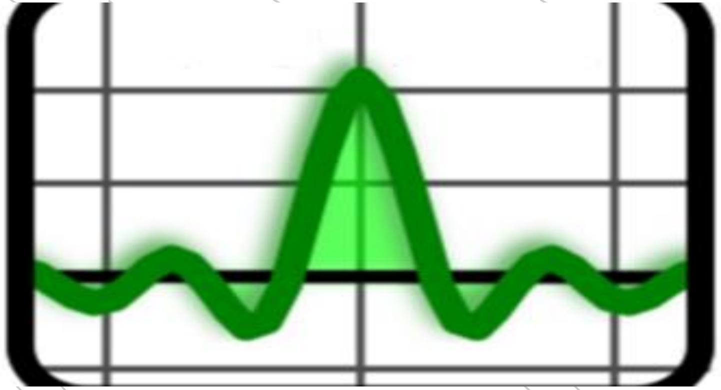

Signals & Systems for Computer Engineering

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6th Week Lecture (cont.)

Implementation of Discrete Time Systems

Transfer function of DT-LTI system is given as,

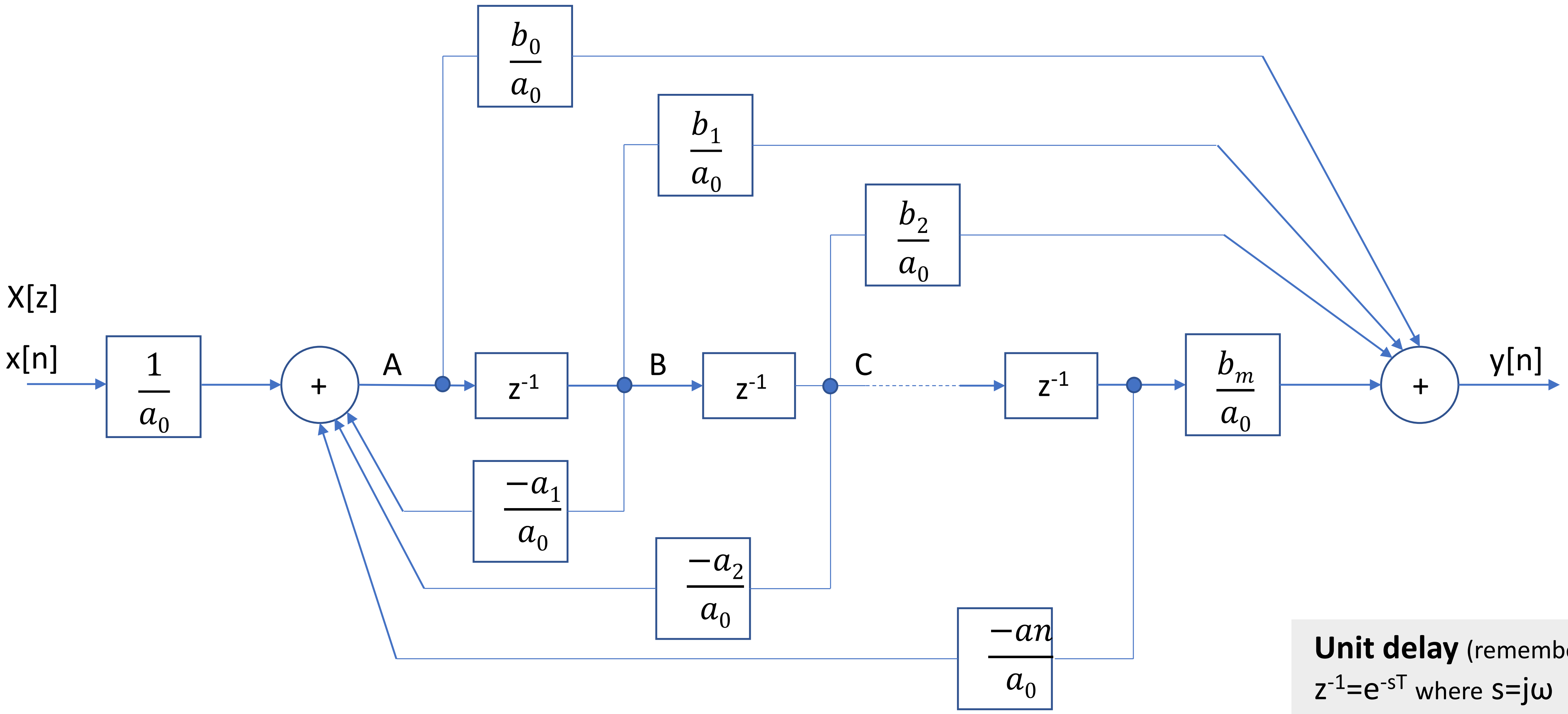
$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \dots\dots + b_m z^{-m}}{a_0 + a_1 z^{-1} + \dots\dots + a_n z^{-n}}$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + \dots\dots + b_m z^{-m}}{a_0 + a_1 z^{-1} + \dots\dots + a_n z^{-n}} \cdot \frac{A(z)}{A(z)} = \frac{Y(z)}{X(z)}$$

$$Y(z) = \frac{1}{a_0} \left(b_0 + b_1 z^{-1} \dots\dots + b_m z^{-m} \right) A(z)$$

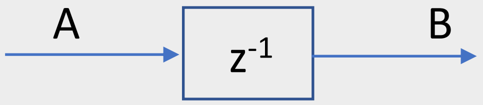
$$A(z) = \frac{1}{a_0} X(z) - \frac{1}{a_0} \left(a_1 z^{-1} + a_2 z^{-2} + \dots\dots\dots + a_n z^{-n} \right) A(z)$$

Direct Programming
(canonical form)



Unit delay (remember):

$z^{-1}=e^{-sT}$ where $s=j\omega$



$B=A\cdot z^{-1} \Rightarrow B(k)=A(k-1)$

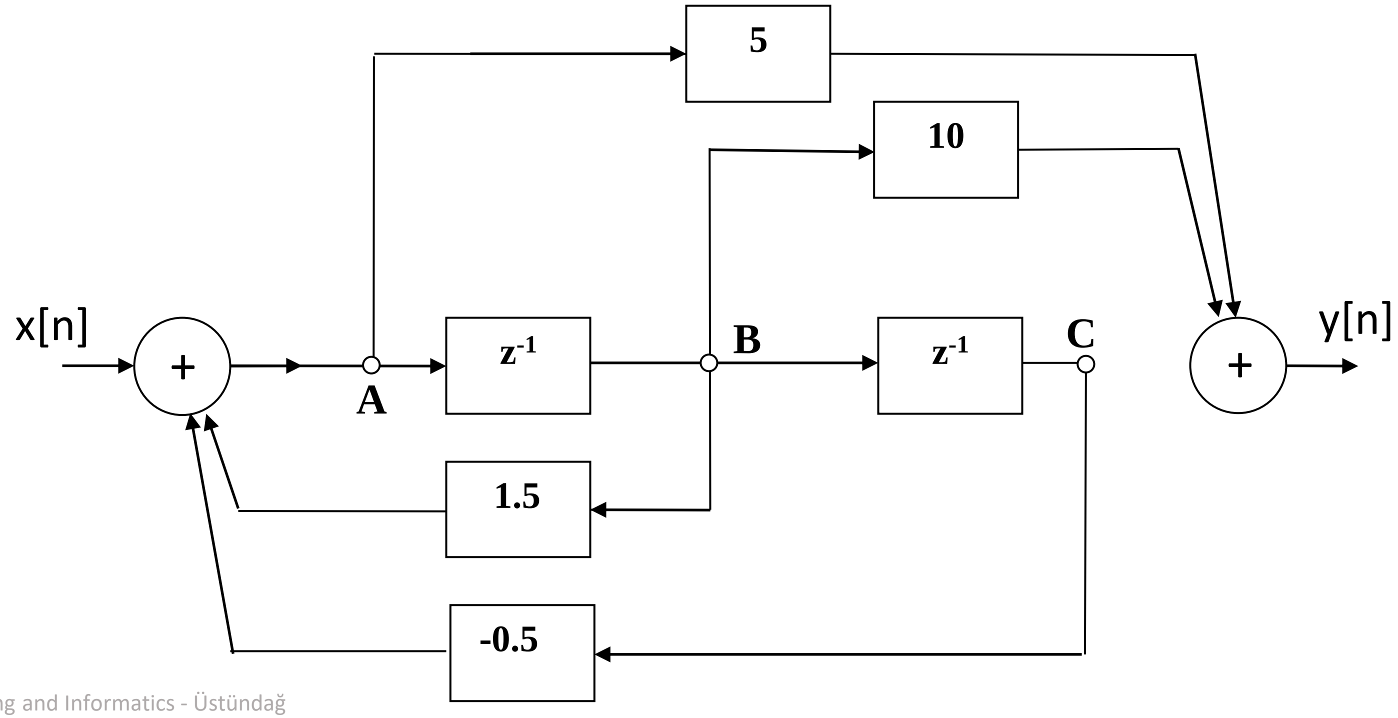
Example:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{5(1+2z^{-1})}{(1-z^{-1})(1-0.5z^{-1})}$$

Write the Psuedo code H(z) for the signal sampled at f_s

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(5+10z^{-1})A(z)}{(1-1.5z^{-1}+0.5z^{-2})A(z)}$$

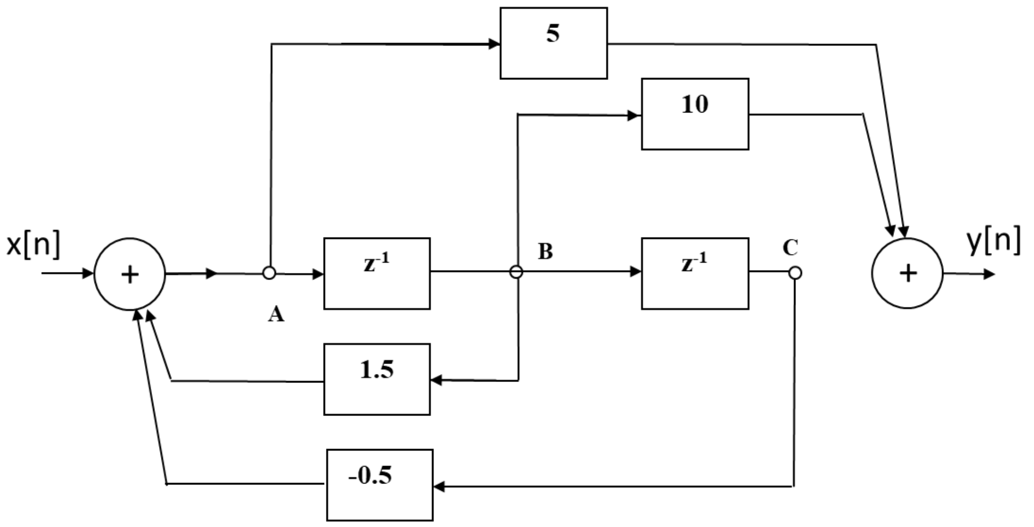
$$Y(z) = (5 + 10 z^{-1}) A(z)$$
$$A(z) = X(z) + 1.5 z^{-1} A(z) - 0.5z^{-2} A(z)$$



Pseudo code:
Timer Interrupt @Ts=1/f_s

X = READ (ADC)
A = X + 1.5 B – 0.5 C
Y = 5 A + 10 B
Output (Y)
C=B
B=A
Return

Proof:



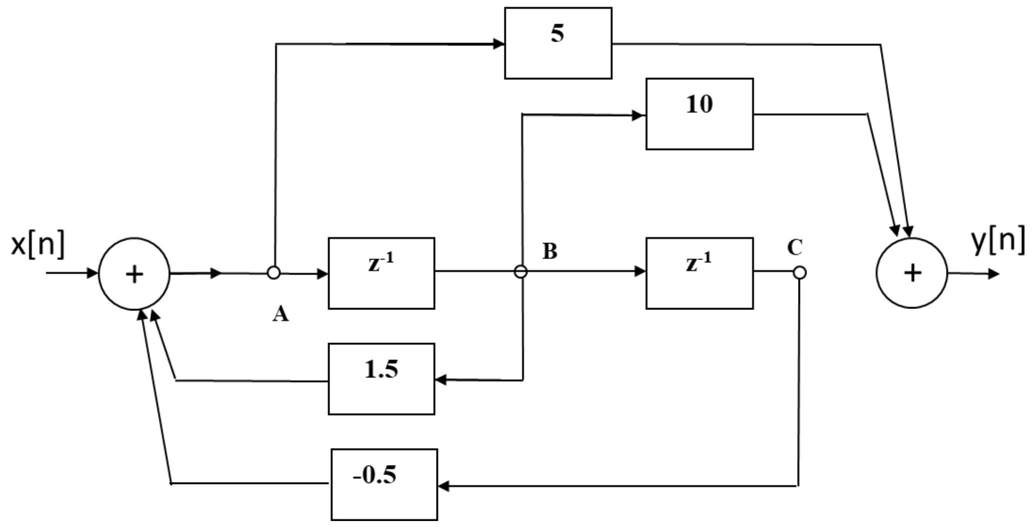
$$Y=5A+10B=$$
$$=5A+10Az^{-1}=(5+10z^{-1})A$$

$$A=X+1.5B-0.5C$$
$$A=X+1.5Az^{-1}-0.5Az^{-2} \Rightarrow \quad A=\frac{X}{1-1.5z^{-1}+0.5z^{-2}}$$

$$Y=\frac{(5+10z^{-1})X}{1-1.5z^{-1}+0.5z^{-2}} \Rightarrow \quad T(z)=\frac{Y(z)}{X(z)}=\frac{(5+10z^{-1})}{(1-1.5z^{-1}+0.5z^{-2})}$$

Step Response:

$x[n]=u[n] \Rightarrow \begin{cases} \text{for } n<0 \; x[n]=0 \Rightarrow y[n]=0 \\ \text{for } n\geq 0 \; x[n]=1 \end{cases}$



@T_s
$t=n\cdot T_s$

Input X
A = X + 1.5 B – 0.5 C
Y = 5 A + 10 B
C=B
B=A
Output Y
Return

Initial condition

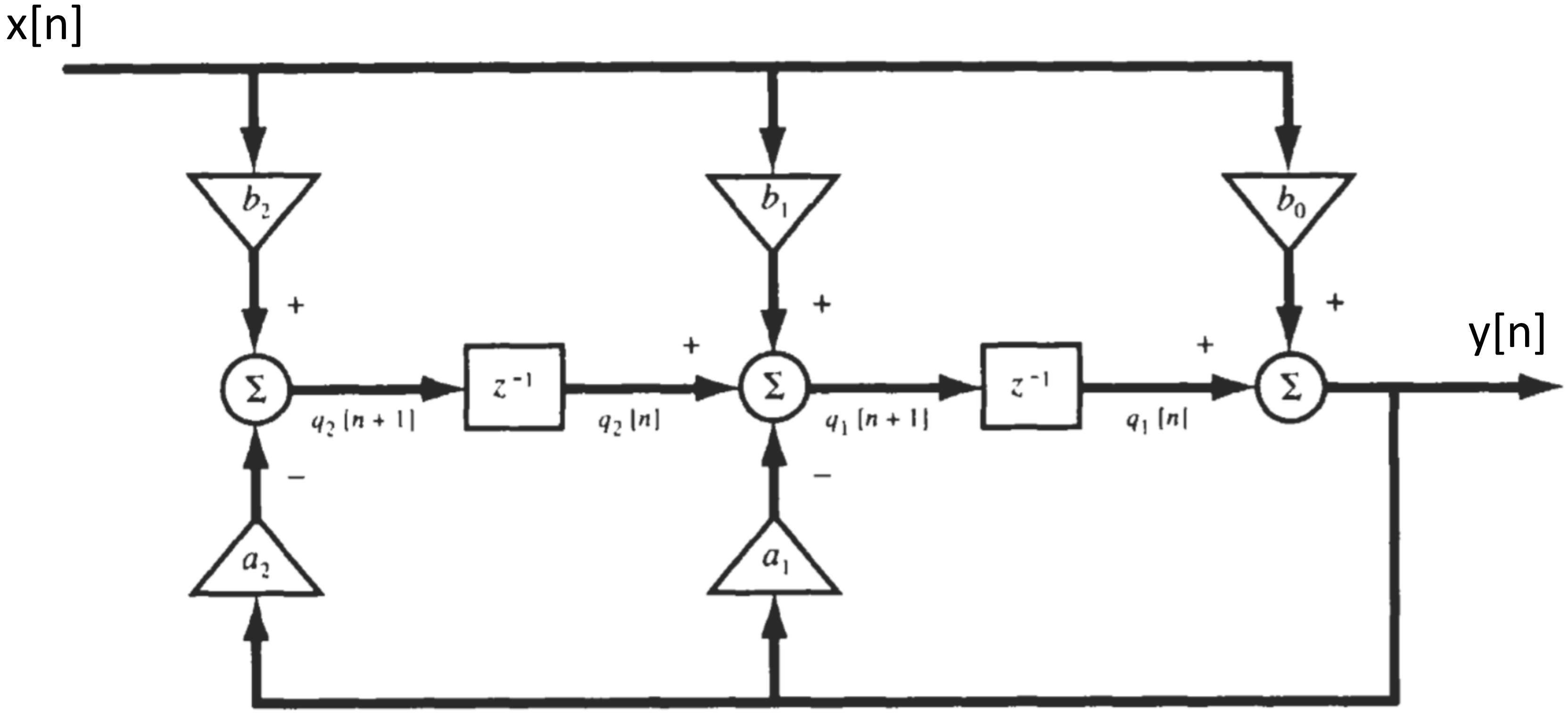
n	x[n]	A	B	C	y[n]
0	1	1	0	0	5
1	1	2.5	1	0	22.5
2	1	4.25	2.5	1	46.25
3	1	6.125	4.25	2.5	73.125

Canonical Simulation
(First form)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

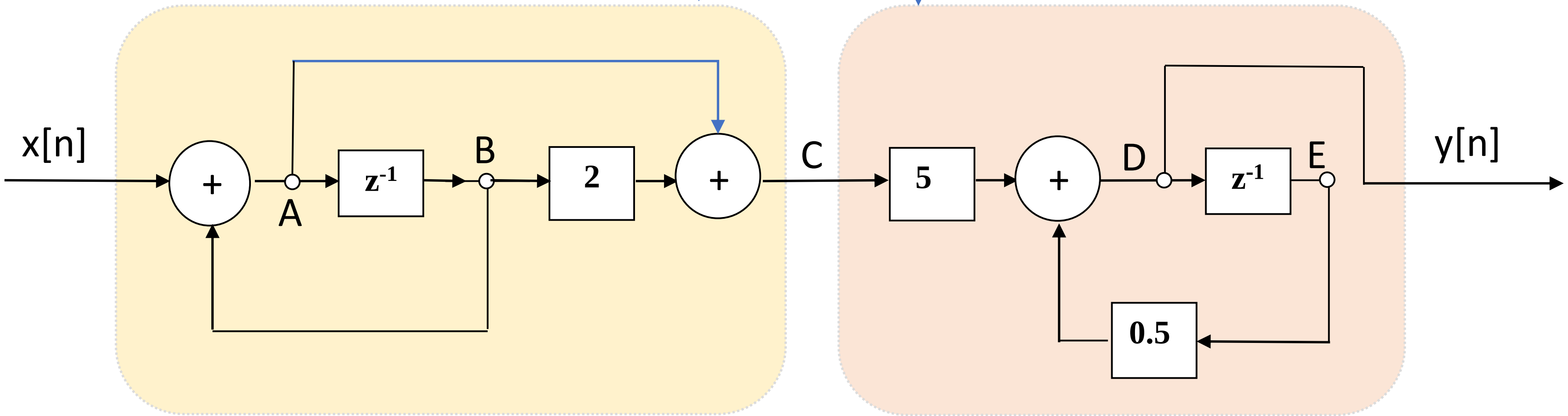
$$\left(1 + a_1 z^{-1} + a_2 z^{-2}\right) Y(z) = \left(b_0 + b_1 z^{-1} + b_2 z^{-2}\right) X(z)$$

$$Y(z) = -a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) + b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z)$$



Programming of Cascaded sub-systems

$$H(z) = \frac{Y(z)}{X(z)} = \underbrace{\frac{1+2z^{-1}}{1-z^{-1}}} \cdot \underbrace{\frac{5}{1-0.5z^{-1}}}$$

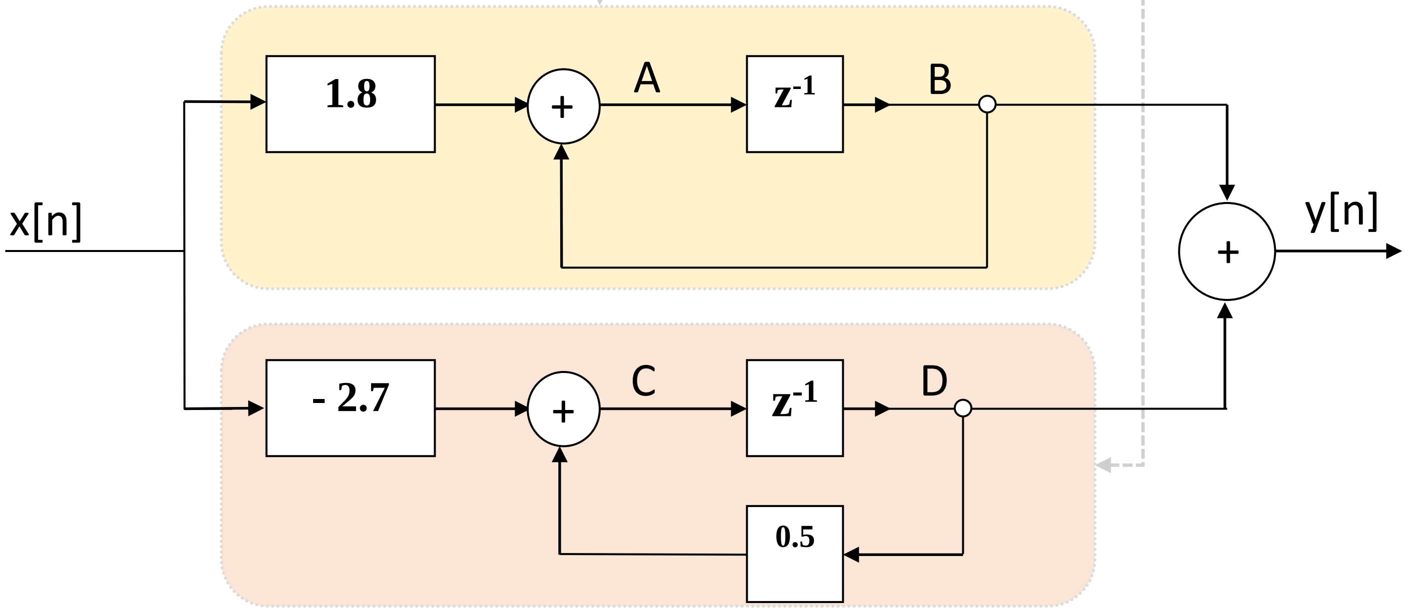


@T_s
$t=k\cdot T_s$

Input X
A=X+B
C=A+2B
D=5C+0.5E
Y=D
Output Y
E=D ; z^{-1}
B=A ; z^{-1}
Return

Programming of Parallel sub-systems

$$H(z) = \frac{Y(z)}{X(z)} = \underbrace{\frac{1.8z^{-1}}{1-z^{-1}}}_{} - \underbrace{\frac{2.7z^{-1}}{1-0.5z^{-1}}}_{}$$

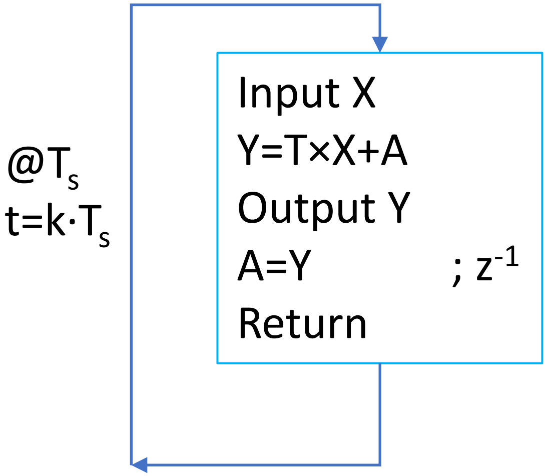


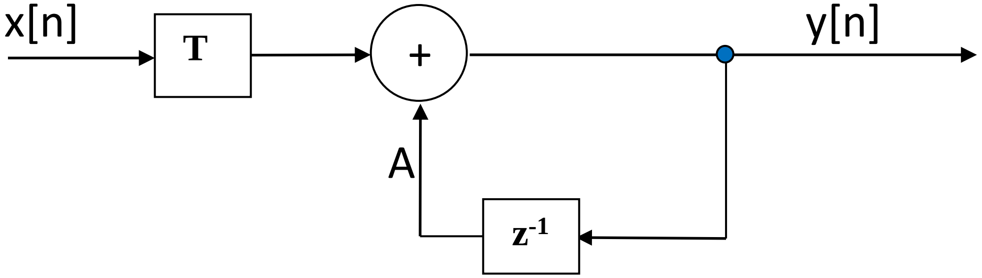
$@T_s$
$t=k \cdot T_s$

```
Input X
A=1.8X+B
C=-2.7X+0.5D
Y=B+D
Output Y
B=A         ; z^-1
D=C         ; z^-1
Return
```

Example:

Digital integrator without delay: $$H(z) = \frac{Y(z)}{X(z)} = T\left(\frac{1}{1-z^{-1}}\right)$$



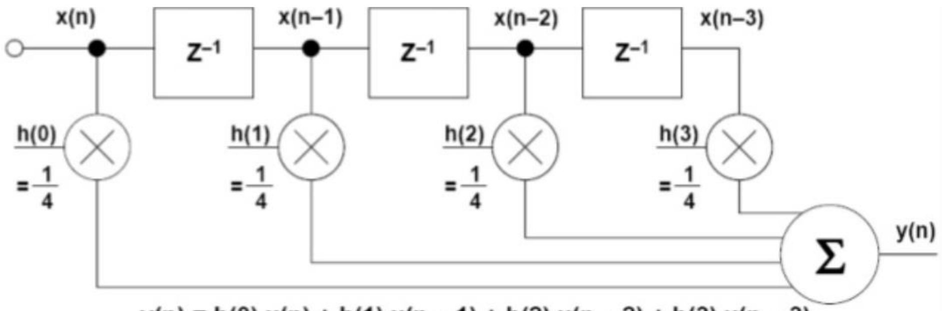


Initial condition: If y(kT)=0 for k<0 then A=0

Y[n]=y(kT)=T[x(0)+x(T)+x(2T)+…+x(kT)]

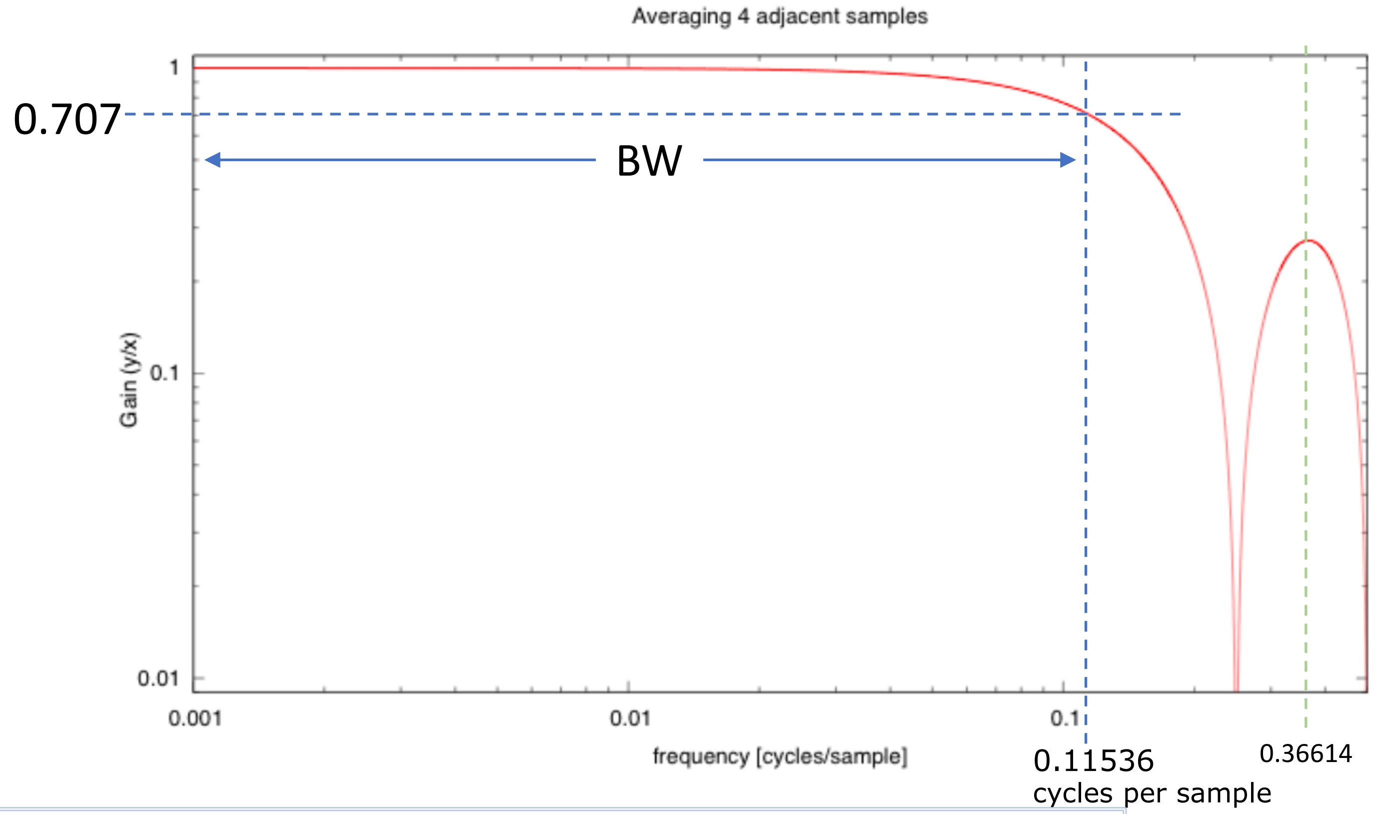
1) Simulate the system for x[n]=u[n-2]-u[n-4]+δ[n-3]
2) What are the differences between integrator with and without Delay ?
3) Write the pseudo code for digital integrator with delay

Additional concerns:



$$y(n) = h(0)\,x(n) + h(1)\,x(n-1) + h(2)\,x(n-2) + h(3)\,x(n-3)$$
$$= \frac{1}{4}x(n) + \frac{1}{4}x(n-1) + \frac{1}{4}x(n-2) + \frac{1}{4}x(n-3)$$
$$= \frac{1}{4}\left[x(n) + x(n-1) + x(n-2) + x(n-3)\right]$$

$$H(z) = 1/4(1 + z^{-1} + z^{-2} + z^{-3}).$$



gnuplot s

Question:

<u>Design and Analysis problem</u>

How can we find the discrete time transfer function and write the real time computational code

If real time characteristic of the target system is given first

(e.g. if BW is given how to find h(i))

```
j=sqrt(-1)
amplitude(omega) = abs(H(exp(j*omega)))
phase(omega) = imag(log(H(exp(j*omega)))) * 180/pi   # phase in degrees

plot amplitude(2*pi*x) notitle
```