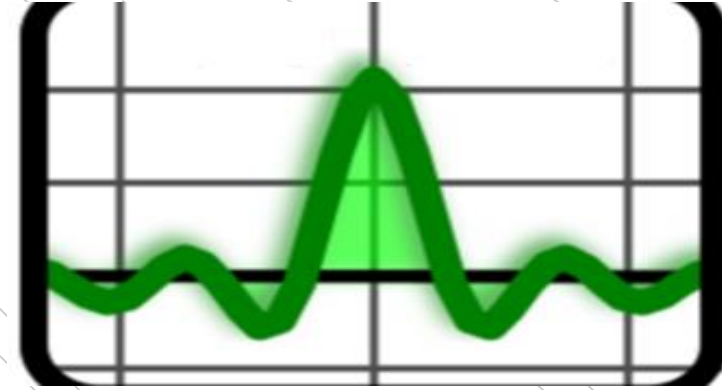


İTÜ



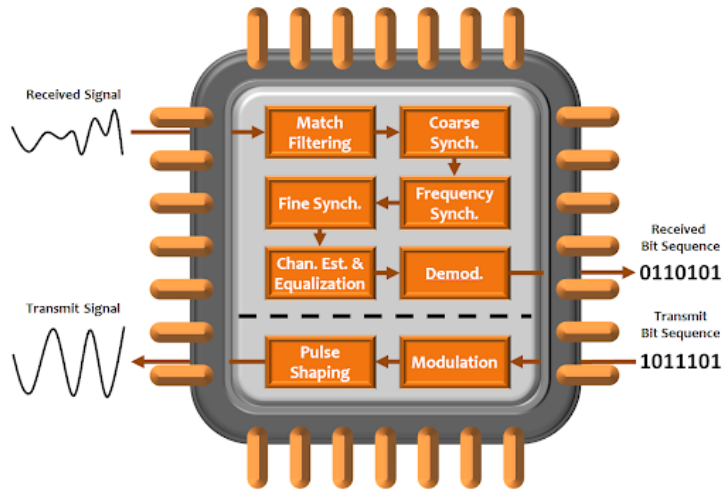
Signals & Systems for Computer Engineering

Prof.Dr. B.Berk ÜSTÜNDAĞ
Istanbul Technical University
Faculty of Computer Engineering and Informatics

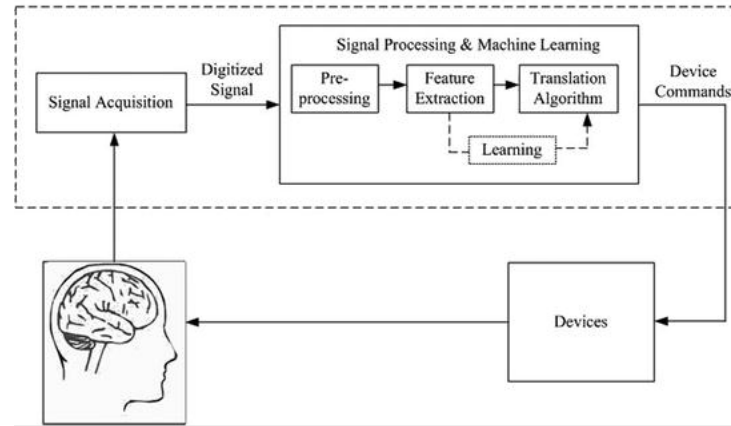
bustundag@itu.edu.tr

BLG354E / CRN: 21560
1st Week Lecture

Introduction:



signal processing for communications



Neuro science and machine learning

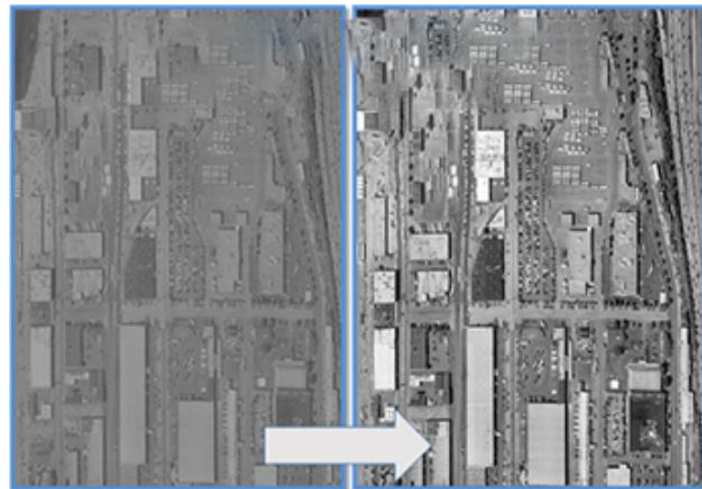


Financial analysis and predictions

Autonomous Navigation



Image processing

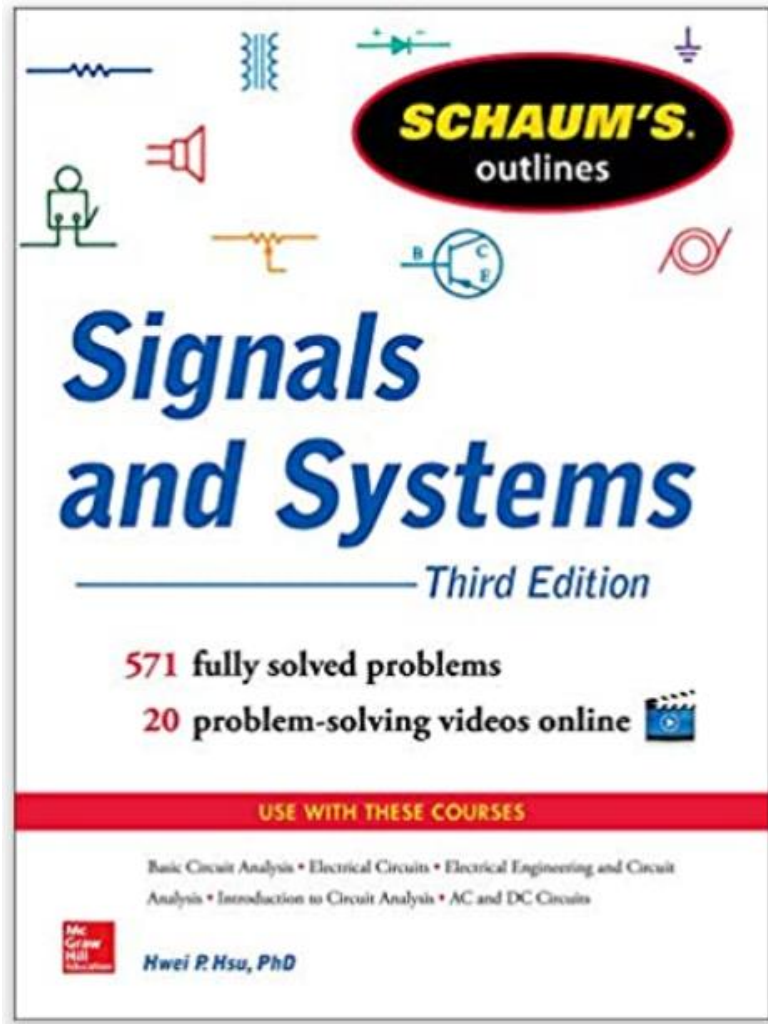


Sensor fusion and control systems



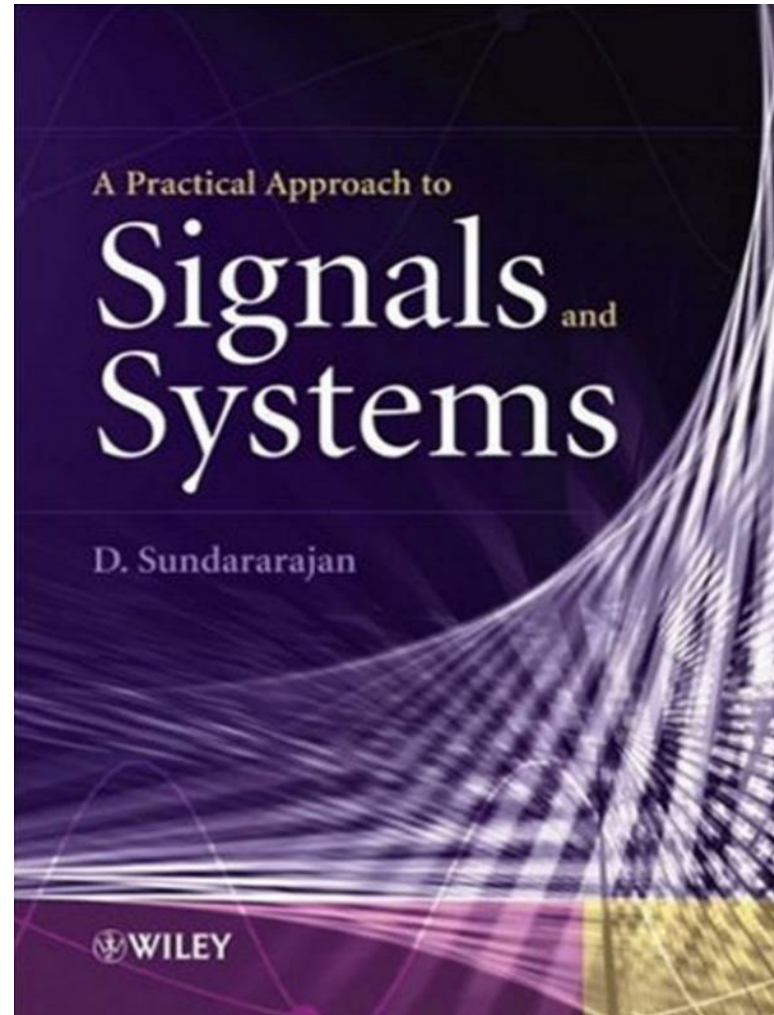
Text Books of the Course:

Main Text Book



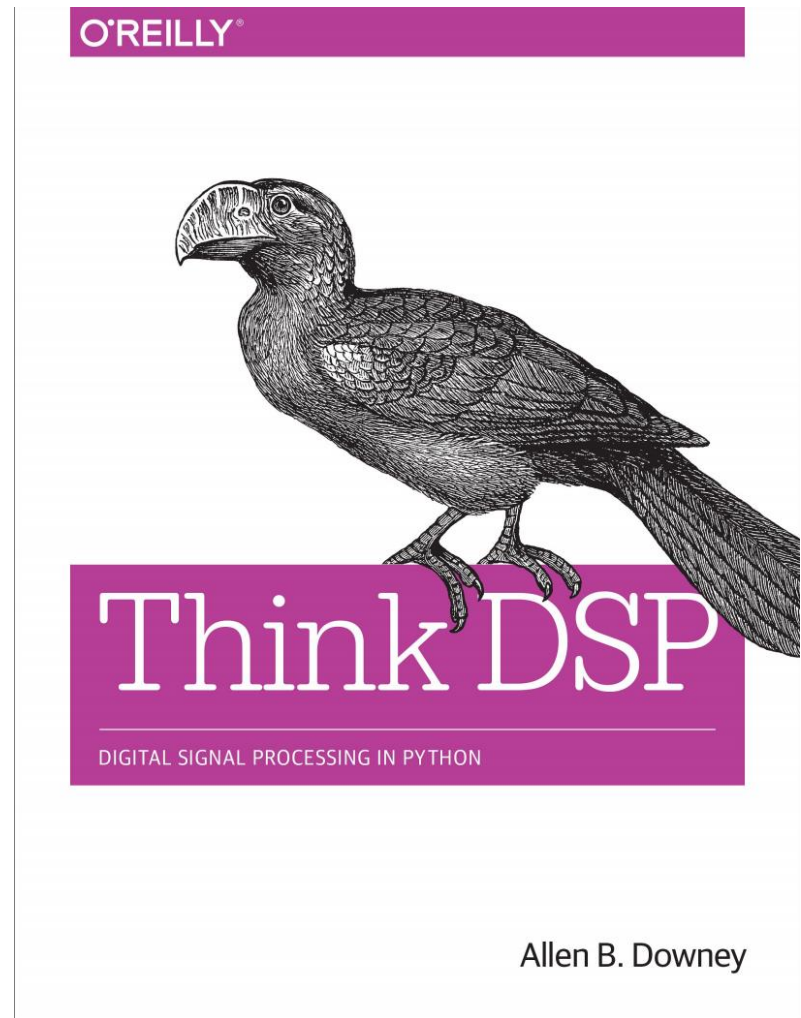
Hwei Hsu, 2013, McGraw-Hill

Additional Application Examples



D. Sundararajan, 2008, JOHN WILEY

Programming Examples with Python



Allen B. Downey, 2016, O'REILLY

<https://github.com/AllenDowney/ThinkDSP>

Course Plan (2020-2021 Spring):

- 1st week: Introduction, Course Outline, Signals and Classification of the Signals
- 2nd week: Properties of Continuous Time Signals and Discrete Time Signals, Classification of the Systems
- 3rd week: Convolution Integral, Introduction to Linear Time Invariant (LTI) Systems
- 4th week: Properties and application examples of CT LTI Systems
- 5th week: Properties and application examples of DT LTI systems, FIR Filters
- 6th week: Real Time Programming and Simulation of DT Systems
- 7th week: Signal Processing Domains, Transforms and Laplace Transform of LTI Systems
- 8th week: Frequency Response of the Systems
- 9th week: Fourier Transform and its Applications, Spectral Analysis with Fourier Transform, Modulation
- 10th week: Convolution in frequency domain, Z Transform of DT Systems and convolution
- 11th week: Fourier Analysis of Discrete-Time Signals and Systems
- 12th week: Fast Fourier Transform
- 13th week: Digital Filter Design and Software Implementation
- 14th week: Generalized System design, analysis and programming applications

Grading Policy:

Quiz – 5 Weeks *	10%
Homework Assignments (3)	20%
Midterm	30%
Final exam	40%

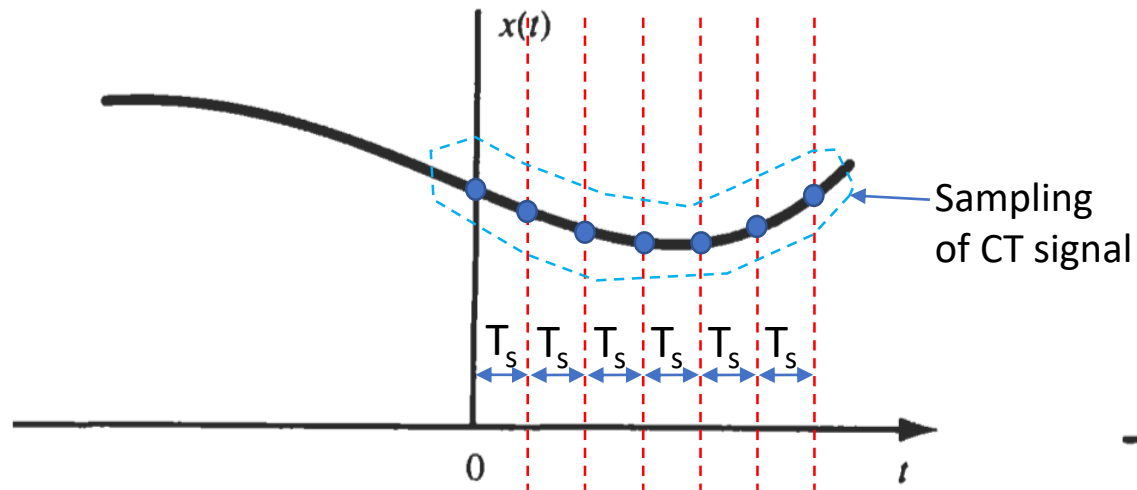
* 4 best quiz scores out of 5 quizzes will be considered in average and the grading.

VF Condition:

- **At least 70% Attendance, Minimum of 4 out of 5 quizzes should be attended.**
- **Overall visa score must be at least 30** ($0.1 \times \text{Quiz} + 0.2 \times \text{Homework} + 0.3 \times \text{Midterm} \geq 18$)

TAs: İlknur Çelik celikil17@itu.edu.tr
Asel Menekşe menekse16@itu.edu.tr

Continuous Time Signal

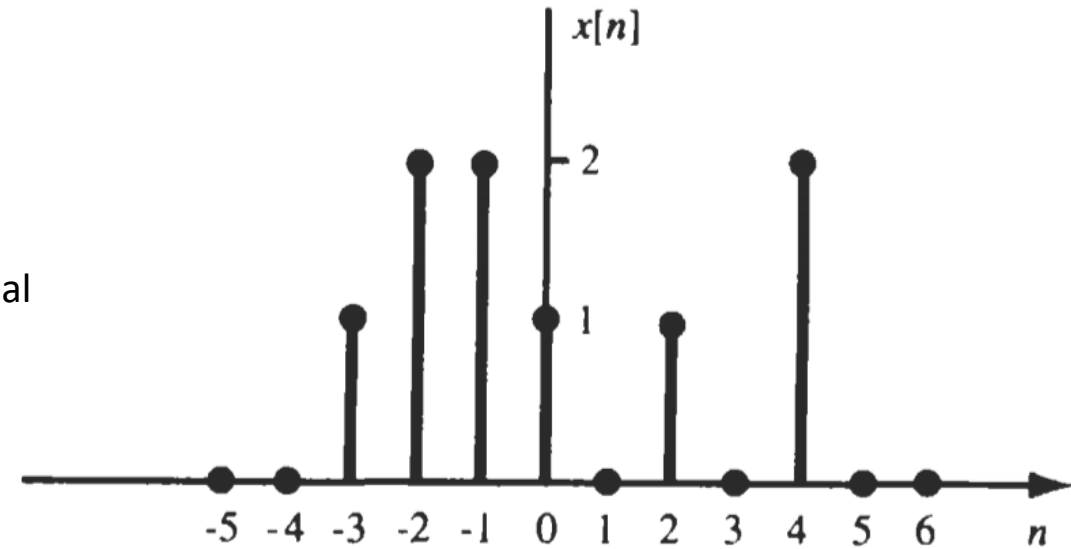


A signal $x(t)$ is a continuous-time signal if t is a continuous variable.

Time interval between sampling instants T_s is called the sampling period. When the sampling intervals are equal (uniform sampling) then:

$$x_n = x[n] = x(t = nT_s)$$

Discrete Time Signal



If t is a discrete variable, that is, $x(t)$ is defined at discrete times, then $x(t)$ is a discrete-time signal.

$$\{x_n\} = \{\dots, 0, 0, 1, 2, 2, 1, 0, 1, 0, 2, 0, 0, \dots\}$$

↑

Notations for discrete time signal sequences:

a) Analytic expression:

$$x[n] = x_n = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ 0 & n < 0 \end{cases} \quad \Rightarrow \quad \{x_n\} = \left\{1, \frac{1}{2}, \frac{1}{4}, \dots, \left(\frac{1}{2}\right)^n, \dots\right\}$$

b) Sequential expression:

$$\{x_n\} = \{\dots, 0, 0, 1, 2, 2, 1, 0, 1, 0, 2, 0, 0, \dots\} \quad \Rightarrow \quad \{x_n\} = \{1, 2, 2, 1, 0, 1, 0, 2\}$$

\uparrow $n=0, x_0=1$ \uparrow

Sum and product of two sequences:

$$\{c_n\} = \{a_n\} + \{b_n\} \rightarrow c_n = a_n + b_n$$

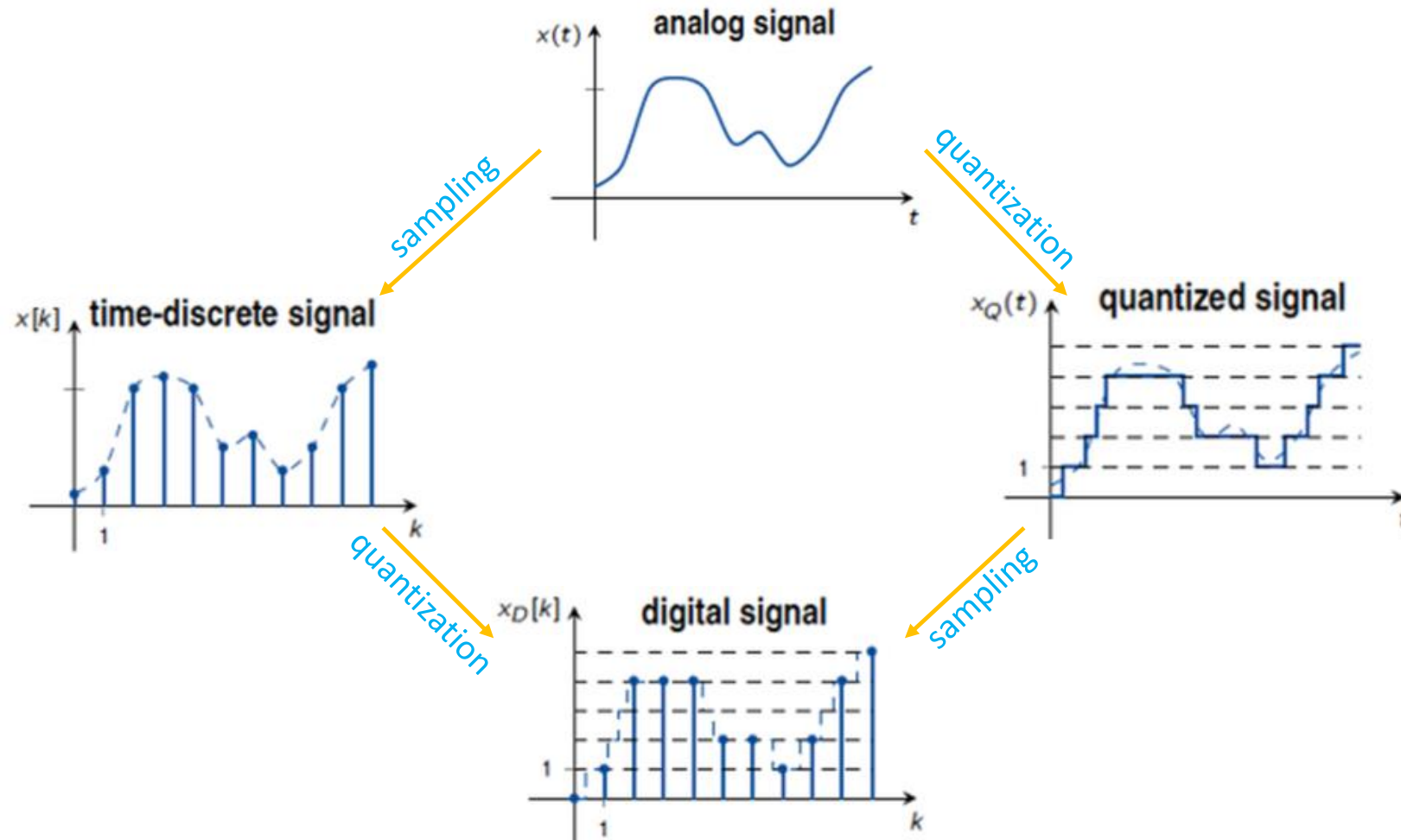
$$\{c_n\} = \{a_n\}\{b_n\} \rightarrow c_n = a_n b_n$$

$$\{c_n\} = \alpha\{a_n\} \rightarrow c_n = \alpha a_n$$

where $\alpha = \text{constant}$

Analog and Digital Signals:

If a continuous-time signal $x(t)$ can take on any value in the continuous interval (a, b) , where a may be $-\infty$ and b may be $+\infty$, then the continuous-time signal $x(t)$ is called an analog signal.



If a discrete-time signal $x[n]$ can take on only a finite number of distinct values, then we call this signal a digital signal.

Real and Complex Signals:

A signal $x(t)$ is a real signal if its value is a real number.

A signal $x(t)$ is a complex signal if its value is a complex number such as $x(t)=x_1(t)+jx_2(t)$ where $j=\sqrt{-1}$

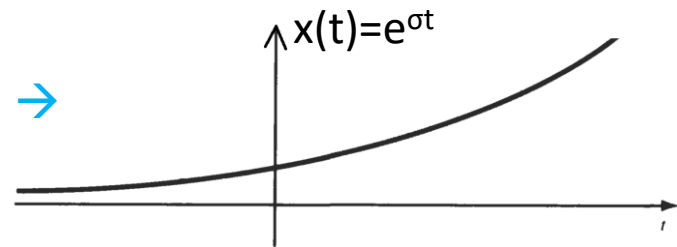
$$s \stackrel{\text{def}}{=} \sigma + j\omega$$

Real part
Imaginary part

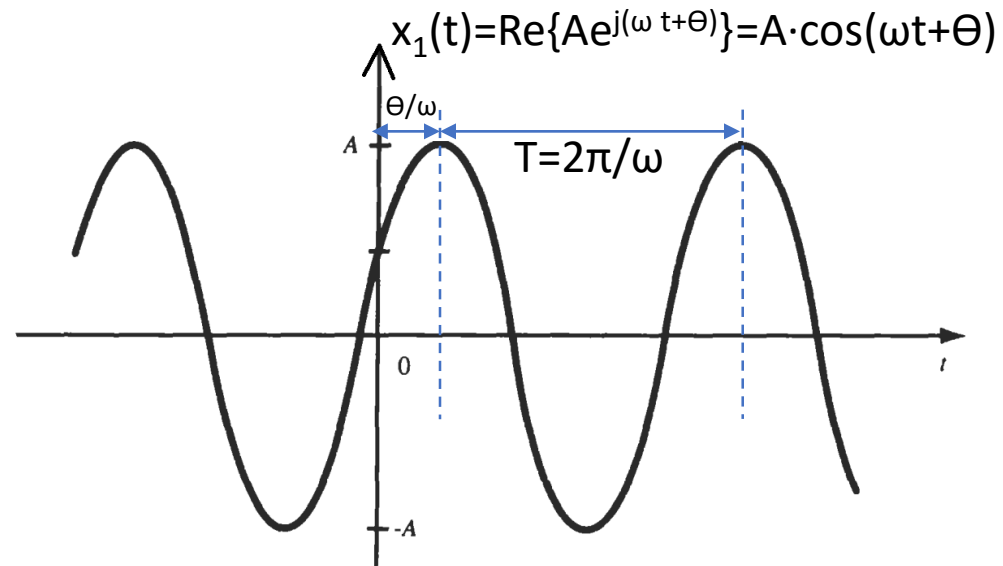
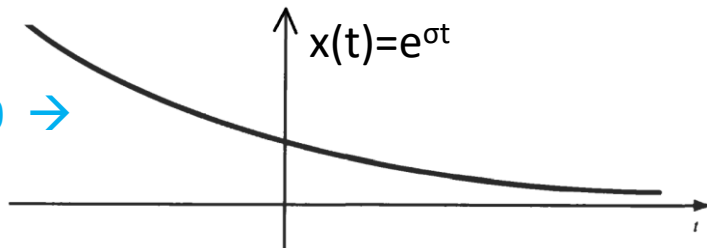
$$x(t) = e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t} (\cos \omega t + j \sin \omega t)$$

If $x(t)=A \cdot e^{j(\omega t + \Theta)}$ then $x(t) = A \cdot \text{Re}\{e^{j(\omega t + \Theta)}\} + A \cdot \text{Im}\{e^{j(\omega t + \Theta)}\}$
 $x(t) = A \cdot \cos(\omega t + \Theta) + j \cdot A \cdot \sin(\omega t + \Theta)$
 $x_1(t) + jx_2(t)$

$\omega=0, \sigma>0 \rightarrow$



$\omega=0, \sigma<0 \rightarrow$



Discrete time exponential signal: $x[n] = e^{j\Omega_0 n} = \cos \Omega_0 n + j \sin \Omega_0 n$

Waveform generation, draw and play in Python

Sinusoidal and Square waveform sound generator

```
import numpy as np
```

```
import sounddevice as sd
```

```
import matplotlib.pyplot as plt
```

```
fs = 44100
```

```
time = np.arange(0, 4, 1/fs)
```

1- Fixed frequency sinusoidal sound at 1kHz

```
sound=np.sin(2*np.pi*1000*time)
```

$\omega=2\pi f$

2- square waveform:

```
#sound=np.sign(np.sin(2*np.pi*1000*time))
```

3- variable frequency sinusoidal waveform:

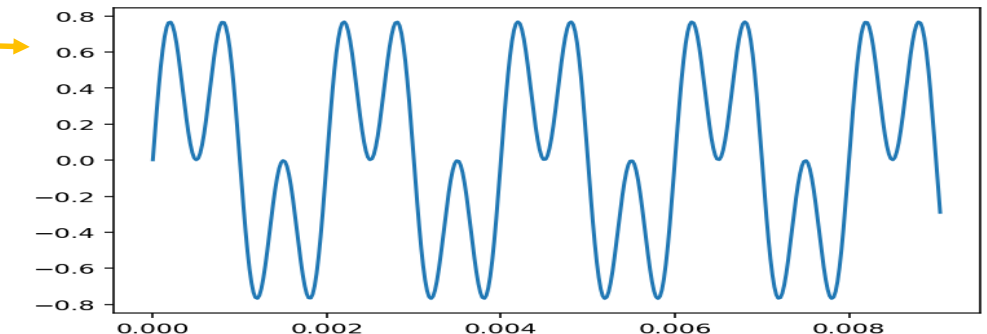
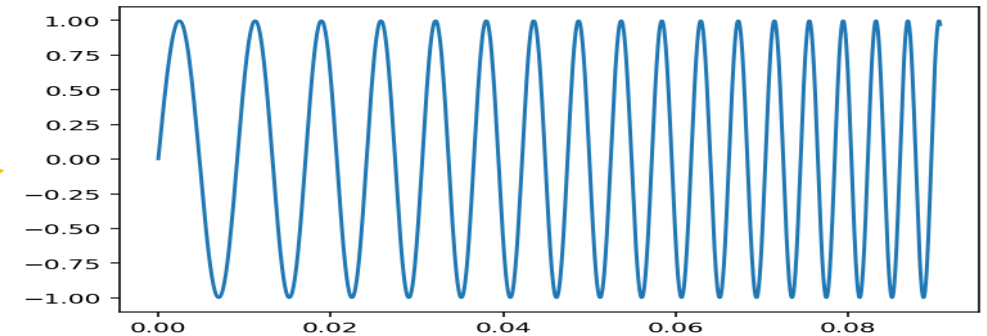
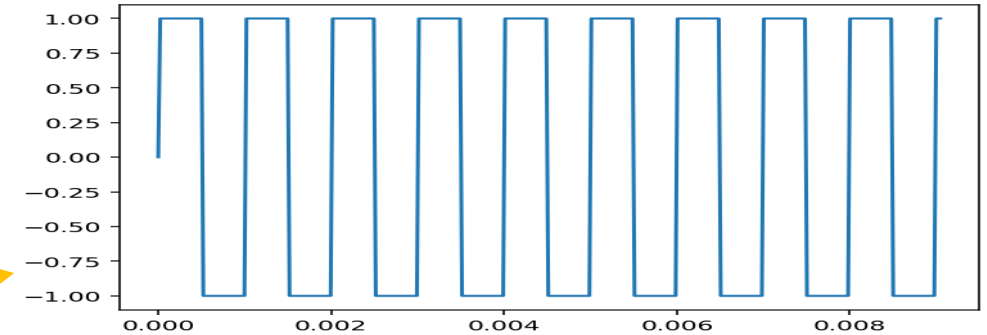
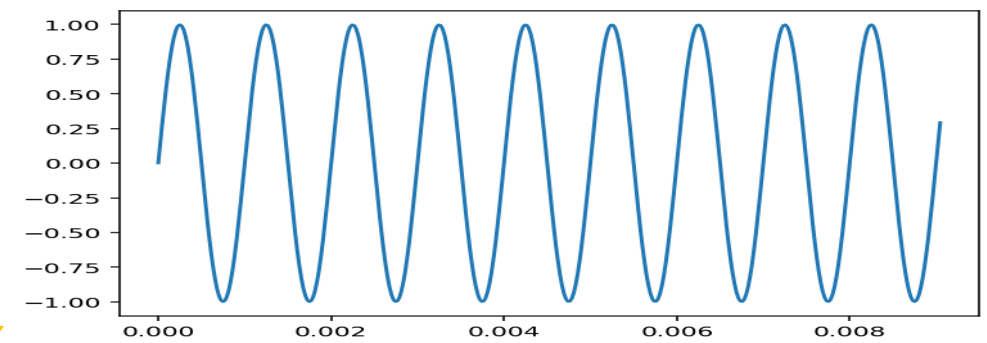
```
# sound=np.sin(2*np.pi*(100+1000*time)*time)
```

4- Multitone sinusoidal waveform:

```
#sound=0.5*(np.sin(2*np.pi*500*time)+np.sin(2*np.pi*1500*time))
```

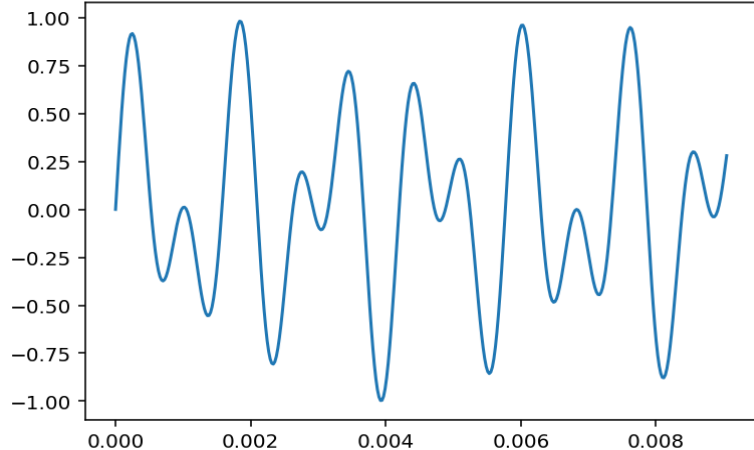
```
plt.plot(time[0:400],sound[0:400])
```

```
sd.play(sound, fs)
```



DTMF (Dual-Tone Multi-Frequency) :

	1209 Hz	1336 Hz	1477 Hz
697 Hz	1	2	3
770 Hz	4	5	6
852 Hz	7	8	9
941 Hz	*	0	#



Tone code for "1" → 697Hz + 1209Hz
(sound1)

```
# DTMF Tone Code Sequence
import numpy as np
import sounddevice as sd
import matplotlib.pyplot as plt
```

```
fs = 44100
time = np.arange(0, 1, 1/fs)
```

```
# DTMF waveform for the key sequence "1", "5", "9", "4"
sound1=0.5*(np.sin(2*np.pi*697*time)+np.sin(2*np.pi*1209*time))
sound2=0.5*(np.sin(2*np.pi*770*time)+np.sin(2*np.pi*1336*time))
sound3=0.5*(np.sin(2*np.pi*852*time)+np.sin(2*np.pi*1477*time))
sound4=0.5*(np.sin(2*np.pi*770*time)+np.sin(2*np.pi*1209*time))
```

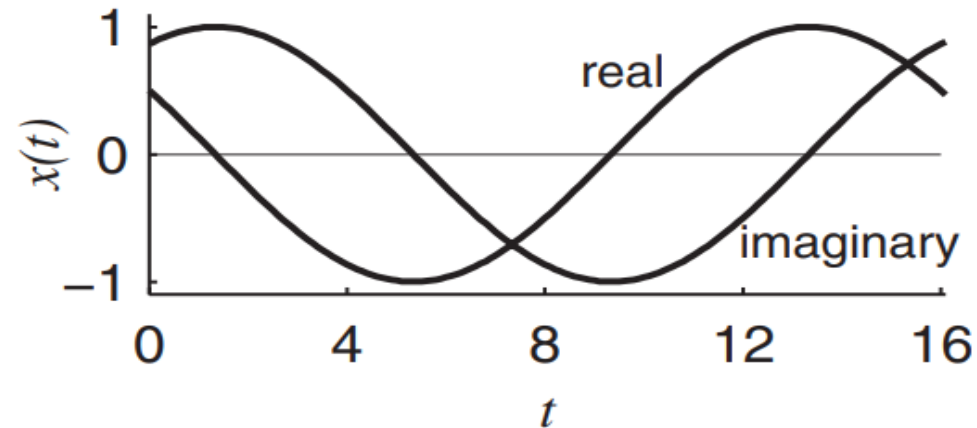
```
sound=np.concatenate((sound1,sound2,sound3,sound4))
plt.plot(time[0:400],sound1[0:400])
sd.play(sound, fs)
```

Example:

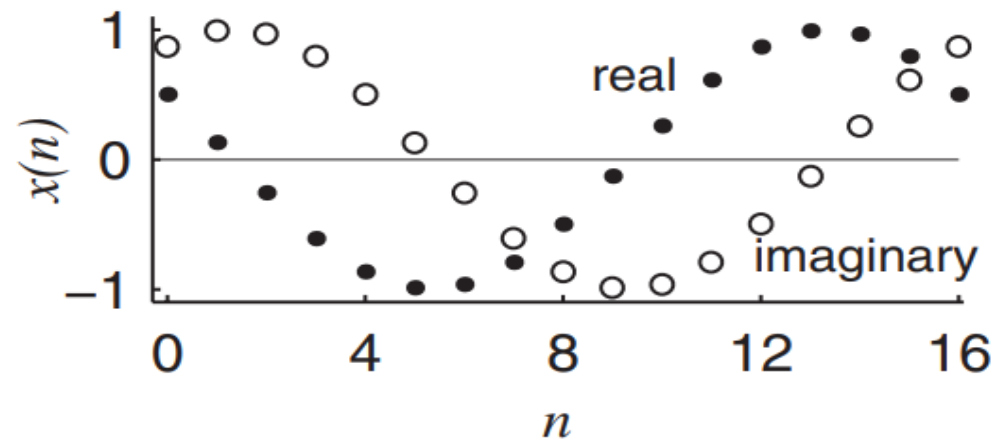
Illustrate the signal $x(t) = e^{j(\frac{2\pi}{16}t + \frac{\pi}{3})}$ as CT signal and for the case if it is discretized at $f_s=1\text{Hz}$

$$x(t) = e^{j(\frac{2\pi}{16}t + \frac{\pi}{3})} = \cos\left(\frac{2\pi}{16}t + \frac{\pi}{3}\right) + j \sin\left(\frac{2\pi}{16}t + \frac{\pi}{3}\right)$$

$$x(t) = e^{j(\frac{2\pi}{16}t + \frac{\pi}{3})}$$



$$x[n] = e^{j(\frac{2\pi}{16}n + \frac{\pi}{3})}$$



Plot of Complex CT Signal in Python

Continuous time Complex signal @50Hz

import numpy as np

import matplotlib.pyplot as plt

Duration=0.05 # 50ms time window

Resolution=1000

f=50 # Frequency=50Hz

t = np.linspace(0, Duration, Resolution)

plt.subplot(2,1,1);

plt.plot(t, np.exp(2j*np.pi*f*t).real);

plt.xlabel('t(s)');

plt.ylabel('Re x(t)');

plt.subplot(2,1,2);

plt.plot(t, np.exp(2j*np.pi*f*t).imag);

plt.xlabel('t(s)');

plt.ylabel('Im x(t)');

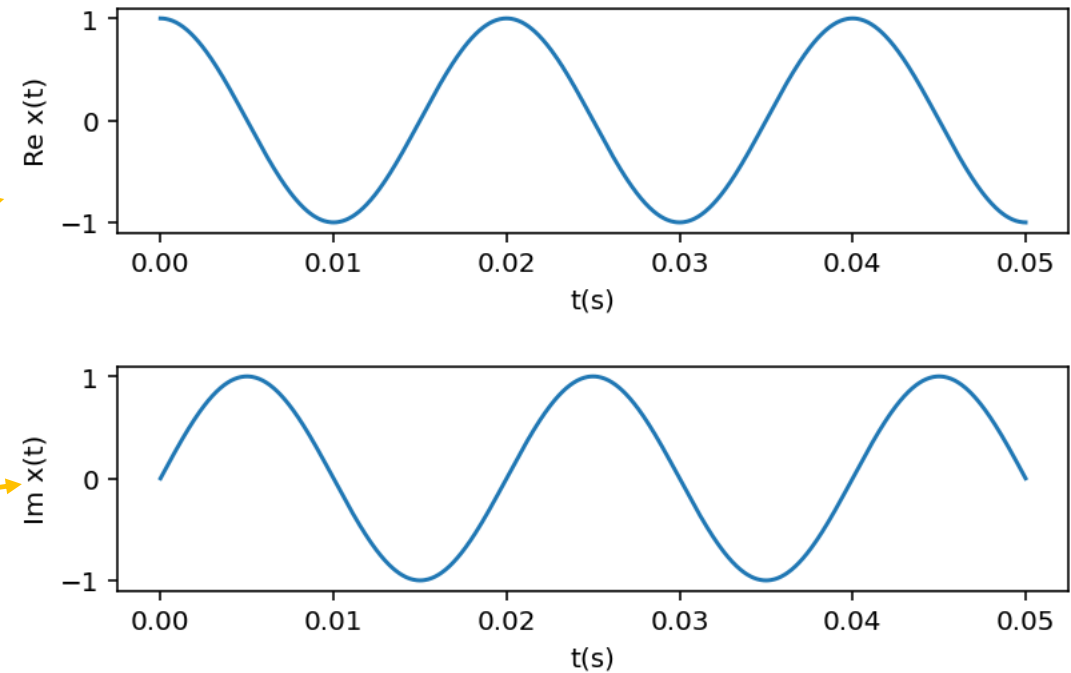
plt.tight_layout(pad=2.0)

plt.show()

$$x(t)=e^{j\omega t}=e^{j2\pi ft}$$

Real(x(t))

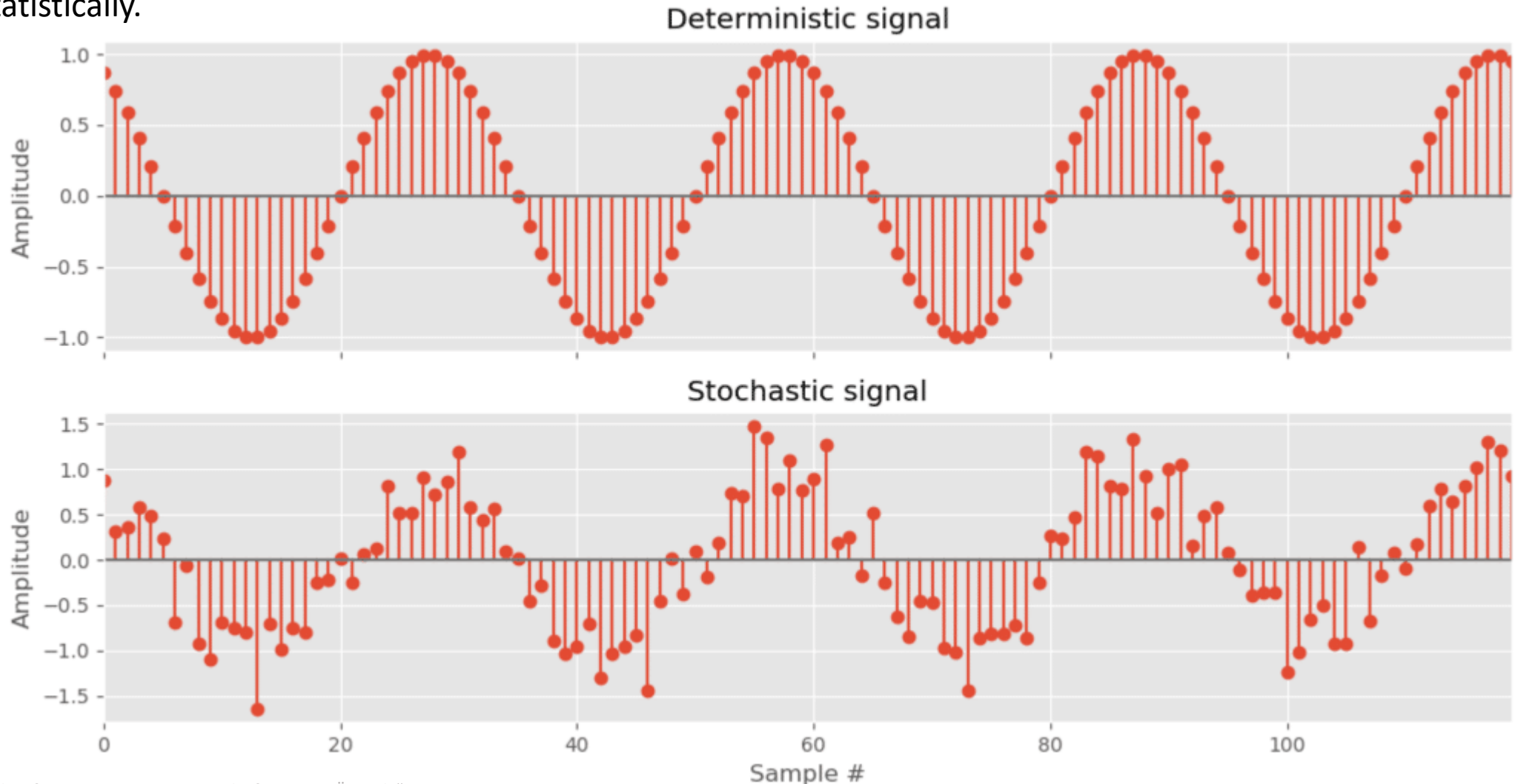
Imaginary(x(t))



Deterministic and Stochastic Signals:

Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a known function of time

Random/Stochastic signals are those signals that take random values at any given time and must be characterized statistically.



Plot of Discrete Time Signal in Python

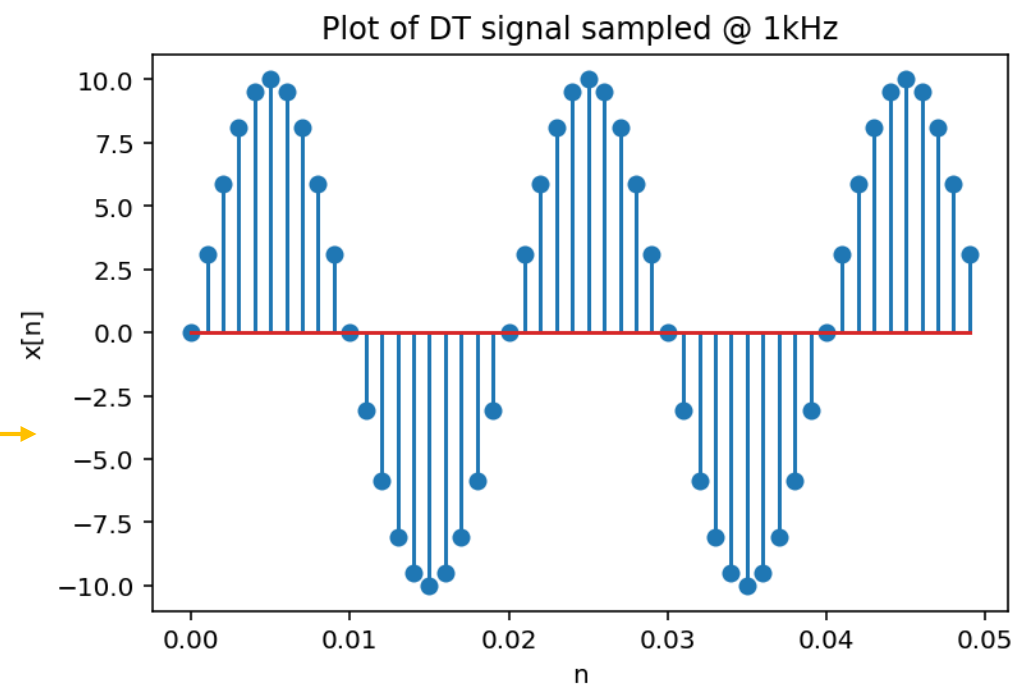
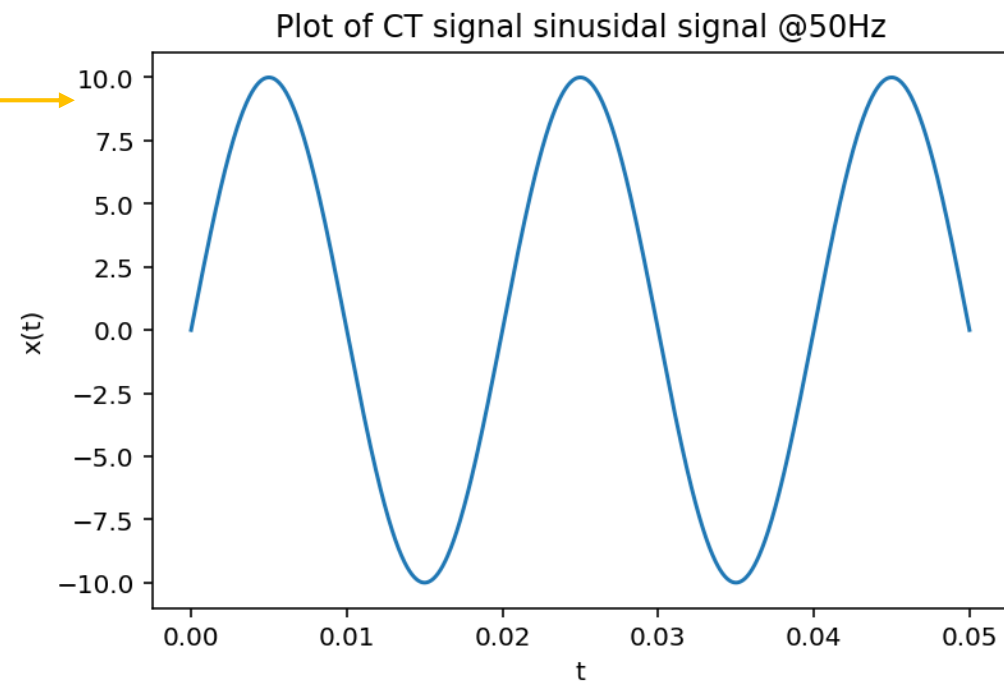
#Continuous time sinusoidal signal @50Hz
and its discrete form when it is sampled @1000kHz

```
import numpy as np
import matplotlib.pyplot as plt
Duration=0.05 # 50ms time window
Resolution=1000
t = np.linspace(0, Duration, Resolution)
plt.plot(t, 10 * np.sin(2*np.pi*50*t));
plt.xlabel('t');
plt.ylabel('x(t)');
plt.title('Plot of CT signal sinusidal signal @50Hz');
plt.show()
```

Sample at 1ms and draw the DT signal:

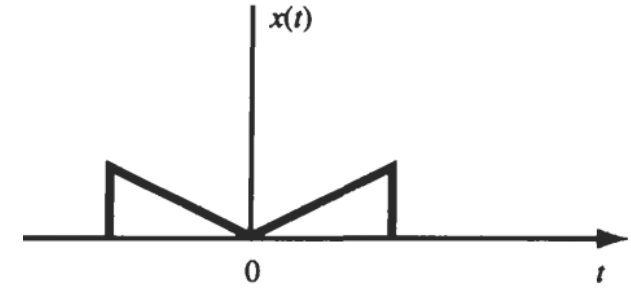
```
Ts = 0.001 # Sampling period
n = np.arange(0,Duration,Ts);
```

```
x = 10*np.sin(2 * np.pi * 50 * n)
plt.xlabel('n');
plt.ylabel('x[n]');
plt.title('Plot of DT signal sampled @ 1kHz');
plt.stem(n, x);
```

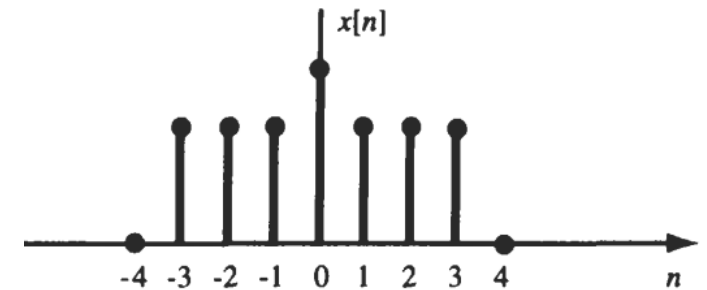


Even and Odd Signals:

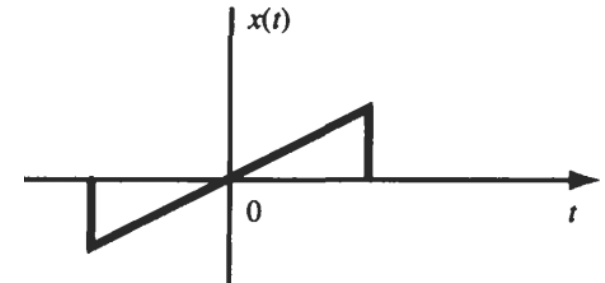
A CT signal $x(t)$ is referred to as an even signal if $x(-t)=x(t)$



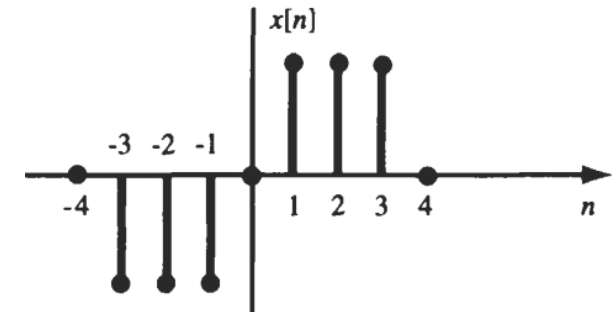
A DT signal $x[n]$ is referred to as an even signal if $x[-n]=x[n]$



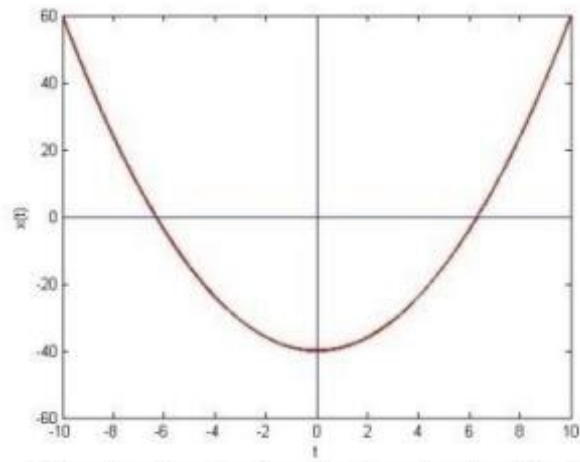
A CT signal $x(t)$ is referred to as an odd signal if $x(-t)=-x(t)$



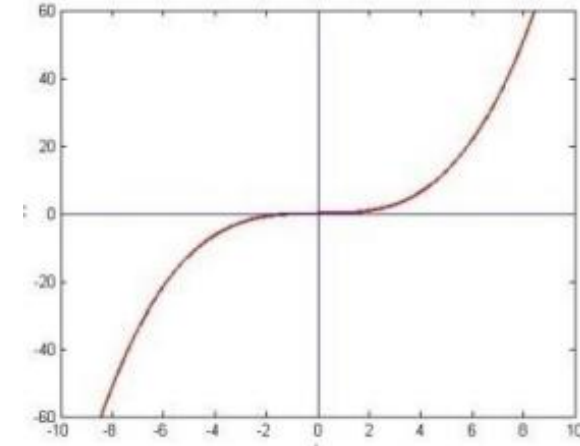
A DT signal $x[n]$ is referred to as an odd signal if $x[-n]=-x[n]$



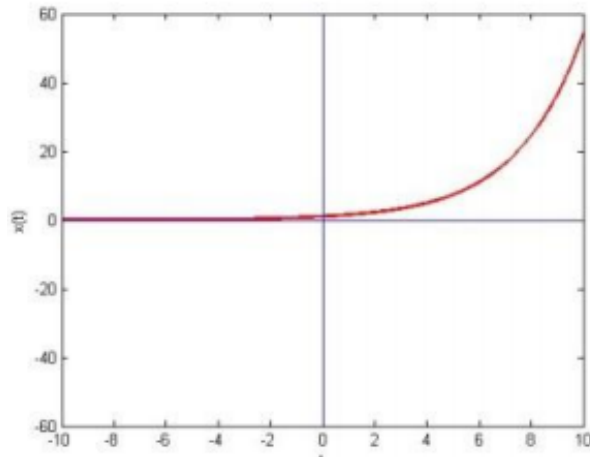
Example:



$x(t) = t^2 - 40$ is even.



$x(t) = 0.1t^3$ is odd.



$x(t) = e^{0.4t}$ is neither even nor odd.

Decomposition Theorem:

Any signal $x(t)$ or $x[n]$ can be expressed as a sum of two signals, one of which is even and one of which is odd.

$$\begin{aligned} x(t) &= x_e(t) + x_o(t) \\ x[n] &= x_e[n] + x_o[n] \end{aligned} \quad \text{where} \quad \left\{ \begin{array}{ll} x_e(t) = \frac{1}{2}\{x(t) + x(-t)\} & \text{even part of } x(t) \\ x_e[n] = \frac{1}{2}\{x[n] + x[-n]\} & \text{even part of } x[n] \\ x_o(t) = \frac{1}{2}\{x(t) - x(-t)\} & \text{odd part of } x(t) \\ x_o[n] = \frac{1}{2}\{x[n] - x[-n]\} & \text{odd part of } x[n] \end{array} \right.$$

Example:

$$x(n) = \cos\left(\frac{2\pi}{8}n + \frac{\pi}{3}\right) \stackrel{?}{\Rightarrow} \frac{\cos\left(\frac{2\pi}{8}n\right)}{2} - \frac{\sqrt{3}}{2} \sin\left(\frac{2\pi}{8}n\right)$$

The even-symmetric component of $x(n)$:

$$x_e(n) = \frac{x(n) + x(-n)}{2} = \frac{\cos\left(\frac{2\pi}{8}n + \frac{\pi}{3}\right) + \cos\left(\frac{2\pi}{8}(-n) + \frac{\pi}{3}\right)}{2} = \frac{2 \cos\left(\frac{2\pi}{8}n\right) \cos\left(\frac{\pi}{3}\right)}{2} = \frac{\cos\left(\frac{2\pi}{8}n\right)}{2}$$

The odd-symmetric component of $x(n)$:

$$x_o(n) = \frac{x(n) - x(-n)}{2} = \frac{\cos\left(\frac{2\pi}{8}n + \frac{\pi}{3}\right) - \cos\left(\frac{2\pi}{8}(-n) + \frac{\pi}{3}\right)}{2} = \frac{-2 \sin\left(\frac{2\pi}{8}n\right) \sin\left(\frac{\pi}{3}\right)}{2} = -\frac{\sqrt{3}}{2} \sin\left(\frac{2\pi}{8}n\right)$$

Integral of decomposed signals:

Integral of an odd-symmetric signal $x_o(t)$ over symmetric limits is zero $\Rightarrow \int_{-t_0}^{t_0} x_o(t) dt = 0$

$$\int_{-t_0}^{t_0} x(t) dt = \int_{-t_0}^{t_0} x_e(t) dt = 2 \int_0^{t_0} x_e(t) dt \quad \text{This property will be used for a simplification of Fourier Series}$$

Example: Express the signal $x(t) = e^{j(\frac{2\pi}{16}t + \frac{\pi}{3})}$ in terms of its symmetric components

$$x_e(t) = \frac{x(t) + x(-t)}{2} = e^{j(\frac{\pi}{3})} \frac{e^{j(\frac{2\pi}{16}t)} + e^{j(\frac{2\pi}{16}(-t))}}{2} = e^{j(\frac{\pi}{3})} \cos\left(\frac{2\pi}{16}t\right)$$

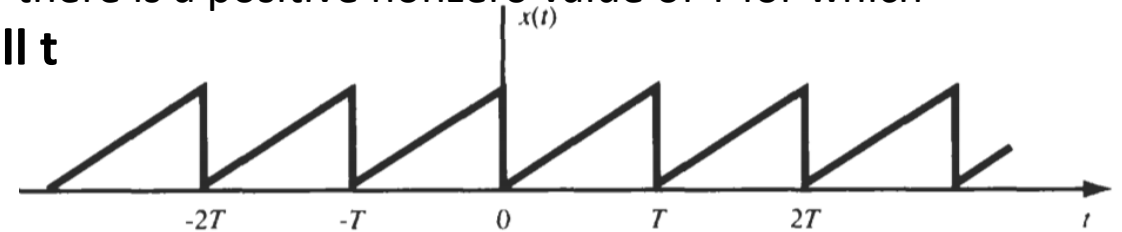
$$x_o(t) = \frac{x(t) - x(-t)}{2} = e^{j(\frac{\pi}{3})} \frac{e^{j(\frac{2\pi}{16}t)} - e^{j(\frac{2\pi}{16}(-t))}}{2} = je^{j(\frac{\pi}{3})} \sin\left(\frac{2\pi}{16}t\right)$$

$$x(t) = x_e(t) + x_o(t) = e^{j(\frac{\pi}{3})} \cos\left(\frac{2\pi}{16}t\right) + je^{j(\frac{\pi}{3})} \sin\left(\frac{2\pi}{16}t\right)$$

Periodic and Nonperiodic Signals:

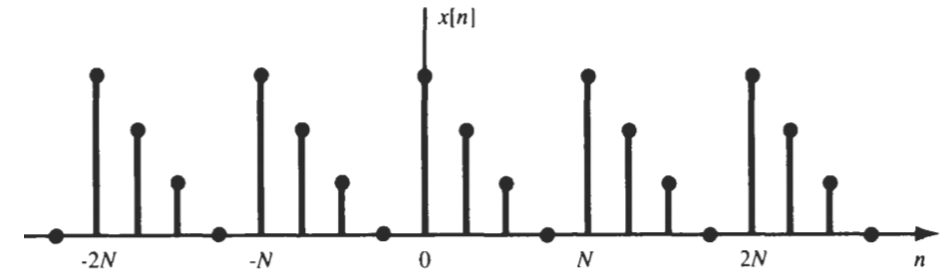
A continuous-time signal $x(t)$ is said to be periodic with period T if there is a positive nonzero value of T for which

$$x(t+T)=x(t) \quad \text{for all } t$$



A discrete-time signal $x[n]$ is said to be periodic with period N if there is a positive nonzero value of N for which

$$x[n+N]=x[n] \quad \text{for all } n$$



Smallest value of T or N that satisfies the above condition is called fundamental period

Signals do not satisfy periodicity conditions are called non-periodic signals

Example:

Determine the fundamental period of $x(t) = e^{j\frac{3\pi t}{5}}$

$$x(t+T)=x(t)$$

\rightarrow

$$\begin{cases} e^{j3\pi t/5} = e^{j3\pi(t+T)/5} \\ 1 = e^{j3\pi T/5} \\ e^{j2k\pi} = e^{j3\pi T/5} \\ T = \frac{10}{3} \quad (k = 1) \end{cases}$$

Determine the fundamental period of $x[n] = e^{j\frac{3\pi n}{5}}$

$$x[n+N]=x[n]$$

\rightarrow

$$\begin{cases} e^{j3\pi n/5} = e^{j3\pi(n+N)/5} \\ 1 = e^{j3\pi N/5} \\ e^{j2k\pi} = e^{j3\pi N/5} \\ T = 10 \quad (k = 3) \end{cases}$$

Proof: Prove that $x(t) = e^{j\omega_0 t}$ is periodic and find its fundamental period

If $x(t)$ is periodic then $e^{j\omega_0(t+T)} = e^{j\omega_0 t}$

$$e^{j\omega_0(t+T)} = e^{j\omega_0 t} e^{j\omega_0 T} \rightarrow e^{j\omega_0 T} = 1 \quad \left\{ \begin{array}{l} \omega_0 = 0 \rightarrow x(t) = 1 \text{ and it is periodic for all } T \\ \omega_0 \neq 0 \rightarrow \omega_0 T = m2\pi \end{array} \right.$$

$$T = m \frac{2\pi}{\omega_0} \quad \text{and the fundamental period } T_0 \text{ is the smallest positive } T \rightarrow T_0 = \frac{2\pi}{\omega_0}$$

If we consider the periodicity criteria for the DT case:

$$x[n] = e^{j\Omega_0 n} \quad \text{is periodic if} \quad e^{j\Omega_0(n+N)} = e^{j\Omega_0 n} e^{j\Omega_0 N} = e^{j\Omega_0 n} \rightarrow e^{j\Omega_0 N} = 1$$

$$\Omega_0 N = m2\pi \quad \text{where } m \text{ is a positive integer} \rightarrow \frac{\Omega_0}{2\pi} = \frac{m}{N}$$

$$x[n] \text{ is periodic only if } \frac{\Omega_0}{2\pi} \text{ is a rational number}$$

Example:

$x(t)$ is a CT signal given as $x(t)=\cos(15t)$. Find the fundamental period of the DT signal $x[n]$ if $x[n]$ is discretized by sampling $x(t)$ at the sampling frequency $f_s = \frac{10}{\pi}$ Hz

$$x[n] = x(nT_s) \quad T_s = 1/f_s = 0.1\pi \text{ seconds} \quad x(t) = \cos(\omega t) = \cos(15t) \rightarrow \omega = 15 \text{ rad/s}$$

The fundamental period of $x(t)$: $T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{15}$

$x[n]$ is periodic if $\frac{T_s}{T_0} = \frac{T_s}{2\pi/15} = \frac{m}{N_0}$

Since m and N_0 are positive integers $\rightarrow T_s = \frac{m}{N_0} T_0 = \frac{m}{N_0} \frac{2\pi}{15} \rightarrow \frac{T_s}{T_0} = \frac{\pi/10}{2\pi/15} = \frac{15}{20} = \frac{3}{4}$

If $x[n]$ is periodic then $N_0 = m \frac{T_0}{T_s} = m \frac{4}{3}$

$m=3$ provides N_0 to be the smallest positive integer. Fundamental period of $x[n]$ is $N_0=4$

Example: (Case study for the signal having multiple periodic components)

Find the fundamental frequency of the signal $x[n] = e^{j\frac{2\pi}{3}n} + e^{j\frac{3\pi}{4}n}$

The diagram illustrates the relationship between the signal components and the fundamental frequency equation. Red arrows point from the coefficients of the exponentials in the signal equation to the variables in the equation below. Specifically, an arrow points from $\frac{2\pi}{3}$ to $\frac{\omega_0}{2\pi}$, and another from $\frac{3\pi}{4}$ to $\frac{k}{N}$.

$$\frac{\omega_0}{2\pi} = \frac{k}{N}$$
$$\frac{\frac{2\pi}{3}}{2\pi} = \frac{1}{3} \qquad \frac{\frac{3\pi}{4}}{2\pi} = \frac{3}{8}$$

Fundamental period of the first exponential is 3

Fundamental period of the second exponential is 8

Since the least common multiple of the periods of the two signals is 24,
 $x[n]$ is periodic with $N_0=24$

Example:

Find the fundamental frequency of $x(t) = \cos t + \sin \sqrt{2} t$

$$x(t) = x_1(t) + x_2(t)$$

$$\cos t = \cos \omega_1 t \quad \sin \sqrt{2} t = \sin \omega_2 t$$

Periods of x_1 and x_2 :

$$T_1 = 2\pi / \omega_1 = 2\pi$$

$$T_2 = 2\pi / \omega_2 = \sqrt{2} \pi$$

$$T_1 / T_2 = \sqrt{2} \text{ is irrational}$$

Therefore $x(t)$ is not periodic and there is no fundamental frequency

Example:

Find the fundamental period of the DT signal $x[n] = \sin\left(\frac{5\pi}{6}n\right) + \cos\left(\frac{3\pi}{4}n\right) + \sin\left(\frac{\pi}{3}n\right)$

The least common multiple of the denominators is 12 $\rightarrow x[n] = \sin\left(\frac{10\pi}{12}n\right) + \cos\left(\frac{9\pi}{12}n\right) + \sin\left(\frac{4\pi}{12}n\right)$

Fundamental frequency is $\omega_0 = \pi/12 \rightarrow$ The fundamental period is $T = 2\pi/\omega_0 = 24$ and the three terms are the 4th, 9th and 10th harmonic of ω_0

Example:

Find the fundamental frequency of the CT signal $x(t) = \sin\left(\frac{5\pi}{6}t\right) + \cos\left(\frac{3\pi}{4}t\right) + \sin\left(\frac{\pi}{3}t\right)$

$$x(t) = \underbrace{\sin\left(\frac{5\pi}{6}t\right)}_{x_1(t)} + \underbrace{\cos\left(\frac{3\pi}{4}t\right)}_{x_2(t)} + \underbrace{\sin\left(\frac{\pi}{3}t\right)}_{x_3(t)}$$

The frequencies and periods of $x_1(t)$, $x_2(t)$ and the $x_3(t)$ are:

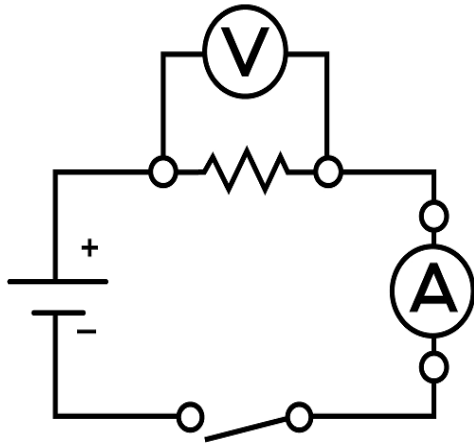
$$\left. \begin{array}{l} \omega_1 = \frac{5\pi}{6}, f_1 = \frac{5}{12}, T_1 = \frac{12}{5} \\ \omega_2 = \frac{3\pi}{4}, f_2 = \frac{3}{8}, T_2 = \frac{8}{3} \\ \omega_3 = \frac{\pi}{3}, f_3 = \frac{1}{6}, T_3 = 6 \end{array} \right\} f_0 = GCD\left(\frac{5}{12}, \frac{3}{8}, \frac{1}{6}\right) = GCD\left(\frac{10}{24}, \frac{9}{24}, \frac{4}{24}\right) = \frac{1}{24}$$

The fundamental angular frequency is $\omega_0 = \pi/12$
and the fundamental period is $T_0 = 2\pi/\omega = 24$

$$f_0 = 1/24$$

$$\rightarrow x(t) = \sin\left(\frac{10\pi}{12}t\right) + \cos\left(\frac{9\pi}{12}t\right) + \sin\left(\frac{4\pi}{12}t\right)$$

Energy and Power Signals:



If $v(t)$ is the voltage drop across the resistor R while the current is $i(t)$ then the instantaneous power $p(t)$ per ohm is defined as

$$p(t) = \frac{v(t)i(t)}{R} = i^2(t)$$

Then the total energy E and average power P on a per-ohm basis are

$$\left\{ \begin{array}{l} E = \int_{-\infty}^{\infty} i^2(t) dt \quad \text{joules} \\ P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} i^2(t) dt \quad \text{watts} \end{array} \right.$$

If we make an analogy to 1 ohm basis Energy and Power definition, then normalized energy content E of an arbitrary continuous-time signal $x(t)$ is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

and the normalized average power P of $x(t)$ is

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

Similarly the normalized energy content E of a discrete-time signal $x[n]$ can be expressed as
$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Then the normalized average power P of $x[n]$ is given as
$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x[n]|^2$$

Energy signal: $x(t)$ is said to be an energy signal or $x[n]$ is said to be an energy sequence if and only if $0 < E < \infty$, and so the power $P=0$.

Power signal: $x(t)$ is said to be a power signal or $x[n]$ is said to be a power sequence if and only if $0 < P < \infty$, and so the energy $E = \infty$

If a signal do not satisfy any of these properties then it is neither an energy signal nor a power signal.

Example: $x[n] = (-0.5)^n u[n]$ Determine if $x[n]$ is an energy or power sequence

$$E = \sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=0}^{\infty} 0.25^n = \frac{1}{1-0.25} = \frac{4}{3} < \infty$$

$$P = \lim_{n \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N x^2[n] = \lim_{n \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^N 0.25^n = \frac{1}{2\infty+1} \sum_{n=0}^N 0.25^n = 0$$

Hence it is an **energy signal** since we got finite energy and zero power