EEE 202 CIRCUIT THEORY
LAB 4

Maximum Power Transfer

Şevval ERBAY

22202658

Section 3

SW Implementation

Introduction:

The purpose of this lab is to design two passive linear circuits to transfer the maximum power to the 180 Ω load from a voltage source with impedance 50 Ω. The chosen configuration of the source is a sinusoidal wave with magnitude 5 V peak-to-peak and frequency 8 MHz. Figure.1 is taken from the lab prompt and illustrates the task visually.

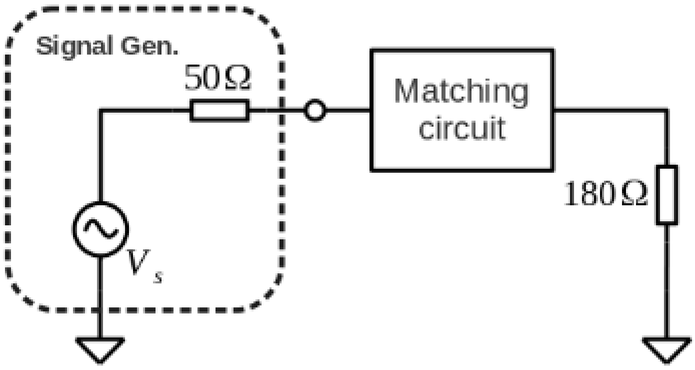

Figure.1 Illustrated circuit (complete)

Analysis:

In order to obtain the maximum power transferred to the load, the total impedance of the source and the load must be complex conjugates of each other and cancel each other. The average power delivered to a resistance is found by using the following formula:

$$P_{\text{average}} = \frac{\mathrm{I}^2 \cdot \mathrm{R}}{2} \tag{1}$$

$$P_{average} = \frac{V^z}{2(R_S + R_L)^2} \cdot R_L \tag{2}$$

For this instance, $\mathrm{R_L} = \mathrm{R_S} = R$. Inserting this to equation (2), we obtain:

$$P_{\text{average}} = \frac{\mathrm{V}^2}{8\mathrm{R}} \tag{3}$$

As requested in the lab prompt, two different methods are used.

- ***Method 1 - T-Section:***

A generic T-Section circuit can be seen in Figure.2. The peak voltage V = 5 V and the frequency is 8 MHz.

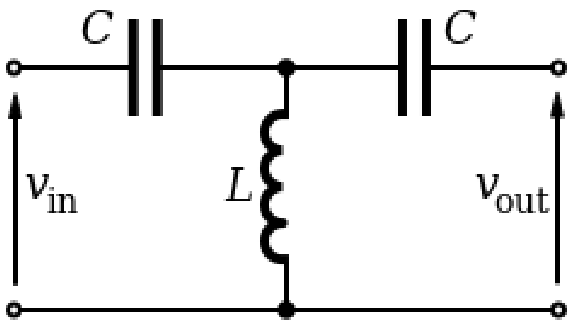

Figure.2 T-Section circuit[1]

Since $Z_s = R_s = 50\,\Omega$ & $Z_L = R_l = 180\,\Omega$ and f = 8 MHz, to calculate the inductance and the capacitance of the values of C and L in Figure.2, impedance inverter formula is used.

$$Z_{input} = Z_S = \frac{X^2}{Z_L}$$

$$X^2 = R_L * R_S \Rightarrow X = \mp 94.87$$

Take X = 94.87, compute L and C by the following formulae and insert into the circuit.

$$jX = j\omega L$$

$$-jX = \frac{1}{j\omega C}$$

$$L = \frac{X}{2\pi f} = \frac{94.87}{2\pi * 8 * 10^6} \approx 1.89\,\mu H$$

$$C = \frac{1}{2\pi f X} = \frac{1}{2\pi * 8 * 10^6 * 94.87} \approx 210\,pF$$

[1] https://tr.m.wikipedia.org/wiki/Dosya:Highpass_T_Filter.svg

- ***Method 2 - L-Section:***

A generic L-Section circuit can be seen in Figure.3. The peak voltage V = 5 V and the frequency is 8 MHz.

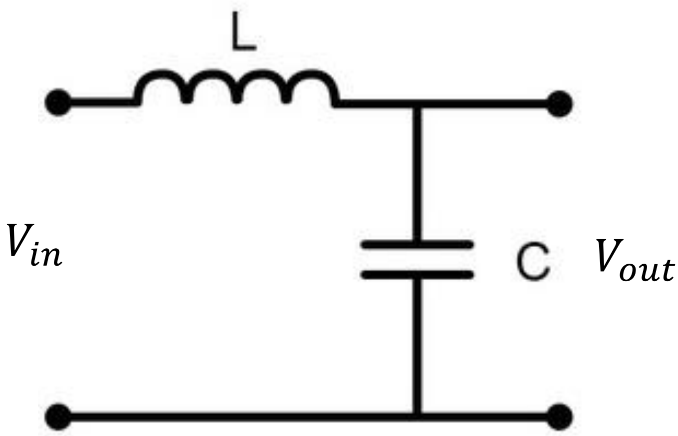

Figure.3 L-Section circuit[2]

Since $Z_S = R_S = 50\ \Omega$ & $Z_L = R_l = 180\ \Omega$ and f = 8 MHz, to calculate the inductance and the capacitance values L and C in Figure.3, the quality factor Q will be used.

$$R_L = R_S * (Q^2 + 1) \Rightarrow Q = \sqrt{\frac{R_L}{R_S} - 1} = \sqrt{\frac{180}{50} - 1} \approx 1.61$$

$$Q = \frac{2\pi f L}{R_S} \Rightarrow L = \frac{R_S Q}{2\pi f} = \frac{50 * 1.61}{2\pi * 8 * 10^6} \approx 1.6\ \mu H$$

$$\omega = 2\pi f = \sqrt{\frac{1}{LC\left(1 + \frac{1}{Q^2}\right)}} \Rightarrow C = \frac{1}{4\pi^2 f^2 L\left(1 + \frac{1}{Q^2}\right)} \approx 179\ pF$$

Since the calculated inductance values for both of the methods are not standard values, T38-8/90 toroidal core will be used to achieve the right inductances. The relationship between the inductance L and the number of turns is expressed as:

$$L = A \cdot N^2$$

where A is the inductance per turn of the toroidal core. This core's A value is 20 nH/N^2. Then, for the T-Section circuit:

$$N = \sqrt{\frac{L}{A}} = \sqrt{\frac{1.89\ \mu H}{20\ nH\ /\ N^2}} \approx 10$$

[2] https://www.quora.com/How-do-you-compare-RC-and-LC-filters

For the L-Section circuit:

$$N = \sqrt{\frac{L}{A}} = \sqrt{\frac{1.6\ \mu H}{20\ nH\ /\ N^2}} \approx 9$$

Simulations:

To find the maximum power that can be delivered, equation (3) is used. V = 5 V and R = 50 Ω are the specifics.

$$\mathrm{P}_{\mathrm{average,max}} = \frac{5^2}{8 * 50} = 62.5\ mW$$

To find the power delivered to the 180 Ω load without the matching circuit, the circuit in Figure.4 is implemented. Figure.5 shows the corresponding power graph.

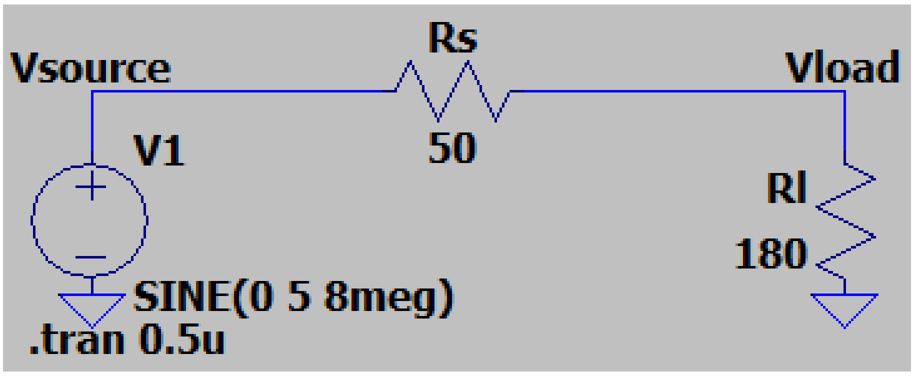

Figure.4 Circuit implementation without the matching circuit

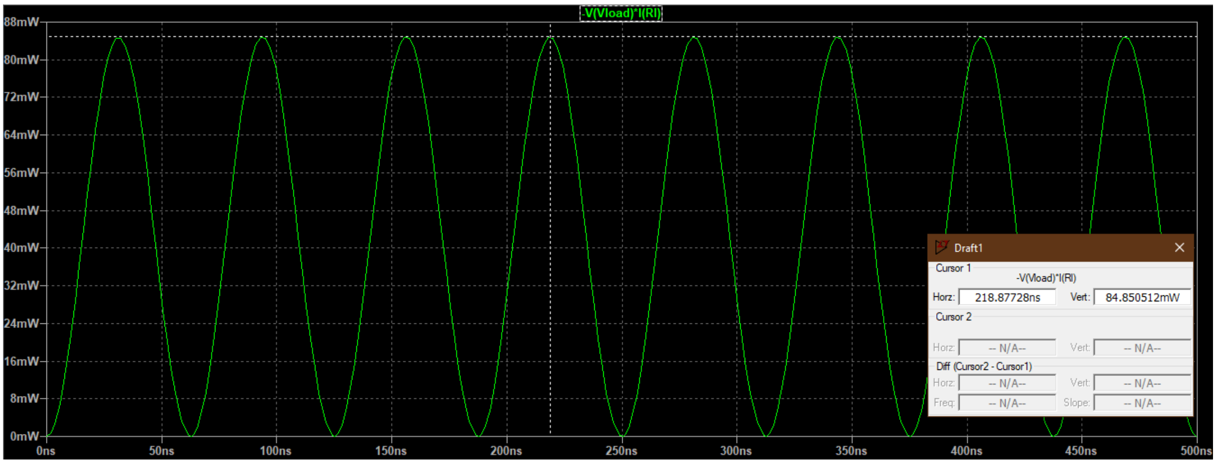

Figure.5 Power graph of Figure.4

$$\mathrm{P}_{\mathrm{average}} = \frac{\mathrm{P}_{\mathrm{maximum}}}{2} = \frac{84.85}{2} = 42.425\ mW$$

The power deliver ratio with respect to the theoretical value is %67.88.

T-Section circuit is implemented as shown in Figure.6. L-Section circuit is implemented as shown in Figure.7. Power graph of T-Section is shown in Figure.8. Power graph of L-Section is shown in Figure.9. The results are discussed in Table.1 and Table.2.

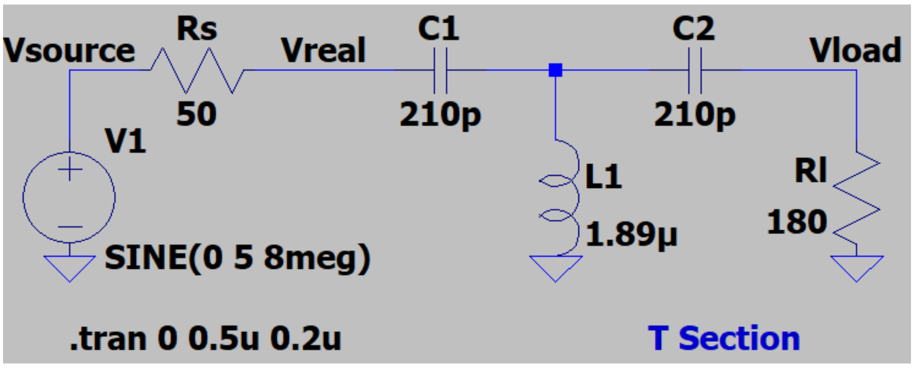

Figure.6 T-Section circuit

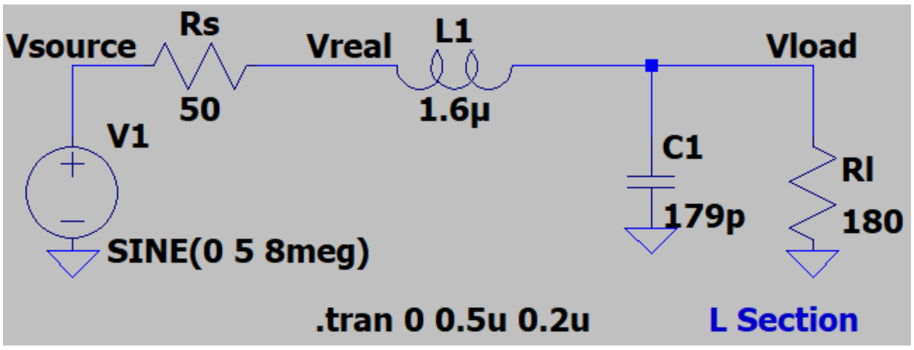

Figure.7 L-Section circuit

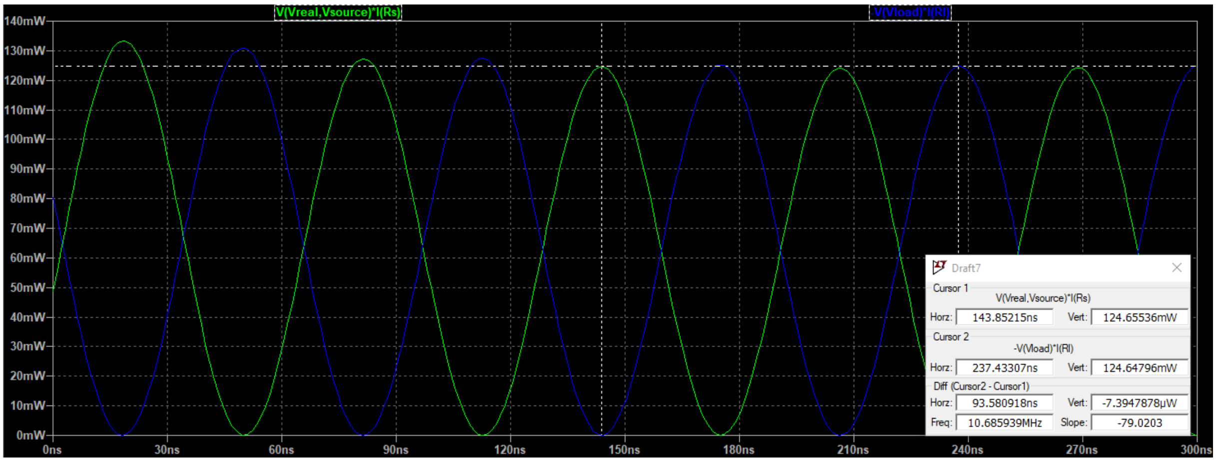

Figure.8 Power graph of (Figure.6) T-Section circuit

$$P_{average} = \frac{P_{maximum}}{2} = \frac{124.65}{2} = 62.325\ mW$$

The power deliver ratio with respect to the theoretical value is %99.72. The power deliver ratio with respect to the power of the source is %99.99.

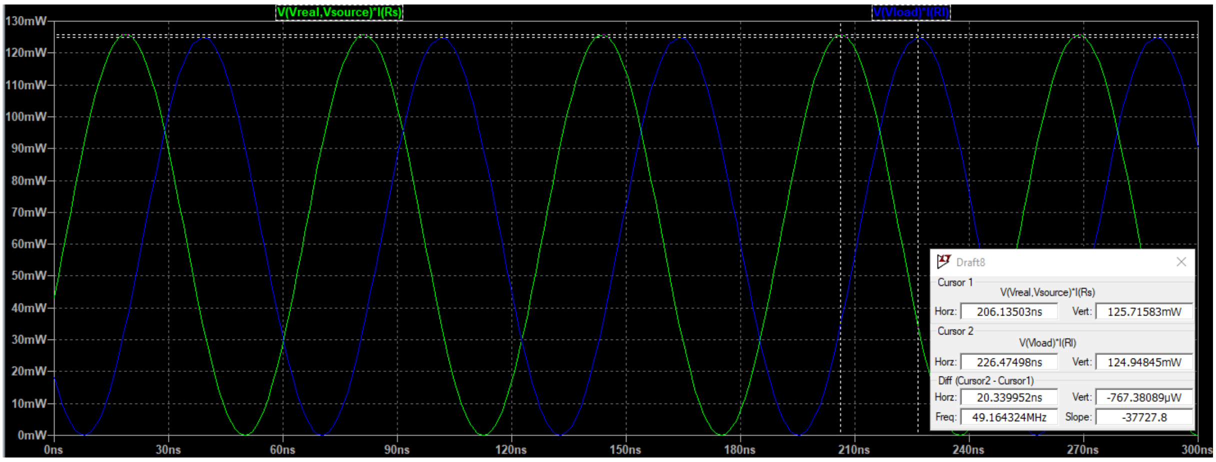

Figure.9 Power graph of (Figure.7) L-Section circuit

$$P_{\text{average}} = \frac{P_{\text{maximum}}}{2} = \frac{124.95}{2} = 62.475\ mW$$

The power deliver ratio with respect to the theoretical value is %99.96. The power deliver ratio with respect to the power of the source is %99.39.

	Maximum power that can be delivered (theoretical)	**Maximum power that is being delivered (simulated)**	**Power deliver ratio with respect to the theoretical value**
Without the matching circuit	62.5 mW	42.425 mW	%67.88

Table.1 Results of Figures.4-5 (implementation without the matching circuit)

	T-Section	**L-Section**
Maximum power that can be delivered (theoretical)	62.5 mW	62.5 mW
Power of the source	62.33 mW	62.86 mW
Maximum power that is being delivered (simulated)	62.325 mW	62.475 mW
Power deliver ratio with respect to the theoretical value	%99.72	%99.96
Power deliver ratio with respect to the power of the source	%99.99	%99.39
Error	%0.01	%0.61

Table.2 Results of Figures.6-9 (implementation with matching circuits)

HW Implementation:

First, a 47 Ω resistor is connected between the terminals of the signal generator which was generating a sinusoidal signal with 5V amplitude and 8 MHz frequency. The voltage passing through the resistor is measured by using an oscilloscope and the power is calculated according to the formula (4). Also, formula (4) is used for all of the power calculations in the hardware lab. Figure.10 shows the setup and Figure.11 holds the corresponding waveform.

$$P_{\text{average}} = \frac{\mathrm{V}^2}{2\mathrm{R}} \tag{4}$$

Figure.10 47 Ω connected between the terminals of the signal generator

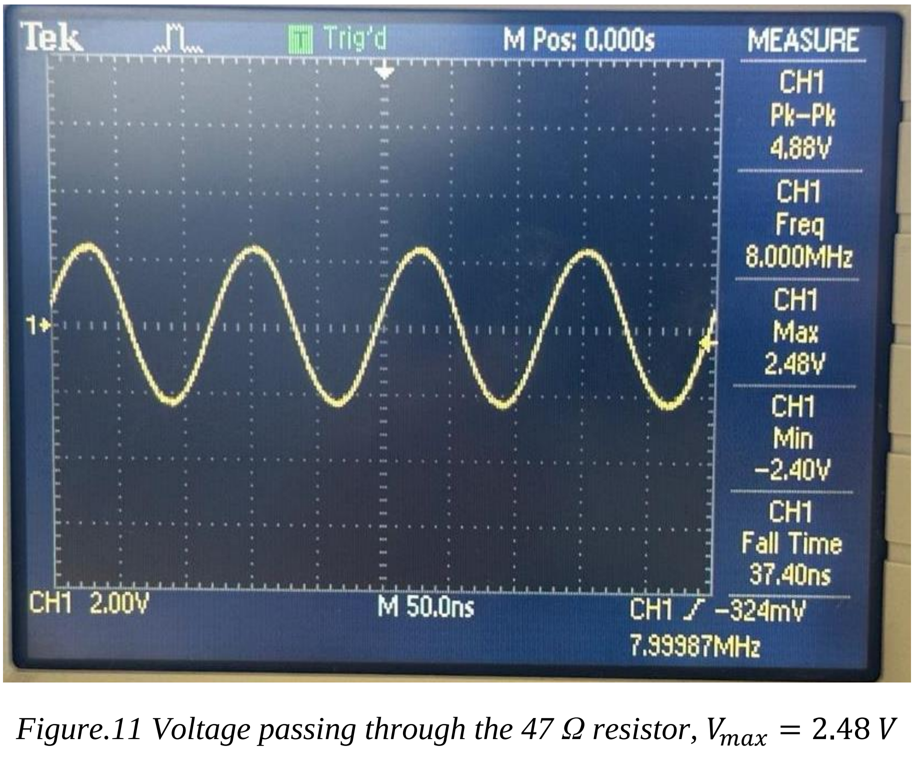

Figure.11 Voltage passing through the 47 Ω resistor, $V_{max} = 2.48\ V$

The power of the source is calculated by taking R = 47 Ω and V = 2.48 V. Thus, power of the source is 65.43 mW.

T-Section circuit is implemented by the winded inductor shown in Figure.12. The circuit is shown in Figure.14. The voltage passing through the 180 Ω resistor is measured by an oscilloscope and the corresponding waveform can be seen in Figure.15. Then the power is calculated by formula (4) by inserting R = 180 V and V = $V_{peak-to-peak} = 9.40\,V$. Power deliver ratios are calculated and the results are discussed in Table.3.

Lastly, L-Section circuit is implemented by the winded inductor shown in Figure.13. The circuit is shown in Figure.16. The voltage passing through the 180 Ω resistor is measured by an oscilloscope and the corresponding waveform can be seen in Figure.17. Then the power is calculated by formula (4) by inserting R = 180 V and V = $V_{peak-to-peak} = 9.60\,V$. Power deliver ratios are calculated and the results are discussed in Table.3.

Figure.12 Inductance of the winded inductor used in T-Section implementation

Figure.13 Inductance of the winded inductor used in L-Section implementation

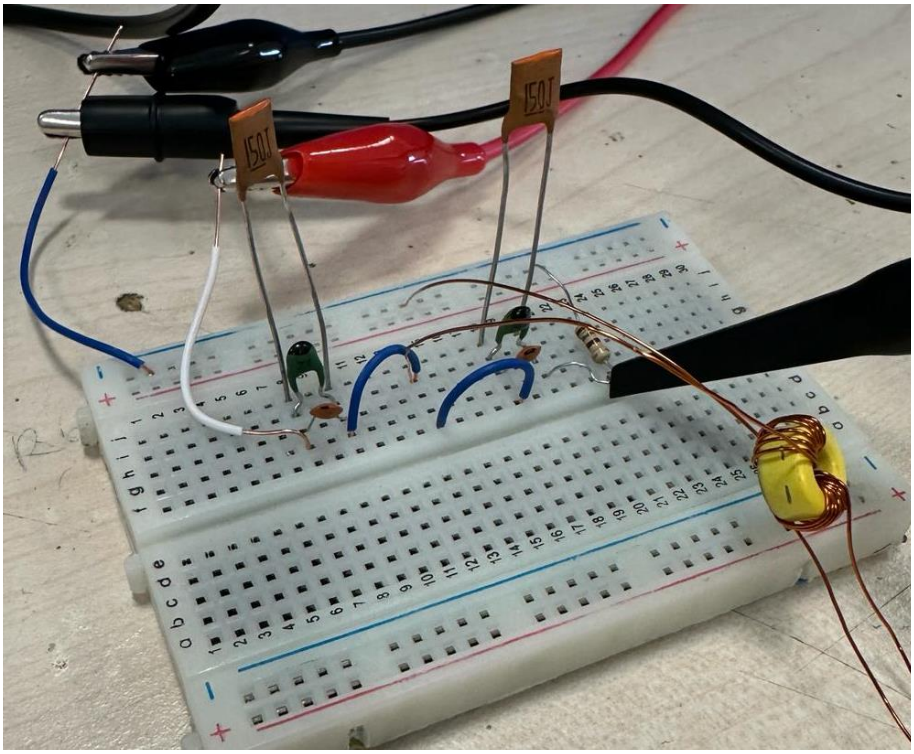

Figure.14 T-Section implementation on breadboard

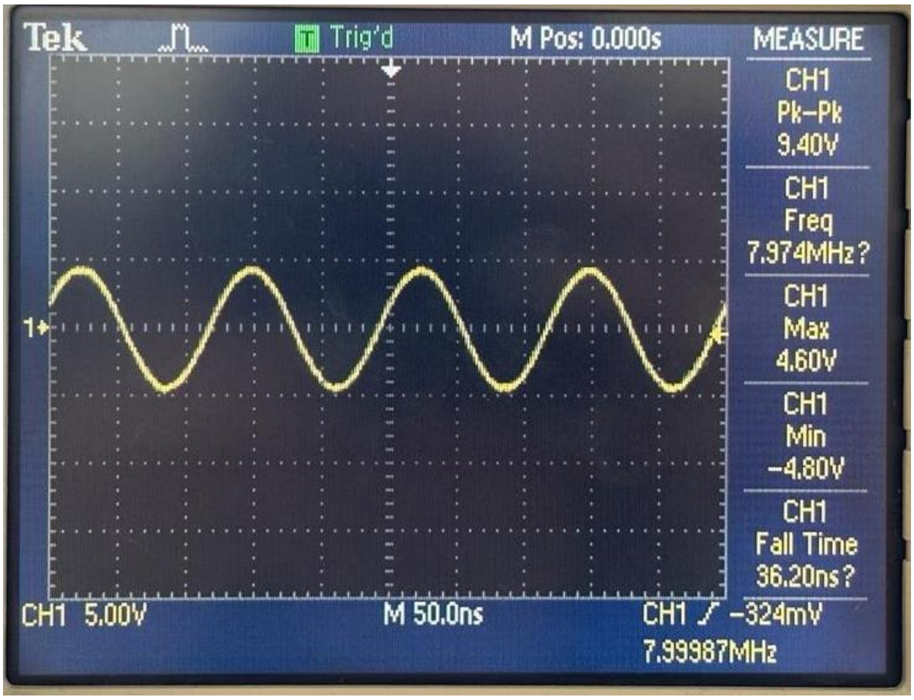

Figure.15 Measurements of the T-Section implementation, $V_{peak-to-peak} = 9.40\ V$

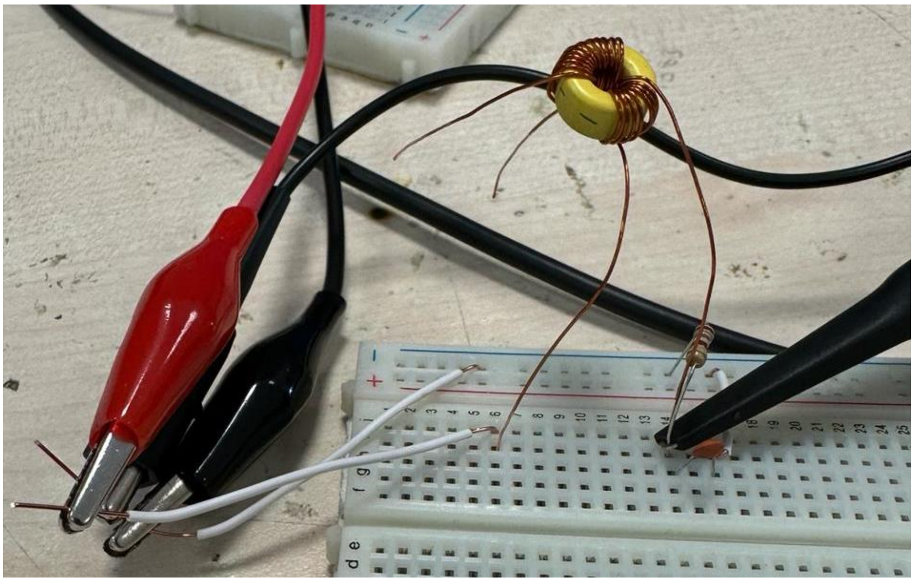

Figure.16 L-Section implementation on breadboard

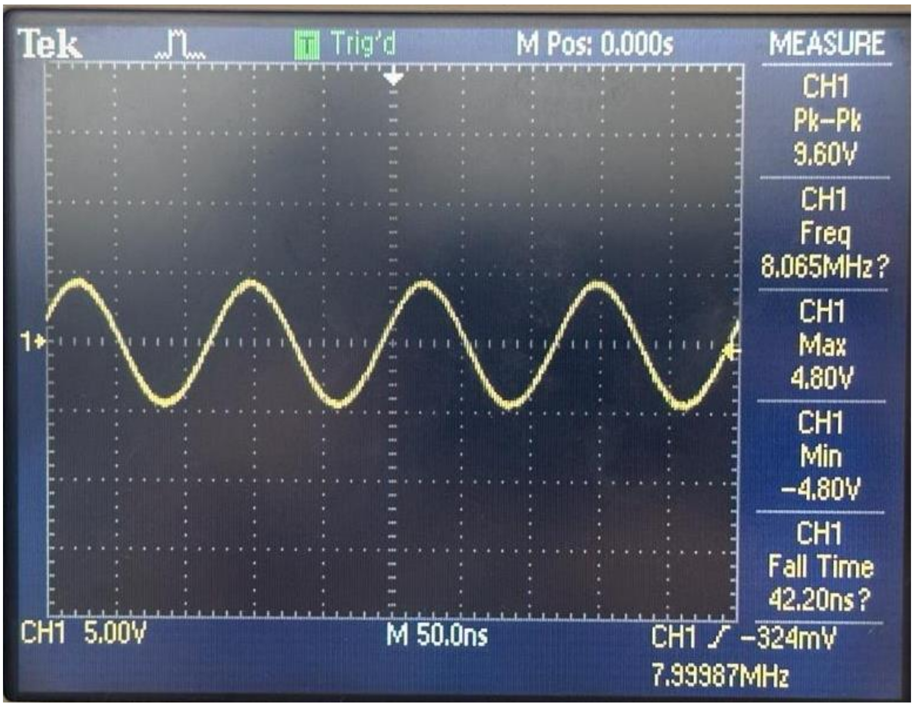

Figure.17 Measurements of the L-Section implementation, $V_{peak-to-peak} = 9.60\ V$

	T-Section	L-Section
Maximum power that can be delivered (theoretical)	62.5 mW	62.5 mW
Power of the source (with 47 Ω resistor)	65.43 mW	65.43 mW
Maximum power that is being delivered (measured)	58.78 mW	64 mW
Power deliver ratio with respect to the theoretical value	%94.05	%102.4
Error with respect to the theoretical value	%5.95	%2.4
Power deliver ratio with respect to the power of the source (with 47 Ω resistor)	%89.84	%97.81
Error with respect to the power of the source (with 47 Ω resistor)	%10.16	%2.19

Table.3 Results of the hardware implementations

Conclusion:

In the software implementation part, for the T-Section circuit the error was %0.01. Power deliver ratio with respect to the theoretical value is %99.72. Power deliver ratio with respect to the power of the source is %99.99. For the L-Section circuit the error was %0.61. Power deliver ratio with respect to the theoretical value is %99.96. Power deliver ratio with respect to the power of the source is %99.39. Since LTSpice does not have any wire resistances and every component is ideal, the errors were minimal. Since the values of the capacitors and the inductors are rounded off, there is still an error. T-Section method was more effective.

In the hardware implementation part, for the T-Section circuit the error was %10.16. Power deliver ratio with respect to the theoretical value is %94.05. Power deliver ratio with respect to the power of the source is %89.84. For the L-Section circuit the error was %2.19. Power deliver ratio with respect to the theoretical value is %102.4. Power deliver ratio with respect to the power of the source is %97.81. Compared to the software lab's errors, one can see the errors are significantly higher in the hardware implementation. The reason for that can be the inner resistances of the components, tolerance values of the capacitors and the resistors and possible leakages of the winded inductors. L-Section method was more effective. The reason for that could be the winding of the inductors. Since the L-Section circuit required less amount of windings, it's inductor was easier to construct and shape to achieve the required inductance. However, considering that both of the number of turns were rounded, the errors occurred in both methods. Although the errors were higher, they were still in the boundaries of the lab prompt.

The lab was effective in teaching us to how to construct circuits to have maximum power transferred to a load (in this case, a resistor). Impedance matching circuits were designed and calculated, then implemented on both LTSpice and on breadboards. All in all, the lab was successful.