BLG 336E - Analysis of Algorithms II

Recitation 5

29 March 2022

Greedy Algorithms:
Kruskal’s and Prim’s
Algorithms

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Outline

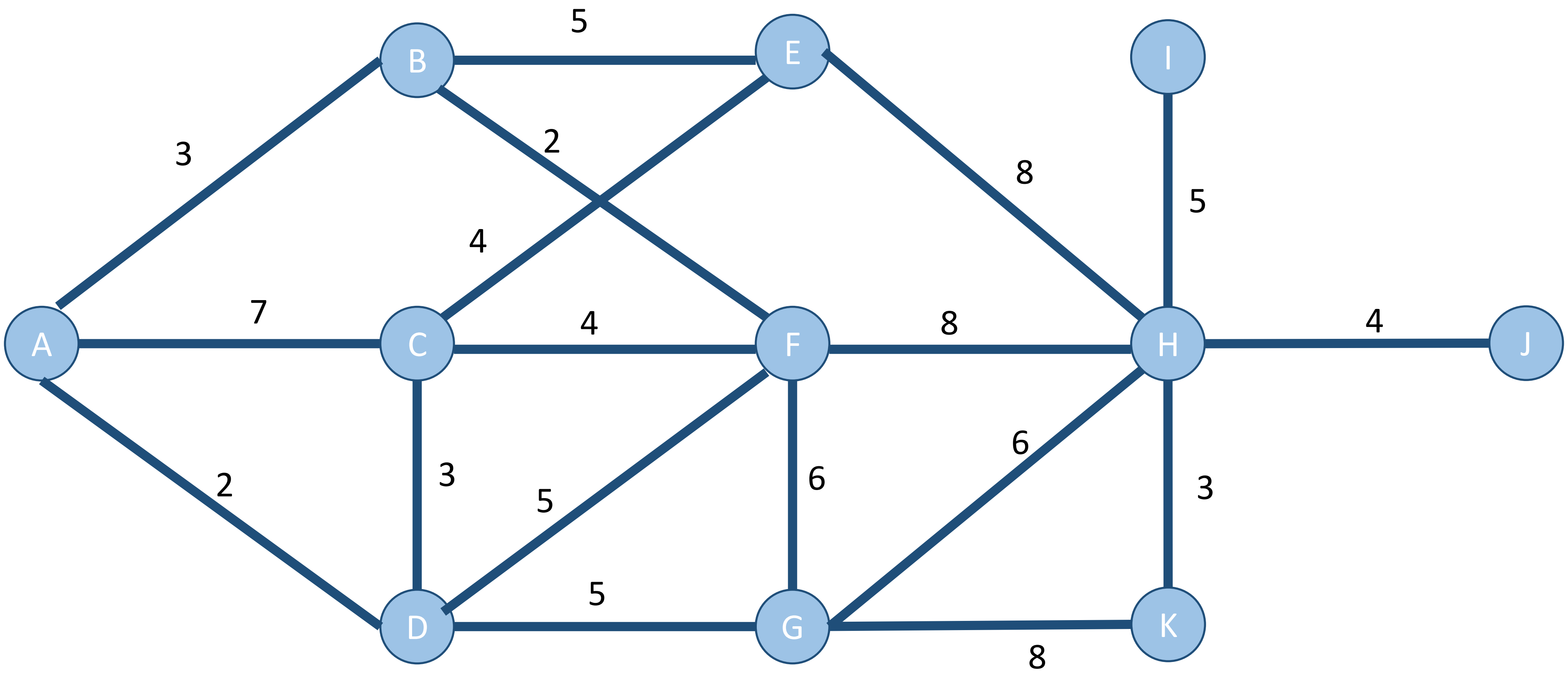
1. Kruskal’s Algorithm
2. Prim’s Algorithm
3. Brain Storming With Questions

Joseph Kruskal (1928 - 2010)

Robert C. Prim (1921 -)

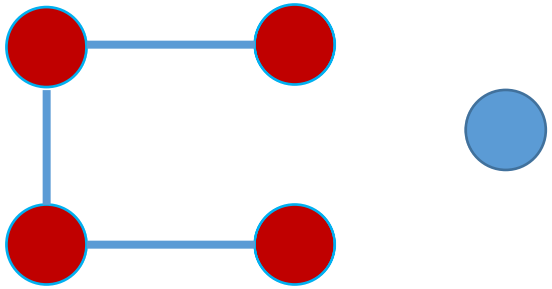
Today's Big Question:

What is the Minimum Spanning Tree (MST) of the following graph?

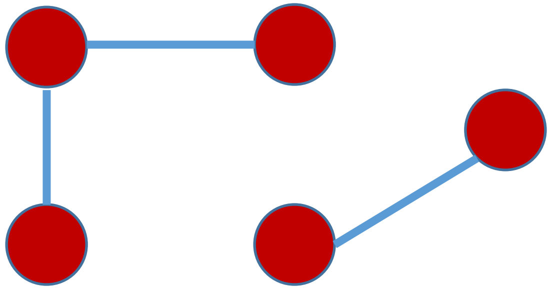


Kruskal’s Algorithm

- Sort the edges by their weights in ascending order
- Pick the lightest edge and add to the MST
- While there exists unvisited nodes of the graph OR resulting MST is not connected:
    - Add the next lightest edge to the tree UNLESS adding it causes a cycle on the tree
- Halt



Continue. Because not every node is visited.



Continue. Because the tree is not connected.

Kruskal’s Algorithm

- Sort the edges by their weights in ascending order
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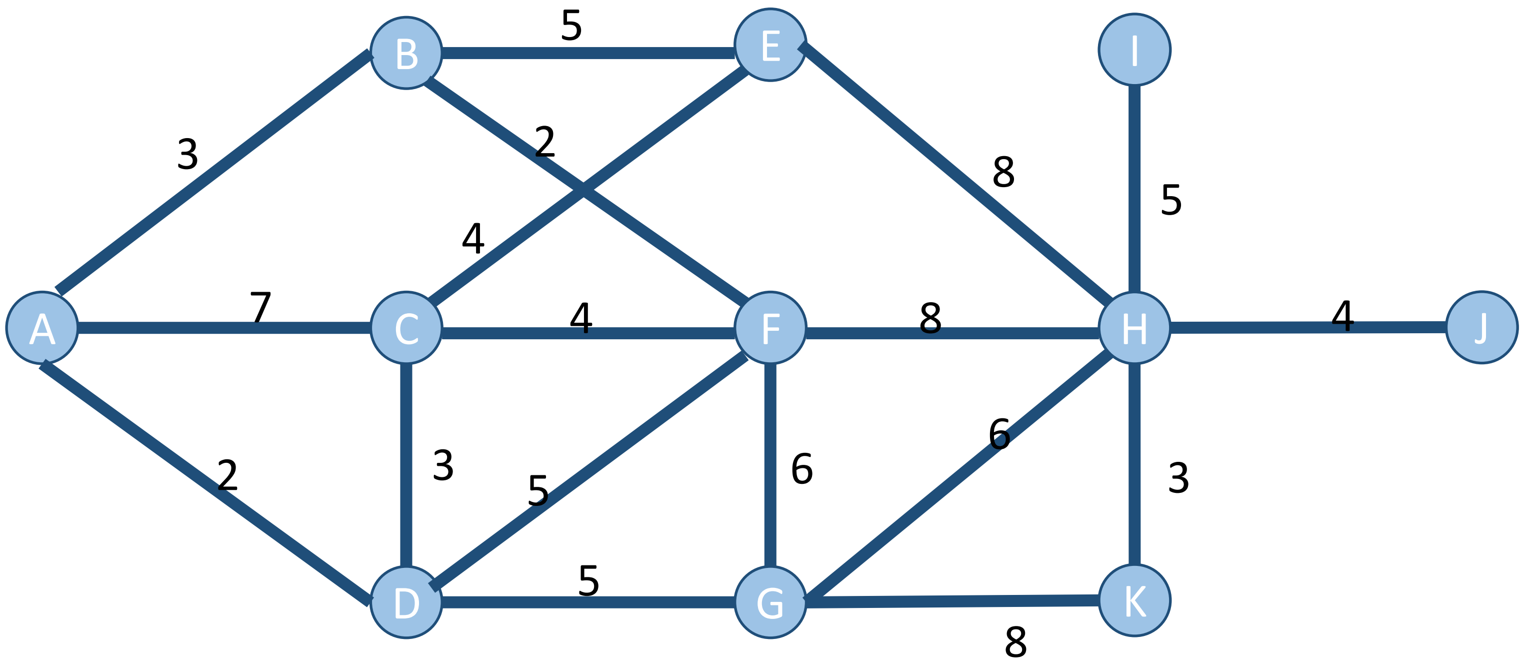


Vertex 1	Vertex 2	Weight
A	B	3
A	C	7
A	D	2
B	E	5
B	F	2
C	E	4
C	F	4
C	D	3
D	F	5
D	G	5
E	H	8
F	H	8
F	G	6
G	K	8
G	H	6
H	I	5
H	J	4
H	K	3

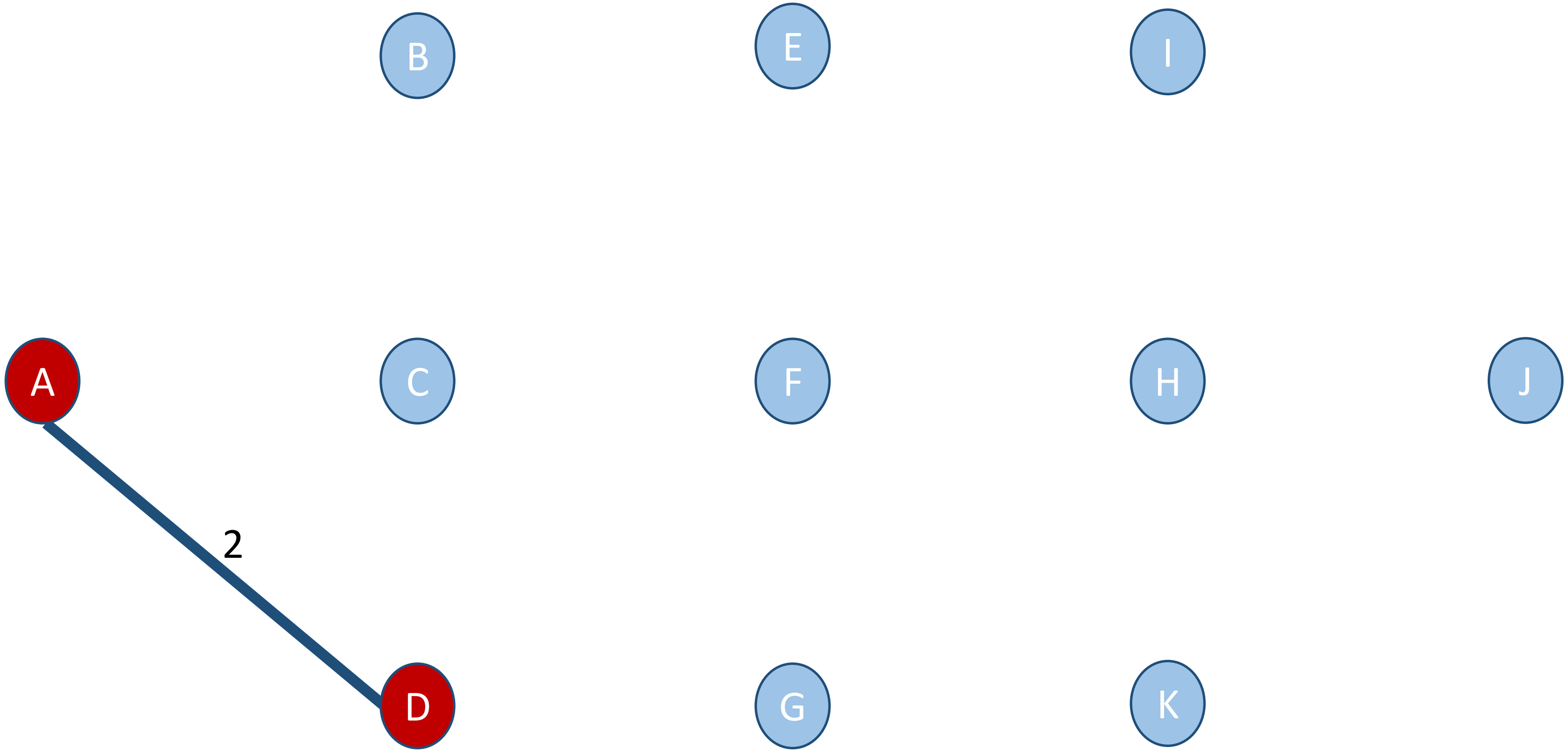
SORT BY EDGE WEIGHT

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

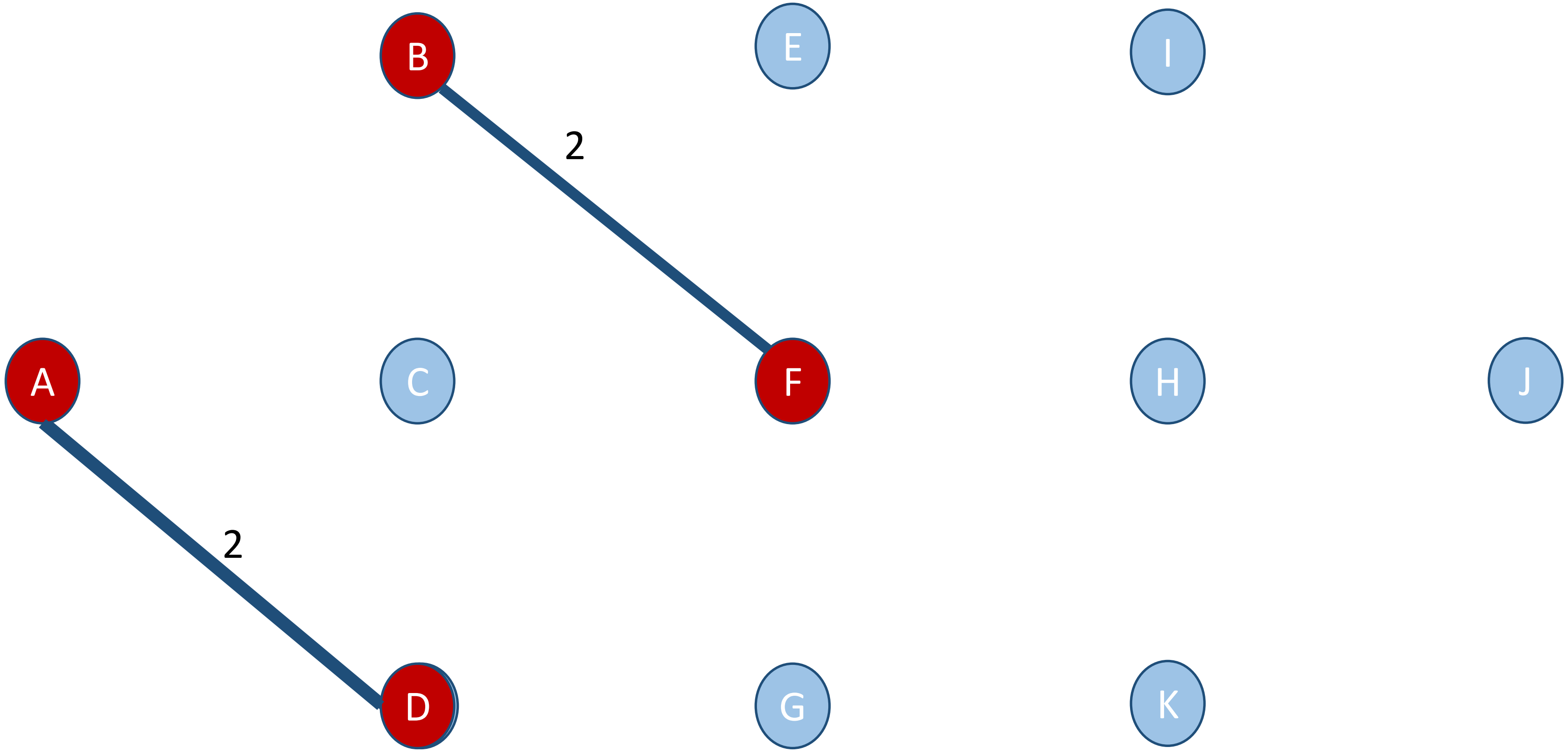
Does adding (A,D) cause a cycle?



KRUSKAL’S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

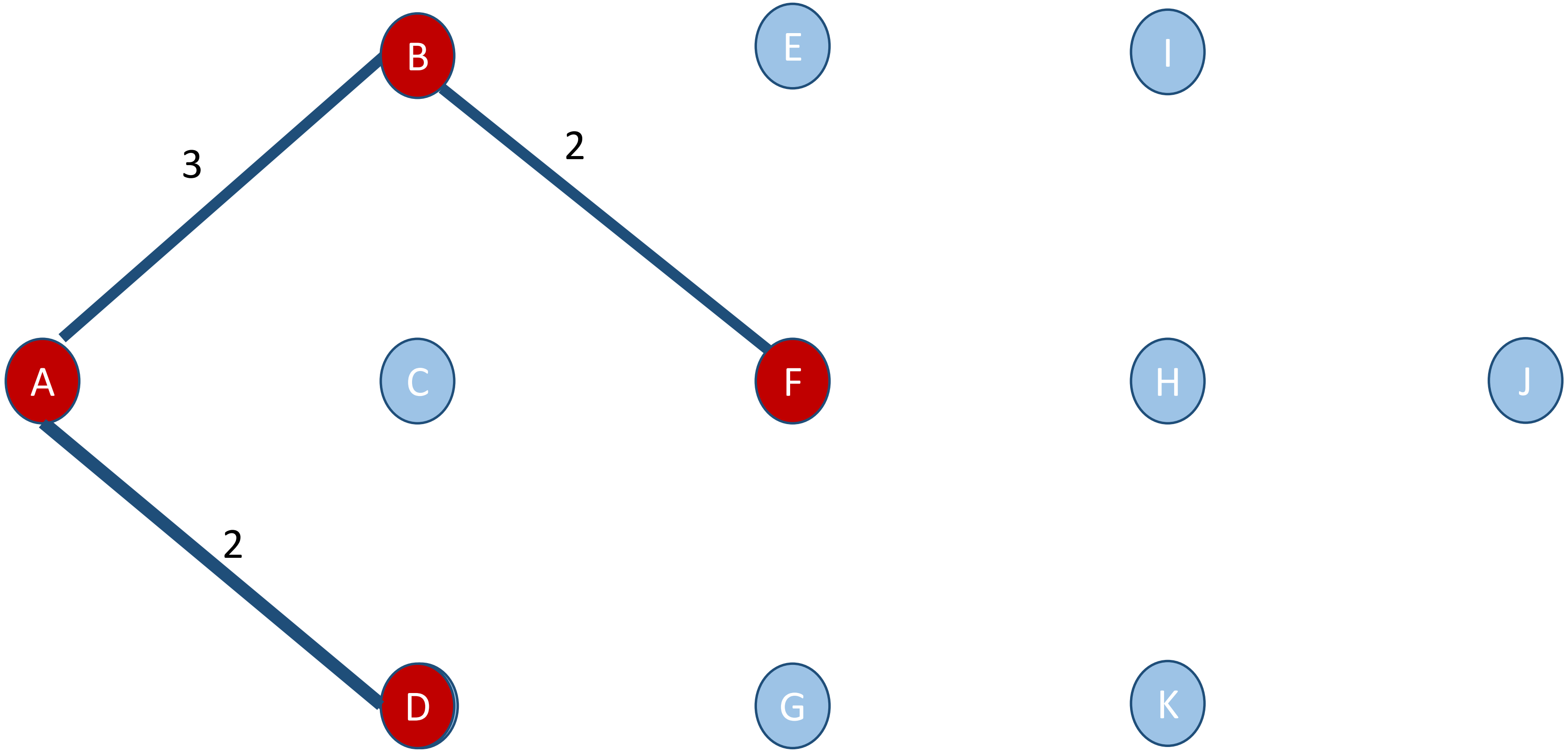
Does adding (B,F) cause a cycle?



KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

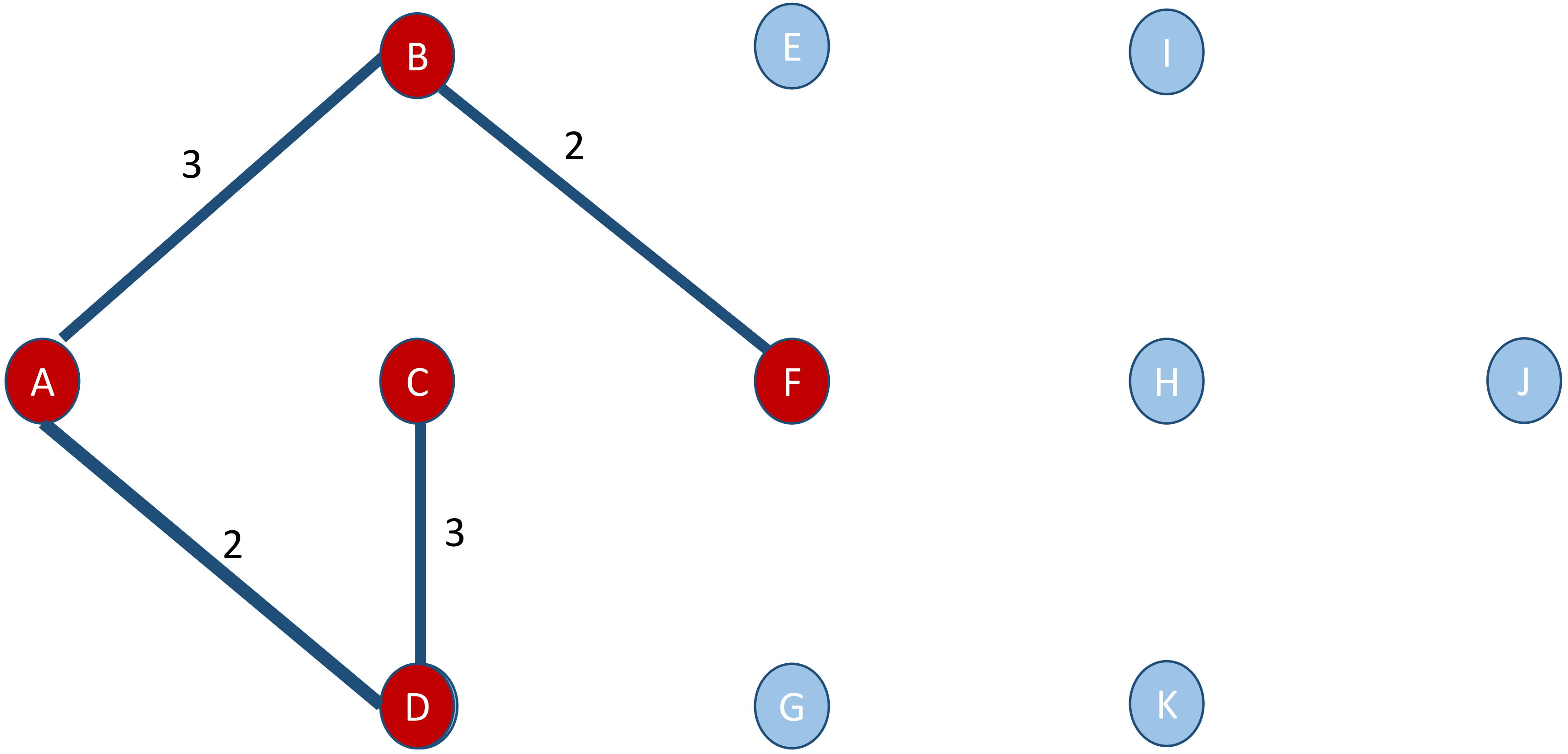
Does adding (A,B) cause a cycle?



KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

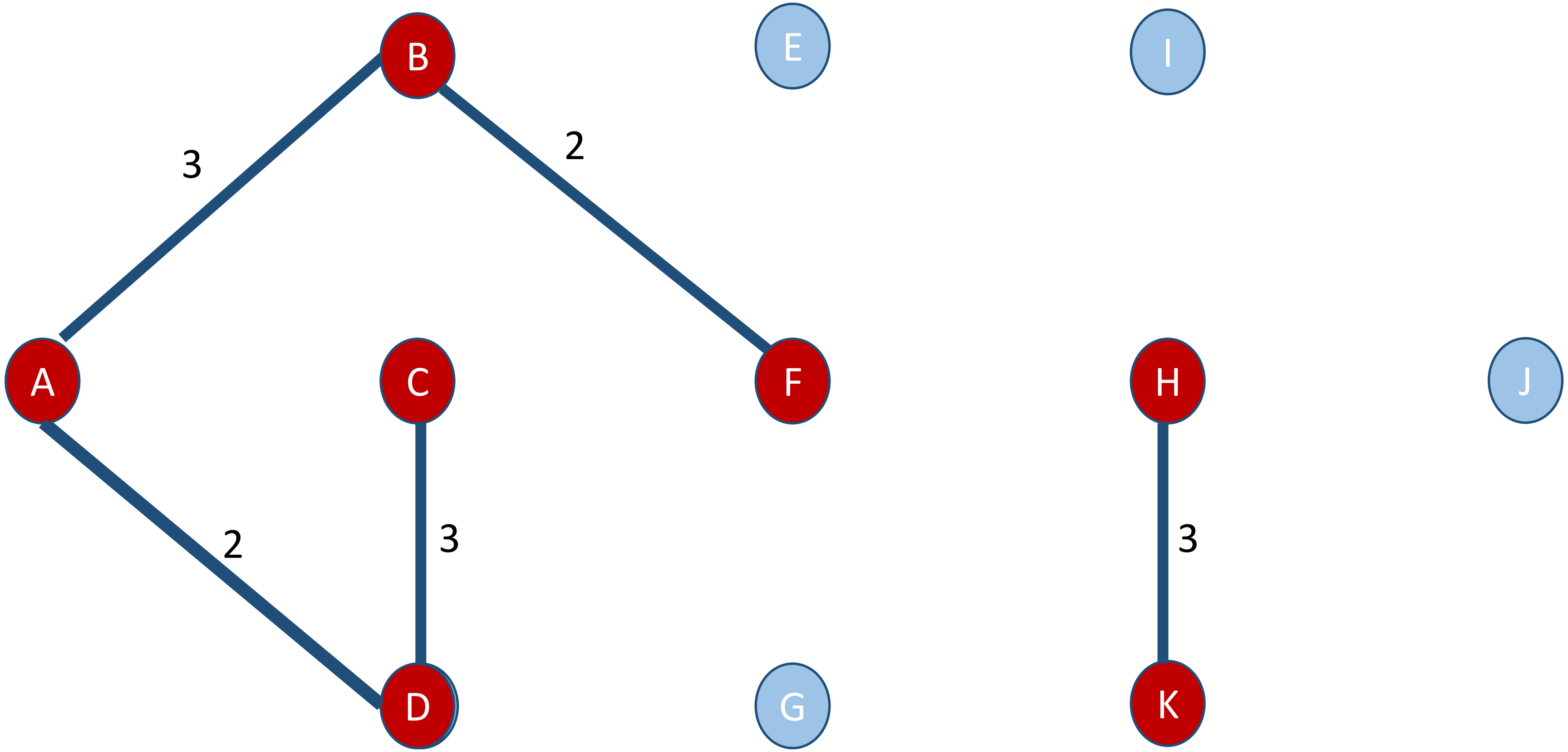
Does adding (C,D) cause a cycle?



KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

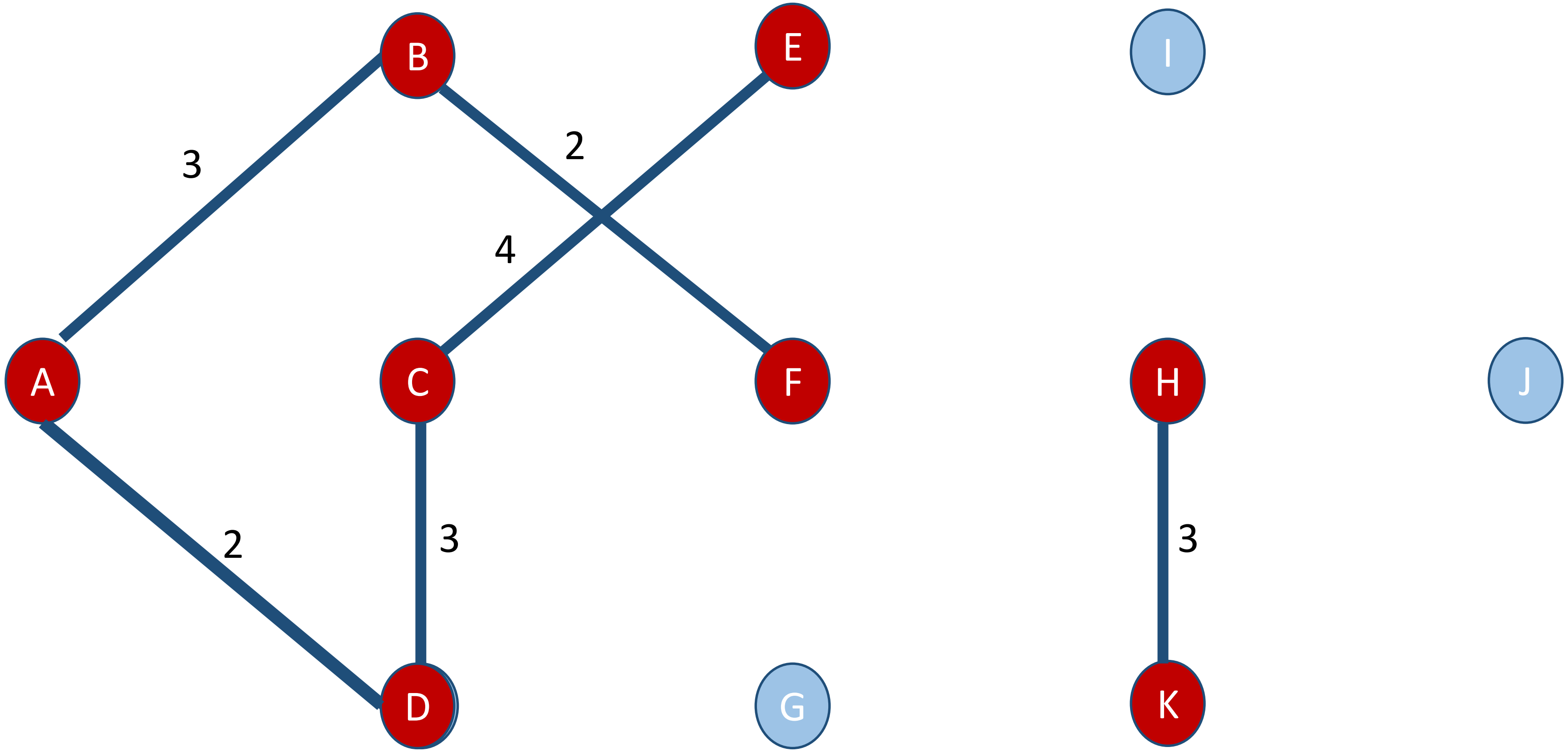
Does adding (H,K) cause a cycle?



KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

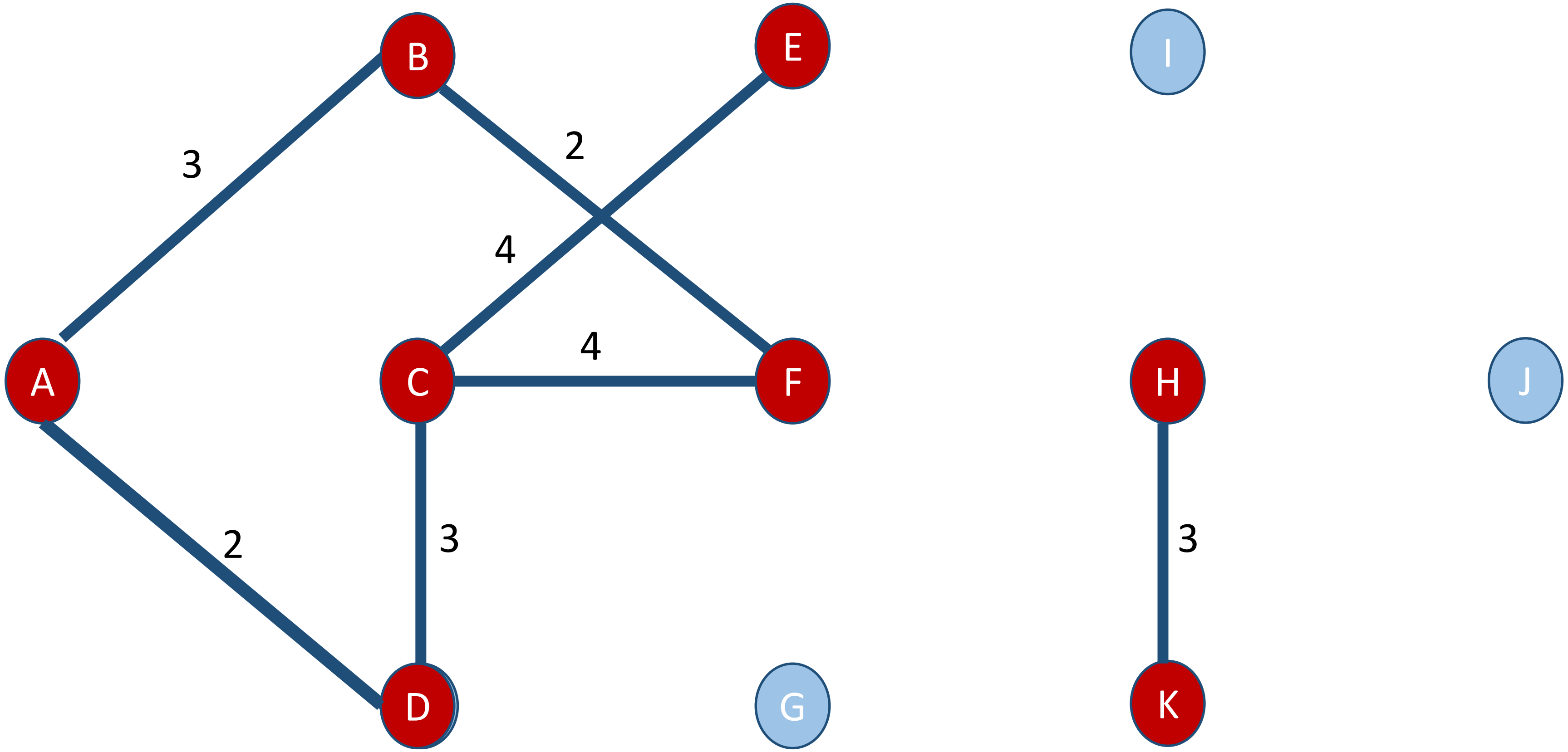
Does adding (C,E) cause a cycle?



KRUSKAL’S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

Does adding (C,F) cause a cycle?



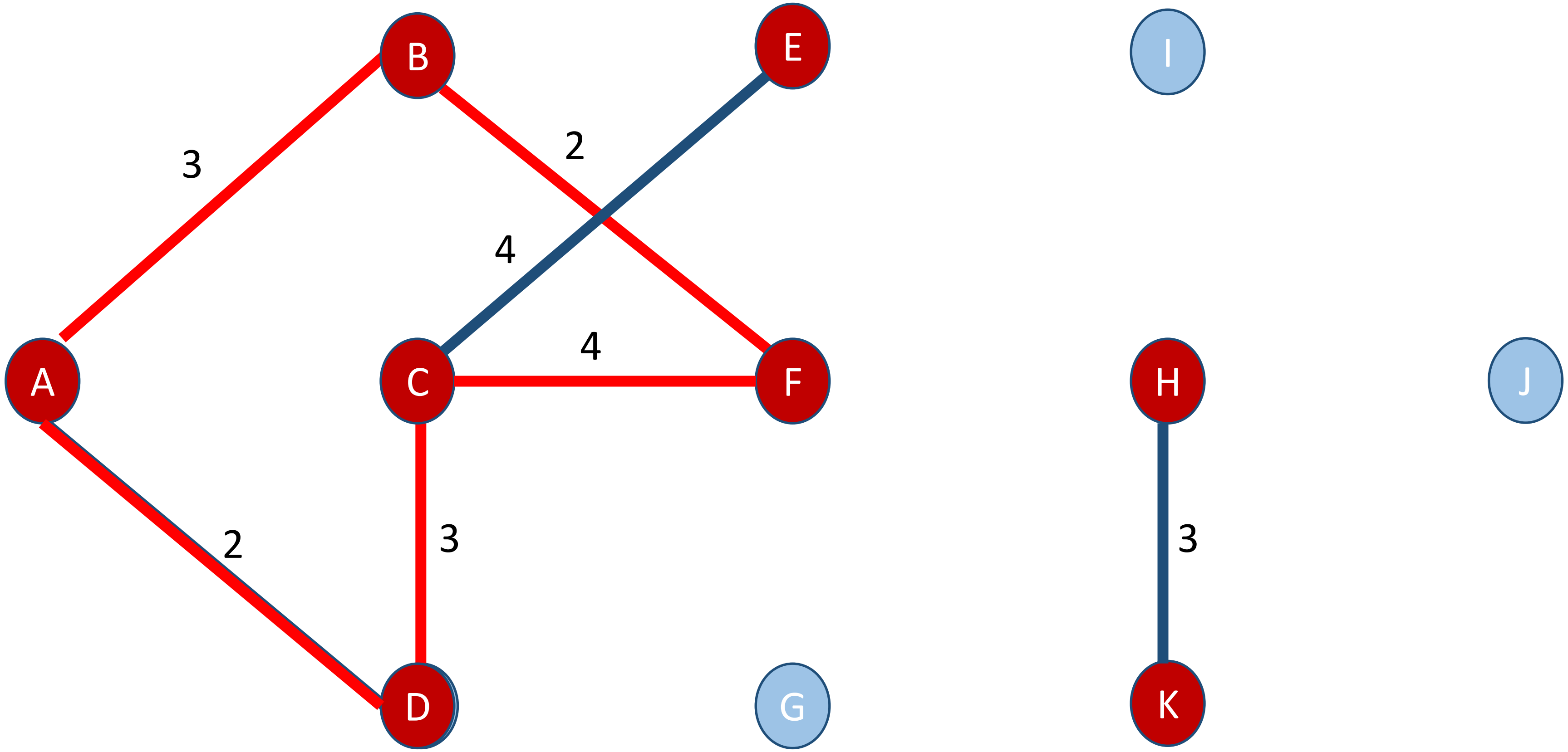
KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

Does adding (C,F) cause a cycle?

CYCLE!

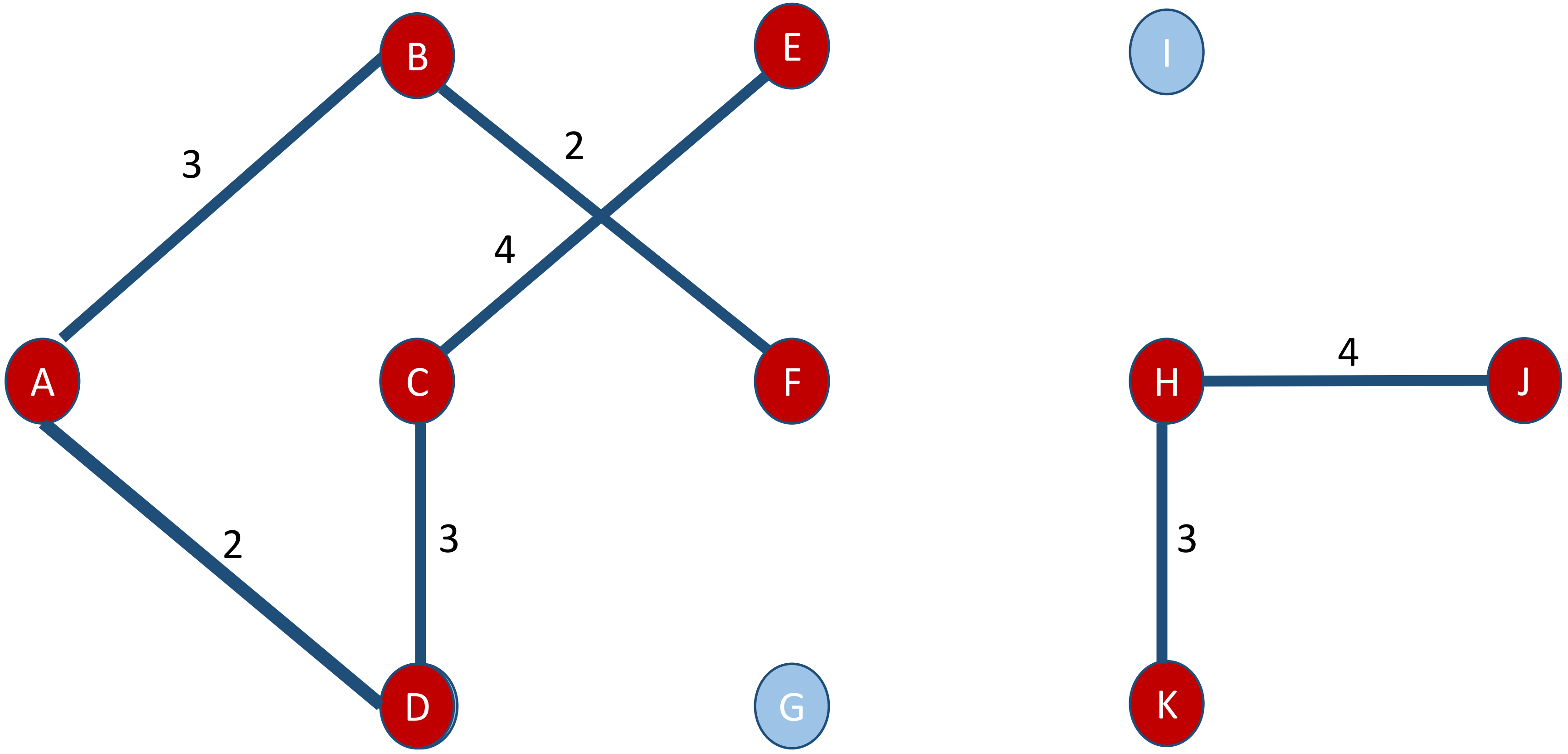
Don't add this edge to the minimum spanning tree!



KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

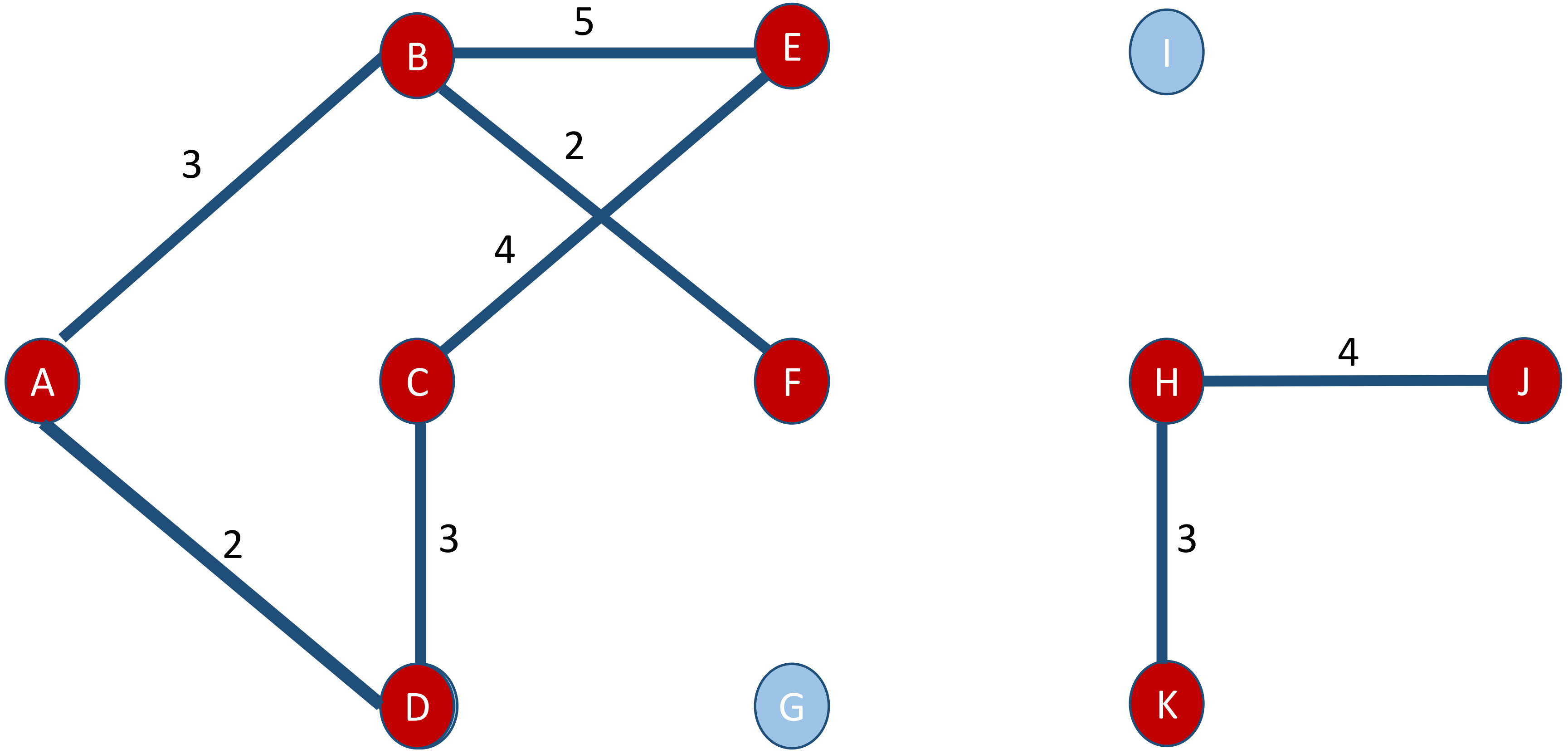
Does adding (H,J) cause a cycle?



KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

Does adding (B,E) cause a cycle?



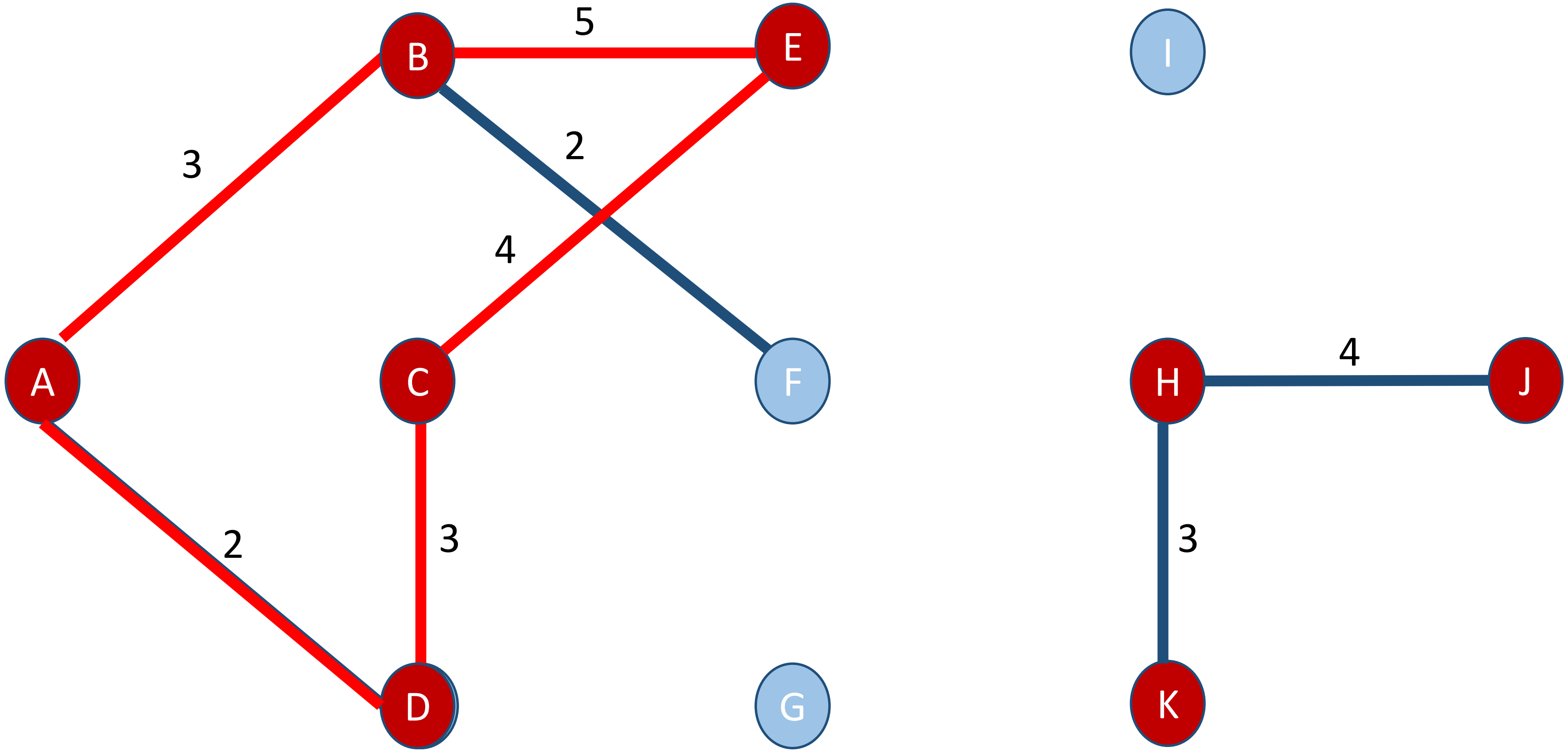
KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

Does adding (B,E) cause a cycle?

CYCLE!

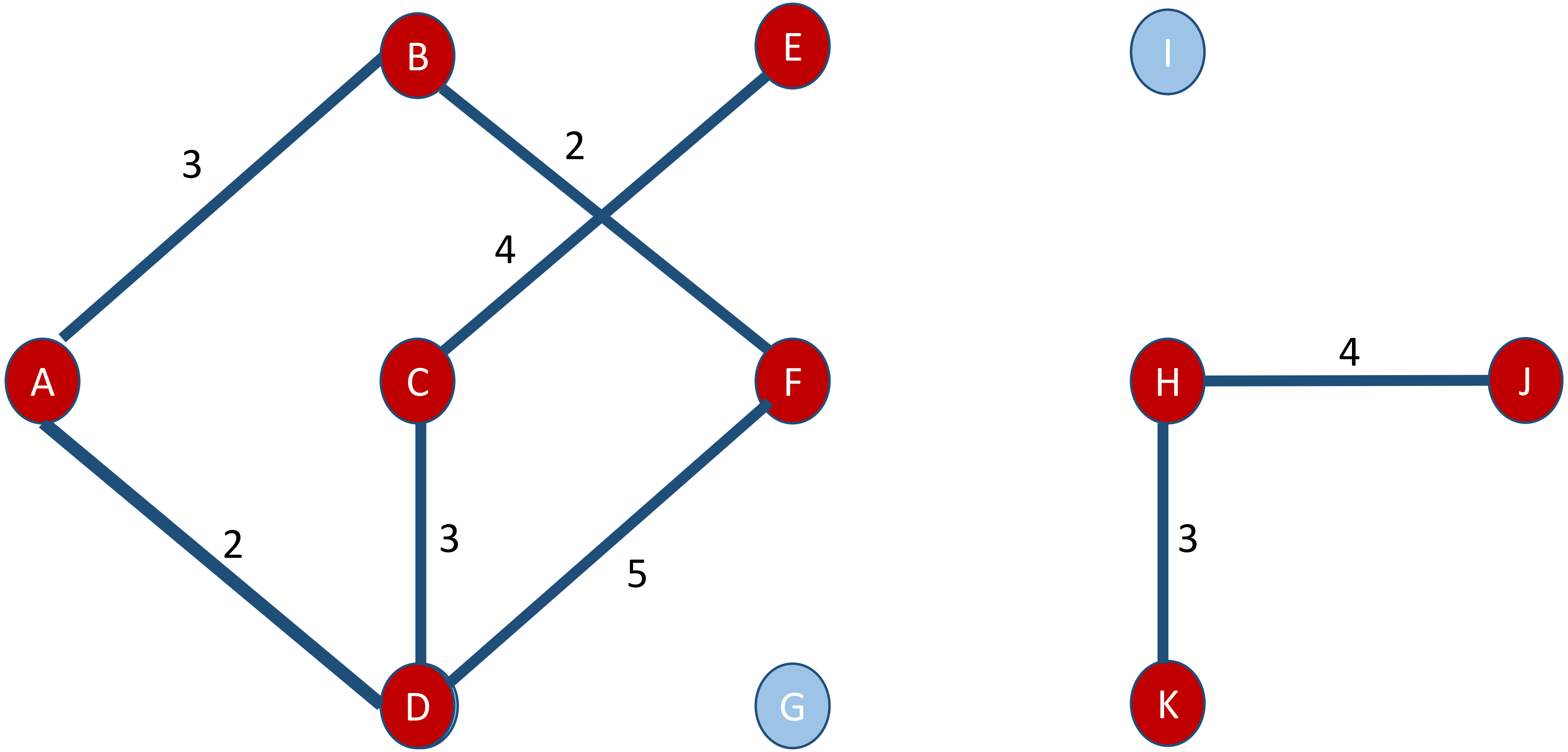
Don't add this edge to the minimum spanning tree!



KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

Does adding (D,F) cause a cycle?



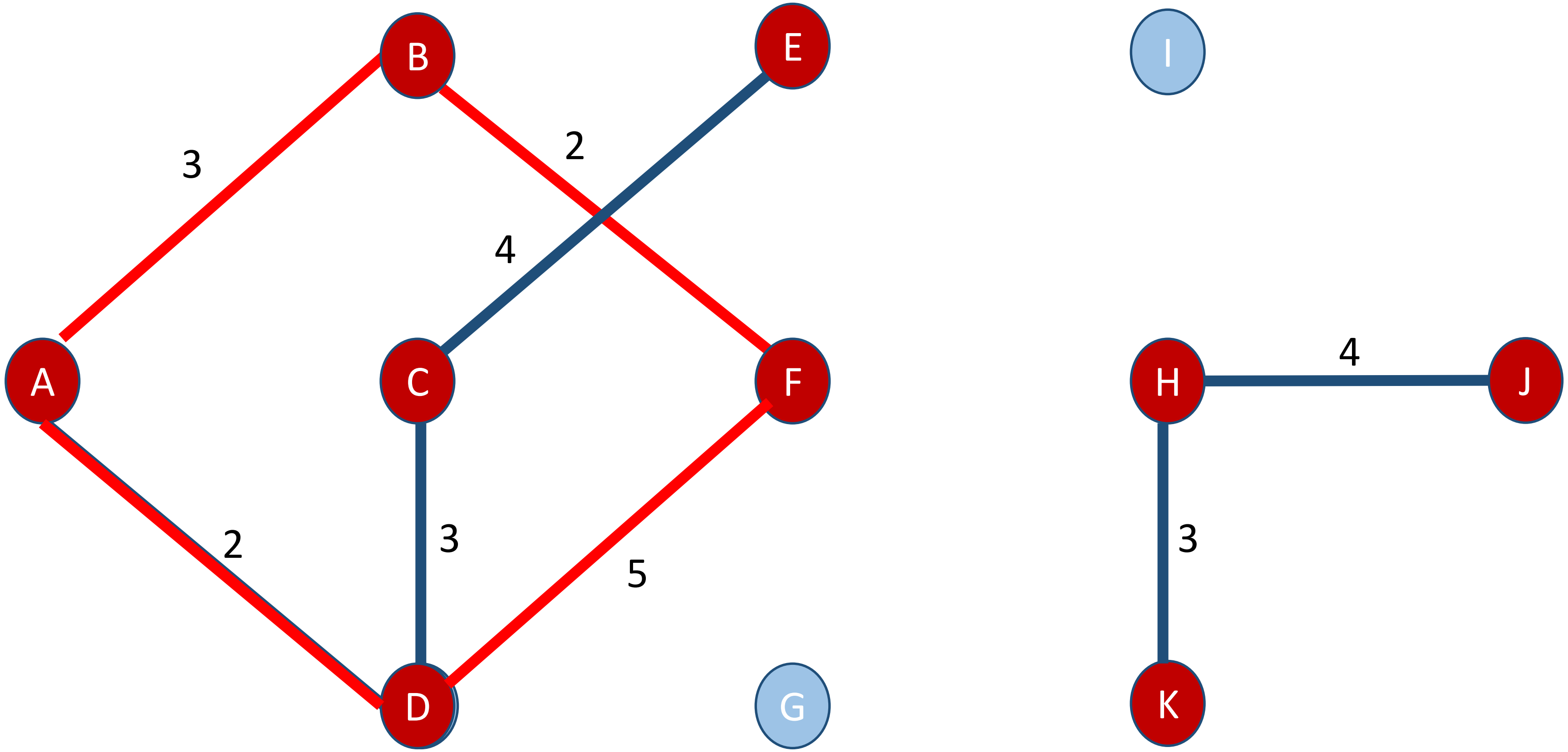
KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

Does adding (D,F) cause a cycle?

CYCLE!

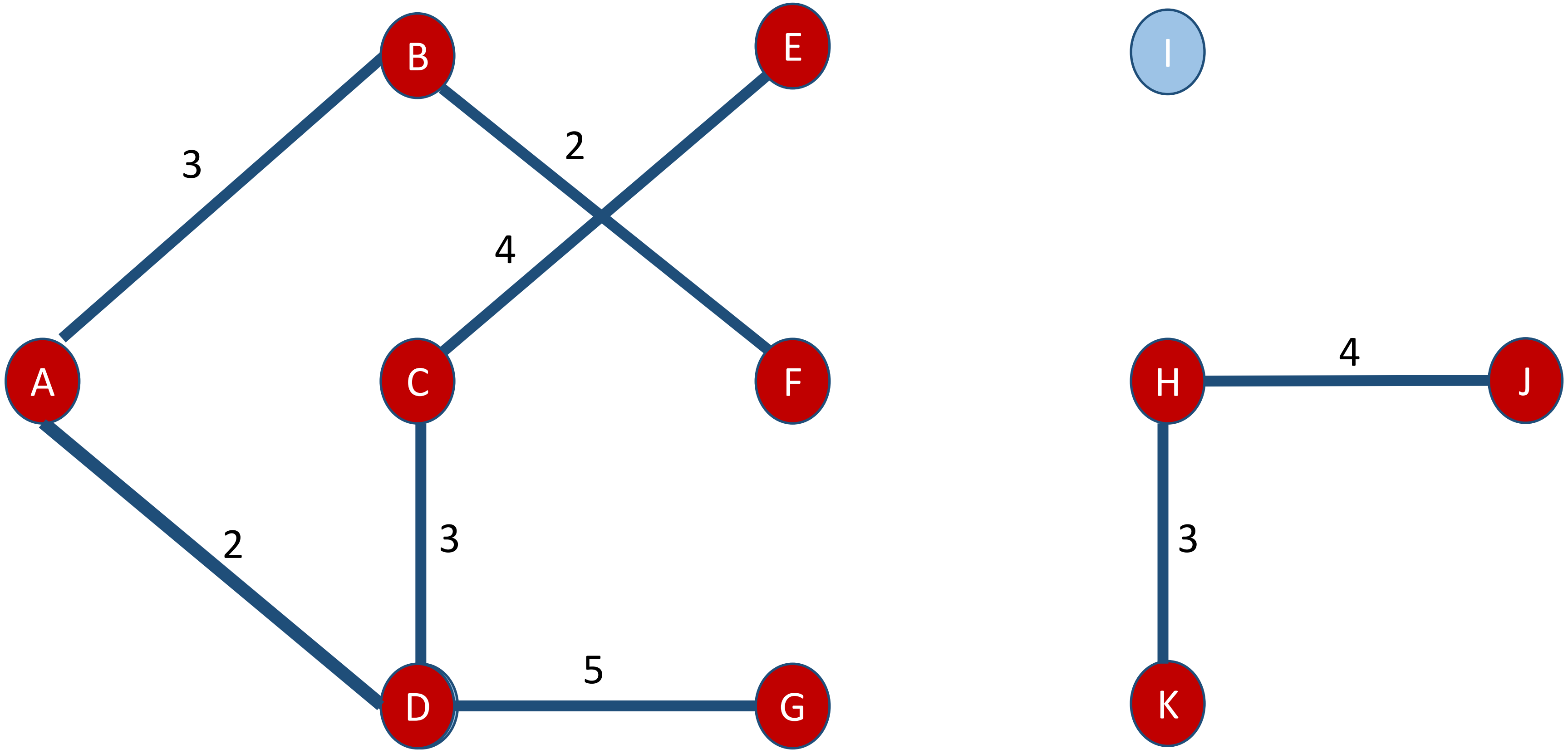
Don't add this edge to the minimum spanning tree!



KRUSKAL’S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

Does adding (D,F) cause a cycle?



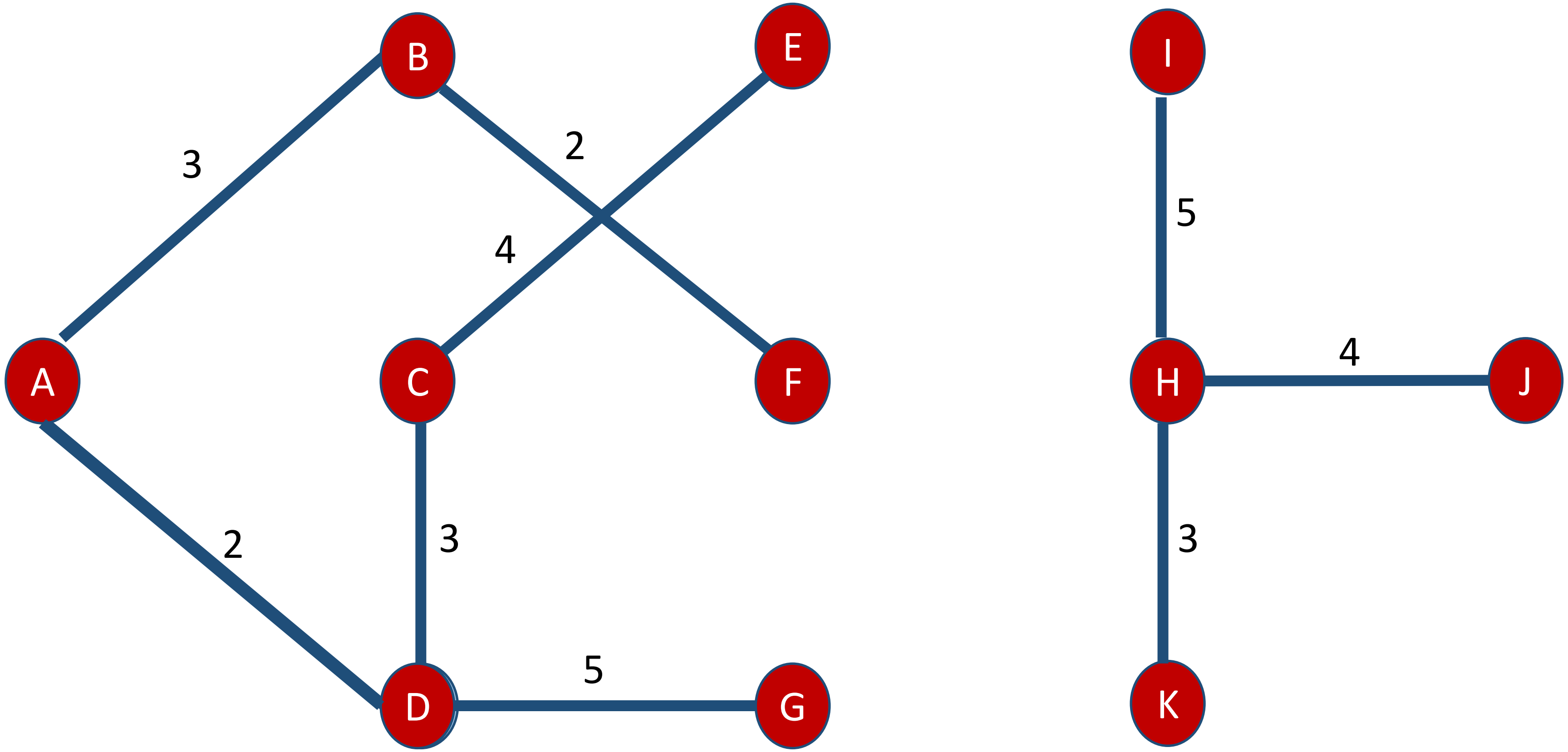
KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

Does adding (H,I) cause a cycle?

ARE WE DONE?

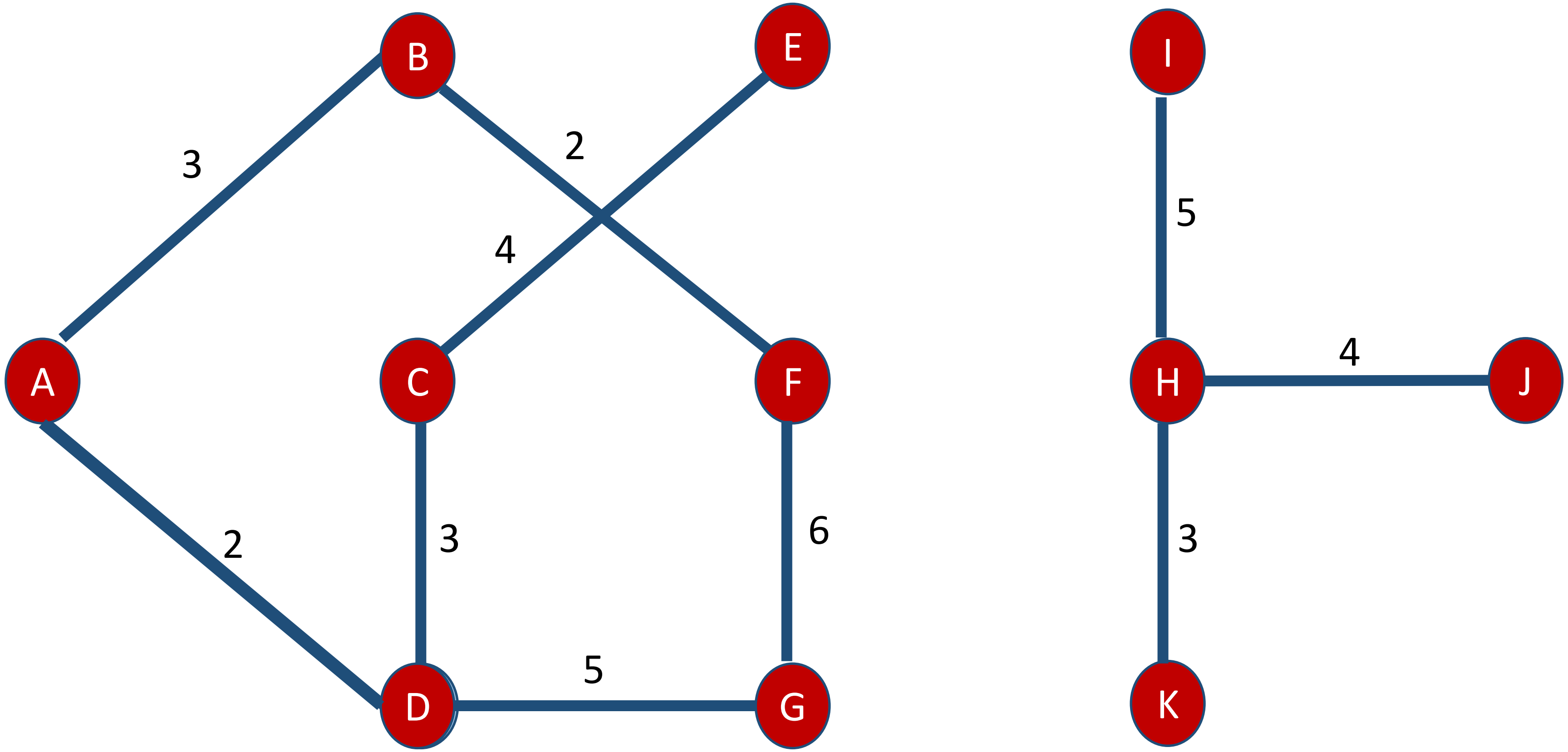
No, the tree is not connected right now. Continue!



KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

Does adding (F,G) cause a cycle?



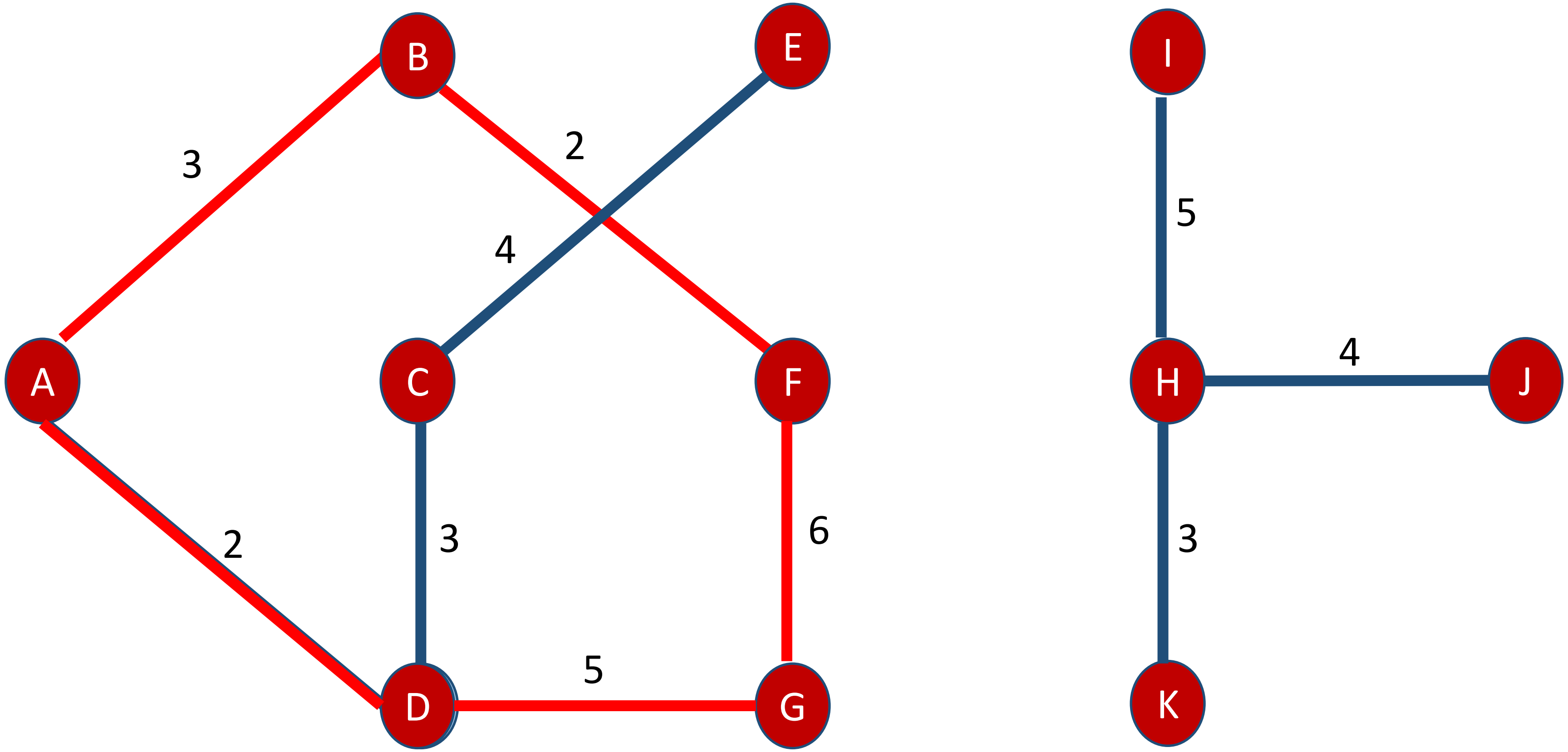
KRUSKAL'S ALGORITHM

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

Does adding (F,G) cause a cycle?

CYCLE!

Don't add this edge to the minimum spanning tree!



KRUSKAL'S ALGORITHM

ARE WE DONE?

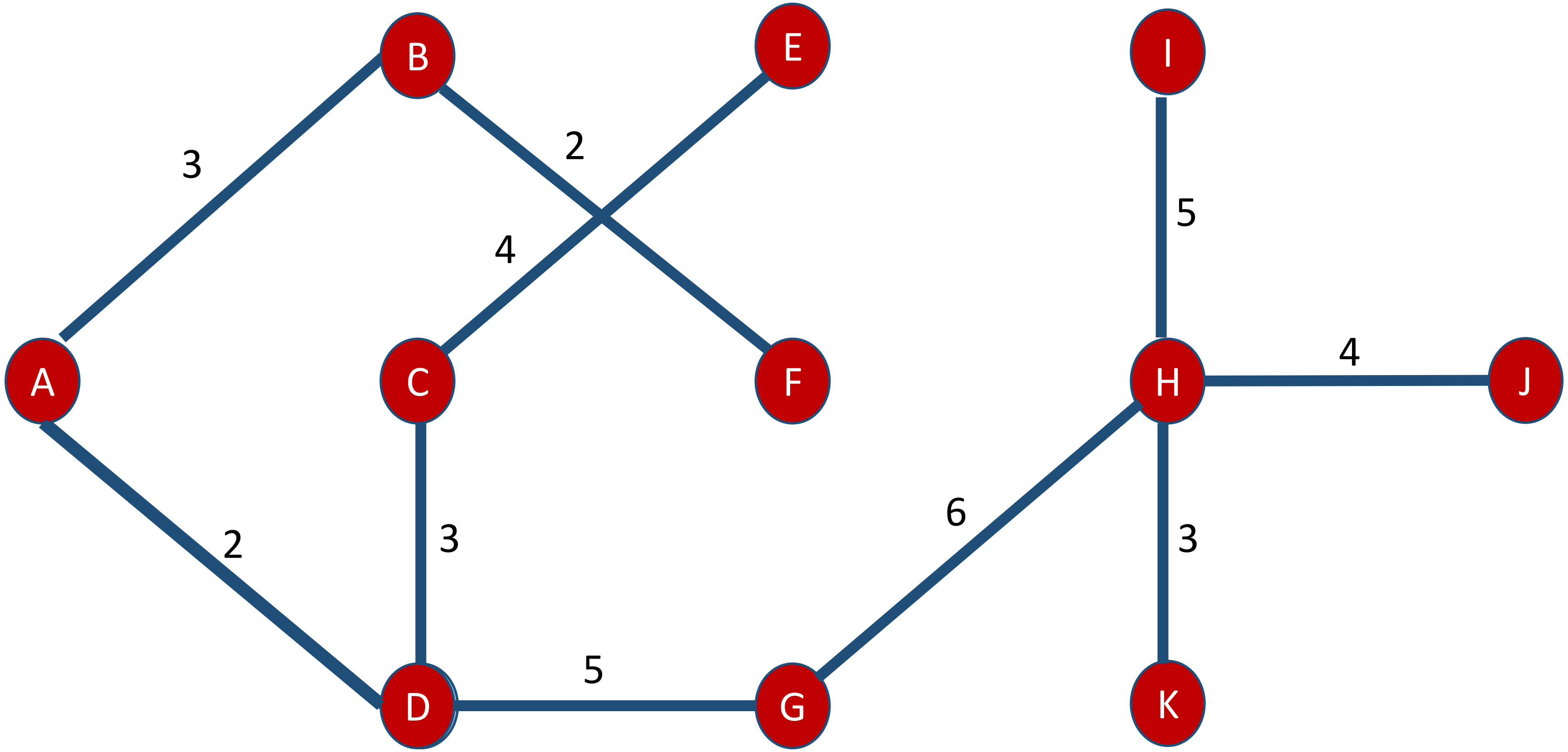
YES!

The graph is connected and all nodes are visited.

Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

The rest would just add cycles.

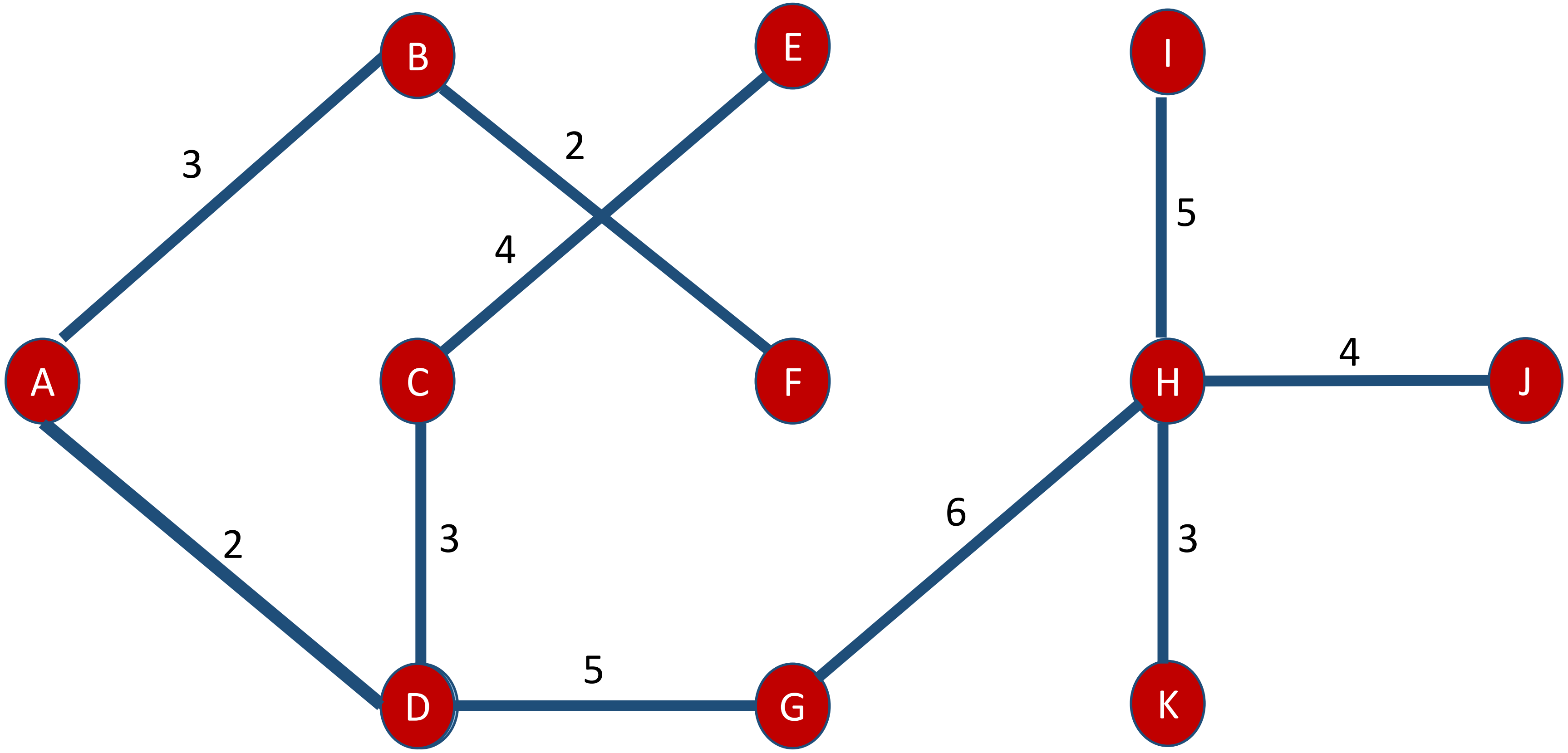
Does adding (G,H) cause a cycle?



KRUSKAL'S ALGORITHM

Weight of the minimum spanning tree (MST) = ?

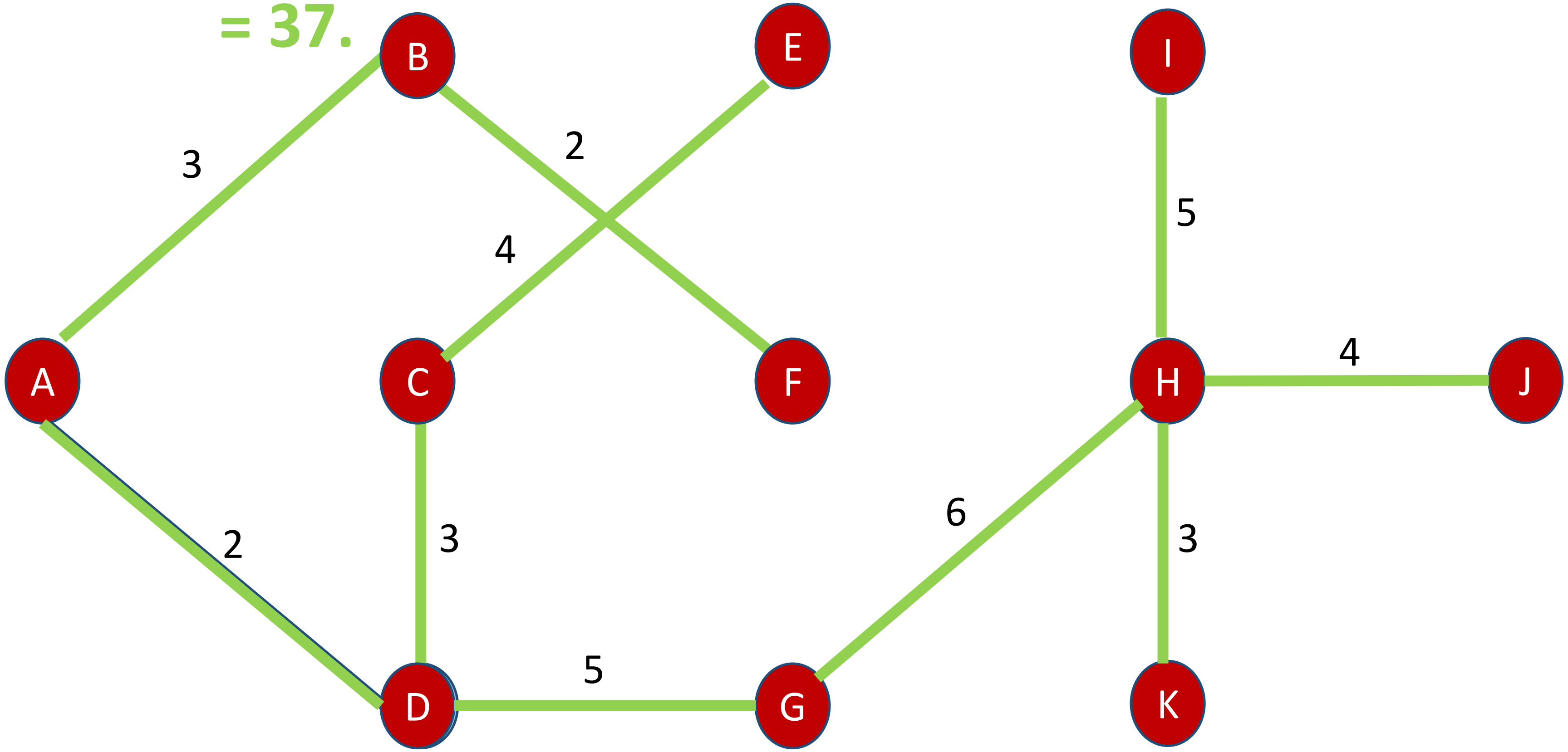
Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8



KRUSKAL’S ALGORITHM

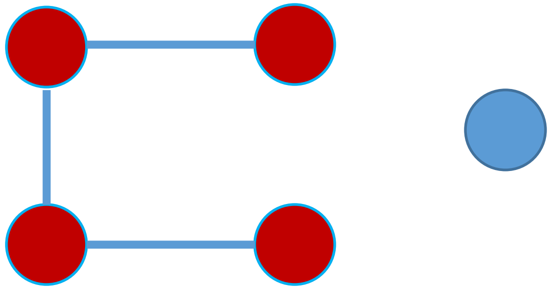
Vertex 1	Vertex 2	Weight
A	D	2
B	F	2
A	B	3
C	D	3
H	K	3
C	E	4
C	F	4
H	J	4
B	E	5
D	F	5
D	G	5
H	I	5
F	G	6
G	H	6
A	C	7
E	H	8
F	H	8
G	K	8

Weight of the minimum spanning tree (MST)
= Sum of all weights of these edges
= 3 + 2 + 2 + 4 + 3 + 5 + 6 + 3 + 5 + 4
= 37.

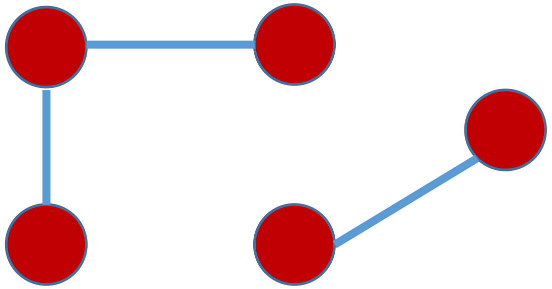


Prim’s Algorithm

- Pick a node at random
- Pick the lightest edge that has a connection with the selected node
- While there exists unvisited nodes of the graph OR resulting MST is not connected:
  - Pick the lightest edge which has a direct connection to one of the nodes on the existing MST
  - Add it to the MST UNLESS it causes a cycle on the tree
- Halt



Continue. Because not every node is visited.



Continue. Because the tree is not connected.

Prim’s Algorithm

- Pick a node at random
- Pick the lightest edge that has a connection with the selected node
- While there exists unvisited nodes of the graph OR resulting MST is not connected:
    - Pick the lightest edge which has a direct connection to one of the nodes on the existing MST
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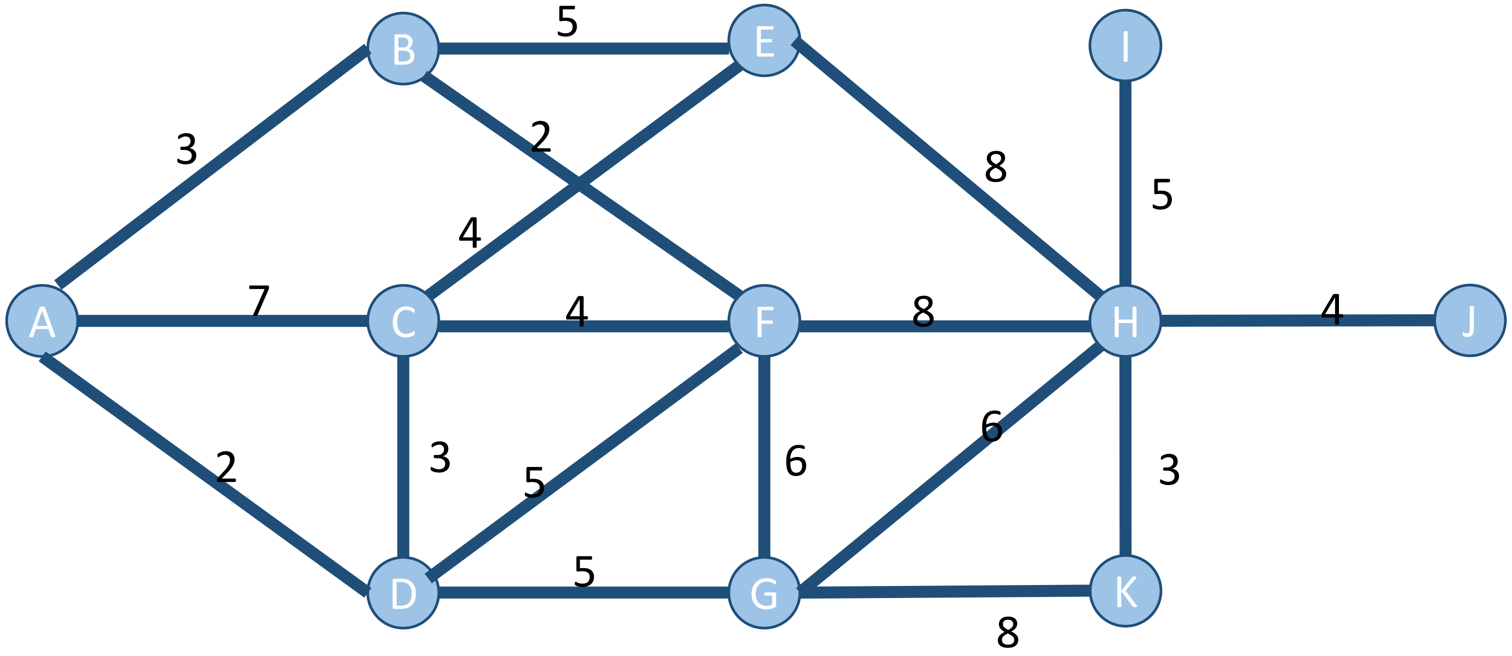


Vertex 1	Vertex 2	Weight
A	B	3
A	C	7
A	D	2
B	E	5
B	F	2
C	E	4
C	F	4
C	D	3
D	F	5
D	G	5
E	H	8
F	H	8
F	G	6
G	K	8
G	H	6
H	I	5
H	J	4
H	K	3

GROUP BY EDGE,
SORT BY WEIGHT

V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
I	H	5

V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

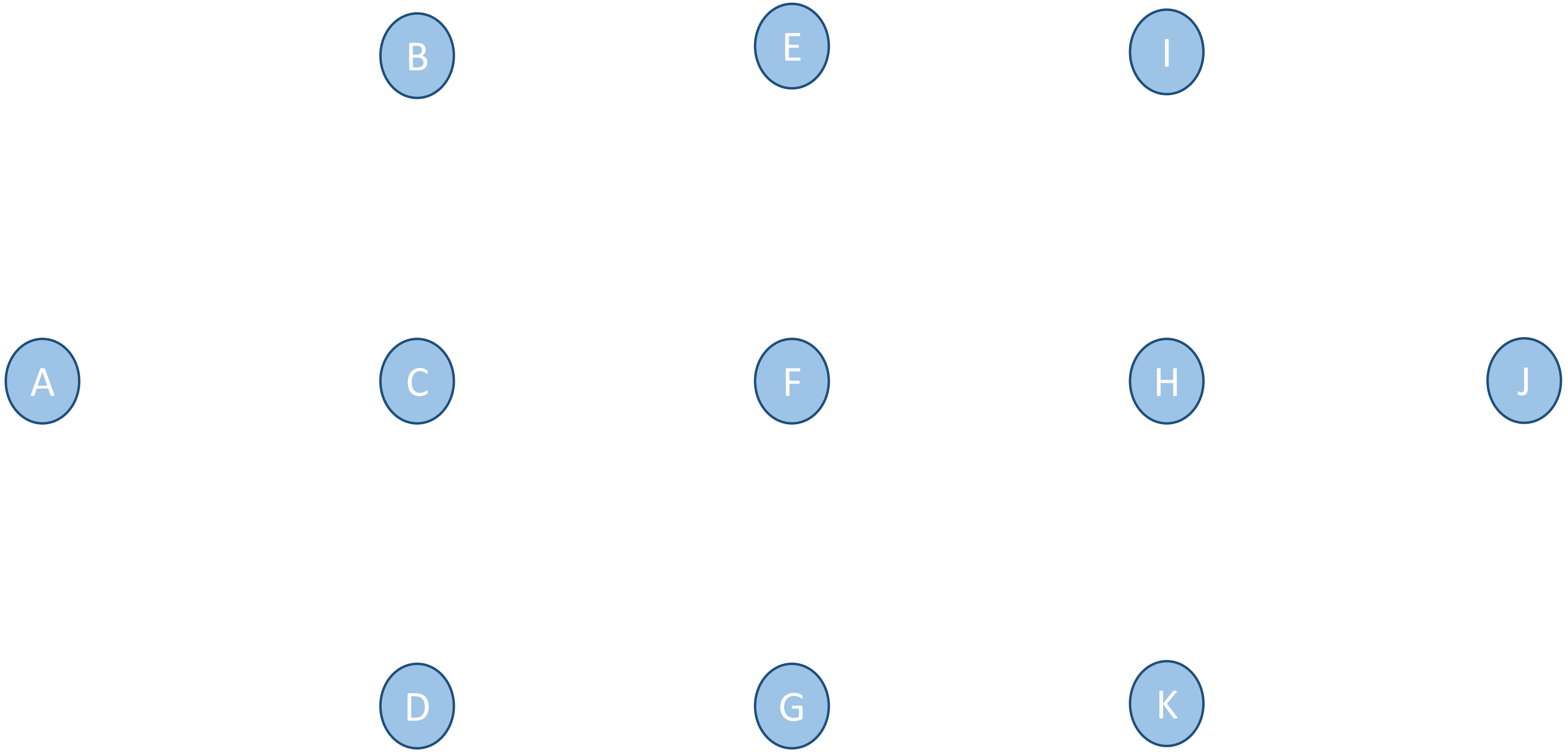
V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Start with the minimum weight edge.

If there are several, select one randomly.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

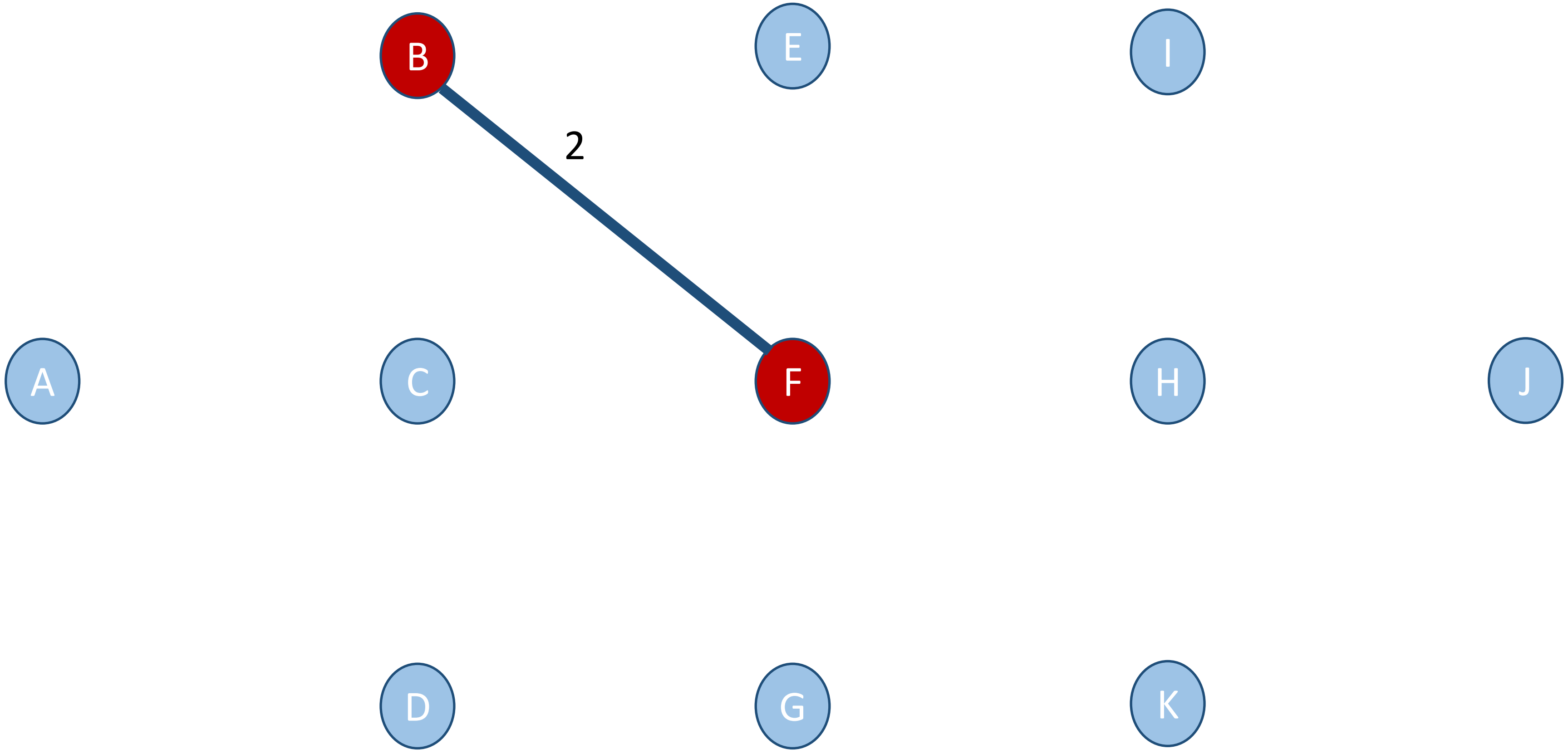
V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Start with the minimum weight edge.

If there are several, select one randomly.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

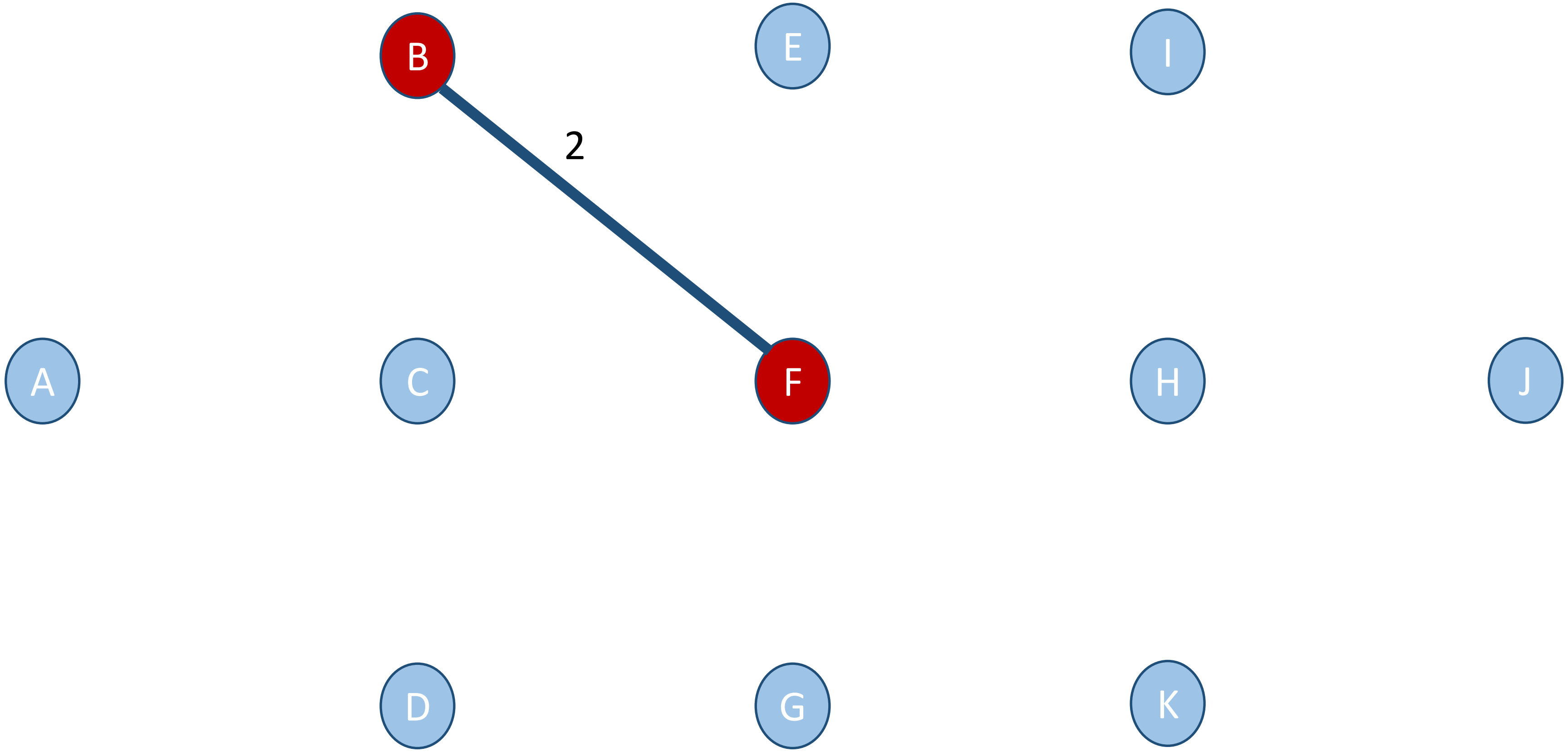
V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B and F. Find the next minimum edge weight.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

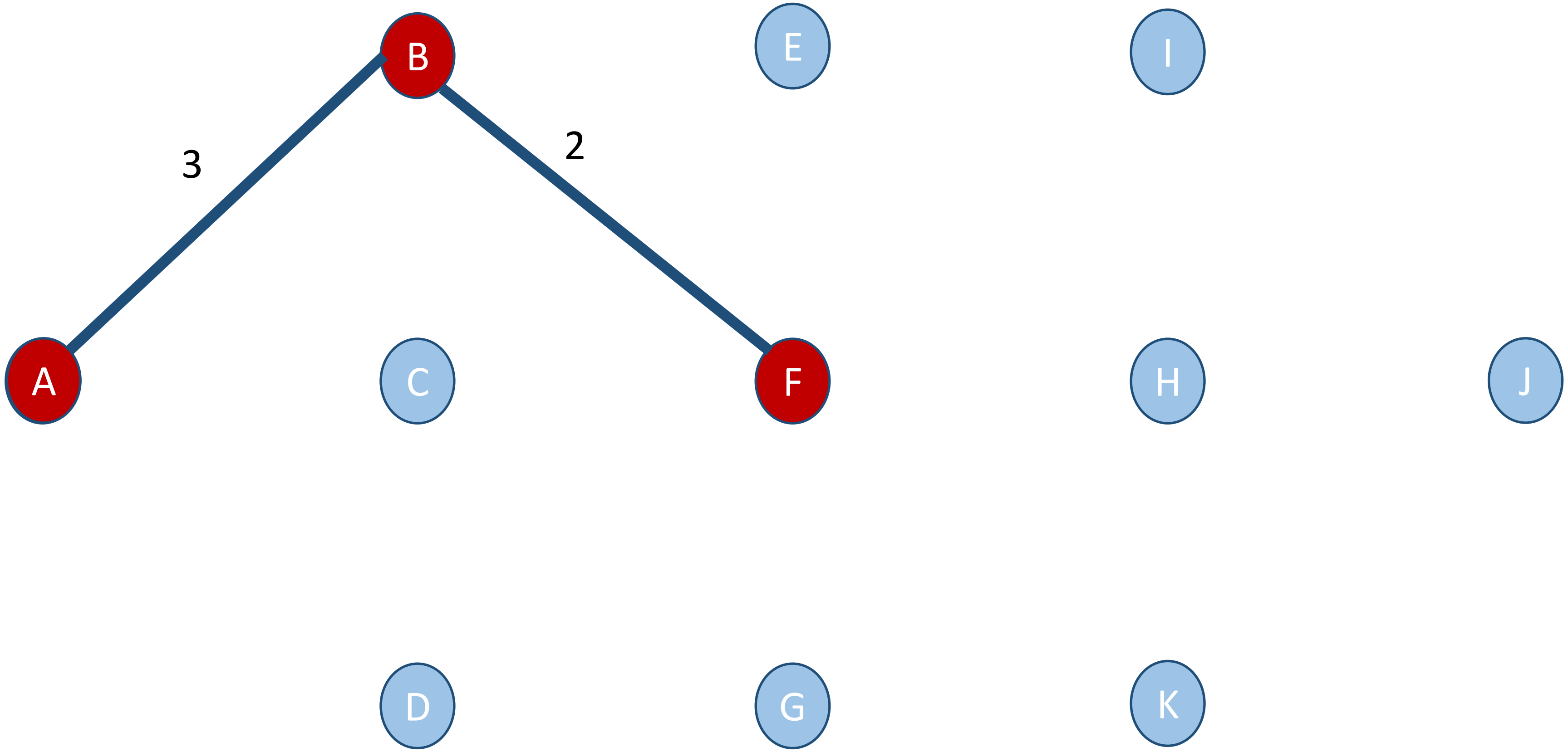
V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B and F. Find the next minimum edge weight.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

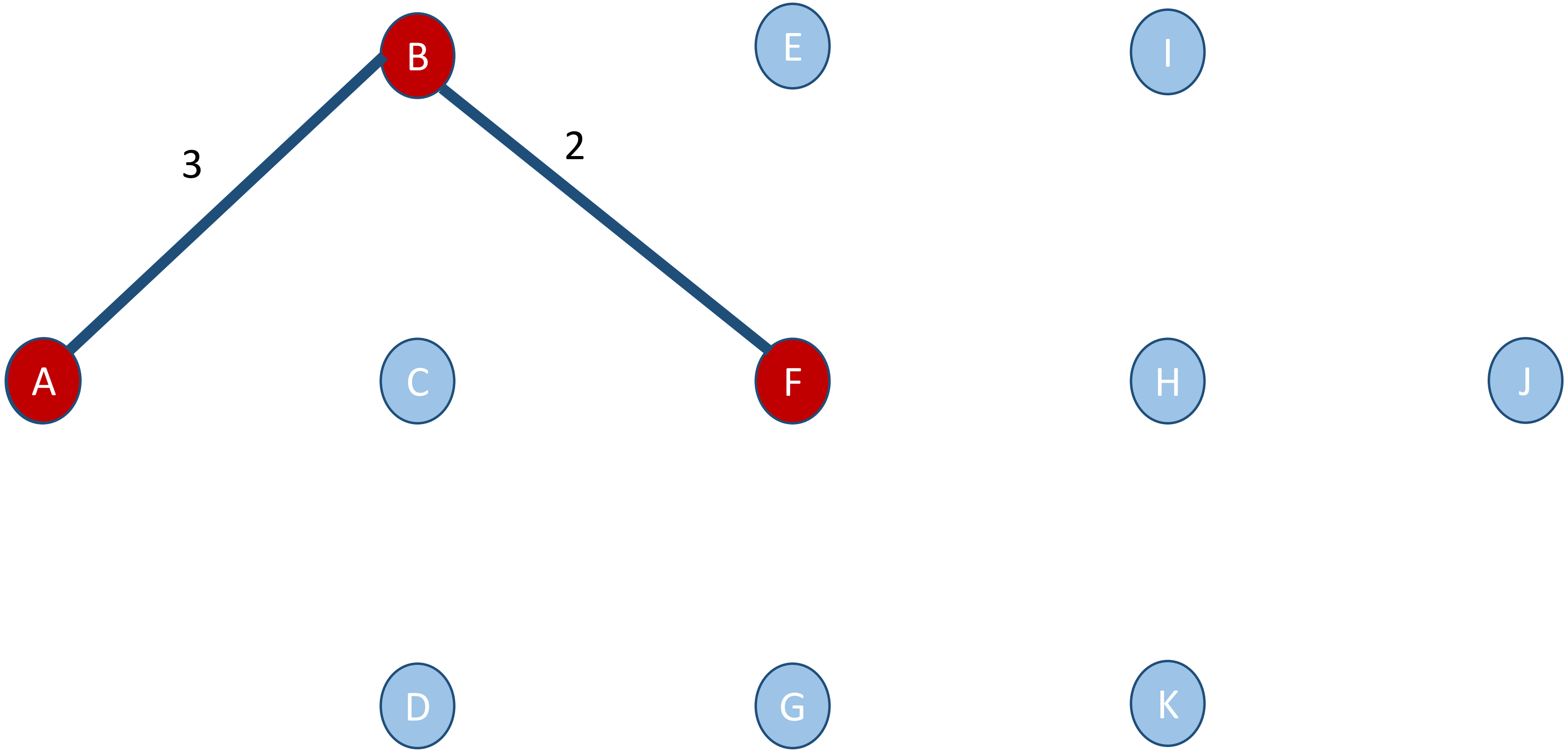
V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F and A. Find the next minimum edge weight.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

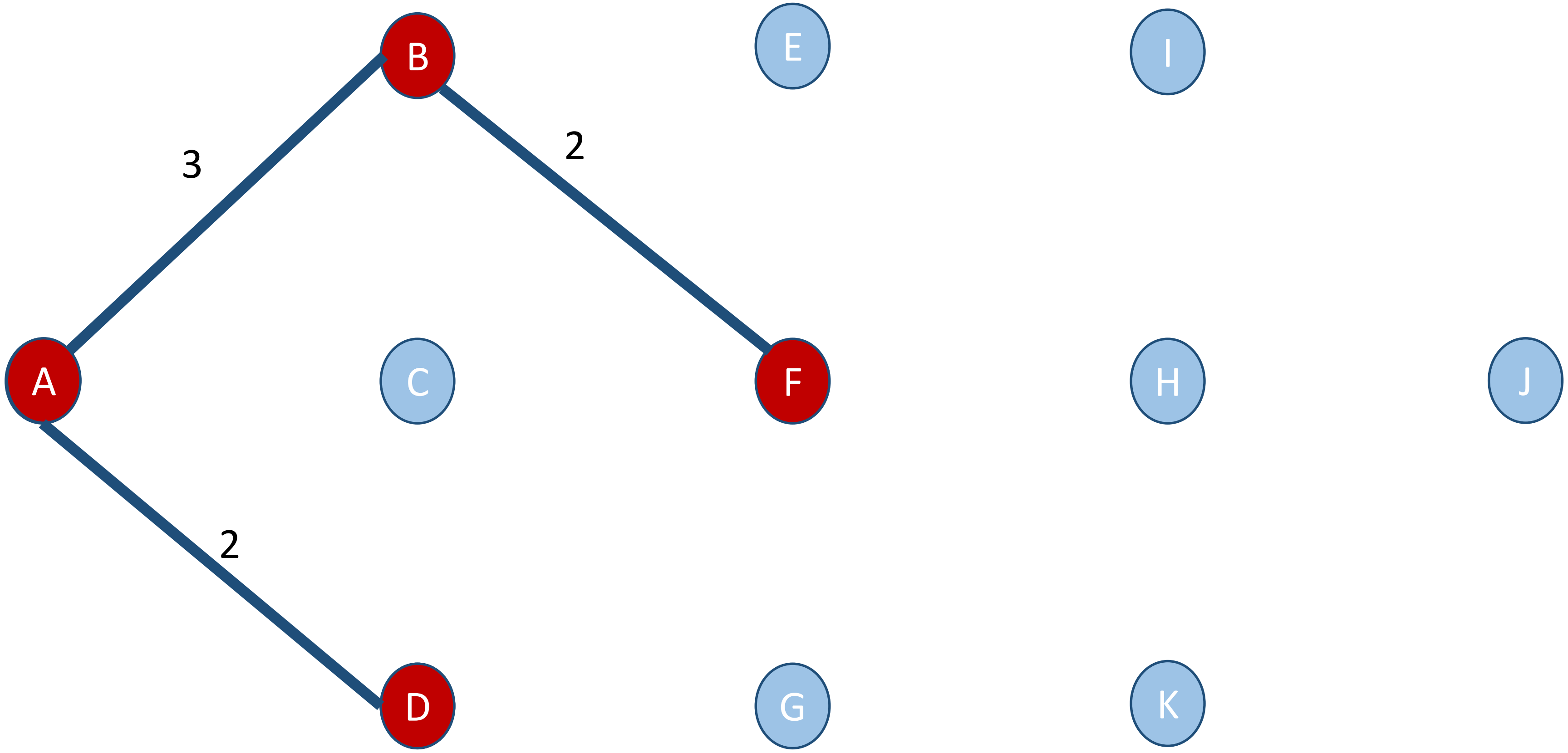
V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F and A. Find the next minimum edge weight.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

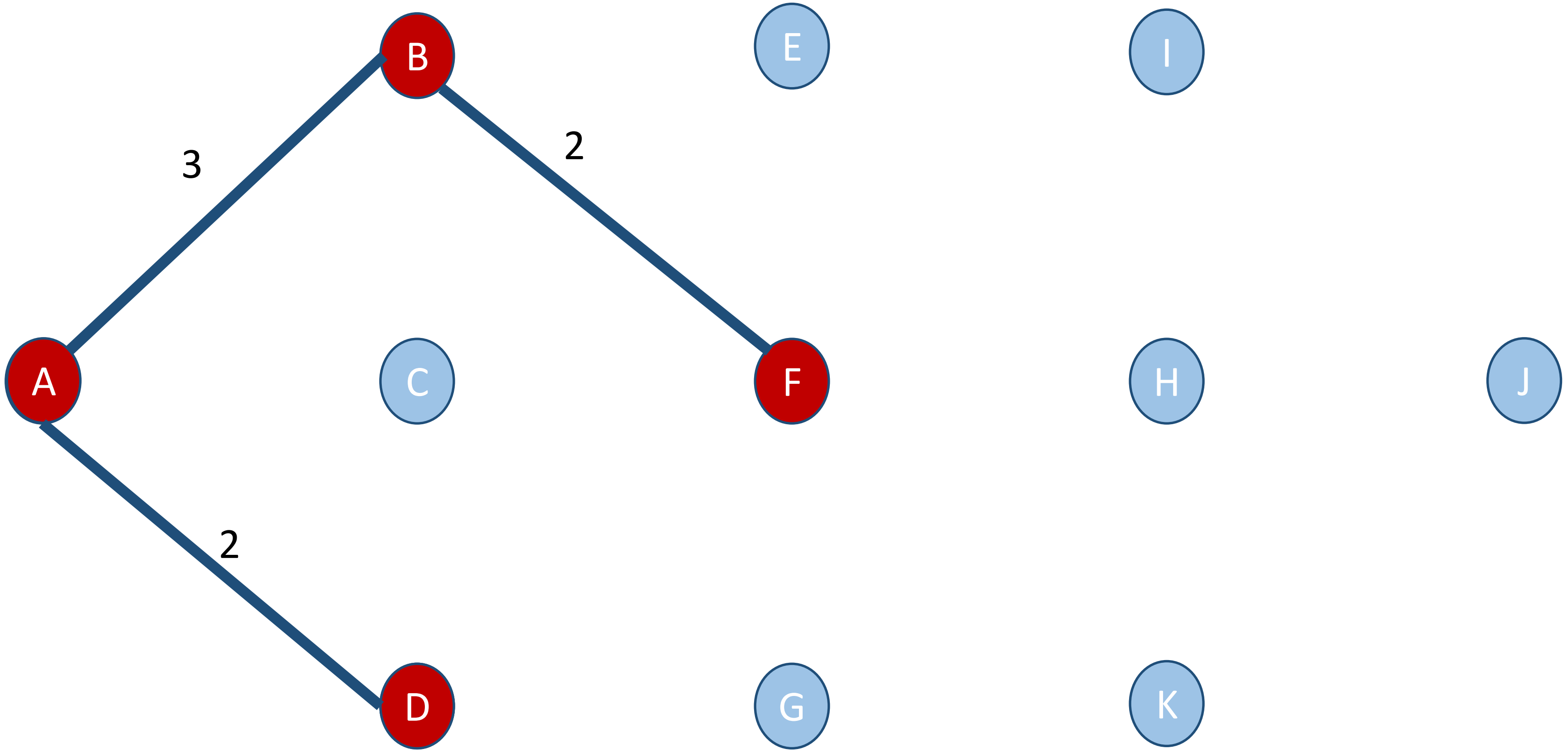
V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A and D. Find the next minimum edge weight.



Check the tables of B, F, A and D. Find the next minimum edge weight.

V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

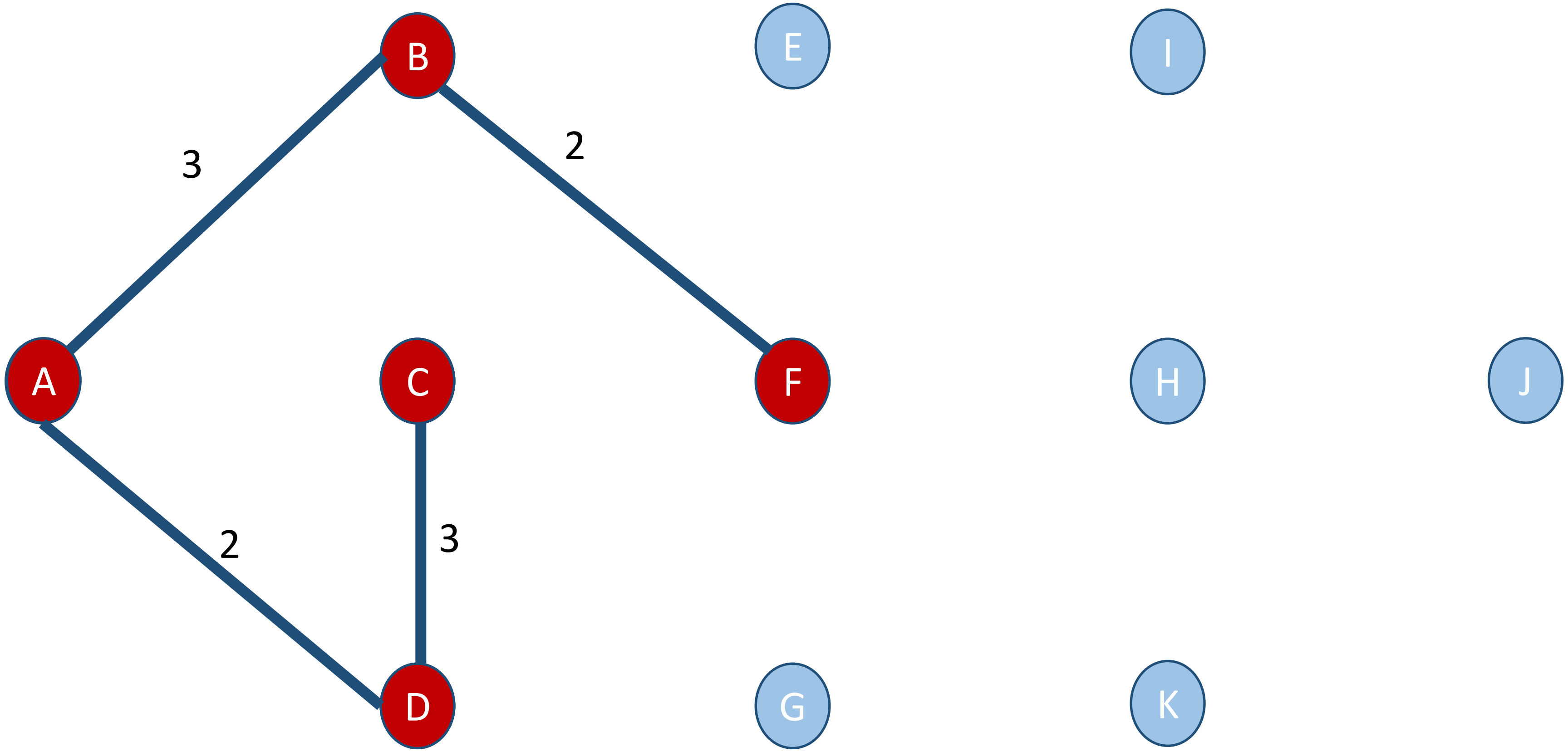
V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5



Check the tables of B, F, A, D and C. Find the next minimum edge weight.

V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

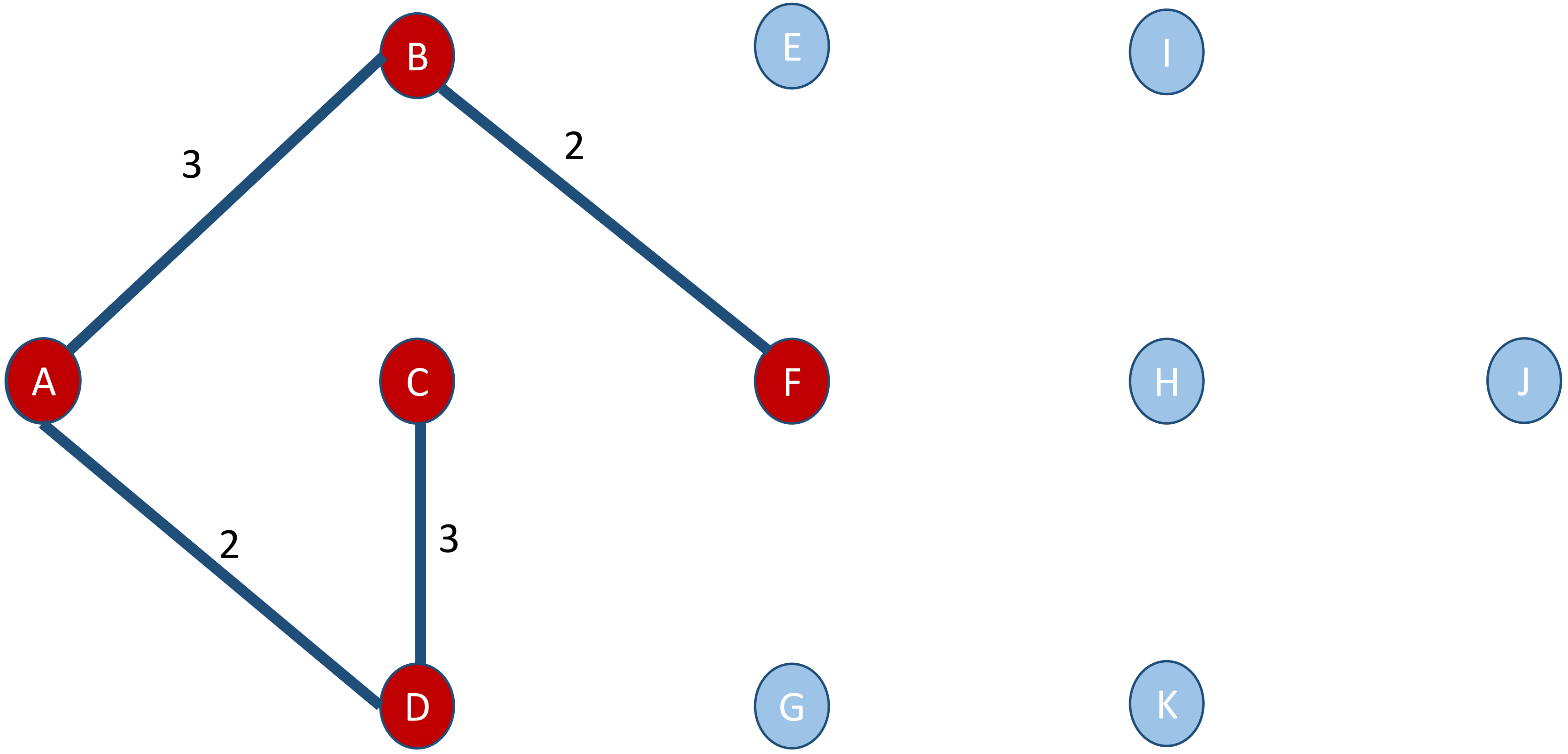
V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5



Check the tables of B, F, A, D and C. Find the next minimum edge weight.

V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

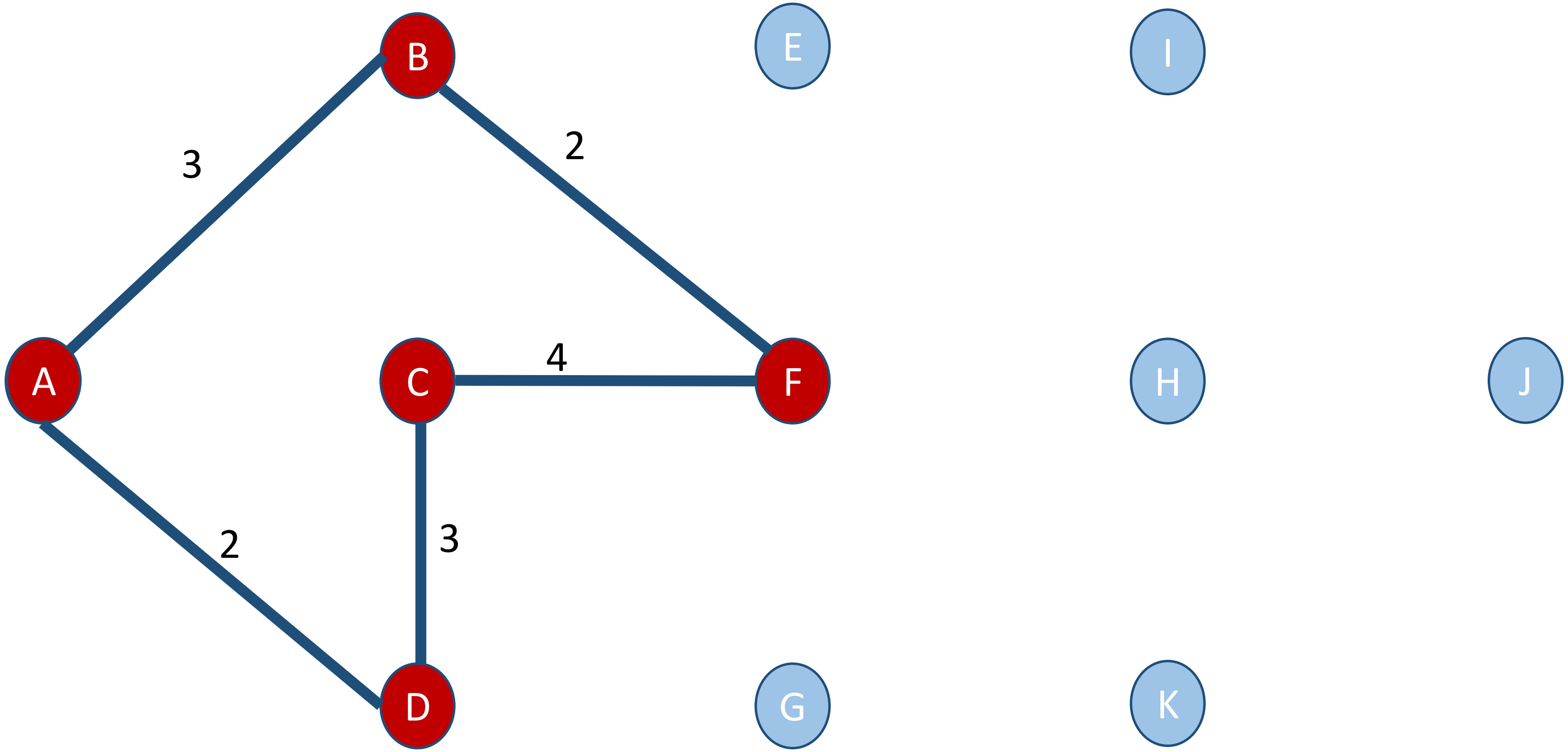
V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

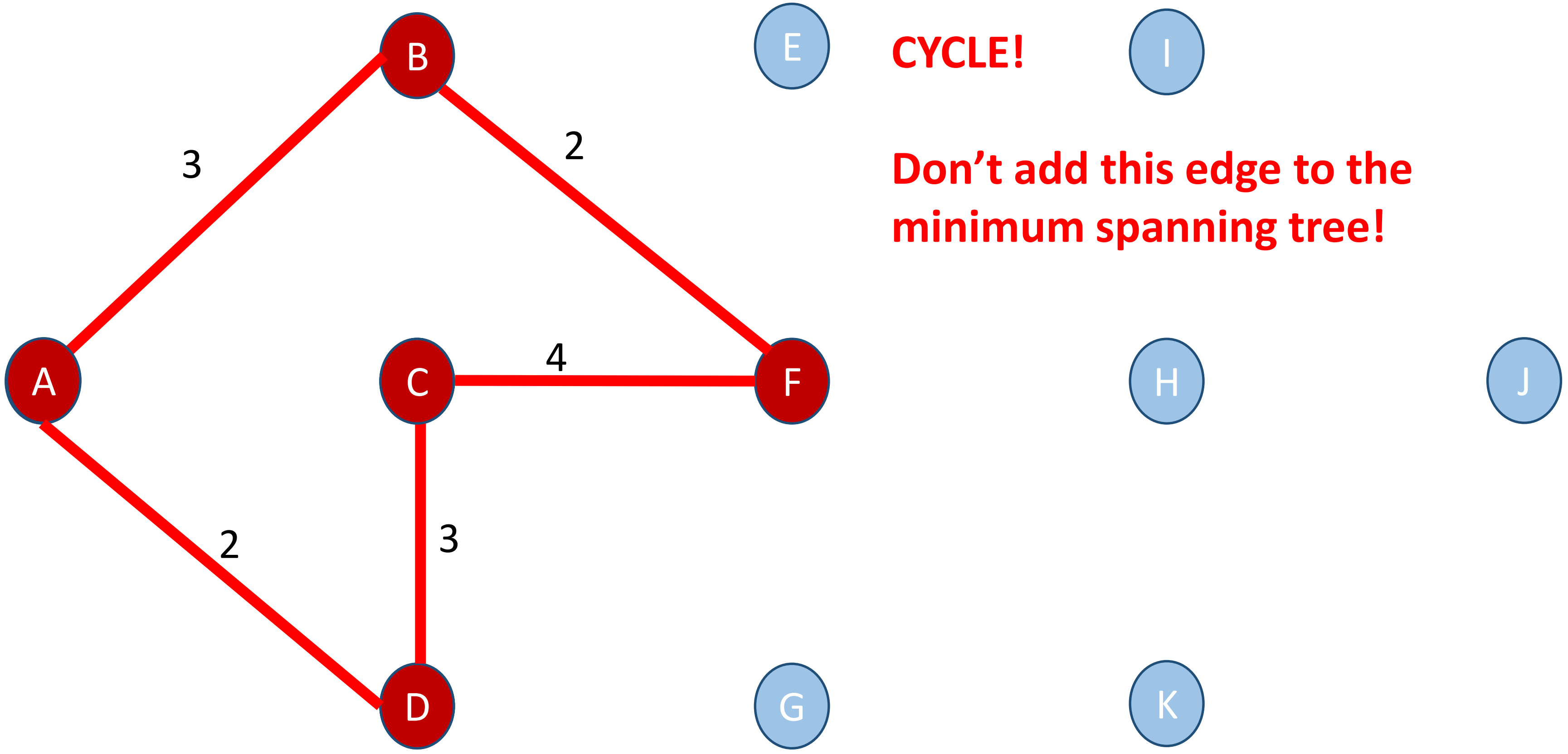
V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D and C. Find the next minimum edge weight.



CYCLE!

Don’t add this edge to the minimum spanning tree!

Check the tables of B, F, A, D and C. Find the next minimum edge weight.

V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

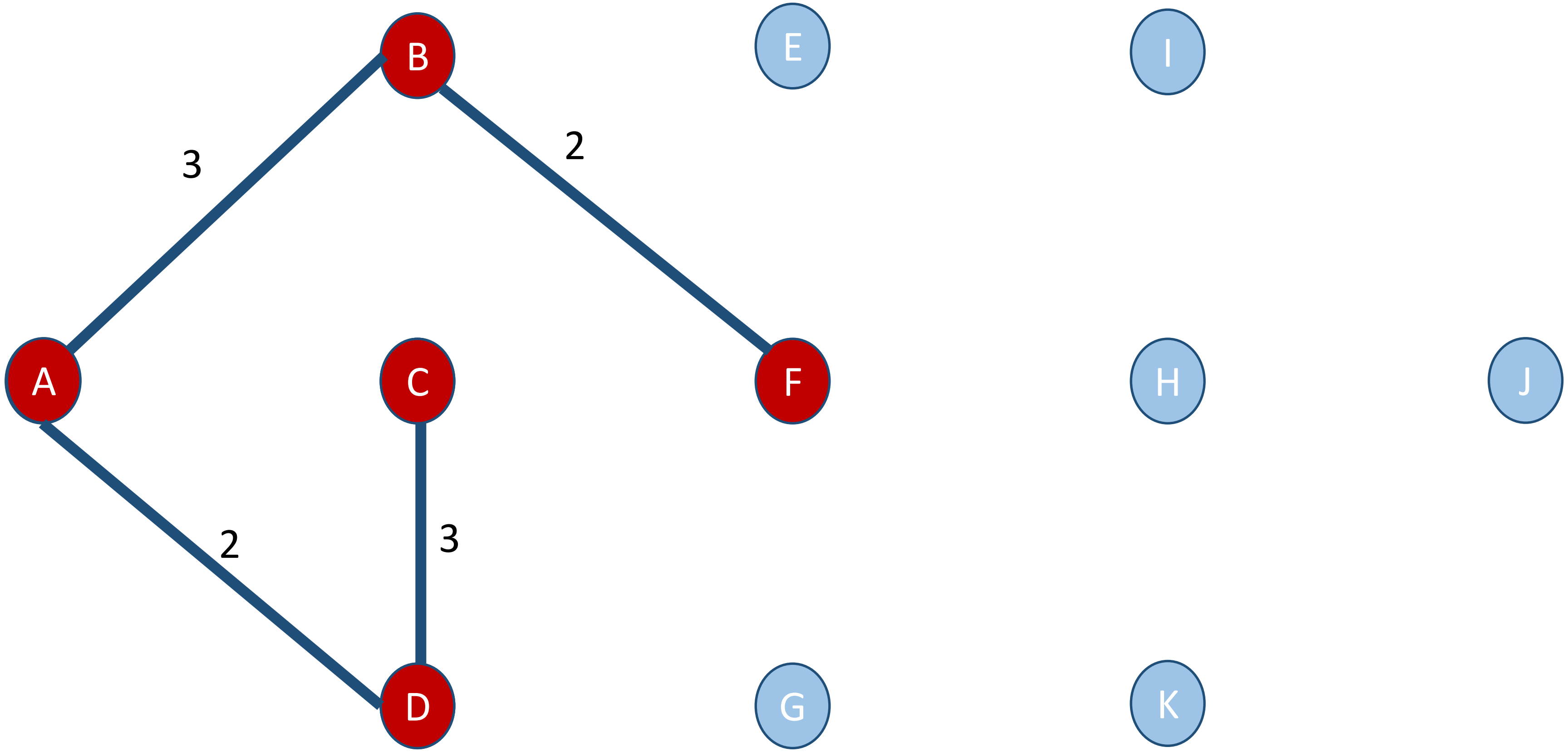
V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

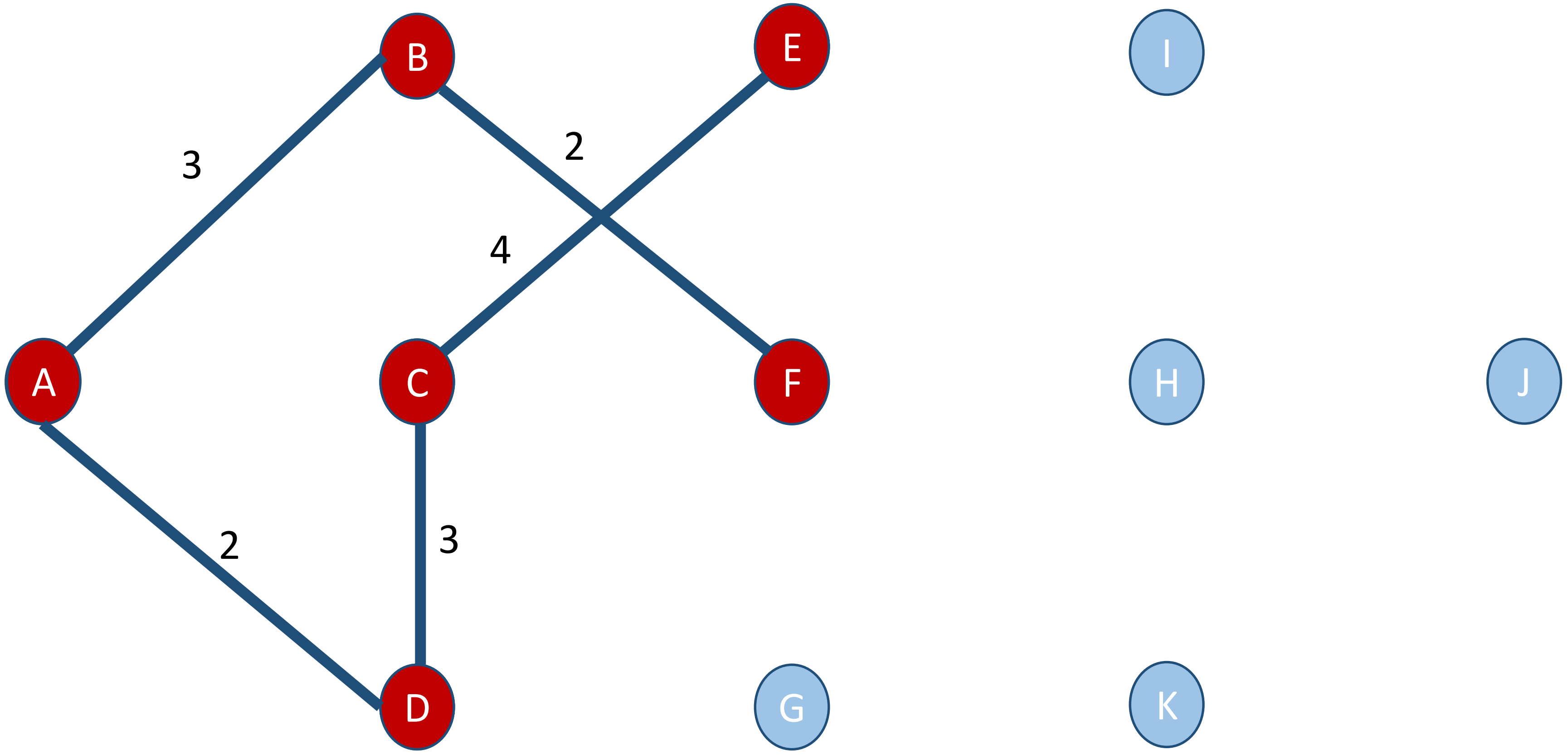
V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D and C. Find the next minimum edge weight.



Check the tables of B, F, A, D, C and E. Find the next minimum edge weight.

V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

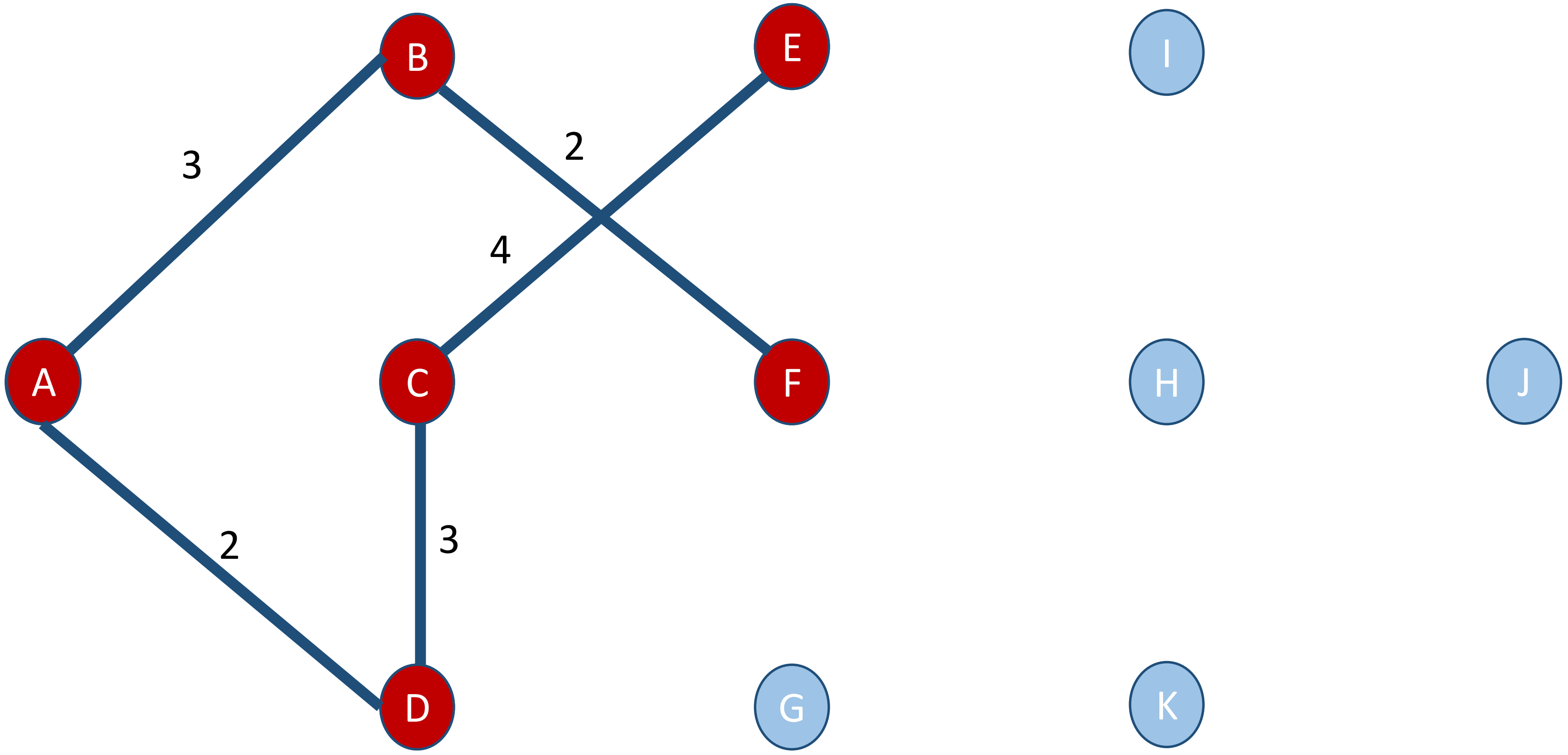
V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

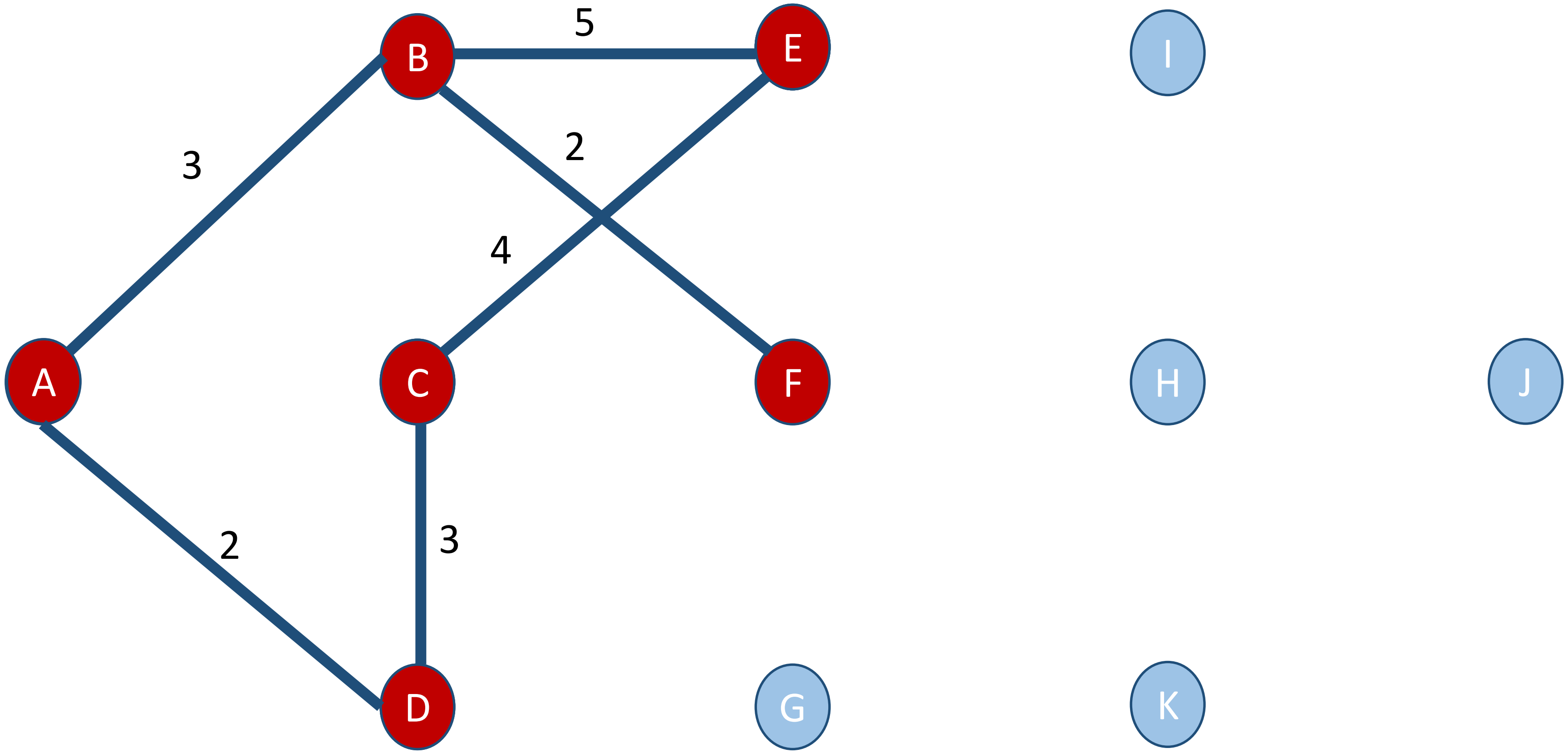
V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D, C and E. Find the next minimum edge weight.



Check the tables of B, F, A, D, C and E. Find the next minimum edge weight.

V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

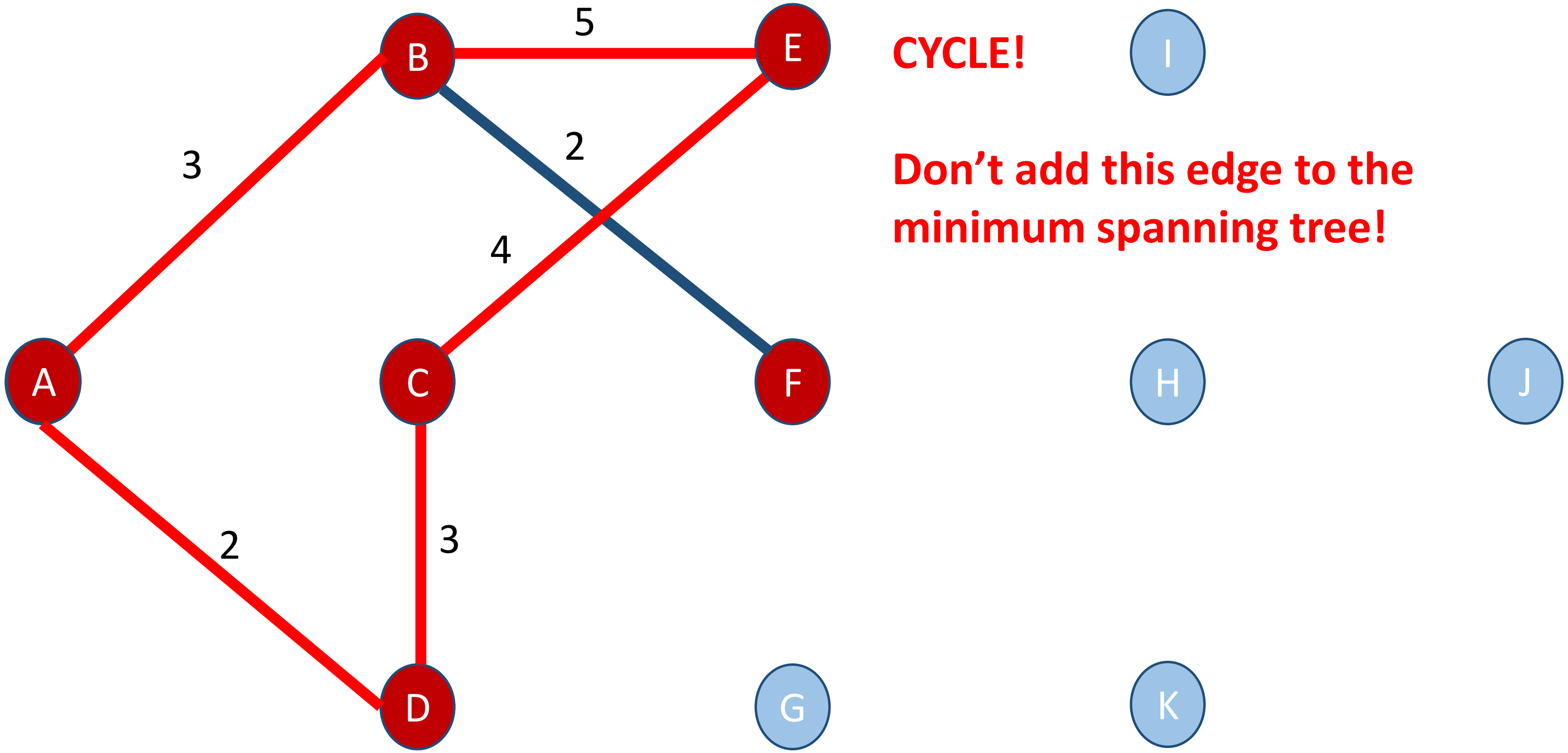
V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5



CYCLE!

Don’t add this edge to the minimum spanning tree!

Check the tables of B, F, A, D, C and E. Find the next minimum edge weight.

V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

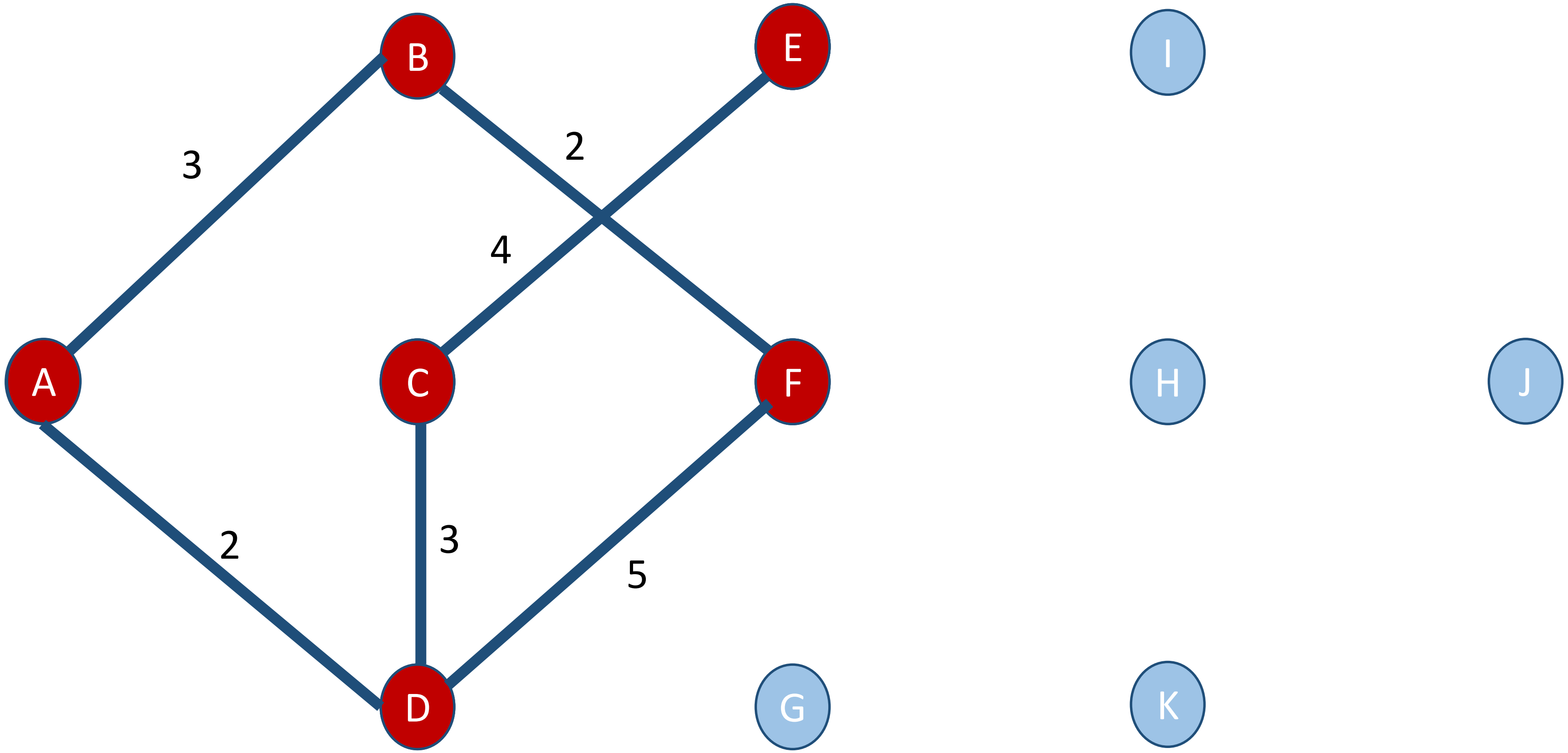
V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

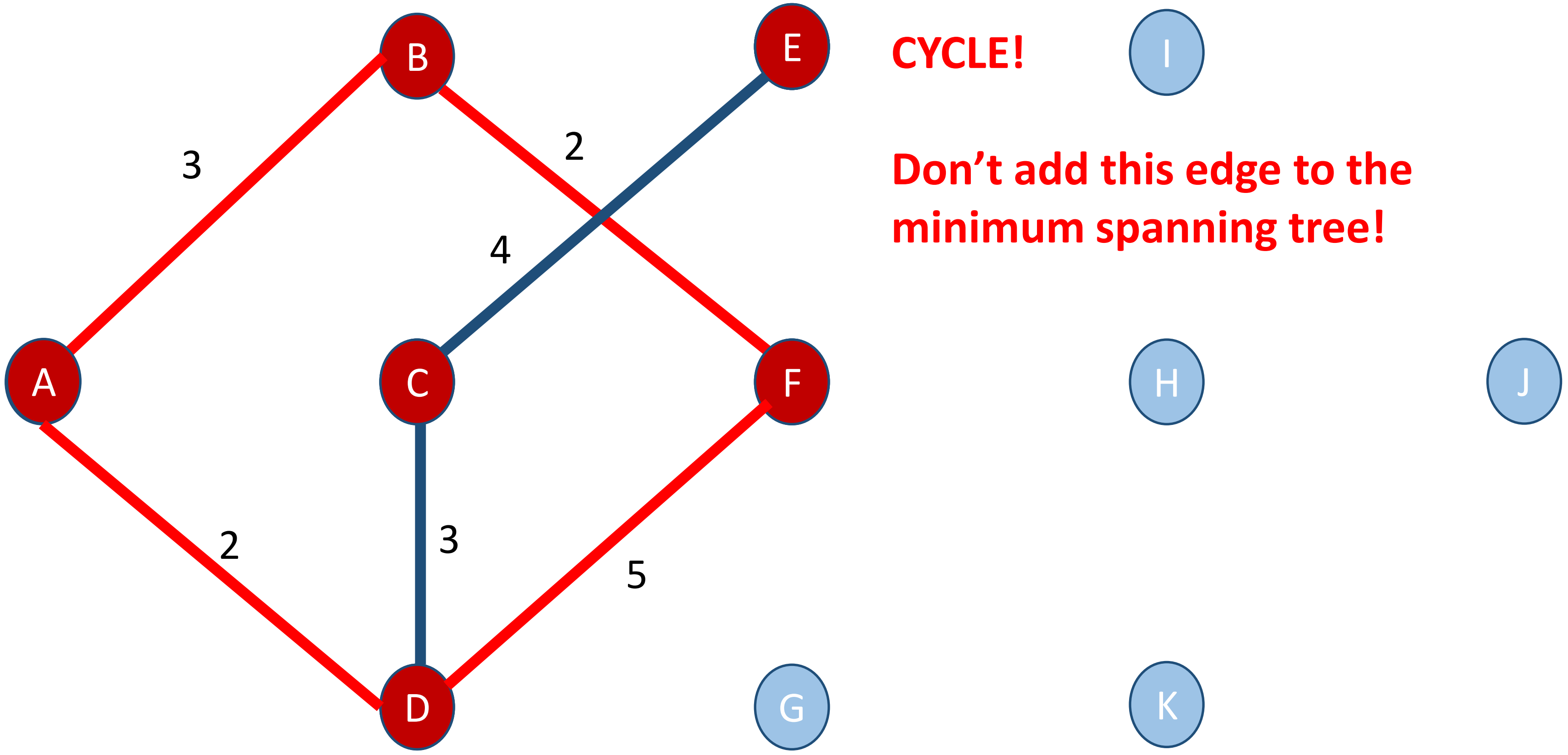
V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D, C and E. Find the next minimum edge weight.



CYCLE!

Don’t add this edge to the minimum spanning tree!

V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

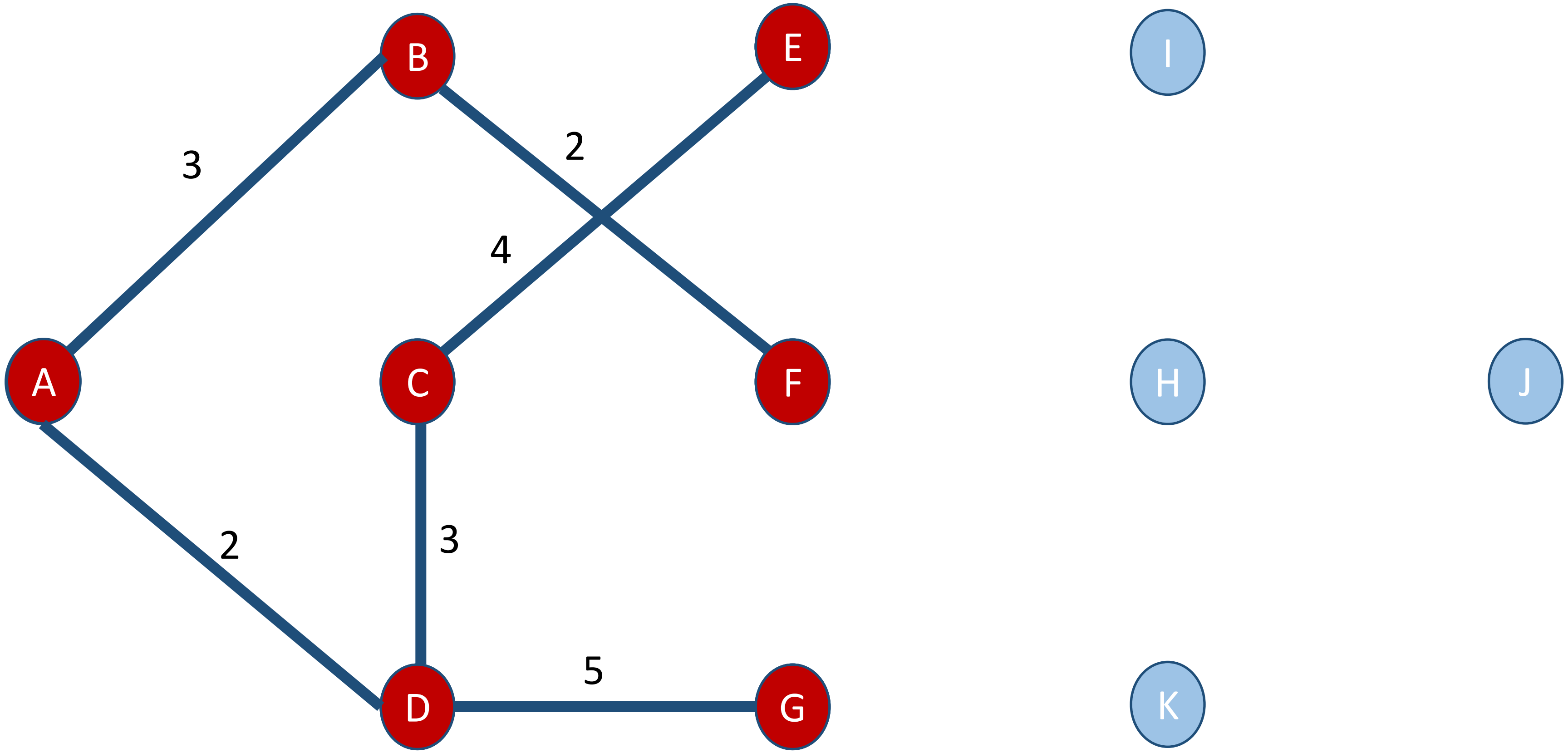
V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D, C and E. Find the next minimum edge weight.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

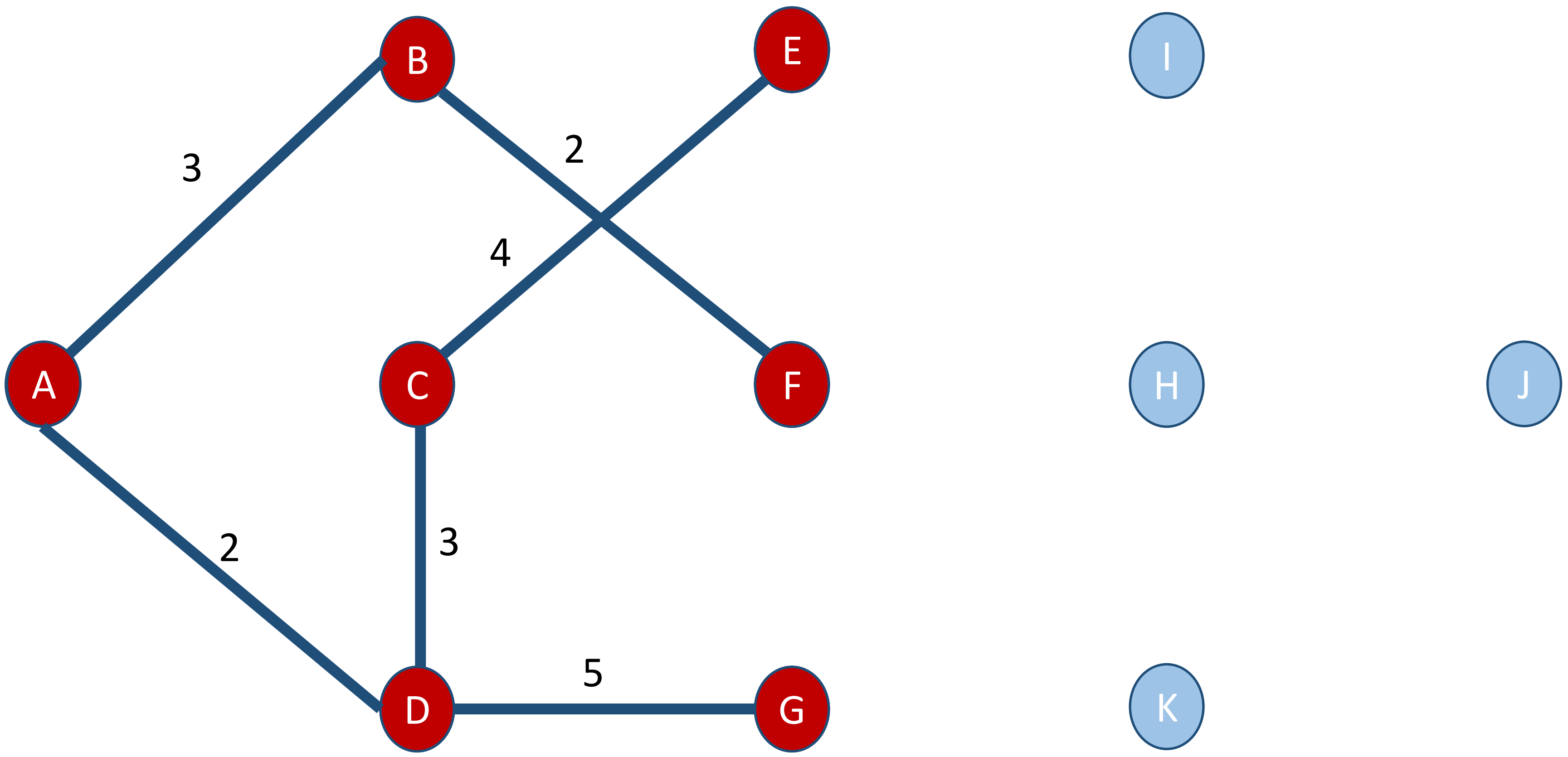
V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D, C, E and G.

Find the next minimum edge weight.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

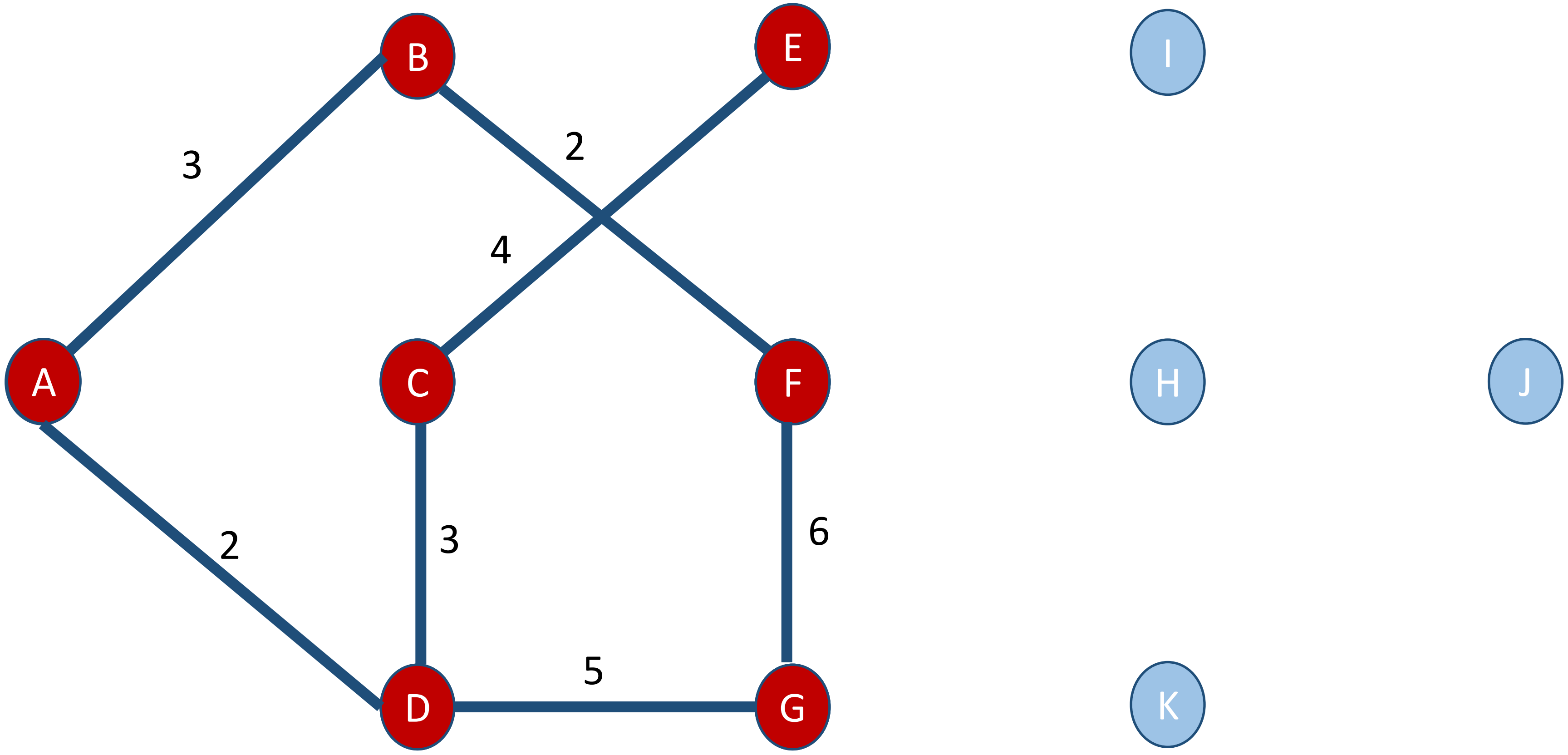
V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D, C, E and G.

Find the next minimum edge weight.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

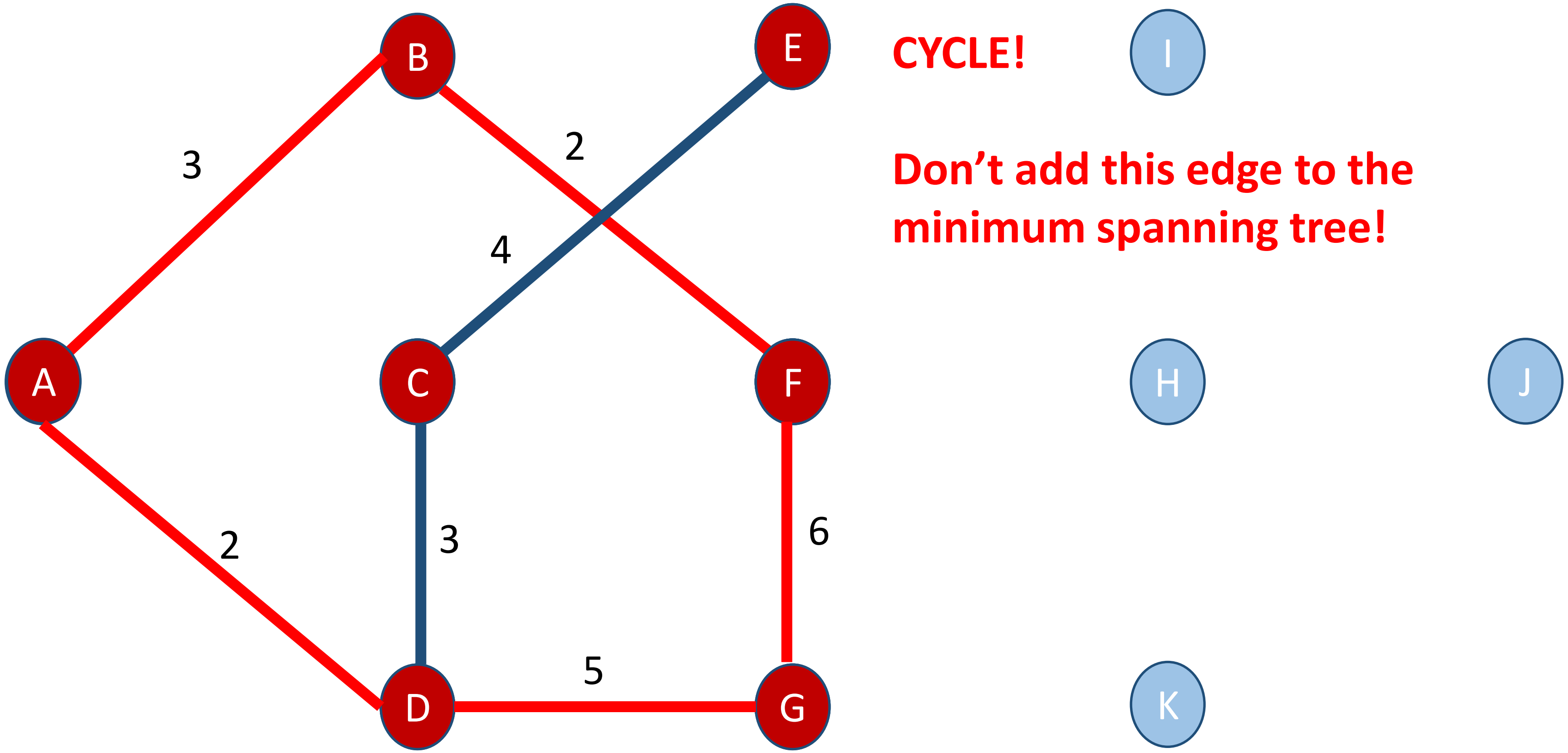
V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D, C, E and G.

Find the next minimum edge weight.



CYCLE!

Don’t add this edge to the minimum spanning tree!

V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

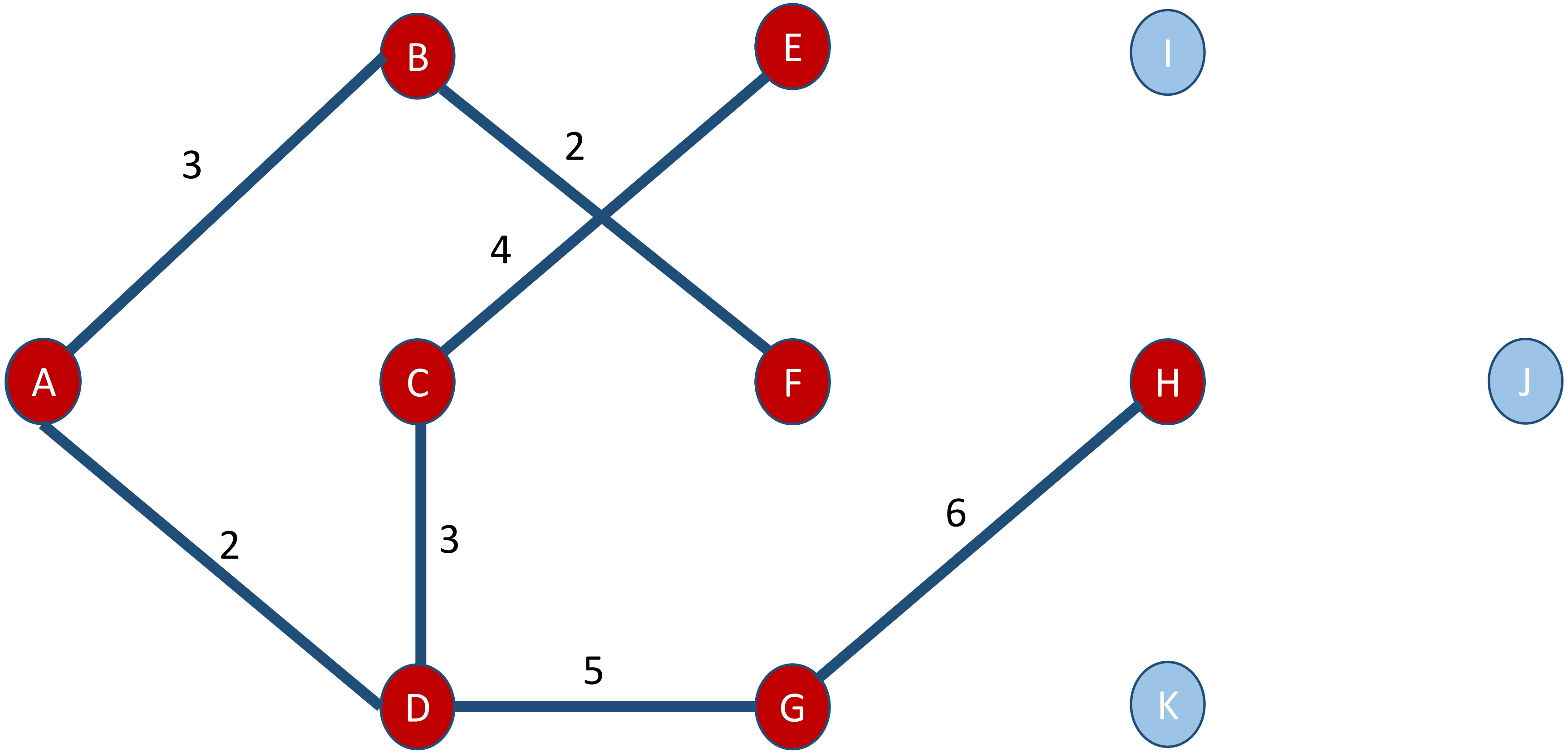
V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D, C, E and G.

Find the next minimum edge weight.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

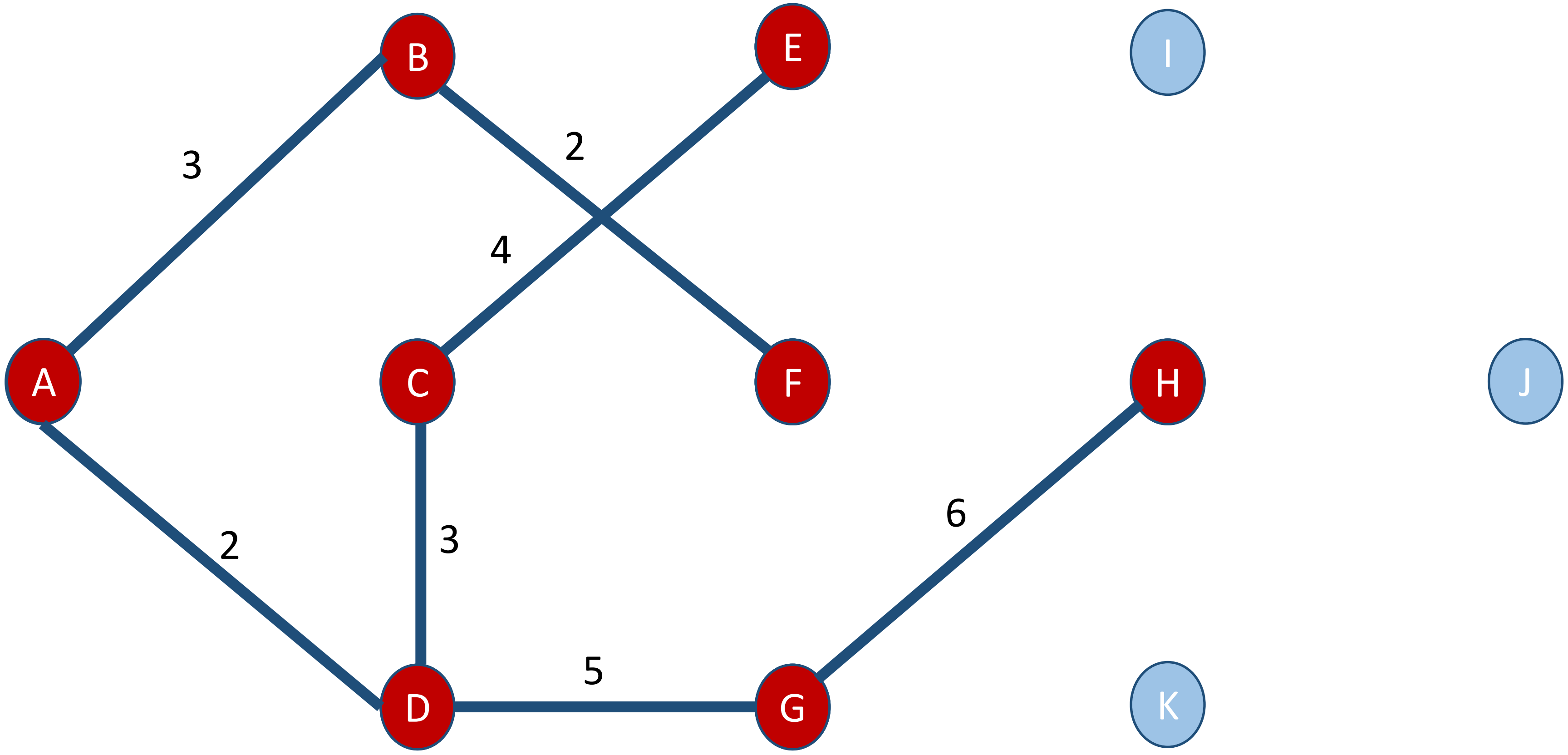
V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D, C, E, G and H.
Find the next minimum edge weight.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

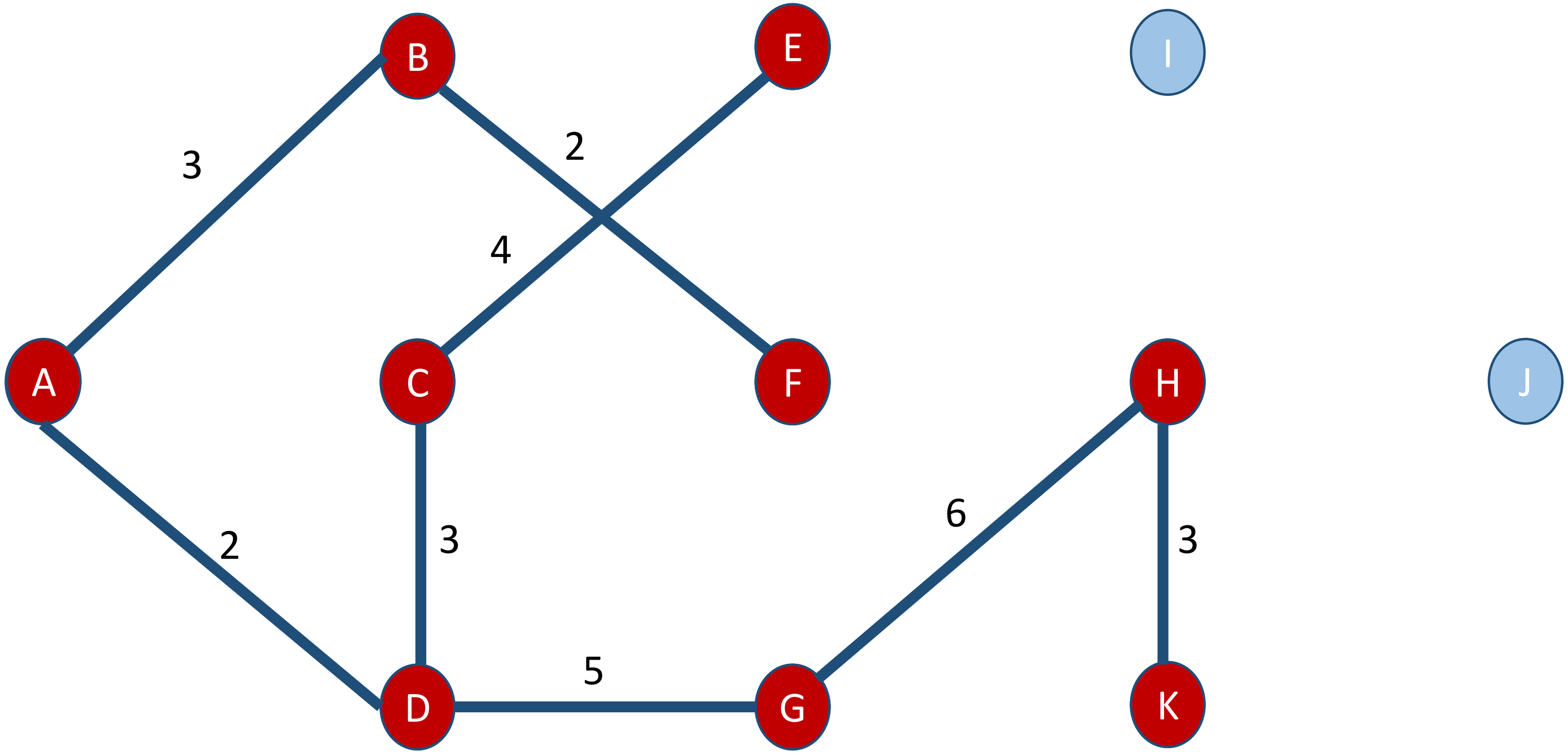
V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D, C, E, G and H.

Find the next minimum edge weight.



Check the tables of B, F, A, D, C, E, G, H and K. Find the next minimum edge weight.

V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

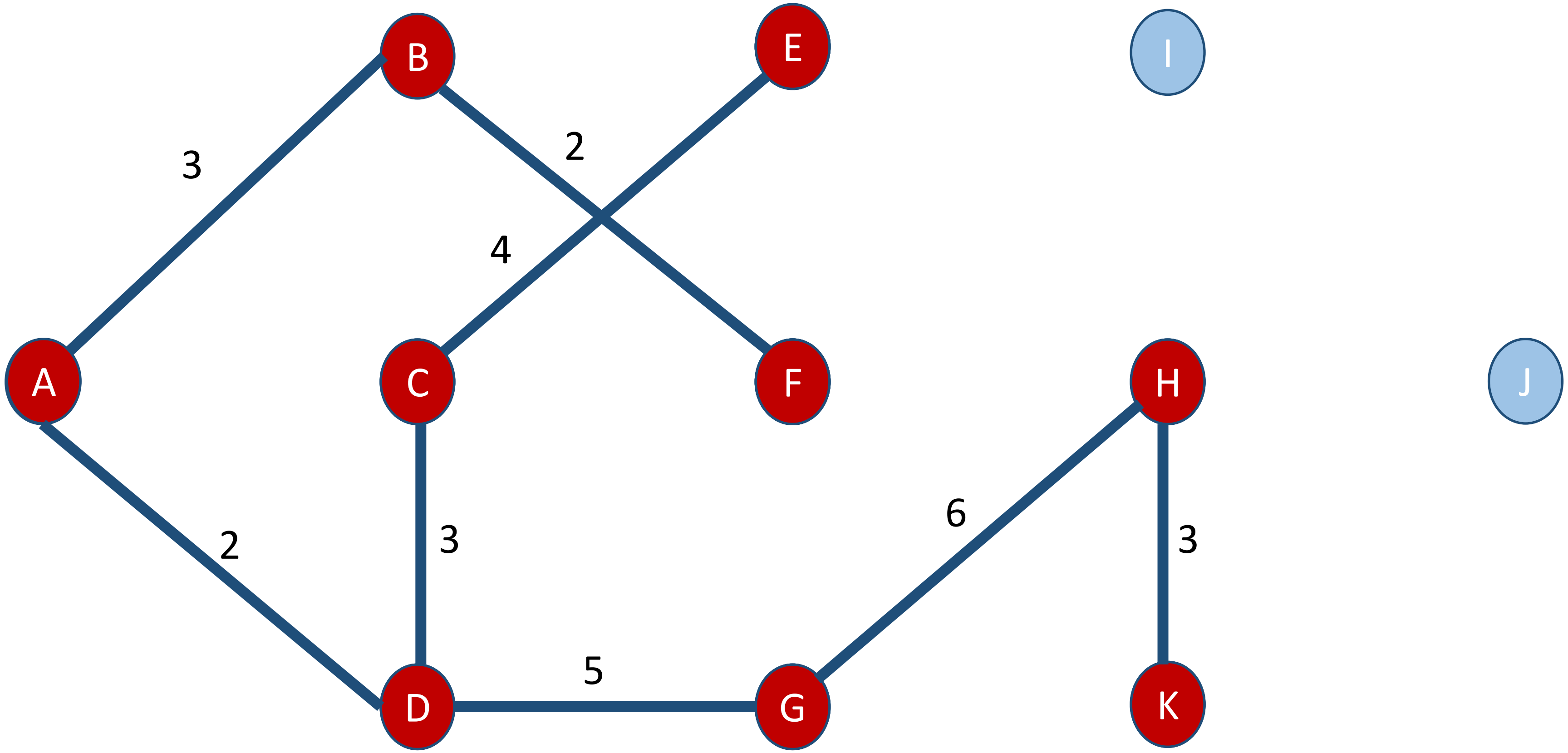
V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

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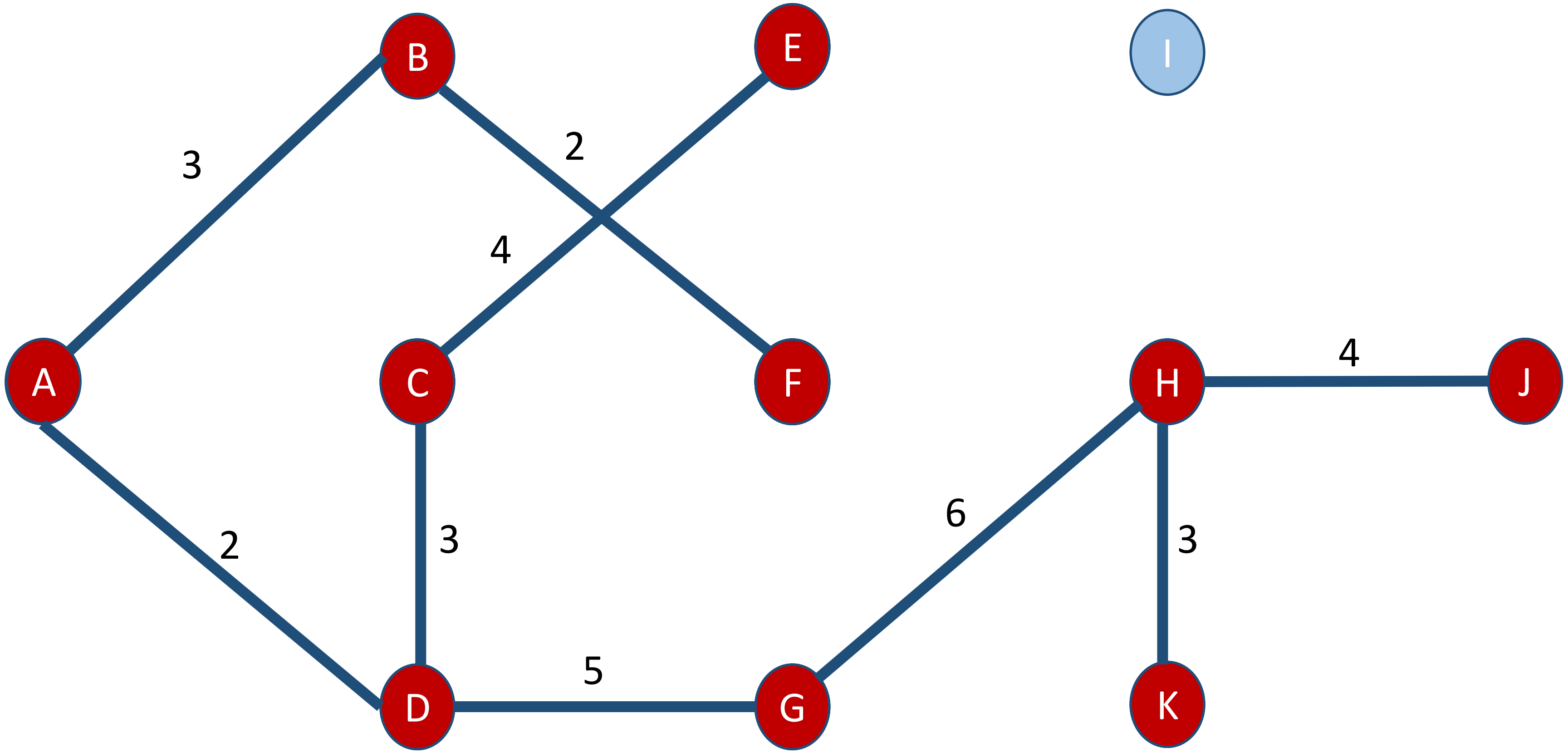
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D, C, E, G, H and K. Find the next minimum edge weight.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

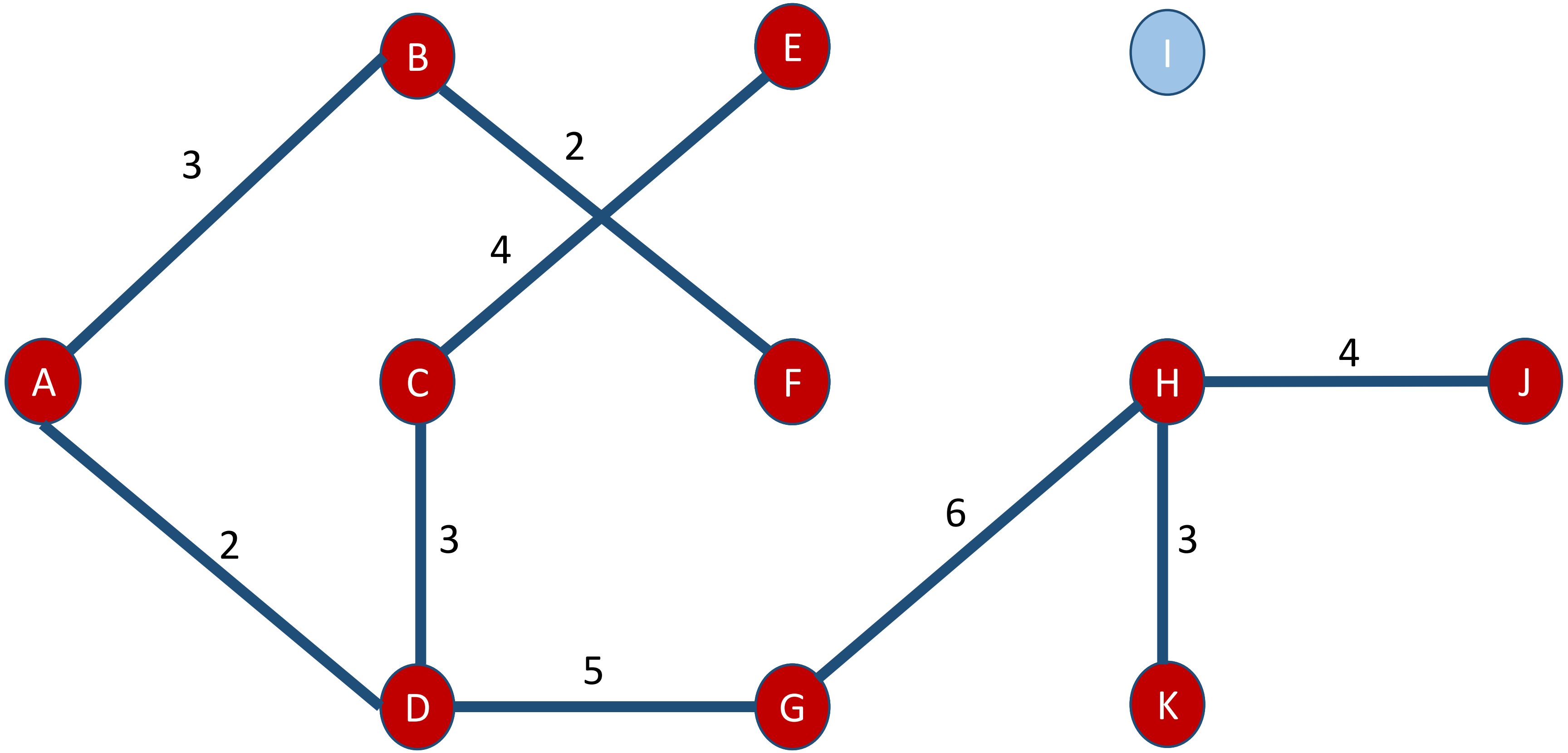
V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D, C, E, G, H, K and J. Find the next minimum edge weight.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

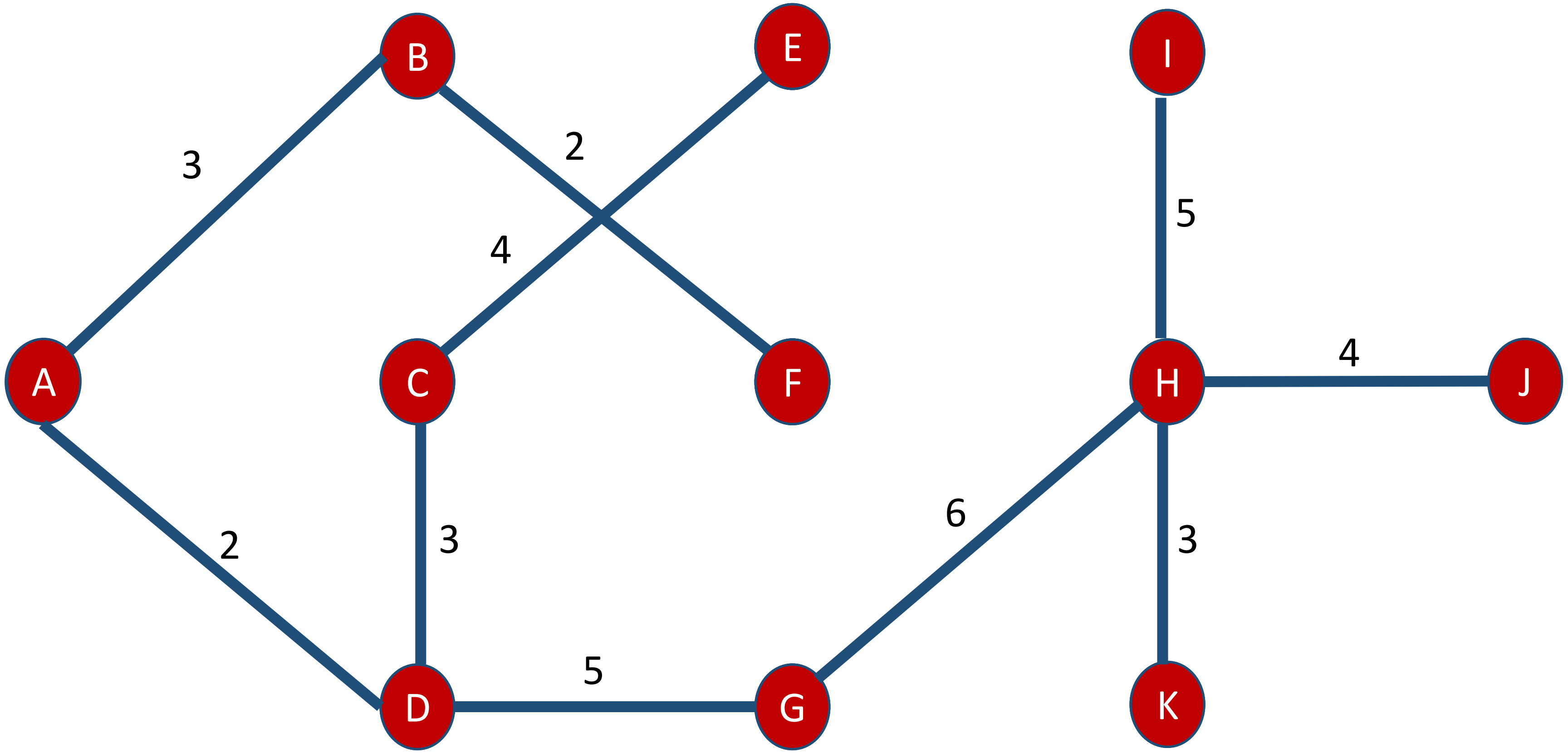
V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

Check the tables of B, F, A, D, C, E, G, H, K and J. Find the next minimum edge weight.



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

V1	V2	W
K	H	3
K	G	8

V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

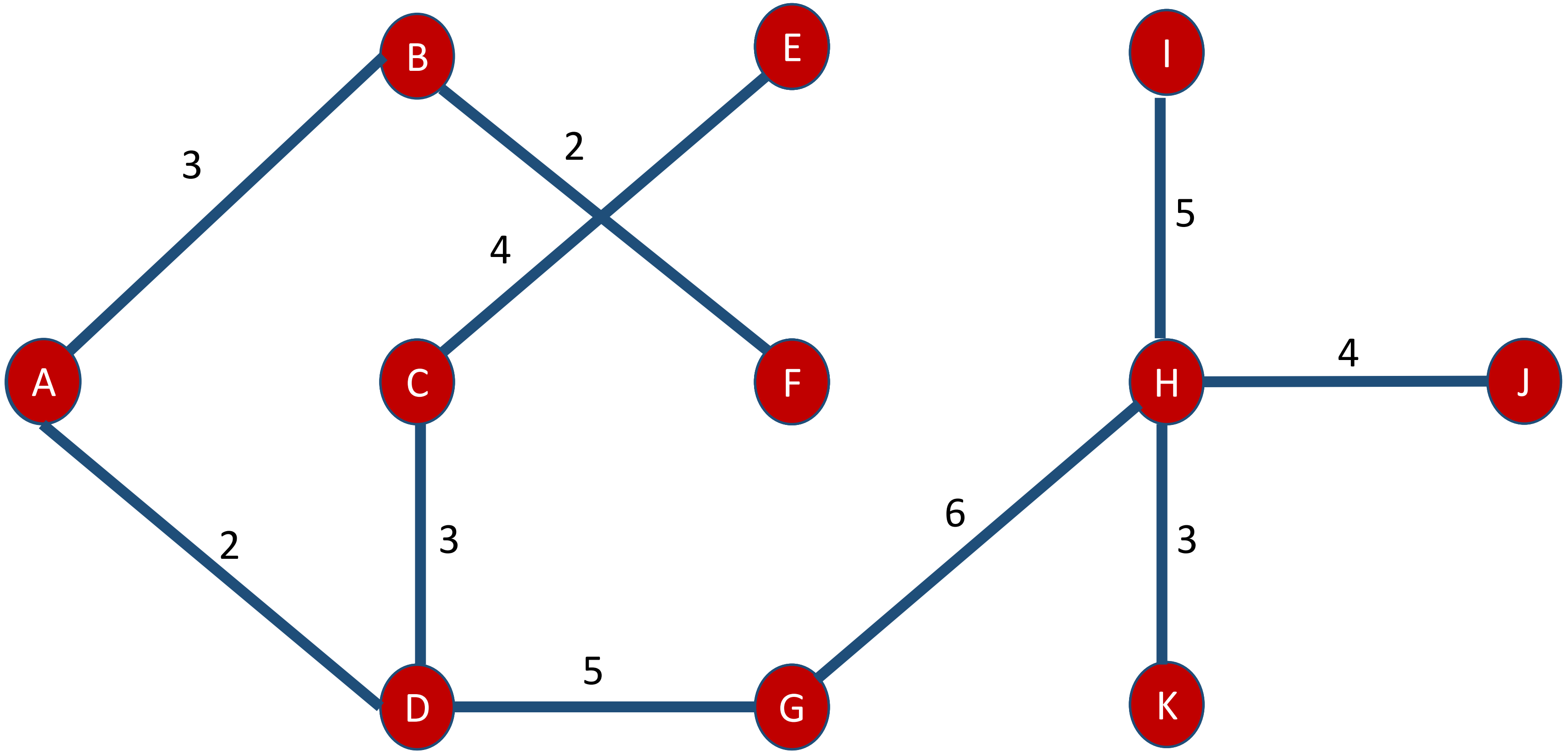
V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

All nodes are visited.

Resulting tree is connected.

Algorithm halts!



V1	V2	W
A	D	2
A	B	3
A	C	7

V1	V2	W
E	C	4
E	B	5
E	H	8

V1	V2	W
I	H	5

V1	V2	W
H	K	3
H	J	4
H	I	5
H	G	6
H	F	8
H	E	8

V1	V2	W
B	F	2
B	A	3
B	E	5

V1	V2	W
F	B	2
F	C	4
F	D	5
F	G	6
F	H	8

V1	V2	W
J	H	4

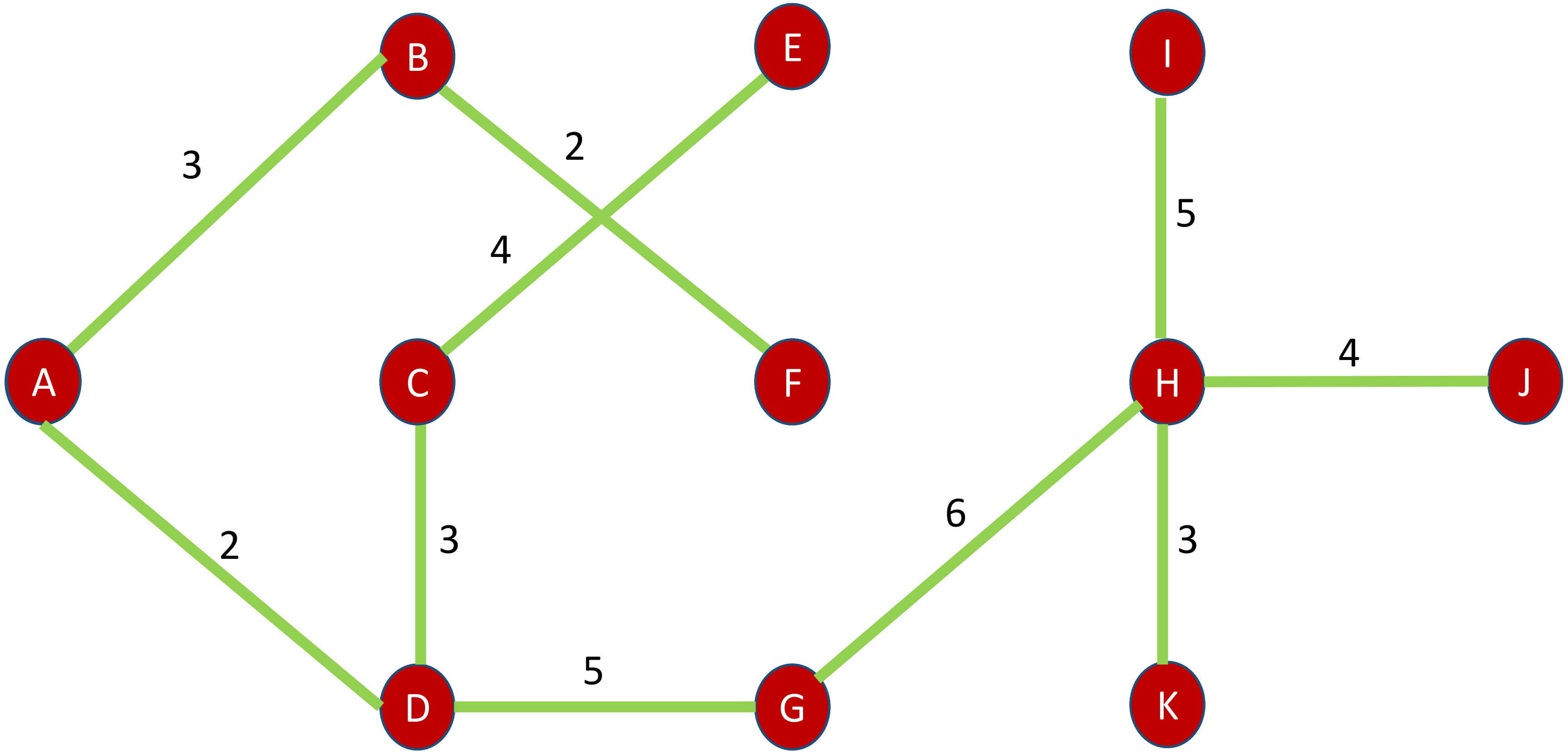
V1	V2	W
K	H	3
K	G	8

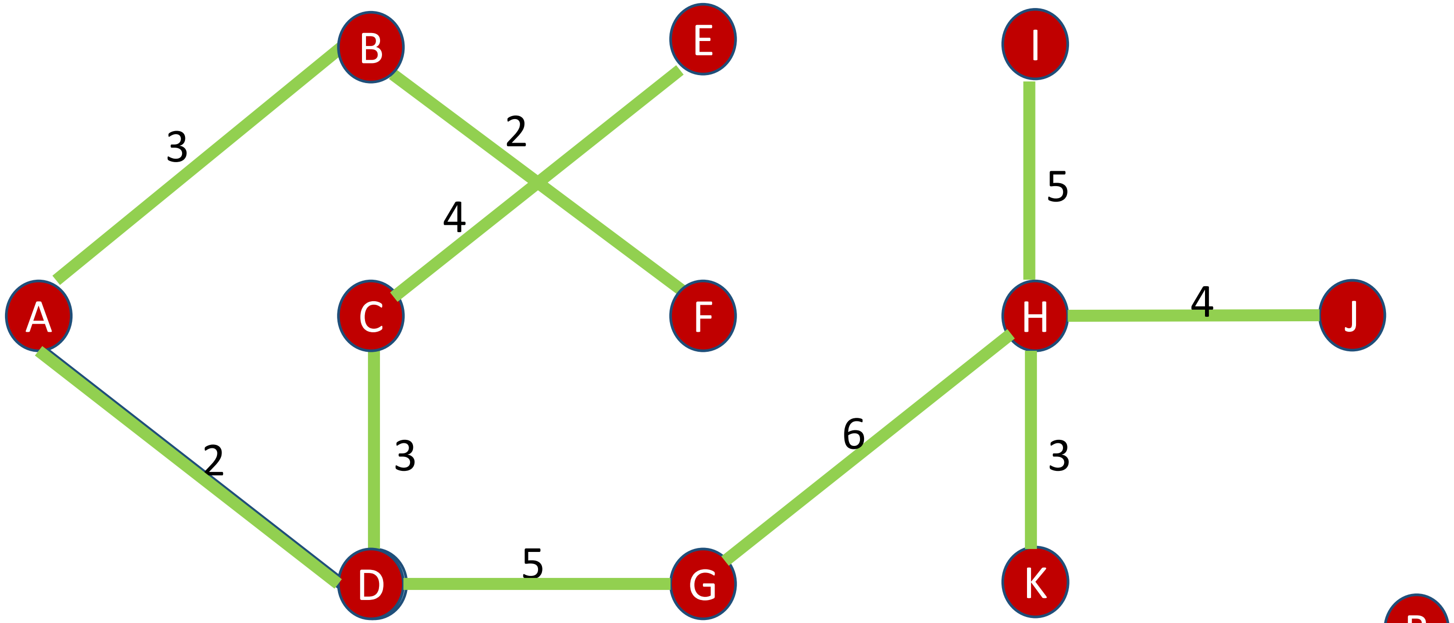
V1	V2	W
C	D	3
C	E	4
C	F	4
C	A	7

V1	V2	W
G	D	5
G	F	6
G	H	6
G	K	8

V1	V2	W
D	A	2
D	C	3
D	F	5
D	G	5

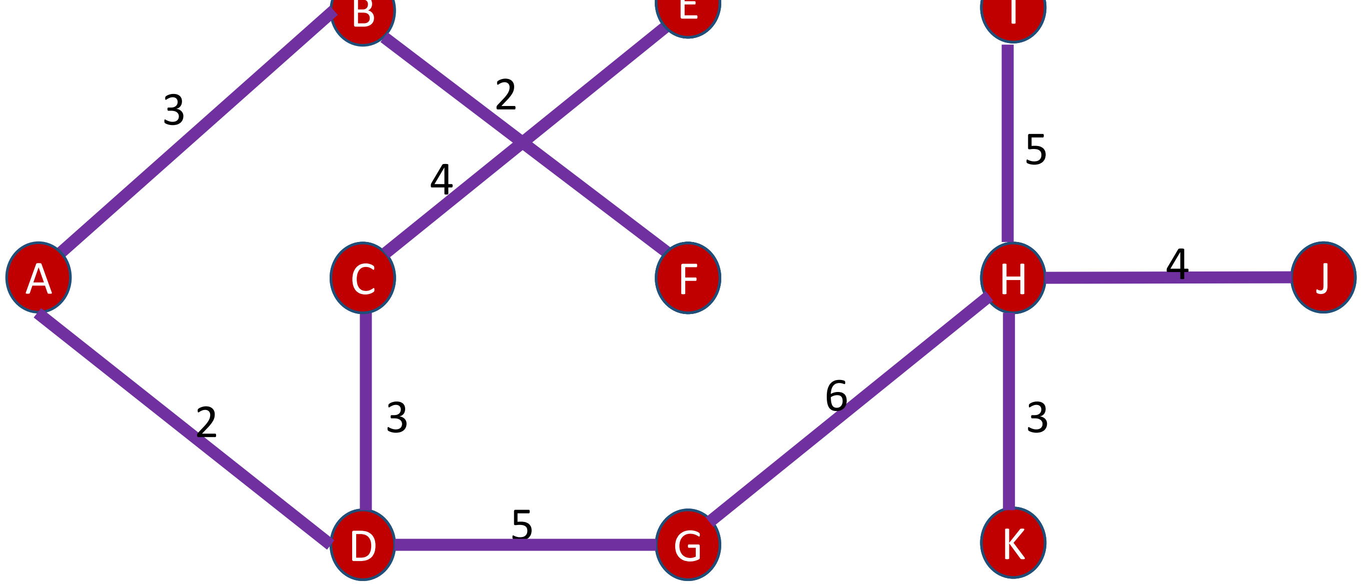
Weight of the minimum spanning tree (MST)
= Sum of all weights of these edges
= 3 + 2 + 2 + 4 + 3 + 5 + 6 + 3 + 5 + 4
= 37.





With KRUSKAL'S Algorithm

With PRIM'S Algorithm



Question 1:

Do Kruskal’s and Prim’s algorithms always yield the same tree?

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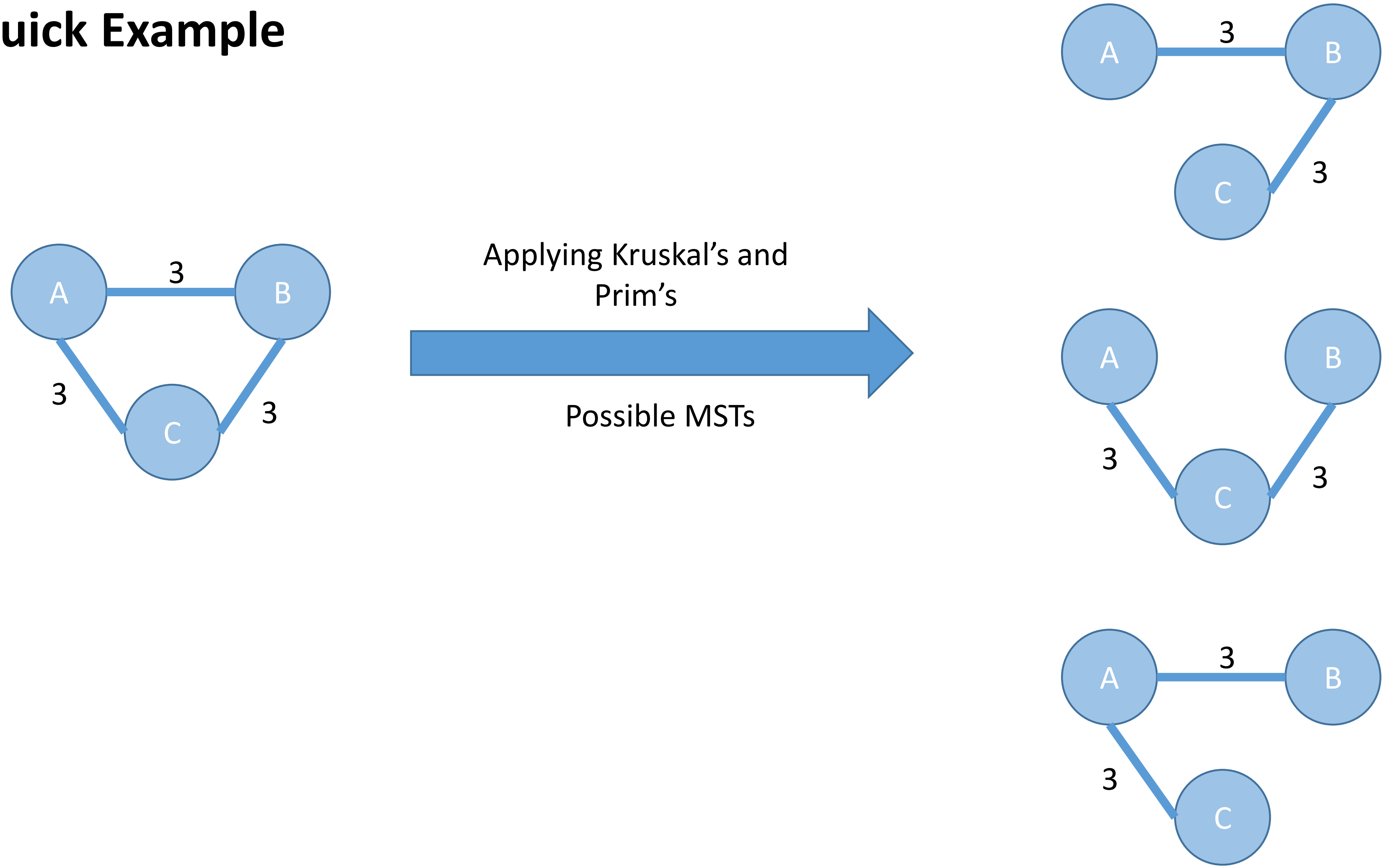
Do Kruskal’s and Prim’s algorithms always yield the same tree?

Answer: DEPENDS!

If the edge weights in the tree are all different from each other, it is guaranteed that they will come up with the same minimum spanning tree.

However, if one or more edges have the same weight, they may yield different MSTs. Because there is *randomness* in the algorithms.

Quick Example



Applying Kruskal’s and Prim’s

Possible MSTs

Question 2:

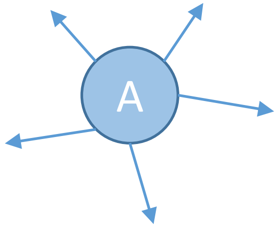
Can we use Prims’s and Kruskal’s for directed graphs?

Question 2:

Can we use Prims's and Kruskal's for directed graphs?

Answer: No!

For Prim's:
Because Prim's algorithm assumes that every node is reachable from at least a node. If there is a node in the graph which only has edges going out from it, the Prim's can't be applied on that tree.

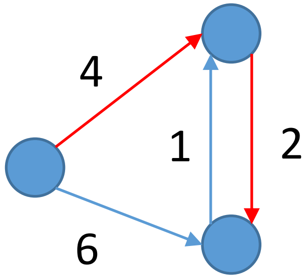


Question 2:

Can we use Prims’s and Kruskal’s for Directed graphs?

Answer: No!

For Kruskal’s:
Let’s explain this with a counter-example:



Heuristically, minimum spanning tree is 4 + 2 = 6

However, the Kruskal’s will start with the lightest edge (Edge 1) by definition. This will lead to a dead end.

Question 3:

Which is better? Prim’s or Kruskal’s algorithms?

Question 3:

Which is better? Prim’s or Kruskal’s algorithms?

Answer: DEPENDS!

Number of vertices

Prim’s has a time complexity of $O(V^2)$

Kruskal’s has a time complexity of $O(E \log V)$.

Number of edges

Therefore,

Prim performs better with dense graph (When E is much bigger than V).

Kruskal performs better with sparse graphs (When E is relatively smaller than V).

THANK YOU FOR ATTENDING THIS RECITATION!

If you have any questions, ask now or contact us later by:
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erolm15@itu.edu.tr