30-variant

1. $\begin{vmatrix} 0 & -2 & 1 & 2 \\ 1 & -2 & -5 & -4 \\ 2 & -4 & 2 & -3 \\ 3 & 1 & -1 & 0 \end{vmatrix}$. **2.** $A = \begin{pmatrix} 1 & -4 \\ 3 & -3 \\ 2 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 5 & -3 & 1 \\ 2 & 3 & 0 \end{pmatrix}$.

3. $\begin{cases} 4x_1 + 2x_2 - x_3 + 2x_4 = 2, \\ x_1 - 3x_2 + x_3 - x_4 = 5, \\ 2x_1 - x_2 + 2x_3 = 7, \\ x_1 + 6x_2 - 4x_3 + 3x_4 = -8. \end{cases}$

1-MUSTAQIL ISH

1. Berilgan determinantni hisoblang: a) $i-$satr elementlari bo‘yicha yoyib; b) $j-$ ustun elementlari bo‘yicha yoyib; c) $j-$ ustundagi bittadan boshqa elementlarni nolga aylantirib va shu ustun elementlari bo‘yicha yoyib.
2. A, B matritsalar va α, β sonlari berilgan. $\alpha A + \beta B$, AB, A^{-1} matritsalarni toping va $AA^{-1} = E$ ekanini tekshiring.
3. Tenglamalar sistemalarini tekshiring. Birgalikda bo‘lgan sistemani Kramer formulalari orqali, matritsalar va Gauss usullari bilan yeching.
4. Bir jinsli tenglamalar sistemalarini yeching.

1-variant

1. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & -4 & 3 \\ 3 & 4 & -1 & 2 \\ 4 & 3 & -2 & 1 \end{vmatrix}, i = 1, j = 2.$ **2.** $A = \begin{pmatrix} 5 & 4 & 2 \\ 3 & 2 & 4 \\ 1 & 0 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 4 & -5 \\ 3 & -7 & 1 \\ 1 & 2 & 2 \end{pmatrix}$,

$\alpha = -1, \quad \beta = 4$.

3. a) $\begin{cases} 2x_1 - x_2 - 3x_3 = 4, \\ 3x_1 + 2x_2 - 3x_3 = 15, \\ x_1 - 4x_2 - 3x_3 = 6. \end{cases}$ **b)** $\begin{cases} 3x_1 + x_2 + 2x_3 = 1, \\ x_1 + 3x_2 + 2x_3 = 7, \\ 2x_1 + x_2 + 3x_3 = 6. \end{cases}$

4. a) $\begin{cases} 2x_1 - 3x_2 + \ \ x_3 = 0, \\ \qquad 5x_2 + 2x_3 = 0, \\ 4x_1 - x_2 + 4x_3 = 0. \end{cases}$ **b)** $\begin{cases} x_1 + 3x_2 - \ \ x_3 = 0, \\ 4x_1 - 5x_2 + \ \ x_3 = 0, \\ 3x_1 - x_2 + 4x_3 = 0. \end{cases}$

2-variant

1. $\begin{vmatrix} -1 & 1 & -2 & 3 \\ 1 & 2 & 2 & 3 \\ -2 & 3 & 1 & 0 \\ 2 & 3 & -2 & 0 \end{vmatrix}, \ i = 3, \ j = 2.$ **2.** $A = \begin{pmatrix} 3 & -1 & 0 \\ 3 & 5 & 1 \\ 4 & -7 & 5 \end{pmatrix}, \ B = \begin{pmatrix} -1 & 0 & 2 \\ 1 & -8 & 5 \\ 3 & 0 & 2 \end{pmatrix},$

$\alpha = -3, \ \beta = 5.$

3. a) $\begin{cases} 4x_1 - \ \ x_2 + 2x_3 = 1, \\ 2x_1 - 3x_2 - \ \ x_3 = 7, \\ -2x_1 + 8x_2 + 5x_3 = 10. \end{cases}$ **b)** $\begin{cases} 2x_1 - x_2 + 2x_3 = \ \ 3, \\ x_1 + x_2 + 2x_3 = -4, \\ 4x_1 + x_2 + 4x_3 = -3. \end{cases}$

4. a) $\begin{cases} 4x_1 - 2x_2 + \ \ x_3 = 0, \\ 3x_1 + \ \ x_2 - 3x_3 = 0, \\ 2x_1 + 4x_2 - 7x_3 = 0. \end{cases}$ **b)** $\begin{cases} 4x_1 - 3x_2 - \ \ x_3 = 0, \\ 3x_1 + \ \ x_2 - 2x_3 = 0, \\ x_1 + 6x_2 \qquad\ \ = 0. \end{cases}$

3-variant

1. $\begin{vmatrix} 2 & -2 & 0 & 3 \\ 3 & 2 & 1 & -1 \\ 1 & 1 & -2 & 1 \\ 3 & 4 & -4 & 0 \end{vmatrix}, \quad i = 3, \quad j = 4.$ **2.** $A = \begin{pmatrix} 5 & -8 & -4 \\ 7 & 0 & -5 \\ 4 & 1 & 0 \end{pmatrix}, \ B = \begin{pmatrix} 1 & 5 & 5 \\ 1 & 2 & 1 \\ 2 & -1 & -3 \end{pmatrix},$

$\alpha = 5, \ \beta = -1.$

3. a) $\begin{cases} 3x_1 + x_2 - \ \ 5x_3 = 0, \\ 2x_1 + x_2 + \ \ 3x_3 = 7, \\ 4x_1 + x_2 - 13x_3 = 2. \end{cases}$ **b)** $\begin{cases} 3x_1 + \ \ x_2 - 2x_3 = \ \ 6, \\ 5x_1 - 3x_2 + 2x_3 = -4, \\ 4x_1 - 2x_2 - 3x_3 = -2. \end{cases}$

4. a) $\begin{cases} 2x_1 + \ \ 5x_2 - \ \ x_3 = 0, \\ 2x_1 + 11x_2 - 5x_3 = 0, \\ 2x_1 - \ \ \ x_2 + 3x_3 = 0. \end{cases}$ **b)** $\begin{cases} 2x_1 - \ \ x_2 + 3x_3 = 0, \\ 3x_1 + 2x_2 - 2x_3 = 0, \\ x_1 - 3x_2 + 4x_3 = 0. \end{cases}$

4-variant

1. $\begin{vmatrix} 6 & 0 & -1 & 1 \\ 2 & -2 & 0 & 1 \\ 1 & 1 & -3 & 3 \\ 4 & 1 & -1 & 2 \end{vmatrix}, i=2,\ \ j=2.$

2. $A=\begin{pmatrix} 5 & -8 & -4 \\ 7 & 0 & -5 \\ 4 & 1 & 0 \end{pmatrix},\ B=\begin{pmatrix} 1 & 5 & 5 \\ 1 & 2 & 1 \\ 2 & -1 & -3 \end{pmatrix},\quad \alpha=-3,\ \ \beta=1.$

3. a) $\begin{cases} 4x_1+2x_2-\ x_3=11, \\ 3x_1-\ x_2+4x_3=-6, \\ 5x_1+5x_2-6x_3=26. \end{cases}$ **b)** $\begin{cases} 3x_1-\ x_2+\ x_3=-11, \\ 5x_1+\ x_2+2x_3=\ \ 8, \\ x_1+2x_2+4x_3=\ 16. \end{cases}$

4. a) $\begin{cases} 5x_1-\ x_2-3x_3=0, \\ 3x_1+2x_2+\ x_3=0, \\ x_1+5x_2+5x_3=0. \end{cases}$ **b)** $\begin{cases} x_1+7x_2-3x_3=0, \\ 4x_1-\ x_2+3x_3=0, \\ 6x_1+4x_2-2x_3=0. \end{cases}$

5-variant

1. $\begin{vmatrix} 1 & -1 & 0 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 2 & -1 & 3 \\ 4 & 0 & 1 & 2 \end{vmatrix},\ i=3,\ \ j=1.$

2. $A=\begin{pmatrix} 1 & 2 & 1 \\ 1 & -2 & 4 \\ 3 & -5 & 3 \end{pmatrix},\ \ B=\begin{pmatrix} 7 & 5 & 1 \\ 5 & 3 & -1 \\ 1 & 2 & 3 \end{pmatrix},\qquad \alpha=-1,\ \beta=-3.$

3. a) $\begin{cases} 2x_1+4x_2-\ 5x_3=10, \\ 3x_1-3x_2+\ 4x_3=\ 1, \\ x_1+11x_2-14x_3=18. \end{cases}$ **b)** $\begin{cases} x_1-3x_2-\ x_3=\ 1, \\ 2x_1+x_2+\ x_3=-7, \\ 2x_1-x_2-3x_3=\ 5. \end{cases}$

4. a) $\begin{cases} 4x_1+\ x_2-3x_3=0, \\ 5x_1+2x_2-\ x_3=0, \\ x_1+\ x_2+2x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1+3x_2-\ x_3=0, \\ x_1-\ x_2+3x_3=0, \\ 3x_1+5x_2+\ x_3=0. \end{cases}$

6-variant

1. $\begin{vmatrix} 5 & 0 & -4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & -1 & 1 \end{vmatrix}$, $i=2, \quad j=4.$

2. $A=\begin{pmatrix} 3 & 1 & 2 \\ -1 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix}, \; B=\begin{pmatrix} 0 & -1 & 2 \\ 2 & 1 & 1 \\ 3 & 7 & 1 \end{pmatrix}, \quad \alpha=1, \; \beta=1.$

3. a) $\begin{cases} 5x_1-4x_2+x_3=6, \\ 3x_1+2x_2-x_3=3, \\ x_1+8x_2-3x_3=2. \end{cases}$ **b)** $\begin{cases} x_1+2x_2+x_3=8, \\ 4x_1-3x_2-2x_3=-1, \\ 2x_1-x_2+3x_3=1. \end{cases}$

4. a) $\begin{cases} 5x_1+x_2-4x_3=0, \\ 2x_1-3x_2+2x_3=0, \\ x_1-10x_2+10x_3=0. \end{cases}$ **b)** $\begin{cases} 4x_1+2x_2-3x_3=0, \\ x_1+x_2+2x_3=0, \\ 3x_1+2x_2-2x_3=0. \end{cases}$

7-variant

1. $\begin{vmatrix} 1 & 8 & 2 & -3 \\ 3 & -2 & 0 & 4 \\ 5 & -3 & 7 & -1 \\ 3 & 2 & 0 & 2 \end{vmatrix}$, $i=1, \; j=4.$

2. $A=\begin{pmatrix} 6 & 7 & 3 \\ 3 & 1 & 0 \\ 2 & 2 & 1 \end{pmatrix}, \; B=\begin{pmatrix} 2 & 0 & 5 \\ 4 & -1 & 2 \\ 4 & 3 & 7 \end{pmatrix}, \; \alpha=1, \; \beta=3.$

3. a) $\begin{cases} 4x_1+x_2-3x_3=3, \\ 5x_1+2x_2-x_3=5, \\ x_1+x_2+2x_3=-2. \end{cases}$ **b)** $\begin{cases} 2x_1+3x_2-x_3=2, \\ x_1-x_2+3x_3=-4, \\ 3x_1+5x_2+x_3=4. \end{cases}$

4. a) $\begin{cases} 2x_1-x_2-3x_3=0, \\ 3x_1+2x_2-3x_3=0, \\ x_1-4x_2-3x_3=0. \end{cases}$ **b)** $\begin{cases} 3x_1+x_2+2x_3=0, \\ x_1+3x_2+2x_3=0, \\ 2x_1+x_2+3x_3=0. \end{cases}$

8-variant

1. $\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & -3 & 2 \\ 3 & 0 & 2 & 1 \\ 3 & -1 & -4 & 3 \end{vmatrix}$, $i=2, \quad j=4.$

2. $A=\begin{pmatrix} -2 & 3 & 4 \\ 3 & -1 & -4 \\ -1 & 2 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 3 & 1 \\ 0 & 6 & 2 \\ 1 & 9 & 2 \end{pmatrix}$, $\alpha=2, \quad \beta=-2$.

3. a) $\begin{cases} 5x_1+x_2-4x_3=-3, \\ 2x_1-3x_2+2x_3=13, \\ x_1-10x_2+10x_3=30. \end{cases}$ **b)** $\begin{cases} 4x_1+2x_2-3x_3=-2, \\ x_1+x_2+2x_3=5, \\ 3x_1+2x_2-2x_3=-1. \end{cases}$

4. a) $\begin{cases} 4x_1-x_2+2x_3=0, \\ 2x_1-3x_2-x_3=0, \\ -2x_1+8x_2+5x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1-x_2+2x_3=0, \\ x_1+x_2+2x_3=0, \\ 4x_1+x_2+4x_3=0. \end{cases}$

9-variant

1. $\begin{vmatrix} 0 & 4 & 1 & 1 \\ -4 & 2 & 1 & 3 \\ 0 & 1 & 2 & -2 \\ 1 & 3 & 4 & -3 \end{vmatrix}$, $i=4, \quad j=3.$

2. $A=\begin{pmatrix} -3 & 4 & 2 \\ 1 & 5 & 3 \\ 0 & 1 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 4 & 4 \\ 1 & 3 & 2 \\ 4 & 1 & 2 \end{pmatrix}$, $\alpha=-5, \ \beta=1$.

3. a) $\begin{cases} 2x_1+6x_2-3x_3=-3, \\ 3x_1-2x_2+x_3=12, \\ x_1+14x_2-7x_3=-8. \end{cases}$ **b)** $\begin{cases} 2x_1-x_2+5x_3=27, \\ 5x_1+2x_2+13x_3=70, \\ 3x_1-x_3=-2. \end{cases}$

4. a) $\begin{cases} 4x_1+x_2-3x_3=0, \\ x_1-2x_2+x_3=0, \\ 5x_1-x_2-2x_3=0. \end{cases}$ **b)** $\begin{cases} 5x_1+x_2-2x_3=0, \\ 2x_1-x_2+3x_3=0, \\ 2x_1+7x_3=0. \end{cases}$

10-variant

1. $\begin{vmatrix} 0 & -2 & 1 & 7 \\ 4 & -8 & 2 & -3 \\ 10 & 1 & -5 & 4 \\ -8 & 3 & 2 & -1 \end{vmatrix}$, $i=4,\ \ j=2.$

2. $A=\begin{pmatrix} -1 & 0 & 2 \\ 2 & 3 & 2 \\ 3 & 7 & 1 \end{pmatrix},\ \ B=\begin{pmatrix} 3 & 0 & 1 \\ -3 & 1 & 7 \\ 1 & 3 & 2 \end{pmatrix},\ \ \ \alpha=-1,\ \ \beta=4.$

3. a) $\begin{cases} 3x_1-2x_2+\ x_3=\ -6, \\ 7x_1-9x_2+5x_3=-10, \\ 2x_1+3x_2-2x_3=2. \end{cases}$ **b)** $\begin{cases} 4x_1+\ x_2-3x_3=-6, \\ 8x_1+3x_2-6x_3=-15, \\ x_1+\ x_2-\ x_3=-4. \end{cases}$

4. a) $\begin{cases} 4x_1-x_2+\ 3x_3=0, \\ 5x_1\qquad -\ 7x_3=0, \\ x_1+x_2-10x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1-\ x_2-3x_3=0, \\ x_1+5x_2+\ x_3=0, \\ 3x_1+4x_2+2x_3=0. \end{cases}$

11-variant

1. $\begin{vmatrix} 5 & -3 & 7 & -1 \\ 3 & 2 & 0 & 2 \\ 2 & 1 & 4 & -6 \\ 3 & -2 & 9 & -4 \end{vmatrix}$, $i=3,\ \ j=4.$

2. $A=\begin{pmatrix} 1 & 7 & 3 \\ -4 & 9 & 4 \\ 0 & 3 & 2 \end{pmatrix},\ \ B=\begin{pmatrix} 6 & 5 & 2 \\ 1 & 9 & 2 \\ 4 & 5 & 2 \end{pmatrix},\ \ \alpha=-3,\ \ \beta=-2.$

3. a) $\begin{cases} 2x_1-3x_2+\ x_3=-1, \\ \qquad 5x_2+2x_3=2, \\ 4x_1-x_2+4x_3=-3. \end{cases}$ **b)** $\begin{cases} x_1+3x_2-\ x_3=0, \\ 4x_1-5x_2+\ x_3=7, \\ 3x_1-x_2+4x_3=-4. \end{cases}$

4. a) $\begin{cases} 2x_1+3x_2-\ x_3=0, \\ 5x_1-\ x_2+2x_3=0, \\ x_1-7x_2+4x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1-\ x_2+2x_3=0, \\ x_1+\ x_2+2x_3=0, \\ 4x_1+\ x_2+4x_3=0. \end{cases}$

12-variant

1. $\begin{vmatrix} 4 & -1 & 1 & 5 \\ 0 & 2 & -2 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 1 & 2 \end{vmatrix}, i=1, \ j=2.$

2. $A=\begin{pmatrix} 2 & 6 & 1 \\ 1 & 3 & 2 \\ 0 & 1 & 1 \end{pmatrix}, \ B=\begin{pmatrix} 4 & -3 & 2 \\ -4 & 0 & 5 \\ 3 & 2 & -3 \end{pmatrix}, \ \alpha=1, \ \beta=2.$

3. a) $\begin{cases} 2x_1+5x_2-x_3=1, \\ 2x_1+11x_2-5x_3=3, \\ 2x_1-x_2+3x_3=1. \end{cases}$ **b)** $\begin{cases} 2x_1-x_2+3x_3=1, \\ 3x_1+2x_2-2x_3=1, \\ x_1-3x_2+4x_3=3. \end{cases}$

4. a) $\begin{cases} 3x_1-2x_2+x_3=0, \\ 4x_1-x_2-2x_3=0, \\ 2x_1-3x_2+4x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1+x_2+3x_3=0, \\ x_1-5x_2-x_3=0, \\ 3x_1+4x_2+x_3=0. \end{cases}$

13-variant

1. $\begin{vmatrix} 2 & 1 & 2 & 0 \\ 3 & 4 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & -3 & -2 \end{vmatrix}, \ i=2, \ j=3.$

2. $A=\begin{pmatrix} 6 & 9 & 4 \\ -1 & -1 & 1 \\ 10 & 1 & 7 \end{pmatrix}, \ B=\begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 0 & 5 & 2 \end{pmatrix}, \alpha=5, \ \beta=2.$

3. a) $\begin{cases} 3x_1+x_2-4x_3=-4, \\ x_1+2x_2-x_3=-4, \\ x_1+7x_2=10. \end{cases}$ **b)** $\begin{cases} 4x_1-7x_2=1, \\ 2x_1+x_2-3x_3=-1, \\ 3x_1+5x_3=16. \end{cases}$

4. a) $\begin{cases} 3x_1+x_2-5x_3=0, \\ 2x_1+x_2+3x_3=0, \\ 4x_1+x_2-13x_3=0. \end{cases}$ **b)** $\begin{cases} 3x_1+x_2-2x_3=0, \\ 5x_1-3x_2+2x_3=0, \\ 4x_1-2x_2-3x_3=0. \end{cases}$

14-variant

1. $\begin{vmatrix} 3 & 2 & 0 & -2 \\ 1 & -1 & 2 & 3 \\ 4 & 5 & 1 & 0 \\ -1 & 2 & 3 & -3 \end{vmatrix}$, $i=3,\ \ j=1.$

2. $A=\begin{pmatrix} 1 & 0 & 3 \\ 3 & 1 & 7 \\ 2 & 1 & 8 \end{pmatrix},\ B=\begin{pmatrix} 3 & 5 & 4 \\ -3 & 0 & 1 \\ 5 & 6 & -4 \end{pmatrix},\ \ \alpha=-5,\ \ \beta=-2.$

3. a) $\begin{cases} 4x_1+x_2-3x_3=-4, \\ 2x_1-3x_2+x_3=6, \\ 2x_1-10x_2+6x_3=10. \end{cases}$ **b)** $\begin{cases} 5x_1+7x_2-x_3=1, \\ x_1+7x_3=6, \\ 2x_1-4x_2+5x_3=-1. \end{cases}$

4. a) $\begin{cases} 2x_1+6x_2-3x_3=0, \\ 3x_1-2x_2+x_3=0, \\ x_1+14x_2-7x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1-x_2+5x_3=0, \\ 5x_1+2x_2+13x_3=0, \\ 3x_1-x_3=0. \end{cases}$

15-variant

1. $\begin{vmatrix} 3 & 1 & 2 & -3 \\ 4 & -1 & 2 & 4 \\ 1 & -1 & 1 & 1 \\ 4 & -1 & 2 & 5 \end{vmatrix}$, $i=1,\ \ j=3.$

2. $A=\begin{pmatrix} 5 & 1 & -2 \\ 1 & 3 & -1 \\ 8 & 4 & -1 \end{pmatrix},\ B=\begin{pmatrix} 3 & 5 & 5 \\ 7 & 1 & 2 \\ 1 & 6 & 0 \end{pmatrix},\ \ \alpha=-2,\ \beta=-2.$

3. a) $\begin{cases} 3x_1+7x_2-x_3=1, \\ 2x_1+15x_2+x_3=10, \\ 4x_1-x_2-3x_3=10. \end{cases}$ **b)** $\begin{cases} 3x_1+2x_2-x_3=6, \\ x_1+3x_2+2x_3=9, \\ 4x_1-5x_2+x_3=5. \end{cases}$

4. a) $\begin{cases} 3x_1-2x_2+x_3=0, \\ 7x_1-9x_2+5x_3=0, \\ 2x_1+3x_2-2x_3=0. \end{cases}$ **b)** $\begin{cases} 4x_1+x_2-3x_3=0, \\ 8x_1+3x_2-6x_3=0, \\ x_1+x_2-x_3=0. \end{cases}$

16-variant

1. $\begin{vmatrix} 3 & 1 & 2 & 0 \\ 5 & 0 & -6 & 1 \\ -2 & 2 & 1 & 3 \\ -1 & 3 & 2 & 1 \end{vmatrix}, i=3, \quad j=2.$

2. $A=\begin{pmatrix} 1 & -2 & 5 \\ 3 & 0 & 6 \\ 4 & 3 & 4 \end{pmatrix}, \quad B=\begin{pmatrix} -1 & -1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & -1 \end{pmatrix}, \quad \alpha=-1, \ \beta=-2.$

3. a) $\begin{cases} 5x_1 - x_2 - x_3 = 3, \\ x_1 + 3x_2 + 7x_3 = 8, \\ 3x_1 + x_2 + 3x_3 = 7. \end{cases}$ **b)** $\begin{cases} 2x_1 + x_2 - 3x_3 = 11, \\ 4x_1 + 8x_3 = -4, \\ 5x_1 - 6x_2 = 21. \end{cases}$

4. a) $\begin{cases} x_1 - 2x_2 - 3x_3 = 0, \\ x_1 + 3x_2 - 5x_3 = 0, \\ 2x_1 + x_2 - 8x_3 = 0. \end{cases}$ **b)** $\begin{cases} 3x_1 - x_2 + x_3 = 0, \\ 5x_1 + x_2 + 2x_3 = 0, \\ x_1 + 2x_2 + 4x_3 = 0. \end{cases}$

17-variant

1. $\begin{vmatrix} 3 & 5 & 3 & 2 \\ 2 & 4 & 1 & 0 \\ 1 & -2 & 2 & 1 \\ 5 & 1 & -2 & 4 \end{vmatrix}, \ i=2, \quad j=4$

2. $A=\begin{pmatrix} 2 & -1 & -3 \\ 8 & -7 & -6 \\ -3 & 4 & 2 \end{pmatrix}, \quad B=\begin{pmatrix} 2 & -1 & -2 \\ 3 & -5 & 4 \\ 1 & 2 & 1 \end{pmatrix}, \quad \alpha=1, \ \beta=2.$

3. a) $\begin{cases} 4x_1 + x_2 - 3x_3 = 5, \\ x_1 - 7x_2 + x_3 = 14, \\ 2x_1 + 15x_2 - 5x_3 = -20. \end{cases}$ **b)** $\begin{cases} 3x_1 - x_2 + 3x_3 = 2, \\ 3x_1 + 6x_2 = 3, \\ 2x_1 - 5x_3 = -12. \end{cases}$

4. a) $\begin{cases} 3x_1 + x_2 - 2x_3 = 0, \\ x_1 + 3x_2 - 5x_3 = 0, \\ 5x_1 - x_2 + x_3 = 0. \end{cases}$ **b)** $\begin{cases} 2x_1 - 3x_2 + 4x_3 = 0, \\ 3x_1 + x_2 - 5x_3 = 0, \\ 4x_1 + x_2 + 6x_3 = 0. \end{cases}$

18-variant

1. $\begin{vmatrix} 3 & 2 & 0 & -5 \\ 4 & 3 & -5 & 0 \\ 1 & 0 & -2 & 3 \\ 0 & 1 & -3 & 4 \end{vmatrix}$, $i=1,\ j=2$

2. $A=\begin{pmatrix} 3 & 1 & 0 \\ 4 & 3 & 2 \\ 2 & 2 & -7 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 1 & 0 \\ 4 & 3 & 2 \\ 2 & 2 & -7 \end{pmatrix}$, $\alpha=2,\ \beta=5$.

3. a) $\begin{cases} 3x_1+5x_2-x_3=7, \\ 2x_1+11x_2-5x_3=6, \\ 4x_1-x_2+3x_3=6. \end{cases}$ **b)** $\begin{cases} 2x_1+4x_2-x_3=7, \\ 4x_1-x_2+5x_3=-11, \\ x_1+3x_2-x_3=6. \end{cases}$

4. a) $\begin{cases} 4x_1+2x_2-x_3=0, \\ 3x_1-x_2+4x_3=0, \\ 5x_1+5x_2-6x_3=0. \end{cases}$ **b)** $\begin{cases} 3x_1-x_2+x_3=0, \\ 5x_1+x_2+2x_3=0, \\ x_1+2x_2+4x_3=0. \end{cases}$

19-variant

1. $\begin{vmatrix} 6 & 2 & 10 & 4 \\ 5 & 7 & -4 & 1 \\ 2 & 4 & -2 & -6 \\ 3 & 0 & -5 & 4 \end{vmatrix}$, $i=2, j=3$.

2. $A=\begin{pmatrix} -3 & 4 & 0 \\ 4 & 5 & 1 \\ -2 & 3 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 7 & -1 \\ 0 & 2 & 6 \\ 2 & -1 & 1 \end{pmatrix}$, $\alpha=1,\ \beta=3$.

3. a) $\begin{cases} 3x_1-x_2+2x_3=0, \\ 4x_1+3x_3=4, \\ x_1+x_2+x_3=-2. \end{cases}$ **b)** $\begin{cases} 3x_1+5x_2-x_3=1, \\ 2x_1+x_2+x_3=-3, \\ x_1+4x_2-3x_3=2. \end{cases}$

4. a) $\begin{cases} 2x_1+4x_2-5x_3=0, \\ 3x_1-3x_2+4x_3=0, \\ x_1+11x_2-14x_3=0. \end{cases}$ **b)** $\begin{cases} x_1-3x_2-x_3=0, \\ 2x_1+x_2+x_3=0, \\ 2x_1-x_2-3x_3=0. \end{cases}$

20-variant

1. $\begin{vmatrix} -1 & 2 & 4 & 1 \\ 2 & 3 & 0 & 6 \\ 2 & 2 & 1 & 4 \\ 3 & 1 & 2 & -1 \end{vmatrix}, i=4, j=3.$

2. $A=\begin{pmatrix} -3 & 4 & -3 \\ 1 & 2 & 3 \\ 5 & 0 & -1 \end{pmatrix}, \quad B=\begin{pmatrix} 2 & -2 & 0 \\ 5 & 4 & 1 \\ 1 & 1 & 2 \end{pmatrix}, \quad \alpha=4, \beta=5.$

3. a) $\begin{cases} 4x_1 + x_2 - 3x_3 = 1, \\ x_1 - 2x_2 + x_3 = 2, \\ 5x_1 - x_2 - 2x_3 = -5. \end{cases}$ **b)** $\begin{cases} 5x_1 + x_2 - 2x_3 = 7, \\ 2x_1 - x_2 + 3x_3 = 2, \\ 2x_1 + 7x_3 = 16. \end{cases}$

4. a) $\begin{cases} 5x_1 - 4x_2 + x_3 = 0, \\ 3x_1 + 2x_2 - x_3 = 0, \\ x_1 + 8x_2 - 3x_3 = 0. \end{cases}$ **b)** $\begin{cases} x_1 + 2x_2 + x_3 = 0, \\ 4x_1 - 3x_2 - 2x_3 = 0, \\ 2x_1 - x_2 + 3x_3 = 0. \end{cases}$

21-variant

1. $\begin{vmatrix} 1 & 1 & -2 & 0 \\ 3 & 6 & -2 & 5 \\ 1 & 0 & 6 & 4 \\ 2 & 3 & 5 & -1 \end{vmatrix}, \quad i=4, \quad j=1.$

2. $A=\begin{pmatrix} 3 & 5 & -6 \\ 2 & 4 & 3 \\ -3 & 1 & 1 \end{pmatrix}, \quad B=\begin{pmatrix} 2 & 8 & -5 \\ -3 & -1 & 0 \\ 4 & 5 & 3 \end{pmatrix}, \quad \alpha=3, \quad \beta=2.$

3. a) $\begin{cases} 4x_1 - x_2 + 3x_3 = -8, \\ 5x_1 - 7x_3 = -3, \\ x_1 + x_2 - 10x_3 = 3. \end{cases}$ **b)** $\begin{cases} 2x_1 - x_2 - 3x_3 = -9, \\ x_1 + 5x_2 + x_3 = 20, \\ 3x_1 + 4x_2 + 2x_3 = 15. \end{cases}$

4. a) $\begin{cases} 5x_1 - x_2 - 2x_3 = 0, \\ 3x_1 - 4x_2 + x_3 = 0, \\ 2x_1 + 3x_2 - 3x_3 = 0. \end{cases}$ **b)** $\begin{cases} 7x_1 - 5x_2 + x_3 = 0, \\ 4x_1 + x_3 = 0, \\ 2x_1 + 3x_2 + 4x_3 = 0. \end{cases}$

22-variant

1. $\begin{vmatrix} 2 & 0 & -1 & -3 \\ 6 & 3 & -9 & 0 \\ 0 & 2 & -1 & 3 \\ 4 & 2 & 0 & 6 \end{vmatrix}$, $i=3,\ \ j=3$.

2. $A=\begin{pmatrix} 2 & -1 & 0 \\ 3 & 3 & 1 \\ 4 & -4 & -5 \end{pmatrix},\ B=\begin{pmatrix} -3 & 0 & -2 \\ 1 & -6 & 3 \\ 2 & 0 & 2 \end{pmatrix},\ \ \alpha=2,\ \ \ \beta=-3.$

3. a) $\begin{cases} 2x_1 + 3x_3 = -2, \\ x_1 - x_2 + 2x_3 = -5, \\ x_1 + x_2 + x_3 = 1. \end{cases}$ **b)** $\begin{cases} 4x_1 - x_2 - x_3 = 10, \\ 2x_1 + 6x_2 = 38, \\ 3x_1 - 7x_3 = 5. \end{cases}$

4. a) $\begin{cases} 5x_1 - 5x_2 - 4x_3 = 0, \\ 4x_1 - 4x_2 - 9x_3 = 0, \\ 3x_1 - 3x_2 - 14x_3 = 0. \end{cases}$ **b)** $\begin{cases} x_1 + x_2 + 2x_3 = 0, \\ 4x_1 + x_2 + 4x_3 = 0, \\ 2x_1 - x_2 + 2x_3 = 0. \end{cases}$

23-variant

1. $\begin{vmatrix} -1 & 2 & 0 & 4 \\ 2 & -3 & 1 & 1 \\ 3 & -1 & 2 & 4 \\ 2 & 0 & 1 & 3 \end{vmatrix}$, $i=4,\ \ j=4$.

2. $A=\begin{pmatrix} 2 & -1 & -4 \\ 4 & -9 & 3 \\ 2 & -7 & 1 \end{pmatrix},\ B=\begin{pmatrix} 0 & 0 & -4 \\ 5 & -6 & 4 \\ 7 & -4 & 1 \end{pmatrix},\ \ \alpha=-5,\ \beta=1.$

3. a) $\begin{cases} x_1 - 2x_2 - 3x_3 = 3, \\ x_1 + 3x_2 - 5x_3 = 0, \\ 2x_1 + x_2 - 8x_3 = 4. \end{cases}$ **b)** $\begin{cases} 3x_1 - x_2 + x_3 = 12, \\ 5x_1 + x_2 + 2x_3 = 3, \\ x_1 + 2x_2 + 4x_3 = 6. \end{cases}$

4. a) $\begin{cases} 3x_1 - x_2 + 2x_3 = 0, \\ 4x_1 + 3x_3 = 0, \\ x_1 + x_2 + x_3 = 0. \end{cases}$ **b)** $\begin{cases} 3x_1 + 5x_2 - x_3 = 0, \\ 2x_1 + x_2 + x_3 = 0, \\ x_1 + 4x_2 - 3x_3 = 0. \end{cases}$

24-variant

1. $\begin{vmatrix} 4 & 1 & 2 & 0 \\ -1 & 2 & 1 & -1 \\ 3 & 1 & 2 & 1 \\ 5 & 0 & 4 & 4 \end{vmatrix}, \quad i=3, \quad j=2.$

2. $A=\begin{pmatrix} 8 & 5 & -1 \\ 1 & 5 & 3 \\ 1 & 1 & 0 \end{pmatrix}, \quad B=\begin{pmatrix} 4 & -7 & -6 \\ 3 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}, \quad \alpha=-1, \beta=-2.$

3. a) $\begin{cases} 3x_1 + x_2 - 2x_3 = 5, \\ x_1 + 3x_2 - 5x_3 = 3, \\ 5x_1 - x_2 + x_3 = 1. \end{cases}$ **b)** $\begin{cases} 2x_1 - 3x_2 + 4x_3 = 3, \\ 3x_1 + x_2 - 5x_3 = 10, \\ 4x_1 + x_2 + 6x_3 = 1. \end{cases}$

4. a) $\begin{cases} 3x_1 + x_2 - 4x_3 = 0, \\ x_1 + 2x_2 - x_3 = 0, \\ x_1 + 7x_2 = 0. \end{cases}$ **b)** $\begin{cases} 4x_1 - 7x_2 = 0, \\ 2x_1 + x_2 - 3x_3 = 0, \\ 3x_1 + 5x_3 = 0. \end{cases}$

25-variant

1. $\begin{vmatrix} 4 & 3 & -2 & -1 \\ 2 & 1 & -4 & 3 \\ 0 & 4 & 1 & -2 \\ 5 & 0 & 1 & -1 \end{vmatrix}, \quad i=2, \quad j=3.$

2. $A=\begin{pmatrix} 2 & 1 & -1 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, \quad B=\begin{pmatrix} 3 & 6 & 0 \\ 2 & 4 & 6 \\ 1 & -2 & 3 \end{pmatrix}, \quad \alpha=3, \quad \beta=5.$

3. a) $\begin{cases} 5x_1 - x_2 - 2x_3 = 1, \\ 3x_1 - 4x_2 + x_3 = 7, \\ 2x_1 + 3x_2 - 3x_3 = 4. \end{cases}$ **b)** $\begin{cases} 7x_1 - 5x_2 + x_3 = -33, \\ 4x_1 + x_3 = -7, \\ 2x_1 + 3x_2 + 4x_3 = 12. \end{cases}$

4. a) $\begin{cases} 4x_1 + x_2 - 3x_3 = 0, \\ x_1 - 7x_2 + x_3 = 0, \\ 2x_1 + 15x_2 - 5x_3 = 0. \end{cases}$ **b)** $\begin{cases} 3x_1 - x_2 + 3x_3 = 0, \\ 3x_1 + 6x_2 = 0, \\ 2x_1 - 5x_3 = 0. \end{cases}$

26-variant

1. $\begin{vmatrix} 3 & 5 & 1 & 2 \\ 0 & 1 & -1 & -2 \\ 3 & 1 & -3 & 0 \\ 1 & 2 & -1 & 2 \end{vmatrix}, i=4, j=1.$

2. $A=\begin{pmatrix} -6 & 1 & 11 \\ 9 & 2 & 5 \\ 0 & 3 & 7 \end{pmatrix}, \ B=\begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 7 \\ 1 & -3 & 2 \end{pmatrix},\ \ \alpha=2,\ \beta=-1.$

3. a) $\begin{cases} 5x_1-5x_2-\ 4x_3=-3, \\ 4x_1-4x_2-\ 9x_3=0, \\ 3x_1-3x_2-14x_3=1. \end{cases}$ **b)** $\begin{cases} x_1+x_2+2x_3=-4, \\ 4x_1+x_2+4x_3=-3, \\ 2x_1-x_2+2x_3=3. \end{cases}$

4. a) $\begin{cases} 3x_1+5x_2-\ x_3=0, \\ 2x_1+11x_2-5x_3=0, \\ 4x_1-\ x_2+3x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1+4x_2-x_3=0, \\ 4x_1-\ x_2+5x_3=0, \\ x_1+3x_2-\ x_3=0. \end{cases}$

27-variant

1. $\begin{vmatrix} 2 & 7 & 2 & 1 \\ 1 & 1 & -1 & 0 \\ 3 & 4 & 0 & 2 \\ 0 & 5 & -1 & -3 \end{vmatrix},\ i=4,\ \ j=1$

2. $A=\begin{pmatrix} 3 & 1 & 2 \\ -1 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix},\ B=\begin{pmatrix} 0 & -1 & 2 \\ 2 & 1 & 1 \\ 3 & 7 & 1 \end{pmatrix},\ \alpha=3,\ \ \beta=-1.$

3. a) $\begin{cases} 2x_1+3x_2-\ x_3=-7, \\ 5x_1-\ x_2+2x_3=12, \\ x_1-7x_2+4x_3=20. \end{cases}$ **b)** $\begin{cases} 2x_1-\ x_2+2x_3=0, \\ x_1+\ x_2+2x_3=4, \\ 4x_1+\ x_2+4x_3=6. \end{cases}$

4. a) $\begin{cases} 2x_1\qquad+3x_3=0, \\ x_1-\ x_2+2x_3=0, \\ x_1+\ x_2+\ x_3=0. \end{cases}$ **b)** $\begin{cases} 4x_1-\ x_2-\ x_3=0, \\ 2x_1+6x_2\qquad=0, \\ 3x_1\qquad-7x_3=0. \end{cases}$

28-variant

1. $\begin{vmatrix} 4 & -5 & 1 & -5 \\ -3 & 2 & 8 & -2 \\ 5 & 3 & -1 & 3 \\ -2 & 4 & 6 & 8 \end{vmatrix}$, $i=1,\ j=3$

2. $A=\begin{pmatrix} 8 & -1 & -1 \\ 5 & -5 & -1 \\ 10 & 3 & 2 \end{pmatrix},\ B=\begin{pmatrix} 3 & 2 & 5 \\ 3 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix},\qquad \alpha=4,\quad \beta=-4.$

3. a) $\begin{cases} 4x_1-2x_2+x_3=5, \\ 3x_1+x_2-3x_3=5, \\ 2x_1+4x_2-7x_3=4. \end{cases}$ **b)** $\begin{cases} 4x_1-3x_2-x_3=5, \\ 3x_1+x_2-2x_3=-2, \\ x_1+6x_2=-5. \end{cases}$

4. a) $\begin{cases} 4x_1+x_2-3x_3=0, \\ 2x_1-3x_2+x_3=0, \\ 2x_1-10x_2+6x_3=0. \end{cases}$ **b)** $\begin{cases} 5x_1+7x_2-x_3=0, \\ x_1+7x_3=0, \\ 2x_1-4x_2+5x_3=0. \end{cases}$

29-variant

1. $\begin{vmatrix} -1 & -2 & 3 & 4 \\ 2 & 0 & 1 & -1 \\ 3 & -3 & 1 & 0 \\ 4 & 2 & 1 & 2 \end{vmatrix}$, $i=4, j=4.$

2. $A=\begin{pmatrix} 3 & -7 & 2 \\ 1 & -8 & 3 \\ 4 & -2 & 3 \end{pmatrix},\ B=\begin{pmatrix} 0 & 5 & -3 \\ 2 & 4 & 1 \\ 2 & 1 & -5 \end{pmatrix},\qquad \alpha=-1,\quad \beta=2.$

3. a) $\begin{cases} 5x_1-x_2-3x_3=19, \\ 3x_1+2x_2+x_3=-2, \\ x_1+5x_2+5x_3=-20. \end{cases}$ **b)** $\begin{cases} x_1+7x_2-3x_3=9, \\ 4x_1-x_2+3x_3=-8, \\ 6x_1+4x_2-2x_3=0. \end{cases}$

4. a) $\begin{cases} 3x_1+7x_2-x_3=0, \\ 2x_1+15x_2+x_3=0, \\ 4x_1-x_2-3x_3=0. \end{cases}$ **b)** $\begin{cases} 3x_1+2x_2-x_3=0, \\ x_1+3x_2+2x_3=0, \\ 4x_1-5x_2+x_3=0. \end{cases}$

30-variant

1. $\begin{vmatrix} -4 & 1 & 2 & 0 \\ 2 & -1 & 2 & 3 \\ -3 & 0 & 1 & 1 \\ 2 & 1 & 2 & 3 \end{vmatrix}, i=2,\ j=2.$

2. $A=\begin{pmatrix} 4 & 1 & -4 \\ 2 & -4 & 6 \\ 1 & 2 & -1 \end{pmatrix},\quad B=\begin{pmatrix} 0 & -1 & 1 \\ 2 & 5 & 0 \\ 1 & 1 & 2 \end{pmatrix},\ \alpha=-4,\ \beta=4.$

3. a) $\begin{cases} 3x_1-2x_2+x_3=3, \\ 4x_1-x_2-2x_3=6, \\ 2x_1-3x_2+4x_3=2. \end{cases}$ **b)** $\begin{cases} 2x_1+x_2+3x_3=-3, \\ x_1-5x_2-x_3=-10, \\ 3x_1+4x_2+x_3=4. \end{cases}$

4. a) $\begin{cases} 5x_1-x_2-x_3=0, \\ x_1+3x_2+7x_3=0, \\ 3x_1+x_2+3x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1+x_2-3x_3=0, \\ 4x_1+8x_3=0, \\ 5x_1-6x_2=0. \end{cases}$

NAMUNAVIY VARIANT YECHIMI

1.30. $\begin{vmatrix} -4 & 1 & 2 & 0 \\ 2 & -1 & 2 & 3 \\ -3 & 0 & 1 & 1 \\ 2 & 1 & 2 & 3 \end{vmatrix}, i=2,\ j=2.$

a) Determinantni $i=2-$satr elementlari bo‘yicha yoyamiz. Determinantning 9^o xossasiga ko‘ra

$$\Delta=a_{21}A_{21}+a_{22}A_{22}+a_{23}A_{23}+a_{24}A_{24}=-a_{21}M_{21}+a_{22}M_{22}-a_{23}M_{23}+a_{24}A_{24}=.$$

$$=-2\cdot\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}-1\cdot\begin{vmatrix} -4 & 2 & 0 \\ -3 & 1 & 1 \\ 2 & 2 & 3 \end{vmatrix}-2\cdot\begin{vmatrix} -4 & 1 & 0 \\ -3 & 0 & 1 \\ 2 & 1 & 3 \end{vmatrix}+3\cdot\begin{vmatrix} -4 & 1 & 2 \\ -3 & 0 & 1 \\ 2 & 1 & 2 \end{vmatrix}=$$

$$=-2\cdot(3+2+0-0-2-0)-(-12+4+0-0+8+18)-2\cdot(0+2+0-0+4+9)+$$

$$+3(0+2-6-0+4+6)=-6-18-30+18=-36.$$

b)Determinantni $j=2-$ustun elementlari bo‘yicha yoyamiz:

$$\Delta = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32} + a_{42}A_{42} = -a_{12}M_{12} + a_{22}M_{22} - a_{32}M_{32} + a_{42}A_{42} =$$

$$= -1\cdot\begin{vmatrix} 2 & 2 & 3 \\ -3 & 1 & 1 \\ 2 & 2 & 3 \end{vmatrix} - 1\cdot\begin{vmatrix} -4 & 2 & 0 \\ -3 & 1 & 1 \\ 2 & 2 & 3 \end{vmatrix} - 0 + 1\cdot\begin{vmatrix} -4 & 2 & 0 \\ 2 & 2 & 3 \\ -3 & 1 & 1 \end{vmatrix} =$$

$$= -(6+4-18-6-4+18) - (-12+4+0-0+8+18) +$$

$$+(-8-18+0-0+12-4) = -0-18-18 = -36.$$

c) Determinantni $j=2-$ustundagi bittadan boshqa elementlarni nolga aylantirib va shu ustun elementlari bo‘yicha yoyib hisoblaymiz.

Buning uchun:

– 1-satr elementlarini 2- satrning mos elementlariga qo‘shamiz;

– 1-satr elementlarini (-1) ga ko‘paytirib 4-satrning mos elementlariga qo‘shamiz;

– determinantni 2-ustun elementlari bo‘yicha yoyamiz

$$\Delta = \begin{vmatrix} -4 & 1 & 2 & 0 \\ -2 & 0 & 4 & 3 \\ -3 & 0 & 1 & 1 \\ 6 & 0 & 0 & 3 \end{vmatrix} = 1\cdot(-1)^{1+2}\cdot\begin{vmatrix} -2 & 4 & 3 \\ -3 & 1 & 1 \\ 6 & 0 & 3 \end{vmatrix} = -\begin{vmatrix} -2 & 4 & 3 \\ -3 & 1 & 1 \\ 6 & 0 & 3 \end{vmatrix}.$$

Uchinchi tartibli determinantda $2-$ustunning $2-$satri elementidan boshqa elementlarini nolga aylantiramiz. Bunda a_{32} element nolga teng bo‘lgani uchun faqat a_{12} elementni nolga aylantiramiz. Buning uchun 1-satrga (-4) ga ko‘paytirilgan 2-satrni qo‘shamiz, hosil bo‘lgan determinantni $2-$ustun elementlari bo‘yicha yoyamiz va kelib chiqqan ikkinchi tartibli determinantni hisoblaymiz:

$$\Delta = -\begin{vmatrix} 10 & 0 & -1 \\ -3 & 1 & 1 \\ 6 & 0 & 3 \end{vmatrix} = -1\cdot(-1)^{2+2}\cdot\begin{vmatrix} 10 & -1 \\ 6 & 3 \end{vmatrix} = -36.$$

2.30. $A=\begin{pmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{pmatrix}$, $\quad B=\begin{pmatrix}0 & -1 & 1\\ 2 & 5 & 0\\ 1 & 1 & 2\end{pmatrix}$, $\quad \alpha=-4, \quad \beta=4$.

a) $\alpha A+\beta B$ matritsani topish uchun A matritsa elementlarini α ga, B matritsa elementlarini β ga ko‘paytiramiz va hosil qilingan αA va βB matritsalarning mos elementlarini qo‘shamiz:

$$\alpha A+\beta B=(-4)\cdot\begin{pmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{pmatrix}+4\cdot\begin{pmatrix}0 & -1 & 1\\ 2 & 5 & 0\\ 1 & 1 & 2\end{pmatrix}=$$

$$=\begin{pmatrix}-16 & -4 & 16\\ -8 & 16 & -24\\ -4 & -8 & 4\end{pmatrix}+\begin{pmatrix}0 & -4 & 4\\ 8 & 20 & 0\\ 4 & 4 & 8\end{pmatrix}=$$

$$=\begin{pmatrix}-16+0 & -4+(-4) & 16+4\\ -8+8 & 16+20 & -24+0\\ -4+4 & -8+4 & 4+8\end{pmatrix}=\begin{pmatrix}-16 & -8 & 20\\ 0 & 36 & -24\\ 0 & -4 & 12\end{pmatrix}.$$

b) AB martitsani matritsalarni ko‘paytirish qoidasi asosida topamiz:

$$AB=\begin{pmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{pmatrix}\cdot\begin{pmatrix}0 & -1 & 1\\ 2 & 5 & 0\\ 1 & 1 & 2\end{pmatrix}=$$

$$=\begin{pmatrix}0+2-4 & -4+5-4 & 4+0-8\\ 0-8+6 & -2-20+6 & 2+0+12\\ 0+4-1 & -1+10-1 & 1+0-2\end{pmatrix}=\begin{pmatrix}-2 & -3 & -4\\ -2 & -16 & 14\\ 3 & 8 & -1\end{pmatrix}.$$

c) A matritsa determinantini hisoblaymiz:

$$|A|=\begin{vmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{vmatrix}=16+6-16-16-48+2=-56\neq 0.$$

A_{ij} algebraik to‘ldiruvchilarni topamiz:

$$A_{11}=\begin{vmatrix}-4 & 6\\ 2 & -1\end{vmatrix}=-8, \qquad A_{12}=-\begin{vmatrix}2 & 6\\ 1 & -1\end{vmatrix}=8, \qquad A_{13}=\begin{vmatrix}2 & -4\\ 1 & 2\end{vmatrix}=8,$$

$$A_{21}=-\begin{vmatrix}1 & -4\\ 2 & -1\end{vmatrix}=-7,\qquad A_{22}=\begin{vmatrix}4 & -4\\ 1 & -1\end{vmatrix}=0,\qquad A_{23}=-\begin{vmatrix}4 & 1\\ 1 & 2\end{vmatrix}=-7,$$

$$A_{31}=\begin{vmatrix}1 & -4\\ -4 & 6\end{vmatrix}=-10,\quad A_{32}=-\begin{vmatrix}4 & -4\\ 2 & 6\end{vmatrix}=-32,\quad A_{33}=\begin{vmatrix}4 & 1\\ 2 & -4\end{vmatrix}=-18.$$

Bundan

$$A^{-1}=\frac{1}{|A|}\begin{pmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{pmatrix}=\frac{1}{-56}\begin{pmatrix}-8 & -7 & -10\\ 8 & 0 & -32\\ 8 & -7 & -18\end{pmatrix}=\begin{pmatrix}\frac{1}{7} & \frac{1}{8} & \frac{5}{28}\\ -\frac{1}{7} & 0 & \frac{16}{28}\\ -\frac{1}{7} & \frac{1}{8} & \frac{9}{28}\end{pmatrix}.$$

$AA^{-1}=E$ ekanini tekshiramiz:

$$AA^{-1}=\begin{pmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{pmatrix}\cdot\begin{pmatrix}\frac{1}{7} & \frac{1}{8} & \frac{5}{28}\\ -\frac{1}{7} & 0 & \frac{16}{28}\\ -\frac{1}{7} & \frac{1}{8} & \frac{9}{28}\end{pmatrix}=\begin{pmatrix}\frac{4-1+4}{7} & \frac{4+0-4}{8} & \frac{20+16-36}{28}\\ \frac{2+4-6}{7} & \frac{2-0+6}{8} & \frac{10-64+54}{28}\\ \frac{1-2+1}{7} & \frac{1+0-1}{8} & \frac{5+32-9}{28}\end{pmatrix}=E.$$

3.30. a) $\begin{cases}3x_1-2x_2+x_3=3,\\ 4x_1-x_2-2x_3=6,\\ 2x_1-3x_2+4x_3=2.\end{cases}$ **b)** $\begin{cases}2x_1+x_2+3x_3=-3,\\ x_1-5x_2-x_3=-10,\\ 3x_1+4x_2+x_3=4.\end{cases}$

a) Sistemaning kengaytirilgan matritsasi ustida elementar almashtirishlar bajaramiz:

$$C=\left(\begin{array}{ccc|c}3 & -2 & 1 & 3\\ 4 & -1 & -2 & 6\\ 2 & -3 & 4 & 2\end{array}\right)\sim\left(\begin{array}{ccc|c}1 & 3 & -2 & 3\\ -2 & 4 & -1 & 6\\ 4 & 2 & -3 & 2\end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccc|c}1 & 3 & -2 & 3\\ 0 & 10 & -5 & 12\\ 0 & -10 & 5 & -10\end{array}\right)\sim\left(\begin{array}{ccc|c}1 & 3 & -2 & 3\\ 0 & 10 & -5 & 12\\ 0 & 0 & 0 & 2\end{array}\right).$$

$r(A)=2\neq 3=r(C)$. Demak, sistema birgalikda emas.

b) Sistemaning kengaytirilgan matritsasi ustida elementar almashtirishlar bajaramiz:

$$C=\left(\begin{array}{ccc|c} 2 & 1 & 3 & -3\\ 1 & -5 & -1 & -10\\ 3 & 4 & 1 & 4\end{array}\right)\sim \begin{array}{c} \\ -2 \\ -3\end{array}\left(\begin{array}{ccc|c}1&-5&-1&-10\\2&1&3&-3\\3&4&1&4\end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccc|c}1&-5&-1&-10\\0&11&5&17\\0&19&4&34\end{array}\right)\sim :11\left(\begin{array}{ccc|c}1&-5&-1&-10\\0&1&\frac{5}{11}&\frac{17}{11}\\0&19&4&34\end{array}\right)\sim$$

$$\sim \begin{array}{c} \\ \\ -19\end{array}\left(\begin{array}{ccc|c}1&-5&-1&-10\\0&1&\frac{5}{11}&\frac{17}{11}\\0&0&-\frac{51}{11}&\frac{51}{11}\end{array}\right)\sim :\left(-\frac{51}{11}\right)\left(\begin{array}{ccc|c}1&-5&-1&-10\\0&1&\frac{5}{11}&\frac{17}{11}\\0&0&1&-1\end{array}\right).$$

$r(A)=3=3=r(C)$. Demak, sistema aniq sistema.

1) *Sistemani Kramer formulalari bilan yechamiz.*
Sistemaning determinantini va yordamchi determinantlarni hisoblaymiz:

$$\Delta=\begin{vmatrix}2&1&3\\1&-5&-1\\3&4&1\end{vmatrix}=51;\qquad \Delta x_1=\begin{vmatrix}-3&1&3\\-10&-5&-1\\4&4&1\end{vmatrix}=-51;$$

$$\Delta x_2=\begin{vmatrix}2&-3&3\\1&-10&-1\\3&4&1\end{vmatrix}=102;\qquad \Delta x_3=\begin{vmatrix}2&1&-3\\1&-5&-10\\3&4&4\end{vmatrix}=-51;$$

Tenglamaning yechimini Kramer formulalari bilan topamiz:

$$x_1=\frac{\Delta x_1}{\Delta}=\frac{-51}{51}=-1;\quad x_2=\frac{\Delta x_2}{\Delta}=\frac{102}{51}=2;\quad x_3=\frac{\Delta x_3}{\Delta}=\frac{-51}{51}=-1.$$

2) *Sistemani matritsalar usuli bilan yechamiz.*
Sistema uchun $\Delta=51$.

Sistema determinantining algebraik to‘ldiruvchilarini topamiz:

$$A_{11}=\begin{vmatrix}-5 & -1\\ 4 & 1\end{vmatrix}=-1; \qquad A_{12}=-\begin{vmatrix}1 & -1\\ 3 & 1\end{vmatrix}=-4; \qquad A_{13}=\begin{vmatrix}1 & -5\\ 3 & 4\end{vmatrix}=19;$$

$$A_{21}=-\begin{vmatrix}1 & 3\\ 4 & 1\end{vmatrix}=11; \qquad A_{22}=\begin{vmatrix}2 & 3\\ 3 & 1\end{vmatrix}=-7; \qquad A_{23}=-\begin{vmatrix}2 & 1\\ 3 & 4\end{vmatrix}=-5;$$

$$A_{31}=\begin{vmatrix}1 & 3\\ -5 & -1\end{vmatrix}=14; \qquad A_{32}=-\begin{vmatrix}2 & 3\\ 1 & -1\end{vmatrix}=5; \qquad A_{33}=\begin{vmatrix}2 & 1\\ 1 & -5\end{vmatrix}=-11.$$

U holda

$$A^{-1}=\frac{1}{51}\begin{pmatrix}-1 & 11 & 14\\ -4 & -7 & 5\\ 19 & -5 & -11\end{pmatrix}.$$

Tenglamaning yechimini $X=A^{-1}B$ formula bilan topamiz:

$$X=A^{-1}B=\frac{1}{51}\begin{pmatrix}-1 & 11 & 14\\ -4 & -7 & 5\\ 19 & -5 & -11\end{pmatrix}\cdot\begin{pmatrix}-3\\ -10\\ 4\end{pmatrix}=\frac{1}{51}\begin{pmatrix}3-110+56\\ 12+\ 70+20\\ -57+\ 50-44\end{pmatrix}=\frac{1}{51}\begin{pmatrix}-51\\ 102\\ -51\end{pmatrix}=\begin{pmatrix}-1\\ 2\\ -1\end{pmatrix}.$$

Demak, $x_1=-1$, $x_2=2$, $x_3=-1$.

3) *Sistemani Gauss usuli bilan yechamiz.*

Gauss usulining 1-bosqichi yuqorida sistemani tekshirishda uning kengaytirilgan matritsasida bajarildi va quyidagi ko‘rinish hosil qilindi:

$$\left(\begin{array}{ccc|c}1 & -5 & -1 & -10\\ 0 & 1 & \frac{5}{11} & \frac{17}{11}\\ 0 & 0 & 1 & -1\end{array}\right).$$

Gauss usulining 2-bosqichini bajaramiz:

$$\begin{cases}x_1-5x_2-\ x_3=-10,\\ \quad x_2+\frac{5}{11}x_3=\ \frac{17}{11},\\ \quad\quad x_3=-\ 1\end{cases}\Rightarrow\begin{cases}\quad\quad x_3=\ -1,\\ \quad x_2+\frac{5}{11}\cdot(-1)=\ \frac{17}{11},\\ x_1-5x_2-\quad(-1)=-10\end{cases}\Rightarrow$$

$$\begin{cases} \quad\quad\quad x_3 = -1, \\ \quad\quad\quad x_2 = \ \ 2, \\ x_1 - 5\cdot 2_2 = -11 \end{cases} \Rightarrow \begin{cases} x_1 = -1, \\ x_2 = \ \ 2, \\ x_3 = -1. \end{cases}$$

4.30. a) $\begin{cases} 5x_1 - \ x_2 - \ x_3 = 0, \\ \ x_1 + 3x_2 + 7x_3 = 0, \\ 3x_1 + \ x_2 + 3x_3 = 0. \end{cases}$ **b)** $\begin{cases} 2x_1 + x_2 - 3x_3 = 0, \\ 4x_1 \quad\quad + 8x_3 = 0, \\ 5x_1 - 6x_2 \quad\quad = 0. \end{cases}$

a) Sistema matritsasi ustida elementar almashtirishlar bajaramiz:

$$A = \begin{array}{c} \\ -5 \\ -3 \end{array} \begin{pmatrix} 5 & -1 & -1 \\ 1 & 3 & 7 \\ 3 & 1 & 3 \end{pmatrix} \sim \begin{array}{c} \\ \\ -2 \end{array} \begin{pmatrix} 0 & -16 & -36 \\ 1 & 3 & 7 \\ 0 & -8 & -18 \end{pmatrix} \sim \begin{pmatrix} 0 & -16 & -36 \\ 1 & 3 & 7 \\ 0 & 0 & 0 \end{pmatrix}.$$

$r(A) = 2,\ n = 3,\ r < n.$ Demak, sistema cheksiz ko‘p yechimga ega.

Ularni topamiz:

$$\begin{cases} 5x_1 - \ x_2 - \ x_3 = 0, \\ \ x_1 + 3x_2 + 7x_3 = 0 \end{cases} \Rightarrow \begin{cases} 5x_1 - \ x_2 = x_3, \\ \ x_1 + 3x_2 = -7x_3. \end{cases}$$

$$\Delta = \begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix} = 16, \quad \Delta x_1 = \begin{vmatrix} x_3 & -1 \\ -7x_3 & 3 \end{vmatrix} = -4x_3, \quad \Delta x_2 = \begin{vmatrix} 5 & x_3 \\ 1 & -7x_3 \end{vmatrix} = -36x_3.$$

$$x_1 = \frac{\Delta x_1}{\Delta} = -\frac{x_3}{4}, \quad x_2 = \frac{\Delta x_2}{\Delta} = -\frac{9x_3}{4}.$$

Erkin noma’lumni $x_3 = -4k$ $(k -$ ixtiyoriy son) deb, sistemaning umumiy yechimini topamiz: $x_1 = k,\ x_2 = 9k,\ x_3 = -4k.$

b) Sistema matritsasi ustida elementar almashtirishlar bajaramiz:

$$A = \begin{array}{c} \\ :4 \\ \\ \end{array} \begin{pmatrix} 2 & 1 & -3 \\ 4 & 0 & 8 \\ 5 & -6 & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & -3 \\ 1 & 0 & 2 \\ 5 & -6 & 0 \end{pmatrix} \sim \begin{array}{c} \\ -2 \\ -5 \end{array} \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & -3 \\ 5 & -6 & 0 \end{pmatrix} \sim$$

$$\sim \begin{array}{c} \\ 6 \\ \\ \end{array} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -7 \\ 0 & -6 & -10 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -7 \\ 0 & 0 & -52 \end{pmatrix}.$$

$r(A) = 3 = n.$ Demak, sistema yagona $x_1 = 0, x_2 = 0, x_3 = 0$ yechimga ega.