

### Tugas 3

X: Tekanan dalam kg/cm yang dipakai untuk melebarkan kepingan besi

Y: Pelebaran keping besi diukur dalam cm<sup>2</sup>.

Hasil terhadap delapan pengamatan adalah sbb:

X 1 2 3 4 5 6 7 8

Y 6,0 8,3 8,5 9,2 10,3 11,5 14,0 15,6

Tentukanlah:

a. persamaan regresi  $\hat{y}$  atas  $x$

b. varkov dari  $\beta$

c. Varians dari  $\hat{y}$  jika  $x_F = 6,5$

Penyelesaian

a.

Y =	6,0	X =	1
	8,3		2
	8,5		3
	9,2		4
	10,3		5
	11,5		6
	14,0		7
	15,6		8

$$Y^2 Y = [6,0 \cdot 8,3 \cdot 8,5 \cdot 9,2 \cdot 10,3 \cdot 11,5 \cdot 14,0 \cdot 15,6] \begin{matrix} 6,0 \\ 8,3 \\ 8,5 \\ 9,2 \\ 10,3 \\ 11,5 \\ 14,0 \\ 15,6 \end{matrix} = 939,48$$

$$(X^T X) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 8 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \\ 1 & 7 \\ 1 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 36 \\ 36 & 204 \end{bmatrix} (X^T X)^{-1} = \begin{bmatrix} \frac{204}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{8}{336} \end{bmatrix}$$

$$(X^T Y) \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} 6,0 \\ 8,3 \\ 8,5 \\ 9,2 \\ 10,3 \\ 11,5 \\ 14,0 \\ 15,6 \end{bmatrix} = \begin{bmatrix} 83,4 \\ 428,2 \end{bmatrix}$$

$$\alpha : (X^T X)^{-1} (X^T Y) = \begin{bmatrix} \frac{204}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{8}{336} \end{bmatrix} \begin{bmatrix} 83,4 \\ 428,2 \end{bmatrix} = \begin{bmatrix} 4,76 \\ 1,26 \end{bmatrix}$$

Sehingga persamaan regresinya adalah  
 $Y = 4,76 + 1,26x$



$$b. \text{ Jika regresinya } \hat{\beta} = B^{-1} (X^T Y) = \begin{bmatrix} 4.76 & 1.26 \end{bmatrix} \begin{bmatrix} 834 \\ 428.2 \end{bmatrix} = 930.516$$

$$\text{JK Kekeliruan} = \text{JK total} - \text{JK regresi } \hat{\beta}$$

$$= 930.48 - 930.516 = 2.964$$

$$\text{RJK Kekeliruan} = \text{JK Kekeliruan} / (n-2)$$

$$= \frac{2.964}{6} = 0.494$$

$$\text{Var}(\hat{\beta}) = \sigma^2 (X^T X)^{-1}$$

$$= 0.494 \begin{bmatrix} \frac{204}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{8}{336} \end{bmatrix} = \begin{bmatrix} 0.2999 & -0.0529 \\ -0.0529 & 0.0118 \end{bmatrix}$$

$$\text{Var}(\hat{y}) = X_F (X^T X)^{-1} X_F^T \sigma^2$$

$$= \begin{bmatrix} 1 \\ 6.5 \end{bmatrix} \begin{bmatrix} \frac{204}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{8}{336} \end{bmatrix} \begin{bmatrix} 1.65 \end{bmatrix} 0.494$$

$$= \begin{bmatrix} \frac{-30}{336} & \frac{16}{336} \end{bmatrix} \begin{bmatrix} 1.65 \end{bmatrix} 0.494$$

$$= \frac{79}{336} 0.494$$

$$= 0.1188 //$$

Sumber Variasi	DK	JK
Regresi $\beta$	1	936,516
Kekeliruan	6	939,48 - 936,516 = 2,964
Total	7	939,48

$$\begin{aligned}
 F \text{ Perhitungan} &= \frac{(JK \text{ Regresi}) / 1}{(JK \text{ Kekeliruan}) / (n-2)} \\
 &= \frac{936,516 / 1}{2,964 / 6} \\
 &= 1895,78
 \end{aligned}$$

$$\begin{aligned}
 F \text{ Perhitungan} &> F_{\alpha} ; (1 : n-2) \\
 p_{\alpha} &= 1 : 6 = 5,99
 \end{aligned}$$

$H_0$  : ditolak maka  $H_1$  diterima