

# BLG 231E - Digital Circuits

## Assignment 3

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The function  $f(a, b, c, d)$  is given below:

$$f(a, b, c, d) = \sum 1(0, 2, 4, 5, 8, 10, 11, 13, 15) + \sum \Phi(1, 6)$$

1. Find the set of all prime implicants using the two different methods below:

a. A Karnaugh map. (20 points)

F		CD			
	F	00	01	11	10
AB	00	1	1	0	1
	01	1	1	0	1
	11	0	1	1	0
	10	1	0	1	1

Instead of “ $\Phi$ ” we write “1”

We will find set of all prime implicants.

F		CD			
	F	00	01	11	10
AB	00	1	1	0	1
	01	1	1	0	1
	11	0	1	1	0
	10	1	0	1	1

The Karnaugh map shows the function  $f(a, b, c, d)$  with prime implicants circled in blue, pink, yellow, and green. The blue circles represent the prime implicants  $\overline{a}\overline{b}$ ,  $\overline{a}b$ ,  $a\overline{b}$ , and  $a\overline{c}$ . The pink circle represents the prime implicant  $\overline{a}\overline{b}$ . The yellow circle represents the prime implicant  $\overline{a}\overline{b}$ . The green circle represents the prime implicant  $a\overline{c}$ .

### SET OF ALL PRIME IMPLICANTS

(0,4,5)  $a'c'$  // we assumed  $\phi$  to be 1 but now we eliminate them .

(0,2,4,)  $a'd'$  // We assumed  $\Phi$  to be 1 but now we eliminate them.

(0,2,8,10)  $b'd'$

(5,13)  $bc'd$

(11,15)  $acd$

(13,15)  $abd$

(11,10)  $ab'c$

**b.** The Quine-McCluskey method (Sort the minterms into groups according to the number of 1's in each term. Compare pairs of terms in adjacent groups and combine terms where possible. Check off terms which have been combined.). (30 points)

Num	abcd
0	0000 ✓
1	0001 ✓
2	0010 ✓
4	0100 ✓
8	1000 ✓
5	0101 ✓
6	0110 ✓
10	1010 ✓
11	1011 ✓
13	1101 ✓
15	1111 ✓

F		CD			
	F	00	01	11	10
AB	00	1	1	0	1
	01	1	1	0	1
	11	0	1	1	0
	10	1	0	1	1

Num	abcd
0,1	000- ✓
0,2	00-0 ✓
0,4	0-00 ✓
0,8	-000 ✓
1,5	0-01 ✓
2,6	0-10 ✓
2,10	-010 ✓
4,5	010- ✓
4,6	01-0 ✓
8,10	10-0 ✓
5,13	-101 ✗
10,11	101- ✗
11,15	1-11 ✗
13,15	11-1 ✗

Num	abcd
0,1,4,5	0 - 0 - <del>X</del>
0,2,4,6	0 - - 0 <del>X</del>
0,2,8,10	- 0 - 0 <del>X</del>
0,4,1,5	0 - 0 - same
0,4,2,6	0 - - 0 same
0,8,2,10	- 0 - 0 same

We are finding prime implicants for the terms which are not combined.

**Prime Implicants:**  $bc'd$  ,  $ab'c$  ,  $acd$  ,  $abd$  ,  $a'c'$  ,  $a'd'$  ,  $b'd'$

2. Construct the prime implicant chart using the cost criteria given below, then simplify the chart to identify all essential prime implicants. Show and explain each step of the simplification. Write out the expression for the function with the lowest cost, and give the total cost. (50 points)

**Cost criteria:** 2 units for each variable and 1 unit for each complement sign.

	$bc'd$	$ab'c$	$acd$	$abd$	$a'c'$	$a'd'$	$b'd'$
Symbols	A	B	C	D	E	F	G
Costs	7	7	6	6	6	6	6
Covered Points	5,13	10,11	11,15	13,15	0,1,4,5	0,2,4,6	0,2,8,10

“Don’ care”(Φ)(1,6) are ignored .

	0	2	4	5	8	10	11	13	15	Cost
A				X				X		7
B						X	X			7
C							X		X	6
D								X	X	6
E	X		X	X						6
F	X	X	X							6
G	X	X			<span style="border: 2px solid blue;">X</span>	X				6

G is essential. Rows and columns covered by G is removed. **Essential**

	4	5	11	13	15	Cost
A		X		X		7
B			X			7
C			X		X	6
D				X	X	6
E	X	X				6
F	X					6

B is covered by C and the cost of C is cheaper so B is not essential.  
 F is covered by E and the cost of E is same so F is not essential.

	4	5	11	13	15	Cost
A		X		X		7
C			X		X	6
D				X	X	6
E	X	X				6

E is essential. Rows and columns covered by E is removed.  
 C is essential. Rows and columns covered by C is removed.

	13	Cost
A	X	7
D	X	6

Since D is cheaper we choose D.

Cheapest set of prime implicants

$G+C+E+D: 6+6+6+6=24$

Cheapest expression :  $acd + abd + a'c' + b'd'$

F		CD			
	F	00	01	11	10
A	00	1	0	1	
	01	1	1	0	
	11	0	1	1	0
	10	1	0	1	1