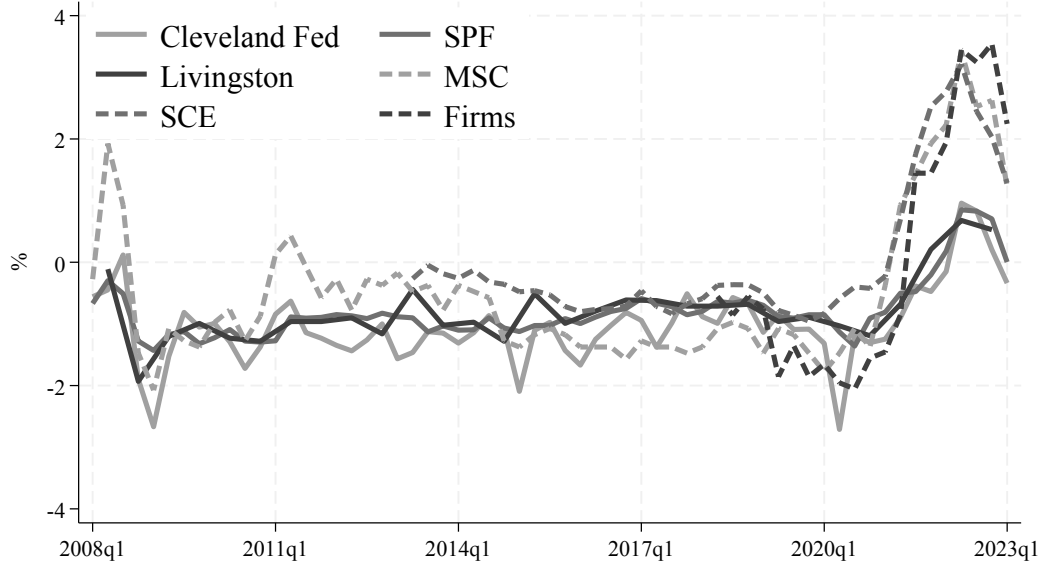
From the above analysis, we see only one specification suggests the Phillips Curve appears to become steeper in recent years. That is when using $\ln V/U$ as a measure of labor market tightness, SPF as the measure of expected inflation and sample 1981Q3-2007Q4. Once we use the Michigan Survey of Consumers as the measure of expected inflation, such a pattern disappears. This is not surprising given the differences between the expected inflations from the survey of professionals and central banks, and those from the households and firms. In Appendix B we show the differences between various surveys of expectations and repeat our analysis with them.

B Baseline Phillips Curve with Various Measures of Inflation Expectations

We first show the differences between various surveys of expectations. Figure B.1 depicts the demeaned series of expected inflation from different surveys. A stark pattern is that households and firms have similar expectations of inflation that are drastically different from those of professional and central banks. In particular, the households and firms expect much

higher inflation after 2020.

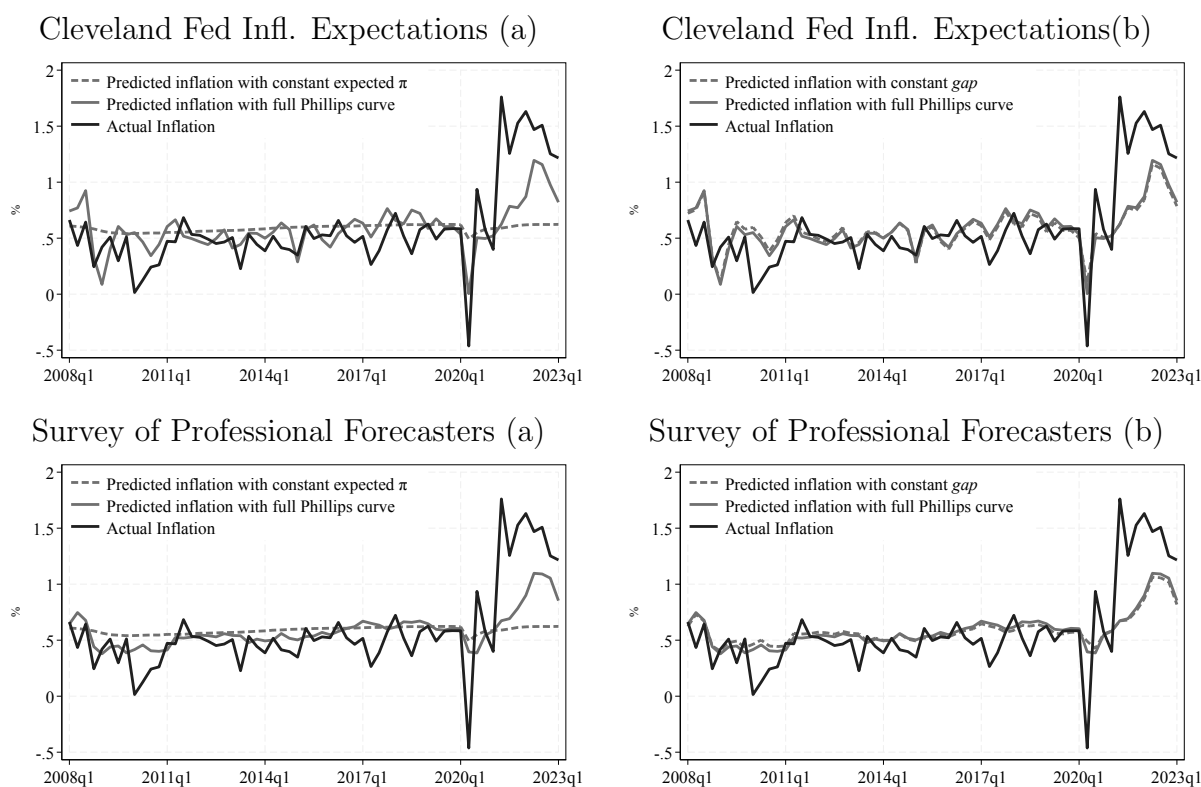
Figure B.1: Different measures of inflation expectations



Notes: Demeaned average expected inflation from various surveys. Solid lines are from the surveys of professionals and central banks. The dashed lines are from surveys of households and firms.

Here we repeat the exercise of Section 1 with various measures of expectations. Key result is that the run-up of inflation expectations can account for inflation increase post-COVID-19 when households or firms survey are used (Survey of Consumer Expectations (New York Fed) or Survey of Firms' Inflation Expectations (Cleveland Fed) or Michigan Survey of Consumers in the main text). The professional forecasts (Cleveland Fed Inflation Expectations, Survey of Professional Forecasters (Philadelphia Fed) or Livingston Survey (Philadelphia Fed)) increase less post-COVID-19, and fail to contribute fully to the increase in actual headline inflation. Note the the baseline Phillips curve (1) is used to do the counterfactual simulations.

Figure B.2: Counterfactual Simulations from the Estimated Phillips Curve, Using Various Measure of Inflation Expectations (a) With inflation expectations at their mean, (b) With gap at its mean



Notes: These counterfactual simulations are done using the baseline Phillips curve (1), using either Cleveland Fed Inflation Expectations or the Survey of Professional Forecasters (Philadelphia Fed) as a measure of expectations.

Figure B.3: Counterfactual Simulations from the Estimated Phillips Curve, Using Various Measure of Inflation Expectations (a) With inflation expectations at their mean, (b) With gap at its mean



Notes: These counterfactual simulations are done using the baseline Phillips curve (1), using either Livingston Survey (Philadelphia Fed), Survey of Consumer Expectations (New York Fed) or Survey of Firms' Inflation Expectations (Cleveland Fed) as a measure of expectations.

Finally, we do a “horse-race” regression to estimate our augmented Phillips curve (2) including both MSC and SPF as potential measures of expected inflation. The results are reported in Table B.1. In column (1) we use *minus* unemployment gap as the measure of the gap and freely estimate it. In column (2) we fix the slope of the negative unemployment gap at the estimate of Hazell, Herreño, Nakamura, and Steinsson [2022] and in column (3) we use the $\ln V/U$ as the measure of the gap. The results suggest that MSC is the more relevant measure for expected inflation in the Phillips Curve.

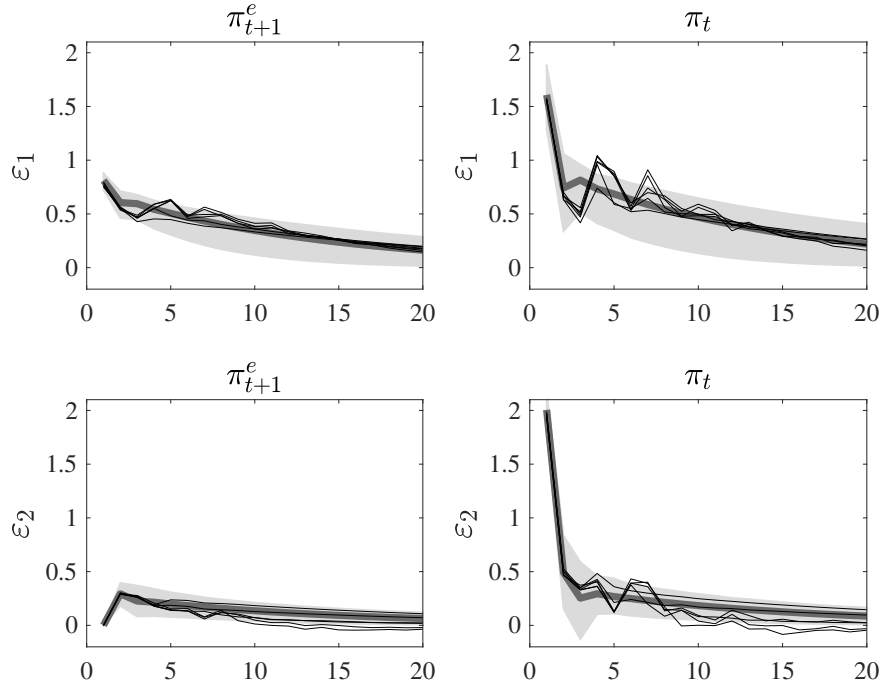
Table B.1: Horse Race using SPF and MSC

Gap	-ugap or $\ln V/U$		
π	Sample 1981q3-2007q4		
	(1) -ugap free slope	(2) -ugap fixed slope	(3) $\ln V/U$
MSC	0.40* (0.177)	0.62* (0.161)	0.47* (0.173)
SPF	0.29 (0.272)	-0.02 (0.233)	0.25 (0.267)
γ_g	0.08* (0.019)	0.0138 (-)	0.19* (0.071)
γ_r	0.30* (0.082)	0.28* (0.054)	0.24* (0.080)
Observations	106	106	106
J Test (jp)	10.655 (0.979)	11.939 (0.971)	10.636 (0.980)
Weak ID Test	135.594	121.041	39.712

Notes: this table reports estimates of the augmented Phillips curve (2). All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. The measure of inflation is Core CPI and the gap is measured with minus unemployment gap or $\log(V/U)$. All regressors are instrumented using six lags of Romer and Romer’s [2004] shocks (as extended by Wieland and Yang [2020]) and their squares as instruments. A \star indicates significance at 5%. Sample is 1981Q3-2007Q4.

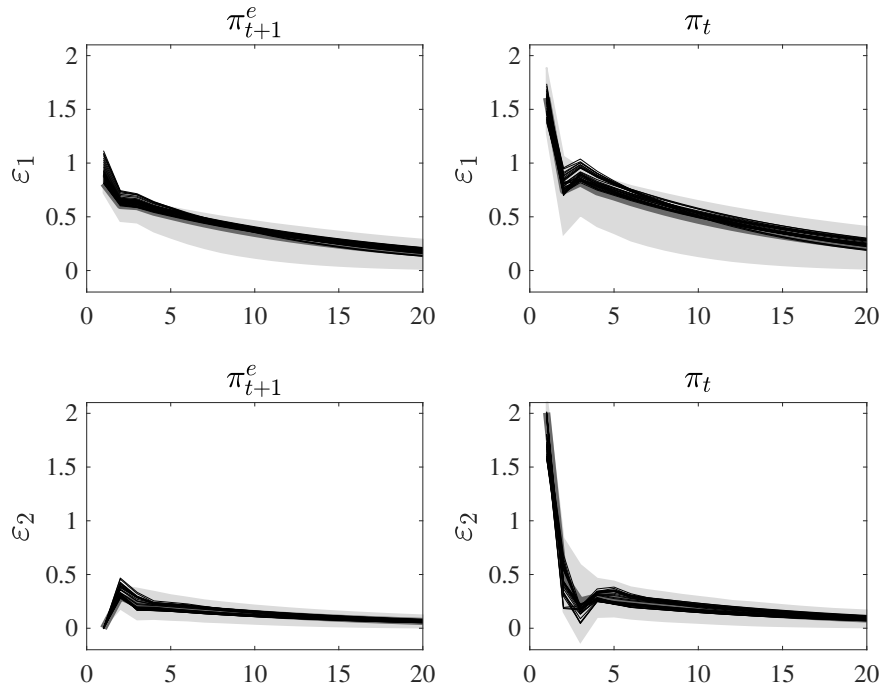
C Robustness of the 2-VAR to Lags and Sample

Figure C.1: Impulse Responses in the 2-VAR (π_t, π_{t+1}^e) , with 2 to 8 Lags



Notes: this Figure plots the impulse responses to a one standard deviation shocks ε_1 and ε_2 . These shocks are obtained from a Choleski orthogonalization. The estimated VAR with two lags of Headline CPI inflations and the MSC inflation expectations are in gray. Sample is 1969Q1-2023Q1. Shaded area represents the 95% confidence band. The black lines corresponds to an estimation with 3 to 8 lags.

Figure C.2: Impulse Responses in the 2-VAR (π_t, π_{t+1}^e) , Starting with First 20 Years and Adding Years One by One



Notes: this Figure plots the impulse responses to a one standard deviation shocks ε_1 and ε_2 . These shocks are obtained from a Choleski orthogonalization. The estimated VAR with two lags of Headline CPI inflations and the MSC inflation expectations are in gray. Sample is 1969Q1-2023Q1. Shaded area represents the 95% confidence band. The black lines corresponds to an estimation samples 1969Q1-1989Q1, 1969Q1-1990Q1, 1969Q1-1991Q1,... etc.

D Using Disaggregated Prices

Table D.1 display the 25 expenditures categories we use, as obtained from BLS and used to compute the CPI. Figure D.1 plots categorical inflations and Table D.2 displays the estimated parameters α_i , σ_i and ρ when we estimate the model

$$\pi_{i,t} = \alpha_i \mathcal{C}_t + e_{i,t},$$

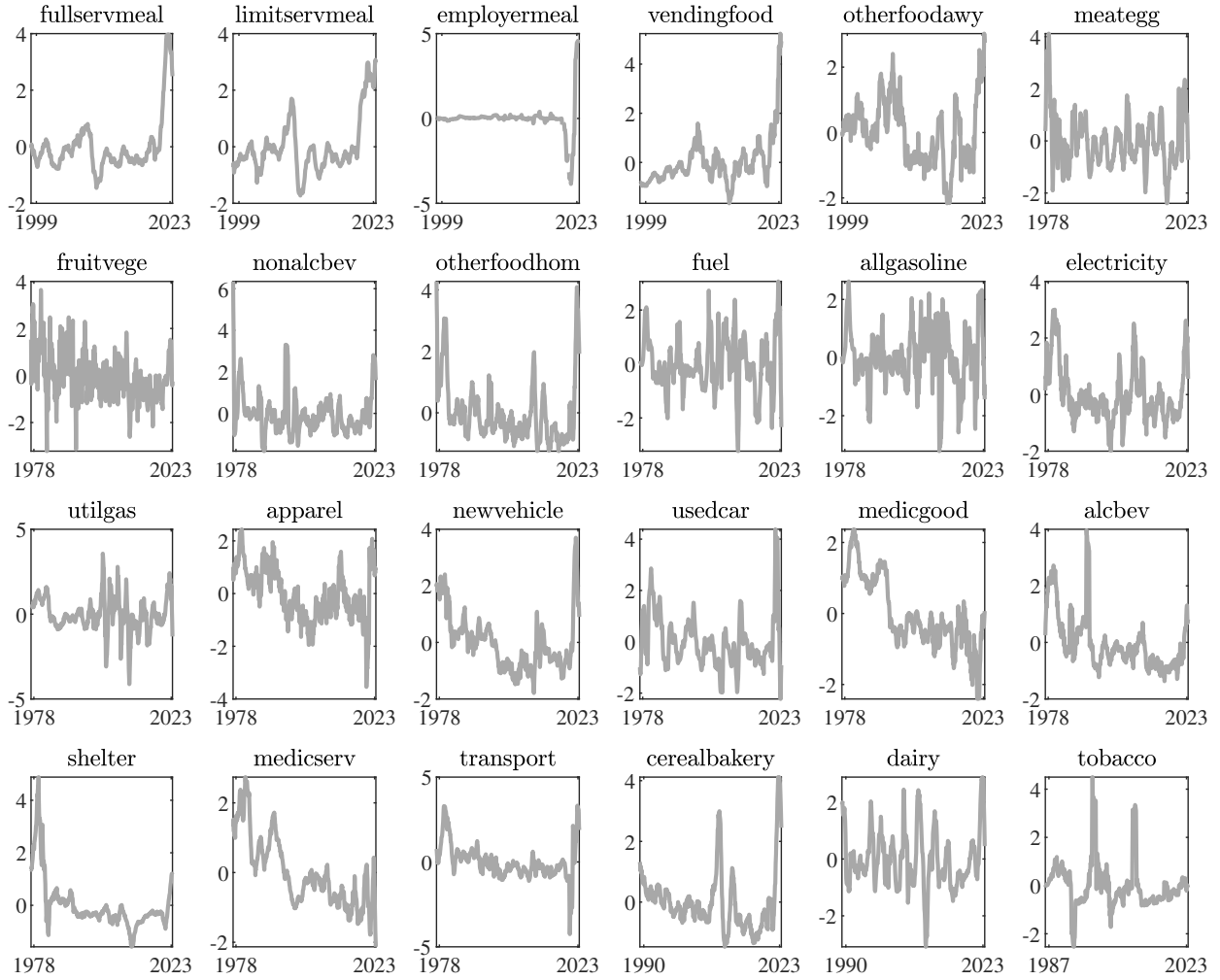
$$\mathcal{C}_t = \rho_C \mathcal{C}_{t-1} + v_t.$$

Table D.1: Summary of Categories

Categories and items	Available sample	Categories and items	Available sample
Food at Home		Food away from Home	
Cereals and bakery products	1990m1-2023m5	Full service meals and snacks	1999m1-2023m5
Meats, poultry, fish, and eggs	1978m1-2023m5	Limited service meals and snacks	1998m12-2023m5
Dairy and related products	1990m1-2023m5	Food at employee sites and schools	1999m1-2023m5
Fruits and vegetables	1978m1-2023m5	Food from vending machines and mobile vendors	1998m12-2023m5
Nonalcoholic beverages and beverage materials	1978m1-2023m5	Other food away from home	1999m1-2023m5
Other food at home	1978m1-2023m5		
Energy commodities		Energy services	
Fuel oil	1978m1-2023m5	Electricity	1978m1-2023m5
Motor fuel (gasoline)	1978m1-2023m5	Utility gas	1978m1-2023m5
less food and energy commodity		less energy services	
Apparel	1978m1-2023m5	Shelter	1978m1-2023m5
New vehicles	1978m1-2023m5	Medical care service	1978m1-2023m5
Used cars and trucks	1978m1-2023m5	Transportation service	1978m1-2023m5
Medical care commodities	1978m1-2023m5		
Alcoholic beverages	1978m1-2023m5		
Tobacco and smoking products	1987m1-2023m5		

Notes: the main sample starts from 1978m1 due to the availability of the monthly Michigan Survey of Consumers. The disaggregated data are level 3 categories from BLS.

Figure D.1: Inflation for the Components in the CPI Basket



Notes: this Figure displays the time series of the 25 expenditures categories we use, as obtained from BLS and used to compute the CPI.

Table D.2: Estimated parameters

Parameter	Estimate		Parameter	Estimate	
ρ	0.997				
	α_i	σ_i		α_i	σ_i
fullservmeal	0.13	0.32	limitservmeal	0.12	0.57
employermeal	0.03	0.97	vending	0.11	0.65
otherfoodawy	0.09	0.79	meategg	0.05	0.90
fruitvege	0.06	0.84	nonalcohol	0.07	0.82
otherfoodhome	0.09	0.55	fuel	0.04	0.93
gasoline	0.03	0.96	electricity	0.09	0.64
utility gas	0.04	0.92	apparel	0.08	0.74
new vehicle	0.09	0.65	used car	0.03	0.96
medical good	0.06	0.84	alcoholbev	0.08	0.67
shelter	0.10	0.53	medical service	0.07	0.80
transportation	0.09	0.61	cereal bakery	0.14	0.51
dairy product	0.08	0.86	tobacco	0.01	1.00

Notes: we normalize $\sigma_v = 1$, and we input series that are normalized (demeaned and standard deviations normalized to 1) because the price series have different volatilities. But the normalization doesn't qualitatively change the results.

E Average Signal from Disaggregate Price Indices

We show that the signal-extraction problem with multiple disaggregate price indices is equivalent to one with an average signal across these disaggregate indices. The disaggregate signals the agent faces are given by (23). Without loss of generality, consider the m disaggregate signals the agent uses to form expectations:

$$X_t \equiv \begin{pmatrix} \pi_{1,t} \\ \pi_{2,t} \\ \dots \\ \pi_{m,t} \end{pmatrix} = \ell_m \tilde{z}_t + \begin{pmatrix} \tilde{e}_{1,t} \\ \tilde{e}_{2,t} \\ \dots \\ \tilde{e}_{m,t} \end{pmatrix}, \quad (\text{E.1})$$

with $\tilde{e}_{j,t} \sim N(0, \tilde{\sigma}_j^2)$, and where we denote the vector of signals as X_t and ℓ_m is an $m \times 1$ vector of ones. Denote the prior of z_t in $t - 1$ as:

$$z_t^{\text{prior}} \sim N(z_{t|t-1}, \sigma^2), \quad (\text{E.2})$$

where $z_{t|t-1}$ denotes the prior mean and σ^2 the stationary prior variances. The posterior mean of the nowcast is:

$$z_{t|t} = z_{t|t-1} + \kappa(X_t - \ell_m z_{t|t-1}), \quad (\text{E.3})$$

where κ is the Kalman Gain:

$$\kappa = \sigma^2 \ell'_m (\sigma^2 \ell_m \ell'_m + V)^{-1}, \quad (\text{E.4})$$

where V is a diagonal matrix with entries $\tilde{\sigma}_j^2$ on the main diagonal. The Kalman Gain can then be explicitly written as a function of σ^2 and $\tilde{\sigma}_j^2$'s:

$$\begin{aligned} \kappa &= \sigma^2 \ell'_m \left(V^{-1} - \frac{V^{-1} \sigma^2 \ell_m \ell'_m V^{-1}}{1 + \sigma^2 \ell'_m V \ell_m} \right) \\ &= \sigma^2 \left[\begin{pmatrix} \frac{1}{\tilde{\sigma}_1^2} & \frac{1}{\tilde{\sigma}_2^2} & \cdots & \frac{1}{\tilde{\sigma}_m^2} \end{pmatrix} - \begin{pmatrix} \frac{1}{\tilde{\sigma}_1^2} \sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2} & \frac{1}{\tilde{\sigma}_2^2} \sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2} & \cdots & \frac{1}{\tilde{\sigma}_m^2} \sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2} \end{pmatrix} \frac{1}{\frac{1}{\sigma^2} + \sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2}} \right] \\ &= \begin{pmatrix} \frac{1}{\tilde{\sigma}_1^2} & \frac{1}{\tilde{\sigma}_2^2} & \cdots & \frac{1}{\tilde{\sigma}_m^2} \end{pmatrix} \frac{1}{\frac{1}{\sigma^2} + \sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2}}. \end{aligned} \quad (\text{E.5})$$

Now consider an average signal:

$$x_t = \tilde{z}_t + \underbrace{\frac{\sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2} \tilde{e}_{j,t}}{\sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2}}}_{\equiv \epsilon_t}, \quad (\text{E.6})$$

$$\epsilon_t \sim N\left(0, \underbrace{\frac{1}{\sum_{j=1}^m \frac{1}{\tilde{\sigma}_j^2}}}_{\equiv \sigma_\epsilon^2}\right). \quad (\text{E.7})$$

With the same prior, the Kalman Gain is given by:

$$\hat{\kappa} = \frac{\frac{1}{\sigma_\epsilon^2}}{\frac{1}{\sigma^2} + \frac{1}{\sigma_\epsilon^2}}. \quad (\text{E.8})$$

Using (E.5) and (E.8), it is then straightforward to show that the posterior mean $z_{t|t}$ is the same when the agent uses disaggregate signals (E.1) with the aggregate signal (E.6).

Moreover, the stationary posterior variances are the same as well. As a result, the stationary posterior belief formed by observing multiple signals (E.1) is equivalent to one formed using an average signal (E.6)

F Models with Rational Expectations

F.1 Incomplete Information Rational Expectations

Take the IIRE model described by (9)-(12). The agent forms expectations about gap_t .

Denoting the nowcasts of which as $\text{gap}_{t|t}$, expectations are given by:

$$\begin{aligned} E_t[\pi_{t+1}] &= E_t[\beta E_{t+1}\pi_{t+2} + \gamma_g \text{gap}_{t+1} + e_{t+1}] \\ &= \frac{\gamma_g \rho}{1 - \beta \rho} \text{gap}_{t|t}. \end{aligned} \tag{F.1}$$

The nowcast $\text{gap}_{t|t}$ is given by:²⁴

$$\text{gap}_{t|t} = \text{gap}_{t|t-1} + k_i(s_t - \gamma_g \text{gap}_{t|t-1}), \tag{F.2}$$

$$\mathcal{K} = \sigma_k^2 \gamma_g (\gamma_g^2 \sigma_k^2 + \sigma_\epsilon^2)^{-1}, \tag{F.3}$$

$$\sigma_k^2 = \rho^2 (\sigma_k^2 - \mathcal{K} \gamma_g \sigma_k^2) + \sigma_v^2, \tag{F.4}$$

where \mathcal{K} is the stationary Kalman Gain and σ_k^2 is the stationary posterior variance. This leads to equations (13) and (14) in the main text. Note that as $\sigma_\epsilon \rightarrow 0$, $\mathcal{K} \rightarrow 1/\gamma_g$ and $w_t \rightarrow e_t$. The IIRE case converges to the FIRE case.

²⁴Note in (F.3) the agent uses correct σ_ϵ because when the agent is rational, she can easily back-out the correct ρ , $\gamma_g \sigma_v$ and σ_ϵ with the variance-covariance structure of s_t . As a result, the agent's information will not support any subjective $\tilde{\sigma}_\epsilon \neq \sigma_\epsilon$.

F.2 Hybrid Phillips Curve or Adaptive Expectations

One related question is whether a hybrid Phillips Curve or adaptive expectations can help to explain the joint dynamics between expected and actual inflation. We consider a Phillips Curve taking the following hybrid form:

$$\pi_t = \beta \underbrace{\left(\tau \pi_{t-1} + (1 - \tau) E_t^{FIRE} \pi_{t+1} \right)}_{\text{observed in MSC}} + \gamma_g \text{gap}_t + e_t, \quad (\text{F.5})$$

$$\text{gap}_t = \rho \text{gap}_{t-1} + v_t, \quad (\text{F.6})$$

where τ can represents the indexation, the fraction of people using adaptive expectations, or motivated by k-level thinking as in Beaudry, Carter, and Lahiri [2023]. The expectations formed under FIRE takes into account that there are backward looking component in inflation. It is easy to show that inflation takes the following form using undetermined coefficients:

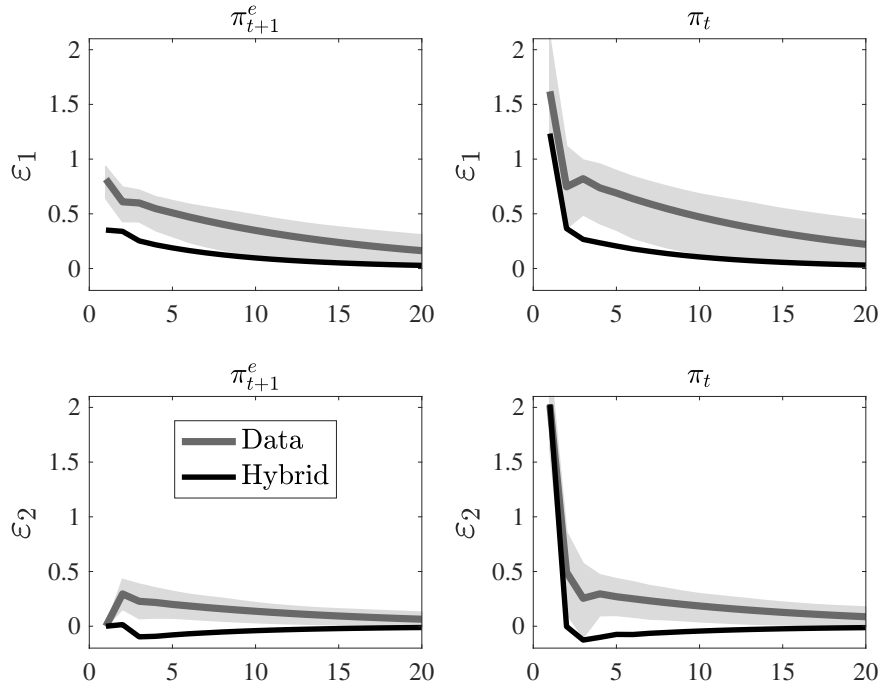
$$\pi_t = a \pi_{t-1} + b \text{gap}_t + c e_t, \quad (\text{F.7})$$

with

$$\begin{cases} a &= \frac{\beta \tau}{1 - a \beta (1 - \tau)}, \\ b &= \frac{\beta (1 - \tau) b \rho + \gamma_g}{1 - a \beta (1 - \tau)}, \\ c &= \frac{1}{1 - a \beta (1 - \tau)}. \end{cases}$$

Following our approach in section 2.2, we set β , ρ , σ_v and γ_g at values consistent with our Phillips Curve, and we estimate the free parameter τ to match the IRFs from the bivariate VAR(2). Our estimate is $\hat{\tau} = 0.05$, and the above model cannot match the empirical IRFs well:

Figure F.1: The Joint Process of π and π_{t+1}^e in the Data and Under Models with Rational Expectations

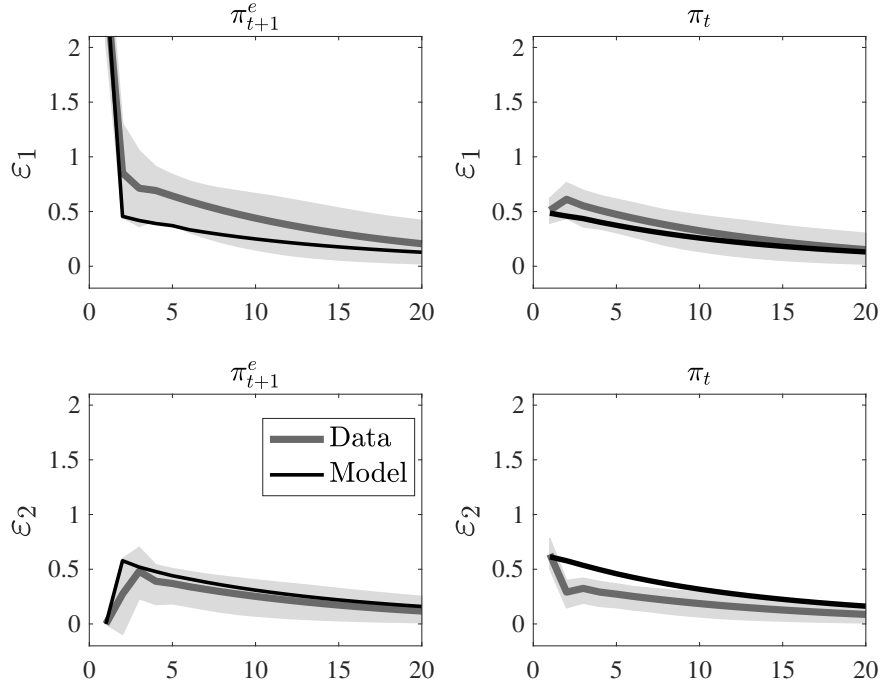


Notes: on this Figure, the grey line plots the impulse responses to a one standard deviation shocks ε_1 and ε_2 estimated with data from MSC and Headline CPI. Sample is 1969Q1-2023Q1. Shaded area represents the 95% confidence band. The black line plot the average impulse responses (over 200 simulations of length 216) obtained from the same VAR estimated on simulated data, when the Data Generating Process is the estimated hybrid model (F.5).

G Alternative Order of Cholesky VAR

We present the IRFs from Cholesky VAR(2) where we order headline inflation first and expected inflation the second. This change of ordering reflects an alternative identification assumption that the first underlying shock affects π_t and π_{t+1}^e simultaneously and the second shock affects π_{t+1}^e on impact then π_t with a lag. Note our Cholesky VAR is just method to summarize joint dynamics of inflation and expectations. If our model is close to the true data generating process for inflation and expected inflation, we would expect IRF from this VAR with alternative (but possibly incorrect) ordering to have similar result. Figure G.1 shows the results from this exercise. Our model matches very well with the VAR using alternative ordering.

Figure G.1: IRF from data and model simulation



Notes: thick gray line is IRF from bi-VAR with actual data; marked black line is average IRF from simulated data across 200 random samples. The estimated VAR is VAR(2) with Cholesky Decomposition ordering $\pi_t - \pi_{t+1}^e$. The shaded area represents 95% CI.

H State-space Representation of Model

H.1 Model

The two observables $\pi_{t+1|t,0}$ and π_t can be written as system of equations of latent states $z_{t|t-1,1}$ and gap_t .

$$\begin{aligned}
\pi_{t+1|t,0} &= \tilde{\rho} z_{t|t,0} = \tilde{\rho}((1-K)z_{t|t-1,1} + K s_t) \\
&= \tilde{\rho}((1-K)z_{t|t-1,1} + K(\beta\pi_{t+1|t,0} + \gamma_g \text{gap}_t + \epsilon_t)) \\
&= \frac{\tilde{\rho}(1-K)}{1-K\beta\tilde{\rho}} z_{t|t-1,1} + \frac{\tilde{\rho}K\gamma_g}{1-K\beta\tilde{\rho}} \text{gap}_t + \frac{\tilde{\rho}K}{1-K\beta\tilde{\rho}} \epsilon_t.
\end{aligned} \tag{H.1}$$

From (27):

$$\begin{aligned}
s_t &= \pi_{t+1|t,0} + \gamma_g \text{gap}_t + \epsilon_t \\
&= \frac{\beta\tilde{\rho}(1-K)}{1-K\beta\tilde{\rho}} z_{t|t-1,1} + \frac{\gamma_g}{1-K\beta\tilde{\rho}} \text{gap}_t + \frac{1}{1-K\beta\tilde{\rho}} \epsilon_t.
\end{aligned} \tag{H.2}$$

Inflation is given by:

$$\begin{aligned}
\pi_t &= \beta\pi_{t+1|t,0} + \gamma_g \text{gap}_t + \gamma\epsilon_t + w_t \\
&= \frac{\beta\tilde{\rho}(1-K)}{1-K\beta\tilde{\rho}} z_{t|t-1,1} + \frac{\gamma_g}{1-K\beta\tilde{\rho}} \text{gap}_t + \left(\frac{K\beta\tilde{\rho}}{1-K\beta\tilde{\rho}} + \gamma \right) \epsilon_t + w_t.
\end{aligned} \tag{H.3}$$

From (33) and expression of s_t , we get recursion of $z_{t|t-1,1}$:

$$\begin{aligned}
z_{t+1|t,1} &= \tilde{\rho} z_{t|t,1} = \tilde{\rho}(1-K-k)z_{t|t-1,1} + \tilde{\rho}K s_t + \tilde{\rho}k\pi_t \\
&= \tilde{\rho} \left(1 - (K+k) \frac{1-\beta\tilde{\rho}}{1-K\beta\tilde{\rho}} \right) z_{t|t-1,1} + \tilde{\rho} \frac{(K+k)\gamma_g}{1-K\beta\tilde{\rho}} \text{gap}_t \\
&\quad + \tilde{\rho} \left(\frac{K+\tilde{\rho}Kk\beta}{1-K\beta\tilde{\rho}} + \gamma k \right) \epsilon_t + \tilde{\rho}k w_t.
\end{aligned} \tag{H.4}$$

We could write the state-space representation as:

$$X_t \equiv \begin{pmatrix} \pi_{t+1|t,0} \\ \pi_t \\ z_{t+1|t,1} \\ \text{gap}_t \end{pmatrix} = F X_{t-1} + B \begin{pmatrix} \epsilon_t \\ w_t \\ v_t \end{pmatrix}, \quad (\text{H.5})$$

where

$$F = \begin{pmatrix} 0 & 0 & \frac{\beta\tilde{\rho}(1-K)}{1-K\beta\tilde{\rho}} & \rho \frac{\tilde{\rho}K\gamma_g}{1-K\beta\tilde{\rho}} \\ 0 & 0 & \frac{\beta\tilde{\rho}(1-K)}{1-K\beta\tilde{\rho}} & \rho \frac{\gamma_g}{1-K\beta\tilde{\rho}} \\ 0 & 0 & \tilde{\rho} \left(1 - (K+k) \frac{1-\beta\tilde{\rho}}{1-K\beta\tilde{\rho}} \right) & \tilde{\rho} \frac{(K+k)\gamma_g}{1-K\beta\tilde{\rho}} \rho \\ 0 & 0 & 0 & \rho \end{pmatrix}, \quad (\text{H.6})$$

$$B = \begin{pmatrix} \frac{\tilde{\rho}K}{1-K\beta\tilde{\rho}} & 0 & \frac{\tilde{\rho}K\gamma_g}{1-K\beta\tilde{\rho}} \\ \frac{K\beta\tilde{\rho}}{1-K\beta\tilde{\rho}} + \gamma & 1 & \frac{\gamma_g}{1-K\beta\tilde{\rho}} \\ \left(\frac{K+\tilde{\rho}Kk\beta}{1-K\beta\tilde{\rho}} + \gamma k \right) \tilde{\rho} & \tilde{\rho}k & \tilde{\rho} \frac{(K+k)\gamma_g}{1-K\beta\tilde{\rho}} \\ 0 & 0 & 1 \end{pmatrix}. \quad (\text{H.7})$$

The observational equation is given by:

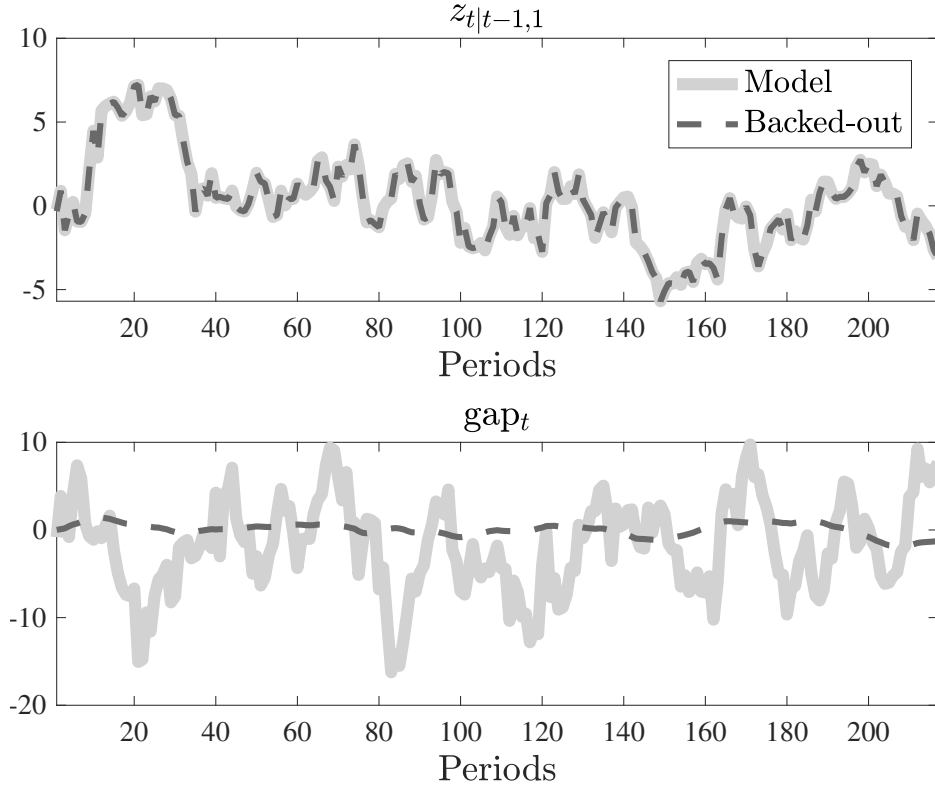
$$O_t = \begin{pmatrix} I_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} \end{pmatrix} X_t. \quad (\text{H.8})$$

We can then estimate the hidden states $z_{t|t-1,1}$ and gap_t using Kalman Filter Smoothing, then get the corresponding shocks implied by the state-space representation.

The precision of the estimated states and shocks depend on the parameters in F and B . To illustrate the performance of this approach in the context of our estimated model, we simulate data using parameters from Table 4. We then plot the time series of the estimated states $E_t\pi_{t+1}$, π_t , $z_{t|t-1,1}$ and gap_t in Figure H.1. The blue solid lines are the actual hidden states from simulated data and the red dash lines are the estimated ones from the above approach. Not surprisingly, the hidden state $z_{t|t-1,1}$ is very well recovered where the estimated values are almost identical to the actual ones. Whereas the gap_t is very illy recovered. This is because the observables $\{E_t\pi_{t+1}, \pi_t\}$ contain a lot more information for $z_{t|t-1,1}$ and almost

no information about gap_t due to the fact γ_g is very small in the actual DGP. As a result, the Kalman Smoothing algorithm puts low weights on the observables when making predictions about this hidden state. Figure H.2 depicts the backed-out shocks and compares them with

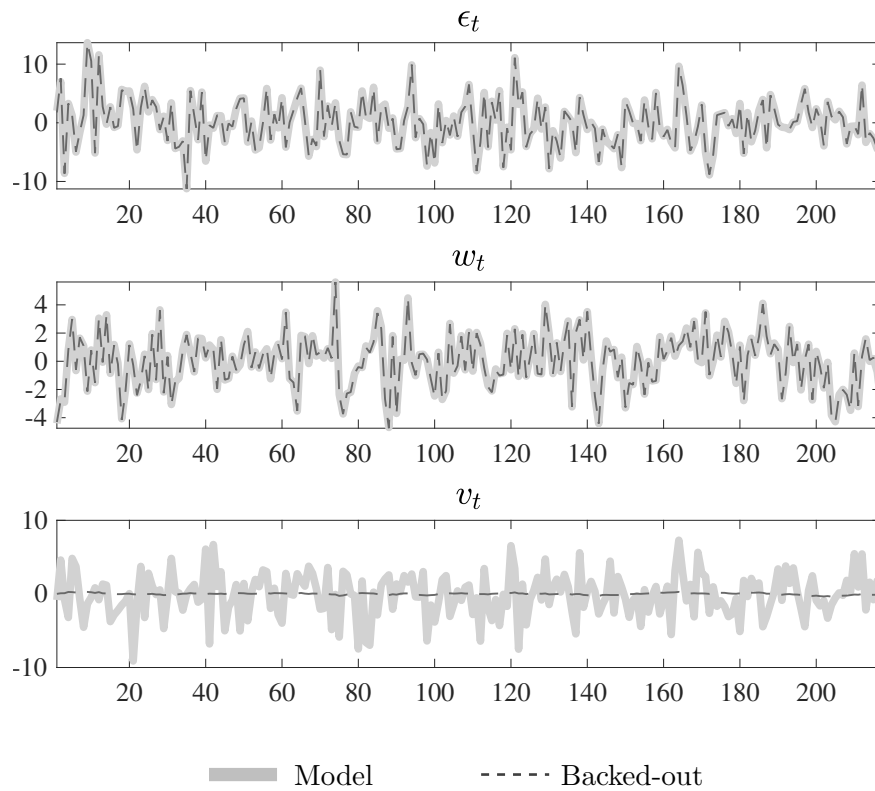
Figure H.1: Estimated v.s. actual latent states $z_{t|t-1,1}$ and gap_t from simulated data



Notes: the blue solid lines are actual latent states from simulated data. The red dash lines are the recovered latent states using the Kalman Smoothing.

actual shock series simulated. For the same reason as the latent states, the observations are quite informative about the broad-base shocks ϵ_t and the aggregate shock w_t , but they are not informative about the common shock v_t . As a result, the recovered v_t series are much less volatile and not really comparable to the actual shock series.

Figure H.2: Estimated v.s. actual latent states $z_{t|t-1,1}$ and gap_t from simulated data

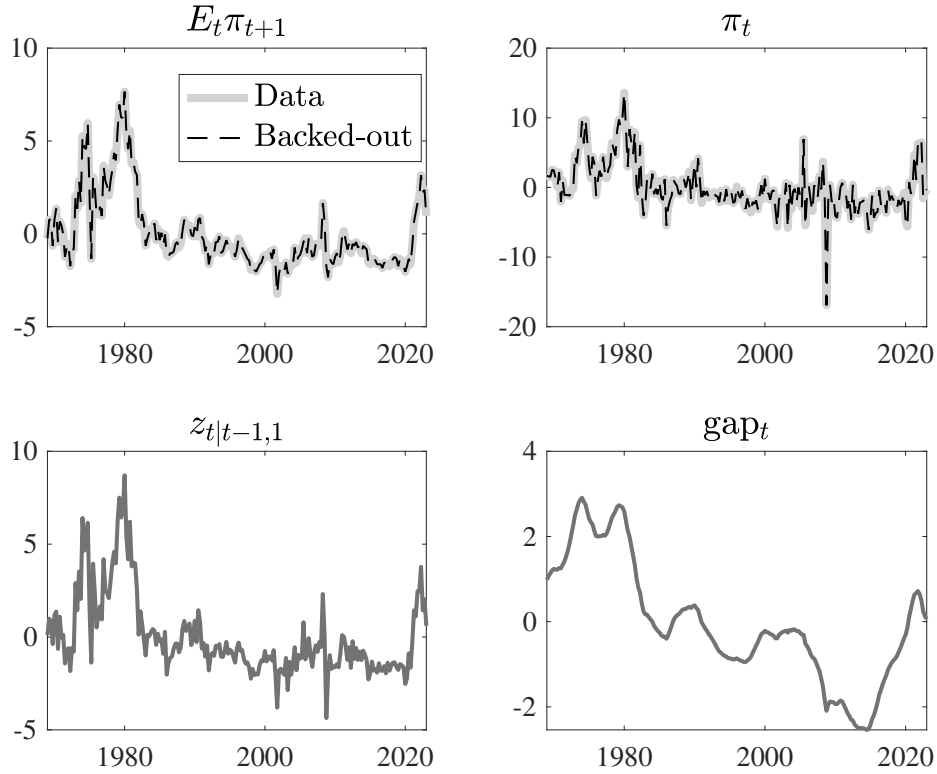


Notes: the dark gray lines are model simulated latent states. The light gray lines are the recovered latent states using the Kalman Smoothing.

H.2 Backed-out Series from Real Data

The figure H.3 depicts the recovered states $\{E_t\pi_{t+1}, \pi_t, z_{t|t-1,1}, \text{gap}_t\}$ applying Kalman Smoothing. By construction, the expected inflation and aggregate inflation are observable so they coincide with each other. Because the actual values for $z_{t|t-1,1}$ and gap_t are treated as unobserved, there are no corresponding actual values plotted for these two variables.

Figure H.3: Estimated v.s. actual states from actual data



Notes: the states π_t and $E_t\pi_{t+1}$ are observables. In the two upper panels, the thick grey line plots actual states and the black dashed line is the recovered latent states using the Kalman Smoothing. The two lines coincide by construction as these two states are observable. Because the actual values for $z_{t|t-1,1}$ and gap_t are treated as unobserved, there are no corresponding actual values plotted for these two variables.

I The Baseline Model without the Timing Restriction

In this appendix, we reestimate the model under the assumption that inflation expectations are reported *after* observing current inflation, meaning that $\pi_{t+1}^e = \pi_{t+1|t,1}$. The estimated

parameters become:

Table I.1: Estimated parameters

From the Phillips Curve			
β	0.99	ρ	0.89
σ_v	3.02	γ_g	0.0138
From Minimum Distance			
$\tilde{\sigma}_v$	0.46	σ_ϵ	3.31
$\tilde{\rho}$	0.98	$\tilde{\sigma}_w$	2.46
γ	0.05	$\tilde{\sigma}_\epsilon$	1.76
σ_w	2.23		

Notes: The Phillips curve estimates use the baseline estimation with Hazell, Herreño, Nakamura, and Steinsson's [2022] estimate of γ_g . σ_w is implied by the variance of the residual e from the Phillips curve, and from σ_ϵ and γ : $\sigma_w = \sqrt{\sigma_\epsilon^2 - \gamma^2 \sigma_\epsilon^2}$.

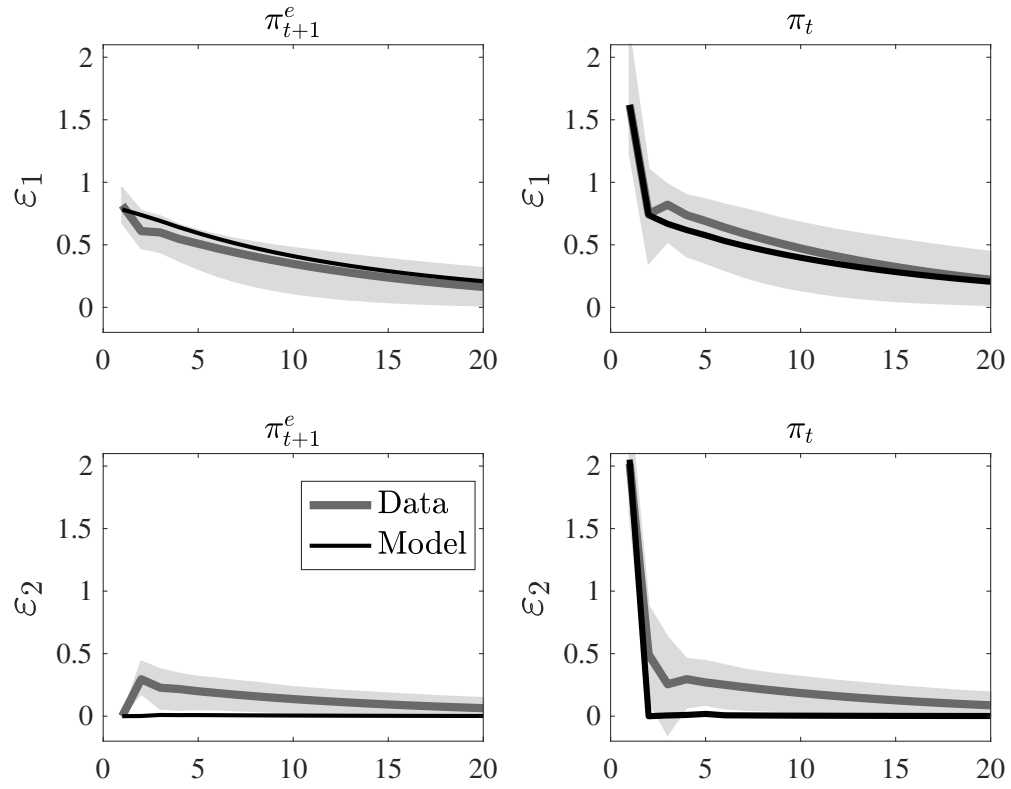
In Figure I.1, we see the model cannot match the IRF of the second shock in the Cholesky VAR because the shock w_t affecting π_t on impact will have no impact on expectations at all in the model without timing restriction. The variance-covariance structure from the model without timing restriction is:

Table I.2: Variance and Auto-covariance of Inflation

	Data	PLM	Model
$var(\pi_{t,h})$	12.52	11.14	11.14
$cov(\pi_{t,h}, \pi_{t-1,h})$	7.70	5.11	4.98
$cov(\pi_{t,h}, \pi_{t-2,h})$	6.99	4.74	4.87

Notes: PLM stands for "Perceived Law of Motion". "The PLM and "Model" moments are the average moment across 200 random samples. In the estimations, we penalize distance between data and model 2-VAR responses as well as distance between these three moments in the PLM and in the model.

Figure I.1: IRF from data and simulated data from model without timing restriction



Notes: The thick gray line is the IRF from the 2-VAR with actual data; the black line is the average IRF from simulated data across 200 random samples. The estimated VAR is a VAR(2) with Cholesky decomposition, ordering π_{t+1}^e first. The shaded area represents the 95% confidence interval.