BLG 372E ANALYSIS OF ALGORITHMS
FINAL – MAY 21, 2013, 9:00-11:00 AM (2hours)- PART B

B

Q4 (25pt)	Q5 (25 pt)	Total (50 pt)

On my honor, I declare that I neither give nor receive any unauthorized help on this exam.

Student Signature:____________________

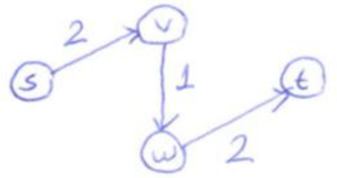
Write your name on each sheet. Write your answers neatly (in English) in the space provided for them. You must show all your work for credit. Books and notes are closed. Good Luck!

Q4)[25pts] Network Flow

True/False questions. Decide whether the following statement is true or false. If it is true, give a short explanation. If it is false, give a counter example:

a) **[5 pts]** Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e. If f is a maximum s-t flow in G, then f saturates every edge out of s with flow (i.e., for all edges e out of s, we have $f(e) = c_e$).

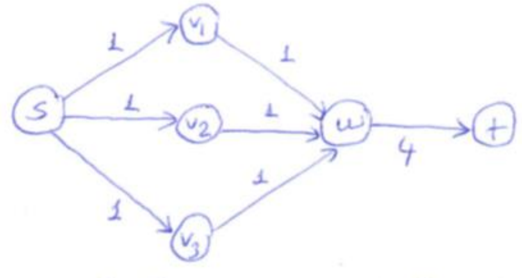
False.



⇐ Max. flow has value 1, does not saturate the edge out of s

b) **[5 pts]** Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e; and let (A,B) be a minimum s-t cut with respect to these capacities $\{ c_e : e \in E\}$. Now suppose we add 1 to every capacity, then (A,B) is still a minimum s-t cut with respect to these new capacities $\{ 1 + c_e : e \in E\}$.

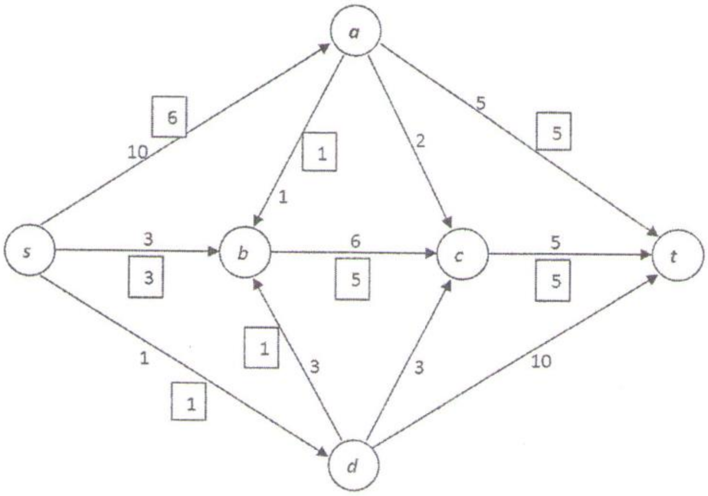
False.



$A = \{s\}$ and $B = V - A$ a min. cut with 3

Add +1 to each this cut has capacity 6, more than the capacity of 5 on the cut with $B = \{t\}$ and $A = V - B$

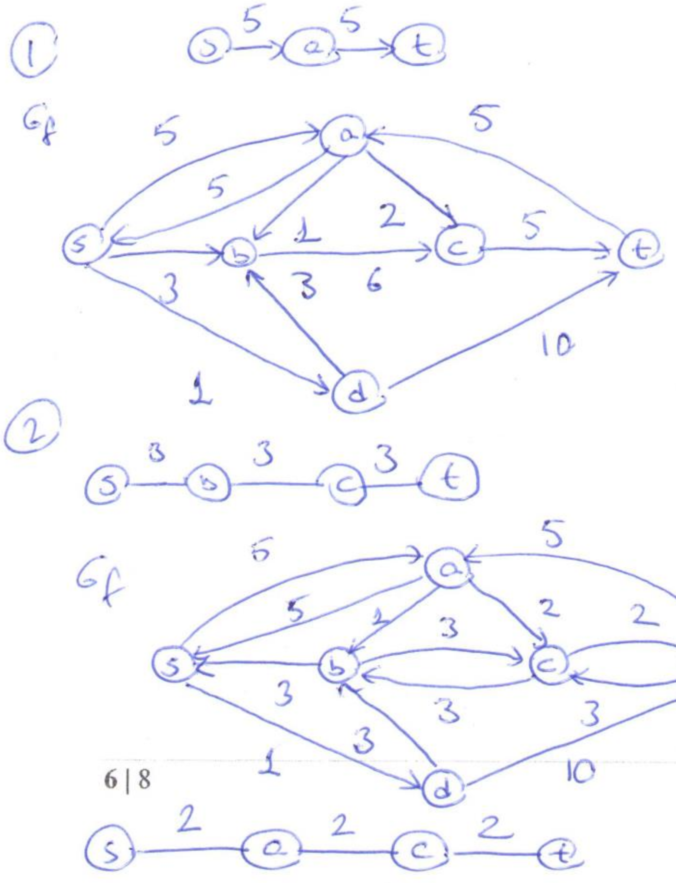
Q4-Cont. Figure below shows a flow network. The capacity of each edge appears as a label next to the edge, and the numbers in boxes give the amount of flow sent on each edge. Edges without boxed numbers have no flow being sent on them.

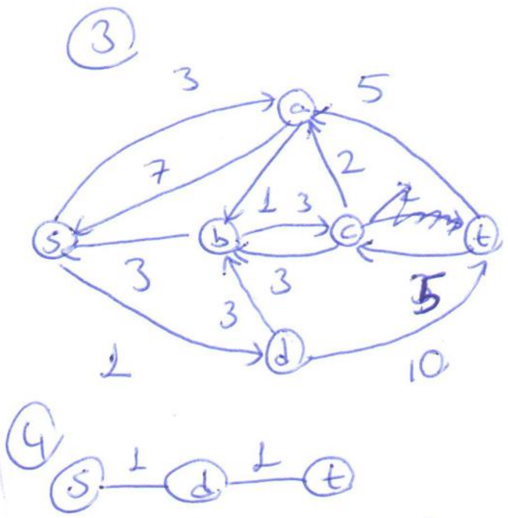


c) **[2 pts]** What is the value of this flow?

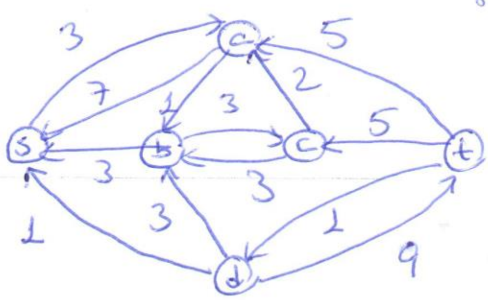
10

d) **[10 pts]** What is the value of the maximum flow? Use Ford-Fulkerson Algorithm to find it. Show your work in detail.





$\Rightarrow 5 + 3 + 2 + 1 = 11$ max. flow



e) [3 pts] What is the minimum s-t cut of this flow network?

$(\{s, a, b, c\}, \{d, t\})$. Capacity = 11

Q5)[25pts] Divide and Conquer-Matrix Multiplication

a) [5 pts] How do you multiply two matrices with the ordinary algorithm? What is the computational complexity?

Input: $A = [a_{ij}]$, $B = [b_{ij}]$, $C = [c_{ij}] = A \cdot B$ $\Big\}$ $i, j = 1, 2, \ldots, n$

$$\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix}$$

$$c_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj}$$

for $i \leftarrow 1$ to n
  do for $j \leftarrow 1$ to n
    do $c_{ij} \leftarrow 0$
      for $k \leftarrow$ ~~[illegible]~~ 1 to n
        do $c_{ij} \leftarrow c_{ij} + a_{ik} \cdot b_{kj}$

Complexity: $\Theta(n^3)$

b) [10 pts] How do you implement Matrix Multiplication with Divide and Conquer approach, explain the steps? What is the computational complexity?

$n \times n$ matrix = 2×2 matrix of $(n/2)\times(n/2)$ submatrices

$$\left[\begin{array}{c|c} r & s \\ \hline t & u \end{array}\right] = \left[\begin{array}{c|c} a & b \\ \hline c & d \end{array}\right]\left[\begin{array}{c|c} e & f \\ \hline g & h \end{array}\right]$$

$$C = A \cdot B$$

$$\left.\begin{array}{l} r = ae + bg \\ s = af + bh \\ t = ce + dg \\ u = cf + dh \end{array}\right\} \begin{array}{l} 8 \text{ multiplications} \\ 4 \text{ additions} \end{array}$$

$$T(n) = 8\,T(n/2) + \Theta(n^2)$$

$$T(n) = \Theta(n^3)$$

c) [10 pts] If the complexity you find at "*a)*" and "*b)*" are the same, how would you design a more efficient algorithm?

Multiply 2x2 matrices with only 7 recursive mults

$P_1 = a.(f-h)$ $\qquad r = P_5 + P_4 - P_2 + P_6$

$P_2 = (a+b).h$ $\qquad s = P_1 + P_2$

$P_3 = (c+d).e$ $\qquad t = P_3 + P_4$

$P(4) = d.(g-e)$ $\qquad u = P_5 + P_1 - P_3 - P_7$

$P_5 = (a+d)(e+h)$ $\qquad$ 7 mults, 18 adds/subs

$P_6 = (b-d)(g+h)$ $\qquad T(n) = 7\,T(n/2) + \Theta(n^2)$

$P_7 = (a-c)(e+f)$ $\qquad T(n) = \Theta(n^{\lg_2 7})$