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Q1.

$$x_1 - x_2 + 3x_3 = 2$$
$$x_1 + x_2 = 4$$
$$3x_1 - 2x_2 + x_3 = 1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 2 \\ 1 & 1 & 0 & 4 \\ 3 & -2 & 1 & 1 \end{array}\right] \xrightarrow{3R_2 - R_3 = R_2} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 2 \\ 0 & 2 & -3 & 2 \\ 3 & -2 & 1 & 1 \end{array}\right] \xrightarrow{3R_1 - R_3 = R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 2 \\ 0 & 2 & -3 & 2 \\ 0 & 1 & -8 & 5 \end{array}\right] \xrightarrow{-\frac{1}{2}R_3 + R_2 = R_3} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 2 \\ 0 & 2 & -3 & 2 \\ 0 & 0 & -6,5 & -5 \end{array}\right]$$

→ backward substitution.

$-6x_3 = -6,5 \rightarrow x_3 = \frac{13}{12}$

$2x_2 - 3x_3 = 2 \quad x_2 = \frac{31}{13}$

$x_1 - x_2 + 3x_3 = 2 \rightarrow x_1 = \frac{21}{13}$

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Q2 $\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$

a) $PA = LU$ $\quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 0 & 0 \\ l_{41} & l_{42} & l_{43} & 0 \end{bmatrix}$

$\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ → changed row3 and row4 → all the L's are "0"

$U = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

→ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = P$ multiplied with (P)

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = L$

b) $b = (26, 9, 1, -3)^T \quad Ax = b$

$\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 26 \\ 9 \\ 1 \\ -3 \end{bmatrix}$

$5x_1 + 6x_2 + 7x_3 + 8x_4 = 26$

$4x_2 + 3x_3 + 2x_4 = 9$

$x_4 = 1$

$-x_3 - 2x_4 = -3$

$x_4 = 1 \quad x_3 = 1 \quad x_2 = 1 \quad x_1 = 1$

$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$