EHB 211E
Basics of Electrical Circuits

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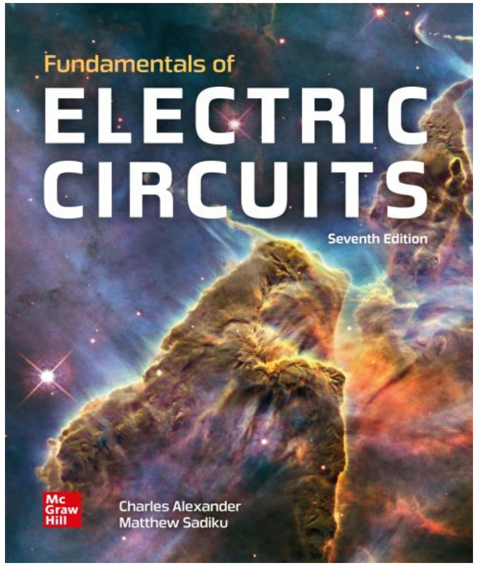
Fundamental Concepts



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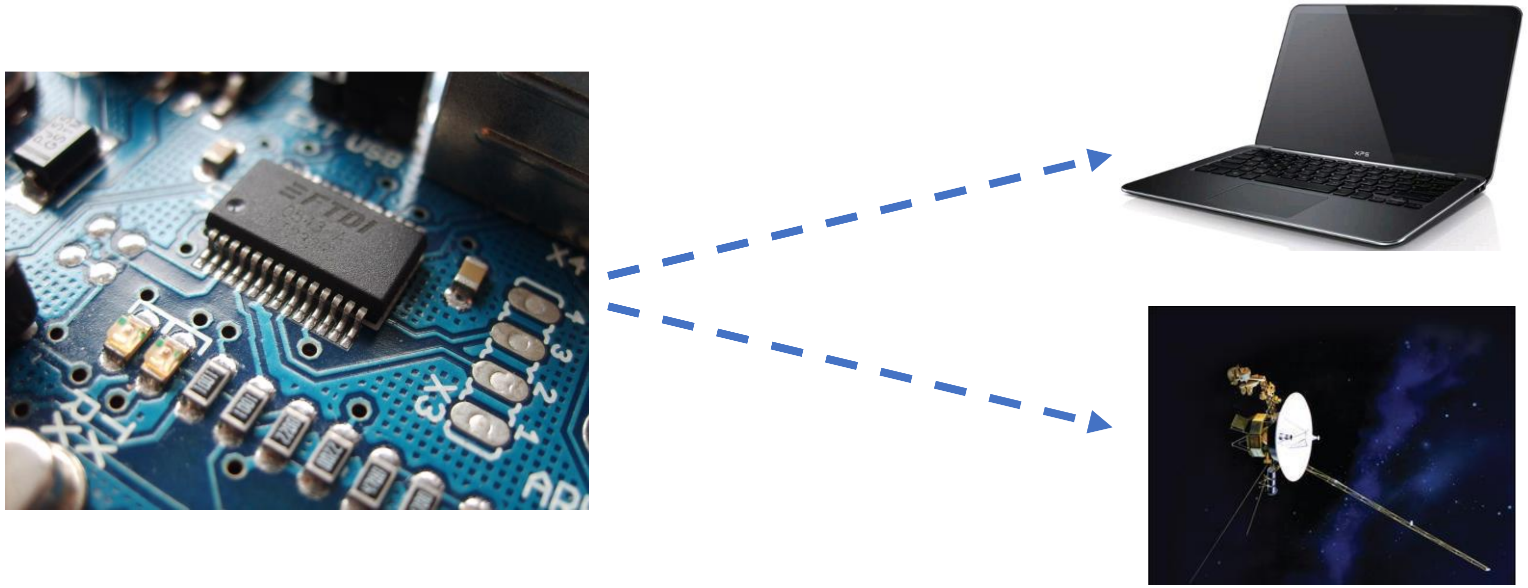
Fundamentals of Electric Circuits by C. Alexander and M. Sadiku, Seventh Edition (McGraw-Hill Education)

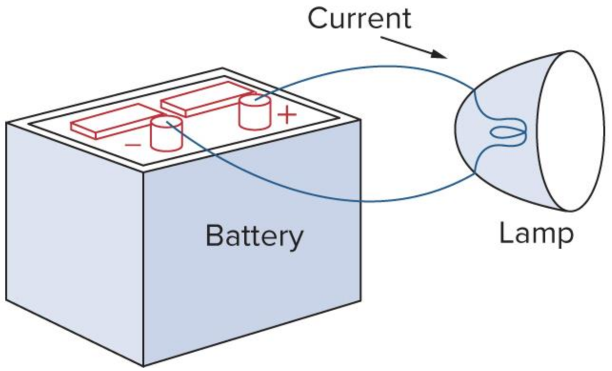
Introduction

- Many branches of electrical engineering, such as power, electric machines, control, electronics, and communications, are based on electric circuit theory.
- Electric circuit theory: Analyzing and designing circuits
- Electrical circuits are present almost everywhere: Computers, TV, Telecommunication systems, and so on.
- Understanding fundamentals → Complex systems → Superior devices
- The purpose of the circuit theory: Predict the electrical behavior of circuits.

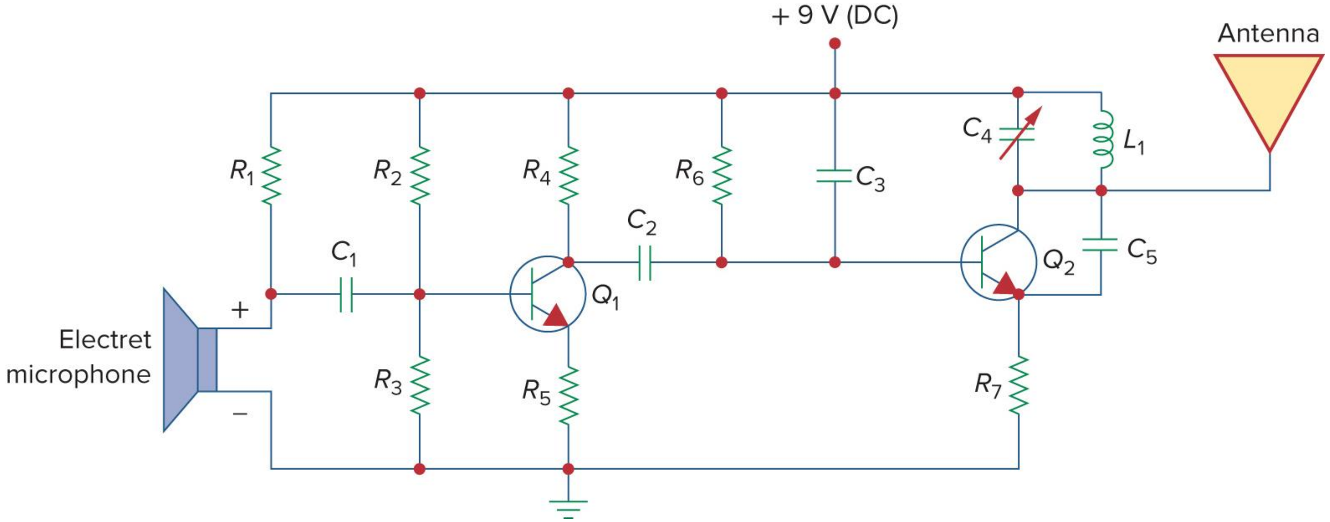


Introduction

- What is an electric (physical) circuit?
  - Interconnection of electrical elements (devices).
  - Electrical elements: Resistors, Capacitors, Diodes, etc.
  - Communicating or transferring energy from one point to another.



A simple electric circuit: a battery, a lamp, and connecting wires



A Complicated real electric circuit: a radio transmitter

- Our goal is to describe the behavior of a circuit as shown above.

Systems of Units

- Different types of system of units: British System of units (F.P.S.), Metric system of units (C.G.S. & M.K.S.), and international system of unit (SI).
- Using international system of unit (SI): Accepted system of units in engineering, science, and industries. It's been adopted since 1960.

Six basic SI units and one derived unit relevant to this text.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Charge	coulomb	C

The SI prefixes.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Charge and Current

- What is charge?
  - Electrical property of the atomic particles of which matter consists, measured in coulombs (C).

- All mater is made of fundamental building blocks known as atoms.

- Atom: electrons, protons, and neutrons.
  - Charge of an electron $= -1.602 \times 10^{-19}$ C
  - Charge of a proton $= 1.602 \times 10^{-19}$ C
  - Neutron = Zero charge (Neutral particles)

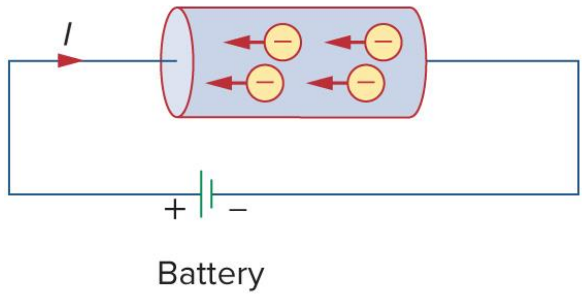
- The following point should be noted about electric charge:
  - In 1 C of charge: $1/(1.602 \times 10^{-19}$ C$) = 6.24 \times 10^{18}$ electrons
  - The law of conservation of charge: Charge can neither be created nor destroyed, only transferred. The algebraic sum of the electric charges in a system does not change.

Charge and Current

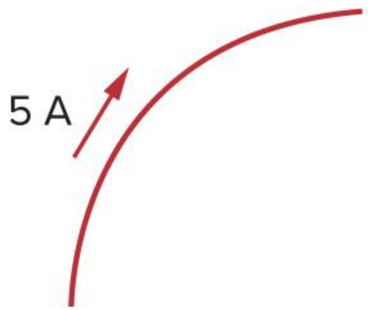
- Unique feature of electric charge or electricity: It is mobile, i.e.,
  - Transfer from one place to another
  - Converted to another form of energy

- When wire is connected to a battery (electromotive force, emf), charges move:
  - Positive charges move in one direction
  - Negative charges move in the opposite direction to create electric current

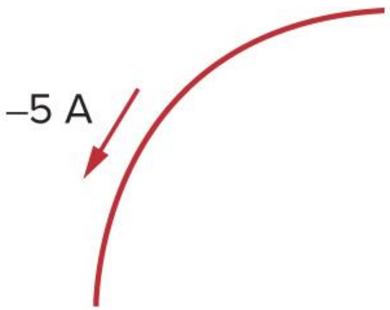
- ❑ Conventionally, direction of current is taken as the movement of positive charges
- ❑ In fact, Electrons move through the wire.



Electric current due to flow of electronic charge in a conductor



Positive current flow



Negative current flow

Charge and Current

- By definition:
  - Electric current is the time rate of change of charge, measured in amperes (A). 1 A = 1 C / sec

- Mathematically, the relationship between current i, charge q, and time t:

$$\boxed{i \triangleq \frac{dq}{dt}}$$

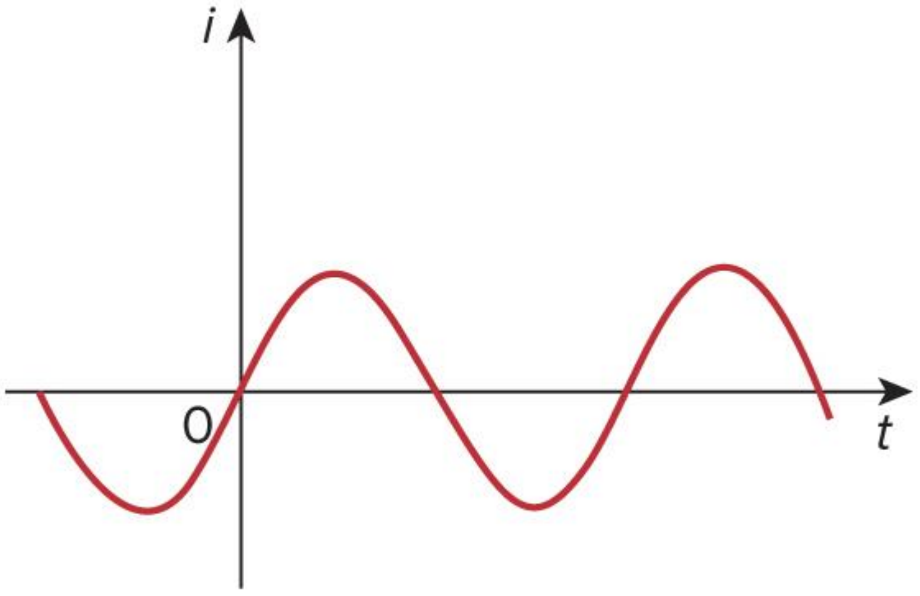
- The charge transferred between time t_0 to t:

$$\boxed{Q \triangleq \int_{t_0}^{t} i dt}$$

Charge and Current

- Several types of current:

Direct current (dc): Current is constant.
Example: Battery



Alternating current (ac): Current varies sinusoidally with time.
Example: Current is used in household to run the electric appliances such as refrigerator, TV, etc.

Example 1:

How much charge is represented by 4,600 electrons?

Solution:

Each electron has -1.602×10^{-19} C. Hence 4,600 electrons will have

$$-1.602 \times 10^{-19}\ \text{C/electron} \times 4{,}600\ \text{electrons} = -7.369 \times 10^{-16}\ \text{C}$$

Example 2:

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

Recall:

Product rule: $f(x)g(x) \Rightarrow \frac{d[f(x)g(x)]}{dx} \Rightarrow \frac{d[f(x)]}{dx}g(x) + f(x)\frac{d[g(x)]}{dx}$

Chain rule: $[f(g(x)] \Rightarrow \frac{d}{dx}[f(g(x)] \Rightarrow f'\big(g(x)\big)g'(x)$

$\frac{d}{dx}[\sin(x)] \Rightarrow \cos(\mathrm{x})$ & $\frac{d}{dx}[\cos(x)] \Rightarrow -sin(\mathrm{x})$

Example 3:

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution:

$$Q = \int_{t=1}^{2} i\,dt = \int_{1}^{2} (3t^2 - t)\,dt$$

$$= \left(t^3 - \frac{t^2}{2}\right)\Big|_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5 \text{ C}$$

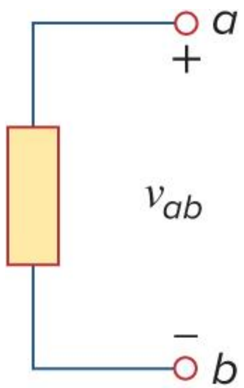
Recall:

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\int a dx = ax$$

Voltage

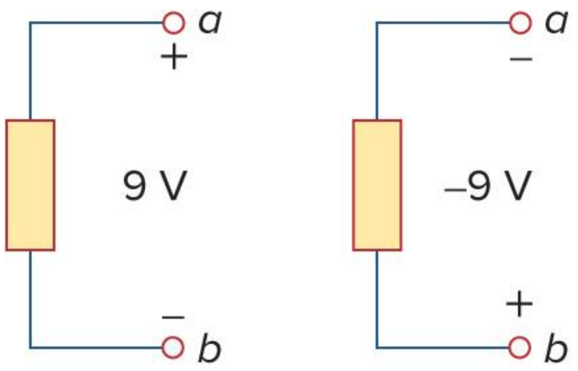
- Work or energy is required to move electrons in a conductor
- Work performed by an external electromotive force (emf), represented by battery
- Voltage (potential difference or emf) is the energy required to move a unit charge through an element, measured in volts (V).



Voltage v_{ab} between point a and b is the energy needed to move a unit charge from a to b.

- Mathematically, $v_{ab} \triangleq \dfrac{d\omega}{dq}$

where ω is energy (J) and q is charge (C).



$$v_{ab} = -v_{ba}$$

- Note that 1 volt = 1 joule/coulomb = 1 newton-meter/coulomb

Power and Energy

- What is power?
  - Time rate of expending or absorbing energy. Unit of power is Watt (W).
- What is energy?
  - Capacity to do work, measured in joules (J)
- Mathematically, $\boxed{p \triangleq \dfrac{d\omega}{dt}}$

where p is power (W), ω is energy (J), and t is time (s).

- Power can be written in terms of voltage and current as:

$$p \triangleq \frac{d\omega}{dt} = \underbrace{\frac{d\omega}{dq}}_{v} \cdot \underbrace{\frac{dq}{dt}}_{i} = vi \Rightarrow \boxed{\mathrm{p} = vi}$$

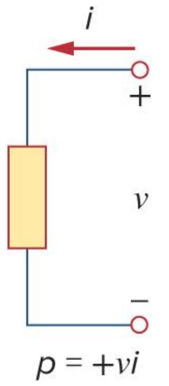
1 Wh=3600 J, where Wh is watt-hours

- The energy absorbed or supplied by an element from time t_0 to time t:

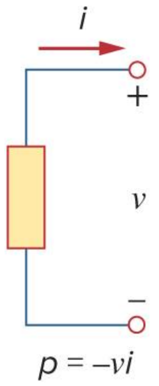
$$\omega = \int_{t_0}^{t} p\, dt = \int_{t_0}^{t} vi\, dt$$

Power and Energy

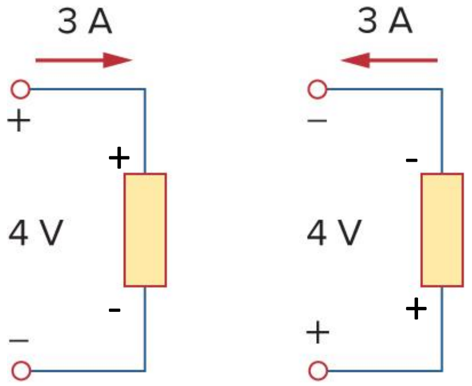
- Power can be absorbed or supplied by an element in a circuit
  - Power is positive if power is delivered to or absorbed by an element.
  - Power is negative if power is supplied by an element.
  - Current direction and voltage polarity play a major role in determining the sign of power



Absorbing power (+)

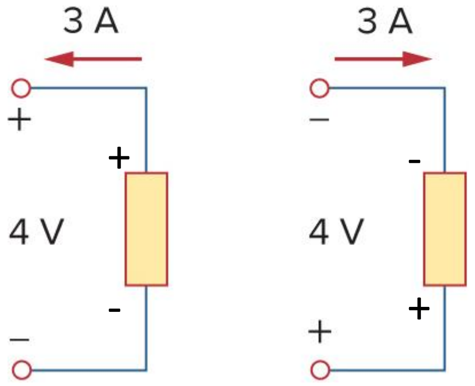


Supplying power (-)



Positive current enters the positive terminal.

$p = 4 \times 3 = 12W$



Positive current enters the negative terminal

$p = -4 \times 3 = -12W$

- Law of conservation of energy (Power): the algebraic sum of power in any circuit must be zero.
- +power absorbed = - power supplied

$\sum p = 0$ (Tellegen’s theorem)

Example 4:

An energy source forces a constant current of 2 A for 10 s to flow through a light bulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:

The total charge is

$$\Delta q = i\,\Delta t = 2 \times 10 = 20\text{ C}$$

The voltage drop is

$$v = \frac{\Delta w}{\Delta q} = \frac{2.3 \times 10^3}{20} = 115\text{ V}$$

Alternatively,

By definition: $p \triangleq \frac{dw}{dt} => dw = p \times dt => \int dw = \int p.dt => \; w = \int_{to}^{t} P.dt$

$w = \int_{to}^{t} P.dt => 2.3 \times 10^3 = \int_0^{10} P.dt => 2.3 \times 10^3 = p.t|_0^{10} => 2.3 \times 10^3 = (10p - 0)$

$\Rightarrow p = 230\,W$

Since $p = vi$, then $230 = v \times 2 => v = 115\text{V}$

Example 5:

Find the power delivered to an element at $t = 3$ ms if the current entering its positive terminal is

$$i = 5 \cos 60\pi t \text{ A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3\, di/dt$.

Solution:

(a) The voltage is $v = 3i = 15 \cos 60\pi t$; hence, the power is

$$p = vi = 75 \cos^2 60\pi t \text{ W}$$

At $t = 3$ ms,

$$p = 75 \cos^2 (60\pi \times 3 \times 10^{-3}) = 75 \cos^2 0.18\pi = 53.48 \text{ W}$$

(b) We find the voltage and the power as

$$v = 3\frac{di}{dt} = 3(-60\pi)5 \sin 60\pi t = -900\pi \sin 60\pi t \text{ V}$$

$$p = vi = -4500\pi \sin 60\pi t \cos 60\pi t \text{ W}$$

At $t = 3$ ms,

$$\begin{aligned} p &= -4500\pi \sin 0.18\pi \cos 0.18\pi \text{ W} \\ &= -14137.167 \sin 32.4^\circ \cos 32.4^\circ = -6.396 \text{ kW} \end{aligned}$$

Recall:

$$\frac{d(cosx)}{dx} = -sinx$$

$$\frac{d(\cos(ax))}{dx} = -a\sin(ax)$$

Example 6:

How much energy does a 100-W electric bulb consume in two hours?

Solution:

We know that $1\, Wh = 3600\, J, then\ 200\, Wh\ (100\, Wh\ \times 2h) = 720\, kJ$

This is same as:

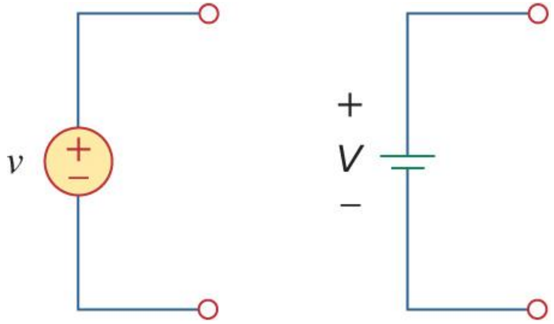
By definition, $p = \frac{w}{t}$

$\Rightarrow w = pt \Rightarrow 100w \times 2h = 200\, Wh$

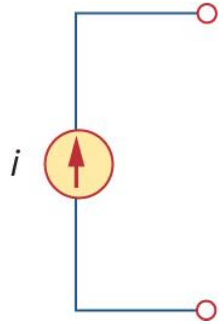
Circuit Elements

- Element: basic building block of a circuit
- Two types of elements in electrical circuits:
  - Passive elements: cannot generate energy. Example: resistors, capacitors, and inductors
  - Active elements: capable of generating energy. Example: batteries, generators, and op amps
- Most important active elements: voltage and current sources
- Two kinds of sources:
  - Ideal independent source: active element that provides a specific voltage or current that is completely independent of other circuit elements
  - Ideal dependent (controlled) source: active element in which the source is controlled by another voltage or current

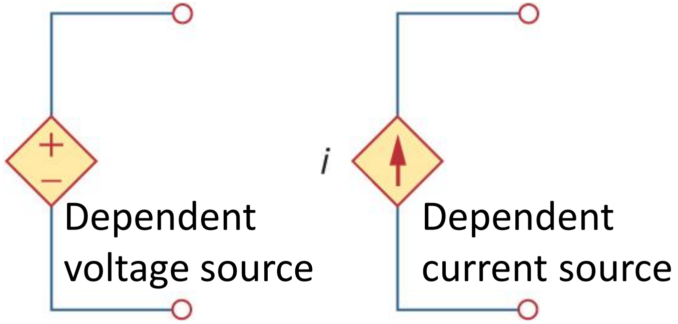
Symbol of independent voltage sources



Symbol of independent current sources



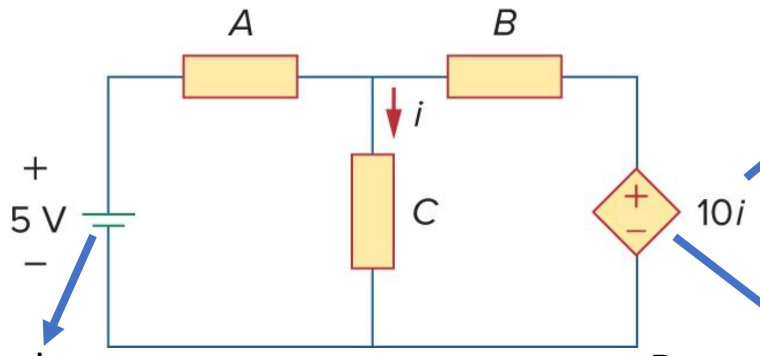
Symbol of dependent sources (Diamond shape)



Circuit Elements

- Four types of dependent sources:
  - A voltage-controlled voltage source (VCVS)
  - A current-controlled voltage source (CCVS)
  - A voltage-controlled current source (VCCS)
  - A current-controlled current source (CCCS)
- Dependent sources are useful in modelling elements such as transistors, operational amplifiers (op amps), and integrated circuits.

Example of a current-controlled voltage source



Independent voltage source

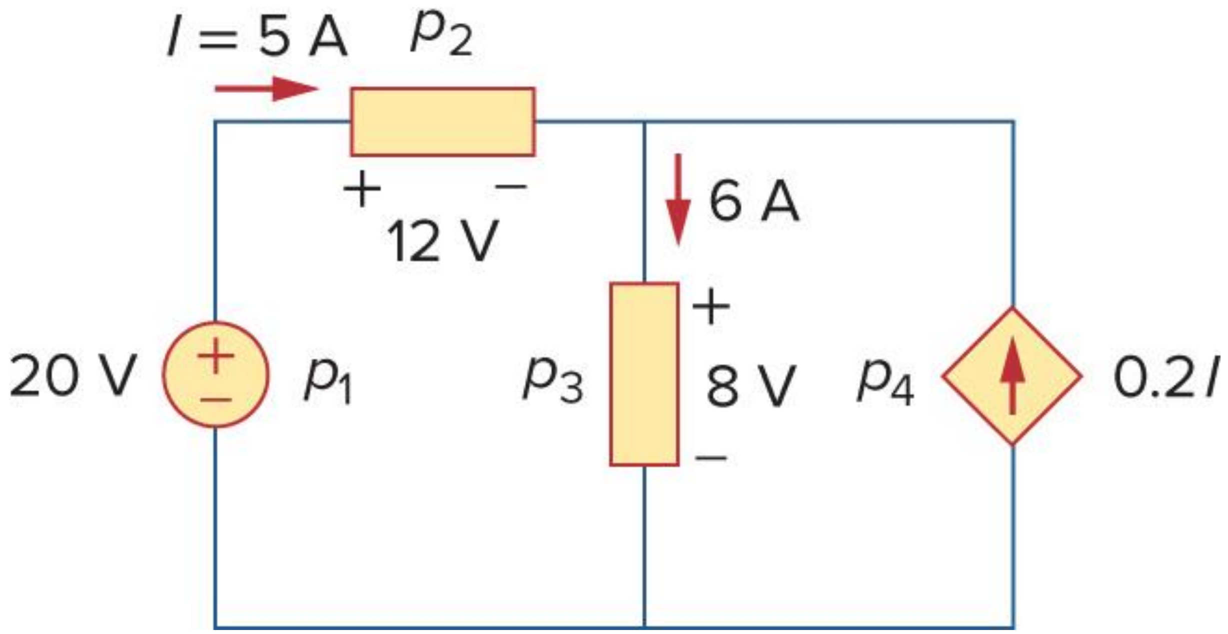
Dependent voltage source (diamond shape)

Voltage 10i of the dependent voltage source depends on the current i through element C

A voltage source→ polarities (+ -)
A current source→ arrow

Example 7:

Calculate the power supplied or absorbed by each element in the figure shown below.



Solution:

- Recall:
  - If power is negative(-), power is supplied by the element.
  - If power is positive (+), power is absorbed by the element.
  - Current direction and voltage polarity play a major role in determining the sign of power

For p_1, the 5-A current is out of the positive terminal (or into the negative terminal); hence,

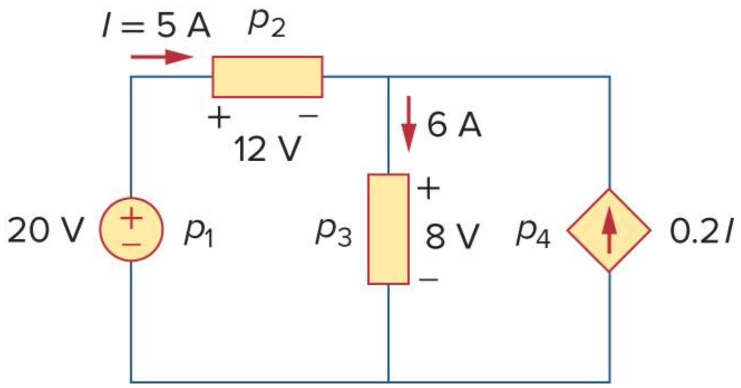
$$p_1 = 20(-5) = -100 \text{ W} \qquad \text{Supplied power}$$

For p_2 and p_3, the current flows into the positive terminal of the element in each case.

$$p_2 = 12(5) = 60 \text{ W} \qquad \text{Absorbed power}$$
$$p_3 = 8(6) = 48 \text{ W} \qquad \text{Absorbed power}$$

For p_4, we should note that the voltage is 8 V (positive at the top), the same as the voltage for p_3, since both the passive element and the dependent source are connected to the same terminals. (Remember that voltage is always measured across an element in a circuit.) Since the current flows out of the positive terminal,

$$p_4 = 8(-0.2I) = 8(-0.2 \times 5) = -8 \text{ W} \qquad \text{Supplied power}$$



$$\boxed{\sum p = 0 \text{ (Tellegen's theorem)}}$$

$$p_1 + p_2 + p_3 + p_4 = -100 + 60 + 48 - 8 = 0$$