x = tekanan dalam kg/cm² yg dipakai untuk melebarkan kepingan besi.

Y = Pelebaran keping besi diukur dalam cm². Hasil terhadap Pengamaran adalah sbg berikut.

$$X = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$$
$$Y = 6{,}0 \quad 8{,}3 \quad 8{,}5 \quad 9{,}2 \quad 10{,}3 \quad 11{,}5 \quad 14{,}0 \quad 15{,}6$$

Tentukan :

a). Persamaan regresi Y atas a

b). Varkov dan β

c). Varians dan $\hat{Y}$ jika $X_k = 6.5$

Penyelesaian :

a)
$$Y = \begin{bmatrix} 6{,}0 \\ 8{,}3 \\ 8{,}5 \\ 9{,}2 \\ 10{,}3 \\ 11{,}5 \\ 14{,}0 \\ 15{,}6 \end{bmatrix} \qquad X = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

$$(Y^T Y) = \begin{bmatrix} 6{,}0 & 8{,}3 & 8{,}5 & 9{,}2 & 10{,}3 & 11{,}5 & 14{,}0 & 15{,}6 \end{bmatrix} \begin{bmatrix} 6{,}0 \\ 8{,}3 \\ 8{,}5 \\ 9{,}2 \\ 10{,}3 \\ 11{,}5 \\ 14{,}0 \\ 15{,}6 \end{bmatrix} = 939{,}48$$

$$(X^T X) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \\ 1 & 7 \\ 1 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 36 \\ 36 & 204 \end{bmatrix} \qquad (X^T X)^{-1} = \begin{bmatrix} \dfrac{204}{336} & \dfrac{-36}{336} \\ \dfrac{-36}{336} & \dfrac{8}{336} \end{bmatrix}$$

$$(X^T Y) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} 6,0 \\ 8,3 \\ 8,5 \\ 9,2 \\ 10,3 \\ 11,5 \\ 14,0 \\ 15,6 \end{bmatrix} = \begin{bmatrix} 83,4 \\ 428,2 \end{bmatrix}$$

$$\lambda = (X^T X)^{-1}(X^T Y) = \begin{bmatrix} \dfrac{204}{336} & \dfrac{-36}{336} \\ \dfrac{-36}{336} & \dfrac{8}{336} \end{bmatrix} \begin{bmatrix} 83,4 \\ 428,2 \end{bmatrix}$$

$$= \begin{bmatrix} 4,76 \\ 1,26 \end{bmatrix}$$

Sehingga Persamaan Regresinya adalah $\hat{Y} = 4,76 + 1,26 X$

b). Jk Regresi $\beta = \beta^T(X^TY) = [4,76 \quad 1,26]\begin{bmatrix} 83,4 \\ 428,2 \end{bmatrix}$

$$= 936,516$$

Jk kekeliruan = Jk total - Jk regresi β

$$= 939,48 - 936,516 = 2,964$$

RJk kekeliruan = Jk kekeliruan / $(n-2)$

$$= \frac{2,964}{6} = 0,494$$

var $(\beta) = \sigma^2 (X^TX)^{-1}$

$$= 0,494 \begin{bmatrix} \dfrac{204}{336} & \dfrac{-36}{336} \\ \dfrac{-36}{336} & \dfrac{8}{336} \end{bmatrix} = \begin{bmatrix} 0,2999 & -0,0529 \\ -0,0529 & 0,0118 \end{bmatrix}$$

var $(\hat{Y}) = X_k (X^TX)^{-1} X_k^T \sigma^2$

$$= \begin{bmatrix} 1 \\ 6,5 \end{bmatrix} \begin{bmatrix} \dfrac{204}{336} & \dfrac{-36}{336} \\ \dfrac{-36}{336} & \dfrac{16}{336} \end{bmatrix} [1 \quad 6,5] \; 0,494$$

$$= \begin{bmatrix} \dfrac{-36}{336} & \dfrac{16}{336} \end{bmatrix} \begin{bmatrix} 1 \\ 6,5 \end{bmatrix} 0,494$$

$$= \frac{74}{336} \; 0,494$$

$$= 0,1088$$

c).

Sumber Variasi	Dk	Jk
Regresi β	1	936,516
Kekeliruan	6	939,48 − 936,516 = 2,964
Total	7	939,48

$$F_{perhitungan} = \frac{(Jk\ Regresi)/1}{(Jk\ kekeliruan)/(n-2)}$$

$$= \frac{936,516/1}{2,964/6}$$

$$= 1895,78$$

$F_{perhitungan} > F_{\alpha} ; (1 : n-2)$

$F_{\alpha} : 1 : 6 = 5,99$

H_0 : ditolak

H_1 : diterima