The Notation of Calculus

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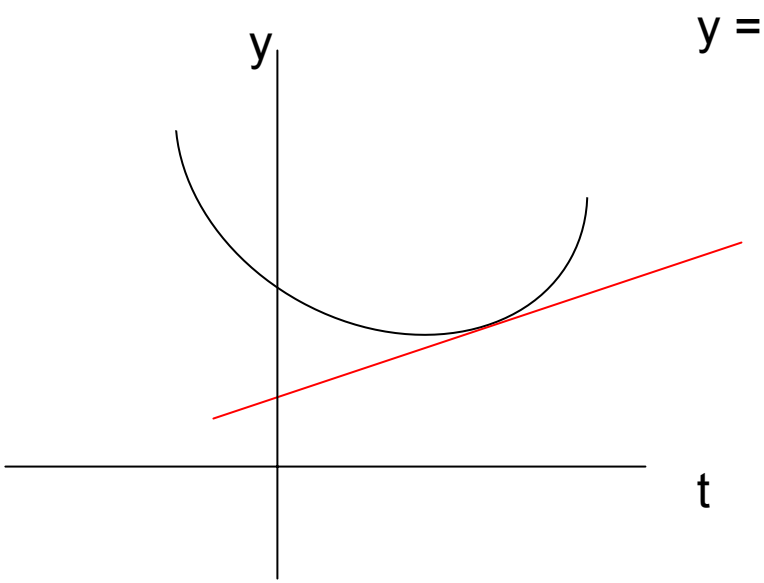
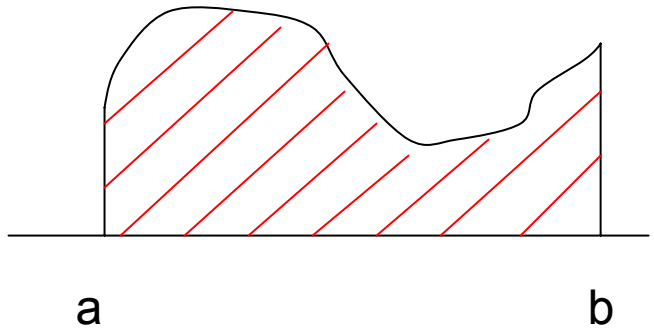
The Calculus Wars

The slope of a line and the area under a curve turned out to be a critical concepts for the development of classical physics.



y = f(t)

tangent



dy/dt

Leibniz (d-ism)

$\dot{y}$

Newton (dot-age)

$\int_a^b f(x)dx$

Leibniz, a few years junior to Newton, and vastly less famous, traveled to England to show his work in calculus to Newton in 1676. Newton stayed with the dots and did not publish until 1704. The "war" began in 1711.

British mathematicians refused to use the "continental" notation. Insults like d-ism and dot-age were hurled across the channel. The honor of inventing the calculus was at stake. Books have been written about the conflict. The outcome was that the rest of the calculus was developed by

L'Hopital, Bernoulli, Euler, Fourier, Lagrange, Legendre, Poisson…

not exactly a list of English names. One hundred years later a secret group at Cambridge met to study the notation that had been developed on the continent.

In a final twist, the Bishop Berkely, who despised the shameless modern disbelief in God, used Leibniz' notation to twit his English opponents (mathematicians were among them) saying "There is more evidence in the existence of God than of being able to divide by zero." He had learned his schoolboy algebra well.

The question is: why was Leibniz right? It is a matter of having discovered useful transformations and then developing the formalism to support them. The simplest of them is

$$(dx/dy).(dy/dt) = dx/dt$$

which acts like algebraic cancellation but represents a limit theorem. Using d-ism meant not having to think through the limit argument every time. Newton's notation, tied to differentiation with respect to time, did not generalize, did not assist thinking about other problems, and led to grotesquely unreadable formulas (with multiple dots). The collected (handwritten) papers of Newton show many examples, which apparently did not keep genius Newton from making progress, but defeated his lesser colleagues.