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Tugas 3

X : Tekanan dlm kg/cm^2 yg dipakai untuk melebarkan kepingan besi..

Y : Pelebaran keping besi diukur dlm cm^2. Hasil terhadap delapan pengamatan adalah sebagai berikut :

X	1	2	3	4	5	6	7	8
Y	6.0	8.3	8.5	9.2	10.3	11.5	14.0	15.6

Tentukanlah :

a. Persamaan regresi Y atas α

b. Varkov dari β

c. Varians dari $\hat{Y}$ jika $X_k = 6.5$

Penyelesaian:

$$Y = \begin{bmatrix} 6.0 \\ 8.3 \\ 8.5 \\ 9.2 \\ 10.3 \\ 11.5 \\ 14.0 \\ 15.6 \end{bmatrix} \qquad X = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

$$(Y^T Y) = \begin{bmatrix} 6.0 & 8.3 & 8.5 & 9.2 & 10.3 & 11.5 & 14.0 & 15.6 \end{bmatrix} \begin{bmatrix} 6.0 \\ 8.3 \\ 8.5 \\ 9.2 \\ 10.3 \\ 11.5 \\ 14.0 \\ 15.6 \end{bmatrix} = 939.48$$

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$$(X^TX) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \\ 1 & 7 \\ 1 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 36 \\ 36 & 204 \end{bmatrix} \quad (X^TX)^{-1} = \begin{bmatrix} \frac{204}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{8}{336} \end{bmatrix}$$

$$(X^TY) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} 6.0 \\ 8.3 \\ 8.5 \\ 9.2 \\ 10.3 \\ 11.5 \\ 14.0 \\ 15.2 \end{bmatrix} = \begin{bmatrix} 83.4 \\ 428.2 \end{bmatrix}$$

$$\alpha = (X^TX)^{-1}(X^TY) = \begin{bmatrix} \frac{204}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{8}{336} \end{bmatrix} \begin{bmatrix} 83.4 \\ 428.2 \end{bmatrix} = \begin{bmatrix} 4.76 \\ 1.26 \end{bmatrix}$$

Sehingga Persamaan regresinya adalah $\hat{Y} = 4.76 + 1.26X$

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b. JK Regresi $\beta = \beta^T (X^T Y) = [4,76 \quad 1,26] \begin{bmatrix} 83,9 \\ 428,2 \end{bmatrix} = 936,51$

JK Kekeliruan = JK Total − JK Regres β

$= 939,48 - 936,516 = 2,964$

RJK Kekeliruan = JK Kekeliruan / $(n-2)$

$= \frac{2,964}{6} = 0,494$

$Var(\beta) = \sigma^2 (X^T X)^{-1}$

$= 0,494 \begin{bmatrix} \frac{209}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{8}{336} \end{bmatrix} = \begin{bmatrix} 0,2999 & -0,0529 \\ -0,0529 & 0,0118 \end{bmatrix}$

$Var(\hat{Y}) = X_k (X^T X)^{-1} X_k^T \sigma^2$

$= \begin{bmatrix} 1 \\ 6,5 \end{bmatrix} \begin{bmatrix} \frac{209}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{16}{336} \end{bmatrix} [1 \cdot 6,5] \; 0,494$

$= \begin{bmatrix} \frac{-30}{336} & \frac{16}{336} \end{bmatrix} [1 \; 6,5) \; 0,494$

$= \frac{79}{336} \; 0,494$

$= 0,1088$

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Sumber Varias	Df	Jk
Regres: β	1	936,516
Kesalahan	6	939,48 - 936,516 = 2,964
Total	7	939,48

$$F \text{ Perhitungan} = \frac{(Jk \text{ Regres}) / 1}{(Jk \text{ Kesalahan}) / (n-2)}$$

$$= \frac{936,516 / 1}{2,064 / 6}$$

$$= 1895,78$$

F Perhitungan > F α ; (1 : n-2)

F α ; 1 : 6 = 5,99

Ho : ditolak maka H1 diterima