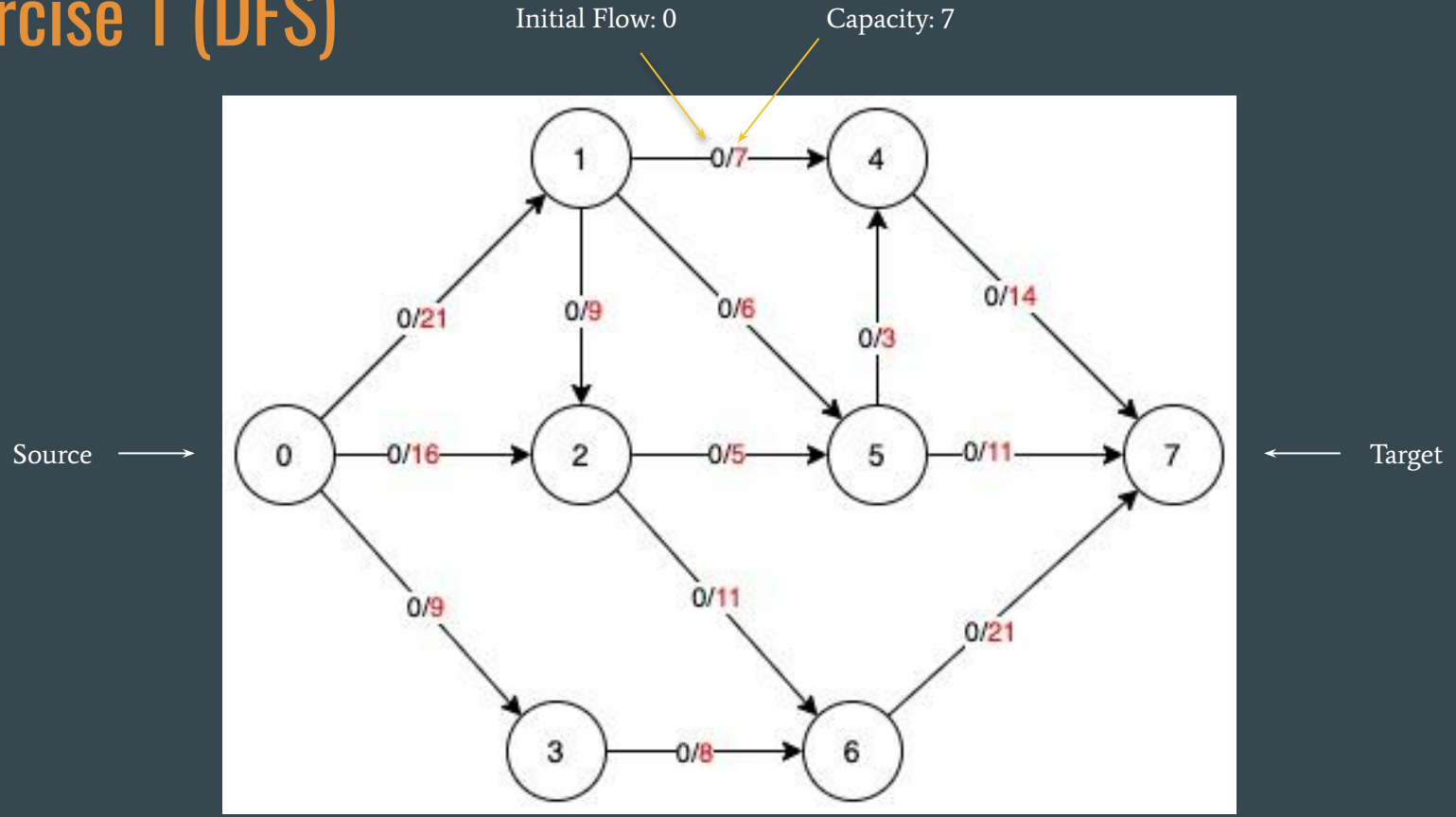
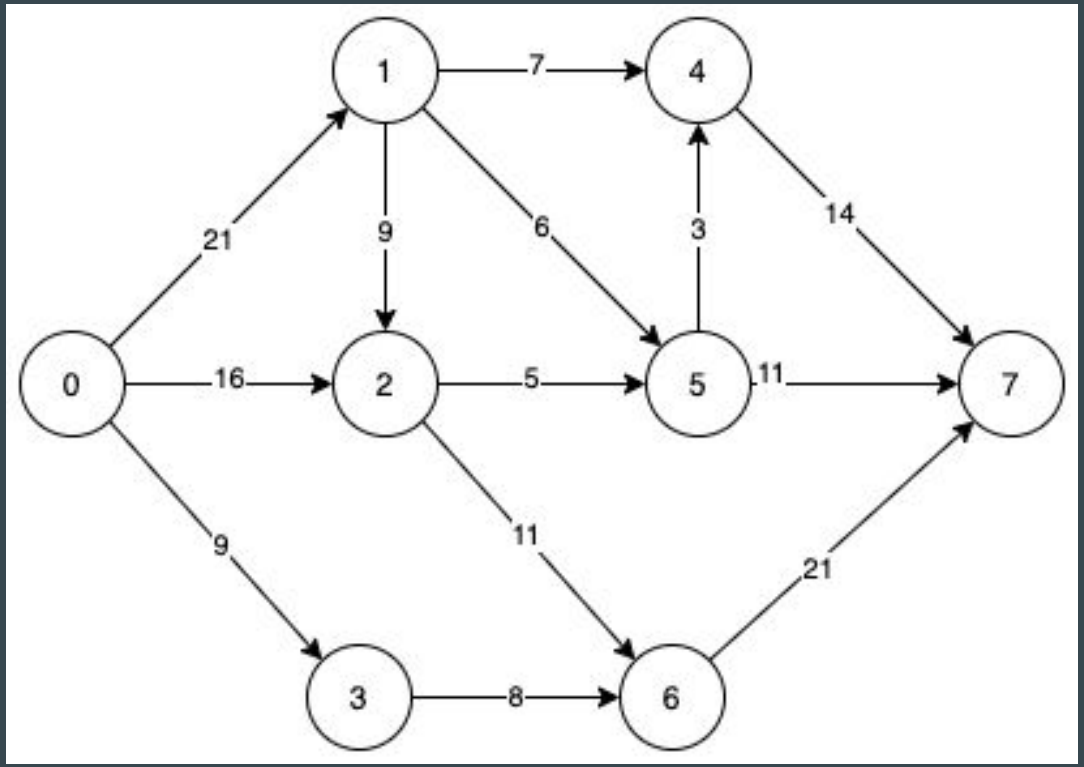
Ford-Fulkerson
Max-Flow Min-Cut
Maximum Bipartite Matching

BLG336E Spring 2022 - Recitation 10
Caner Özer - ozerc@itu.edu.tr

Exercise 1 (DFS)

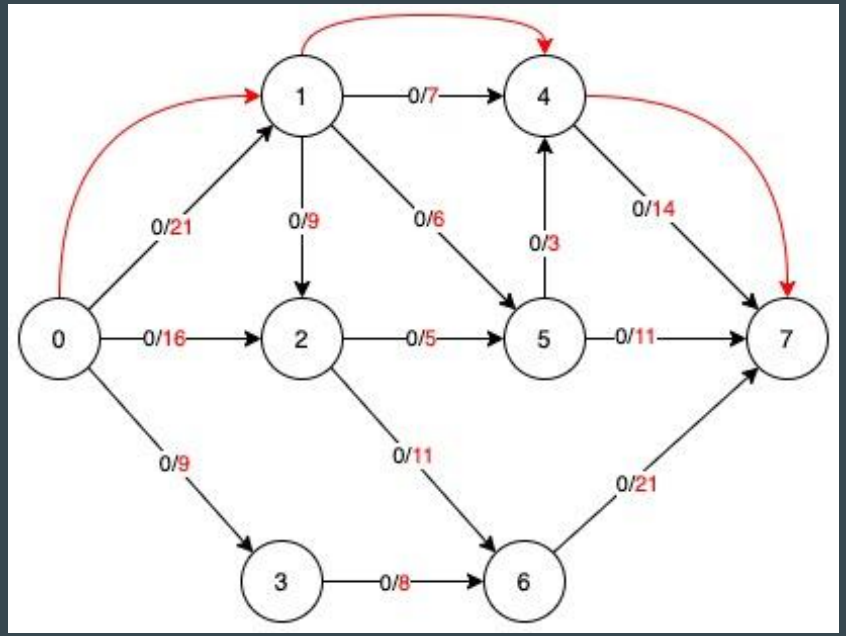


Initial Residual Graph of Exercise 1

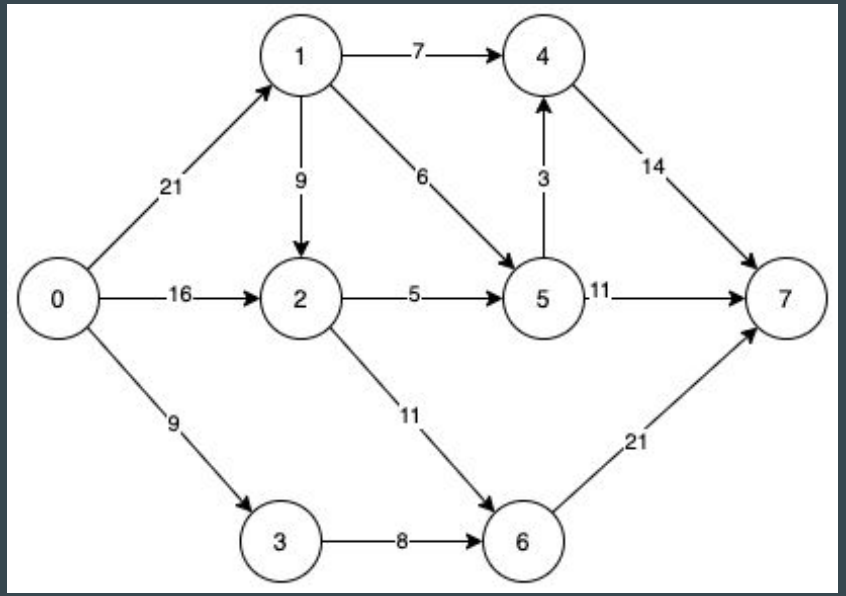


Exercise 1

Search Path: 0-1-4-7



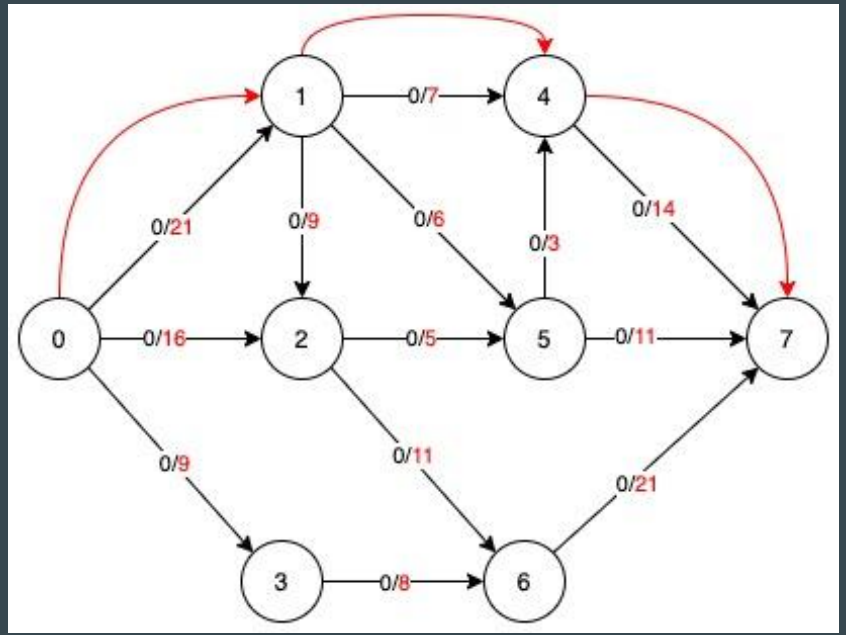
Flow Graph



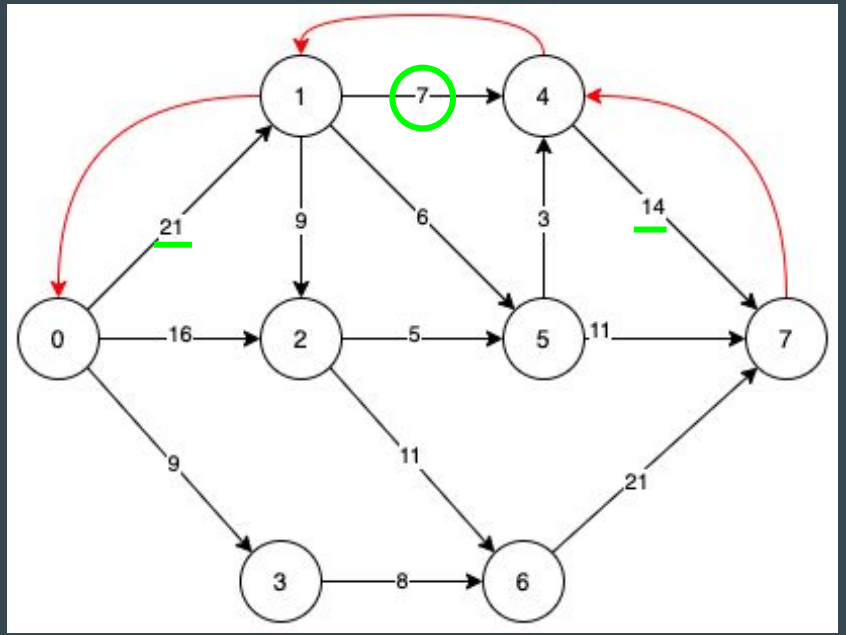
Residual Graph

Exercise 1

Min capacity of the path: 7



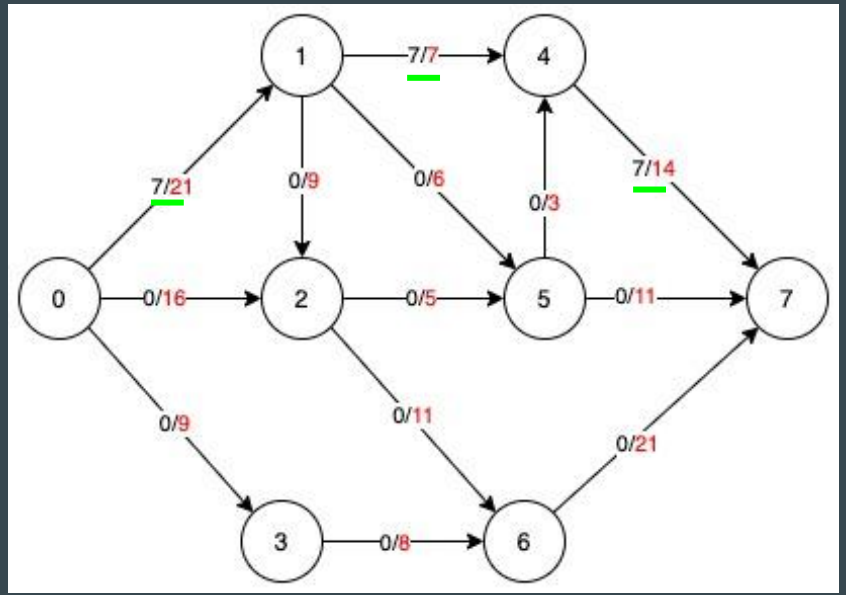
Flow Graph



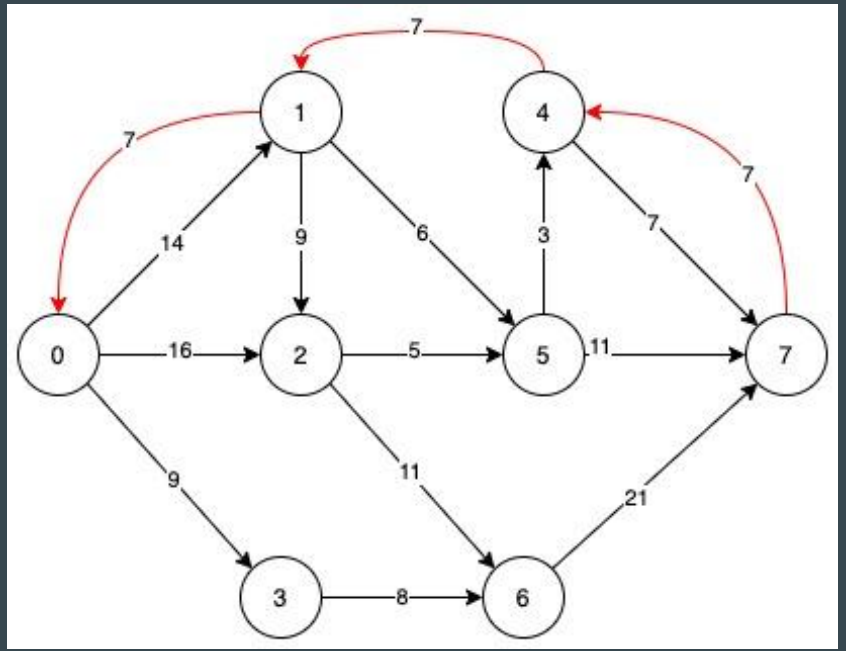
Residual Graph

Exercise 1

Update flow and residual graphs and set MaxFlow=7.



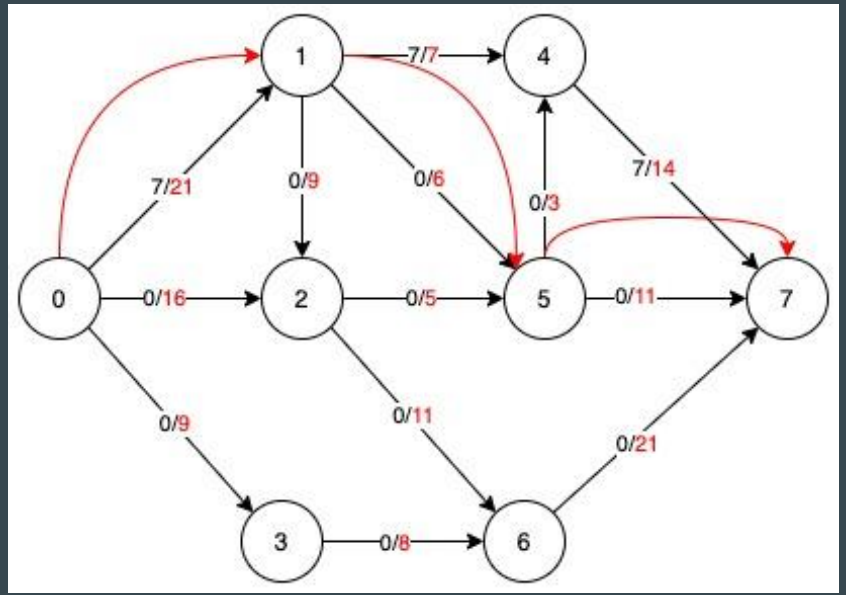
Flow Graph



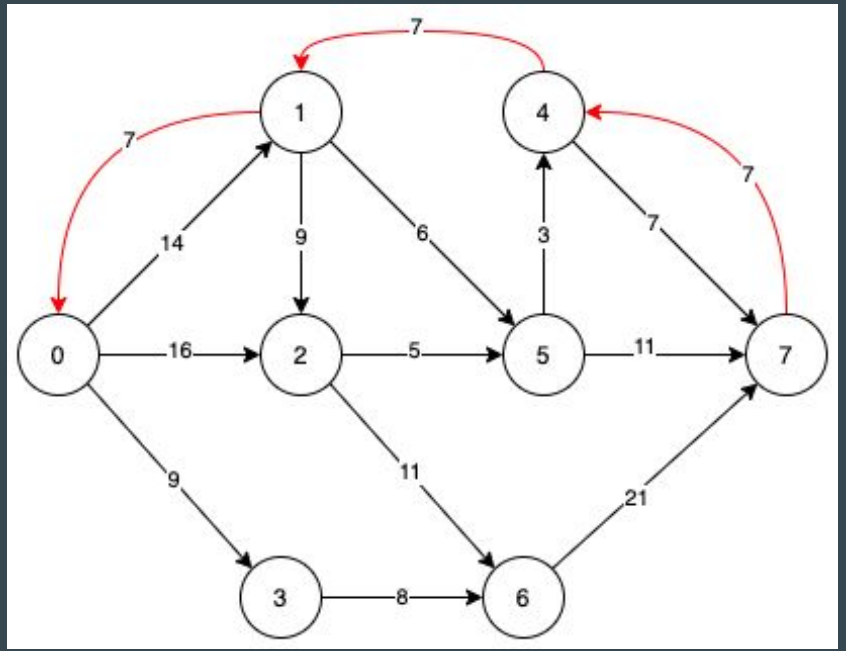
Residual Graph

Exercise 1

Search Path: 0-1-5-7



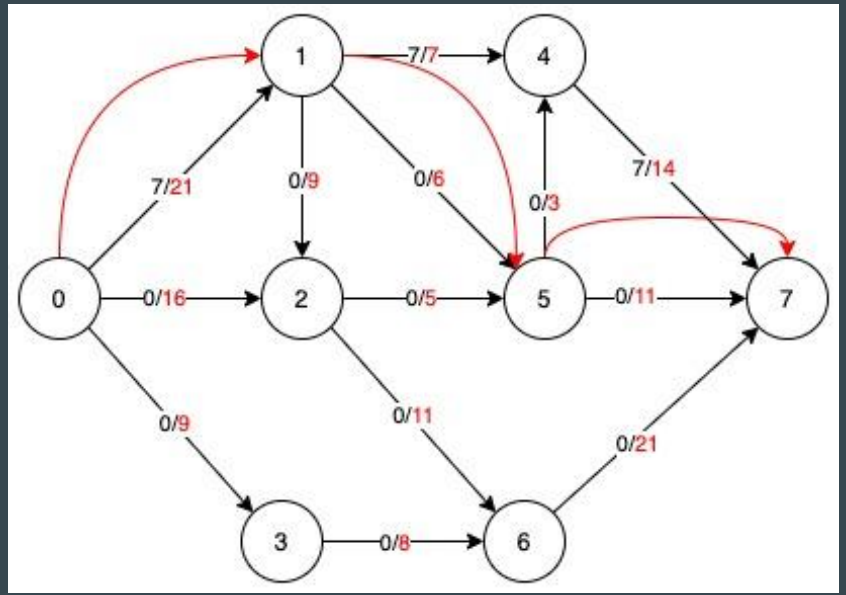
Flow Graph



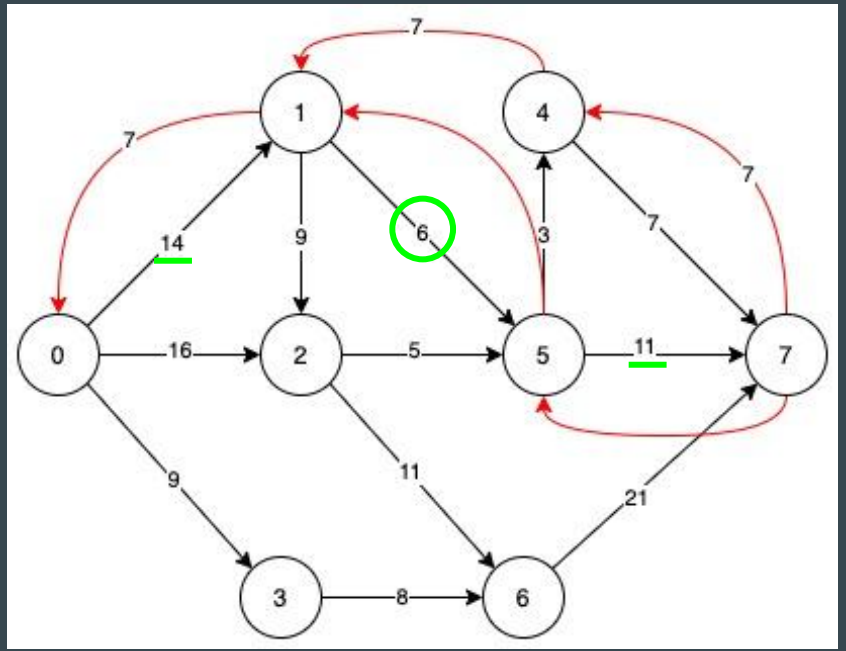
Residual Graph

Exercise 1

Min capacity of the path: 6



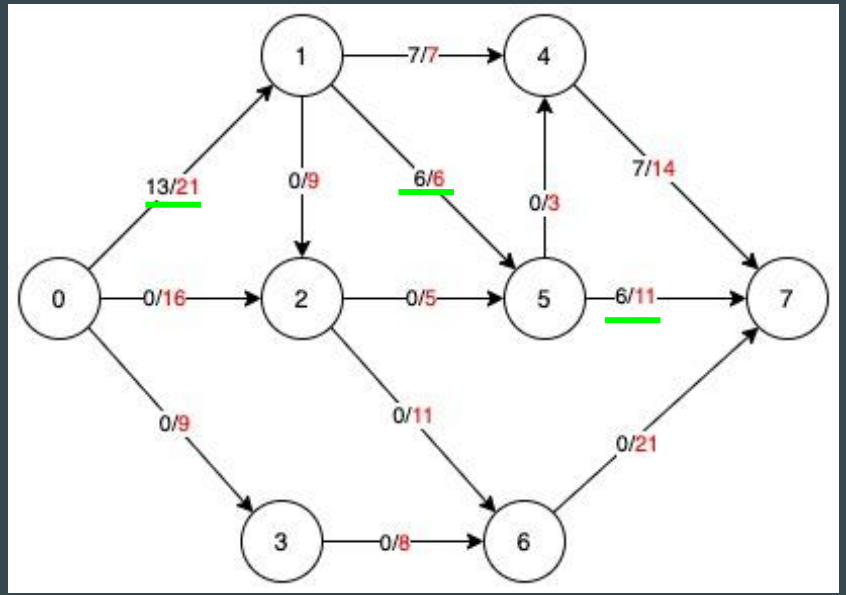
Flow Graph



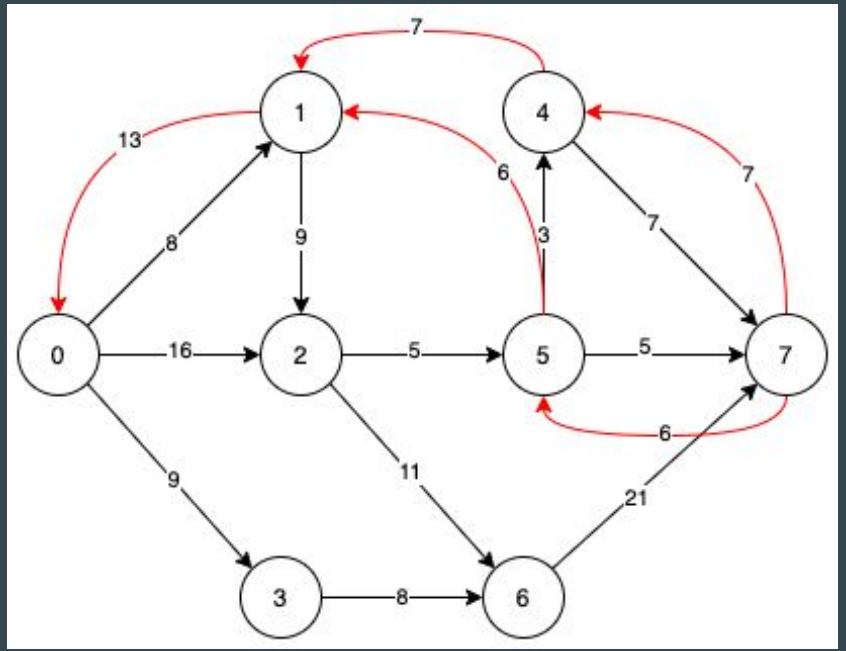
Residual Graph

Exercise 1

Update flow and residual graphs and set MaxFlow=13.



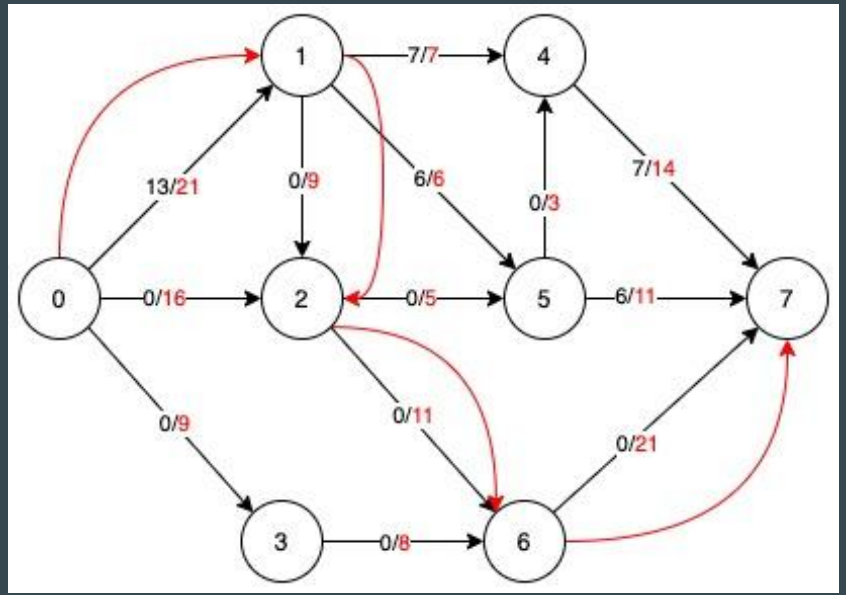
Flow Graph



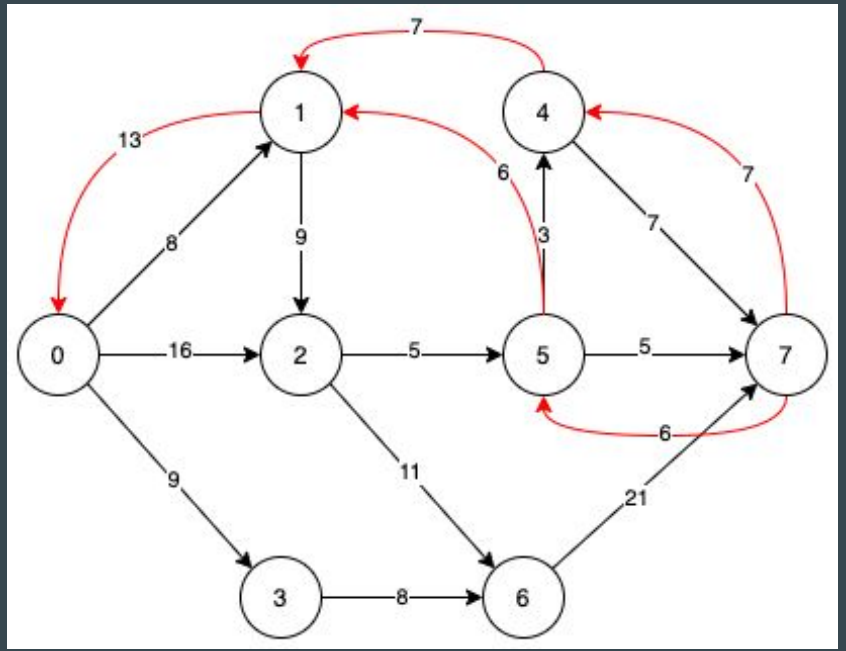
Residual Graph

Exercise 1

Search Path: 0-1-2-6-7



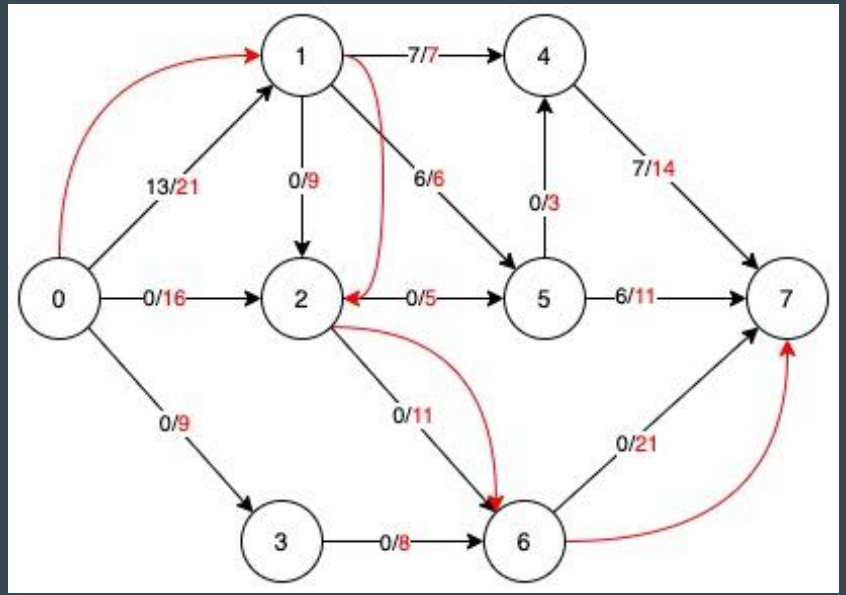
Flow Graph



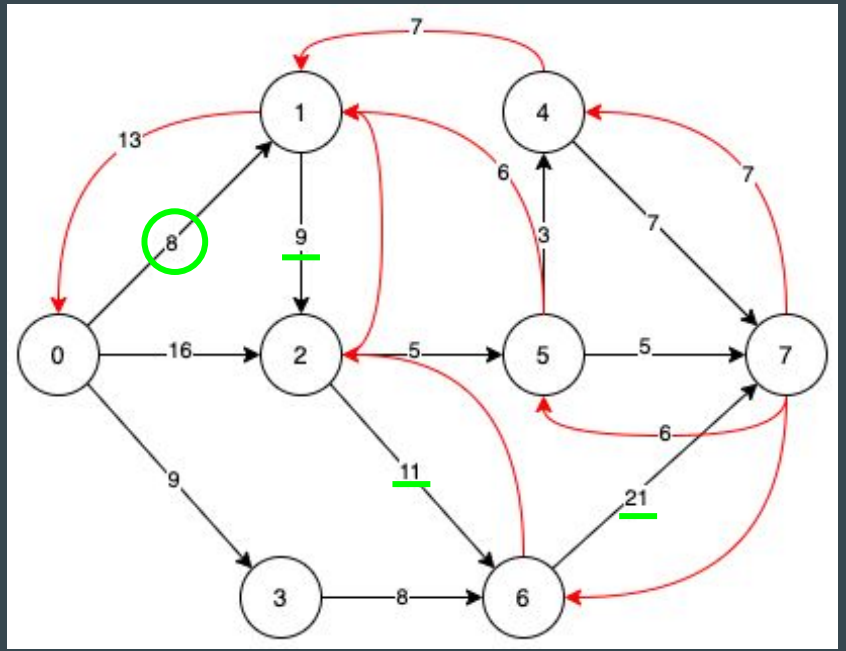
Residual Graph

Exercise 1

Min capacity of the path: 8



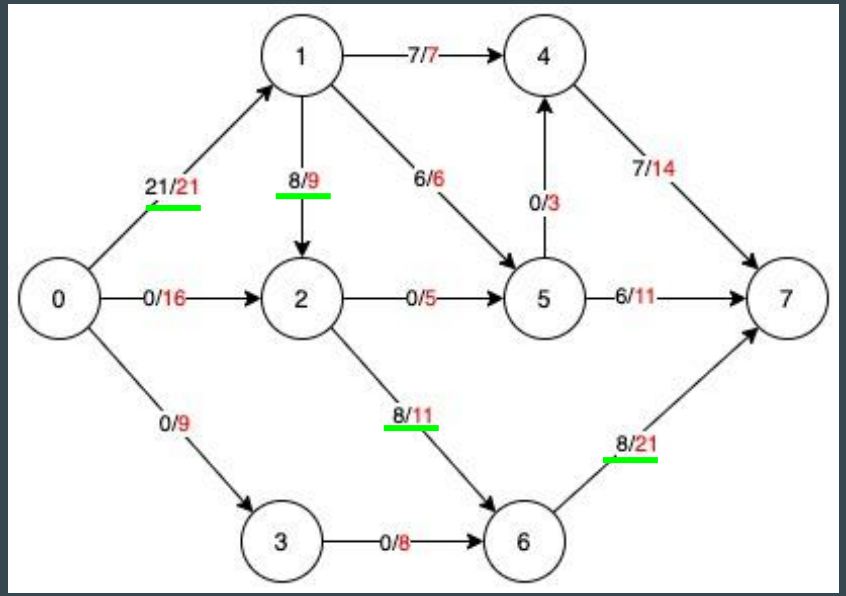
Flow Graph



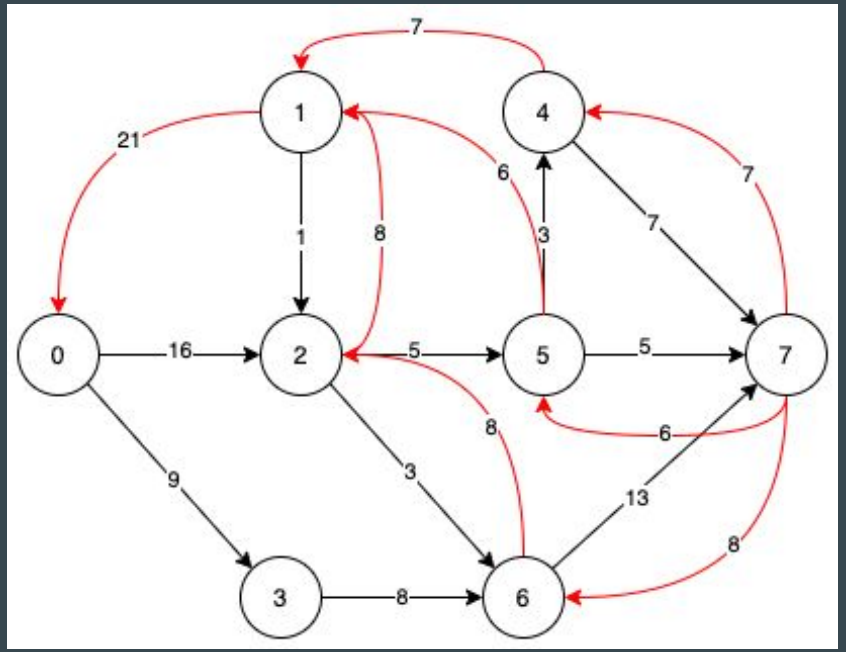
Residual Graph

Exercise 1

Update flow and residual graphs and set MaxFlow=21.



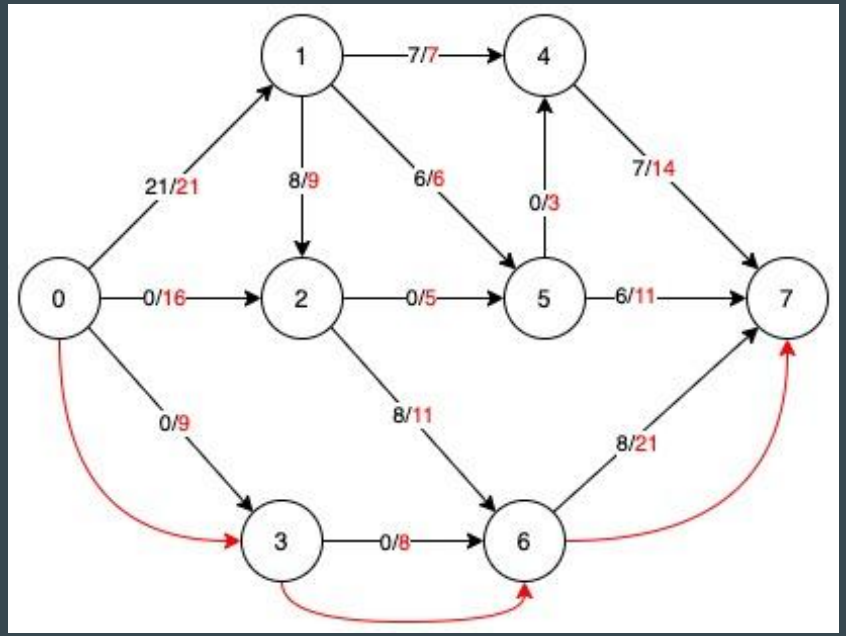
Flow Graph

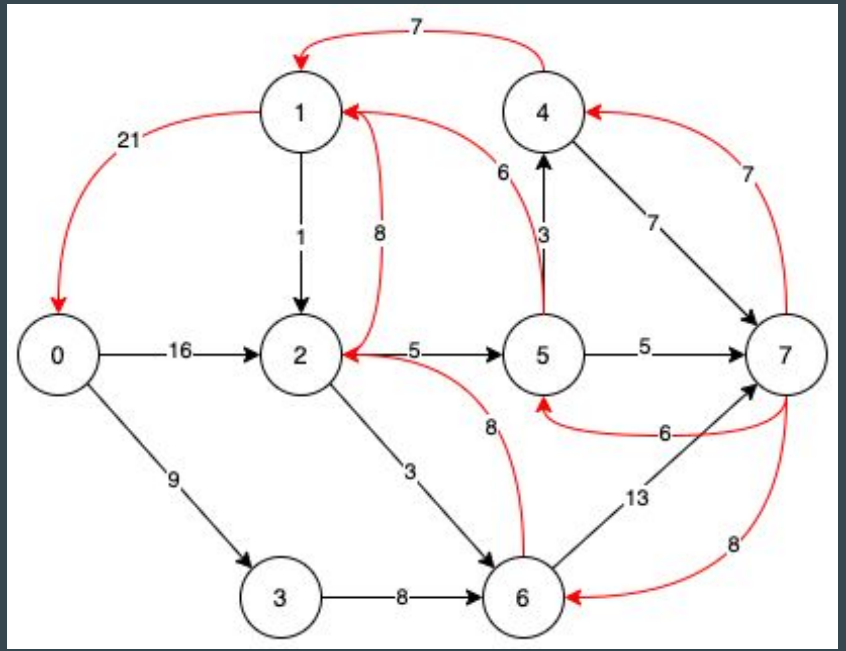


Residual Graph

Exercise 1

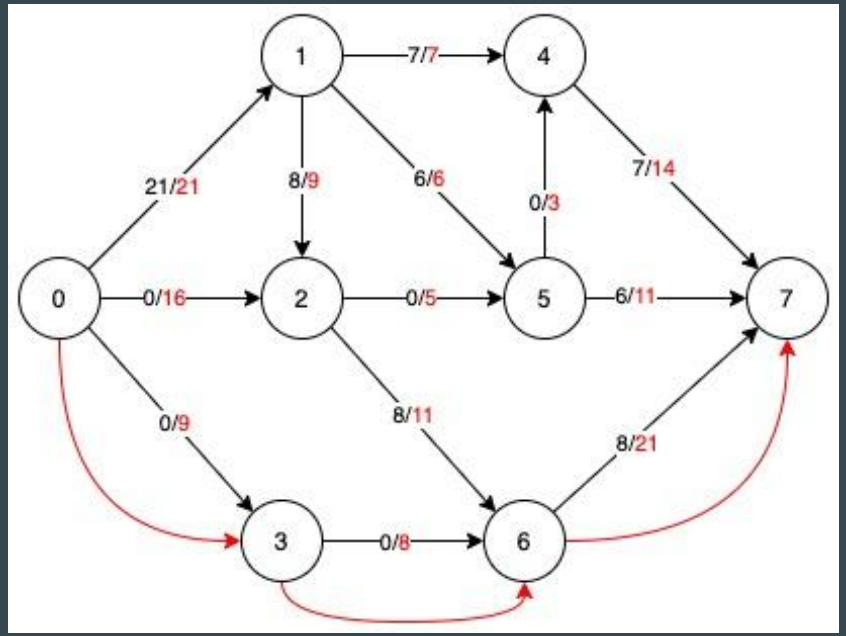
Search Path: 0-3-6-7

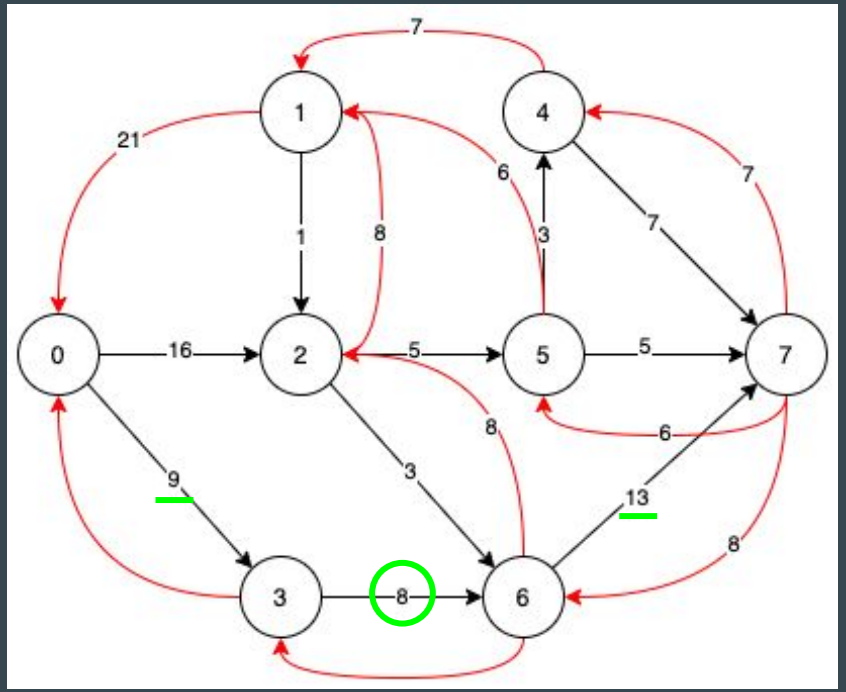




Exercise 1

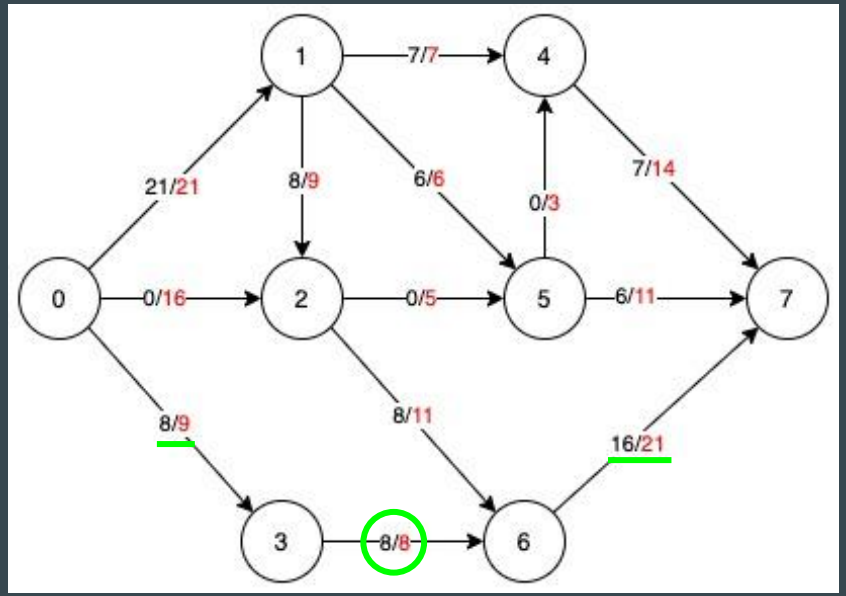
Min capacity of the path: 8

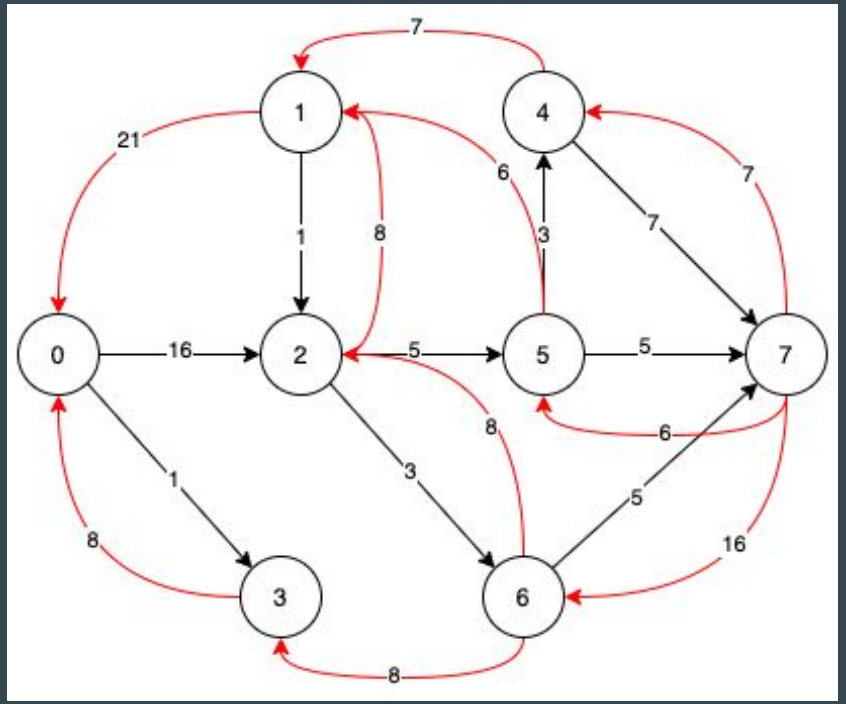




Exercise 1

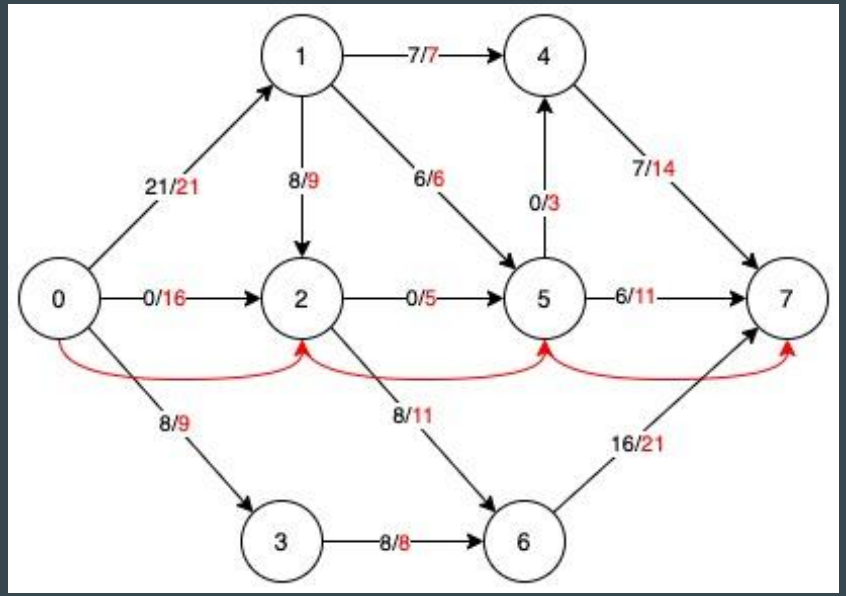
Update flow and residual graphs and set MaxFlow=29.

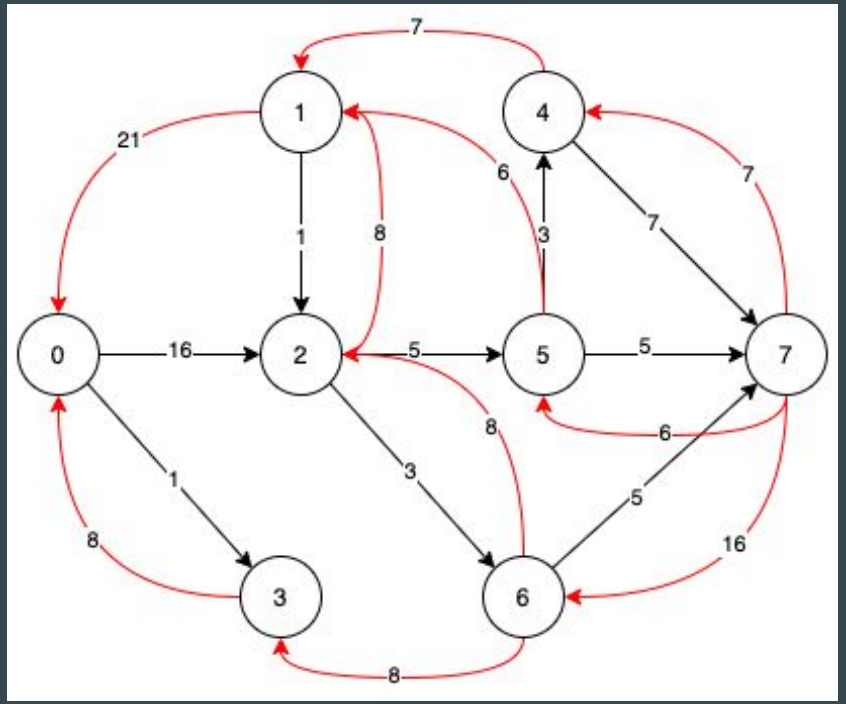




Exercise 1

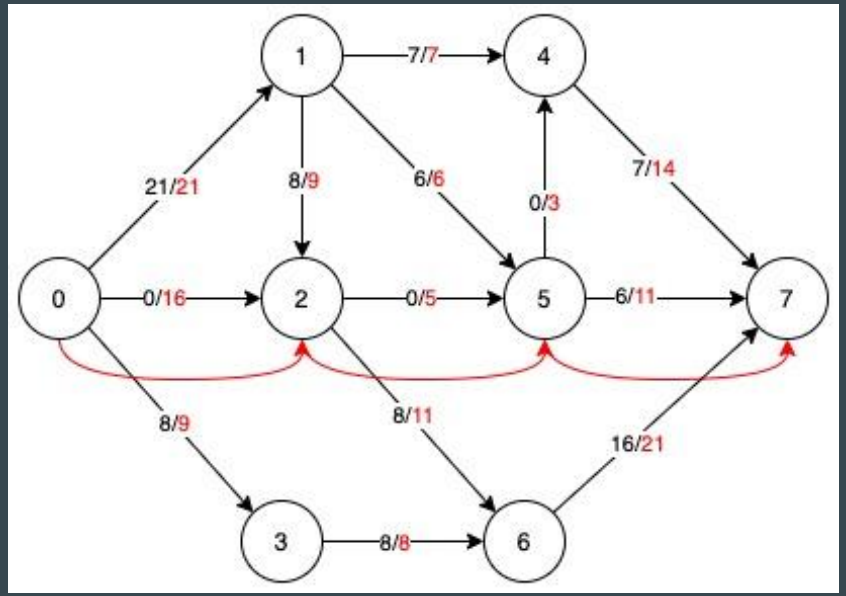
Search Path: 0-2-5-7

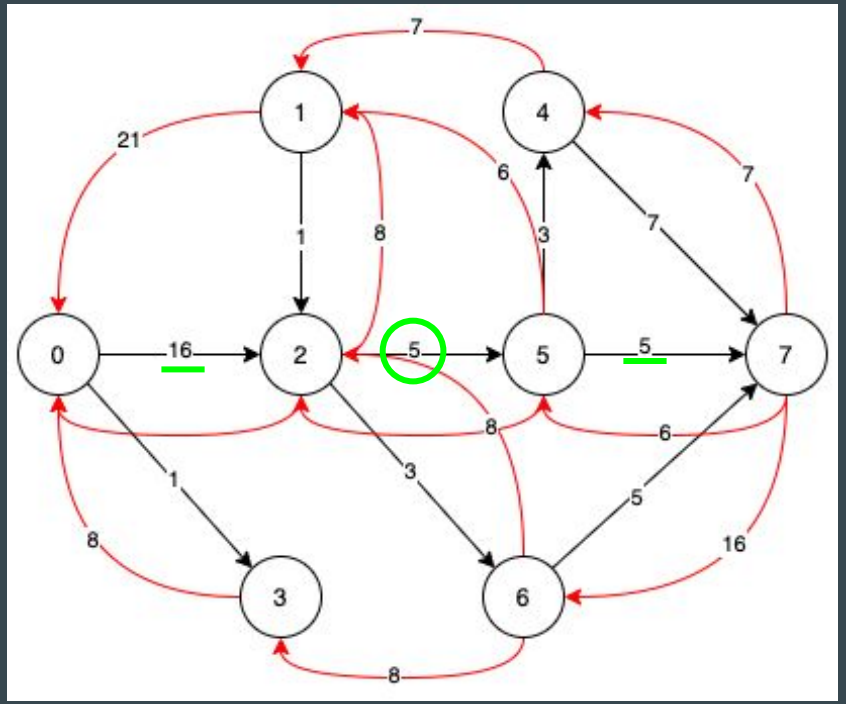




Exercise 1

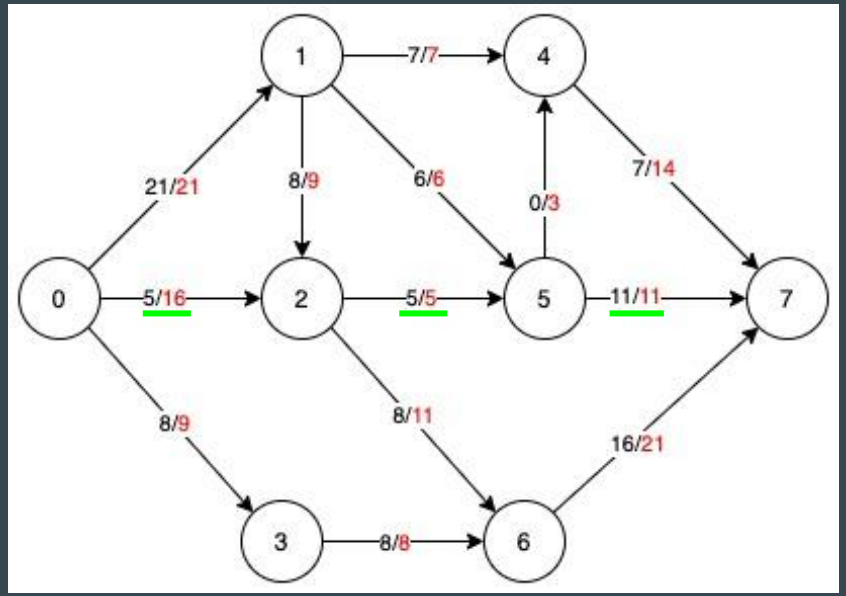
Min capacity of the path: 5

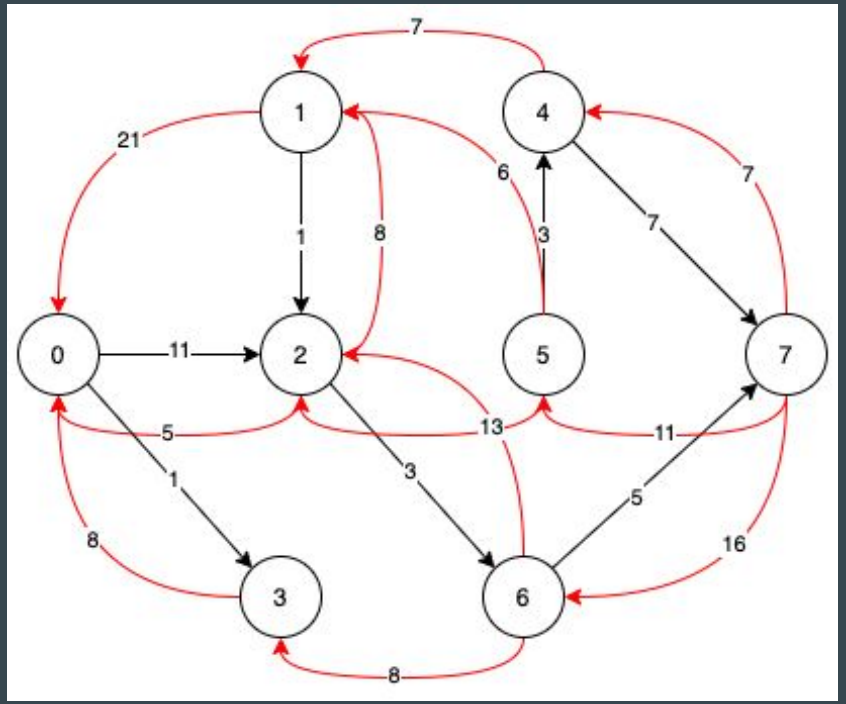




Exercise 1

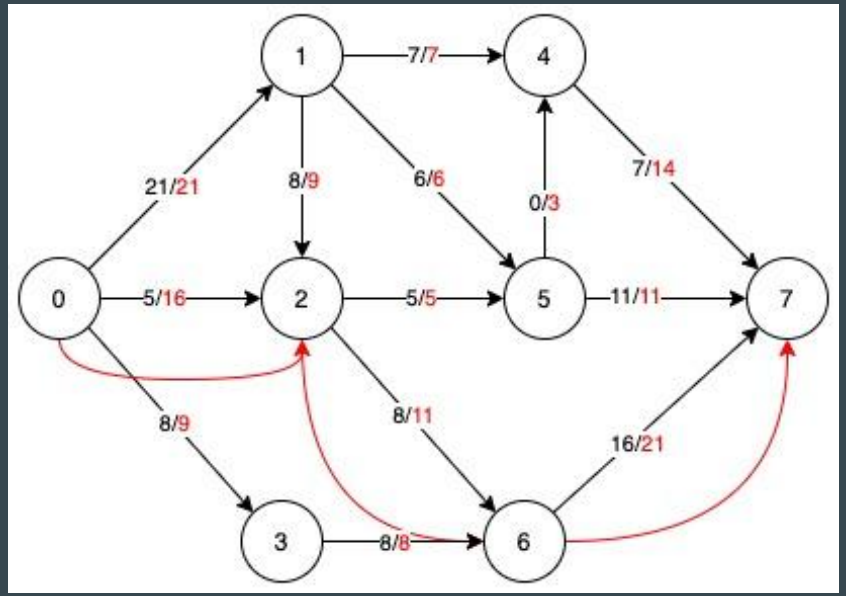
Update flow and residual graphs and set MaxFlow=34.

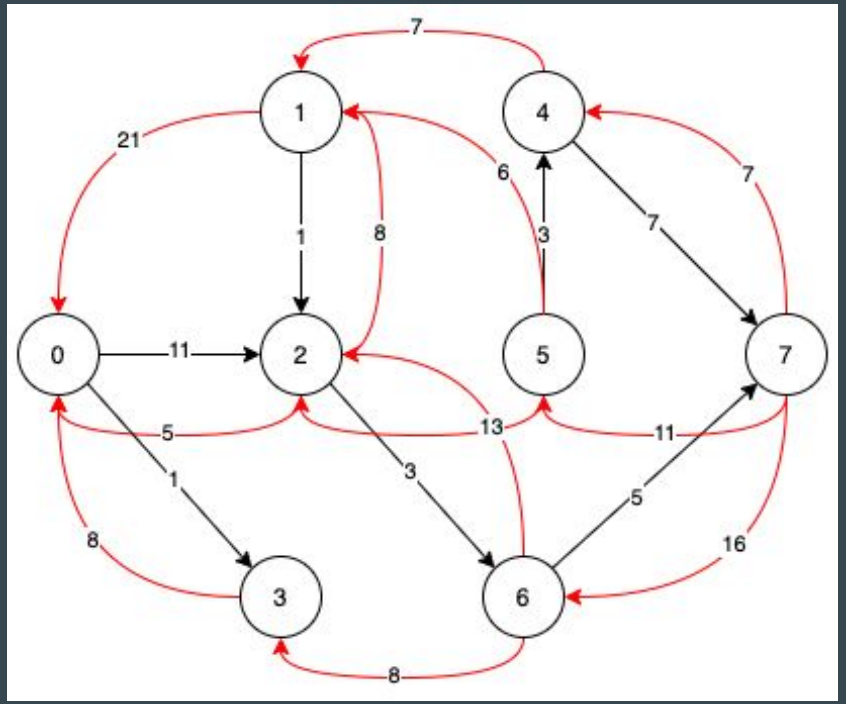




Exercise 1

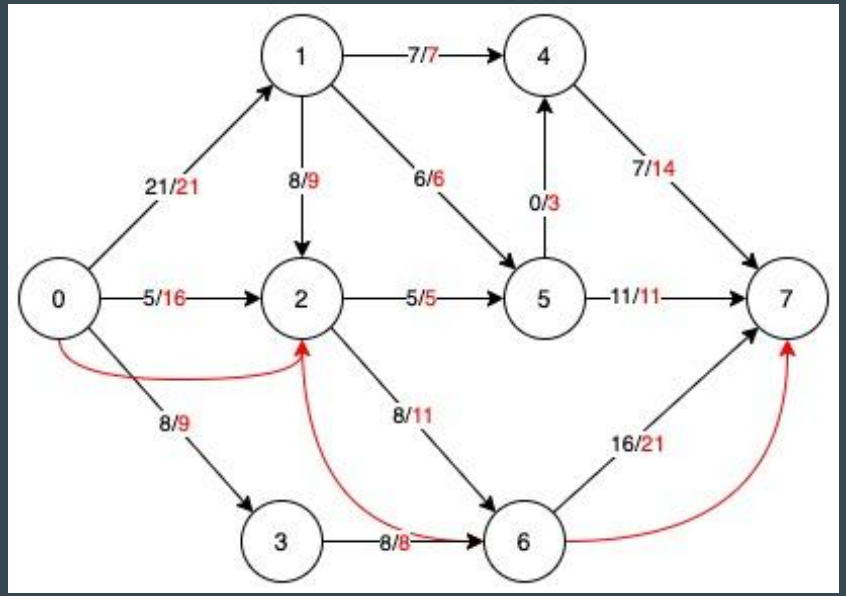
Search Path: 0-2-6-7

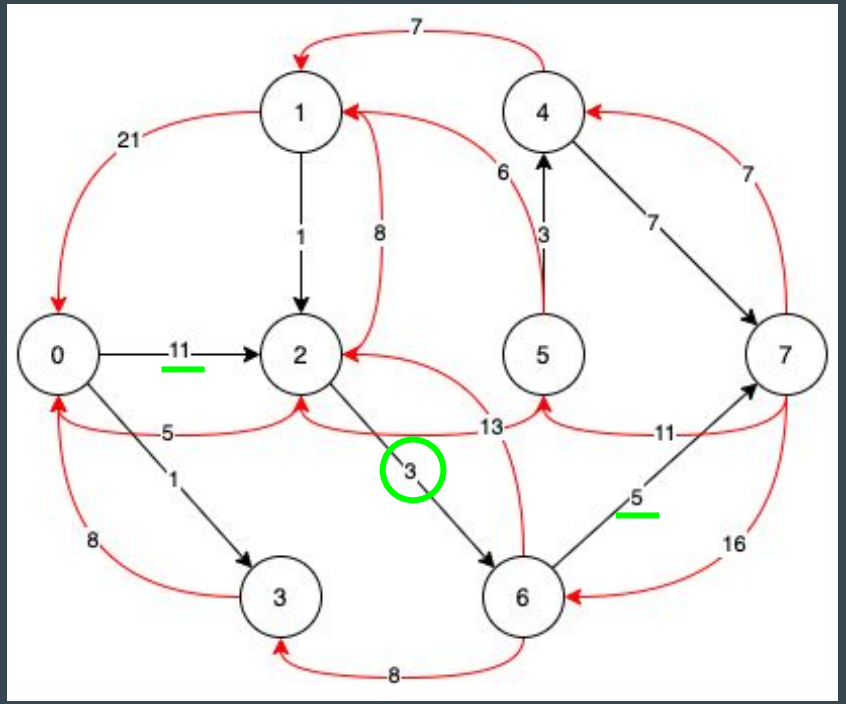




Exercise 1

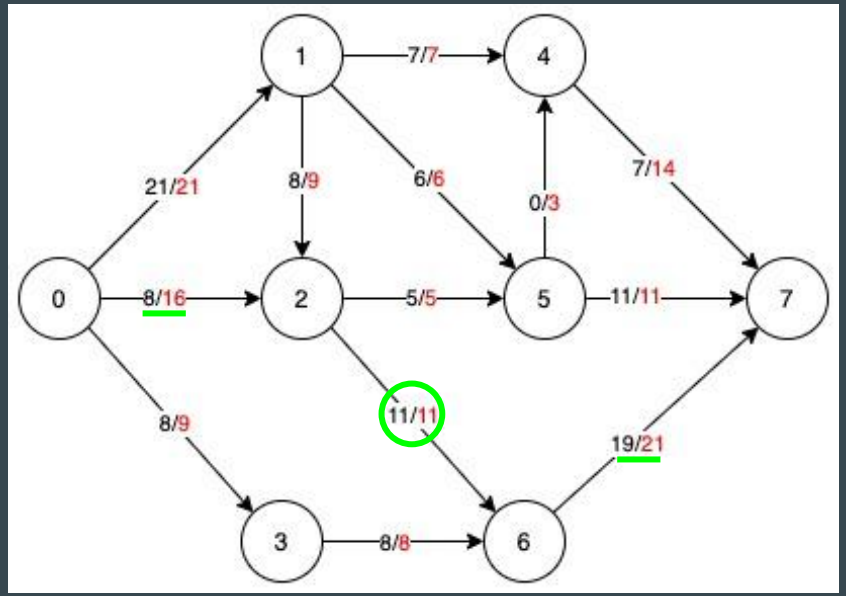
Min capacity of the path: 3

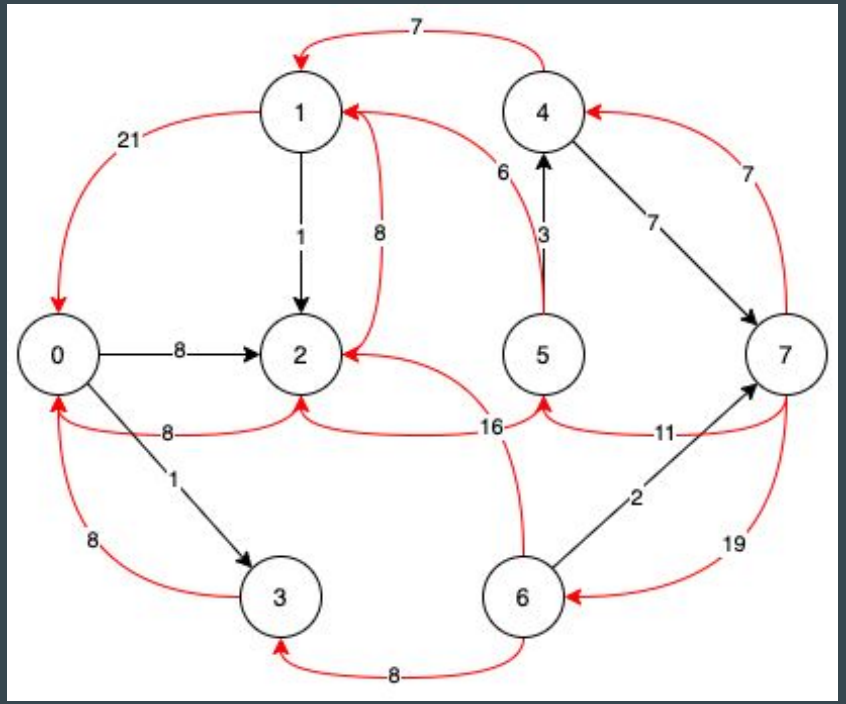




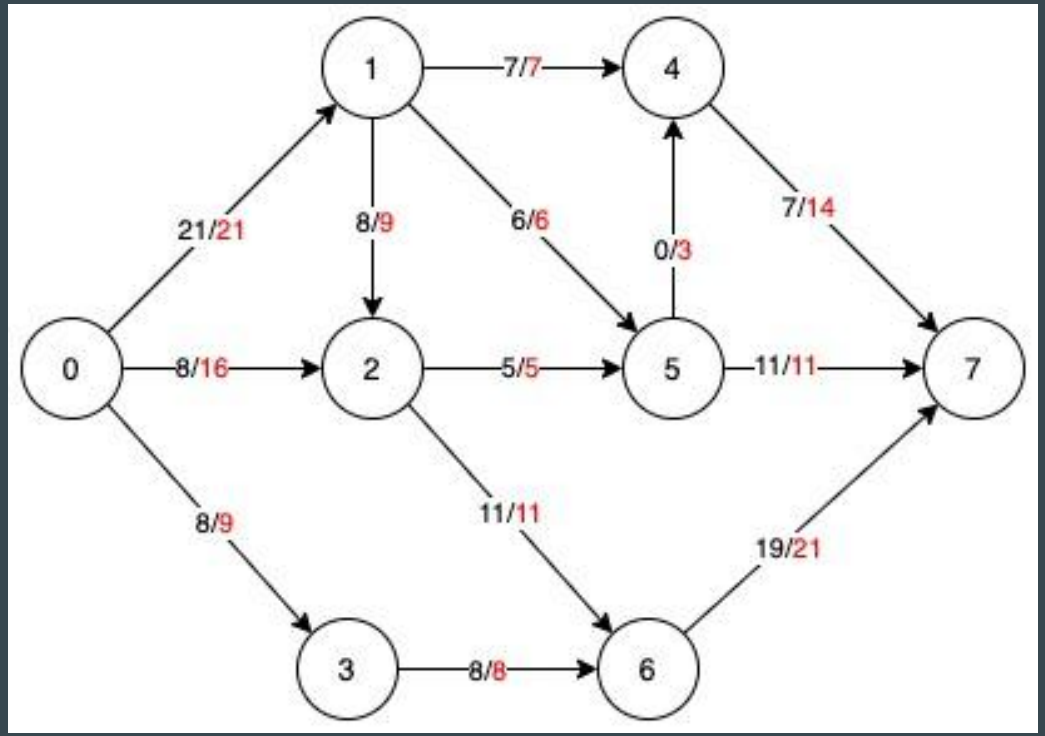
Exercise 1

Update flow and residual graphs and set MaxFlow=37.





Exercise 1



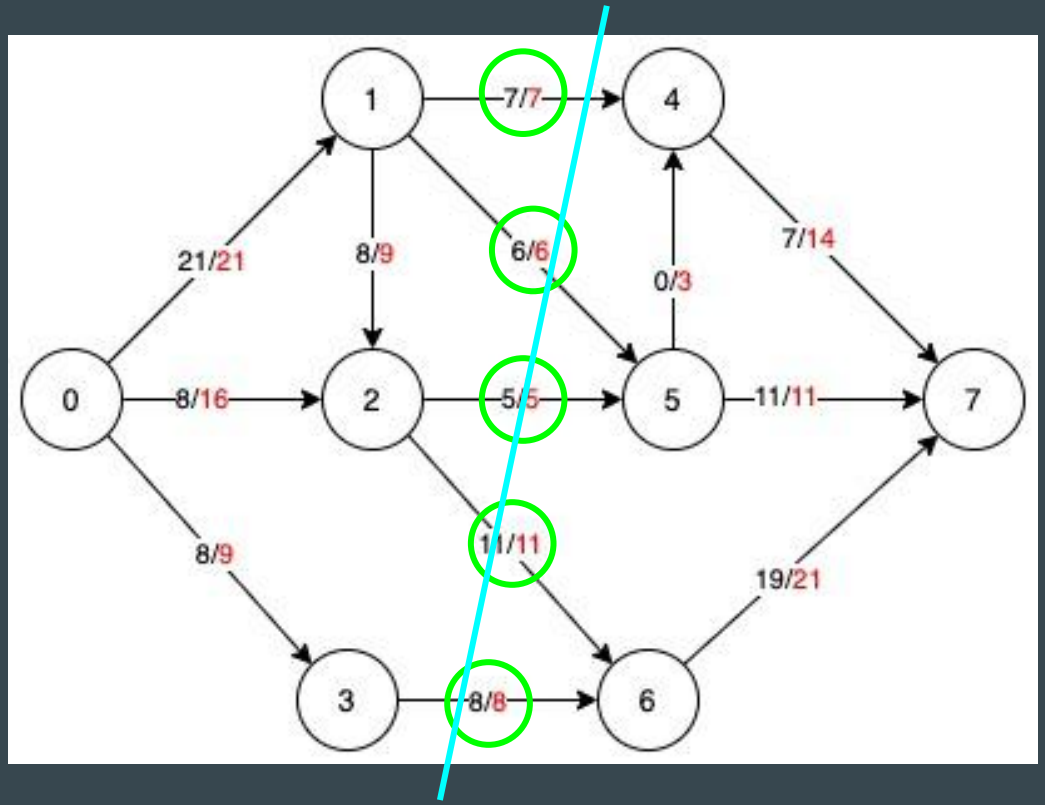
NO MORE POSSIBLE PATHS

Max-flow Min-cut Theorem

“The max-flow min-cut theorem states that the maximum flow through any network from a given source to a given sink is exactly equal to the minimum sum of a cut.”*

*https://www.baeldung.com/cs/minimum-cut-graphs

Back to Exercise 1



Max Flow: 37

Set defining one of the min-cuts:
{1→4,1→5,2→5,2→6,3→6}

Summation of the min cut: 37

*Deliberately showing this cut to show that there are no paths available which can let us to reach from source node to target, while there are other cuts available

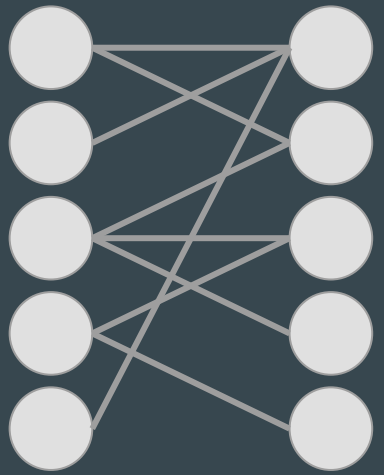
Wait... So, how would the pseudocode be like?

1) Define **a residual graph** which is initially a copy of the original graph
2) Find a path on the residual graph from source to target
3) Extract the bottlenecking edge within the path
4) Add the bottleneck value on to the max-flow
5) Decrement the forward direction by the bottleneck value
6) Increment the backward direction by the bottleneck value
7) Go to step 2 if target node is still reachable through other paths
8) Otherwise, end iterating and return the max-flow

Bipartite Matching

Supposingly in 2 independent sets, which we call one of them as the source and the other is the target, we define a function f so that each of the members in the target set is matched only by a single member from the source set.

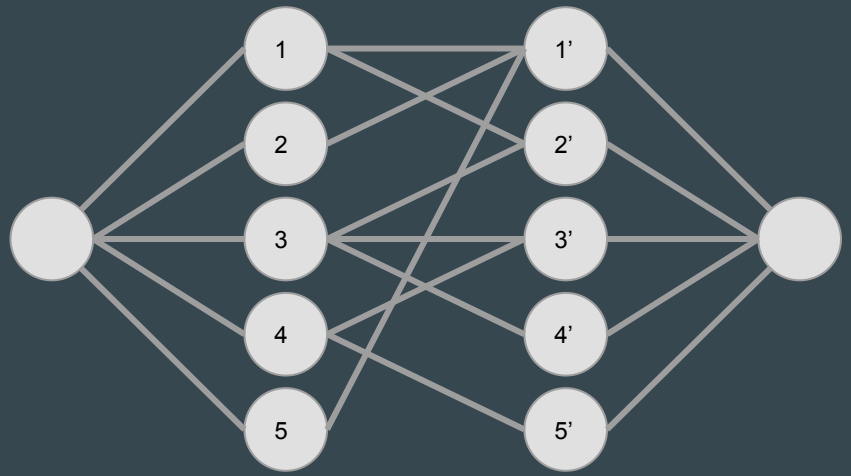
However, it is not always possible to guarantee that every member in both sets are matched



Maximum Bipartite Matching

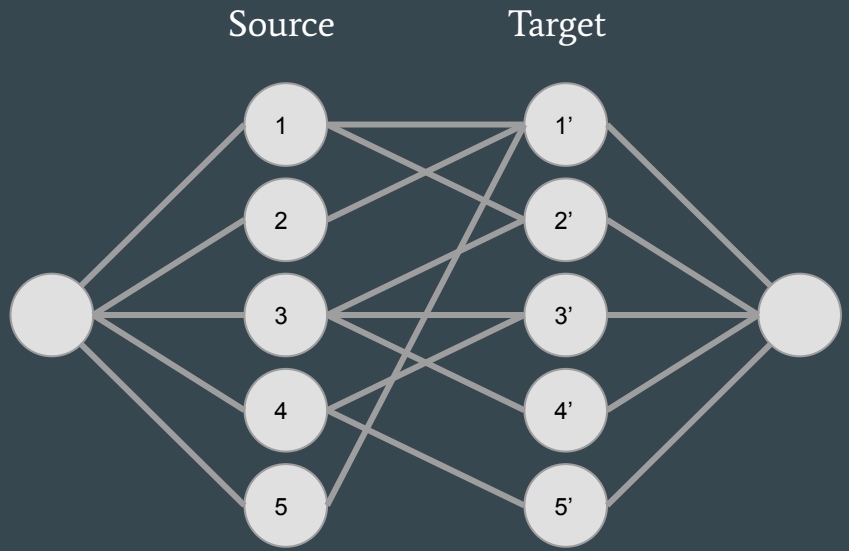
At least, we can maximize the number of matches with maximum flow

Achievable after adding a source and a target node while preserving the connectivity



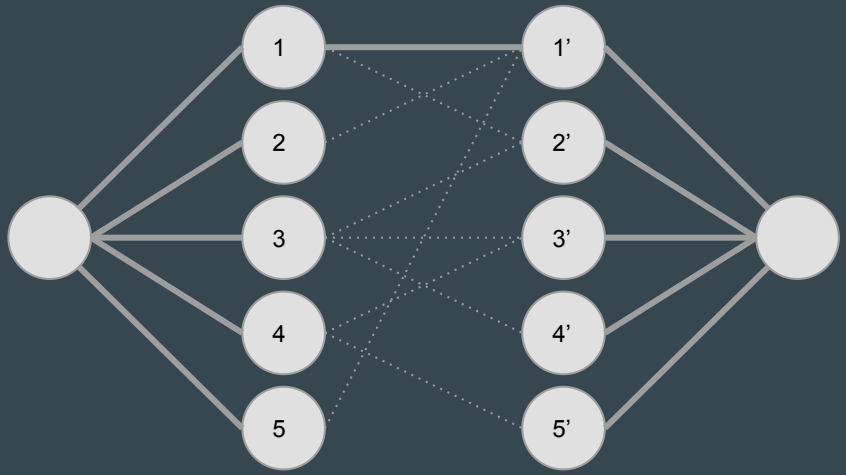
Example 2

Find the maximum number of matches that can be made between Source and Target sets. If more than one solution is feasible, start with descending order. Set edge weights to 1 wherever necessary.



Example 2

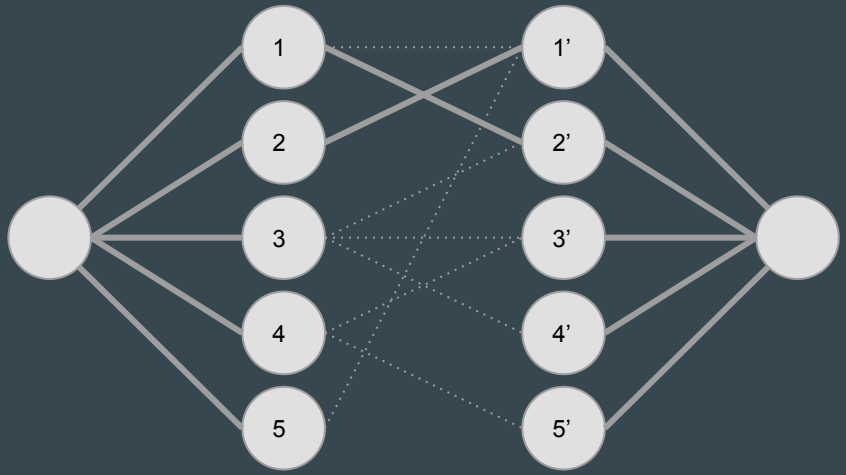
Matching 1 with 1’



Example 2

Trying to match 2 with 1’

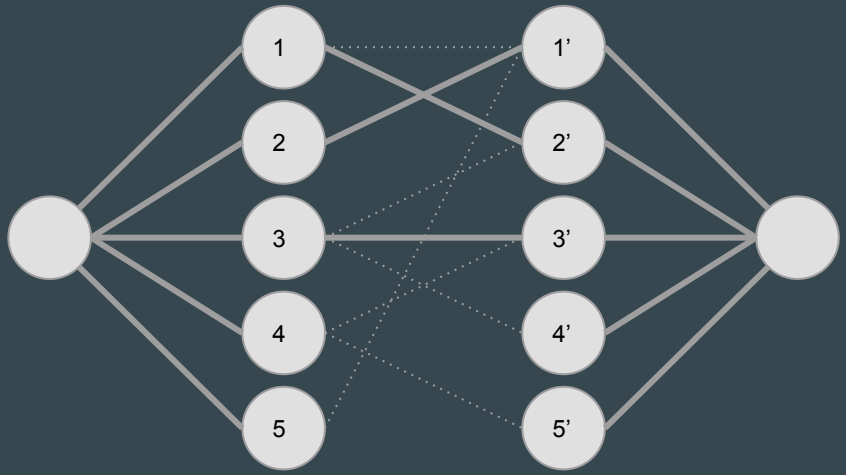
- Since 1’ is matched to 1 but it can be also matched to 2’, 1 becomes matched 2’ and 2 can be matched to 1’



Example 2

3 cannot be matched to 2’

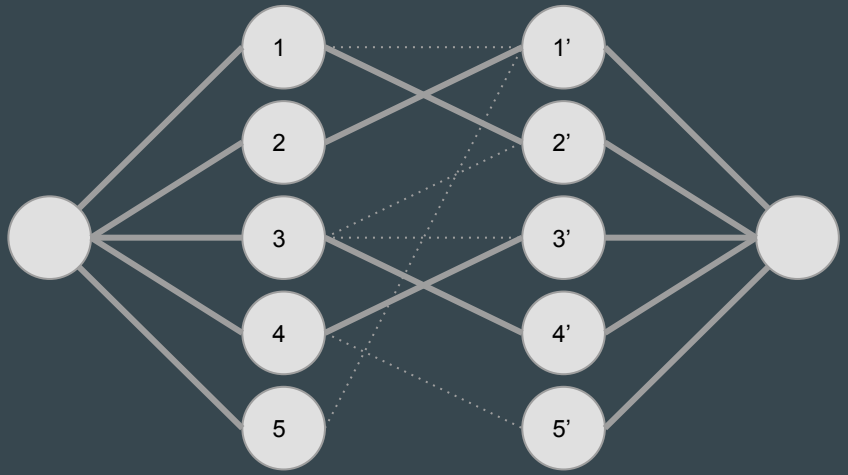
3 can be matched to 3’



Example 2

Trying to match 4 with 3’

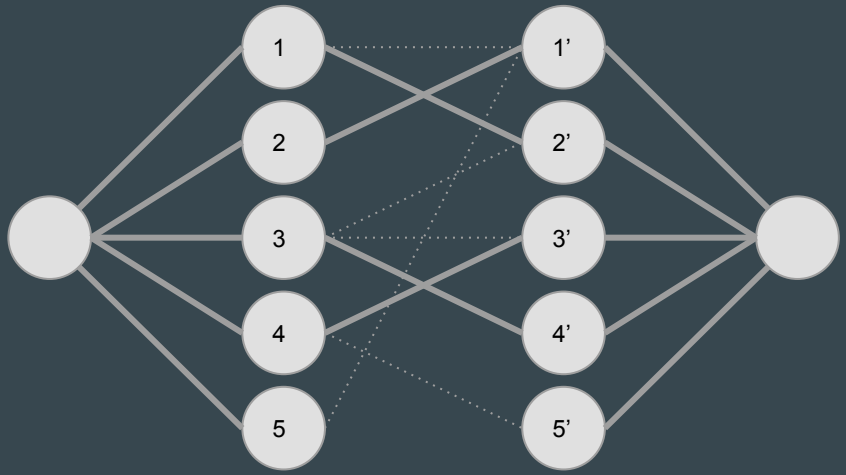
- Since 3’ is matched to 3 but it can be also matched to 4’, 3 becomes matched 4’ and 4’ can be matched to 3’



Example 2

Trying to match 5 with 1’

- 2 does not have any other available opportunity other than 1’, hence we cannot match 5 to any of the members in the target set



Example 2