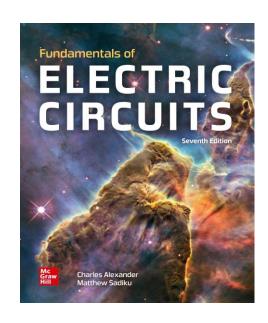
EHB 211E Basics of Electrical Circuits

Asst. Prof. Onur Kurt

Methods of Analysis





Nodal Analysis

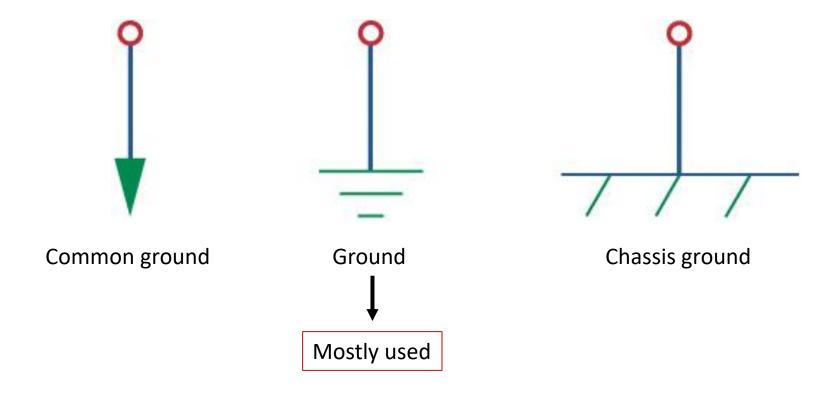


- Provide a general procedure for analyzing circuit using node voltages
- In nodal analysis, node voltages needed to be found
- Steps to determine node voltages:
 - \Box Select a node as the reference node. Assign voltages v_1, v_2, \dots, v_{n-1} to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
 - □ Apply KCL to each of the n-1 nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
 - Solve the resulting simultaneous equations to obtain the unknown node voltages.

Nodal Analysis

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- Choosing a reference (datum) node.
- Reference node: ground node (zero potential)





• For the circuit shown below, express the branch currents in terms of node voltages.

$$\text{KCL: } \sum i_{in} = \sum i_{out}$$

At node 1:
$$I_1 = I_2 + i_1 + i_2$$

At node 2:
$$I_2 + i_2 = i_3$$

Ohm's law:
$$v = iR \Rightarrow i = \frac{v}{R}$$

$$i_1 = \frac{v_1 - 0}{R_1}$$
 or $i_1 = G_1 v_1$

$$i_2 = \frac{v_1 - v_2}{R_2}$$
 or $i_2 = G_2(v_1 - v_2)$

$$i_3 = \frac{v_2 - 0}{R_2}$$
 or $i_3 = G_3 v_2$

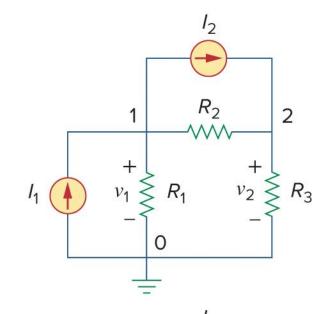
$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$
 or $I_1 = I_2 + G_1 v_1 + G_2 (v_1 - v_2)$

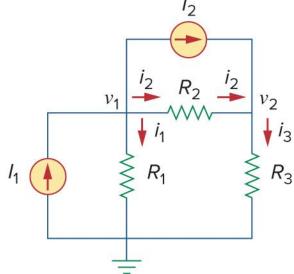
$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_2}$$
 or $I_2 + G_2(v_1 - v_2) = G_3 v_2$

Current flows from higher potential (+) to lower potential (-) in a resistor

$$i = \frac{v_{higher} - v_{lower}}{R}$$

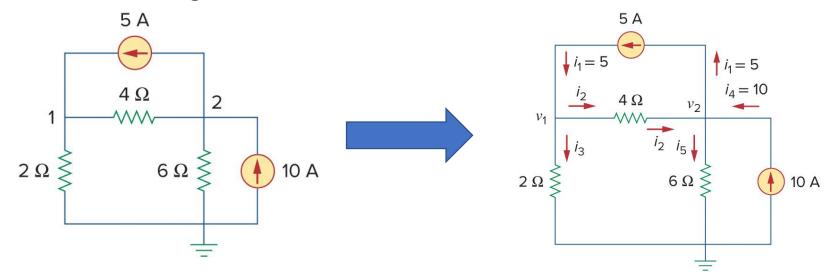
$$G = \frac{1}{R}$$







Calculate the node voltages in the circuit shown below



Solution:

At node 1:
$$i_1 = i_2 + i_3 \Rightarrow 5 = i_2 + i_3$$

$$i_2 = \frac{v_1 - v_2}{4}$$
, $i_3 = \frac{v_1 - 0}{2}$

$$5 = \frac{v_1 - v_2}{4} + \frac{v_1}{2} \longrightarrow Eq \ 1$$

$$\frac{v_1 - v_2}{4} + \frac{v_1}{2} = 5$$

$$\frac{v_1 - v_2}{4} - \frac{v_2}{6} = -5$$

At node 2:
$$i_2 + i_4 = i_1 + i_5 \Rightarrow i_2 + 10 = 5 + i_5$$

$$i_5 = \frac{v_2 - 0}{6}$$

$$\frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2}{6} \Rightarrow \frac{v_1 - v_2}{4} - \frac{v_2}{6} = -5 \longrightarrow Eq \ 2$$

$$\frac{v_1 - v_2}{4} + \frac{v_1}{2} = 5$$

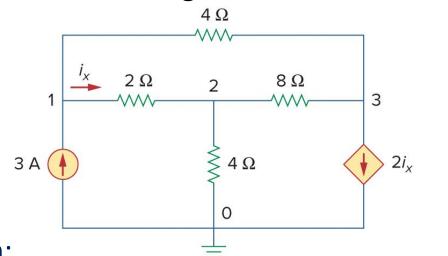
$$v_1 = \frac{40}{3} = 13.333 V$$

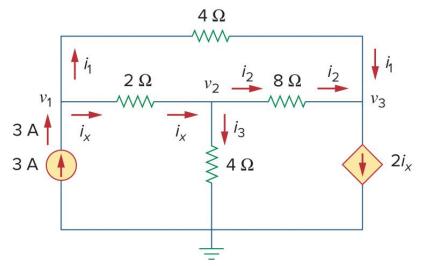
$$\frac{v_1 - v_2}{4} - \frac{v_2}{6} = -5$$

$$v_2 = 20 V$$



Determine the voltages at the nodes in the circuit shown below





Solution:

At node 1:
$$3 = i_1 + i_x$$
 $i_1 = \frac{v_1 - v_3}{4}$ $i_3 = \frac{v_2}{4}$

At node 2:
$$i_x = i_2 + i_3$$

At node 2:
$$i_x - i_2 + i_3$$

At node 3: $i_1 + i_2 = 2i_x$ $i_2 = \frac{v_2 - v_3}{8}$ $i_x = \frac{v_1 - v_2}{2}$

$$i_1 = \frac{v_1 - v_3}{4}$$

$$i_2 = \frac{v_2 - v_3}{Q}$$

$$i_3 = \frac{v_2}{4}$$

$$i_x = \frac{v_1 - v_2}{2}$$

$$3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2} \Rightarrow 3v_1 - 2v_2 - v_3 = 12 \longrightarrow Eq \ 1$$

$$\frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2}{4} \Rightarrow 4v_1 - 7v_2 + v_3 = 0 \longrightarrow Eq \ 2$$

$$\frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = 2\left(\frac{v_1 - v_2}{2}\right) \Rightarrow 6v_1 - 9v_2 + 3v_3 = 0 \longrightarrow Eq \ 3$$

$$v_1 = 4.8 V$$

3 equation and 3 unknowns



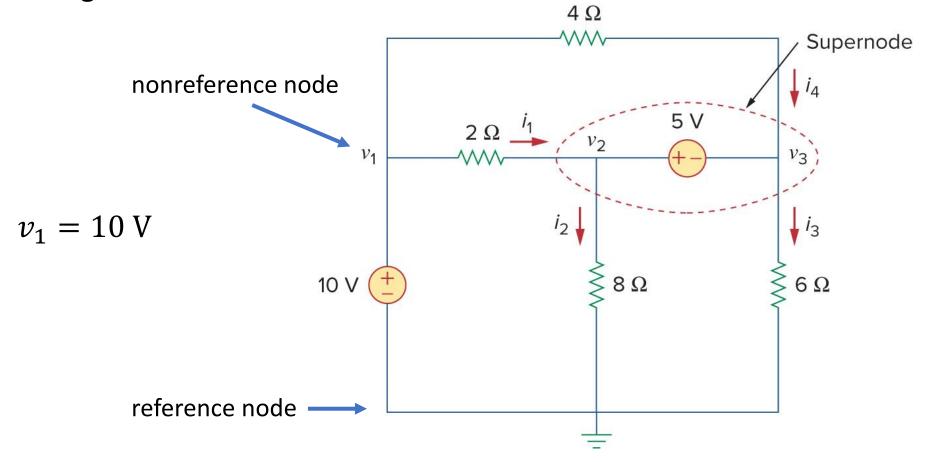
$$v_2 = 2.4 V$$

$$v_3 = -2.4 V$$

Nodal Analysis with Voltage Sources

TO THE TOTAL TOTAL

- How voltage sources affect nodal analysis. There are two cases.
- Case I: If a voltage source is connected between reference node and nonreference node, set the voltage at nonreference node equal to the voltage of the voltage source.



Nodal Analysis with Voltage Sources

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- Case II: If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a **supernode**.
- Nodes 2 and 3 form a supernode

$$ext{KCL: } \sum i_{in} = \sum i_{out}$$

At supernode node: $i_1 + i_4 = i_2 + i_3$

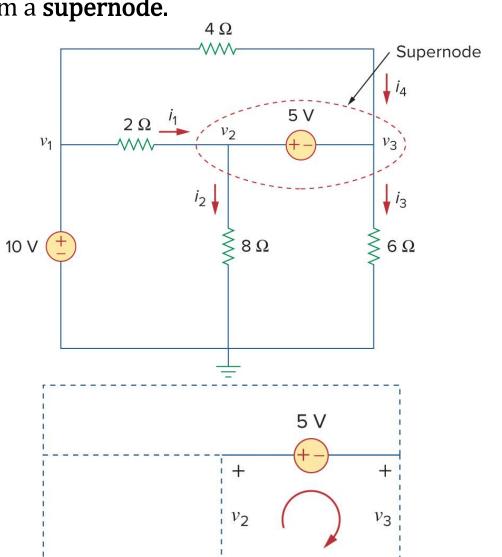
$$i_1 = \frac{v_1 - v_2}{2}$$
, $i_2 = \frac{v_2}{8}$, $i_3 = \frac{v_3}{6}$, $i_4 = \frac{v_1 - v_3}{4}$

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2}{8} + \frac{v_3}{6}$$
 where $v_1 = 10 \text{ V}$

KVL:
$$\sum_{m=1}^{M} V_m = 0$$
, $-v_2 + 5 + v_3 = 0 \Rightarrow v_2 - v_3 = 5$

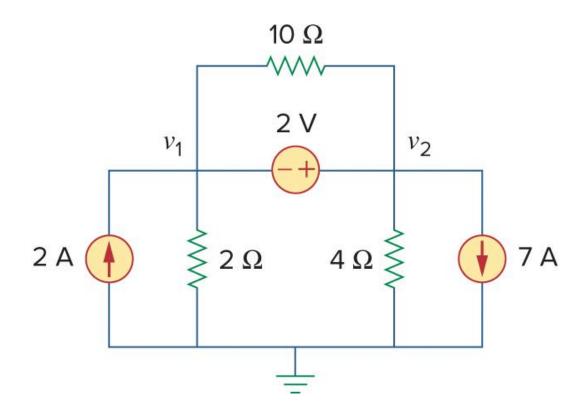
KCL must be satisfied at the supernode. KCL not only applies to node but also closed surface

Supernode requires the application of both KCL and KVL

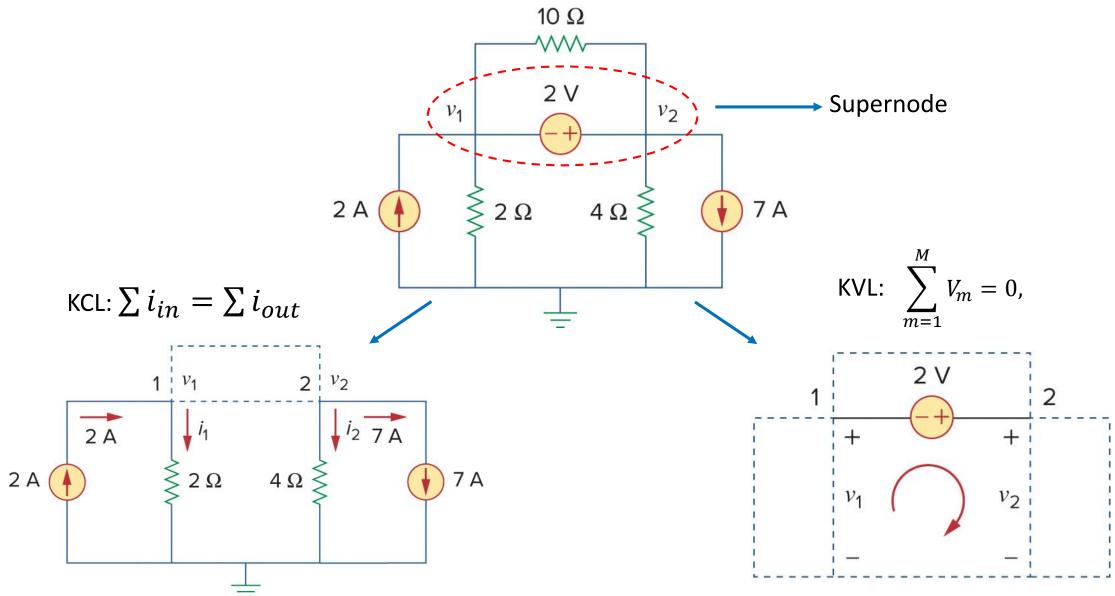




• For the circuit shown below, find the node voltages.







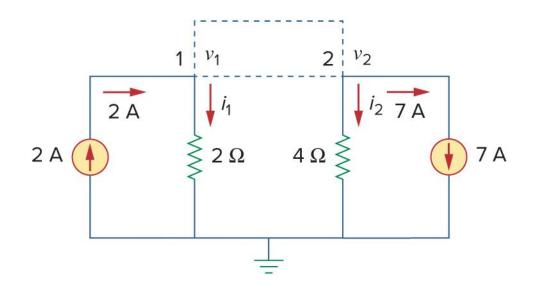


Apply KCL at supernode:

$$2 = i_1 + i_2 + 7 \Rightarrow i_1 + i_2 = -5$$

$$i_1 = \frac{v_1}{2}, \quad i_2 = \frac{v_2}{4}$$
 $\frac{v_1}{2} + \frac{v_2}{4} = -5$

$$\frac{v_1}{2} + \frac{v_2}{4} = -5 \Rightarrow 2v_1 + v_2 = -20 \longrightarrow Eq \ 1$$

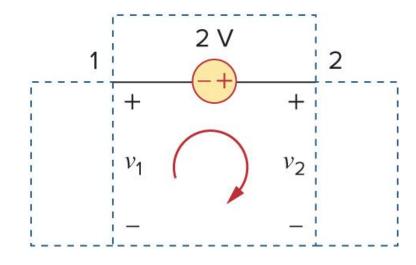


Apply KVL at supernode:

$$-v_1 - 2 + v_2 = 0 \Rightarrow -v_1 + v_2 = 2 \longrightarrow Eq 2$$

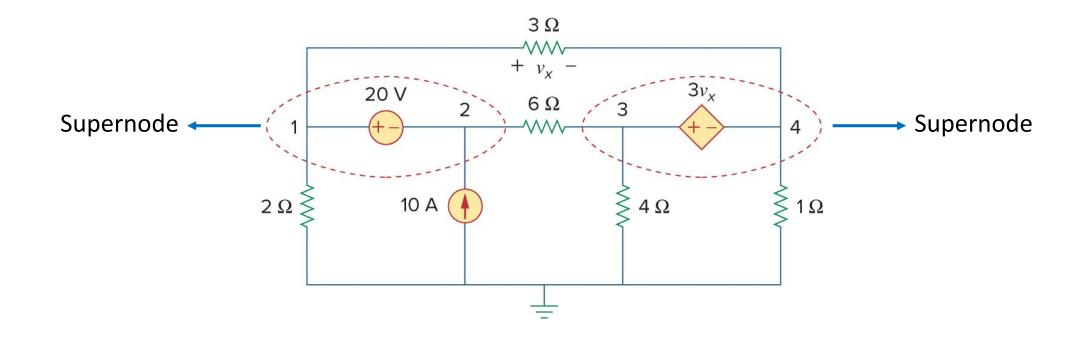
2 eqs and 2 unknows:

$$2v_1 + v_2 = -20$$
 $v_1 = -7.333 V$ $v_2 = -5.333 V$





• Find the node voltages in the circuit shown below





• Node 1 and 2 as well as node 3 and 4 form a supernode:

Apply KCL at supernode 1 and 2:

$$i_3 + 10 = i_1 + i_2$$

$$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$$

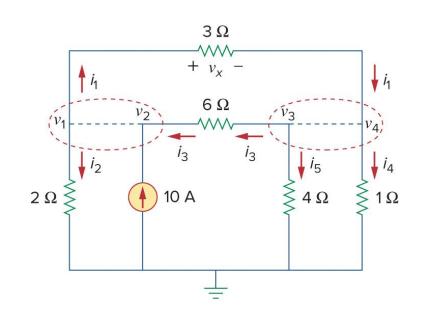
or

$$5v_1 + v_2 - v_3 - 2v_4 = 60$$

Apply KCL at supernode 3 and 4:

$$i_1 = i_3 + i_4 + i_5$$
 \Rightarrow $\frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$ or

$$4v_1 + 2v_2 - 5v_3 - 16v_4 = 0$$





 $3v_{\rm y}$

Apply KVL at loop 1:

$$-v_1 + 20 + v_2 = 0$$
 \Rightarrow $v_1 - v_2 = 20$

Apply KVL at loop 2:

$$-v_3 + 3v_x + v_4 = 0$$

$$v_x = v_1 - v_4$$

$$3v_1 - v_3 - 2v_4 = 0$$

Apply KVL at loop 3:

$$v_x - 3v_x + 6i_3 - 20 = 0$$

$$6i_3 = v_3 - v_2$$
 and $v_x = v_1 - v_4$

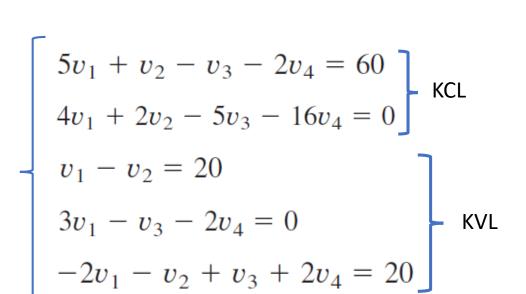
$$-2v_1 - v_2 + v_3 + 2v_4 = 20$$

$$v_1 = 26.67 \text{ V}$$

$$v_2 = 6.667 \text{ V}$$

$$v_3 = 173.33 \text{ V}$$

$$v_4 = -46.67 \text{ V}$$



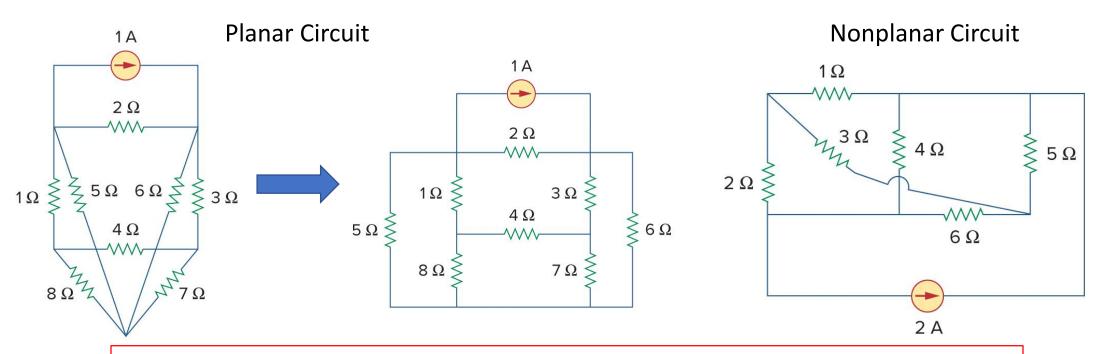
 6Ω

 3Ω

20 V

Mesh Analysis

- Provide a procedure for analyzing circuit using mesh currents
- Mesh analysis is also known as loop analysis or the mesh-current method
- In mesh analysis, apply KVL to find unknown currents
- Mesh analysis can only be applied to a planar circuit.
- Planar circuit: drawn in a plane with no branches crossing one another. Otherwise it nonplanar.



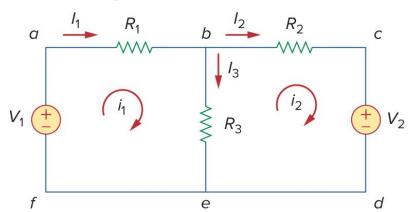
Nodal analysis applies KCL to find unknows voltages (For supernode: apply both KCL & KVL) Mesh analysis applies KVL to find unknown currents (For supermesh: apply both KVL & KCL)

Mesh Analysis



- What is a mesh?
 - □ A loop which does not contain any other loops within it.
 - □ Path abefa: mesh (only one loop)
 - □ Path bcdeb: mesh (only one loop
 - □ Path abcdefa: not a mesh (two loops)
- The current through a mesh is known as mesh current
- Steps to determine mesh current:
 - \square Assign a mesh current $i_1, i_2, ..., i_n$ to the n meshes.
 - □ Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh current
 - □ Solve the resulting n simultaneous equations to get the mesh currents

 i_1 and i_2 : mesh currents I_1 , I_2 , and I_3 : branch currents





- Obtain branch currents using mesh analysis
- 1st: assign mesh currents $(i_1 \text{ and } i_2)$ to meshes 1 and 2
- Direction of mesh currents is chosen arbitrarily (clockwise)
- 2nd: apply KVL to mesh 1:

$$-V_1 + R_1 i_1 + R_3 (i_1 - i_2) = 0$$

$$(R_1 + R_3) i_1 - R_3 i_2 = V_1 \longrightarrow Eq 1$$

Apply KVL to mesh 2:

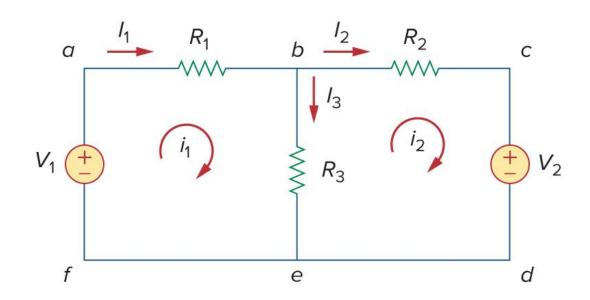
$$R_2 i_2 + V_2 + R_3 (i_2 - i_1) = 0$$

 $-R_3 i_1 + (R_2 + R_3) i_2 = -V_2 \longrightarrow Eq 2$

Last step is to solve for the mesh currents:

Equations in matrix form:

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$



$$I_1 = i_1, \qquad I_2 = i_2, \qquad I_3 = i_1 - i_2$$



For the circuit shown below, find the branch currents using mesh analysis.

Solution:

- 1st: assign mesh currents
- 2nd: apply KVL to mesh 1:

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

 $3i_1 - 2i_2 = 1 \longrightarrow Eq 1$

Apply KVL to mesh 2:

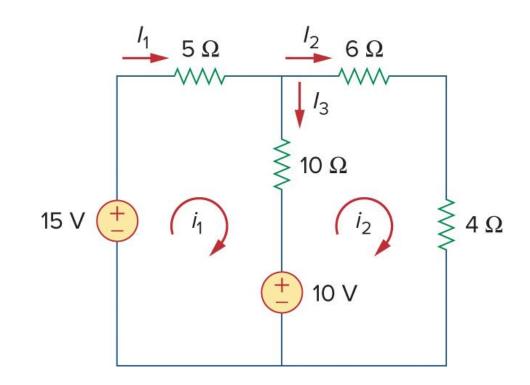
$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

 $i_1 = 2i_2 - 1 \longrightarrow Eq 2$

• Substitute *Eq* 2 into *Eq* 1:

$$6i_2 - 3 - 2i_2 = 1 \implies i_2 = 1 \text{ A}$$

 $i_1 = 2i_2 - 1 = 2 - 1 = 1 \text{ A}$
 $I_1 = i_1 = 1 \text{ A}, \qquad I_2 = i_2 = 1 \text{ A}, \qquad I_3 = i_1 - i_2 = 0$





• Use mesh analysis to find the current I_0 in the circuit shown below

Solution:

- Apply KVL to three meshes
- For mesh 1:

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$
$$11i_1 - 5i_2 - 6i_3 = 12 \longrightarrow Eq 1$$

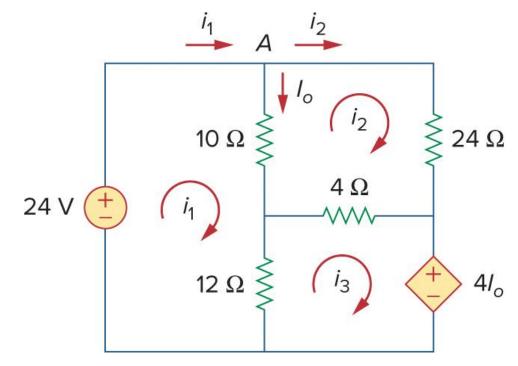
For mesh 2:

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$
$$-5i_1 + 19i_2 - 2i_3 = 0 \longrightarrow Eq 2$$

For mesh 3:

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

• At node A, KCL: $I_o = i_1 - i_2$ $4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$ $-i_1 - i_2 + 2i_3 = 0 \longrightarrow Eq 3$



Equations in matrix form:

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

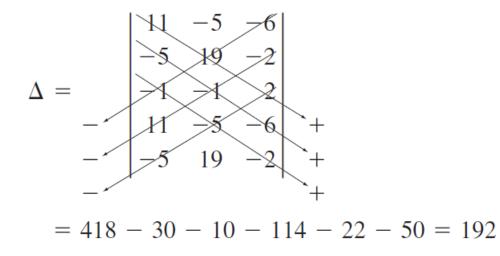


• Cramer's rule

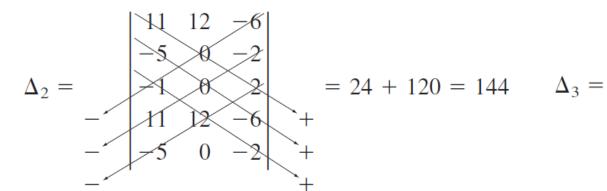
$$i_1 = \frac{\Delta_1}{\Delta}$$
 $i_2 = \frac{\Delta}{\Delta}$

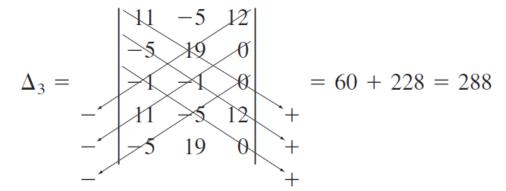
$$i_3 = \frac{\Delta_3}{\Delta}$$

$$i_1 = \frac{\Delta_1}{\Delta}$$
 $i_2 = \frac{\Delta_2}{\Delta}$ $i_3 = \frac{\Delta_3}{\Delta}$ where Δ is determinant
$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$



$$\Delta_{1} = \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ -12 & -5 & -6 \\ -19 & -2 & + \\ -10 & -2 & + \\ -10 & -2$$







$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A},$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

$$I_o = i_1 - i_2 = 1.5 \text{ A}.$$

Mesh Analysis with Current Sources



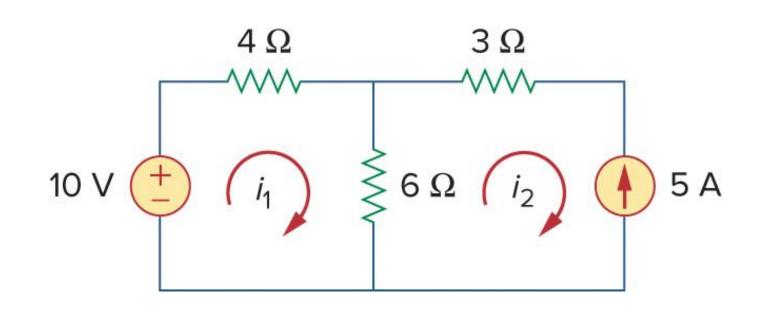
- How current sources affect mesh analysis. There are two cases.
- Case I: when a current source exists only in one mesh
- Set $i_1 = -5$ A
- Write mesh equation for the other mesh

KVL for mesh 1:

$$-10 + 4i_1 + 6(i_1 - i_2) = 0$$

$$-10 + 4i_1 + 6i_1 - 6i_2 = 0$$

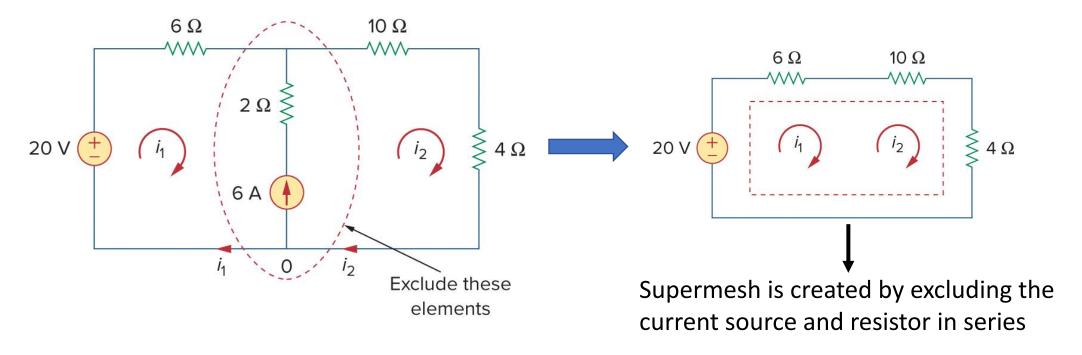
$$10i_1 - 6i_2 = 10 \Rightarrow i_1 = -2 A$$



Mesh Analysis with Current Sources



Case II: When a current source exists between two meshes, create a supermesh
by excluding the current source and any elements connected in series with it

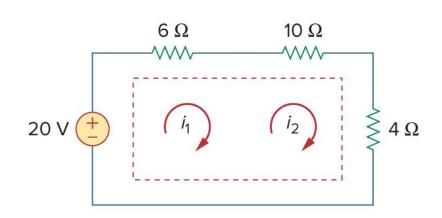


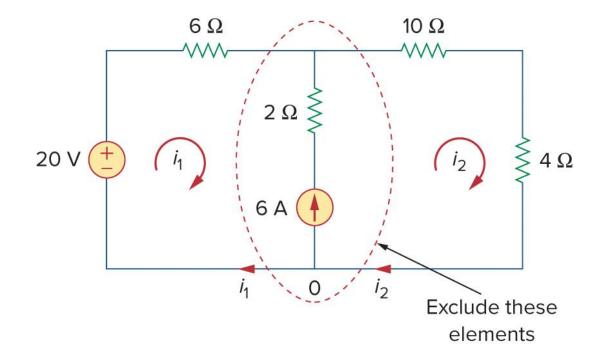
- Supermesh results when two meshes have a (dependent or independent) current source in common.
- If a circuit has two or more supermeshes that intersect, they should be combined to form a larger supermesh

Mesh Analysis with Current Sources



A Supermesh requires the application of both KVL and KCL





Apply KVL to the supermesh:

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$
$$6i_1 + 14i_2 = 20$$

 Apply KCL to node zero (intersection of two meshes):

$$i_2 = i_1 + 6$$

 $i_1 = -3.2 \text{ A}, \qquad i_2 = 2.8 \text{ A}$



• For the circuit shown below, find i_1 to i_4 using mesh analysis.

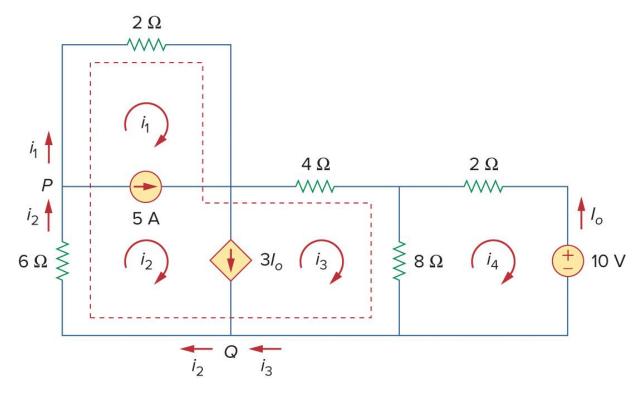
Solution:

- Two supermeshes:
 - 1st one is between mesh 1 & 2
 - 2nd one is between mesh 2 & 3
- Combine them and exclude both current sources
- Apply KVL to the larger supermesh:

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$
$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \longrightarrow Eq 1$$

 For independent current source, apply KCL to node P:

$$i_2 = i_1 + 5 \longrightarrow Eq 2$$



 For dependent current source, apply KCL to node Q:

$$i_2 = i_3 + 3I_o$$
 $I_o = -i_4$
 $i_2 = i_3 - 3i_4 \longrightarrow Eq 3$

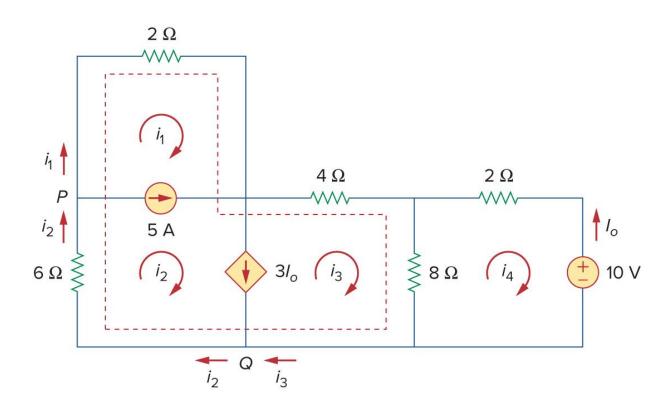


• Apply KVL in mesh 4:

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

$$5i_4 - 4i_3 = -5 \longrightarrow Eq 4$$

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0$$
 $i_2 = i_1 + 5$
 $i_2 = i_3 - 3i_4$
 $5i_4 - 4i_3 = -5$



$$i_1 = -7.5 \text{ A}, \qquad i_2 = -2.5 \text{ A}, \qquad i_3 = 3.93 \text{ A}, \qquad i_4 = 2.143 \text{ A}$$

Nodal versus Mesh Analysis

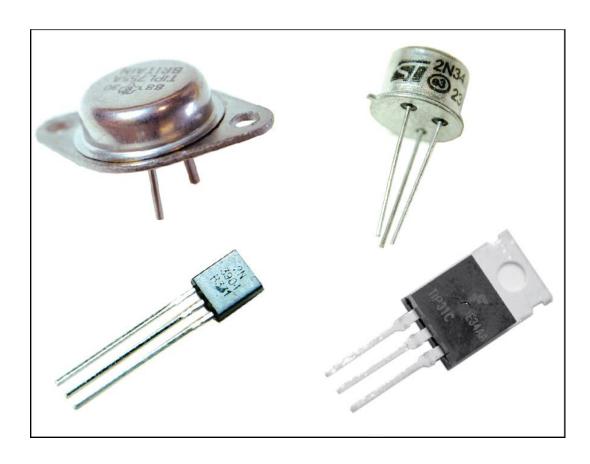


- Both methods provide a systematic way of analyzing a complex network.
- How do we know which method is better or more efficient?
- The choice of the better method is dictated by two factors.
- First factor: the nature of the particular network
 - Mesh analysis: more suitable if network contains many series-connected elements, voltage sources, or supermeshes
 - Nodal analysis: more suitable if network contains parallel-connected elements, current sources, or supernodes
 - Better to use nodal analysis for a circuit with fewer nodes than meshes
 - Better to use mesh analysis for a circuit with fewer meshes than nodes
- Second factor: Based on required information
 - If node voltages are required, apply node analysis
 - If branch or mesh currents are required, apply mesh analysis
- You must learn both methods!



• Transistors play essential role for the design of integrated circuits (IC).

- What is a transistor?
 - □ Current **Trans**ferring res**istor**
 - □ Three terminal semiconductor device
- Two types of transistors:
 - □ Bipolar Junction Transistor (BJT)
 - □ Field-Effect Transistor (FET)

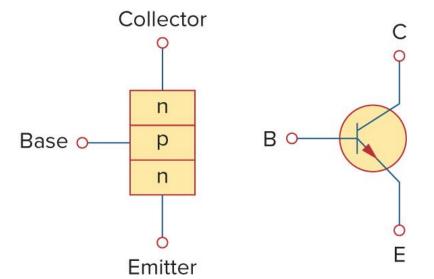


Various types of transistors

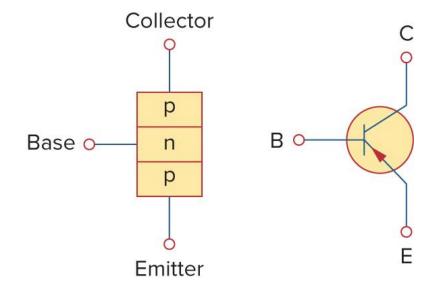


- Two types of Bipolar Junction Transistor (BJT)
 - npn
 - pnp

npn: Arrowhead pointing down

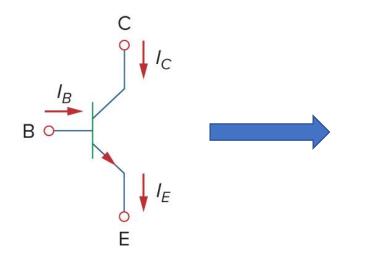


pnp: Arrowhead pointing up



• Three terminals: emitter (E), base (B), and collector (C)





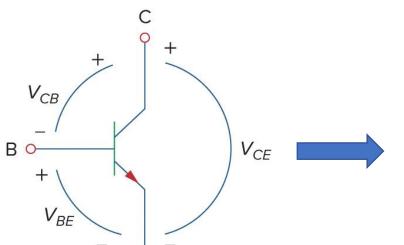
Apply KCL: $\sum i_{in} = \sum i_{out}$

$$I_E = I_C + I_B$$

 I_E : emitter current

I_C: collector current

 I_B : base current



Apply KVL: $\sum_{m=1}^{M} V_m = 0$

$$V_{CE} - V_{BE} - V_{CB} = 0$$
 or $V_{CE} + V_{EB} + V_{BC} = 0$

 V_{CE} : collector-emitter voltage

 V_{EB} : emitter-base voltage

 V_{BC} : base-collector voltage



- BJT: three modes of operation
 - □ Active mode
 - Cutoff mode
 - Saturation mode
- Operation in active mode:

$$V_{BE} \approx 0.7 \ V$$
 $I_C = \alpha I_E$ $I_C = \beta I_B$

$$I_E = I_C + I_B \Rightarrow I_E = \beta I_B + I_B \Rightarrow (1 + \beta) I_B \Rightarrow I_E = (1 + \beta) I_B$$

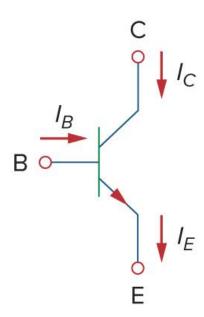
$$\alpha I_{E} = \beta I_{B} \Rightarrow \alpha \frac{I_{E}}{I_{B}} = \beta \Rightarrow \alpha \frac{(1+\beta) I_{B}}{I_{B}} = \beta \Rightarrow \alpha + \alpha \beta = \beta$$

$$\Rightarrow \alpha = \beta - \alpha \beta \Rightarrow \alpha = \beta (1-\alpha) \Rightarrow \beta = \frac{\alpha}{1-\alpha}$$

 α : common-base current gain (in the range of 0.98 to 0.999)

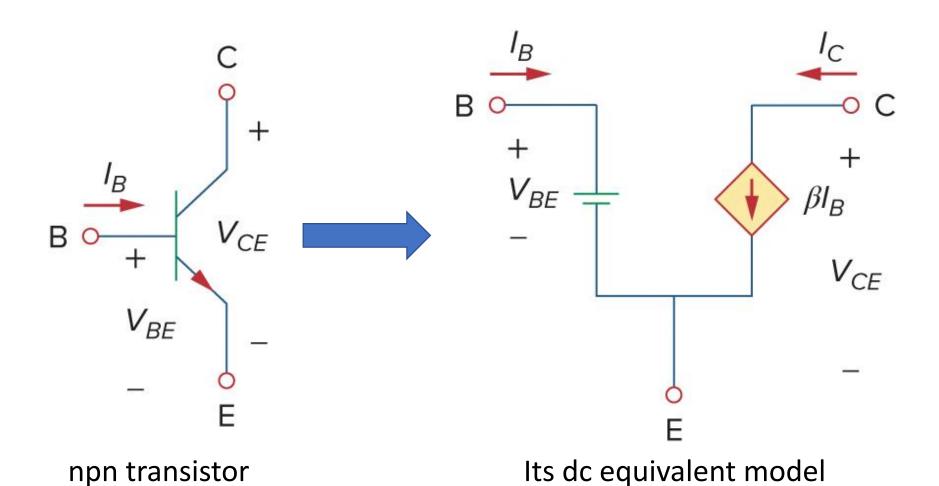
eta: common-emitter current gain (in the range of 50 to 1000)

 α and β is the transistor properties and assume constant





• In active mode, the BJT can be modeled as a dependent current-controlled current source:





Find the I_B , I_C , and v_0 in the transistor shown below. Assume that the transistor operates in the active mode and that $\beta = 50$.

Solution:

For the input loop, KVL gives

$$-4 + I_B(20 \times 10^3) + V_{BE} = 0$$

Since $V_{BE} = 0.7 \text{ V}$ in the active mode,

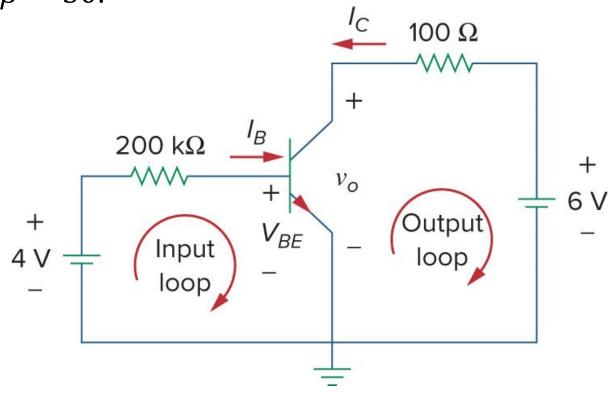
$$I_B = \frac{4 - 0.7}{20 \times 10^3} = 165 \,\mu\text{A}$$

$$I_C = \beta I_B = 50 \times 165 \,\mu\text{A} = 8.25 \,\text{mA}$$

For the output loop, KVL gives

$$-v_o - 100I_C + 6 = 0$$

$$v_o = 6 - 100I_C = 6 - 0.825 = 5.175 \text{ V}$$





 $1 \, \mathrm{k}\Omega$

^

• For the BJT circuit shown below, $\beta=150$ and $V_{BE}=0.7~V$. Find v_0 .

Solution:

• Method 1: Solving with **mesh analysis**

1st loop:

$$-2 + 100kI_1 + 200k(I_1 - I_2) = 0$$

$$3I_1 - 2I_2 = 2 \times 10^{-5} \longrightarrow Eq 1$$

2nd loop:

$$200k(I_2 - I_1) + V_{BE} = 0$$

$$-2I_1 + 2I_2 = -0.7 \times 10^{-5} \longrightarrow Eq 2$$

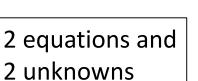
$$I_1 = 1.3 \times 10^{-5} \text{A}$$
 and $I_2 = (-0.7 + 2.6)10^{-5}/2 = 9.5 \,\mu\text{A}$

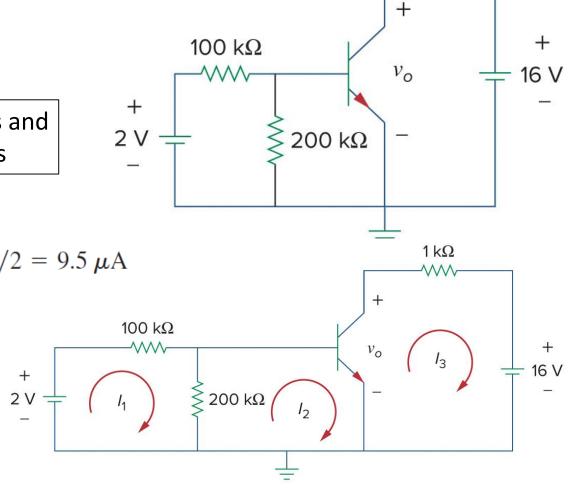
Since
$$I_3 = -150I_2 = -1.425 \text{ mA}$$

3rd loop:

$$-v_o + {}^{1}kI_3 + 16 = 0$$

$$v_o = -1.425 + 16 = 14.575 V$$





- Method 2: Solving with nodal analysis
- Replace transistor with its equivalent circuit

At node number 1: $V_1 = 0.7 \text{ V}$

Apply KCL:

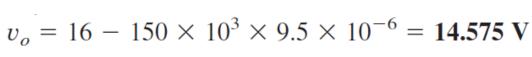
$$(0.7 - 2)/100k + 0.7/200k + I_B = 0$$

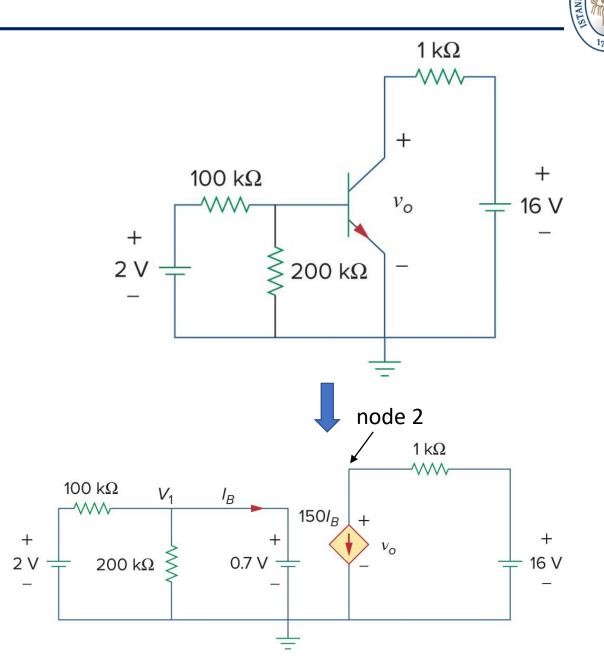
$$I_B = 9.5 \,\mu\text{A}$$

At node number 2 we have:

Apply KCL:

$$150I_B + (v_o - 16)/1k = 0$$





PSpice



- What is PSpice?
 - □ Free computer software circuit analysis program
 - □ Allows you to simulate and analyze a circuit
 - □ Helpful program in determining the voltages and currents in a circuit
 - □ Use online sources (YouTube and google) for a tutorial on how to use the PSpice program

