

Kelompok 5:

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Tugas menghitung ulang contoh soal Y,X1,X2,X3 Modul 4.2 Regresi Multiple dan membandingkan hasil perhitungan manual dengan hasil perhitungan R

- Perhitungan manual**

Regresi Multiple Modul 4.2.
Dan hasil penelitian diperoleh data sbg :

Y	X ₁	X ₂	X ₃
25,5	1,74	5,3	10,8
31,2	6,32	5,42	9,4
25,9	6,22	8,41	7,2
38,4	10,52	4,63	8,5
18,4	1,19	11,6	9,4
26,7	1,22	5,85	9,9
26,4	4,1	6,62	8
25,9	6,32	8,72	9,1
32	4,08	4,42	8,7
25,2	4,19	7,6	9,2
39,7	10,15	4,83	9,4
35,7	1,72	3,12	7,6
26,5	1,7	5,3	8,2

Apakah koefisien regresi mempunyai arti atau tidak?
↳ penyelesaian.
$$\beta = (X^T X)^{-1} (X^T Y)$$

Y =	X ₁ =	X ₂ =	X ₃ =
25,5	1,74	5,3	10,8
31,2	6,32	5,42	9,4
25,9	6,22	8,41	7,2
38,4	10,52	4,63	8,5
18,4	1,19	11,6	9,4
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32	4,08	4,42	8,7
25,2	4,15	7,6	9,2
39,7	10,15	4,83	9,4
35,7	1,72	3,12	7,6
26,5	1,7	5,3	8,2

$$(X^T X) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1,79 & 6,32 & 6,22 & 10,52 & 11,9 & 1,22 & 4,1 & 6,32 & 4,08 & 4,15 & 10,15 & 1,72 & 1,7 \\ 5,3 & 5,42 & 8,91 & 4,63 & 11,6 & 5,85 & 6,62 & 8,72 & 4,42 & 7,6 & 4,83 & 3,12 & 5,3 \\ 10,8 & 9,4 & 7,2 & 8,5 & 9,4 & 9,9 & 8 & 9,1 & 8,7 & 9,2 & 9,4 & 7,6 & 8,2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1,79 & 5,3 & 10,8 \\ 1 & 6,32 & 5,42 & 9,4 \\ 1 & 6,22 & 8,91 & 7,2 \\ 1 & 10,52 & 4,63 & 8,5 \\ 1 & 11,9 & 11,6 & 9,4 \\ 1 & 1,22 & 5,85 & 9,9 \\ 1 & 4,1 & 6,62 & 8 \\ 1 & 6,32 & 8,72 & 9,1 \\ 1 & 4,08 & 4,42 & 8,7 \\ 1 & 4,15 & 7,6 & 9,2 \\ 1 & 10,15 & 4,83 & 9,4 \\ 1 & 1,72 & 3,12 & 7,6 \\ 1 & 1,7 & 5,3 & 8,2 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 59,43 & 81,82 & 115,4 \\ 59,43 & 394,725 & 360,664 & 522,078 \\ 81,82 & 360,664 & 576,726 & 728,31 \\ 115,4 & 522,078 & 728,31 & 1035,196 \end{bmatrix}$$

$$(X^T Y) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1,79 & 6,32 & 6,22 & 10,52 & 11,9 & 1,22 & 4,1 & 6,32 & 4,08 & 4,15 & 10,15 & 1,72 & 1,7 \\ 5,3 & 5,42 & 8,91 & 4,63 & 11,6 & 5,85 & 6,62 & 8,72 & 4,42 & 7,6 & 4,83 & 3,12 & 5,3 \\ 10,8 & 9,4 & 7,2 & 8,5 & 9,4 & 9,9 & 8 & 9,1 & 8,7 & 9,2 & 9,4 & 7,6 & 8,2 \end{bmatrix}$$

$$\begin{bmatrix} 25,5 \\ 31,2 \\ 25,9 \\ 38,4 \\ 18,4 \\ 24,7 \\ 26,4 \\ 25,9 \\ 3,2 \\ 25,2 \\ 33,7 \\ 35,7 \\ 26,5 \end{bmatrix} = \begin{bmatrix} 377,5 \\ 1877,567 \\ 2246,661 \\ 3337,780 \end{bmatrix}$$

Untuk menentukan persamaan regresi multiple, maka

$$\beta = (X^T X)^{-1} (X^T Y)$$

$$= \begin{bmatrix} 13 & 59,43 & 81,82 & 115,4 \\ 59,43 & 394,725 & 360,662 & 522,078 \\ 81,82 & 360,662 & 576,726 & 728,31 \\ 115,4 & 522,078 & 728,31 & 1035,96 \end{bmatrix}^{-1} \begin{bmatrix} 377,5 \\ 1877,567 \\ 2246,661 \\ 3337,780 \end{bmatrix}$$

$$= \begin{bmatrix} 8,064794635 & -0,08259270531 & -0,09419511495 & -0,7905268759 \\ -0,08259270531 & 0,008479816258 & 0,00176687178 & 0,003720020321 \\ -0,09419511495 & 0,00176687178 & 0,01662442431 & -0,002063307812 \\ -0,7905268759 & 0,003720020321 & -0,002063307812 & 0,08860128817 \end{bmatrix}$$

$$\begin{bmatrix} 377,5 \\ 1877,567 \\ 2246,661 \\ 3337,780 \end{bmatrix} = \begin{bmatrix} 39,15734995 \\ 1,016100491 \\ -1,861649203 \\ -0,3432604926 \end{bmatrix}$$

maka persamaan regresinya adalah.

$$\hat{y} = 39,15734995 + 1,016100491x_1 - 1,861649203x_2 - 0,3432604926x_3$$

• ANOVA

$$JK \text{ Regresi} = \beta^T (X^T Y) - (\sum y_i)^2 / n$$

$$= \begin{bmatrix} 39,15734995 & 1,016100491 & -1,861649203 & -0,3432604926 \end{bmatrix}$$

$$\begin{bmatrix} 377,5 \\ 1877,567 \\ 2246,661 \\ 3337,780 \end{bmatrix} - \frac{(377,5)^2}{13}$$

$$= 11361,473595780436 - 10962,019230769$$

$$= 399,45436501193$$

$$- RJK \text{ regresi} = \frac{JK \text{ regresi}}{k} = \frac{399,45436501143}{3} = 133,15145500381$$

$$- JK \text{ ketelitian} = (Y^T Y) - P^T (X^T Y) \\ = 11400,15 - 11361,473595780436 \\ = 38,676404219564$$

$$- RJK \text{ ketelitian} = \frac{JK \text{ ketelitian}}{n-k-1} = \frac{38,676404219564}{9} = 4,2973782466182$$

$$\bullet \text{ Total} = (Y^T Y) - \frac{(\sum Y_i)^2}{n} = 11400,15 - \frac{10962,019230769}{13} = 438,130769231$$

Daftar Anova yang diperoleh

Sumber variasi	Dk	JK	RJK
Regresi pada $\beta_1, \beta_2, \beta_0$	$k=3$	399,45436501143	133,15145500381
Ketelitian	$n-k-1$ $13-3-1=9$	38,676404219564	4,2973782466182
Total	$n-1$ $13-1=12$	438,130769231	

menguji signifikansi keseluruhan model regresi

• Hipotesis

$H_0 = \beta_j = 0$ (variabel dependen tidak mempunyai hub linear dg variabel independen)

$H_1 =$ min ada satu $\beta_j \neq 0$ (variabel dependen punya hub linear dg variabel independen)

• Statistika uji

$$F \text{ hitung} = \frac{RJK \text{ regresi}}{RJK \text{ residu}} \\ = \frac{133,15145500381}{4,2973782466182} = 30,984346120473$$

$F \text{ tabel} = F_{\alpha; 3; 9} = 3,86$

$F \text{ tabel} < F \text{ hitung}$ maka H_1 diterima

untuk menguji apakah koefisien regresi mempunyai arti / tidak
maka hipotesisnya :

- 1) $H_0 : \beta_1 = 0$ $H_1 : \beta_1 \neq 0$
- 2) $H_0 : \beta_2 = 0$ $H_1 : \beta_2 \neq 0$
- 3) $H_0 : \beta_3 = 0$ $H_1 : \beta_3 \neq 0$

• Statistika uji

$$t = \frac{\beta_j}{\sqrt{C(j+1, j+1) \sigma}}$$

$$1) t = \frac{1,0161}{\sqrt{0,00848 \cdot 2,0731}} = 5,32$$

$$2) t = \frac{-1,8616}{\sqrt{0,016629 \cdot 2,0731}} = -6,96$$

$$3) t = \frac{0,3437}{\sqrt{0,088601 \cdot 2,0731}} = 0,52$$

Sedangkan $t_{0.05 : 9} = 1,833$

Untuk 1 dan 2 H_0 ditolak, untuk 3 H_0 diterima, berarti koefisien regresi
untuk x_3 tidak mempunyai arti, sehingga regresinya menjadi :

$$\hat{y} = 39,15734995 + 1,016100441 x_1 - 1,861649203 x_2$$

Konfidensi Interval

1) Rata-rata respon

aktu* respon bila $x_1 = 3, x_2 = 8, x_3 = 9$

dg konfidensi interval 95%.

→ persamaan regresi y_1 diperoleh

$$\hat{y} = 39,15734995 + 1,016100941x_1 - 1,861649203x_2 - 0,3432604926x_3$$

bila disubstitusikan nilai $x_1 = 3, x_2 = 8, x_3 = 9$ diperoleh $\hat{y} = 24,2322$

$$x_0^T (X^T X)^{-1} x_0 = [1 \ 3 \ 8 \ 9]$$

$$\begin{bmatrix} 8,064794655 & -0,00259270531 & -0,09419511495 & -0,7905268759 \\ -0,00259270531 & 0,008979816238 & 0,001716687178 & 0,003720020321 \\ -0,09419511495 & 0,001716687178 & 0,01662942431 & -0,002063307812 \\ -0,7905268759 & 0,003720020321 & -0,002063307812 & 0,08860128617 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 8 \\ 9 \end{bmatrix} = 0,1267$$

$$t^2 = 4,2977 \text{ maka } t = 2,0731$$

$$t_{\frac{\alpha}{2}} = t_{0,025; 9} = 2,262$$

maka konfidensi intervalnya adalah

$$24,2322 - 2,262(2,0731)\sqrt{0,1267} < \mu_y(x_{10}, x_{20}, \dots, x_{k0}) < 24,2322 + 2,262(2,0731)\sqrt{0,1267}$$

atau

$$22,5633 < \mu_y(x_{10}, x_{20}, \dots, x_{k0}) < 25,9011$$

Artinya dengan tingkat kepercayaan 95% dapat kita katakan bahwa nilai μ_y / 3, 8, 9 ada diantara 22,5633 hingga 25,9011

2) Nilai Respon

$$\hat{y} = 24,2322, \quad \sigma = 2,0731, \quad x_0^T (X^T X)^{-1} x_0 = 0,1267, \quad \text{dan } t_{\frac{\alpha}{2}} = t_{0,025; 9}$$

$$= 2,262 \text{ ; maka}$$

$$24,2322 - 2,262(2,0731)\sqrt{0,1267} < \mu_y(x_{10}, x_{20}, \dots, x_{k0}) < 24,2322 + 2,262(2,0731)\sqrt{0,1267}$$

$$\text{atau } 19,2597 < y_0 < 29,2097$$

Artinya dg tingkat kepercayaan 95% kita memprediksi bahwa nilai y_0 dg $x_1 = 3, x_2 = 8, x_3 = 9$ akan berada diantara 19,2597 dan 29,2097.

- **Perhitungan menggunakan Rstudio**

-Input

```
setwd("C:\\Users\\HP\\Documents")
data<-read.csv("regresimultiple.csv")
data
```

```
##analisis regresi linear ganda
g<-lm(Y~X1+X2+X3,data = data)
summary(g)
anova(g)
```

-Output

```
> setwd("C:\\Users\\HP\\Documents")
> data<-read.csv("regresimultiple.csv")
> data
```

```
      Y      X1      X2      X3
1  25.5  1.74  5.30  10.8
2  31.2  6.32  5.42  9.4
3  25.9  6.22  8.41  7.2
4  38.4 10.52  4.63  8.5
5  18.4  1.19 11.60  9.4
6  26.7  1.22  5.85  9.9
7  26.4  4.10  6.62  8.0
8  25.9  6.32  8.72  9.1
9  32.0  4.08  4.42  8.7
10 25.2  4.15  7.60  9.2
11 39.7 10.15  4.83  9.4
12 35.7  1.72  3.12  7.6
13 26.5  1.70  5.30  8.2
```

```
> ##analisis regresi linear ganda
> g<-lm(Y~X1+X2+X3,data = data)
> summary(g)
```

```
Call:
lm(formula = Y ~ X1 + X2 + X3, data = data)
```

```
Residuals:
    Min     1Q   Median     3Q     Max
-1.8532 -1.4495 -0.3219  0.5919  3.2121
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  39.1573     5.8871   6.651 9.36e-05 ***
X1           1.0161     0.1909   5.323 0.000479 ***
X2          -1.8616     0.2673  -6.964 6.58e-05 ***
X3          -0.3433     0.6171  -0.556 0.591572
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 2.073 on 9 degrees of freedom
Multiple R-squared:  0.9117, Adjusted R-squared:  0.8823
F-statistic: 30.98 on 3 and 9 DF, p-value: 4.496e-05
```

```
> anova(g)
Analysis of Variance Table
```

Response: Y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
X1	1	187.312	187.312	43.5875	9.904e-05	***
X2	1	210.813	210.813	49.0561	6.297e-05	***
X3	1	1.330	1.330	0.3095	0.5916	
Residuals	9	38.676	4.297			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1