Find $|a-b|$, $2a+b$ and $3a-4b$ for the following vectors in R^3.

$a=(2,5,-4)\,,\ b=(1,-2,-3)$ $(\sqrt{51}\,,\ (5,8,-11)\,,\ (2,23,0))$

$a=2i-j\,,\ b=j-3k$ $(\sqrt{17}\,,\ 4i-j-3k\,,\ 6i-7j+12k)$

Calculate a determinant to determine whether the given vectors u, v, and w are linearly dependent or independent.

$u = (5, -2, 4)\,,\; v = (2, -3, 5)\,,\; w = (4, 5, -7)$ $\quad$ *(Lin. dep.)*

$u = (1, 1, 0)\,,\; v = (4, 3, 1)\,,\; w = (3, -2, -4)$ $\quad$ *(Lin. indep.)*

Determine whether the given vectors u, v, and w are linearly dependent by reducing the system to echelon form and choosing a real value for free variables, if any, to determine values of other variables.

$u = (5,5,4)\,,\ v = (2,3,1)\,,\ w = (4,1,5)$ $(Lin.\,dep.\,,\ a = -1\,,\ b = 3\,,\ c = 1)$

$u = (1,1,-2)\,,\ v = (-2,-1,6)\,,\ w = (3,7,2)$ $(Lin.\,dep.\,,\ a = 11\,,\ b = 4\,,\ c = -1)$

$u = (1,4,5)\,,\ v = (4,2,5)\,,\ w = (-3,3,-1)$ $(Lin.\,indep.)$

Express the vector t as a linear combination of the vectors u, v, and w.

$t = (5, 30, -21)$, $u = (5, 2, -2)$, $v = (1, 5, -3)$, $w = (5, -3, 4)$ $(t = u + 5v - w)$

$t = (0, 0, 19)$, $u = (1, 4, 3)$, $v = (-1, -2, 2)$, $w = (4, 4, 1)$ $(t = 2u + 6v + w)$

$t = (7, 7, 7)$, $u = (2, 5, 3)$, $v = (4, 1, -1)$, $w = (1, 1, 5)$ $(t = u + v + w)$

Determine whether or not W is a subspace of R^n.

W is a set of all vectors in R^3 such that $x_1 = 5x_2$. (Yes)

W is a set of all vectors in R^3 such that $x_1+x_2+x_3 = 1$. (No)

W is a set of all vectors in R^4 such that $x_1 = 3x_3$. (Yes)

Determine whether or not W is a subspace of R^n.

W is a set of all vectors in R^2 such that $x_1^2 + x_2^2 = 0$. (Yes)

W is a set of all vectors in R^2 such that $x_1 + x_2 = 1$. (No)

W is a set of all vectors in R^4 such that $x_1 x_2 = x_3 x_4$. (No)

Find two solution vectors u and v such that the solution space is the set of all linear combinations of the form $su + tv$.

$$\begin{array}{rcrcrcrcl} x_1 & - & 4x_2 & + & x_3 & - & 4x_4 & = & 0 \\ x_1 & + & 2x_2 & + & x_3 & + & 8x_4 & = & 0 \\ x_1 & + & x_2 & + & x_3 & + & 6x_4 & = & 0 \end{array} \qquad (u = (-1,0,1,0)\,,\ v = (-4,-2,0,1))$$

$$\begin{array}{rcrcrcrcrcl} x_1 & + & 3x_2 & + & 2x_3 & + & 5x_4 & - & x_5 & = & 0 \\ 2x_1 & + & 7x_2 & + & 4x_3 & + & 11x_4 & + & 2x_5 & = & 0 \\ 2x_1 & + & 6x_2 & + & 5x_3 & + & 12x_4 & - & 7x_5 & = & 0 \end{array} \qquad (u = (2,-1,-2,1,0)\,,\ v = (3,-4,5,0,1))$$

Reduce the given system to echelon form to find a single solution vector u such that the solution space is the set of all scalar multiples of u.

$$\begin{array}{rcrcrcrcl} x_1 & - & 3x_2 & - & 5x_3 & - & 6x_4 & = & 0 \\ 2x_1 & + & x_2 & + & 4x_3 & - & 4x_4 & = & 0 \\ x_1 & + & 3x_2 & + & 7x_3 & + & x_4 & = & 0 \end{array} \qquad (u = (1, 2, -1, 0))$$

$$\begin{array}{rcrcrcrcl} x_1 & + & 3x_2 & + & 3x_3 & + & 3x_4 & = & 0 \\ 2x_1 & + & 7x_2 & + & 5x_3 & - & x_4 & = & 0 \\ 2x_1 & + & 7x_2 & + & 4x_3 & - & 4x_4 & = & 0 \end{array} \qquad (u = (-6, 4, -3, 1))$$

Prove that if u is a (fixed) vector in the vector space V, then the set W of all scalar multiples cu of u is a subspace of V.

Suppose that A is an $n \times n$ matrix and that k is a scalar. Show that the set of all vectors x such that $Ax = kx$ is subspace of R^n.

Let A be an $n \times n$ matrix, b be a nonzero vector, and x_0 be a solution vector of the system $Ax = b$. Show that x is a solution of the nonhomogeneous system $Ax = b$ if and only if $y = x - x_0$ is a solution of the homogeneous system $Ay = 0$.

Determine whether the given vectors are linearly independent or not. Do this without solving a linear system of equations.

$v_1 = (4, -2, 6, -4)\,,\ v_2 = (6, -3, 9, -6)$ *(Lin. dep.)*

$v_1 = (1, 0, 0)\,,\ v_2 = (1, 1, 0)\,,\ v_3 = (1, 1, 1)$ *(Lin. indep.)*

$v_1 = (2, 1, 0, 0)\,,\ v_2 = (3, 0, 1, 0)\,,\ v_3 = (4, 0, 0, 1)$ *(Lin. indep.)*

$v_1 = (1, 0, 3, 0)\,,\ v_2 = (0, 2, 0, 4)\,,\ v_3 = (1, 2, 3, 4)$ *(Lin. dep.)*

Express the indicated vector w as a linear combination of the given vectors if it's possible. If not, show that it is impossible.

$w = (3,-1,-2)\,,\ v_1 = (-3,1,-2)\,,\ v_2 = (6,-2,3)$ $(w = 7v_1 + 4v_2)$

$w = (4,-4,3,3)\,,\ v_1 = (7,3,-1,9)\,,\ v_2 = (-2,-2,1,-3)$ $(w = 2v_1 + 5v_2)$

$w = (7,7,9,11)\,,\ v_1 = (2,0,3,1)\,,\ v_2 = (4,1,3,2)\,,\ v_3 = (1,3,-1,3)$ $(w = 6v_1 - 2v_2 + 3v_3)$

$w = (2,-3,2,-3)\,,\ v_1 = (1,0,0,3)\,,\ v_2 = (0,1,-2,0)\,,\ v_3 = (0,-1,1,1)$ $(Impossible)$

Determine whether or not the following vectors in R^3 are linearly dependent or not.

$v_1 = [1 \ \ -2 \ \ 1]^T$, $v_2 = [2 \ \ 1 \ \ -1]^T$, $v_3 = [7 \ \ -4 \ \ 1]^T$ $(Lin.\, dep.)$

If the vectors v_1, v_2, and v_3 are linearly dependent, show this. Otherwise, find a nontrivial linear combination of them that is equal to the zero vector.

$v_1 = (2, 0, -3)\,,\ v_2 = (4, -5, -6)\,,\ v_3 = (-2, 1, 3)$ $(3v_1 + v_2 + 5v_3 = 0)$

$v_1 = (3, 9, 0, 5)\,,\ v_2 = (3, 0, 9, -7)\,,\ v_3 = (4, 7, 5, 0)$ $(7v_1 + 5v_2 - 9v_3 = 0)$

$v_1 = (2, 0, 3, 0)\,,\ v_2 = (5, 4, -2, 1)\,,\ v_3 = (2, -1, 1, -1)$ $(Lin.\, indep.)$

$v_1 = (1, 1, -1, 1)\,,\ v_2 = (2, 1, 1, 1)\,,\ v_3 = (3, 1, 4, 1)$ $(Lin.\, indep.)$

The vectors v_i are known to be linearly independent. Apply the definition of linear independence to show that the vectors u_i are also linearly independent.

$u_1 = v_1 + v_2\,,\ u_2 = 2v_1 + 3v_2$

$u_1 = v_2 + v_3\,,\ u_2 = v_1 + v_3\,,\ u_3 = v_1 + v_2$

$u_1 = v_1\,,\ u_2 = v_1 + 2v_2\,,\ u_3 = v_1 + 2v_2 + 3v_3$

Show that if the (finite) set S of vectors is linearly independent, then any subset T of S is also linearly independent.

If some $k \times k$ submatrix of A is the $k \times k$ identity matrix, then $v_1, v_2, \cdots, v_k$ are linearly independent.

Suppose that $k = n$, that the vectors $v_1, v_2, \cdots, v_k$ are linearly independent, and that B is a nonsingular $n \times n$ matrix. Prove that the column vectors of the matrix AB are linearly independent.

Determine whether or not the given vectors in R^n form a basis for R^n.

$v_1 = (3, -7, 5, 2)\,,\; v_2 = (1, -1, 3, 4)\,,\; v_3 = (7, 11, 3, 13)$ (No)

$v_1 = (0, 7, -3)\,,\; v_2 = (0, 5, 4)\,,\; v_3 = (0, 5, 10)$ (No)

$v_1 = (0, 0, 1)\,,\; v_2 = (0, 1, 2)\,,\; v_3 = (1, 2, 3)$ (Yes)

$v_1 = (2, 0, 0, 0)\,,\; v_2 = (0, 3, 0, 0)\,,\; v_3 = (0, 0, 7, 6)\,,\; v_4 = (0, 0, 4, 5)$ (Yes)

Find basis for indicated subspace of R^3.

The plane with equation $x - 2y + 5z = 0$ $(v_1 = (2,1,0)\,,\ v_2 = (-5,0,1))$

The plane with equation $y = z$ $(v_1 = (1,0,0)\,,\ v_2 = (0,1,1))$

The line of intersection of planes above $(v = (-3,1,1))$

Find basis for indicated subspace of R^4.

The set of all vectors of the form (a, b, c, d) for which $a = b + c + d$

$$(v_1 = (1,1,0,0)\,,\; v_2 = (1,0,1,0)\,,\; v_3 = (1,0,0,1))$$

The set of all vectors of the form (a, b, c, d) for which $a = 3c$ and $b = 4d$

$$(v_1 = (3,0,1,0)\,,\; v_2 = (0,4,0,1))$$

The set of all vectors of the form (a, b, c, d) for which $a + 2b = c + 3d = 0$

$$(v = (-2,1,0,0)\,,\; v_2 = (0,0,3,1))$$

Find a basis for the solution space of the homogeneous system.

$$\begin{array}{rcrcrcl} x_1 & + & 3x_2 & + & 4x_3 & = & 0 \\ 3x_1 & + & 8x_2 & + & 7x_3 & = & 0 \end{array}$$

$((11, -5, 1))$

$$\begin{array}{rcrcrcrcrcl} x_1 & + & 3x_2 & - & 4x_3 & - & 8x_4 & + & 6x_5 & = & 0 \\ x_1 & & & + & 2x_3 & + & x_4 & + & 3x_5 & = & 0 \\ 2x_1 & + & 7x_2 & - & 10x_3 & - & 19x_4 & + & 13x_5 & = & 0 \end{array}$$

$((-1, 3, 0, 1, 0), (-3, -1, 0, 0, 1), (-2, 2, 1, 0, 0))$

Let $\{v_1\,,\,v_2\,,\,\cdots\,,\,v_k\}$ be a basis for the proper subspace W of the vector space V, and suppose that the vector v of V isn't in W. Show that the vectors $v_1\,,\,v_2\,,\,\cdots\,,\,v_k, v$ are linearly independent.

Suppose that the vectors $v_1\,,\,v_2\,,\,\cdots\,,\,v_k\,,\,v_{k+1}$ span the vector space V and that v_{k+1} is a liner combination of $v_1\,,\,v_2\,,\,\cdots\,,\,v_k$. Show that the vectors $v_1\,,\,v_2\,,\,\cdots\,,\,v_k$ span V.

Find both a basis for the row space and a basis for the column space of the following matrix.

$$A = \begin{bmatrix} 1 & -3 & -9 & -5 \\ 2 & 1 & 4 & 11 \\ 1 & 3 & 3 & 13 \end{bmatrix}$$

(Row Basis: the 3 row vectors of E, Column Basis: the first 3 column vectors of A)

$$A = \begin{bmatrix} 1 & 4 & 9 & 2 \\ 2 & 2 & 6 & -3 \\ 2 & 7 & 16 & 3 \end{bmatrix}$$

(Row Basis: the 3 row vectors of E, Column Basis: the 1st, 2nd and 4th column vectors of A)

Find both a basis for the row space and a basis for the column space of the following matrix.

$$A = \begin{bmatrix} 1 & 1 & 3 & 3 & 1 \\ 2 & 3 & 7 & 8 & 2 \\ 2 & 3 & 7 & 8 & 3 \\ 3 & 1 & 7 & 5 & 4 \end{bmatrix}$$

(Row Basis: the 3 row vectors of E, Column Basis: the 1st, 2nd and 5th column vectors of A)

$$A = \begin{bmatrix} 1 & 1 & 3 & 3 & 0 \\ -1 & 0 & -2 & -1 & 1 \\ 2 & 3 & 7 & 8 & 1 \\ -2 & 4 & 0 & 6 & 7 \end{bmatrix}$$

(Row Basis: the 3 row vectors of E, Column Basis: the 1st, 2nd and 5th column vectors of A)

A set S of vectors in R^4 is given. Find a subset of S that forms a basis for the subspace of R^4 spanned by S.

$v_1 = (3,2,2,2)\,,\; v_2 = (2,1,2,1)\,,\; v_3 = (4,3,2,3)\,,\; v_4 = (1,2,3,4)$ $(Lin.\, ind.\, v_1\,,\; v_2\,,\; v_4)$

$v_1 = (5,4,2,2)\,,\; v_2 = (3,1,2,3)\,,\; v_3 = (7,7,2,1)\,,\; v_4 = (1,-1,2,4)\,,\; v_5 = (5,4,6,7)$
$(Lin.\, ind.\, v_1\,,\; v_2\,,\; v_4\,,\; v_5)$

Let V be a subspace of R^4 given by

$$V = \{[\,a \;\; b \;\; c \;\; d]^T \in R^4 \,|\, a+b=0 \;\, and \;\, c-d=0; a\,,b\,,c\,,d \in R\}$$

.

a) Find a basis for V. $([1 \;\; -1 \;\; 0 \;\; 0]^T\,,\,[0 \;\; 0 \;\; 1 \;\; 1]^T)$

b) Find the dimension of V. (2)

Explain why the rank of a matrix A is equal to the rank of its transpose A^T.

Let A be a 3×5 matrix whose 3 row vectors are linearly independent. Prove that for each b in R^3, the nonhomogeneous system $Ax = b$ has a solution.

Let A be a 5×3 matrix that has 3 linearly independent row vectors. Suppose that b is a vector in R^5 such that the nonhomogeneous system $Ax = b$ has a solution. Prove that this solution is unique.

Let A be a $m \times n$ matrix and suppose that the system $Ax = b$ is consistent. Prove that its solution is unique if and only if the rank of A is equal to n.

Determine whether the given vectors are mutually orthogonal.

$v_1 = (5, 2, -4, -1)\,,\; v_2 = (3, -5, 1, 1)\,,\; v_3 = (3, 0, 8, -17)$ (Yes)

$v_1 = (3, -2, 3, -4)\,,\; v_2 = (6, 3, 4, 6)\,,\; v_3 = (17, -12, -21, 3)$ (Yes)

$v_1 = (1, 2, 3, -2, 1)\,,\; v_2 = (3, 2, 3, 6, -4)\,,\; v_3 = (6, 2, -4, , 1, 4)$ (Yes)

The three vertices A, B, and C of a triangle are given. Prove that each triangle is a right triangle by showing that its sides a, b, and c satisfy the Pythagorean relation $a^2 + b^2 = c^2$.

$A(6,6,5,8)\,,\ B(6,8,6,5)\,,\ C(5,7,4,6)$ $(a^2 = 7\,,\ b^2 = 7\,,\ c^2 = 14)$

$A(3,5,1,3)\,,\ B(4,2,6,4)\,,\ C(1,3,4,2)$ $(a^2 = 18\,,\ b^2 = 18\,,\ c^2 = 36)$

$A(2,8,-3,-1,2)\,,\ B(-2,5,6,2,12)\,,\ C(-5,3,2,-3,5)$ $(a^2 = 103\,,\ b^2 = 112\,,\ c^2 = 215)$

Given mutually orthogonal vectors $\{v_1\,,\,v_2\,,\,\cdots\,,\,v_k\}$, show that

$$|v_1 + v_2 + \cdots + v_k|^2 = |v_1|^2 + |v_2|^2 + \cdots + |v_k|^2.$$

Prove that $|u \cdot v| = |u||v|$ if and only if the vectors u and v are linearly dependent.

Determine whether or not the indicated set of 3×3 matrices is a subspace of M_{33}.

The set of all diagonal 3×3 matrices (Yes)

The set of all symmetric 3×3 matrices $(a_{ij} = a_{ji})$ (Yes)

The set of all nonsingular 3×3 matrices (No)

The set of all singular 3×3 matrices (No)

Determine whether or not the indicated set of functions is a subspace of F of all real valued functions on R.

The set of all f such that $f(0) = 0$ (Yes)

The set of all f such that $f(x) \neq 0$ for all x (No)

The set of all f such that $f(0) = 0$ and $f(1) = 1$ (No)

The set of all f such that $f(-x) = -f(x)$for all x (Yes)

Determine whether or not the indicated set of polynomials $a_0 + a_1x + a_2x^2 + a_3x^3$ satisfying the given condition is a subspace of the space P of all polynomials.

$a_3 \neq 0$ (No)

$a_0 = a_1 = 0$ (Yes)

$a_0+a_1+a_2+a_3 = 0$ (Yes)

a_0 , a_1 , a_2 , a_3 are all integers (No)

Determine whether the given functions are linearly independent.

$\sin x$ and $\cos x$ $(Lin.\, ind.)$

e^x and xe^x $(Lin.\, ind.)$

$1{+}x$, $1{-}x$ and $1{-}x^2$ $(Lin.\, ind.)$

Determine whether the given functions are linearly independent.

$1+x$, $x+x^2$ and $1-x^2$ $(Lin.\, dep.)$

$\cos 2x$, $\sin^2 x$ and $\cos^2 x$ $(Lin.\, dep.)$

$2cosx+3\sin x$ and $4\cos x+5\sin x$ $(Lin.\, ind.)$

Let V be the set of all ordered pairs of real numbers. Define addition $\oplus$ and multiplication by a scalar $\odot$ on V by

$$(x_1, x_2) \oplus (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$$
$$\alpha \odot (x_1, x_2) = (\alpha^2 x_1, \alpha^2 x_2).$$

Is V a vector space with these operations? Explain. (No)

Let V be a vector space and W_1 and W_2 be subspaces of V. Show that $W_1 \cap W_2$ is also subspace of V.

Consider R^2 as a set. In this set, for x , $y \in R^2$ and $\alpha \in R$, $\oplus$ and $\odot$ are defined by

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \oplus \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix}$$

$$\alpha \odot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2\alpha x_1 \\ \alpha x_2 \end{bmatrix}$$

if $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$. Determine whether R^2 is a vector space with these operations?

(No)

Let A be a 2×2 matrix. Determine whether the set $S = \{x \in R^2 \,|\, Ax = 0\}$ is a subspace of R^2.

(Yes)

Let V be the set of all positive real numbers. If $x\,,\, y \in V$ and $c \in R$, define addition $\oplus$ and multiplication by a scalar $\odot$ on V by

$$x \oplus y = xy + 1$$
$$c \odot x = x.$$

Is V a vector space with these operations? Explain. (No)

Let S be the set of all polynomials in P_4 having at least one real root. Is S a subspace of P_4? (No)

Let V be the set of all positive real numbers. If $x\,,\ y \in V$ and $c \in R$, define addition $\oplus$ and multiplication by a scalar $\odot$ on V by

$$x \oplus y = xy$$
$$c \odot x = x.$$

Is V a vector space with these operations? Explain. (No)

Let $W = \{(x, y, z) \,|\, z^2 = x^2 + y^2\} \subset R^3$. Is W a subspace of R^3? (No)

Let V be the set of all positive real numbers. If $x\,,\, y \in V$ and $c \in R$, define addition $\oplus$ and multiplication by a scalar $\odot$ on V by

$$x \oplus y = xy$$
$$c \odot x = c + x.$$

Is V a vector space with these operations? Explain. (No)

Let $S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in R^{2\times 2} \,|\, 3a + d = b \right\}$. Show that S is a subspace of $R^{2\times 2}$. Find a basis for the subspace of S and determine its dimension.

$$\left(\left\{ \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right\}, 3 \right)$$

Show that $\{1 + x + x^2\ ,\ 1 - x + x^2\ ,\ 1 - x^2\}$ is a basis for P_3?

Determine whether the vectors $p_1(x) = 2 - x - x^2$, $p_2(x) = 1 + x^2$ and $p_3(x) = 1 - 3x$ are linearly independent in P_3. (Yes)

Let S be the set of polynomials in P_4 of even degree. Is S a subspace of P_4? (No)

Let $S = \{p(x) = ax^3 + bx^2 + cx + d \in P_4 \,|\, p(1) = p(-1) = 0\}$. Find a basis of S and the dimension of S. $(\{x^2 - x\,,\, x^2 - 1\}\,,\, 2)$

Let $\{1\,,\, x - 1\,,\, (x-1)(x-2)\}$ is a basis of P_3 and that $W = \{p(x) \in P_3 \,|\, p(1) = 0$ is a subspace of P_3. Find $dim\, W$. (2)

For what values of the scalar k are the 3 row vectors $(k, 1, 0)$, $(1, k, 1)$ and $(0, 1, k)$ linearly dependent and for what values are they linearly independent? $(\mp\sqrt{2}\, and\, 0\,;\, otherwise)$

Let $S = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in R^3 \,|\, x_1 + 2x_2 + x_3 = 0 \right\}$.

a) Is S a subspace of R^3? (Yes)

b) If so, find a basis for S and its dimension. $((-2,1,0)^T\,,\ (-1,0,1)^T\,,\ 2)$

Let $A = \begin{bmatrix} 3 & 9 & 1 \\ 2 & 6 & 7 \\ 1 & 3 & -6 \end{bmatrix}$.

a) Find a basis for the row space of A and the rank of A. $(\{[1\ 3\ -6]\,,\,[0\ 0\ 1]\}\,,\,2)$

b) Find a basis for the null space $N(A)$ and its dimension. $(\{(-3,1,0)\}\,,\,1)$

Let $A = \begin{bmatrix} 5 & -10 & 25 \\ -1 & 2 & -5 \\ 2 & -4 & 10 \end{bmatrix}$.

a) Find a basis for the row space of A and its dimension. $(\{(1\,,\,-2\,,\,5)\}\,,\,1)$

b) Find a basis for the column space of A and its dimension. $(\{(5\,,\,-1\,,\,2)^T\}\,,\,1)$

c) Find a basis for the null space $N(A)$ and its dimension. $(\{(2\ 1\ 0)^T\,,\,(-5\ 0\ 1)^T\}\,,\,2)$

d) Are the column vectors of A linearly independent in R^3? (No)

Let $A = \begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & -1 & 0 & 0 & 2 \\ 2 & 1 & 6 & 0 & 1 \end{bmatrix}$.

a) Find a basis for the null space $N(A)$ and its dimension. $(\{(-1,1,0,-2,1)\,,\,(-2,-2,1,0,0)\}\,,\,2)$

b) Find the rank of the matrix A. (3)

c) Determine whether the vector $b = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}^T$ belongs to row space of A. (No)

Let $A = \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ -2 & 0 & -1 & 1 & -1 \\ 3 & 0 & 4 & -4 & -1 \end{bmatrix}$.

Find a basis for the null space $N(A)$ and the rank of A.

$(\{(0,0,1,1,0)\,,\,(-1,0,1,0,1)\,,\,(0,1,0,0,0)\}\,,\,2)$

Let $A = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 3 & 0 & 0 & 3 \\ 2 & 1 & 4 & -3 \\ 1 & 0 & 2 & -1 \end{bmatrix}$.

Find a basis for the row space of A and its rank.

$(\{(1\ 0\ 2\ \ -1), (0\ 1\ 0\ \ -1), (0\ 0\ 1\ \ -1)\}, 3)$

Let $A = \begin{bmatrix} 1 & -2 & 2 & -1 \\ -3 & 6 & -1 & -7 \\ 2 & -4 & 5 & -4 \end{bmatrix}$.

a) Find a basis for the null space $N(A)$ and its dimension. $(\{(2,1,0,0), (-3,0,2,1)\}, 2)$

b) Find the rank of the matrix A. (2)

c) Determine whether the vector $b = \begin{pmatrix} 7 \\ 2 \\ -2 \\ -1 \end{pmatrix}$ belongs to the null space of A. (Yes)

Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 1 & -3 & 4 \\ 2 & 5 & 11 & 12 \end{bmatrix}$.

a) Find a basis for the null space $N(A)$ and its dimension. $(\{(2,-3,1,0)\}\,,\,1)$

b)Find a basis for the column space of A and its rank.

$(\{(1,3,2)\,,\,(1,1,5)\,,\,(1,4,12)\}\,,\,3)$

Let $A = \begin{bmatrix} 1 & 1 & 0 & -1 \\ -2 & -1 & 3 & 1 \\ 3 & 2 & -3 & -1 \end{bmatrix}$.

a) Find a basis for the column space of A and the rank of A. $(\{(1,-2,3)\,,\,(1,-1,2)\,,\,(-1,1,-1)\}\,,\,3)$

b)Find a basis for the null space of A and determine its dimension. $(\{(3,-3,1,0)\}\,,\,1)$

Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 6 & 8 & 10 & 12 & 14 \end{bmatrix}$.

a) Find a basis for the column space of A and the rank of A. $\left(\{(1,3,4)\,,\,(2,4,6)\}\,,\,2\right)$

b)Find a basis for the null space of A and determine its dimension.

$\left(\{(1,-2,1,0,0,0)\,,(2,-3,0,1,0,0)\,,(3,-4,0,0,1,0)\,,(4,-5,0,0,0,1)\}\,,\,4\right)$