

LR(1) for

$$E \leftarrow T \mid$$
$$T \leftarrow F$$
$$T \leftarrow T * F$$
$$F \leftarrow i$$
$$F \leftarrow (T)$$

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$E \leftarrow T \mid$

$T \leftarrow F$

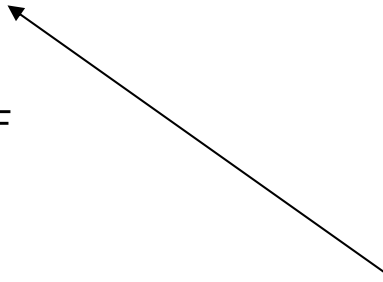
$T \leftarrow T * F$

$F \leftarrow i$

$F \leftarrow (T)$

a simple
expression cfg

end-of-file symbol



$E \leftarrow \square T \mid$



LR start state

$E \leftarrow T \mid$

$T \leftarrow F$

$T \leftarrow T * F$

$F \leftarrow i$

$F \leftarrow (T)$

| | |
|-----------------------|--------|
| $E \leftarrow T \mid$ | |
| $T \leftarrow F$ | \mid |
| $T \leftarrow T * F$ | \mid |

marked rules

lookahead

$E \leftarrow T \mid$
 $T \leftarrow F$
 $T \leftarrow T * F$
 $F \leftarrow i$
 $F \leftarrow (T)$

closure for T

kernel in red

closure in green

| | |
|---------------------------|--------|
| $E \leftarrow T \mid$ | |
| $T \leftarrow F \mid$ | \mid |
| $T \leftarrow T * F \mid$ | \mid |
| $F \leftarrow i \mid$ | \mid |
| $F \leftarrow (T) \mid$ | \mid |

$E \leftarrow T \mid$

$T \leftarrow F$

$T \leftarrow T * F$

$F \leftarrow i$

$F \leftarrow (T)$

closure for F

| | |
|---------------------------|--------|
| $E \leftarrow T \mid$ | |
| $T \leftarrow F \mid$ | \mid |
| $T \leftarrow T * F \mid$ | \mid |
| $F \leftarrow i \mid$ | \mid |
| $F \leftarrow (T) \mid$ | \mid |
| $T \leftarrow F$ | $*$ |
| $T \leftarrow T * F$ | $*$ |

$E \leftarrow T \mid$
 $T \leftarrow F$
 $T \leftarrow T * F$
 $F \leftarrow i$
 $F \leftarrow (T)$

another closure for T

| | |
|---------------------------|--------|
| $E \leftarrow T \mid$ | |
| $T \leftarrow F \mid$ | \mid |
| $T \leftarrow T * F \mid$ | \mid |
| $F \leftarrow i \mid$ | \mid |
| $F \leftarrow (T) \mid$ | \mid |
| $T \leftarrow F$ | * |
| $T \leftarrow T * F$ | * |
| $F \leftarrow i$ | * |
| $F \leftarrow (T)$ | * |

$E \leftarrow T \mid$
 $T \leftarrow F$
 $T \leftarrow T * F$
 $F \leftarrow i$
 $F \leftarrow (T)$

another closure for F

| | |
|---------------------------------|------------|
| $E \leftarrow \square T \vdash$ | |
| $T \leftarrow \square F$ | \vdash^* |
| $T \leftarrow \square T * F$ | \vdash^* |
| $F \leftarrow \square i$ | \vdash^* |
| $F \leftarrow \square (T)$ | \vdash^* |

$E \leftarrow T \vdash$

$T \leftarrow F$

$T \leftarrow T * F$

$F \leftarrow i$

$F \leftarrow (T)$

merge lookaheads

closure complete

| | |
|-----------------------|----------|
| $E \leftarrow T \mid$ | |
| $T \leftarrow F$ | \mid^* |
| $T \leftarrow T * F$ | \mid^* |
| $F \leftarrow i$ | \mid^* |
| $F \leftarrow (T)$ | \mid^* |

| | |
|-----------------------|----------|
| $E \leftarrow T \mid$ | |
| $T \leftarrow T * F$ | \mid^* |

$E \leftarrow T \mid$
 $T \leftarrow F$
 $T \leftarrow T * F$
 $F \leftarrow i$
 $F \leftarrow (T)$



shift T

new kernel

| | |
|-----------------------|----------|
| $E \leftarrow T \mid$ | |
| $T \leftarrow F$ | \mid^* |
| $T \leftarrow T * F$ | \mid^* |
| $F \leftarrow i$ | \mid^* |
| $F \leftarrow (T)$ | \mid^* |

| | |
|-----------------------|----------|
| $E \leftarrow T \mid$ | |
| $T \leftarrow T * F$ | \mid^* |

| | |
|-----------------------|----------|
| $T \leftarrow F \mid$ | \mid^* |
|-----------------------|----------|

$E \leftarrow T \mid$
 $T \leftarrow F$
 $T \leftarrow T * F$
 $F \leftarrow i$
 $F \leftarrow (T)$

shift F

another new kernel

| | |
|-------------------------------|----------|
| $E \leftarrow \square T \mid$ | |
| $T \leftarrow \square F$ | \mid^* |
| $T \leftarrow \square T * F$ | \mid^* |
| $F \leftarrow \square i$ | \mid^* |
| $F \leftarrow \square (T)$ | \mid^* |

| | |
|-------------------------------|----------|
| $E \leftarrow T \square \mid$ | |
| $T \leftarrow T \square * F$ | \mid^* |

| | |
|--------------------------|----------|
| $T \leftarrow F \square$ | \mid^* |
|--------------------------|----------|

| | |
|--------------------------|----------|
| $F \leftarrow i \square$ | \mid^* |
|--------------------------|----------|

$E \leftarrow T \mid$
 $T \leftarrow F$
 $T \leftarrow T * F$
 $F \leftarrow i$
 $F \leftarrow (T)$

shift i

another new kernel

1

| | |
|-------------------------------|----------|
| $E \leftarrow \square T \mid$ | |
| $T \leftarrow \square F$ | \mid^* |
| $T \leftarrow \square T * F$ | \mid^* |
| $F \leftarrow \square i$ | \mid^* |
| $F \leftarrow \square (T)$ | \mid^* |

| | |
|-------------------------------|----------|
| $E \leftarrow T \square \mid$ | |
| $T \leftarrow T \square * F$ | \mid^* |

| | |
|--------------------------|----------|
| $T \leftarrow F \square$ | \mid^* |
|--------------------------|----------|

| | |
|--------------------------|----------|
| $F \leftarrow i \square$ | \mid^* |
|--------------------------|----------|

| | |
|------------------------------|----------|
| $F \leftarrow (\square T)$ | \mid^* |
|------------------------------|----------|

$E \leftarrow T \mid$
 $T \leftarrow F$
 $T \leftarrow T * F$
 $F \leftarrow i$
 $F \leftarrow (T)$

finish state 1

shift (

another new kernel

1

| | |
|-------------------------------|----------|
| $E \leftarrow \square T \mid$ | |
| $T \leftarrow \square F$ | \mid^* |
| $T \leftarrow \square T * F$ | \mid^* |
| $F \leftarrow \square i$ | \mid^* |
| $F \leftarrow \square (T)$ | \mid^* |

| | |
|-------------------------------|----------|
| $E \leftarrow T \square \mid$ | |
| $T \leftarrow T \square * F$ | \mid^* |

| | |
|--------------------------|----------|
| $T \leftarrow F \square$ | \mid^* |
|--------------------------|----------|

| | |
|--------------------------|----------|
| $F \leftarrow i \square$ | \mid^* |
|--------------------------|----------|

| | |
|------------------------------|----------|
| $F \leftarrow (\square T)$ | \mid^* |
| $T \leftarrow \square F$ | $)^*$ |
| $T \leftarrow \square T * F$ | $)^*$ |
| $F \leftarrow \square i$ | $)^*$ |
| $F \leftarrow \square (T)$ | $)^*$ |

$E \leftarrow T \mid$

$T \leftarrow F$

$T \leftarrow T * F$

$F \leftarrow i$

$F \leftarrow (T)$

closure

1

| | |
|-------------------------------|----------|
| $E \leftarrow \square T \mid$ | |
| $T \leftarrow \square F$ | \mid^* |
| $T \leftarrow \square T * F$ | \mid^* |
| $F \leftarrow \square i$ | \mid^* |
| $F \leftarrow \square (T)$ | \mid^* |

| | |
|-------------------------------|----------|
| $E \leftarrow T \square \mid$ | |
| $T \leftarrow T \square * F$ | \mid^* |

| | |
|--------------------------|----------|
| $T \leftarrow F \square$ | \mid^* |
|--------------------------|----------|

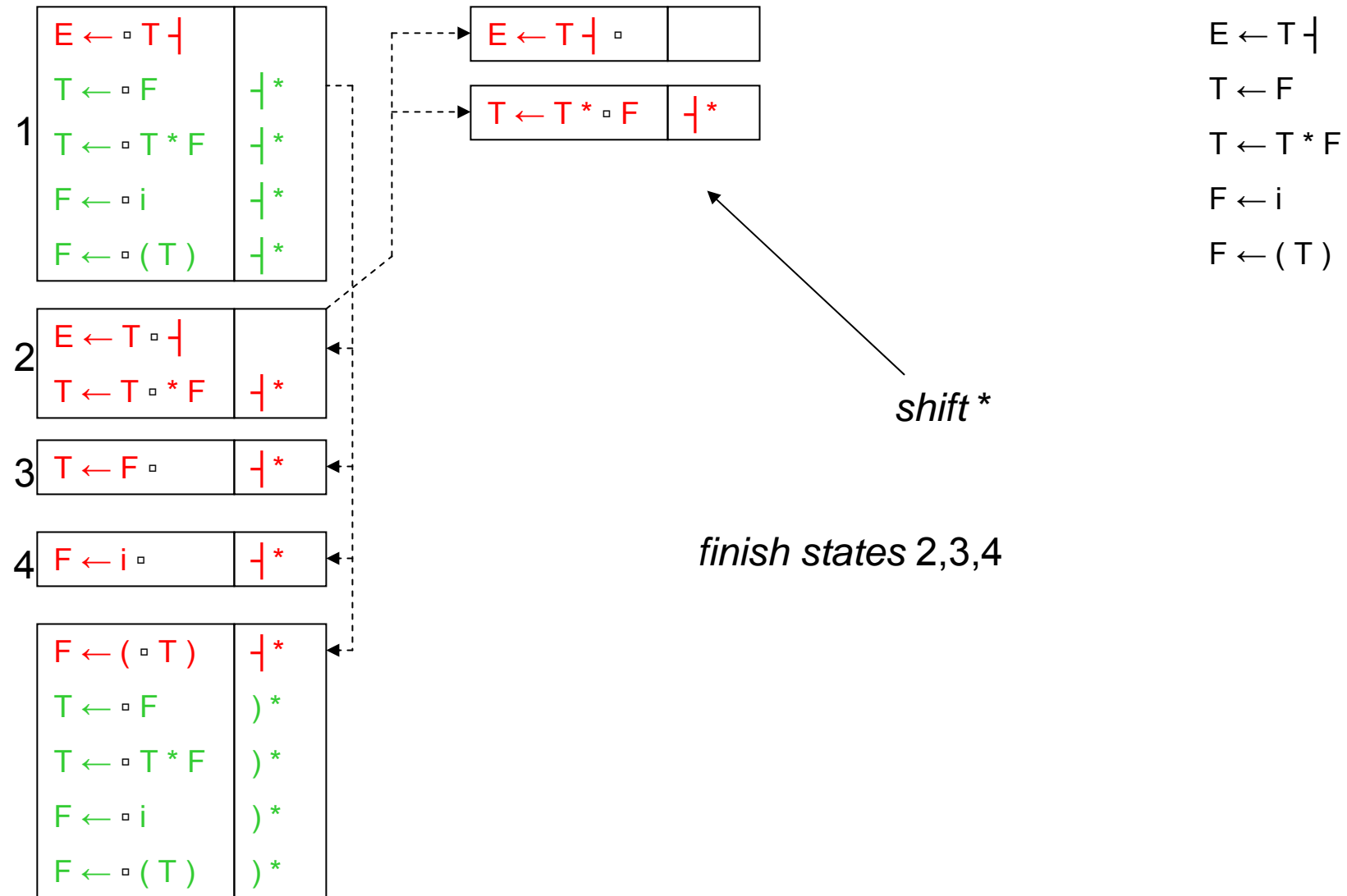
| | |
|--------------------------|----------|
| $F \leftarrow i \square$ | \mid^* |
|--------------------------|----------|

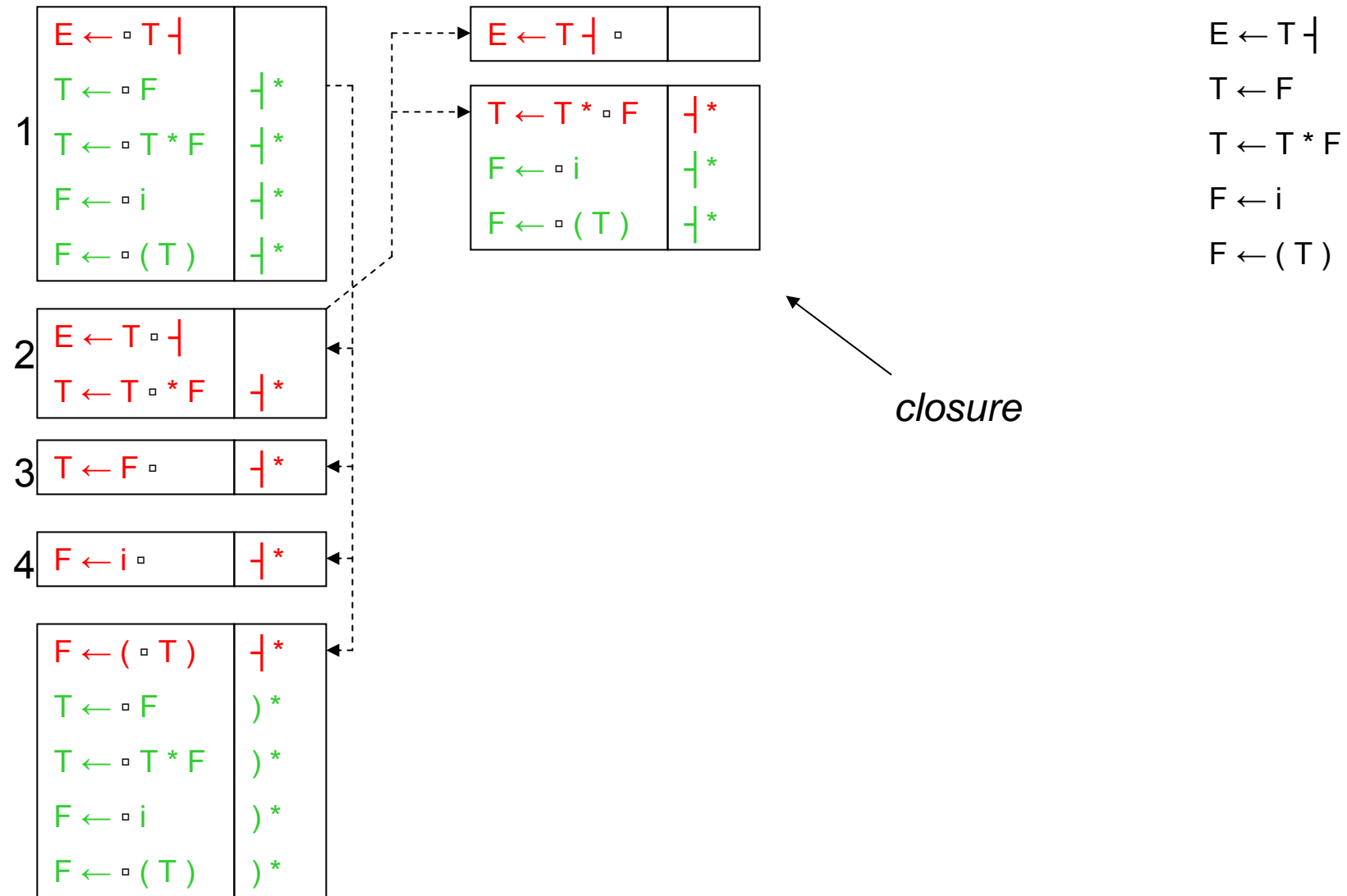
| | |
|------------------------------|----------|
| $F \leftarrow (\square T)$ | \mid^* |
| $T \leftarrow \square F$ | $)^*$ |
| $T \leftarrow \square T * F$ | $)^*$ |
| $F \leftarrow \square i$ | $)^*$ |
| $F \leftarrow \square (T)$ | $)^*$ |

| | |
|-------------------------------|--|
| $E \leftarrow T \mid \square$ | |
|-------------------------------|--|

- $E \leftarrow T \mid$
- $T \leftarrow F$
- $T \leftarrow T * F$
- $F \leftarrow i$
- $F \leftarrow (T)$

shift \mid (final LR state)





1

| | |
|-----------------------|----------|
| $E \leftarrow T \mid$ | |
| $T \leftarrow F$ | \mid^* |
| $T \leftarrow T * F$ | \mid^* |
| $F \leftarrow i$ | \mid^* |
| $F \leftarrow (T)$ | \mid^* |

| | |
|-----------------------|--|
| $E \leftarrow T \mid$ | |
|-----------------------|--|

| | |
|----------------------|----------|
| $T \leftarrow T * F$ | \mid^* |
| $F \leftarrow i$ | \mid^* |
| $F \leftarrow (T)$ | \mid^* |

| | |
|-------------------------|----------|
| $F \leftarrow (T \mid)$ | \mid^* |
| $T \leftarrow T * F$ | $)^*$ |

2

| | |
|-----------------------|----------|
| $E \leftarrow T \mid$ | |
| $T \leftarrow T * F$ | \mid^* |

3

| | |
|-----------------------|----------|
| $T \leftarrow F \mid$ | \mid^* |
|-----------------------|----------|

4

| | |
|-----------------------|----------|
| $F \leftarrow i \mid$ | \mid^* |
|-----------------------|----------|

| | |
|-------------------------|----------|
| $F \leftarrow (T \mid)$ | \mid^* |
| $T \leftarrow F$ | $)^*$ |
| $T \leftarrow T * F$ | $)^*$ |
| $F \leftarrow i$ | $)^*$ |
| $F \leftarrow (T)$ | $)^*$ |

$E \leftarrow T \mid$

$T \leftarrow F$

$T \leftarrow T * F$

$F \leftarrow i$

$F \leftarrow (T)$

shift T

1

| | |
|-----------------------|----------|
| $E \leftarrow T \mid$ | |
| $T \leftarrow F$ | \mid^* |
| $T \leftarrow T * F$ | \mid^* |
| $F \leftarrow i$ | \mid^* |
| $F \leftarrow (T)$ | \mid^* |

2

| | |
|-----------------------|----------|
| $E \leftarrow T \mid$ | |
| $T \leftarrow T * F$ | \mid^* |

3

| | |
|-----------------------|----------|
| $T \leftarrow F \mid$ | \mid^* |
|-----------------------|----------|

4

| | |
|-----------------------|----------|
| $F \leftarrow i \mid$ | \mid^* |
|-----------------------|----------|

| | |
|----------------------|----------|
| $F \leftarrow (T)$ | \mid^* |
| $T \leftarrow F$ | $)^*$ |
| $T \leftarrow T * F$ | $)^*$ |
| $F \leftarrow i$ | $)^*$ |
| $F \leftarrow (T)$ | $)^*$ |

| | |
|-----------------------|--|
| $E \leftarrow T \mid$ | |
|-----------------------|--|

| | |
|----------------------|----------|
| $T \leftarrow T * F$ | \mid^* |
| $F \leftarrow i$ | \mid^* |
| $F \leftarrow (T)$ | \mid^* |

| | |
|------------------------|----------|
| $F \leftarrow (T \mid$ | \mid^* |
| $T \leftarrow T * F$ | $)^*$ |

| | |
|-----------------------|-------|
| $T \leftarrow F \mid$ | $)^*$ |
|-----------------------|-------|

$E \leftarrow T \mid$

$T \leftarrow F$

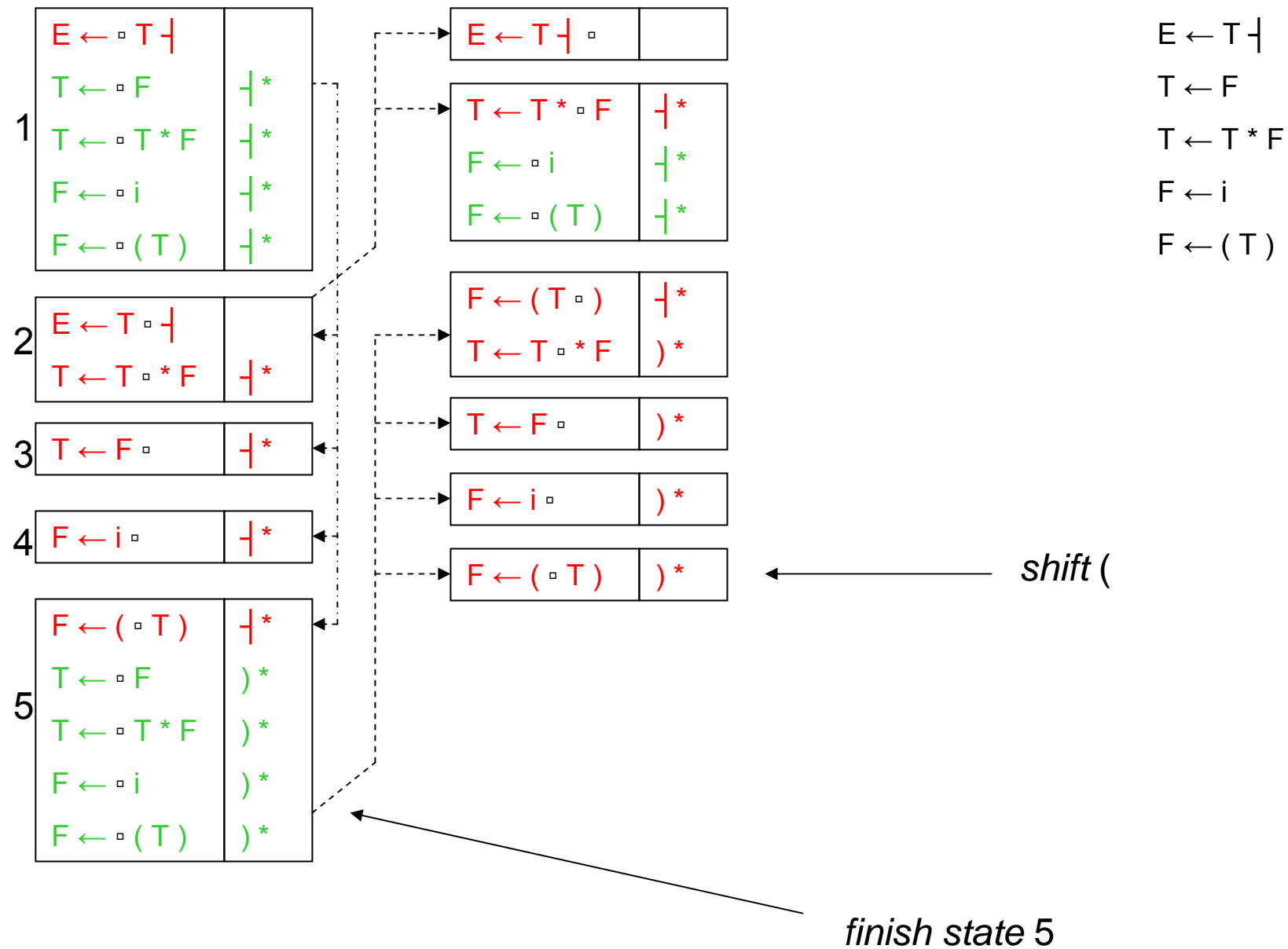
$T \leftarrow T * F$

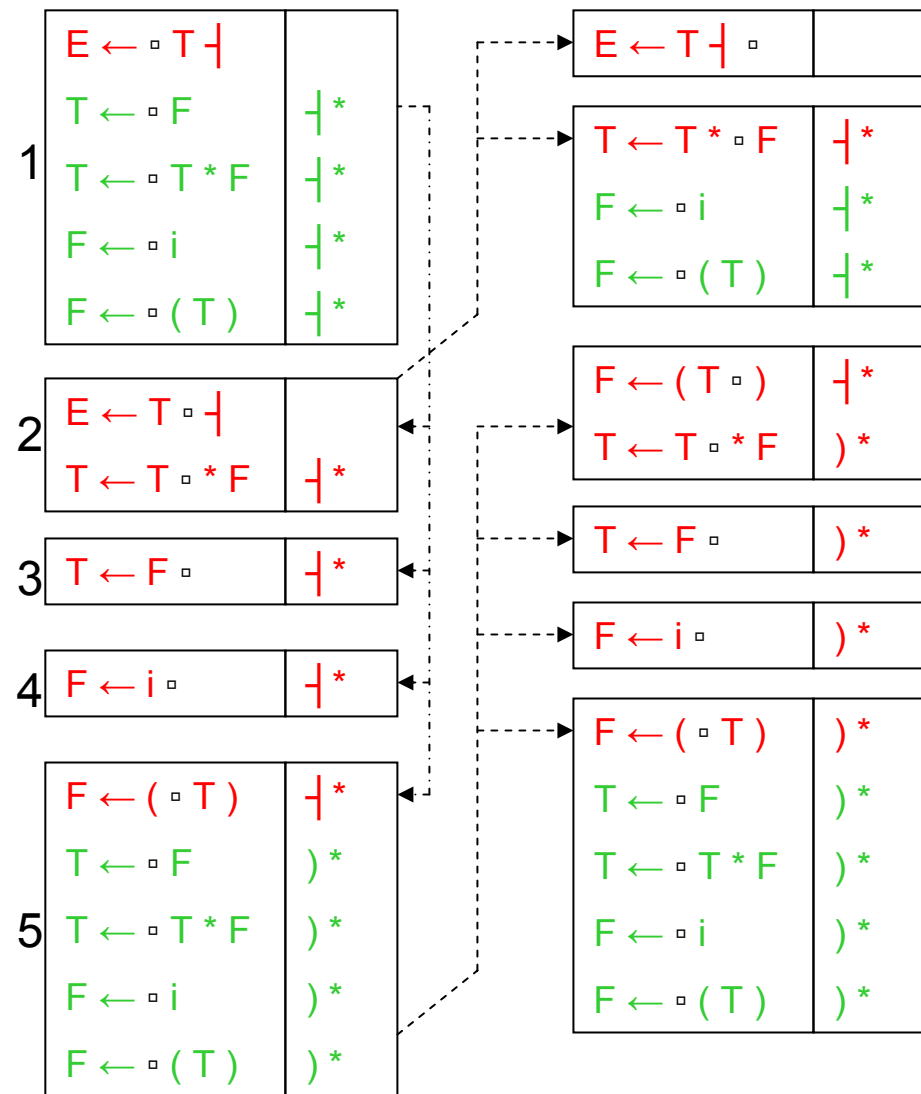
$F \leftarrow i$

$F \leftarrow (T)$

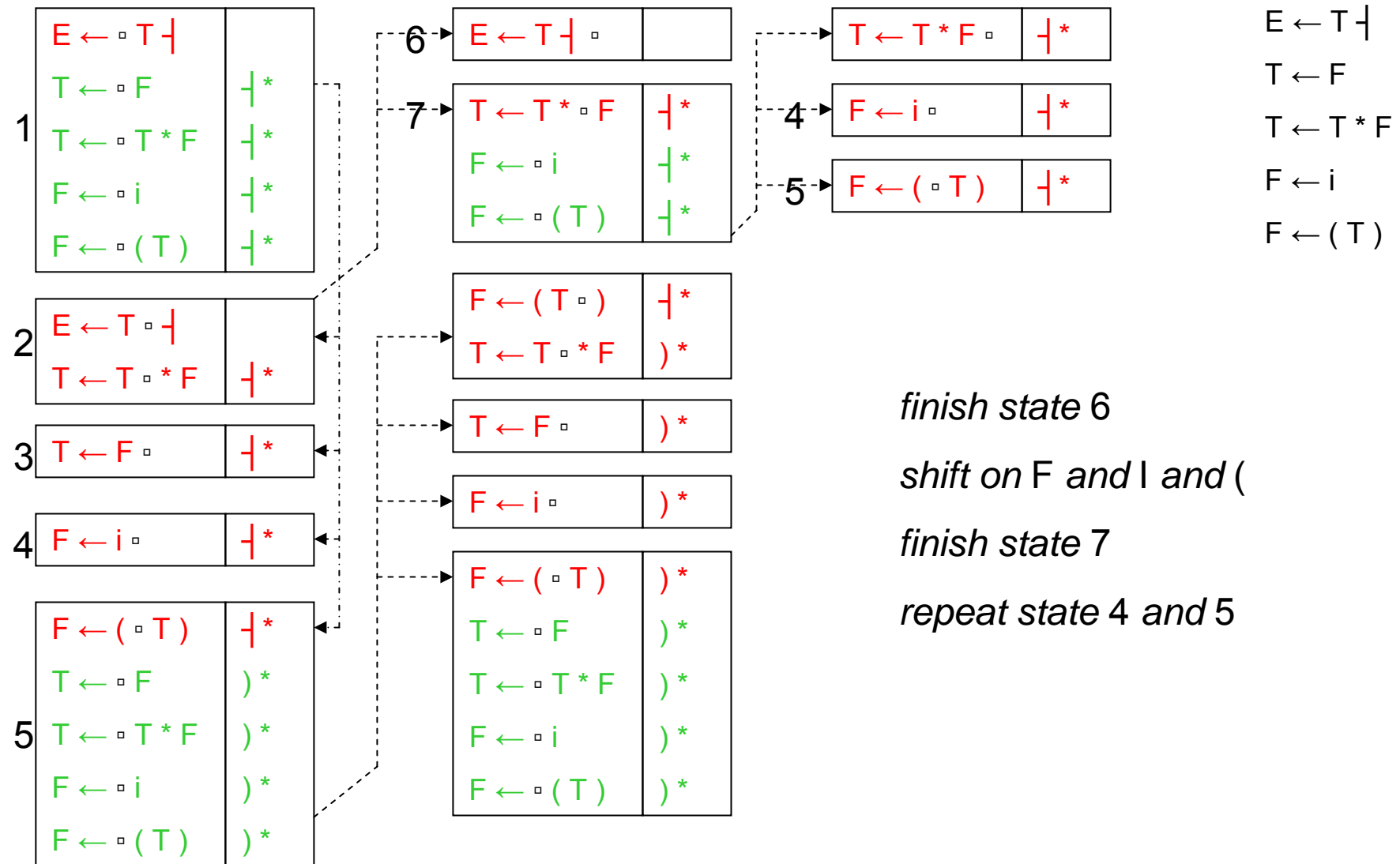
shift F

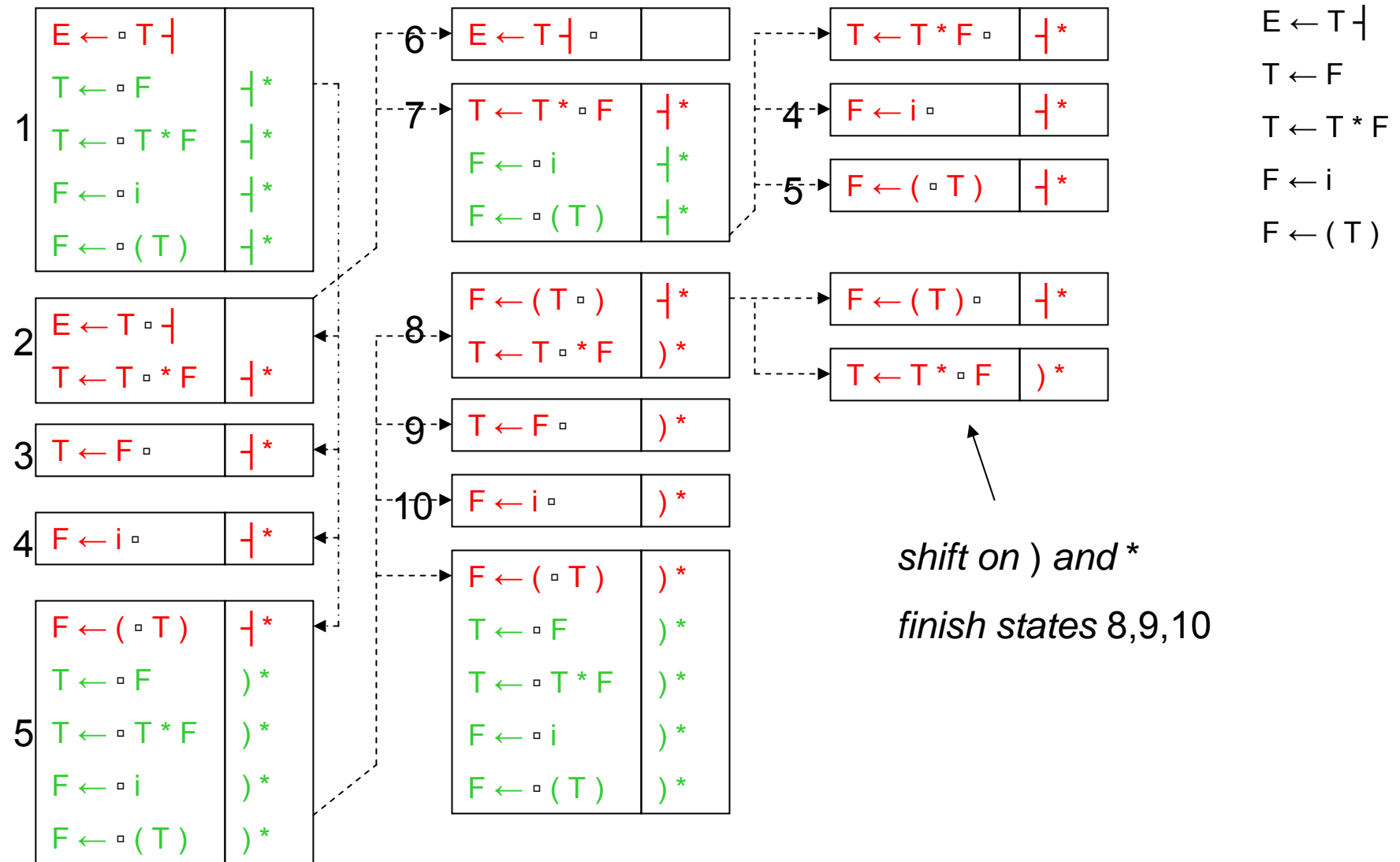


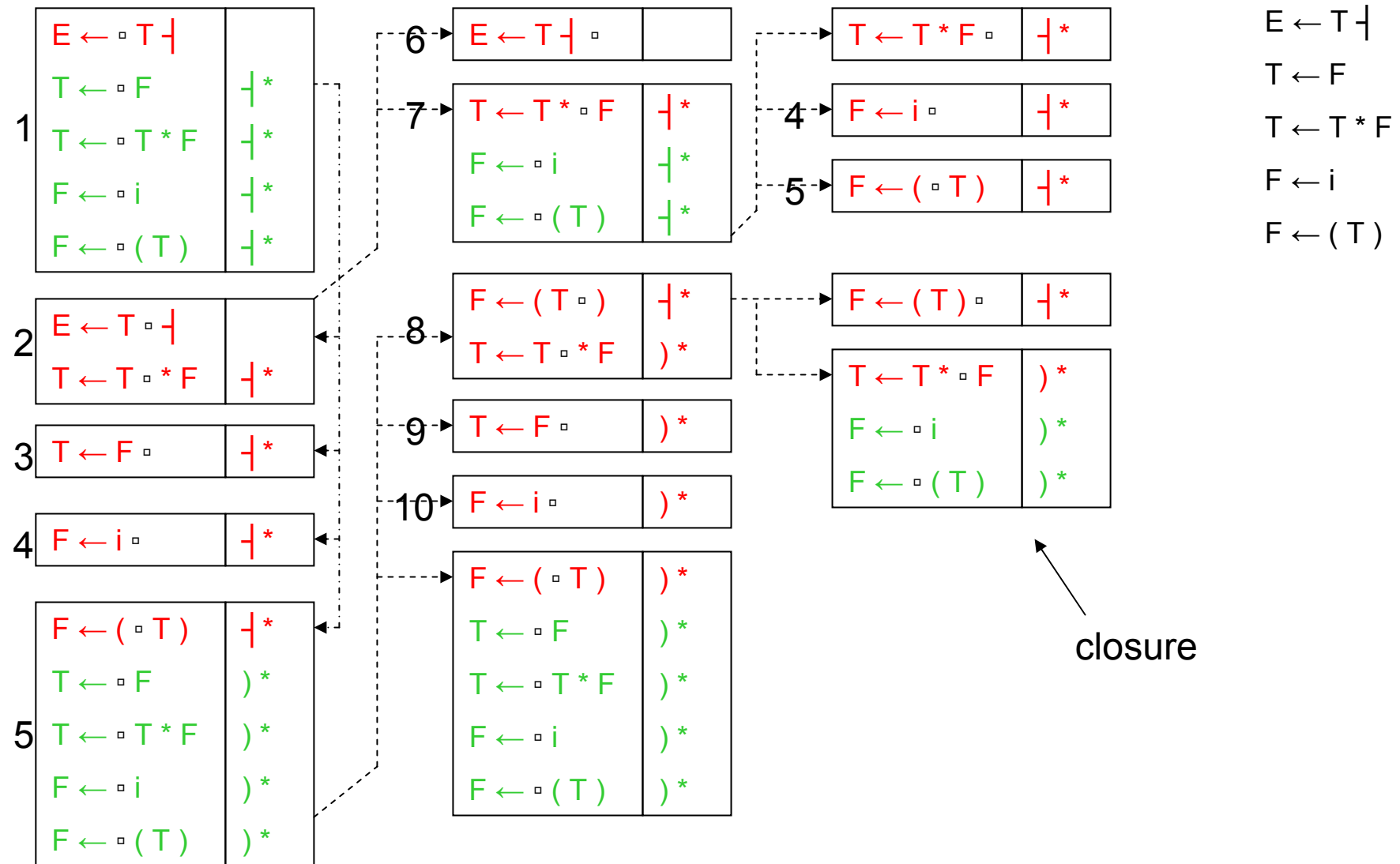


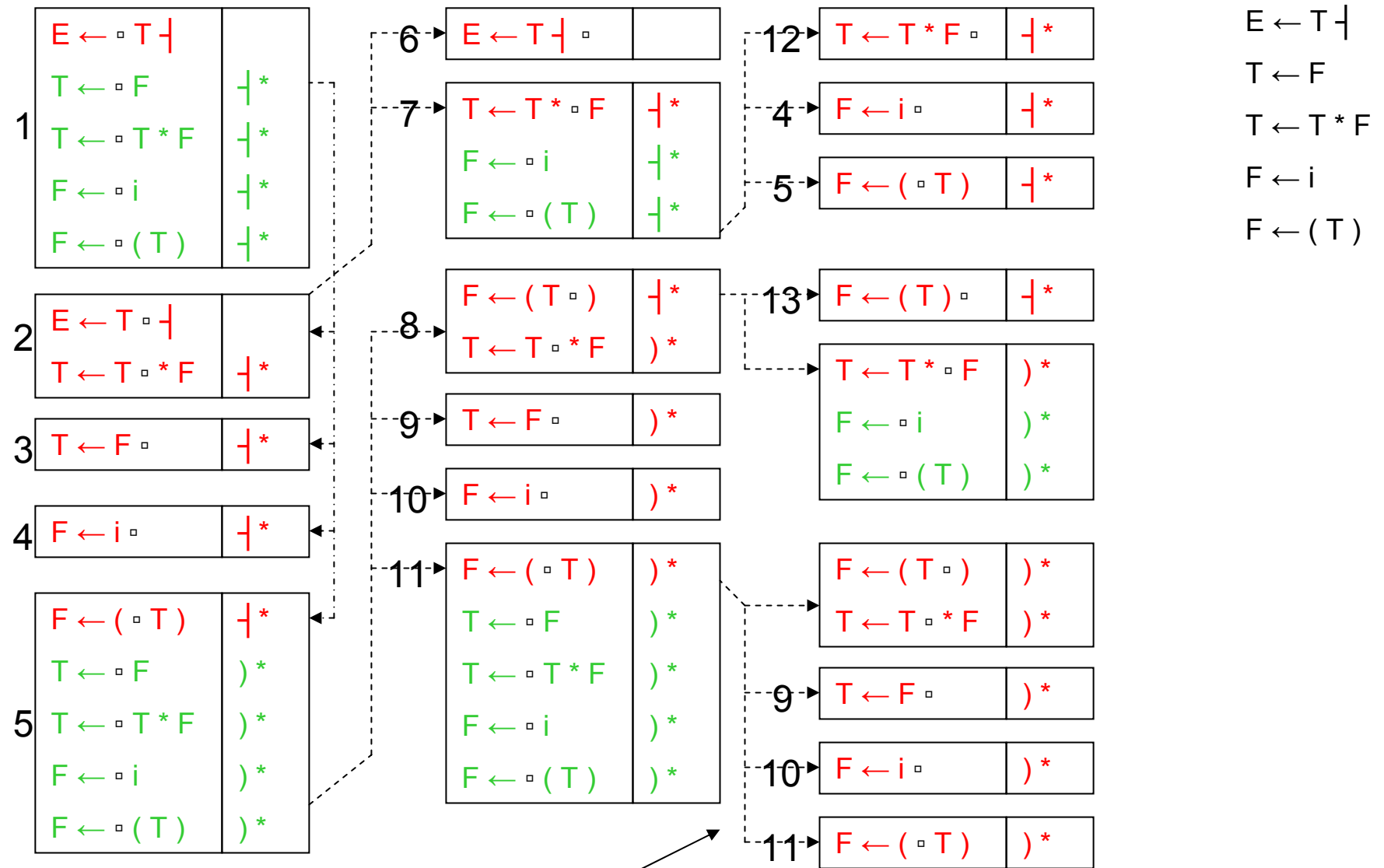


$E \leftarrow T \mid$
 $T \leftarrow F$
 $T \leftarrow T * F$
 $F \leftarrow i$
 $F \leftarrow (T)$



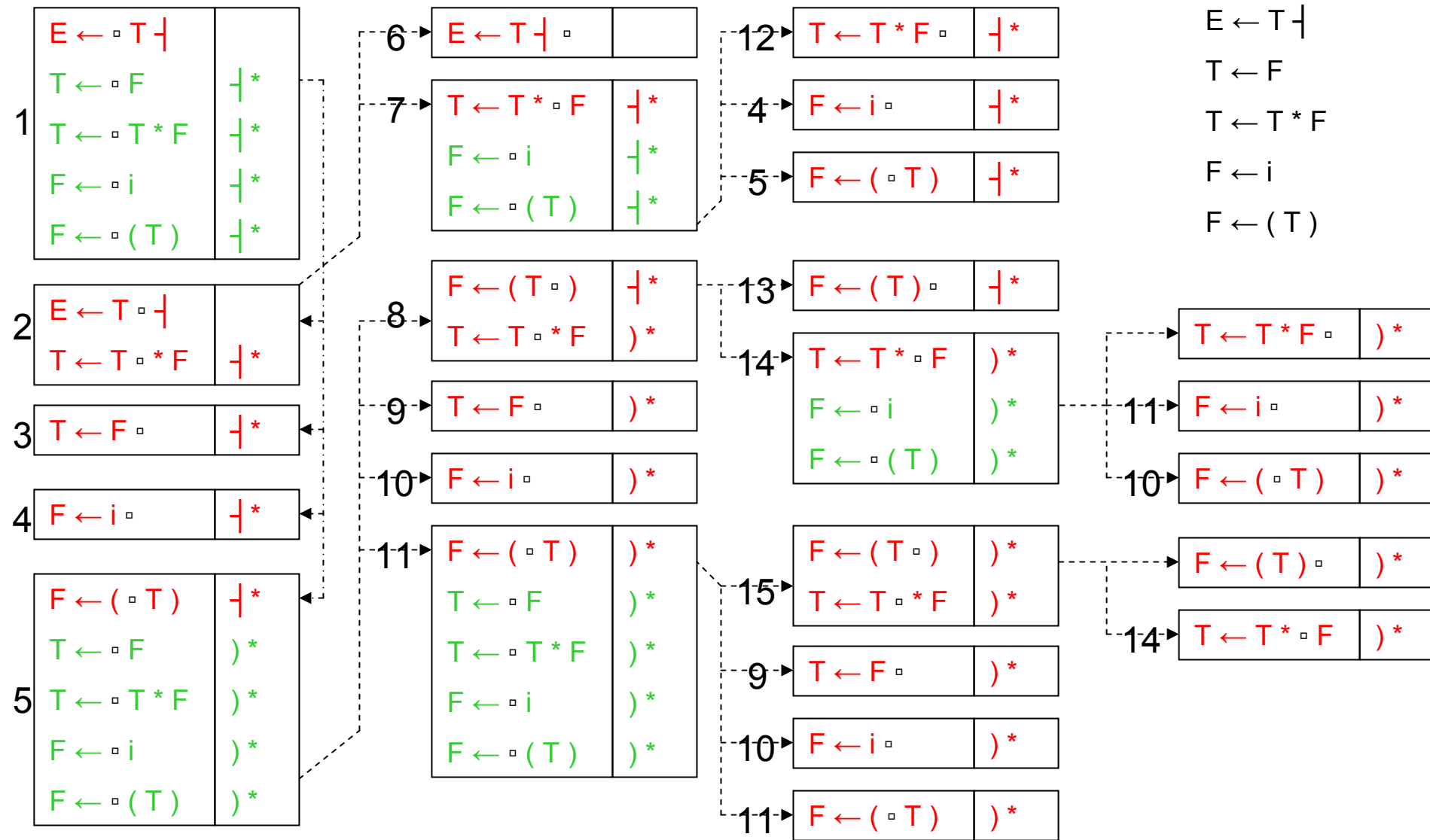




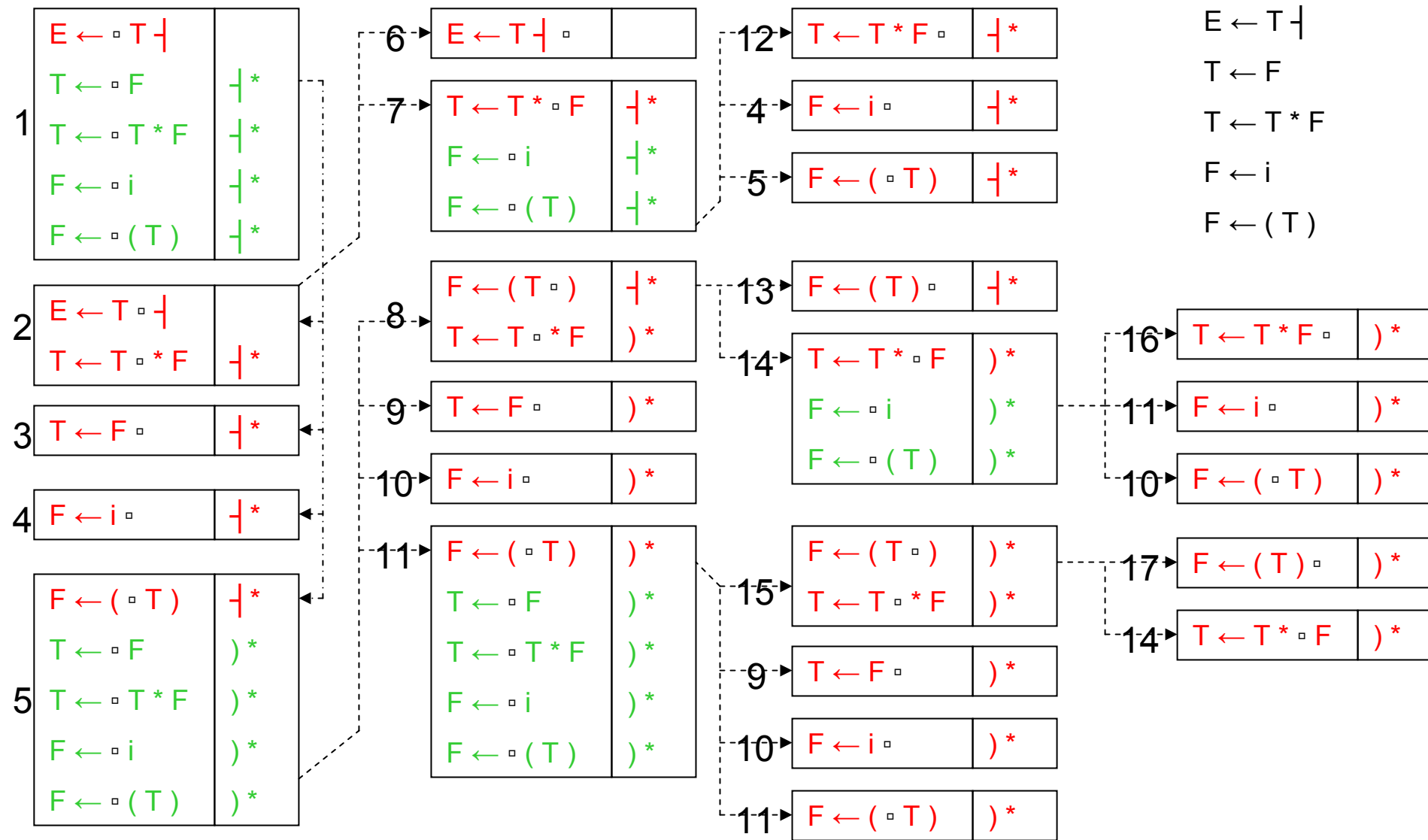


shift on T and F and i and (

finish state 11, repeat states 9,10,11, finish states 12,13



*shift) and *, finish state 15, repeat state 14*



finish states 16,17

states

symbols

The LR Matrix

stack

input

| | E | T | F | | i | * | (|) |
|----|----|----|----|----|----|----|----|----|
| 1 | | 2 | 3 | | 4 | | 5 | |
| 2 | | | | 6 | | 7 | | |
| 3 | | | | -2 | | -2 | | |
| 4 | | | | -4 | | -4 | | |
| 5 | | 8 | 9 | | 10 | | 11 | |
| 6 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 7 | | | 12 | | 4 | | 5 | |
| 8 | | | 14 | | | | | 13 |
| 9 | | | | | | -2 | | -2 |
| 10 | | | | | | -4 | | -4 |
| 11 | | 15 | 9 | | 10 | | 11 | |
| 12 | | | | -3 | | -3 | | |
| 13 | | | | -5 | | -5 | | |
| 14 | | | 16 | | 11 | | 10 | |
| 15 | | | | | | 14 | | 17 |
| 16 | | | | | | -3 | | -3 |
| 17 | | | | | | -5 | | -5 |

-1: $E \leftarrow T |$

-2: $T \leftarrow F$

-3: $T \leftarrow T * F$

-4: $F \leftarrow i$

-5: $F \leftarrow (T)$

$_1$

$_1 i_4$

$_1 F$

$_1 F_3$

$_1 T$

$_1 T_2$

$_1 T_2^* 7$

$_1 T_2^* 7 i_4$

$_1 T_2^* 7 F$

$_1 T_2^* 7 F_{12}$

$_1 T$

$_1 T_2$

$_1 T_2 |_6$

E

$i * i |$

$* i |$

$* i |$

$* i |$

$* i |$

$* i |$

$i |$

$|$

$|$

$|$

$|$

$|$