586/29 $\mathcal{L}^{-1}\left[\dfrac{5-3s}{s^2+25}\right]$

$$= \mathcal{L}^{-1}\left[\frac{5}{s^2+5^2} - 3\,\frac{s}{s^2+5^2}\right]$$

$$= \sin(5t) - 3.\cos(5t)$$

31 $\mathcal{L}^{-1}\left[\dfrac{10s-3}{25-s^2}\right] = ?$

1st solution

$$\frac{10s-3}{25-s^2} = \frac{10s-3}{(5-s)(5+s)} = \frac{A}{5-s} + \frac{B}{5+s}$$

$$A = \frac{10.5-3}{5+5} = \frac{47}{10}, \qquad B = \frac{10.(-5)-3}{5-(-5} = \frac{-53}{10}$$

$$\mathcal{L}^{-1}\left[\frac{10s-3}{25-s^2}\right] = \mathcal{L}^{-1}\left[\frac{\frac{47}{10}}{5-s} + \frac{-\frac{53}{10}}{5+s}\right]$$

$$= \mathcal{L}^{-1}\left[-\frac{47}{10}\,\frac{1}{s-5} - \frac{53}{10}\,\frac{1}{s+5}\right] \qquad \mathcal{L}\left[e^{at}\right] = \frac{1}{s-a}$$

$$= -\frac{47}{10}\,e^{5t} - \frac{53}{10}\,e^{-5t}$$

<u>2nd solution</u> $\mathcal{L}^{-1}\left[\frac{10s-3}{25-s^2}\right]$

$$\mathcal{L}\left[\sinh(kt)\right] = \frac{k}{s^2-k^2}$$

$$\mathcal{L}\left[\cosh(kt)\right] = \frac{s}{s^2-k^2}$$

$$\mathcal{L}^{-1}\left[\frac{10s-3}{25-s^2}\right] = \mathcal{L}^{-1}\left[\frac{3}{s^2-5^2} - \frac{10s}{s^2-5^2}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{3}{5}\,\frac{5}{s^2-5^2} - 10\,\frac{s}{s^2-5^2}\right]$$

$$= \frac{3}{5}\cdot \sinh(5t) - 10\cdot \cosh(5t)$$

$$\cosh x = \frac{e^x+e^{-x}}{2}, \quad \sinh x = \frac{e^x-e^{-x}}{2}$$

598/10

Solve the IVP $x'' + 3x' + 2x = t$

$x(0) = 0, \quad x'(0) = 2$

$\mathcal{L}[x(t)] = X(t)$

$\mathcal{L}[x''(t)] = s^2 X(s) - s\,x(0) - x'(0)$

$\mathcal{L}[x'(t)] = s\,X(s) - x(0)$

$\mathcal{L}[x'' + 3x' + 2x] = \mathcal{L}[t]$

$s^2 X(s) - s\,x(0) - x'(0) + 3[s\,X(s) - x(0)] + 2X(s) = \frac{1}{s^2}$

$s^2 X(s) - s \cdot 0 - 2 + 3[s\,X(s) - 0] + 2X(s) = \frac{1}{s^2}$

$(s^2 + 3s + 2)X(s) - 2 = \frac{1}{s^2}$

$(s^2 + 3s + 2)X(s) = \frac{1 + 2s^2}{s^2}$

$$X(s) = \frac{1+2s^2}{s^2(s^2+3s+2)} = \frac{1+2s^2}{s^2(s+2)(s+1)}$$

$$\frac{1+2s^2}{s^2(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{s+2}$$

$$1+2s^2 = A(s+1)(s+2)s + B(s+1)(s+2) + C(s+2)s^2 + D(s+1)s^2$$

$$s=0: \quad 1+0 = 0 + 2B + 0 + 0 \Rightarrow B = 1/2$$

$$s=-1 \quad 1+2 = 0 + 0 + C(-1+2)1 + 0 \rightarrow C = 3$$

$$s=-2 \quad 1+(-2)^2 = 0 + 0 + 0 + D.(-1).4 \rightarrow D = -\frac{5}{4}$$

$$s=1: \quad \cancel{1+2} = A\cdot 2\cdot 3\cdot 1 + \cancel{\frac{1}{2}\cdot 2\cdot 3} + 3\cdot 3\cdot 1 - \frac{5}{4}\cdot 2\cdot 1$$

$$6A = \frac{5}{2} - 9 = \frac{-13}{2} \rightarrow A = \frac{-13}{12}$$

$$X(s) = \frac{-\frac{13}{12}}{s} - \frac{\frac{1}{2}}{s^2} + \frac{3}{s+1} - \frac{\frac{5}{4}}{s+2}$$

$$x(t) = -\frac{13}{12} \cdot 1 - \frac{1}{2} \cdot t + 3 \cdot e^{-t} - \frac{5}{4} \cdot e^{-2t}$$

608/6 $\quad F(s) = \frac{s-1}{(s+1)^3} \Rightarrow f(t) = ?$

$$\mathcal{L}^{-1}\left[\frac{s-1}{(s+1)^3}\right] = \mathcal{L}^{-1}\left[\frac{s+1-3}{(s+1)^3}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{s+1}{(s+1)^3} - \frac{3}{(s+1)^3}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{1}{(s+1)^2} - 3\frac{1}{(s+1)^3}\right]$$

$$X(s) = \frac{-\frac{13}{12}}{s} - \frac{\frac{1}{2}}{s^2} + \frac{3}{s+1} - \frac{\frac{5}{4}}{s+2}$$

$$x(t) = -\frac{13}{12}\cdot 1 - \frac{1}{2}\cdot t + 3\cdot e^{-t} - \frac{5}{4}\cdot e^{-2t}$$

608/6 $\quad F(s) = \frac{s-1}{(s+1)^3} \Rightarrow f(t) = ?$

$$\mathcal{L}^{-1}\left[\frac{s-1}{(s+1)^3}\right] = \mathcal{L}^{-1}\left[\frac{s+1-2}{(s+1)^3}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{s+1}{(s+1)^3} - \frac{2}{(s+1)^3}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{1}{(s+\underset{a}{1})^2} - \frac{2}{(s+\cancel{1})^3}\right]$$

$$= e^{-t}\,t - e^{-t}\,t^2$$

$$\mathcal{L}\left[e^{at} f(t)\right] = F(s-a)$$

$$\mathcal{L}^{-1}\left[F(s-a)\right] = e^{at} f(t)$$

608/8 $F(s) = \dfrac{s+2}{s^2+4s+5} \Rightarrow f(t) = ?$

$$F(s) = \frac{s+2}{s^2+4s+4+1} = \frac{s+2}{(s+2)^2+1}$$

$$\mathcal{L}^{-1}\left[\frac{s+2}{(s+2)^2+1}\right] = e^{-2t}\cos t$$

$$\mathcal{L}^{-1}\left[F(s-a)\right] = e^{at} f(t)$$

608/10

$$F(s) = \frac{2s-3}{9s^2-12s+20}$$

$$\Delta = (-12)^2 - 4.\ 9.\ 20 = 144 - 36.\ 20 < 20$$

$$9s^2 - 12s + 20 = (3s)^2 - 2.\ 3s.\ 2 + 2^2 + 16$$

$$= (3s-2)^2 + 16$$

$$F(s) = \frac{2s-3}{(3s-2)^2+16} = \frac{2s-3}{\left[3\left(s-\frac{2}{3}\right)\right]^2+16}$$

$$= \frac{2s-3}{3^2\left(s-\frac{2}{3}\right)^2+4^2} = \frac{1}{9}\ \frac{2s-3}{\left(s-\frac{2}{3}\right)^2+\left(\frac{4}{3}\right)^2}$$

$$F(s) = \frac{2}{9} \frac{s - \frac{3}{2}}{\left(s - \frac{2}{3}\right)^2 + \left(\frac{4}{3}\right)^2}$$

$$= \frac{2}{9} \left\{ \frac{s - \frac{2}{3} + \frac{2}{3} - \frac{3}{2}}{\left(s - \frac{2}{3}\right)^2 + \left(\frac{4}{3}\right)^2} \right\}$$

$$= \frac{2}{9} \left\{ \frac{s - \frac{2}{3}}{\left(s - \frac{2}{3}\right)^2 + \left(\frac{4}{3}\right)^2} - \frac{5}{2.4} \frac{\frac{4}{3}}{\left(s - \frac{2}{3}\right)^2 + \left(\frac{4}{3}\right)^2} \right\}$$

$$= \frac{2}{9} \left\{ \cos\left(\frac{4t}{3}\right) \cdot e^{\frac{2}{3}t} - \frac{5}{8} \cdot \sin\left(\frac{4}{3}t\right) \cdot e^{\frac{2}{3}t} \right\}$$

625/6 $\quad F(s) = \dfrac{s}{s^2+\pi^2} e^{-s} \Rightarrow f(t) = ?$

$$\mathcal{L}[u_a(t)\, f(t-a)] = e^{-as} F(s)$$

$$\mathcal{L}[e^{-as} F(s)] = u_a(t)\, f(t-a)$$

$$\mathcal{L}^{-1}\left[e^{-s} \frac{s}{s^2+\pi^2}\right] = u_1(t) \cos \pi(t-1)$$

$a=1$

$$F(s) = \frac{s}{s^2+\pi^2}$$

$$f(t) = \cos(\pi t)$$

625/8

$$F(s) = \frac{s(1-e^{-2s})}{s^2+\pi^2}$$

$$F(s) = \frac{s}{s^2+\pi^2} - e^{-2s}\frac{s}{s^2+\pi^2}$$

$$\mathcal{L}^{-1}\left[\frac{s}{s^2+\pi^2}\right] = \cos(\pi t)$$

$$\mathcal{L}^{-1}\left[e^{-2s}\frac{s}{s^2+\pi^2}\right] = u_2(t)\cos\pi(t-2)$$

$$f(t) = \cos\pi t - u_2(t)\cos\pi(t-2)$$

625/10 $F(s) = 2s\left(\dfrac{e^{-\pi s} - e^{-2\pi s}}{s^2+4}\right)$

$$f(t) = \mathcal{L}^{-1}\left[\frac{2s\,e^{-\pi s}}{s^2+4} - \frac{2s\,e^{-2\pi s}}{s^2+4}\right]$$

$$= \mathcal{L}^{-1}\left[2e^{-\pi s}\frac{s}{s^2+4} - 2e^{-2\pi s}\frac{s}{s^2+4}\right]$$

$$= 2\,u_{\pi}(t)\cos 2(t-\pi) - 2\cdot u_{2\pi}(t)\cos 2(t-2\pi)$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^{5/2}}\right] = ?$$

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$n = 0, 1, 2, 3, \ldots$$

$$\boxed{\mathcal{L}[t^a] = \frac{\Gamma(a+1)}{s^{a+1}}}$$

$$a > -1$$

$$a + 1 = \frac{5}{2} \qquad a = \frac{3}{2}$$

$$\mathcal{L}(t^{3/2}) = \frac{\Gamma(5/2)}{s^{5/2}}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^{5/2}}\right] = \frac{1}{\Gamma(\frac{5}{2})} t^{3/2}$$

$$a = n = 0, 1, 2, \ldots$$

$$L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}}$$

$$\Gamma(x+1) = x\,\Gamma(x)$$

$$\Gamma(n+1) = n\,\Gamma(n) = n(n-1)\,\Gamma(n-1)$$

$$= n.(n-1)(n-1)\ldots = n!$$

210E196F
Jan. 2, 2020

$$u_1' x^2 + u_2' x^2 \ln x = 0$$

$$u_1' 2x + u_2' (2x \ln x + x) = \ln x$$

$$\begin{bmatrix} x^2 & x^2 \ln x \\ 2x & 2x \ln x + x \end{bmatrix} \begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} 0 \\ \ln x \end{bmatrix}$$

$$W = \begin{vmatrix} x^2 & x^2 \ln x \\ 2x & 2x \ln x + x \end{vmatrix} = x^3$$

$$u_1' = \frac{\begin{vmatrix} 0 & x^2 \ln x \\ \ln x & 2x \ln x + x \end{vmatrix}}{\begin{vmatrix} x^2 & x^2 \ln x \\ 2x & 2x \ln x + x \end{vmatrix}} = \frac{W_1}{W}$$

210E 17 GF 3 January 2018

1a $$y^{(5)} - 3y^{(4)} = xe^{3x} + \cos x$$

i $y_c = ?$ $$y^{(5)} - 3y^{(4)} = 0 \qquad y = e^{rx}$$

$$r^5 - 3r^4 = 0 \Rightarrow r^4(r-3) = 0$$

$$r_1 = r_2 = r_3 = r_4 = 0, \qquad r_5 = 3$$

$$y_c = c_1 . \underbrace{e^{0.x}}_{y_1=1} + c_2 . x.1 + c_3 . x^2.1 + c_4 x^3.1 + c_5 e^{3x}$$

ii $$y_p = x(A_1x + A_0)e^{3x} + B_1 \cos x + B_2 \sin x$$

(1b) $y'' + y = \cot x$ find the general solution by using var. of. pars.

(i) $y'' + y = 0 \xrightarrow{y = e^{r \cdot x}} r^2 + 1 = 0$

$r^2 = -1 = i^2 \Rightarrow r = 0 \mp i$ $\alpha = 0$, $\beta = 1$

$$y = e^{\alpha x}\left[C_1 \cos(\beta x) + C_2 \sin(\beta x)\right]$$

$$y = C_1 \cos x + C_2 \sin x$$

$y_1 = \cos x$ $y_2 = \sin x$

$$y = u_1(x)\, y_1(x) + u_2(x)\, y_2(x))$$

$$y_1 = \cos x$$

$$y_2 = \sin x$$

$$\begin{cases} u_1' y_1 + u_2' y_2 = 0 \\ u_1' y_1' + u_2' y_2' = f \end{cases}$$

$$\begin{cases} u_1' \cos x + u_2' \sin x = 0 \\ u_1' (-\sin x) + u_2' \cos x = \cot x \end{cases}$$

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} 0 \\ \cot x \end{bmatrix}$$

$$u_1' = \frac{\begin{vmatrix} 0 & \sin x \\ \cot x & \cos x \end{vmatrix}\ W_1}{\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}\ W} = \frac{-\dfrac{\cos x}{\sin x}\cdot \sin x}{1} = -\cos x$$

$$u_1' = -\cos x \longrightarrow u_1(x) = -\sin x + C_1$$

$$u_2' = \frac{\begin{vmatrix} \cos x & 0 \\ -\sin x & \cot x \end{vmatrix}}{1} = \cos x \cdot \frac{\cos x}{\sin x} = \frac{\cos^2 x}{\sin x}$$

$$u_2' = \frac{1-\sin^2 x}{\sin x} = \frac{1}{\sin x} - \sin x = \operatorname{cosec} x - \sin x$$

$$\int$$

$$u_2(x) = -\ln|\cot x + \operatorname{cosec} x| + \cos x + C_2$$

$$y = u_1 y_1 + u_2 y_2$$

$$= (-\sin x + c_1) . \cos x + (-\ln|\cot x + \operatorname{cosec} x| + \cos x + c_2) \sin x$$

$$= c_1 \cos x + c_2 \sin x - \ln|\cot x + \operatorname{cosec} x|$$

$$\int \operatorname{cosec} x \, dx = \int \operatorname{cosec} x \frac{\operatorname{cosec} x + \cot x}{\operatorname{cosec} x + \cot x} dx$$

$$-du \quad -$$

$$= \int \frac{\operatorname{cosec}^2 x + \operatorname{cosec} x \cot x}{\operatorname{cosec} x + \cot x} dx$$

$$u$$

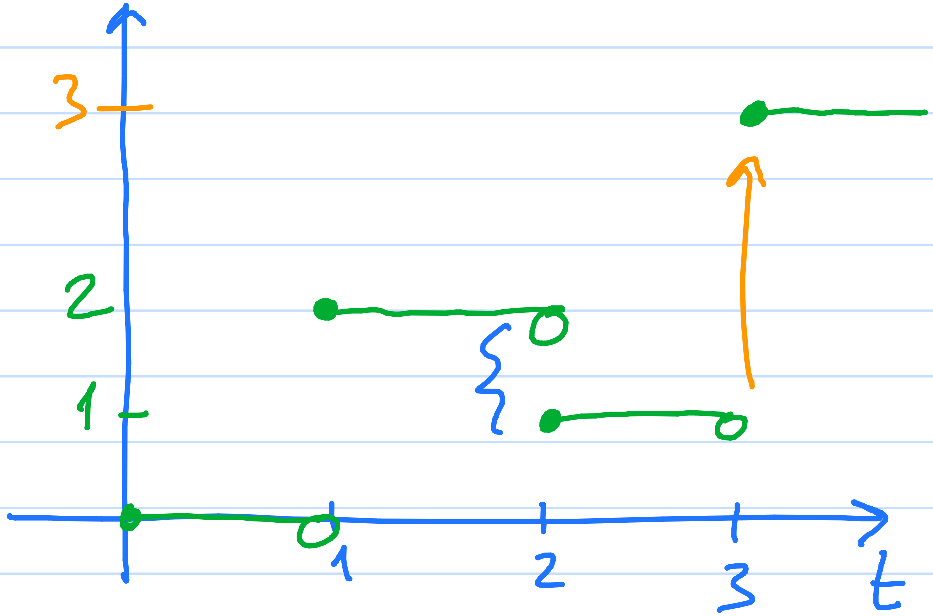
$$u = \operatorname{cosec} x + \cot x$$

$$\frac{du}{dx} = -\operatorname{cosec} x \cot x - \operatorname{cosec}^2 x$$

$$= -\ln|u| + c = -\ln|\operatorname{cosec} x + \cot x| + C$$

②

$$f(t) = \begin{cases} 0 & t < 1 \\ 2 & 1 \leq t < 2 \\ 1 & 2 \leq t < 3 \\ 3 & 3 \leq t \end{cases}$$



$\mathcal{L}[f(t)] = ?$

$$u_a(t) = \begin{cases} 0 & t < a \\ 1 & a \leq t \end{cases}$$

$$f(t) = 2u_1(t) - 1.\,u_2(t) + 2u_3(t)$$

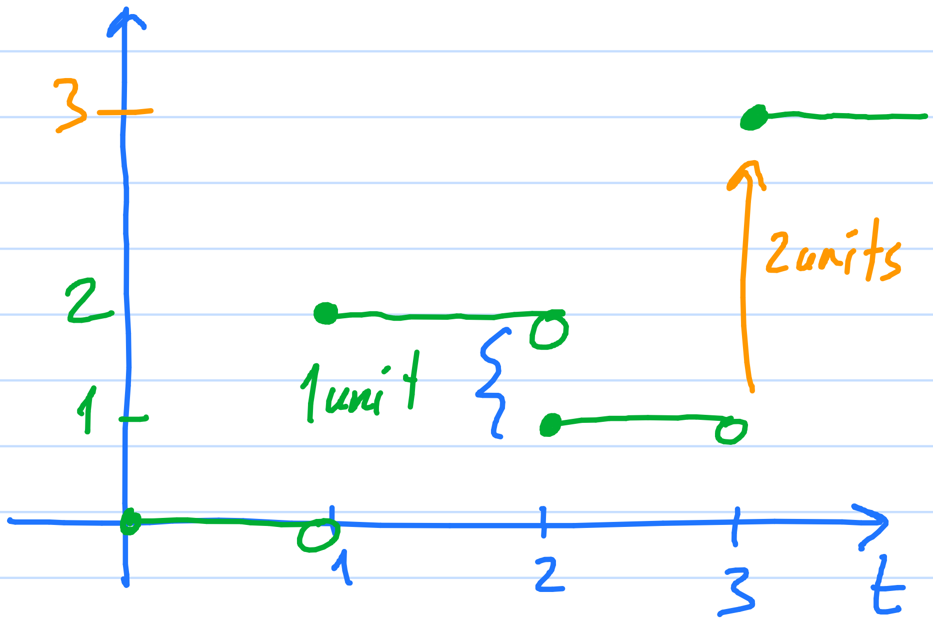
$0 \leq t < 1 \qquad f(t) = 2.0 - 1.0 + 2.0 = 0$

$1 \leq t < 2 \qquad f(t) = 2.1 - 1.0 + 2.0 = 2$

$2 \leq t < 3 \qquad f(t) = 2.1 - 1.1 + 2.0 = 1$

(2)

$$f(t) = \begin{cases} 0 & t < 1 \\ 2 & 1 \leq t < 2 \\ 1 & 2 \leq t < 3 \\ 3 & 3 \leq t \end{cases}$$



$$\mathcal{L}[f(t)] = \mathcal{L}\left[2u_1(t) - u_2(t) + 2u_3(t)\right]$$

$$= 2 \cdot \frac{e^{-1 \cdot s}}{s} - \frac{e^{-2 \cdot s}}{s} + 2 \cdot \frac{e^{-3 \cdot s}}{s}$$

(2b) Solve the IVP

$$y'' + 4y = \sin t - u_{2\pi}(t)\sin(t-2\pi)$$

$$y(0) = y'(0) = 0$$

$$\mathcal{L}[u_a(t)\, f(t-a)] = e^{-as} F(s)$$

$$\mathcal{L}[u_{2\pi}(t)\, \underbrace{\sin(t-2\pi)}] = e^{-2\pi \cdot s} \frac{1}{s^2+1}$$

$$f(t-2\pi) = \sin(t-2\pi)$$

$$f(t) = \sin t \Rightarrow F(s) = \frac{1}{s^2+1}$$

(2b) Solve the IVP

$$y'' + 4y = \sin t - u_{2\pi}(t)\sin(t-2\pi)$$

$$y(0) = y'(0) = 0$$

Let $\mathcal{L}[y(t)] = Y(s)$

$$\mathcal{L}[y'' + 4y] = \mathcal{L}[\sin t - u_{2\pi}(t)\sin(t-2\pi)]$$

$$s^2 Y(s) - s\underbrace{y(0)}_{=0} - \underbrace{y'(0)}_{=0} + 4Y(s) = \frac{1}{s^2+1} - e^{-2\pi s}\frac{1}{s^2+1}$$

$$(s^2+4)Y(s) = \frac{1-e^{-2\pi s}}{s^2+1}$$

$$Y(s) = \frac{1-e^{-2\pi s}}{(s^2+4)(s^2+1)}$$

$$Y(s) = \frac{1}{(s^2+4)(s^2+1)} - e^{-2\pi s}\frac{1}{(s^2+4)(s^2+1)}$$

$$\frac{1}{(r+4)(r+1)} = \frac{A}{r+4} + \frac{B}{r+1} \qquad A = \frac{1}{-4+1} = \frac{-1}{3}$$

$$B = \frac{1}{-1+4} = \frac{1}{3}$$

$$\frac{1}{(s^2+4)(s^2+1)} = \frac{-\frac{1}{3}}{s^2+4} + \frac{\frac{1}{3}}{s^2+1}$$

$$\mathcal{L}^{-1}\left[\frac{1}{(s^2+4)(s^2+1)}\right] = \mathcal{L}^{-1}\left[\frac{1}{3}\,\frac{1}{s^2+1} - \frac{1}{3.2}\,\frac{1.2}{s^2+2^2}\right]$$

$$= \frac{1}{3}\sin t - \frac{1}{6}\sin 2t = f_1(t)$$

$$Y(s) = \frac{1}{(s^2+4)(s^2+1)} - e^{-2\pi s}\frac{1}{(s^2+4)(s^2+1)}$$

$$\mathcal{L}^{-1}\left[e^{-2\pi s}\frac{1}{(s^2+4)(s^2+1)}\right]$$

$$\mathcal{L}^{-1}\left[e^{-as}F(s)\right] = u_a(t)\,f(t-a)$$

$$= u_{2\pi}(t)\left[\frac{1}{3}\sin(t-2\pi) - \frac{1}{6}\sin 2(t-2\pi)\right] = f_2(t)$$

$$y(t) = f_1(t) - f_2(t)$$

(3) $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

(a) find the eigenvalus of A.

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 & 2 \\ 0 & 1-\lambda & 0 \\ 0 & 1 & 3-\lambda \end{vmatrix} = (1-\lambda).(-1)^{1+1}.\begin{vmatrix} 1-\lambda & 0 \\ 1 & 3-\lambda \end{vmatrix}$$

$$= (1-\lambda)(1-\lambda)(3-\lambda) = (1-\lambda)^2(3-\lambda) = 0$$

$$\lambda_1 = \lambda_2 = 1, \qquad \lambda = 3$$

(b) Find the eigenvectors corresponding to the eigenvalues.

$$(A - \lambda I)\underset{\sim}{v} = \underset{\sim}{0}$$

$\boxed{\lambda = 1}$

$$(A - 1.I)\underset{\sim}{v} = 0$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0.a + 1.b + 2.c = 0$

$b + 2c = 0$

$$V = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ -2c \\ c \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$a = 1,\ c = 0:\quad \underset{\sim}{v_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad a = 0,\ c = 1:\quad V_2 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

$$\boxed{\lambda = 3}$$

$$\begin{bmatrix} -2 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$(A - \lambda I)\underset{\sim}{v} = \underset{\sim}{0}$

$* \quad b = 0$

$* \quad -2a + 1 \cdot \underset{0}{\cancel{b}} + 2 \cdot c = 0$

$c = a$

Let $a = 1 \Rightarrow$

$$v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\left\{ v_1, \; v_2, \; v_3 \right\} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

3c Find a diagonal matrix D and an invertible matrix P such that $D = P^{-1}AP$.

$$P = \begin{bmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$D = P^{-1}AP = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\{ v_1, v_2, v_3 \} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$