due: 05.06.2022

Signals and Systems for Computer Engineering

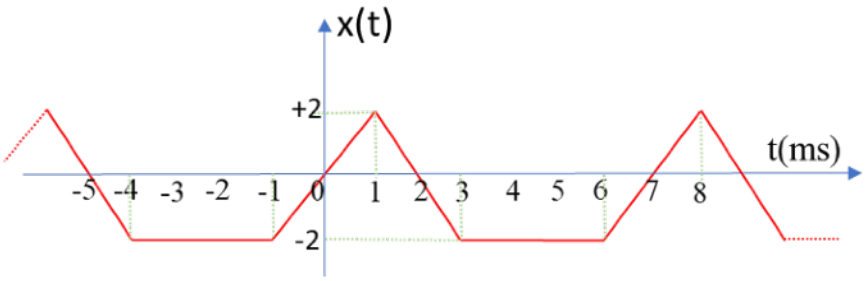
Assignment #4

BLG354E – CRN 21560

1. Find 4-points DFT (Discrete Fourier Transform) of the periodic DT signal $x[n] = \{1, 2, 0, -1\}$ as $X[k] = DFT\{x[n]\}$ where

$$\mathbf{W}_N = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N & W_N^2 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix} \qquad W_N = e^{-j(2\pi/N)}$$

2. $x(t)$ is a periodic continuous-time signal given in the below diagram. $x[n]$ is the discrete time signal that is provided through sampling of $x(t)$ at 1kHz. Find the DFT of $x[n]$ by using decimation-in-time FFT algorithm (8-points FFT).

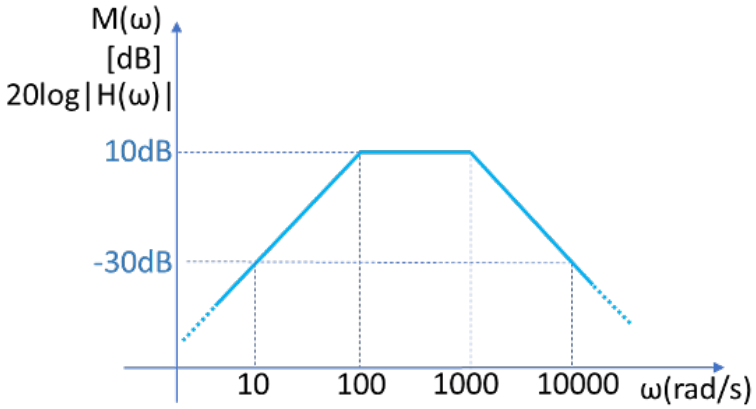


3. Transfer function of a discrete time system H(z) is given as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2z^{-1}}{(1 - 0.5z^{-1})^2}$$

where z^{-1} denotes the unit delay. Find the first 4 values of the output signal sequence $y[n] = \{y[0], y[1], y[2], y[3]\}$ if unit step signal $x[n] = u[n]$ is applied to this system. (initial condition can be considered as zero)

4. Frequency response of a band-pass filter (BPF) is given in the Bode Plot seen below. Find its transfer function $H(s)$.



5. BPF filter defined in question 4 will be implemented on a computer by sampling the input signal at 10kHz. Find the discrete time transfer function $H(z)$ and write the pseudo code that performs it in real-time.

```
Timer interrupt @Ts:
X = Read (ADC)
…




…
Return
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6. Express the output signal $y(t)$ of the BPF system described in question 4 for the input signal $x(t) = 10\ sin(500t)$.
7. The input-output relationship of a discrete time system is given by the difference equation $y[n] = x[n-2] + 2ay[n-1] - a^2y[n-2]$. Find the value interval of “a” that makes this system BIBO stable. (Where x and y denote input and the output respectively)
8. Find the fundamental period of the signal $x[n] = cos^2(\frac{\pi}{12}n)\ sin(\frac{3\pi}{4}n)$.