Problem Session 2

1. A logical expression has been given as follows,

$$\boldsymbol{f(a,b,c,d) = \sum m\,(0,1,6,7,8,9)}$$

- **a.** Simplify the given expression.
- **b.** Implement the expression (found in **a**) by using two-input logical gates.
- **c.** Implement the expression by using only two-input NAND gates.

Answer:

a. The first canonical form of $\boldsymbol{f}$

$$f(a,b,c,d) = \sum m\,(0,1,6,7,8,9)$$

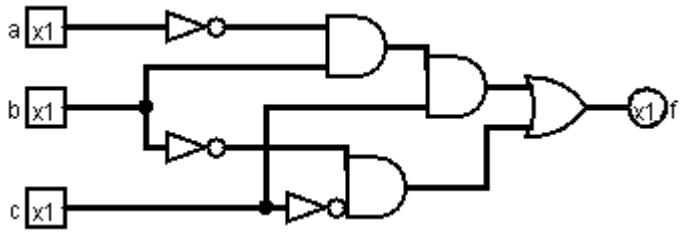
$$= \sum m\,(0,1,6,7,8,9) = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}d + \bar{a}bc\bar{d} + \bar{a}bcd + a\bar{b}\bar{c}\bar{d} + a\bar{b}\bar{c}d$$

$$= \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}d + \bar{a}bc\bar{d} + \bar{a}bcd + a\bar{b}\bar{c}\bar{d} + a\bar{b}\bar{c}d$$

$$= \bar{b}\bar{c}(\bar{a}\bar{d} + \bar{a}d + a\bar{d} + ad) + \bar{a}bc(\bar{d} + d)$$

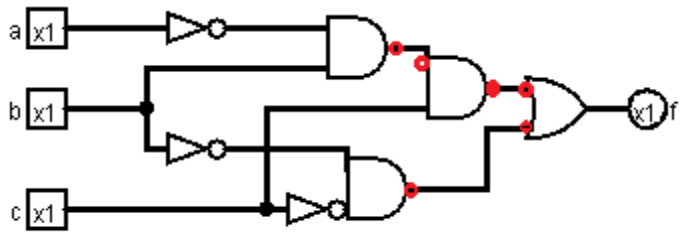
$$= \bar{b}\bar{c} + \bar{a}bc$$

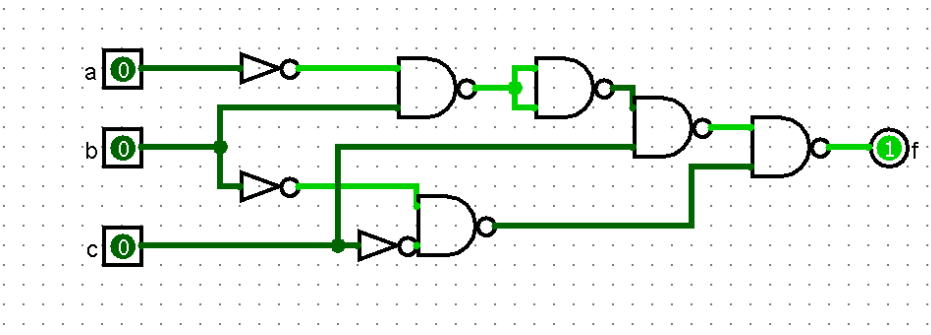
b. The circuit …



c.

---By using circuit design

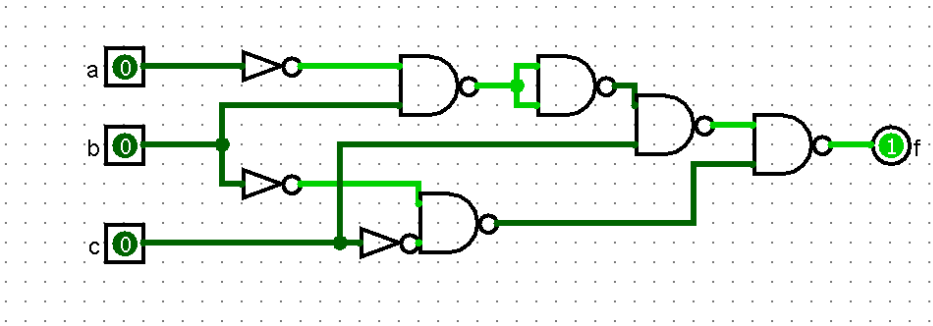




-- by using the expression

Assume that NAND operation has been presented with bowtie.

$$
\begin{aligned}
f &= \overline{b}\overline{c} + \bar{a}bc \\
&= \left(\overline{b} \bowtie \overline{c}\right)' + ((\bar{a}b) \bowtie c)' \\
&= \left(\overline{b} \bowtie \overline{c}\right)' + ((\bar{a}b) \bowtie c)' \\
&= (\overline{b} \bowtie \overline{c}) \bowtie ((\bar{a}b) \bowtie c) \\
&= \left(\overline{b} \bowtie \overline{c}\right) \bowtie ((\bar{a} \bowtie b)' \bowtie c) \\
&= \left(\overline{b} \bowtie \overline{c}\right) \bowtie (\{(\bar{a} \bowtie b) \bowtie (\bar{a} \bowtie b)\} \bowtie c)
\end{aligned}
$$



2. The truth table of expression f is given as follows.

a	b	c	d	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

a. Write the first and second canonical form of f.
b. Minimize the second canonical form of the expression by using the axioms and theorems of the Boolean algebra.
c. Draw the minimized expression in b by only using 2-input NOR.

Answer:

a. The first canonical form of the expression

$$f(a,b,c,d) = \sum m\,(1,4,5,6,7,9,12,13,14,15)$$
$$= \bar{a}\bar{b}\bar{c}d + \bar{a}b\bar{c}\bar{d} + \bar{a}b\bar{c}d + \bar{a}bcd + \bar{a}bcd + a\bar{b}\bar{c}d + ab\bar{c}\bar{d} + ab\bar{c}d + abc\bar{d} + abcd$$

The second canonical form of the expression

$$f(a,b,c,d) = \prod M\,(0,2,3,8,10,11)$$
$$= (a+b+c+d)(a+b+\bar{c}+d)(a+b+\bar{c}+\bar{d})(\bar{a}+b+c+d)(\bar{a}+b+\bar{c}+d)(\bar{a}+b+\bar{c}+\bar{d})$$

b. Minimization of the second canonical form

$$f = (a+b+c+d)(a+b+\bar{c}+d)(a+b+\bar{c}+\bar{d})(\bar{a}+b+c+d)(\bar{a}+b+\bar{c}+d)(\bar{a}+b+\bar{c}+\bar{d})$$

$$= [(a+b+\bar{c})+(d\,\bar{d})][(a\,\bar{a})+(b+c+d)][(a\,\bar{a})+(b+\bar{c}+d)][(\bar{a}+b+\bar{c})+(d\,\bar{d})]$$

$$= (a+b+\bar{c})(b+c+d)(b+\bar{c}+d)(\bar{a}+b+\bar{c})$$

$$= [(a+b+\bar{c})(\bar{a}+b+\bar{c})][(b+c+d)(b+\bar{c}+d)]$$

$$= [(a\,\bar{a})+(b+\bar{c})][(b+d)+(\bar{c}\,c)]$$

$$= (b+\bar{c})(b+d)$$

c. The given expression has operations with only two parameters. Thus, in PoS form, it can be easily implemented with 2-input NOR gates by just replacing the AND, OR, NOT gates with NOR gates.