

Tugas 3

X = tekanan dalam kg/cm^2 yang dipakai untuk melebarkan kepingan besi

Y = pelebaran kepingan besi diukur dalam cm^2 . Hasil terhadap delapan pengamatan adalah sbas berikut :

X	1	2	3	4	5	6	7	8
Y	6.0	8.3	8.5	9.2	10.3	11.5	14.0	15.6

Tentukanlah :

a. Persamaan regresi Y atas X

b. Varkov dan β

c. Varians dari \hat{Y} jika $X_k = 6.5$

Penyelesaian :

a.

$$Y = \begin{bmatrix} 6.0 \\ 8.3 \\ 8.5 \\ 9.2 \\ 10.3 \\ 11.5 \\ 14.0 \\ 15.6 \end{bmatrix} \quad X = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 6.0 \\ 8.3 \\ 8.5 \\ 9.2 \\ 10.3 \\ 11.5 \\ 14.0 \\ 15.6 \end{bmatrix}$$

$$(Y^T Y) = [6.0 \ 8.3 \ 8.5 \ 9.2 \ 10.3 \ 11.5 \ 14.0 \ 15.6] \quad = 939.48$$

$$(X^T X) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \\ 1 & 7 \\ 1 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 36 \\ 36 & 204 \end{bmatrix} \quad (X^T X)^{-1} = \begin{bmatrix} \frac{204}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{8}{336} \end{bmatrix}$$

$$(X^T Y) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} 6,0 \\ 8,3 \\ 8,5 \\ 9,2 \\ 10,3 \\ 11,5 \\ 14,0 \\ 15,6 \end{bmatrix} = \begin{bmatrix} 83,4 \\ 428,2 \end{bmatrix}$$

$$\alpha = (X^T X)^{-1} (X^T Y) = \begin{bmatrix} \frac{204}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{8}{336} \end{bmatrix} \begin{bmatrix} 83,4 \\ 428,2 \end{bmatrix} = \begin{bmatrix} 4,76 \\ 1,26 \end{bmatrix}$$

Sehingga persamaan Regresinya adalah $\hat{Y} = 4,76 + 1,26 X$

$$b. \text{JK Regresi } \beta = \beta^T (X^T Y) = \begin{bmatrix} 4,76 & 1,26 \end{bmatrix} \begin{bmatrix} 83,9 \\ 428,2 \end{bmatrix} = 936,516$$

$$\begin{aligned} \text{JK Kekeliruan} &= \text{JK total} - \text{JK regresi } \beta \\ &= 939,48 - 936,516 = 2,964 \end{aligned}$$

$$\begin{aligned} \text{RJK Kekeliruan} &= \text{JK Kekeliruan} / (n-2) \\ &= \frac{2,964}{6} = 0,494 \end{aligned}$$

$$\text{Var}(\beta) = \sigma^2 (X^T X)^{-1}$$

$$= 0,494 \begin{bmatrix} \frac{204}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{8}{336} \end{bmatrix} = \begin{bmatrix} 0,2999 & -0,0529 \\ -0,0529 & 0,0118 \end{bmatrix}$$

$$\text{Var}(\hat{Y}) = X_k (X^T X)^{-1} X_k^T \sigma^2$$

$$= \begin{bmatrix} 1 \\ 6,5 \end{bmatrix} \begin{bmatrix} \frac{204}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{8}{336} \end{bmatrix} \begin{bmatrix} 1 & 6,5 \end{bmatrix} 0,494$$

$$= \begin{bmatrix} \frac{-30}{336} & \frac{16}{336} \end{bmatrix} \begin{bmatrix} 1 & 6,5 \end{bmatrix} 0,494$$

$$= \frac{74}{336} 0,494$$

$$= 0,1088$$

Sumber Variasi	DK	JK
Regresi β	1	936,516
Kekeliruan	6	939,48 - 936,516 = 2,964
Total	7	939,48

$$F_{\text{perhitungan}} = \frac{(\text{JK Regresi}) / 1}{(\text{JK Kekeliruan}) / (n-2)}$$
$$= \frac{936,516 / 1}{2,964 / 6}$$
$$= 1895,78$$

$$F_{\text{perhitungan}} > F_{\alpha; (1; n-2)}$$
$$F_{\alpha; 1; 6} = 5,99$$

H_0 : ditolak maka H_1 diterima