Analysis of VaR Estimators

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HSBCQA
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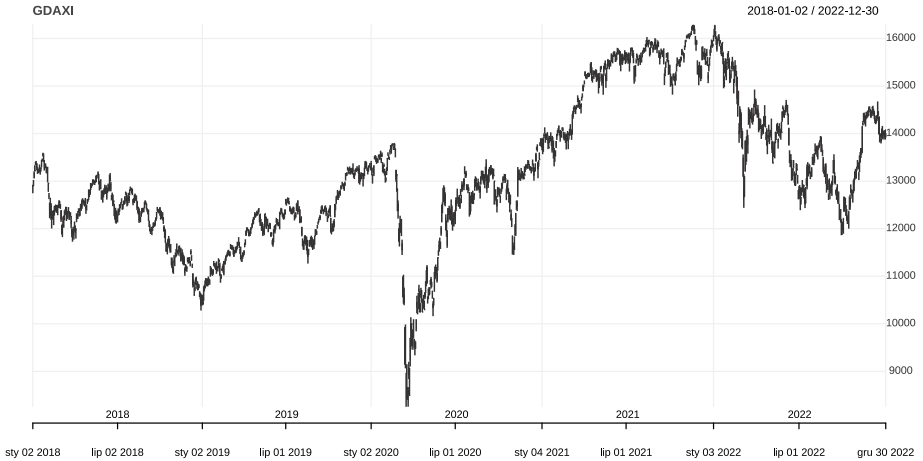
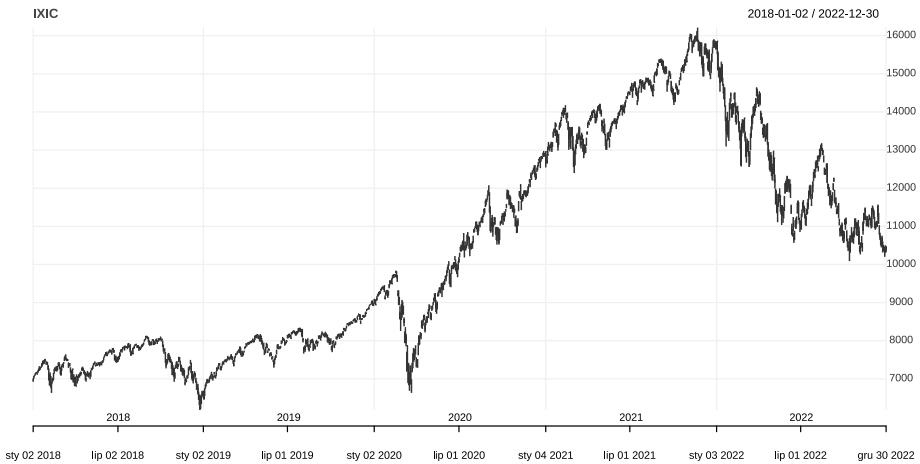
Data


GSPC
2018-01-02 / 2022-12-30
4800
4600
4400
4200
4000
3800
3600
3400
3200
3000
2800
2600
2400
2200
2018
2019
2020
2021
2022
sty 02 2018
lip 02 2018
sty 02 2019
lip 01 2019
sty 02 2020
lip 01 2020
sty 04 2021
lip 01 2021
sty 03 2022
lip 01 2022
gru 30 2022


GDAXI
2018-01-02 / 2022-12-30
16000
15000
14000
13000
12000
11000
10000
9000
2018
2019
2020
2021
2022
sty 02 2018
lip 02 2018
sty 02 2019
lip 01 2019
sty 02 2020
lip 01 2020
sty 04 2021
lip 01 2021
sty 03 2022
lip 01 2022
gru 30 2022


IXIC
2018-01-02 / 2022-12-30
16000
15000
14000
13000
12000
11000
10000
9000
8000
7000
2018
2019
2020
2021
2022
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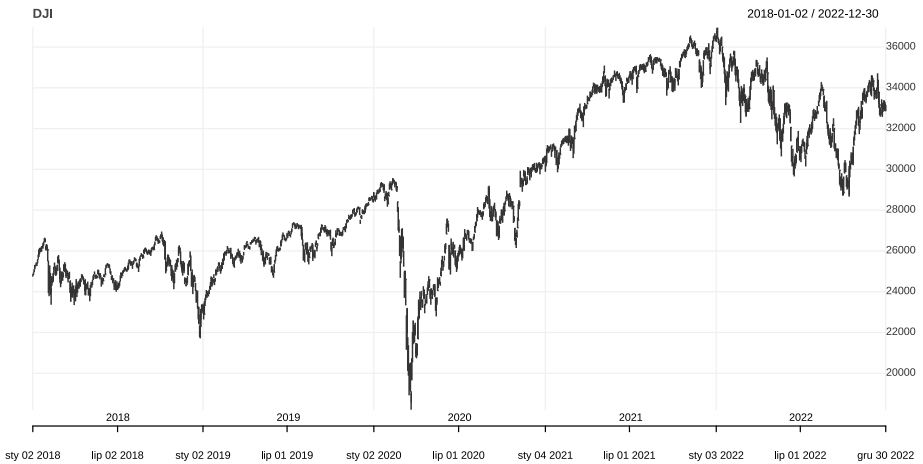
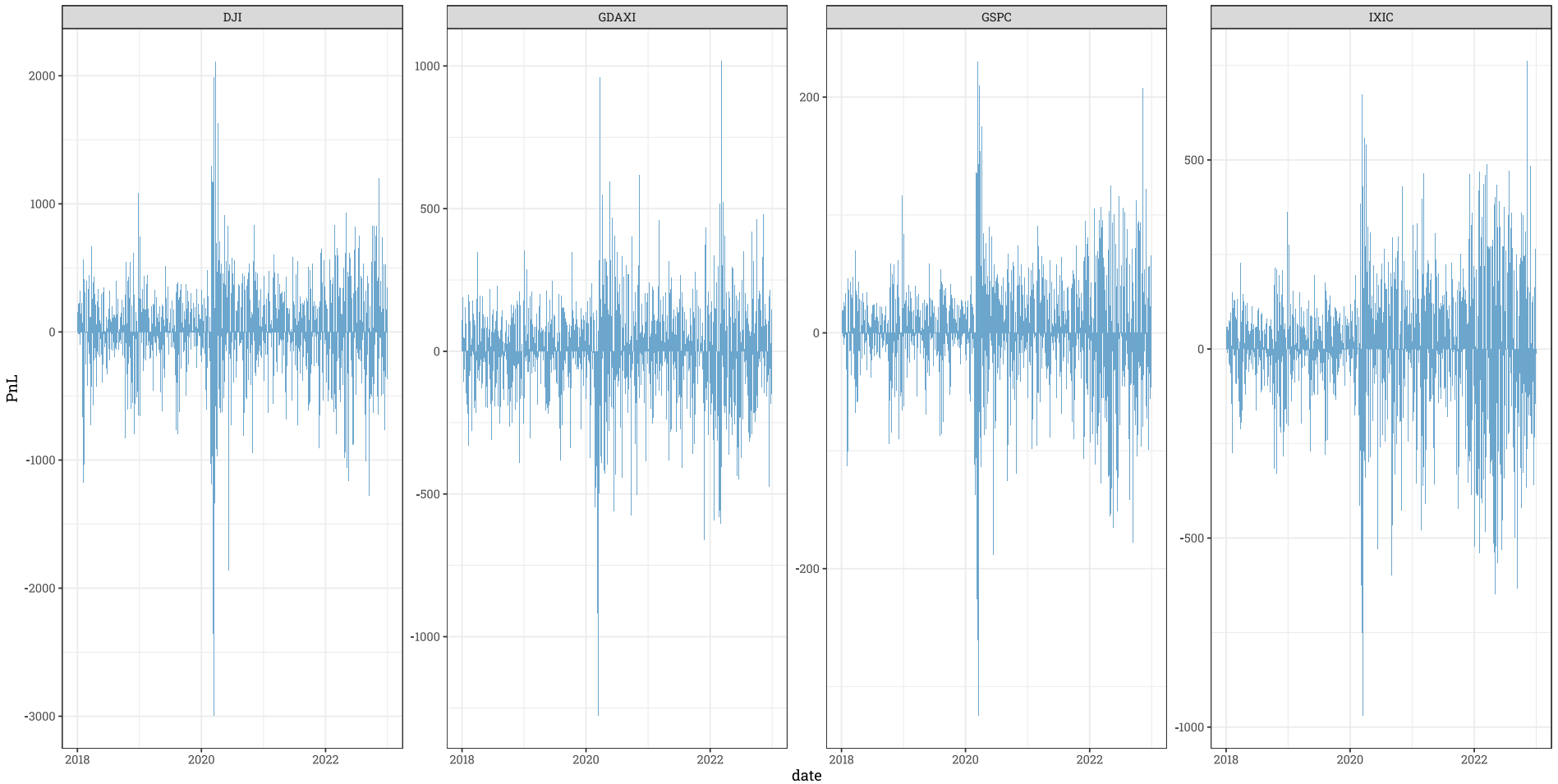
lip 02 2018
sty 02 2019
lip 01 2019
sty 02 2020
lip 01 2020
sty 04 2021
lip 01 2021
sty 03 2022
lip 01 2022
gru 30 2022


DJI
2018-01-02 / 2022-12-30
36000
34000
32000
30000
28000
26000
24000
22000
20000
2018
2019
2020
2021
2022
sty 02 2018
lip 02 2018
sty 02 2019
lip 01 2019
sty 02 2020
lip 01 2020
sty 04 2021
lip 01 2021
sty 03 2022
lip 01 2022
gru 30 2022

PnLs


DJI
GDAXI
GSPC
IXIC
PnL
date
2018
2020
2022
2000
1000
0
-1000
-2000
-3000
500
-500
200
-200

Statistical Properties

Normality

The assumption about normality of returns data should not be made, as we have to account for heavy tails.

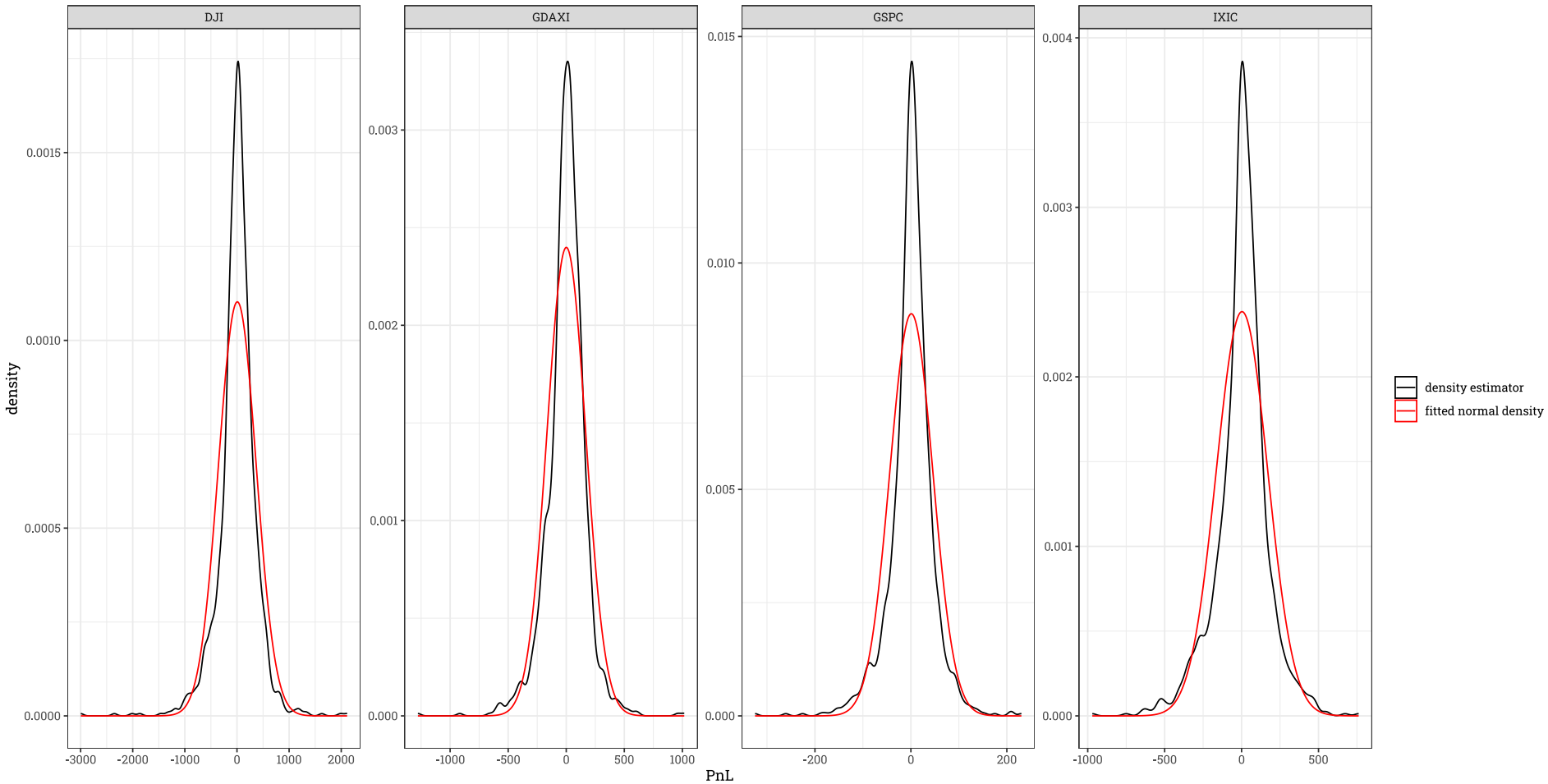
Homogenity

The volatility of Profit and Losses fluctuates over time, which may result in an overestimation or underestimation of the Value at Risk estimates.

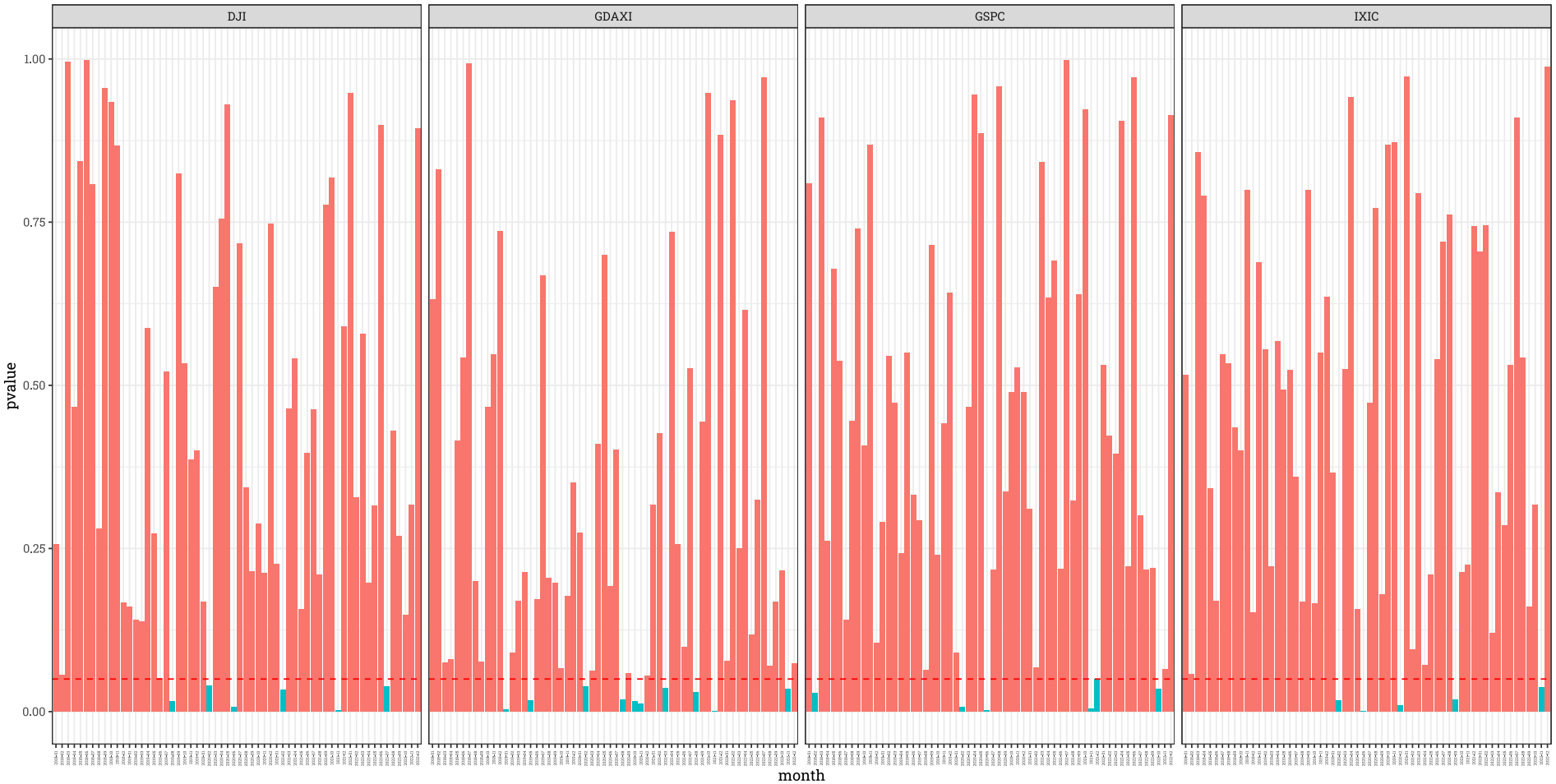
Independence

There is a considerable degree of autocorrelation present in the absolute values of the Profit and Losses, which may suggest the presence of volatility clusters within the data.

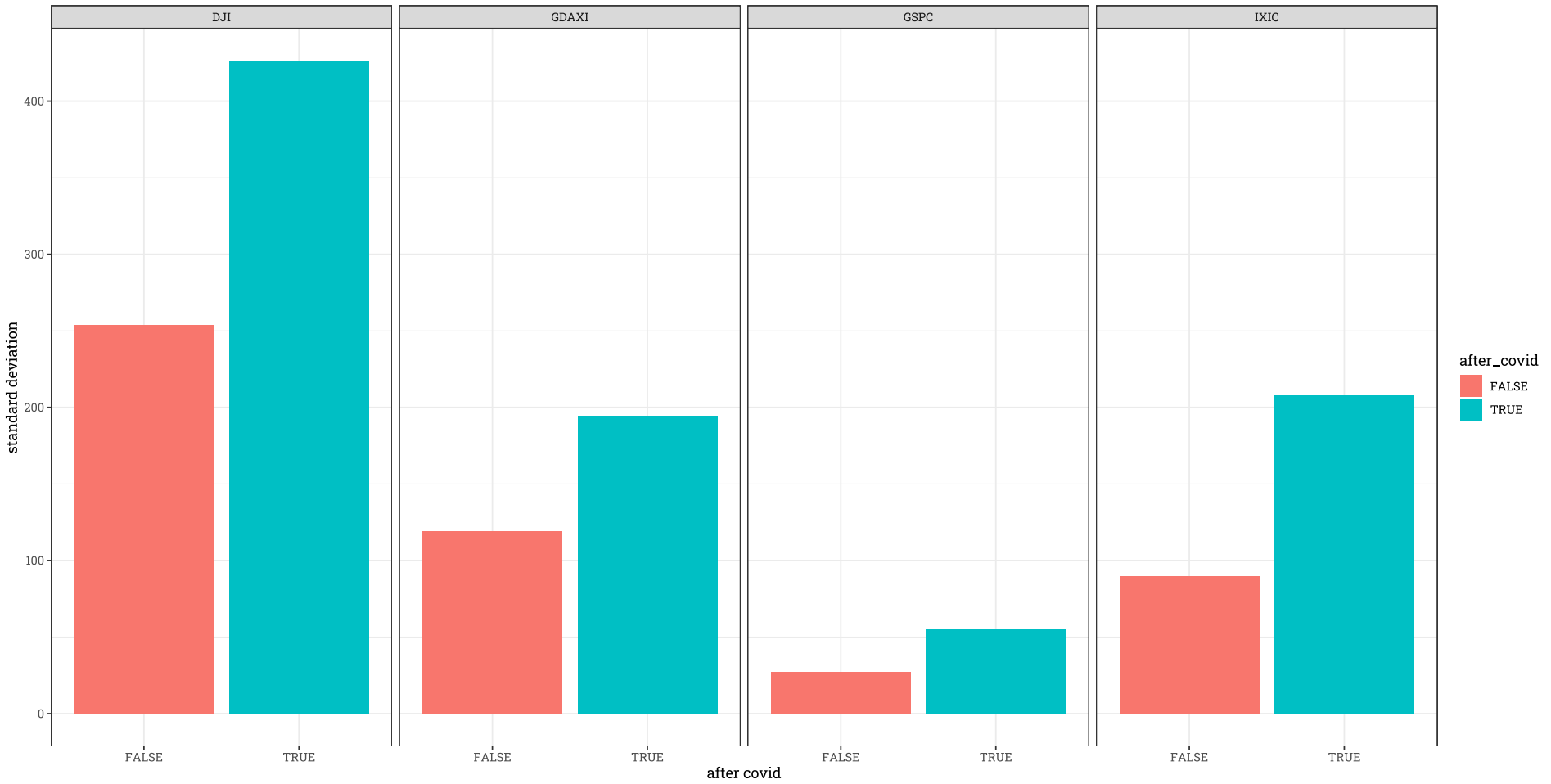
Normality


DJI
GDAXI
GSPC
IXIC
density
PnL
density estimator
fitted normal density

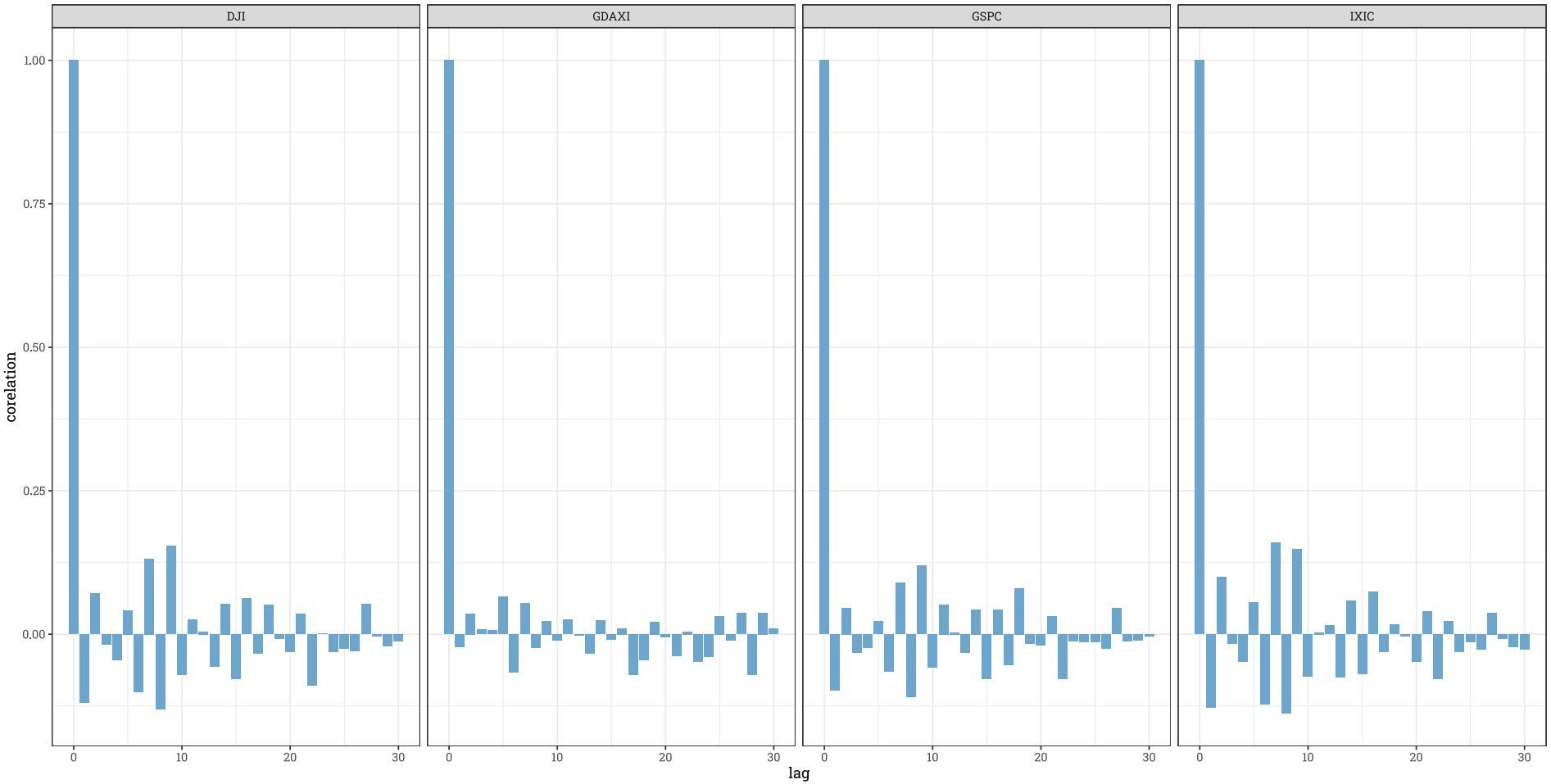
Normality



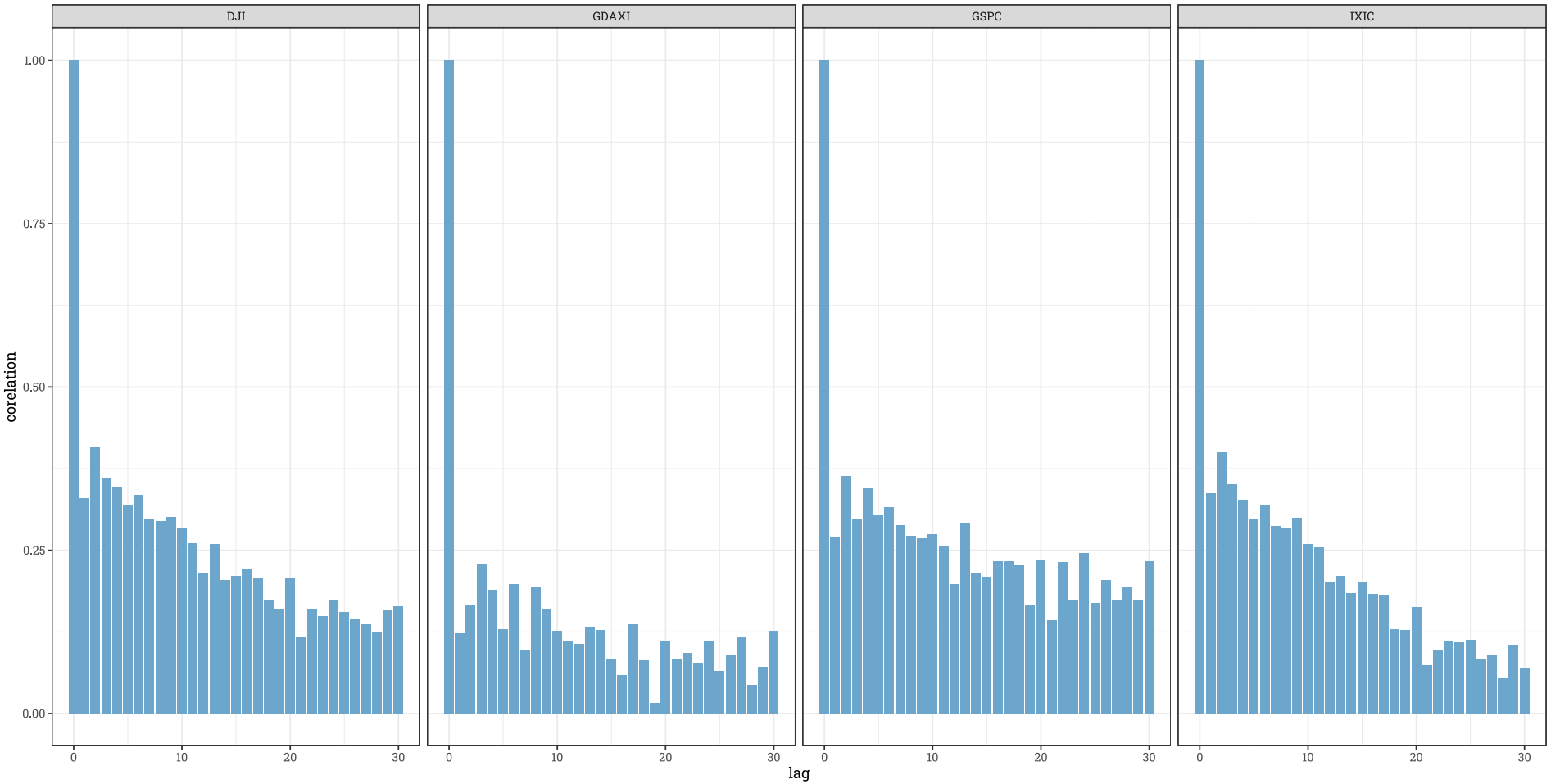
Homogenity


DJI
GDAXI
GSPC
IXIC
standard deviation
400
300
200
100
0
FALSE
TRUE
after covid
after_covid
FALSE
TRUE

Independence



Independence



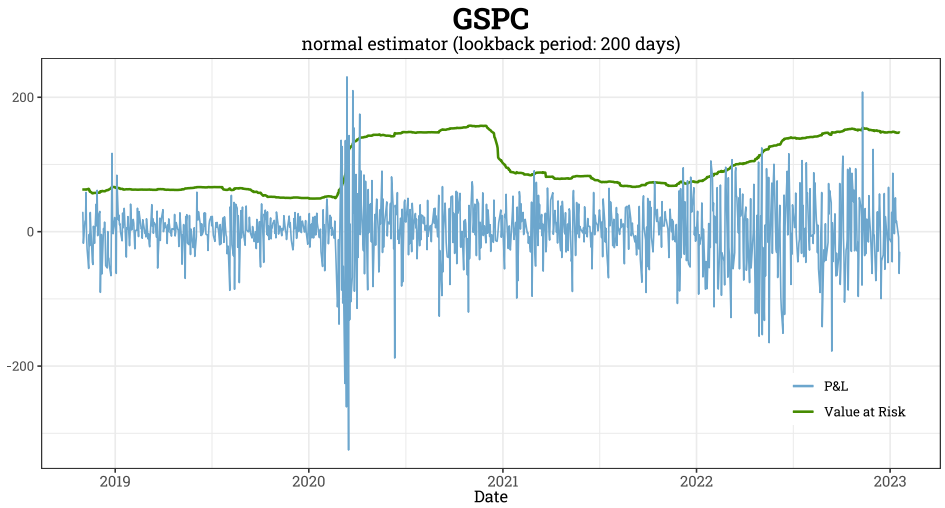
Description of Estimators

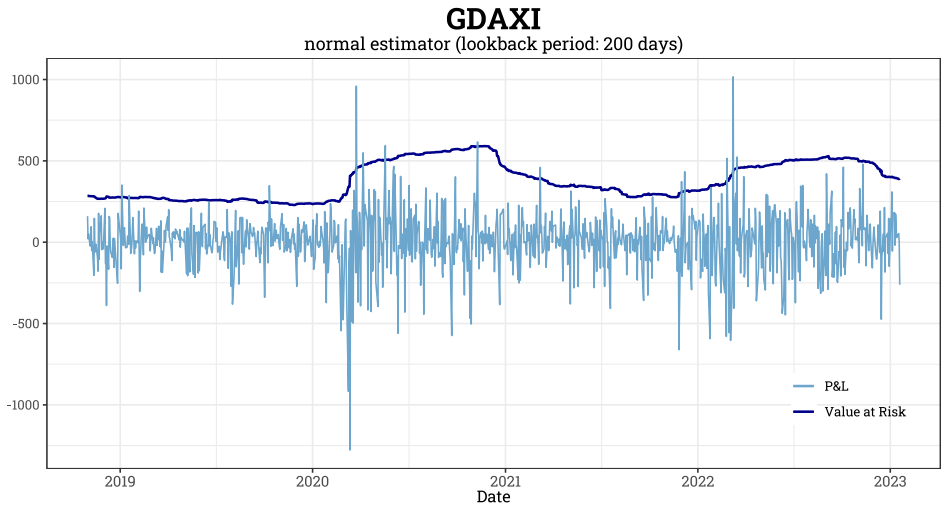
The following three estimators of VaR were used:

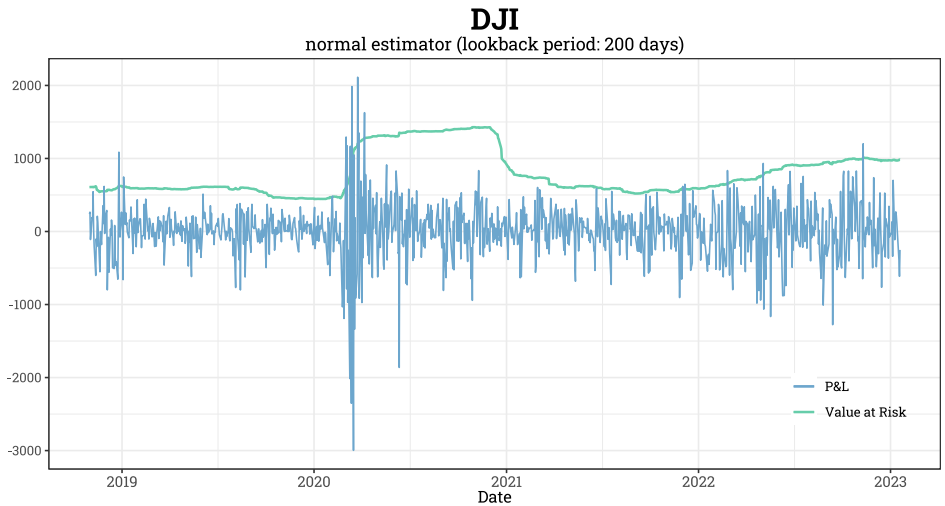
- Normal Estimator: $-(\bar{x} + \bar{\sigma}(x)\Phi^{-1}(\alpha))$
- Unbiased Normal Estimator: $-(\bar{x} + \bar{\sigma}(x)\sqrt{\frac{n+1}{n}}t_{n-1}^{-1}(\alpha))$
- Empirical Estimator: $-x_{(\lfloor n\alpha \rfloor + 1)}$

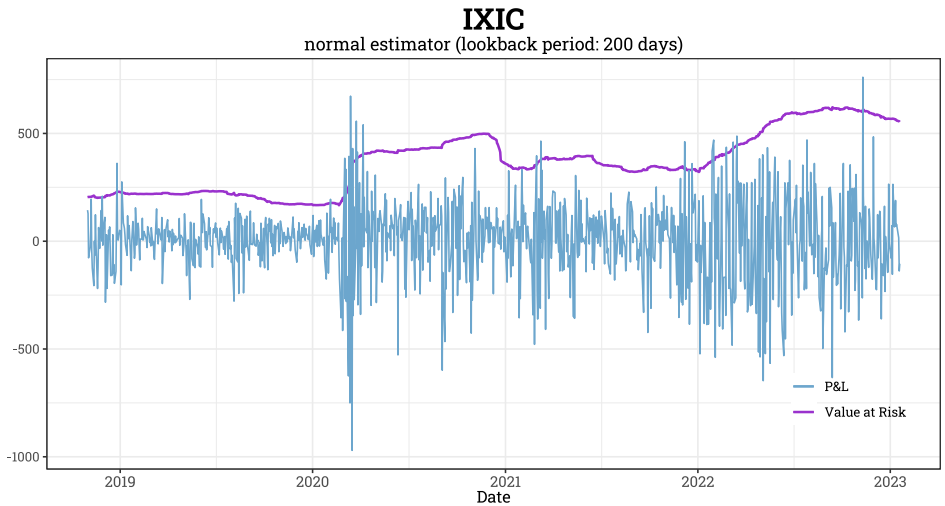
In the following slides, estimators for 200 and 500 days lookback period are shown.

Normal

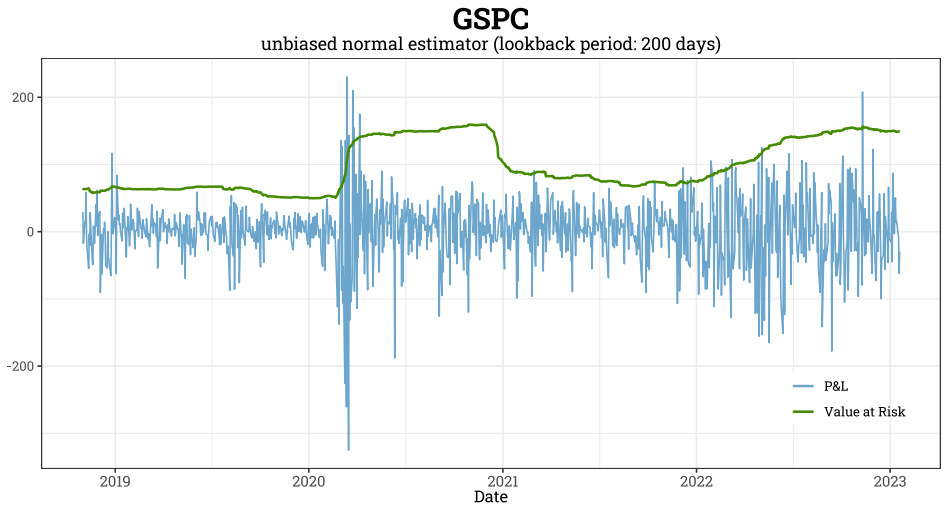

GSPC
normal estimator (lookback period: 200 days)
200
0
-200
2019
2020
2021
2022
2023
Date
P&L
Value at Risk

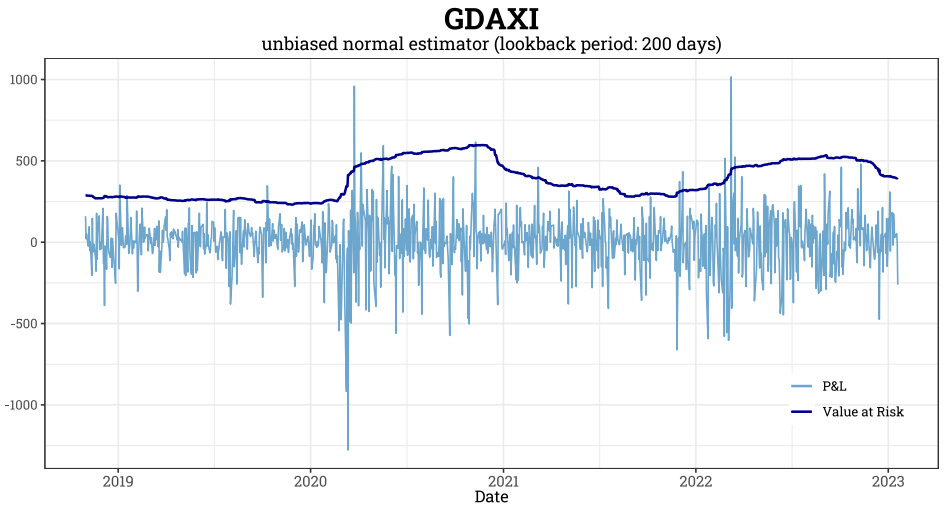

GDAXI
normal estimator (lookback period: 200 days)
1000
500
0
-500
-1000
2019
2020
2021
2022
2023
Date
P&L
Value at Risk

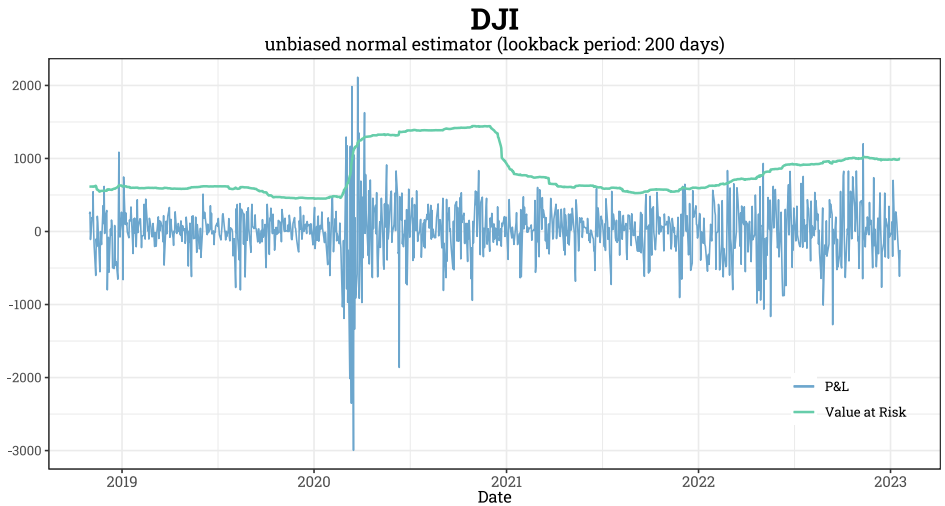

DJI
normal estimator (lookback period: 200 days)
2000
1000
0
-1000
-2000
-3000
2019
2020
2021
2022
2023
Date
P&L
Value at Risk

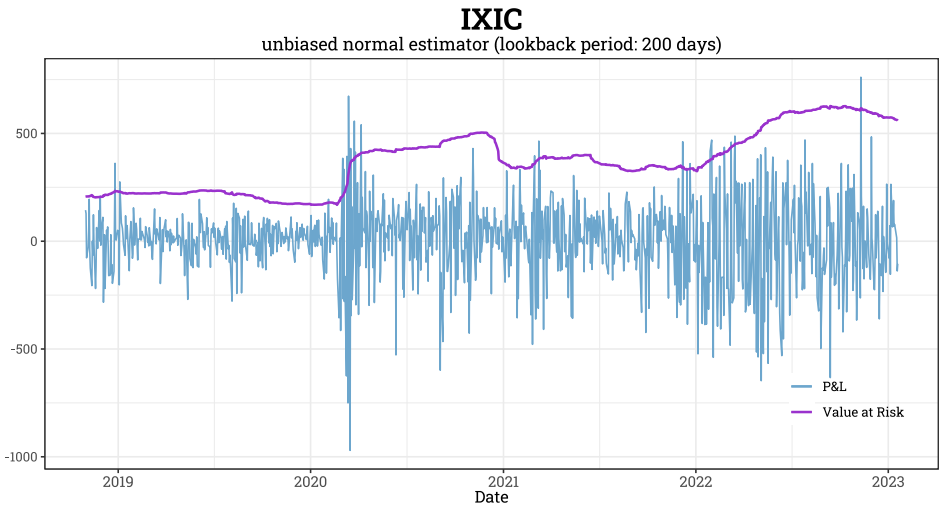

IXIC
normal estimator (lookback period: 200 days)
500
0
-500
-1000
2019
2020
2021
2022
2023
Date
P&L
Value at Risk

Unbiased Normal

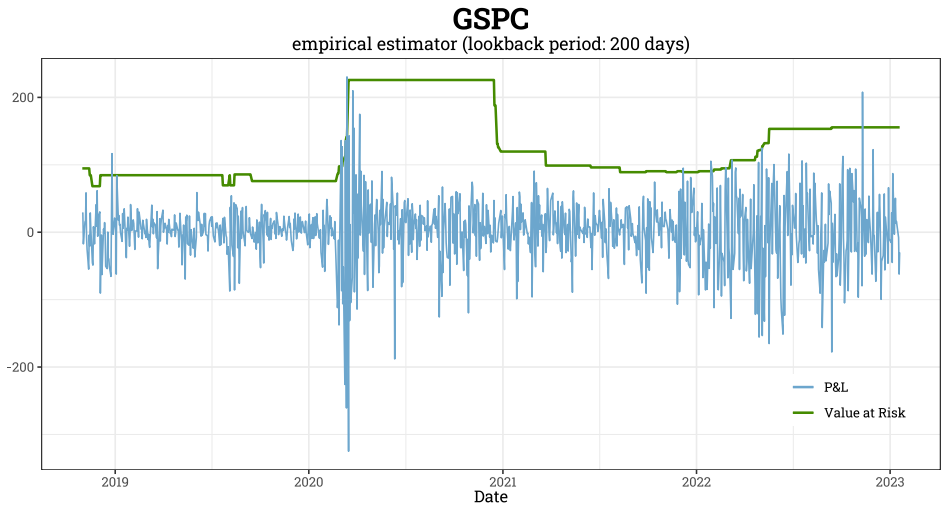


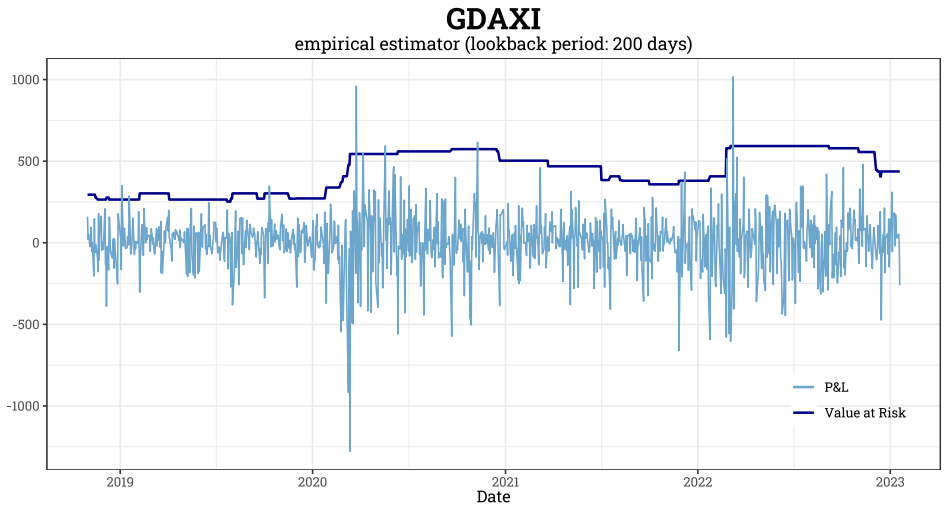


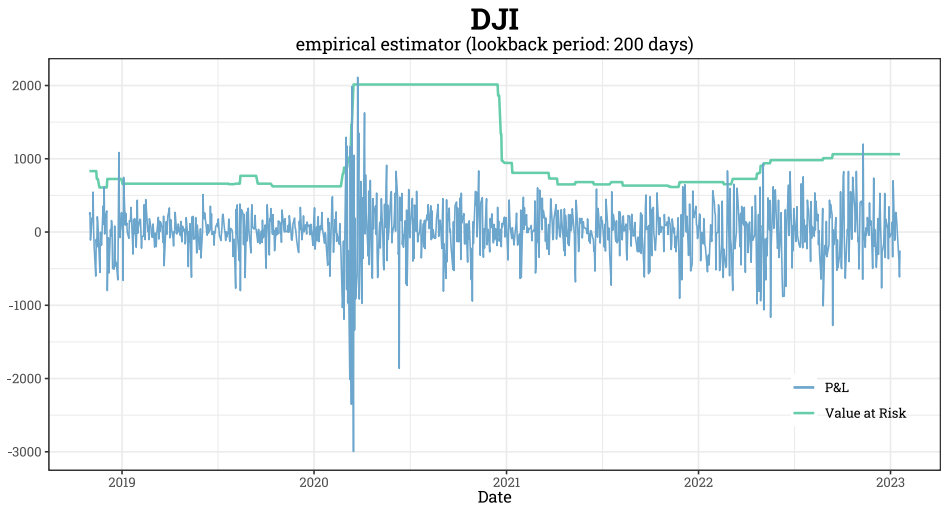


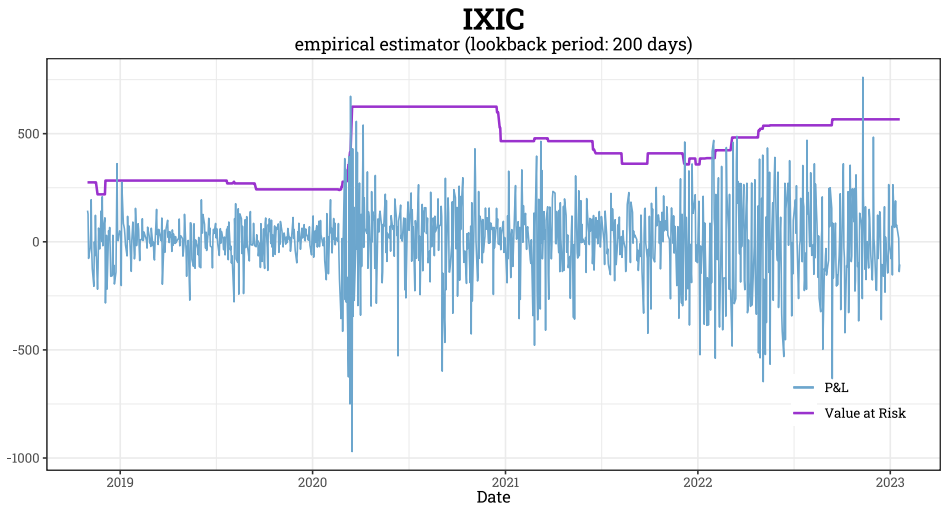


Empirical

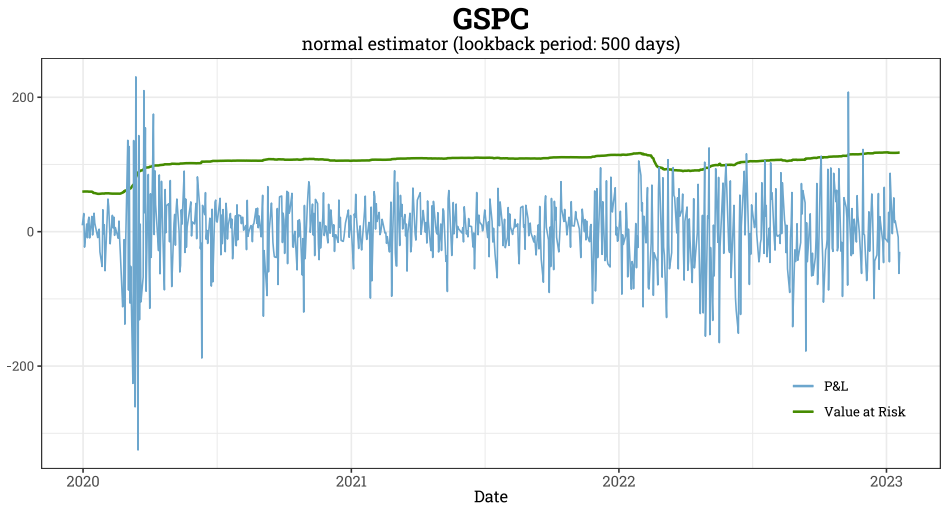

GSPC
empirical estimator (lookback period: 200 days)
200
0
-200
2019
2020
2021
2022
2023
Date
P&L
Value at Risk

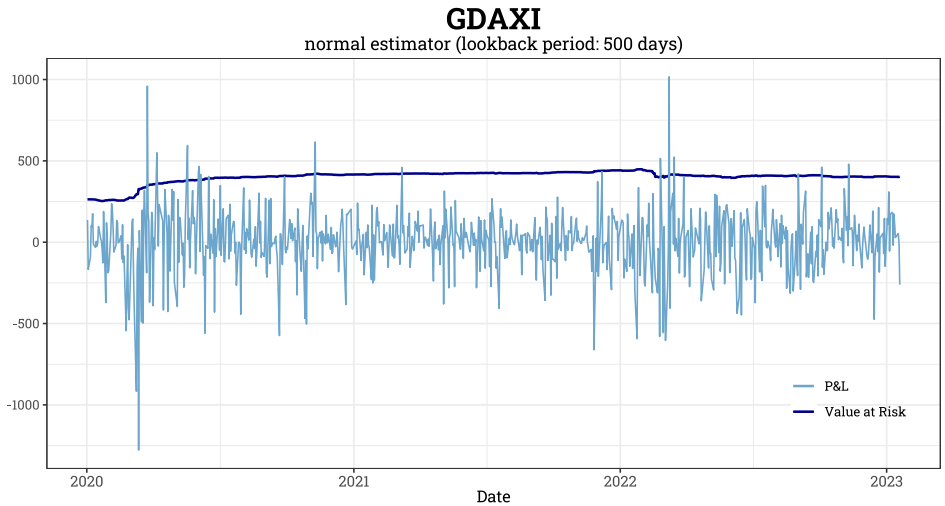

GDAXI
empirical estimator (lookback period: 200 days)
1000
500
0
-500
-1000
2019
2020
2021
2022
2023
Date
P&L
Value at Risk

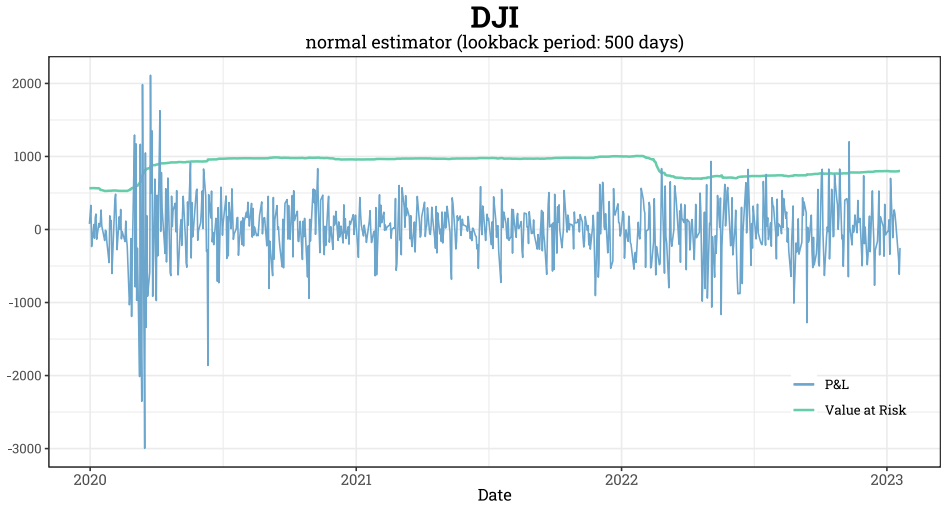

DJI
empirical estimator (lookback period: 200 days)
2000
1000
0
-1000
-2000
-3000
2019
2020
2021
2022
2023
Date
P&L
Value at Risk

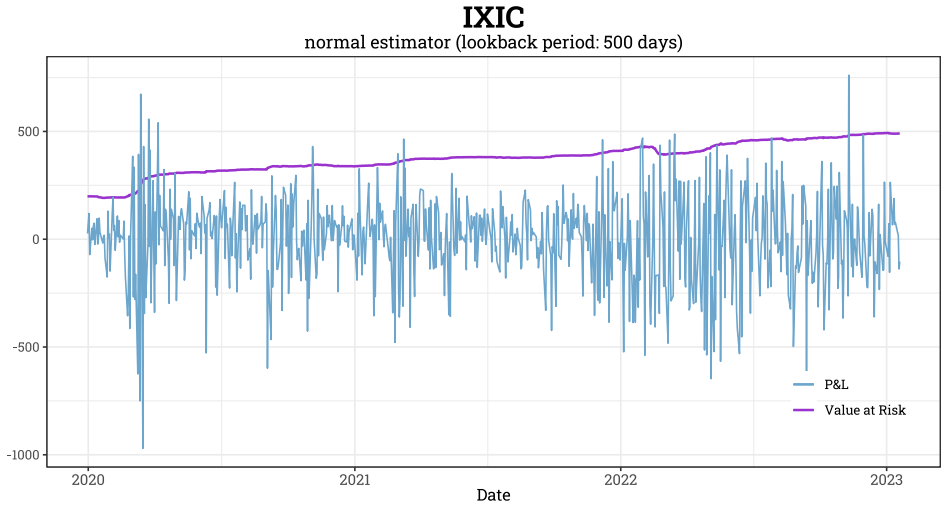

IXIC
empirical estimator (lookback period: 200 days)
500
0
-500
-1000
2019
2020
2021
2022
2023
Date
P&L
Value at Risk

Normal

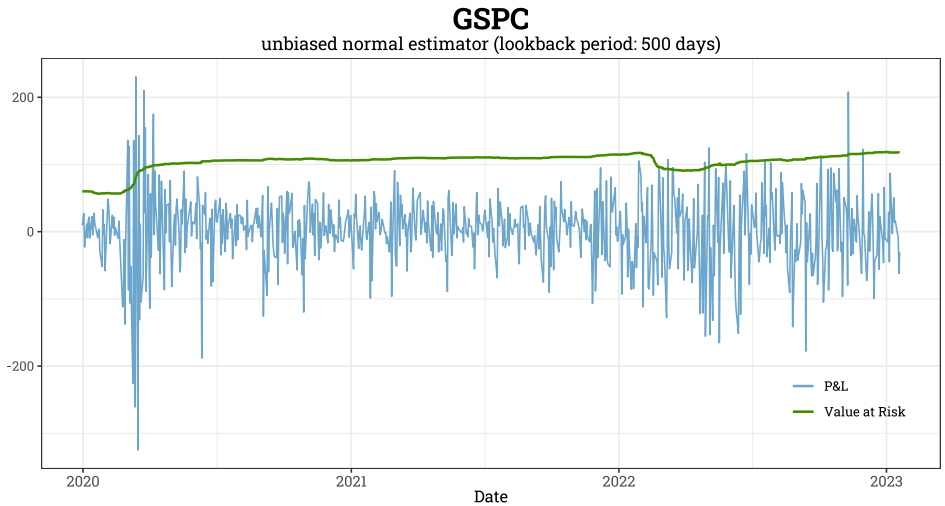

GSPC
normal estimator (lookback period: 500 days)
200
0
-200
2020
2021
2022
2023
Date
P&L
Value at Risk

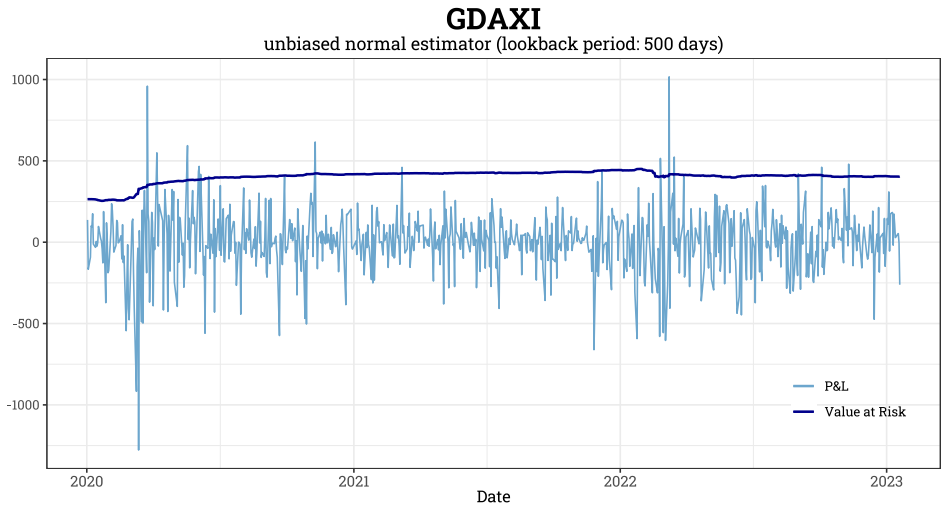

GDAXI
normal estimator (lookback period: 500 days)
1000
500
0
-500
-1000
2020
2021
2022
2023
Date
P&L
Value at Risk

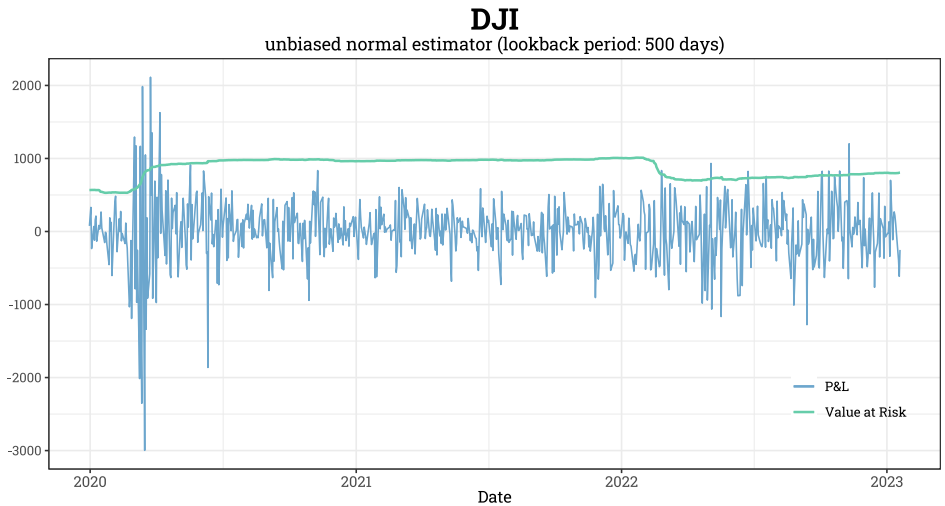

DJI
normal estimator (lookback period: 500 days)
2000
1000
0
-1000
-2000
-3000
2020
2021
2022
2023
Date
P&L
Value at Risk

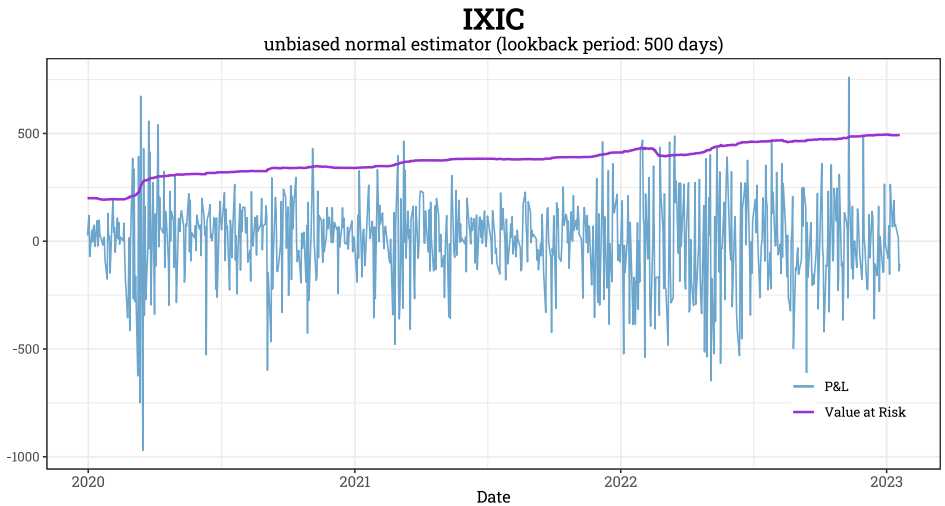

IXIC
normal estimator (lookback period: 500 days)
500
0
-500
-1000
2020
2021
2022
2023
Date
P&L
Value at Risk

Unbiased Normal

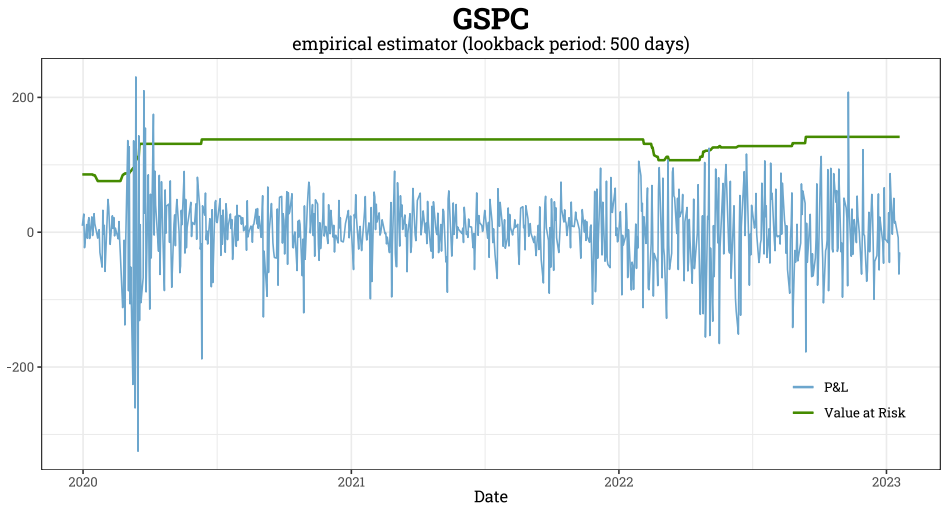


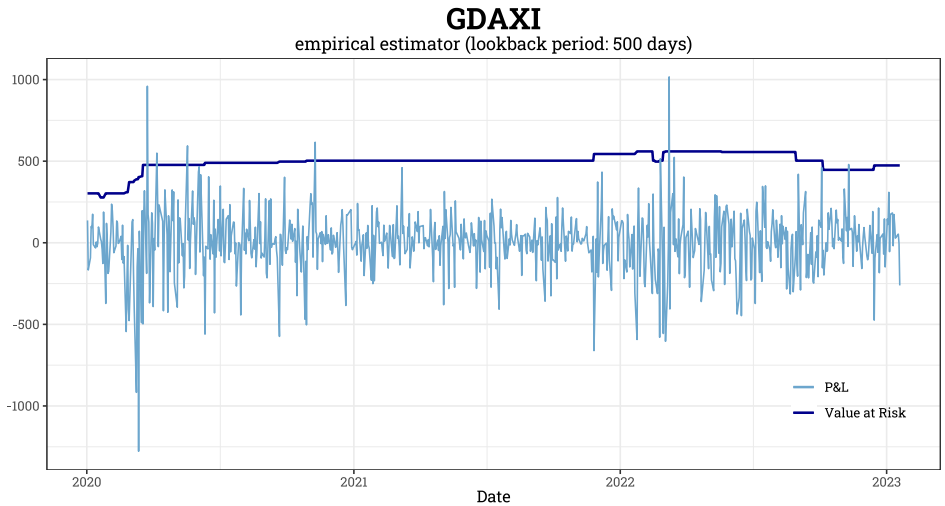






Empirical


GSPC
empirical estimator (lookback period: 500 days)
200
0
-200
2020
2021
2022
2023
Date
P&L
Value at Risk


GDAXI
empirical estimator (lookback period: 500 days)
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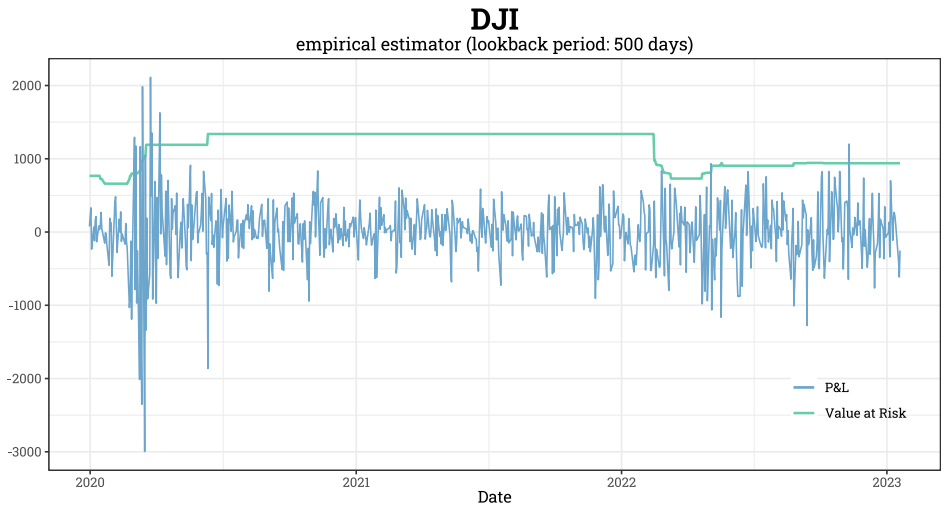
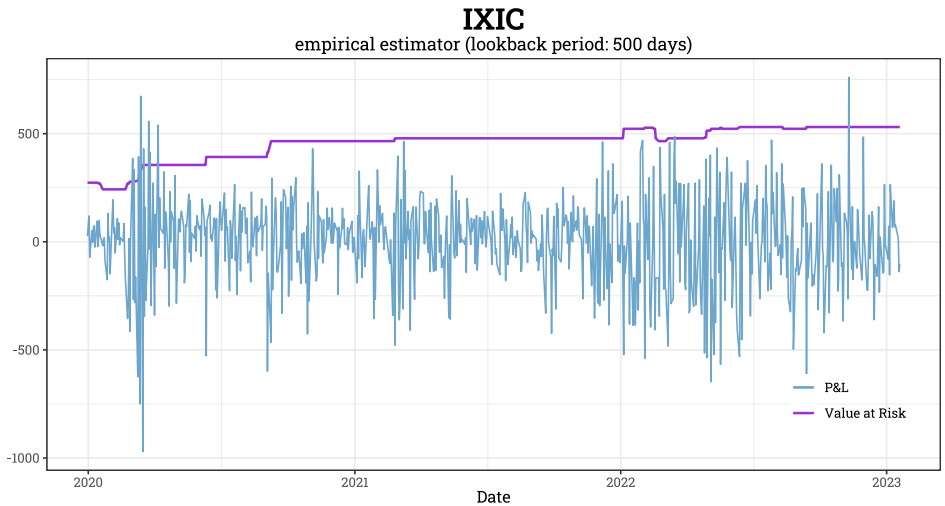
500
0
-500
-1000
2020
2021
2022
2023
Date
P&L
Value at Risk

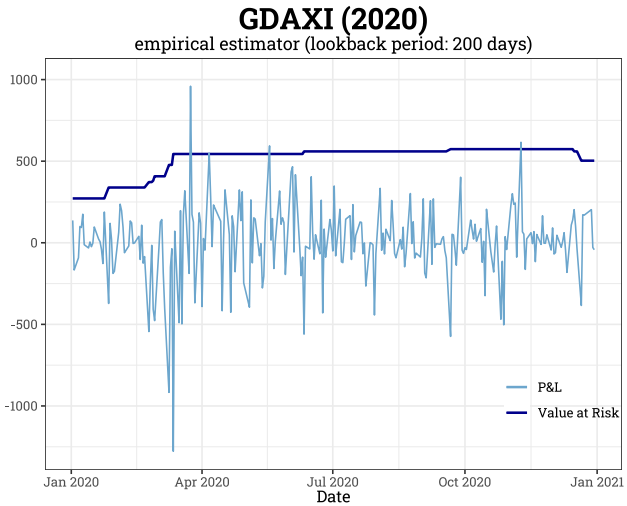

DJI
empirical estimator (lookback period: 500 days)
2000
1000
0
-1000
-2000
-3000
2020
2021
2022
2023
Date
P&L
Value at Risk

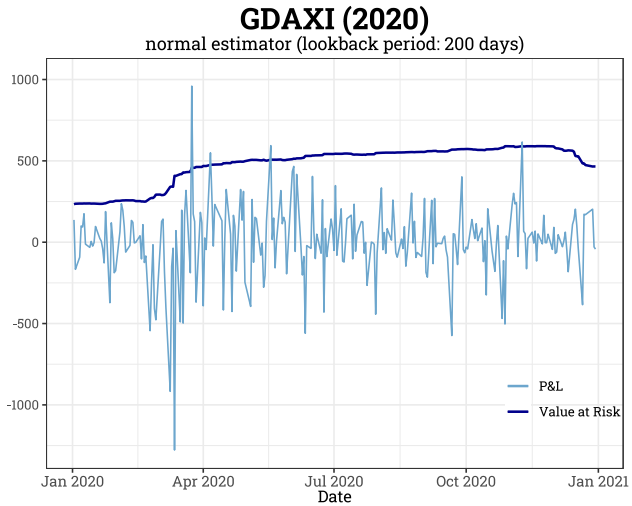

IXIC
empirical estimator (lookback period: 500 days)
500
0
-500
-1000
2020
2021
2022
2023
Date
P&L
Value at Risk

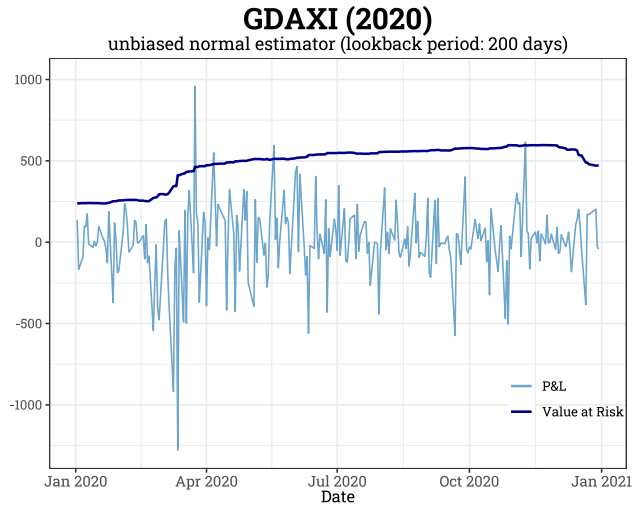
Covid Time

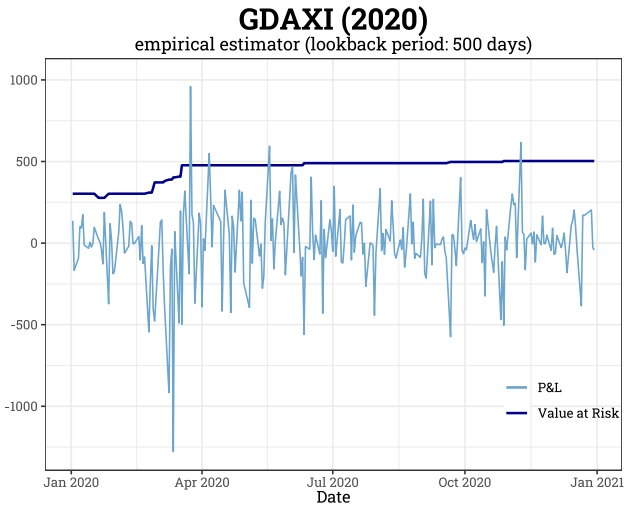
- March 2020 stock market crash triggered by COVID-19.
- Natural gas, food, healthcare, and software stocks earn high positive returns.
- Petroleum, real estate, entertainment, and hospitality stocks fall dramatically.
- Loser stocks exhibit extreme asymmetric volatility.

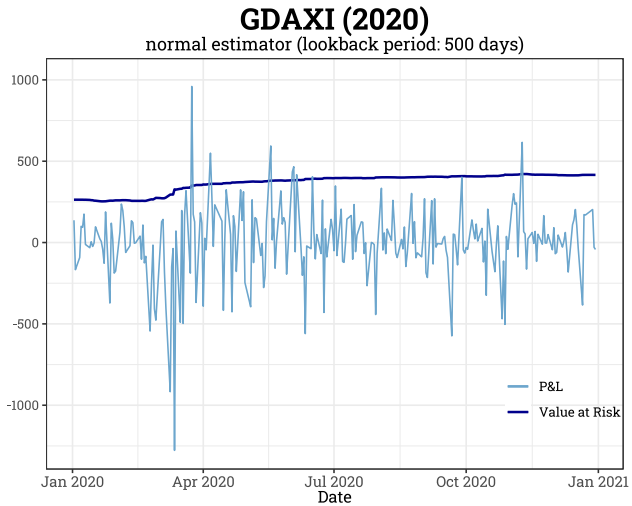
DAX

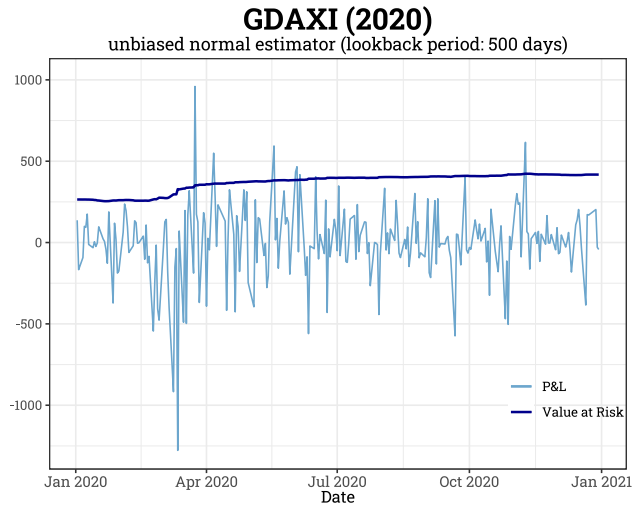

GDAXI (2020)
empirical estimator (lookback period: 200 days)
1000
500
0
-500
-1000
P&L
Value at Risk
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date


GDAXI (2020)
normal estimator (lookback period: 200 days)
1000
500
0
-500
-1000
P&L
Value at Risk
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date

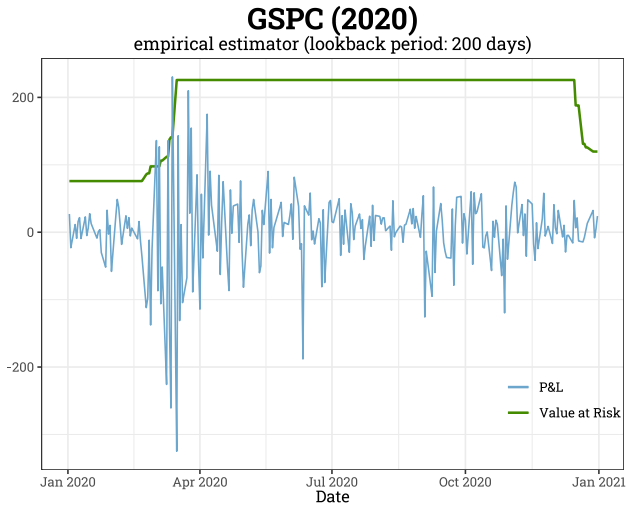

GDAXI (2020)
unbiased normal estimator (lookback period: 200 days)
1000
500
0
-500
-1000
P&L
Value at Risk
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date

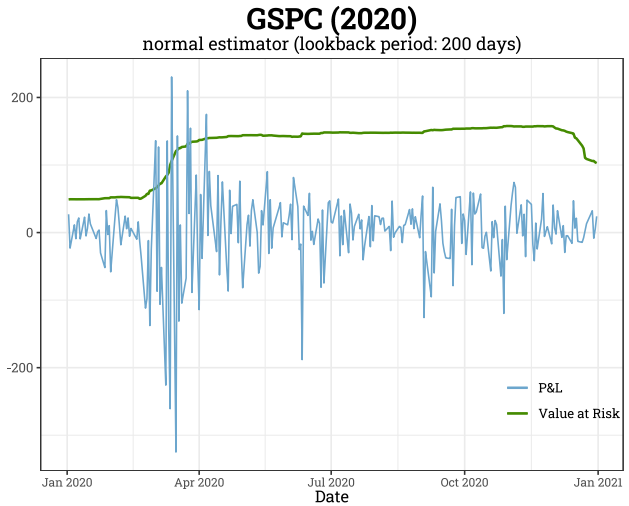

GDAXI (2020)
empirical estimator (lookback period: 500 days)
1000
500
0
-500
-1000
P&L
Value at Risk
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date

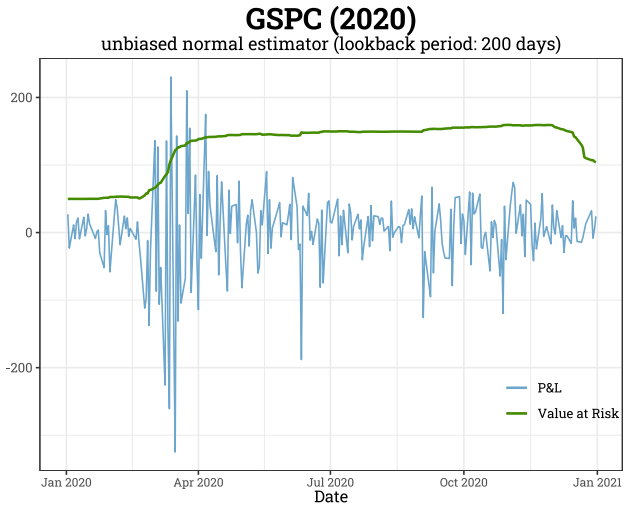

GDAXI (2020)
normal estimator (lookback period: 500 days)
1000
500
0
-500
-1000
P&L
Value at Risk
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date

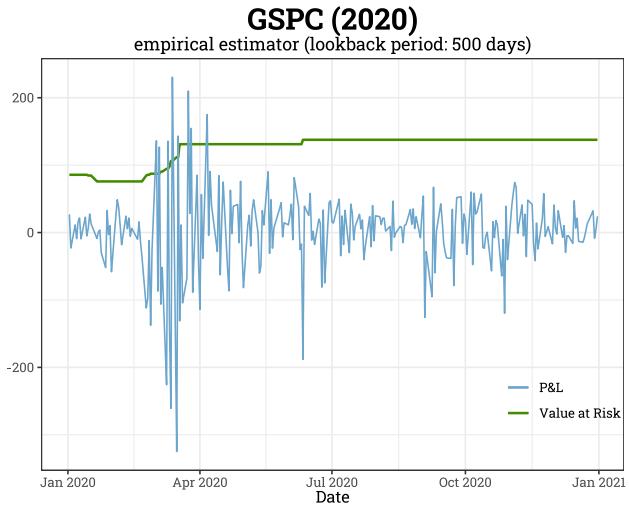

GDAXI (2020)
unbiased normal estimator (lookback period: 500 days)
1000
500
0
-500
-1000
P&L
Value at Risk
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date

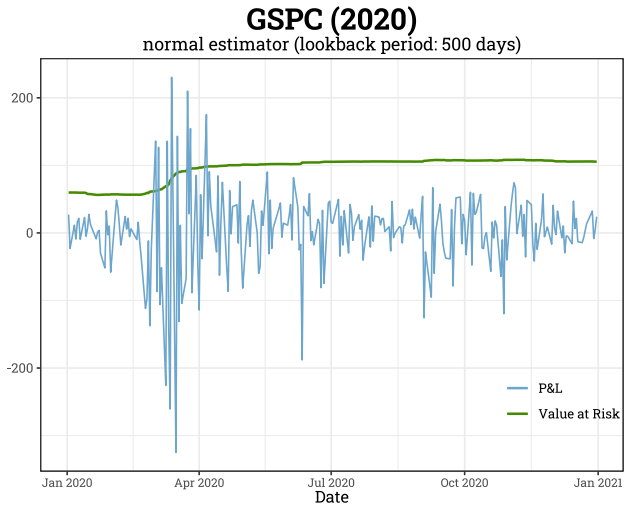
SnP 500


GSPC (2020)
empirical estimator (lookback period: 200 days)
200
0
-200
P&L
Value at Risk
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date


GSPC (2020)
normal estimator (lookback period: 200 days)
200
0
-200
P&L
Value at Risk
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date


GSPC (2020)
unbiased normal estimator (lookback period: 200 days)
200
0
-200
P&L
Value at Risk
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date


GSPC (2020)
empirical estimator (lookback period: 500 days)
200
0
-200
P&L
Value at Risk
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date


GSPC (2020)
normal estimator (lookback period: 500 days)
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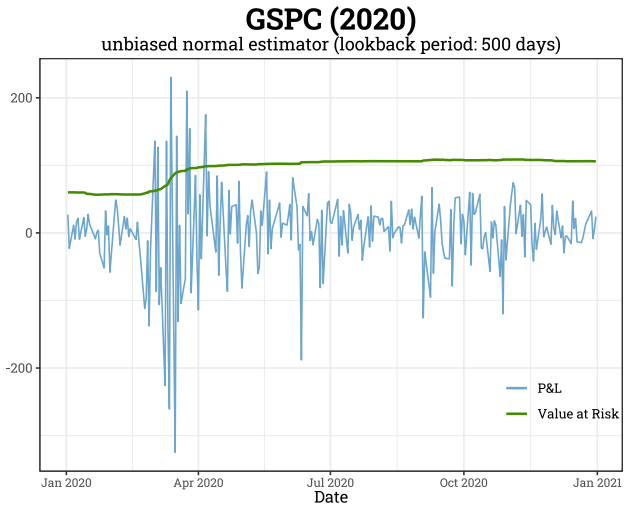
-200
P&L
Value at Risk
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date


GSPC (2020)
unbiased normal estimator (lookback period: 500 days)
200
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P&L
Value at Risk
Jan 2020
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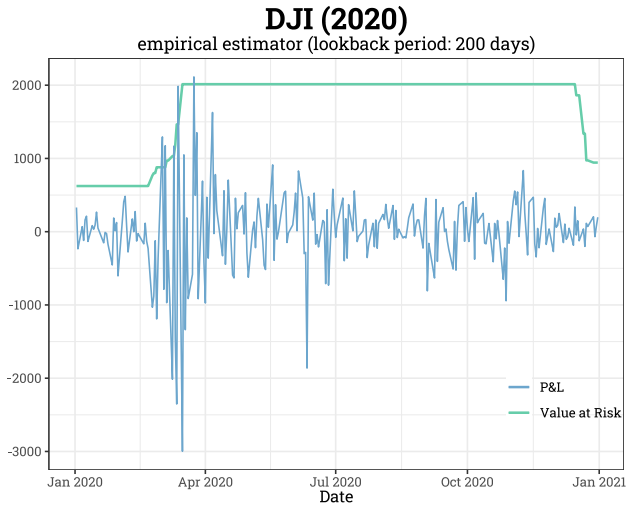
Date

Dow Jones


DJI (2020)
empirical estimator (lookback period: 200 days)
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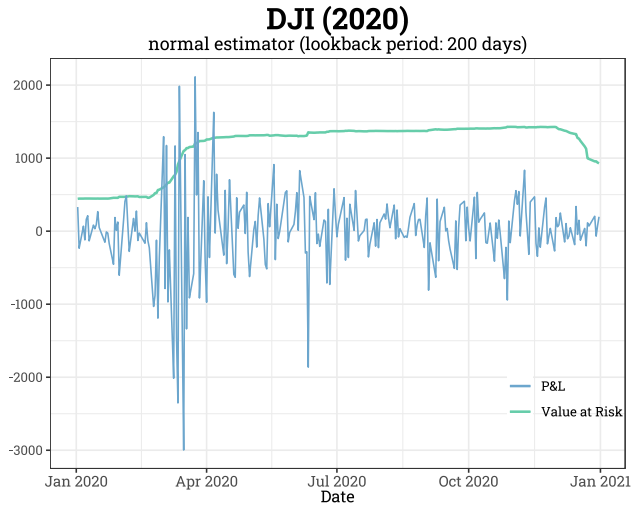
1000
0
-1000
-2000
-3000
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date
P&L
Value at Risk


DJI (2020)
normal estimator (lookback period: 200 days)
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Jan 2020
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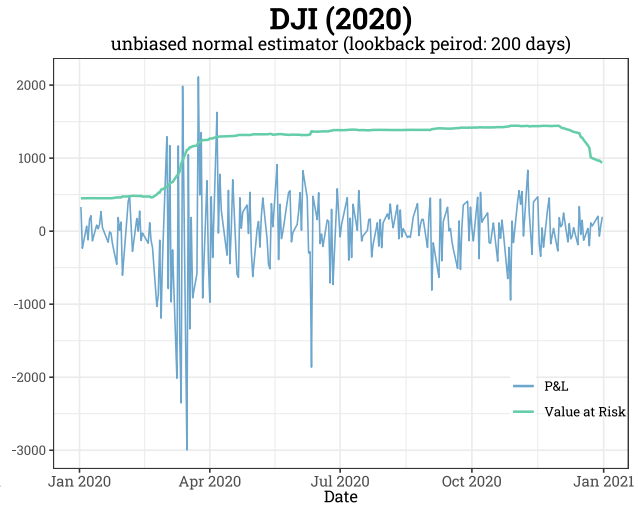
Oct 2020
Jan 2021
Date
P&L
Value at Risk


DJI (2020)
unbiased normal estimator (lookback peirod: 200 days)
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Jan 2020
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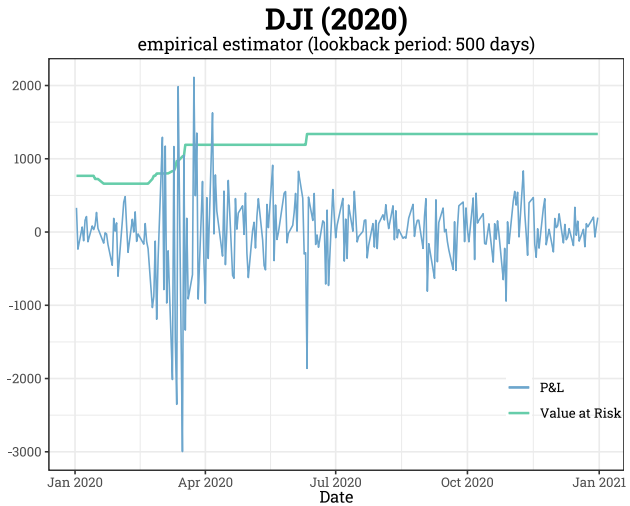
P&L
Value at Risk


DJI (2020)
empirical estimator (lookback period: 500 days)
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Jan 2020
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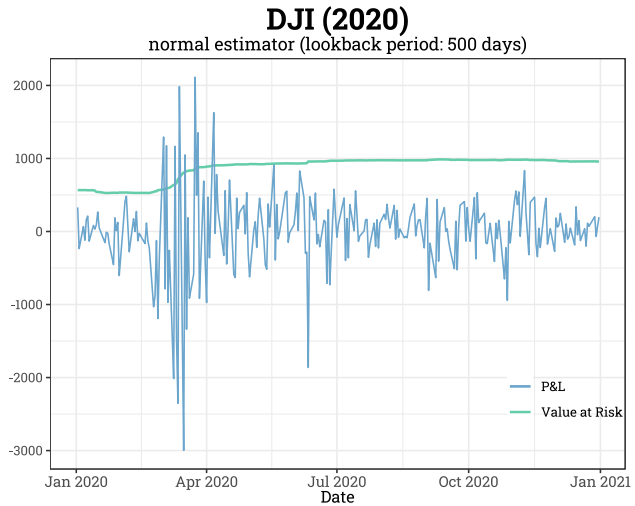
Jul 2020
Oct 2020
Jan 2021
Date
P&L
Value at Risk


DJI (2020)
normal estimator (lookback period: 500 days)
2000
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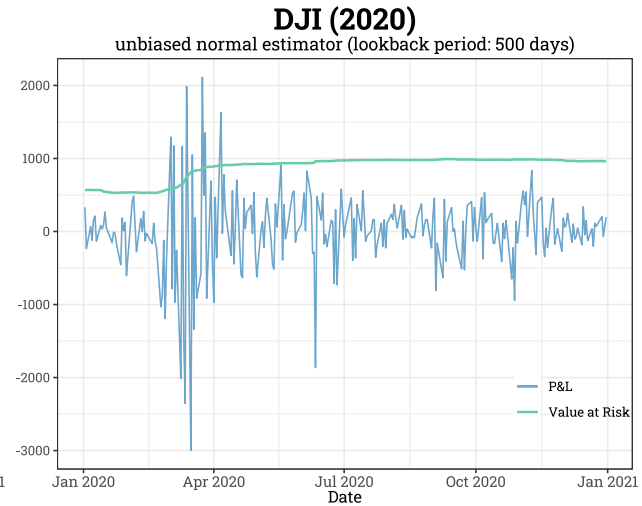
-2000
-3000
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date
P&L
Value at Risk


DJI (2020)
unbiased normal estimator (lookback period: 500 days)
2000
1000
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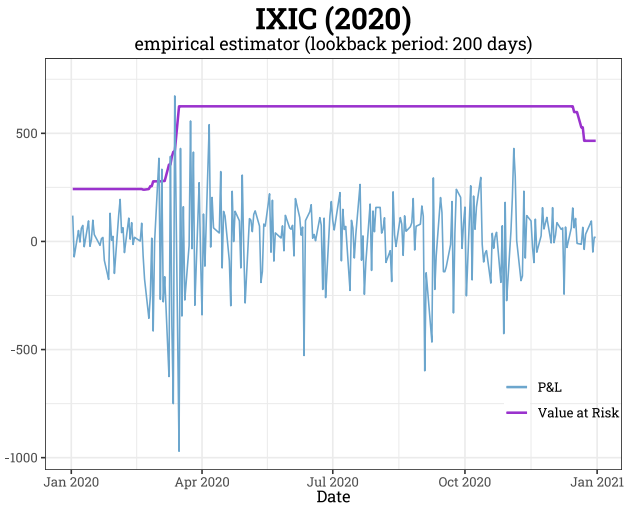
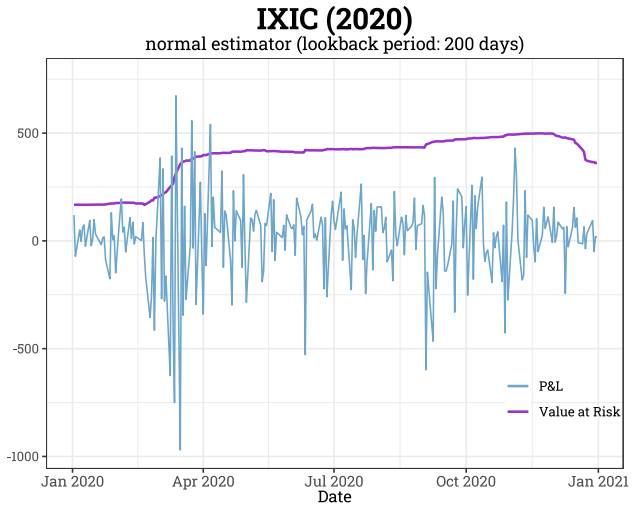
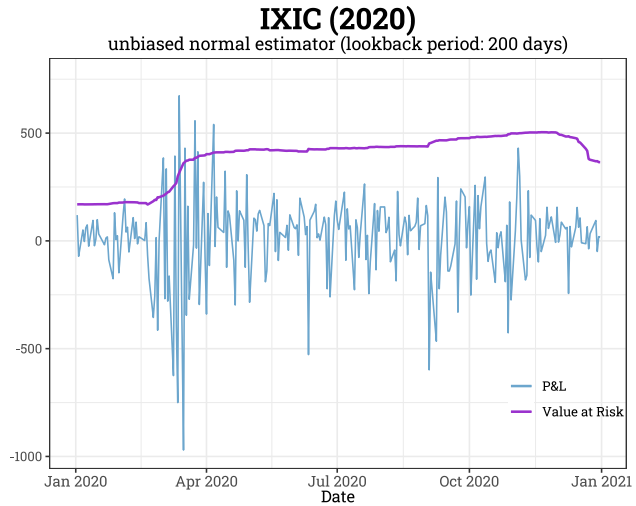
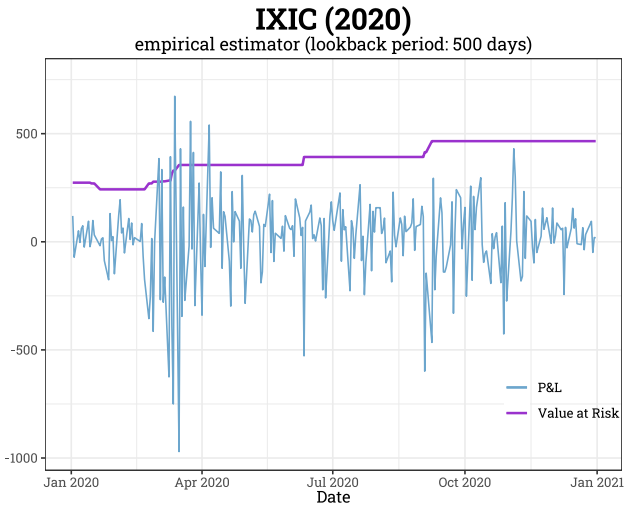
-1000
-2000
-3000
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date
P&L
Value at Risk

Nasdaq


IXIC (2020)
empirical estimator (lookback period: 200 days)
500
0
-500
-1000
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date
P&L
Value at Risk


IXIC (2020)
normal estimator (lookback period: 200 days)
500
0
-500
-1000
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date
P&L
Value at Risk


IXIC (2020)
unbiased normal estimator (lookback period: 200 days)
500
0
-500
-1000
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date
P&L
Value at Risk


IXIC (2020)
empirical estimator (lookback period: 500 days)
500
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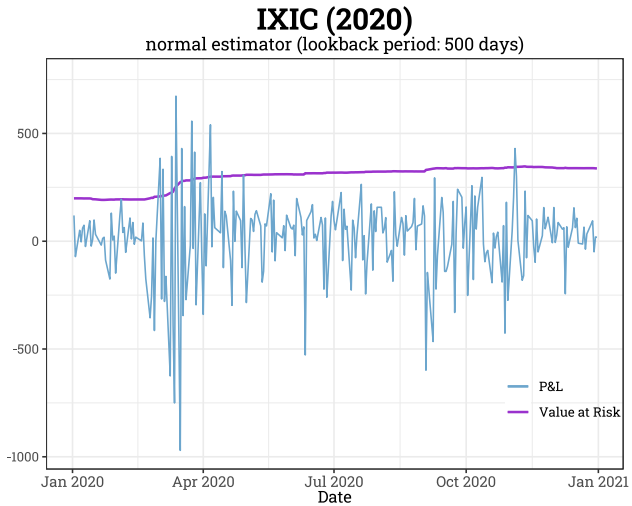
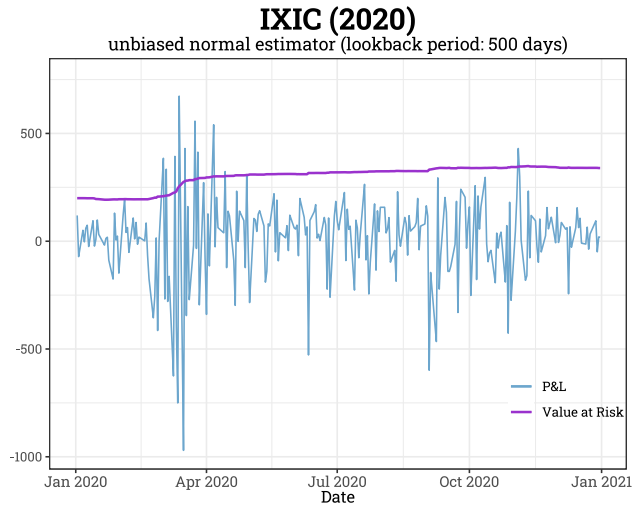
-500
-1000
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date
P&L
Value at Risk


IXIC (2020)
normal estimator (lookback period: 500 days)
500
0
-500
-1000
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date
P&L
Value at Risk


IXIC (2020)
unbiased normal estimator (lookback period: 500 days)
500
0
-500
-1000
Jan 2020
Apr 2020
Jul 2020
Oct 2020
Jan 2021
Date
P&L
Value at Risk

Regulatory VaR backtest

We count the number of capital breaches (exceptions) in the sample to quantify IMA model performance. We can divide by k to get the **exceptions rate** $T_k(y)$:

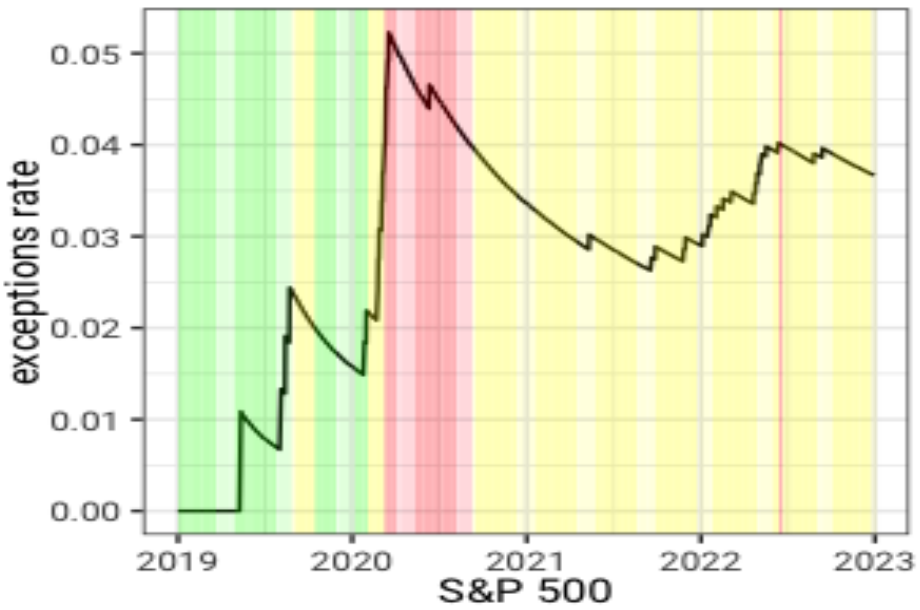
$$T_k(y) := \frac{1}{k} \sum_{i=0}^{k} 1_{\{y_i<0\}}$$

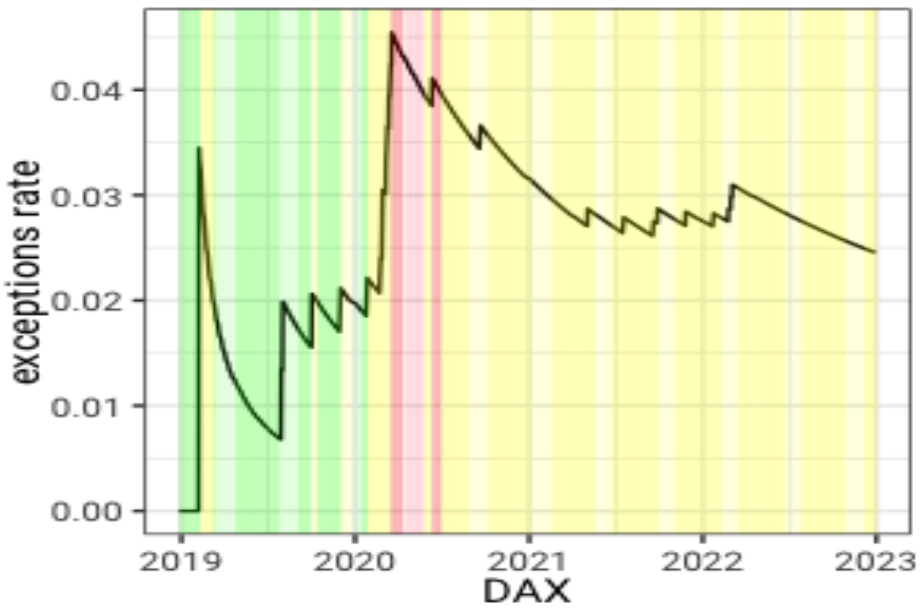
For VaR at level 1% the regulator is using $k = 250$ and the model is said to be in:

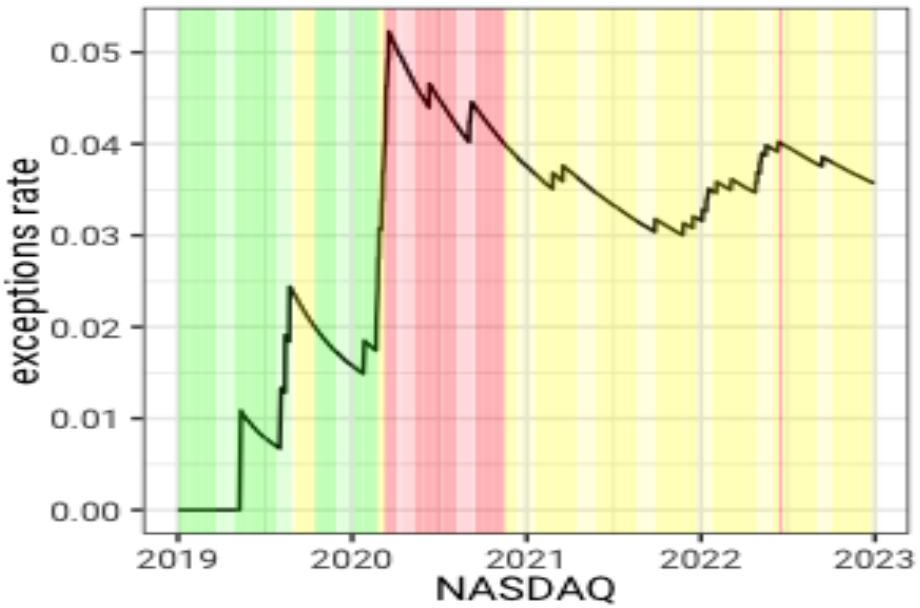
- **green zone**, if there are **less than 5 breaches** ($T_k \in [0.00, 0.02)$)
- **yellow zone**, if there are **between 5 and 10 breaches** ($T_k \in [0.02, 0.04)$)
- **red zone**, of there are **more than 10 breaches** ($T_k \in [0.04, 1.00)$)

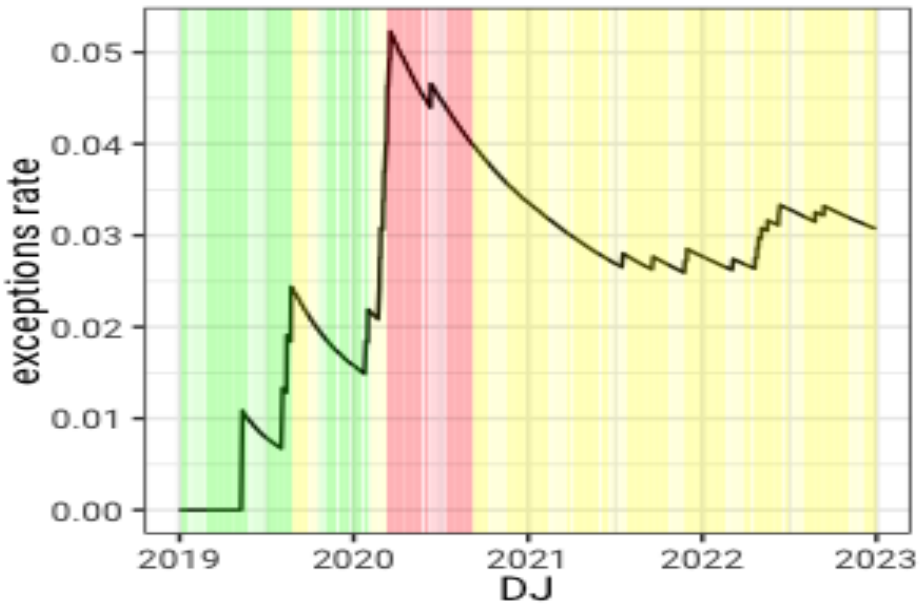
Note: *In **Regulatory VaR backtest** the sample is not an i.i.d. sample. The portfolio profile could be changing, and so can the estimated risks.*

Normal

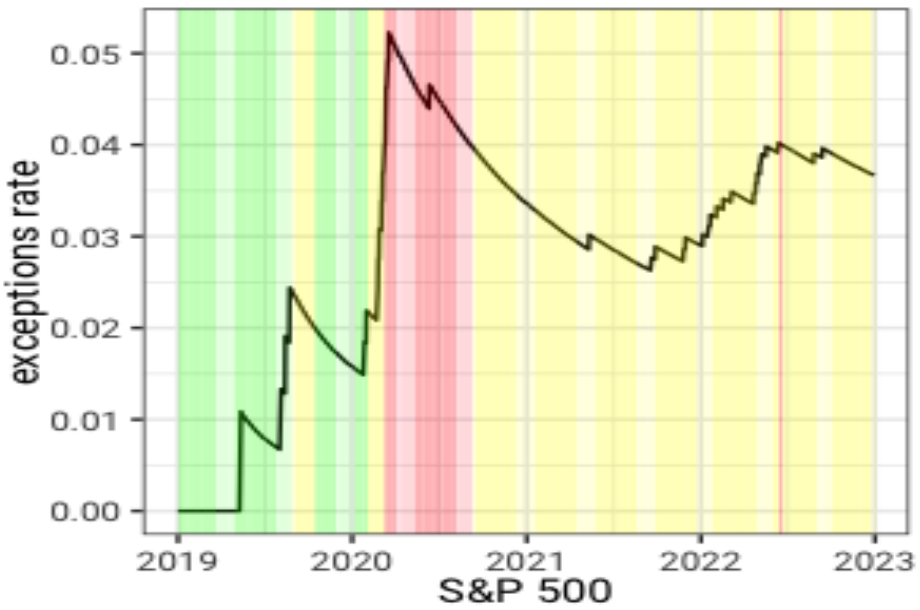

exceptions rate
0.05
0.04
0.03
0.02
0.01
0.00
2019
2020
2021
2022
2023
S&P 500

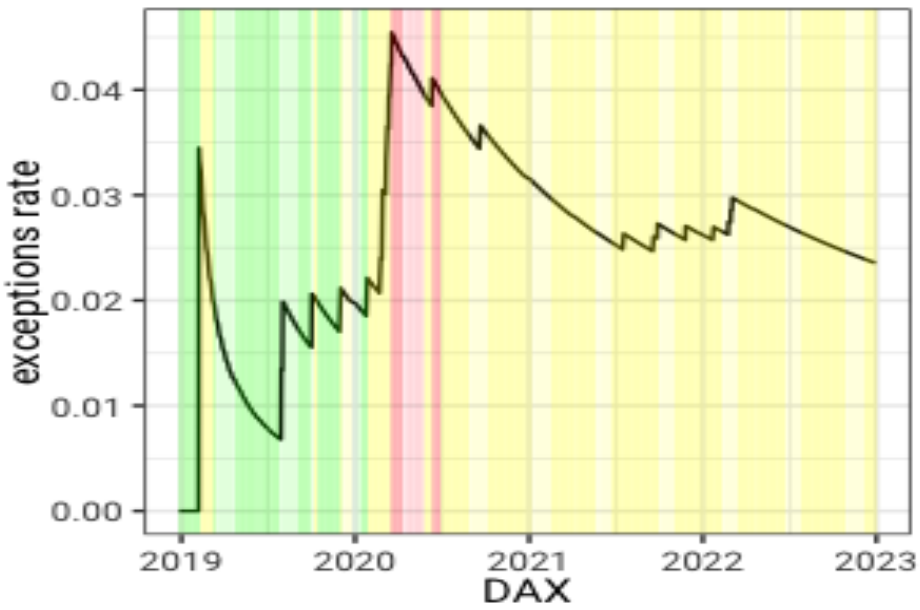

exceptions rate
0.04
0.03
0.02
0.01
0.00
2019
2020
2021
2022
2023
DAX

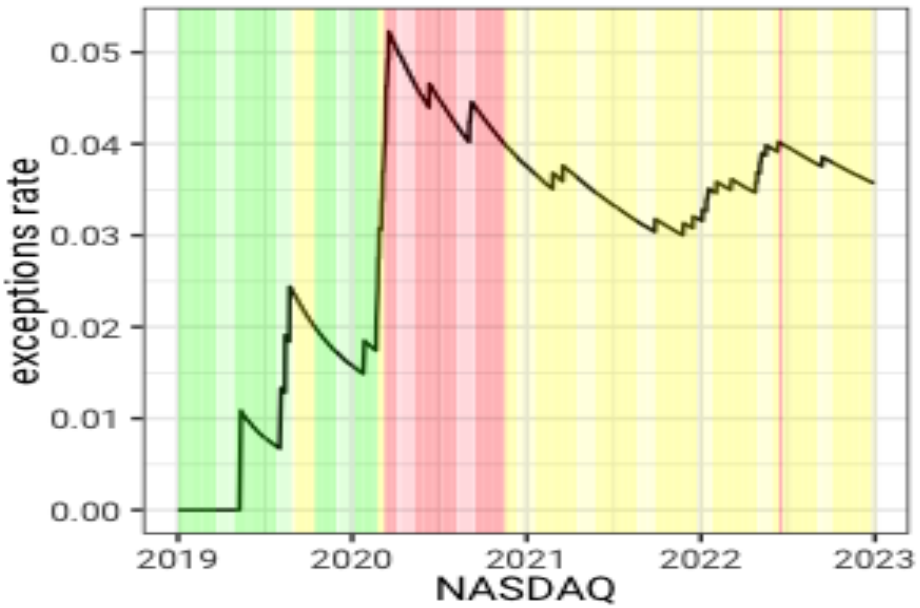

exceptions rate
0.05
0.04
0.03
0.02
0.01
0.00
2019
2020
2021
2022
2023
NASDAQ

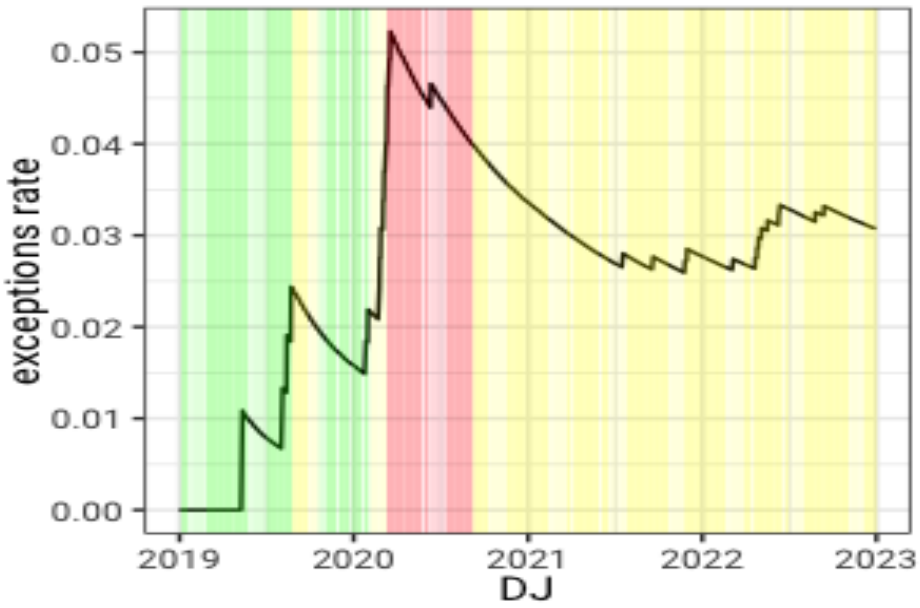

exceptions rate
0.05
0.04
0.03
0.02
0.01
0.00
2019
2020
2021
2022
2023
DJ

Unbiased Normal

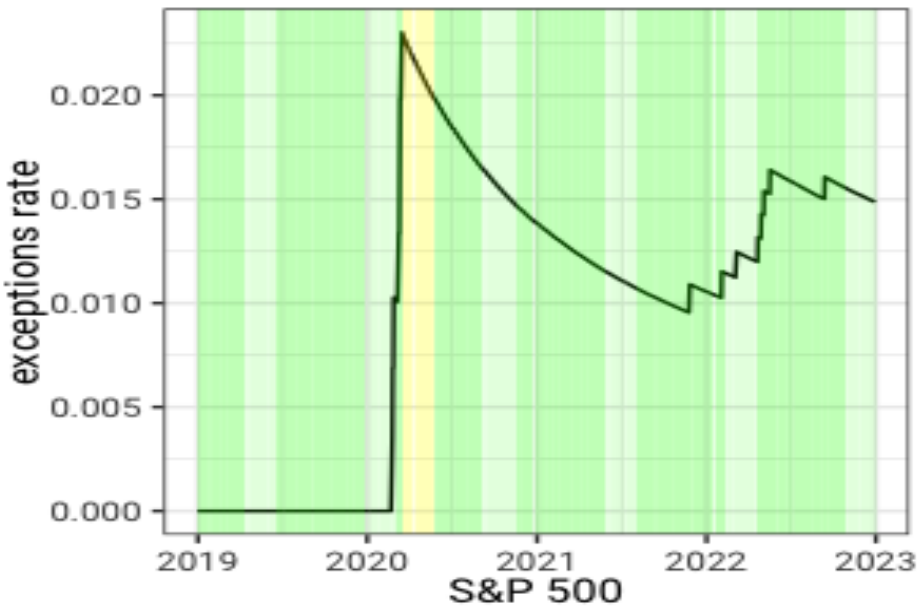


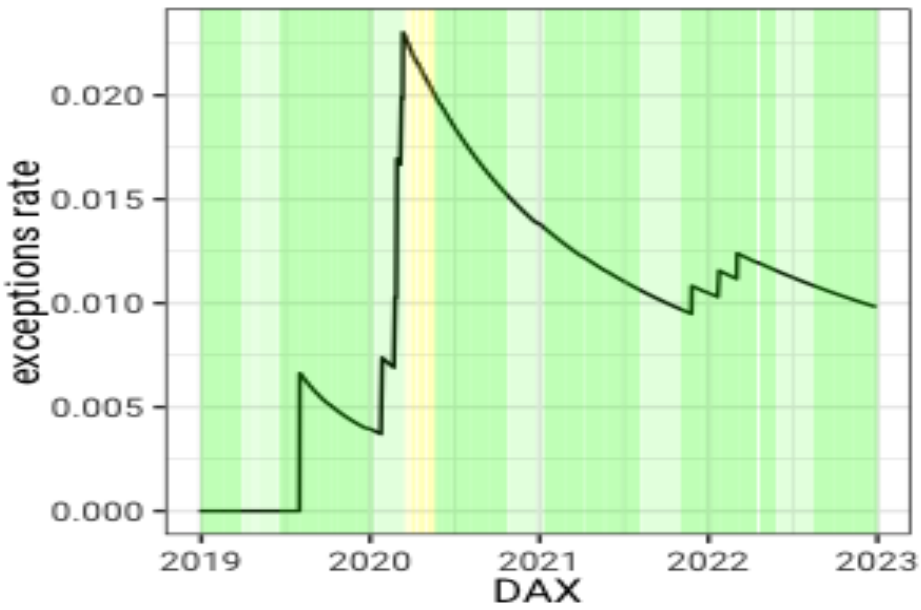


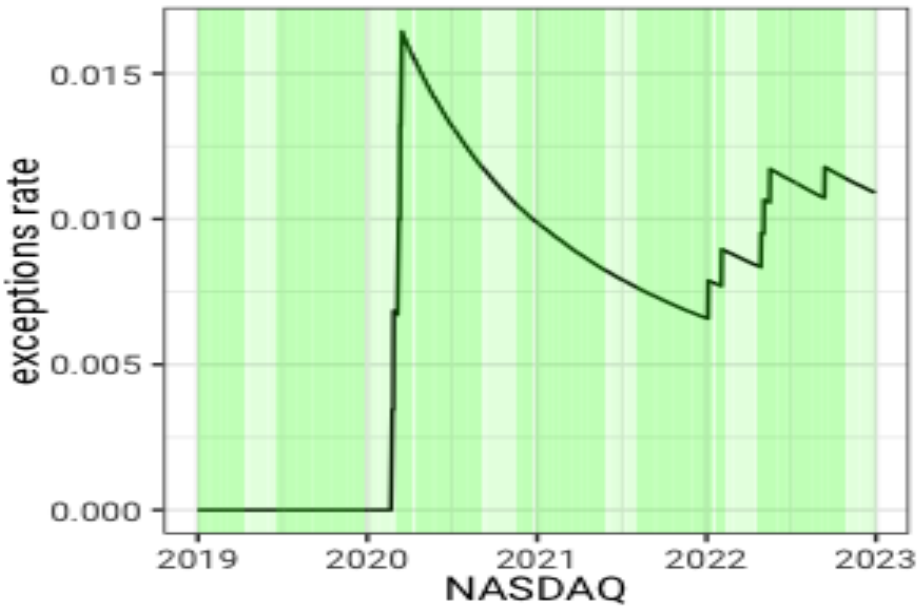


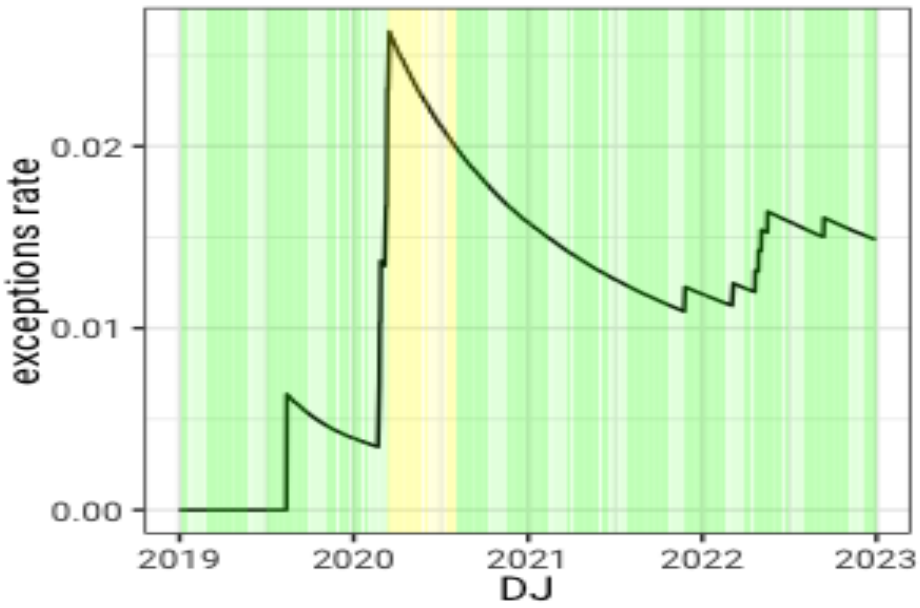


Empirical


exceptions rate
0.020
0.015
0.010
0.005
0.000
2019
2020
2021
2022
2023
S&P 500


exceptions rate
0.020
0.015
0.010
0.005
0.000
2019
2020
2021
2022
2023
DAX


exceptions rate
0.015
0.010
0.005
0.000
2019
2020
2021
2022
2023
NASDAQ


exceptions rate
0.02
0.01
0.00
2019
2020
2021
2022
2023
DJ

Conclusions

Based on the analysis of plots, backtesting and model assumptions we conclude that the empirical estimator seems to be the best.
The following actions can be done to improve the result:

- The clustering of breaches, lack of homogeneity, and dependence observed in the data may imply that time series based estimators would be more appropriate for this analysis.
- We can use another backtests (e.g. *PIT-based* or *mean quantile score evaluation*). For example, comparing Regulatory backtest to PIT-based. Regulatory does not assume a sample to be i.i.d.. Then (in the case of i.i.d. sample) PIT-based backtest will provide us with more accurate picture of backtesting for Normal and Normal Unbiased VaR estimators.

The End

Thank you for your attention!

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Sources

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2. HSBC Quants Academy Lecture Notes
3. The Basel Framework (BSBC)