1) Find the (2, 3)-entry of the product

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 2 & 5 & 1 \\ 4 & -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 5 & 1 & 0 \\ 0 & 4 & 3 \end{bmatrix}$$

A. 4
B. 5
C. 6
D. 7
E. 8

2) Find a matrix A such that $AB = C$, where

$$B = \begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 8 & 4 \\ -3 & -1 \end{pmatrix}.$$

3) Solve the following equation for A:

$$A^t - [1\ 0\ 0]^t[0\ 1] = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$$

A. $\begin{bmatrix} 0 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 2 & 3 \\ 4 & 4 & 6 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 6 \end{bmatrix}$ E. $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

4) Find a matrix A such that $\left(2A^T + \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}\right)^T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

and give its first row.

A. $(2, -1)$

B. $(0, 0)$

C. $(-1/2, 1/2)$

D. $(0, 1/2)$

E. $(1/2, 0)$

5) Let $A = \begin{pmatrix} 3 & 1 & 0 \\ 2 & 3 & -1 \\ 0 & 2 & -1 \end{pmatrix}$. Then the main diagonal of A^{-1} is:

A. $1, 3, -7.$

B. $-1, -3, -6.$

C. $1, -3, -7.$

D. $-1, 3, -6.$

E. $-1, -3, -7.$

6) Find the main diagonal of the inverse of $\begin{bmatrix} 1 & -2 & -3 \\ -2 & 2 & 4 \\ -3 & 0 & 2 \end{bmatrix}$.

A. $(2,\ -7/2,\ -1)$
B. $(5/2,\ 7/2,\ 3/2)$
C. $(2,\ 1,\ -1)$
D. $(-1,\ -7/2,\ 3)$
E. $(7/2,\ 2,\ -1)$

7) Determine for which value(s) of t the matrix $\begin{pmatrix} 1 & 2 & -1 \\ 2 & 0 & t \\ 0 & 1 & 1 \end{pmatrix}$ is invertible.

- ☐ $t \neq 3.$
- ☐ $t \neq -6.$
- ☐ $t = -1.$
- ☐ $t = -6.$
- ☐ $t = 3.$

8) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 9$, find $\begin{vmatrix} 3a-5g & g & d \\ 3b-5h & h & e \\ 3c-5i & i & f \end{vmatrix}$:

A. -45
B. 45
C. -27
D. 9
E. 27

9) Let A and B be two matrices such that $\det A = -2$ and $\det(B^t) = 3$. Find $\det(AB)$.

A. 6
B. 1
C. 5
D. -5
E. -6

10) Find a complex number z such that $\begin{vmatrix} 1 & 1-i & -i \\ 0 & z & 1+i \\ 0 & 0 & 1+i \end{vmatrix} = 3+i.$

A. $z = -i$

B. $z = 2i$

C. $z = 2 - i$

D. $z = 1 - i$

E. $z = 2 + 2i$

11) Let A, B, C be square invertible matrices satisfying $AB = B^2C$. Assume that $\det B = 3$ and $\det C = 2$. Find a formula for A and calculate the determinant of A.

A. $A = BC, \quad \det A = 6.$

B. $A = B^3C, \quad \det A = 11.$

C. $A = B^2CB^{-1}, \quad \det A = 6.$

D. $A = B^2CB^{-1}, \quad \det A = 5.$

E. $A = BC, \quad \det A = 5.$

12) Find scalars $a, b, c \in \mathbb{R}$ such that $au_1 + bu_2 + cu_3 = w$, where

$$u_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad u_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad w = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}.$$

Answer: $a =$ ☐ $b =$ ☐ $c =$ ☐

13) For which values of a and b does the system

$$\begin{cases} -x + 3y + 2z = -8 \\ x + z = 2 \\ 2x + 2y + az = b \end{cases}$$

have more than one solution?

A. if $a = -4$ and $b \neq 0$.
B. if $a \neq -4$ and $b \neq 0$.
C. if $a = 4$ and $b = 0$.
D. if $a \neq 4$ and $b \neq 0$.
E. if $a = 4$ and $b \neq 0$.

14) For a non-homogeneous system of 12 equations in 15 unknowns, answer the following three questions:

- Can the system be inconsistent?
- Can the system have infinitely many solutions?
- Can the system have a unique solution?

A. No, Yes, No.
B. Yes, Yes, Yes.
C. Yes, Yes, No.
D. No, No, No.
E. Yes, No, Yes.

15) Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & t & 0 \\ 1 & 0 & -1 \end{pmatrix}$. Find the set of values of t for which the homogeneous system of linear equations $AX = 0$ has a non-trivial solution.

A. $t = -3$

B. $t \neq 2$

C. $t \neq -3$

D. $t \neq 1$ and $t \neq 3$

E. $t = 2$

16) Given a non-homogeneous system of 5 equations in 7 unknowns, answer by yes or no the following three questions and indicate which combination of answers is right.

- Can the system have no solution?
- Can the system have infinitely many solutions?
- Can the system have a unique solution?

A. No, No, No.
B. Yes, Yes, Yes.
C. No, No, Yes.
D. Yes, Yes, No.
E. No, Yes, Yes.

17) If the coefficient matrix A in a homogeneous system of 12 equations in 16 unknowns is known to have rank 5, how many parameters are there in the general solution?

A. 10
B. 7
C. 11
D. none
E. 12

18) Determine whether or not W is a subspace of $\mathbb{R}^3$ where W consists of all vectors (a, b, c) in $\mathbb{R}^3$ such that:

(a)	$b = a^2$	☐ yes	☐ no
(b)	$a = 2b = 3c$	☐ yes	☐ no
(c)	$a = 3b$	☐ yes	☐ no
(d)	$ab = 0$	☐ yes	☐ no
(e)	$a \leqslant b \leqslant c$	☐ yes	☐ no
(f)	$a + b + c = 0$	☐ yes	☐ no

19) Find all values of c so that $\{(2, -1, 3), (0, c, 2), (8, -1, 8)\}$ is linearly independent.

A. $c \neq \pm 3/2$
B. $c = 0$
C. $c = 3/2$
D. $c \neq -3/2$
E. $c > 0$

20) What is the dimension of the subspace of $\mathbf{R}^3$ spanned by $(1,\ 1,\ 1)$, $(-1,\ 1,\ -1)$, $(1,\ 1,\ 3)$ and $(0,\ 2,\ 1)$?

A. 0
B. 1
C. 2
D. 3
E. 4

21) Let V be an n-dimensional vector space. True or false:

(a) If the vectors $v_1, \ldots, v_m$ span V, then $m < n$. ☐ true ☐ false

(b) Any n vectors which span V are linearly independent. ☐ true ☐ false

(c) Every set of n vectors in V is linearly independent. ☐ true ☐ false

(d) V has a basis consisting of n elements. ☐ true ☐ false

(e) V is spanned by $n-1$ or fewer vectors. ☐ true ☐ false

(f) Any $n+1$ or more vectors in V are linearly dependent. ☐ true ☐ false

22) Let V be a vector space . Which of the following statements are always valid:

(a) Every subset of V is a subspace of V ☐ true ☐ false

(b) Every subspace of V is a subset of V ☐ true ☐ false

(c) $\{0\}$ is a subspace of V ☐ true ☐ false

(d) Let $u, v \in V$ be vectors, and let W be a subspace of V. If W contains the vectors u and v, then W also contains the sum $u+v$. ☐ true ☐ false

23) Which of the following statements are true?

(1) Each spanning set for $\mathbf{R}^n$ has exactly n vectors.
(2) If $\{u, v, w\}$ is linearly independent, then $\{u, v\}$ is also linearly independent.
(3) If A is an $n \times n$ matrix, then $\det A = (-1)^n \det(A^t)$.
(4) If A is an $n \times n$ matrix, then $\dim \operatorname{col} A = n$.
(5) If A is an $n \times n$ matrix, the $\dim \text{Null} A = n - \operatorname{rank} A$.
(6) The set of $n \times n$ diagonal matrices is a subspace of the vector space of of all $n \times n$ matrices.

A. All six are true.
B. (2), (5) and (6).
C. (1), (2) and (4).
D. (3), (2) and (6).
E. (4), (5) and (6).

Let A be an 8×6 matrix such that $Ax = 0$ has only the trivial solution $x = 0$.

24)

- What is the rank of A?
- Do the columns of A span $\mathbf{R}^8$?

A. 0, Yes
B. 6, Yes
C. 6, No
D. 8, Yes
E. 8, No

25) . For which values of a does the matrix $\begin{pmatrix} 1 & -a & 2 \\ 0 & 1 & -2 \\ 2 & 1 & a \end{pmatrix}$ have rank 2?

A. $a = -3/2$ and $a = 1$.

B. $a = 2/5$.

C. No value of a.

D. $a = 3/4$ and $a = -1/2$.

E. $a = -4/3$.

26) A basis for the solution space of the system

$$\begin{aligned} u - 2x + 3y + 4z &= 0 \\ -2u + 4x - 5y - 6z &= 0 \end{aligned}$$ is:

A. { (0, 0, 0, 0) }
B. { (2, 1, 0, 0), (2, 0, -2, 1) }
C. { (1, 2, 0, 0) }
D. { (2, 0, -2, 1) }
E. { (2, 1, 0, 0), (1, -3, -4, 1) }

27) Let $A = \begin{bmatrix} 1 & 1 & 0 & -5 \\ 2 & 1 & 3 & 2 \\ 3 & 1 & 3 & 4 \\ 1 & 0 & 0 & 2 \end{bmatrix}$. The dimension of the solution-space of $Ax = 0$ is:

A. 1
B. 2
C. 4
D. 0
E. 3

28) Consider the following matrix: $$B = \begin{pmatrix} 1 & 5 & 3 & 2 & -1 \\ 4 & -2 & 0 & 1 & 2 \\ 3 & -1 & 1 & 2 & 1 \\ 2 & 6 & 4 & 3 & -1 \\ 4 & 0 & 2 & 3 & 1 \end{pmatrix}$$

(a) What is the rank of B? (b) Find a basis of the Col(B) and Row(B)

29) The eigenvalues of the matrix $\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & -1 \\ 0 & 2 & -3 \end{bmatrix}$ are:

A. 2, 3, 4
B. -3, 3, 4
C. 0, 1, 3
D. -3, 0, 4
E. -1, -2, 1

30) Suppose that a given matrix A satisfies $A^2 - 2A - I = 0$. Give a formula for A^{-1}:

- ☐ $A^{-1} = 2A + I$.
- ☐ $A^{-1} = A + I$.
- ☐ $A^{-1} = A - 2I$.
- ☐ $A^{-1} = 2A - I$.
- ☐ $A^{-1} = A + 2I$.

31) Let $A = \begin{pmatrix} 0 & 2 & -1 \\ -2 & 8 & -5 \\ -3 & 10 & -7 \end{pmatrix}$, $X = \begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}$, $Y = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$. Which of the following statements is true?

A. Y is an eigenvector of A with the eigenvalue 2.

B. Y is an eigenvector of A with the eigenvalue -2.

C. Y is an eigenvector of A with the eigenvalue 3.

D. X is an eigenvector of A with the eigenvalue 3.

E. X is an eigenvector of A with the eigenvalue -2.