

Tugas 3

X = tekanan dalam kg/cm^2 yang dipakai untuk melebarkan kepingan besi

Y = pelebaran keping besi diukur dalam cm^2 . Hasil terhadap pengamatan adl sbg berikut:

X	1	2	3	4	5	6	7	8
Y	6,0	8,3	8,5	9,4	10,3	11,5	14,0	15,6

Tentukan :

- Persamaan regresi Y atas X
- Varians dari β
- Varians dari \hat{Y} jika $X_k = 6,5$

Penyelesaian :

$$a) \quad Y = \begin{bmatrix} 6,0 \\ 8,3 \\ 8,5 \\ 9,4 \\ 10,3 \\ 11,5 \\ 14,0 \\ 15,6 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

$$(Y^T Y) = [6,0 \quad 8,3 \quad 8,5 \quad 9,4 \quad 10,3 \quad 11,5 \quad 14,0 \quad 15,6]$$

$$\begin{bmatrix} 6,0 \\ 8,3 \\ 8,5 \\ 9,4 \\ 10,3 \\ 11,5 \\ 14,0 \\ 15,6 \end{bmatrix}$$

$$= 99,48$$

$$(X^T X) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \\ 1 & 7 \\ 1 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 36 \\ 36 & 209 \end{bmatrix} \quad (X^T X)^{-1} = \begin{bmatrix} \frac{209}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{8}{336} \end{bmatrix}$$

$$(X^T Y) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} 6,0 \\ 8,3 \\ 8,5 \\ 9,2 \\ 10,5 \\ 11,5 \\ 14,0 \\ 15,6 \end{bmatrix} = \begin{bmatrix} 83,9 \\ 428,2 \end{bmatrix}$$

$$\alpha = (X^T X)^{-1} (X^T Y) = \begin{bmatrix} \frac{209}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{8}{336} \end{bmatrix} \begin{bmatrix} 83,9 \\ 428,2 \end{bmatrix} = \begin{bmatrix} 9,76 \\ 1,26 \end{bmatrix}$$

sehingga persamaan regresinya adalah $\hat{y} = 9,76 + 1,26x$

$$b. \text{JK Regresi } \beta = \beta^T (X^T Y) = [4,76 \cdot 1,26] \begin{bmatrix} 183,9 \\ 428,2 \end{bmatrix} = 936,516$$

$$\text{JK Kekeliruan} \rightarrow \text{JK Total} - \text{JK Regresi } \beta \\ = 939,48 - 936,516 = 2,964$$

$$\text{JK Kekeliruan} = \text{JK Kekeliruan} / (n-2) \\ = \frac{2,964}{6} = 0,494$$

$$\text{var}(\beta) = \sigma^2 (X^T X)^{-1}$$

$$= 0,494 \begin{bmatrix} \frac{209}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{16}{336} \end{bmatrix} = \begin{bmatrix} 0,2999 & -0,0529 \\ -0,0529 & 0,0118 \end{bmatrix}$$

$$\text{var}(\hat{Y}) = X_u (X^T X)^{-1} X_u^T \sigma^2$$

$$= \begin{bmatrix} 1 \\ 6,5 \end{bmatrix} \begin{bmatrix} \frac{209}{336} & \frac{-36}{336} \\ \frac{-36}{336} & \frac{16}{336} \end{bmatrix} \begin{bmatrix} 1 & 6,5 \end{bmatrix} 0,494$$

$$= \begin{bmatrix} \frac{-30}{336} & \frac{16}{336} \end{bmatrix} \begin{bmatrix} 1 & 6,5 \end{bmatrix} 0,494$$

$$= \frac{74}{336} 0,494$$

$$= 0,1088$$

c) sumber variasi	Dk	Jk
Regresi P	1	936,516
Residual	6	939,48 - 936,516 = 2,964
Total	7	939,48

$$\begin{aligned}
 F_{\text{hitung}} &= \frac{(Jk \text{ Regresi}) / 1}{(Jk \text{ Residual}) / (n-2)} \\
 &= \frac{936,516 / 1}{2,964 / 6} \\
 &= 1895,78
 \end{aligned}$$

$$F_{\text{hitung}} > F_{\alpha ; (1 : n-2)}$$

$$F_{\alpha ; 1 : 6} = 5,99$$

H_0 : ditolak maka H_1 diterima.