İTÜ MAT 2 1 0 E 2 0 G 1 HOMEWORK 1 November 22, 2020

Due: **Monday, November 23, 2020 at 08.00 a.m.**
Submission Link: `http://veti.itu.edu.tr/form/mathavuz/mat210e20g1`

1 For which value of **a** is the system

$$\begin{aligned} \mathbf{3x_1 - x_2 + 4x_3 - 7x_4} &= \mathbf{3} \\ \mathbf{x_1 - x_2 + 2x_3 - 3x_4} &= \mathbf{1} \\ \mathbf{-x_1 + 2x_2 - 3x_3 + 4x_4} &= \mathbf{a} \end{aligned}$$

consistent?

2 Consider the matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{1} & \mathbf{4} & \mathbf{-1} \\ \mathbf{0} & \mathbf{-5} & \mathbf{7} & \mathbf{2} & \mathbf{0} \\ \mathbf{-1} & \mathbf{0} & \mathbf{2} & \mathbf{3} & \mathbf{1} \\ \mathbf{0} & \mathbf{3} & \mathbf{2} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix}.$$

a) **A** is invertible.
b) **A** is in row-echelon form.
c) dim(Null **A**)=1.
d) rank **A**=3.
e) There is a vector $\mathbf{b} \in \mathbb{R}^{\mathbf{5}}$ such that $\mathbf{Ax} = \mathbf{b}$ is not consistent.
f) det(**A**)=0.
g) dim(Null **A**)+rank **A**=4.

Which of the statements above are true for **A**?
(Submit the corresponding number without parentheses.)

(1) a, d, e.
(2) c, e, f.
(3) c, f, g.
(4) a, b, c, e.
(5) c, e, f, g.
(6) b, c, d, g.
(7) a, b, c, e, f.
(8) b, c, d, f, g.
(9) b, c, e, f, g.

3

a) If $\mathbf{A}$ and $\mathbf{B}$ are invertible $\mathbf{n \times n}$ matrices, then so is $\mathbf{A + B}$.
b) The rank of a $\mathbf{4 \times 5}$ matrix may be $\mathbf{5}$.
c) The rank of a $\mathbf{5 \times 4}$ matrix may be $\mathbf{5}$.
d) If $\mathbf{S}$ and $\mathbf{A}$ are invertible, then $(\mathbf{S^{-1}AS})^{\mathbf{-1}} = \mathbf{S^{-1}A^{-1}S}$.
e) If $\mathbf{A}$ and $\mathbf{B}$ are given $\mathbf{n \times n}$ matrices, then there is a unique $\mathbf{n \times n}$ matrix $\mathbf{X}$ satisfying $(\mathbf{A + X})\mathbf{B} = \mathbf{A}$ if $\mathbf{B}$ invertible.
f) It is possible that a system $\mathbf{Ax = b}$ has a unique solution for some $\mathbf{b}$ if $\mathbf{A}$ is a $\mathbf{4 \times 5}$ matrix.
g) If $\mathbf{A}$ is a $\mathbf{5 \times 5}$ matrix such that the system $\mathbf{Ax = 0}$ has only the trivial solution, then the system $\mathbf{Ax = b}$ is consistent for every $\mathbf{b} \in \mathbb{R}^{\mathbf{5}}$.
h) An $\mathbf{n \times n}$ matrix of rank $\mathbf{n}$ is invertible.
i) The system $\mathbf{Ax = 0}$ has only the trivial solution if and only if there are no free variables.
j) For any two $\mathbf{n \times n}$ matrices $\mathbf{A}$ and $\mathbf{B}$, $(\mathbf{A + B})^{\mathbf{2}} = \mathbf{A^2 + 2AB + B^2}$.
k) For a $\mathbf{4 \times 4}$ matrix $\mathbf{A}$, det(3$\mathbf{A}$)=3det($\mathbf{A}$).

Which of the statements above are true?
(Submit the corresponding number without parentheses.)

(1) b, c, d, f, g.
(2) a, b, d, i, k.
(3) a, c, d, e, h.
(4) d, e, g, h, i.
(5) b, c, d, g, h, j.
(6) a, c, d, f, g, k.
(7) a, b, d, h, j, k.
(8) a, b, c, e, g, h, k.
(9) a, c, d, e, h, i, j.
(10) b, d, e, f, h, j, k.

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4 If $\begin{vmatrix} \mathbf{yz} & \mathbf{x} & \mathbf{x^2} \\ \mathbf{zx} & \mathbf{y} & \mathbf{y^2} \\ \mathbf{xy} & \mathbf{z} & \mathbf{z^2} \end{vmatrix} = \mathbf{-5}$, find $\begin{vmatrix} \mathbf{1} & \mathbf{x^2} & \mathbf{x^3} \\ \mathbf{1} & \mathbf{y^2} & \mathbf{y^3} \\ \mathbf{1} & \mathbf{z^2} & \mathbf{z^3} \end{vmatrix}$.

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5 For which value of $\mathbf{a}$ is the solution set of the following system

$$\mathbf{x + 5y + z = b - 5}$$
$$\mathbf{2x - 4y - z = a + b - 3}$$

a subspace of $\mathbb{R}^{\mathbf{3}}$?

- Submissions through other platforms (e-mail, Ninova etc.) other than the link specified above will not be graded.
- Please be reminded that clicking on the "Save" button alone does not submit your answers. Make sure to click on "**Finish and send my answers**" button to submit. The form may be submitted only once.
- Please do not leave your submission to the last minute as there might be a system overload. If you are having trouble logging into the system, please try again later. Requests for deadline extension will not be considered.
- In the form, write only the final result as **number** and do not use any additional character such as space. Sample question: If $\mathbf{f(x) = x + 1}$, $\mathbf{f(0)}$ =? Correct answer: **1** Examples of answers that would be considered wrong: $\mathbf{f(0) = 1}$ / **1.0** / **1,0** / **01** / one ...