BLG 231E - Digital Circuits

Assignment 1

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Part 1 – Computer Arithmetic

1. ***A*** and ***B*** are two 8-bit binary integers, and ***B*** = **1101 1001**. For the operation ***A*** **-** ***B***, answer the following questions:

a. If ***A*** and ***B*** are *signed* binary integers, what are the **i)** largest and **ii)** smallest decimal values of **A** that yield valid results (that can be represented using 8 bits) after the operation? Explain your answer briefly.

Solution: a. i) The signed integers are in between -128, 127 so the result of the **A-B** should be in between these values, if the value of **A-B** is not in between these values, the overflow will occur. Also, **A** should be between the range of -128 to 127. So to find the largest value of A that yield valid results, we have to find the value of **A** in the case of **A-B** is equal to 127;

B=11011001 _____2's complement____> 00100111=(2^5*1)+(2^2*1)+(2^1*1)+(2^0*1)=39

So B= -39

-128 <= A- B < =127 ---------- -128 <=A+39 <= 127 also -128 <= A <= 127

A-B=+127 ----------> A = B + 127

The largest decimal value of A is +88.

Solution: 1. a. ii) To find the smallest value **A** that yield valid results, **A-B** should be smallest, but at the same time the A need to be between the range of -128 to 127.

-128 <= A- B < =127 ---------- -128 <=A+39 <= 127 also -128 <= A <= 127

A can get the smallest value in range of -128:127 which is -128, because it also fits to -128< A- B < 127

The smallest decimal value of A is -128.

b. Write the binary representation for the largest value of the signed ***A*** you found in **(a.i)**. Carry out the binary operation ***A - B*** using **2's complement**, and show that the result is valid using binary numbers only.
Solution: 1. b.)

A – B = A + B+ 1

B=11011001 B=00100110 B + 1 = 00100111

01011000 : +88 --> A
11011001 : -39 ---> B _____2's complement
-

01011000 : +88 --> A
00100111 : 39--> B + 1
+

01111111 : 127

The result is valid

c. If ***A*** and ***B*** are *unsigned* binary integers, what are the **i)** largest and **ii)** smallest binary values of **A** that yield valid results after the operation? Explain your answer briefly.

Solution: 1. c.) The unsigned binary numbers are in the range of 0 to 255, so the A and A-B need to be in range of 0 to 255.

B=11011001 _____2's complement____> 00100111=(2^5*1)+(2^2*1)+(2^1*1)+(2^0*1)=39
So B= -39

0 <= A- B <= 255 ---------- 0 <= A+39 <= 255 also 0 < =A < =255

A-B=A+39 A+39=255 A=216 in decimal A= 011011000

The largest binary value of A is 11011000

Smallest value of A is "0" since it yields the 0 <= A- B <= 255

A=0 in decimal A= 00000000

The largest binary value of A is 00000000

2. ***A*** and ***B*** are two 8-bit, **signed**, binary integers, and ***A* = 1011 1100**. If we perform the operation ***A* + *B*** ,

a. What are the **i)** largest and **ii)** smallest decimal values of ***B*** that yield valid results after the operation? Explain your answer briefly.

Solution 2. a. i)B and **A+B** should be in between -128 and 127.

A= 1011 1100 _____2's complement____> 01000100=(2^6*1)+(2^2*1)=68

So A is -68

-128<= A+B <= 127 -128<= (-68)+B <= 127 also -128 <= B <= 127

Largest value of B is "127" since it yields the -128 <= (-68)+B <= 127

The largest decimal value of B is 127

Solution 2. a. ii)B and **A+B** should be in between -128 and 127

A= 1011 1100 _____2's complement____> 01000100=(2^6*1)+(2^2*1)=68

So A is -68

-128<= A+B <= 127 -128<= (-68)+B <= 127 also -128 <= B <= 127

-68 + B = -128 B= -60

Smallest value of B is "-60" since it yields the -128<= (-68)+B <= 127

The smallest decimal value of B is -60

b. Write the binary representation for the smallest value of the signed ***B*** you found in **(a.ii)**. Perform the binary operation ***A + B***, and show that the result is valid using binary numbers only.

Solution 2. b.)

60 =00111100 1's complement 11000011 :::: 2'complement 11000100

-60=11000100

B=11000100

```
    10111100  : -68 -->  A
    11000100 : -60  --->  B
 +
_____________________
    110000000   -128
```

the leftmost 1 is ignored the result is 10000000

The result is valid

Part 2 – Boolean Algebra

3. Simplify the following logic expressions using axioms, properties, and theorems of Boolean algebra.a.

a. E(a, b, c) = $a\bar{b}\bar{c} + ab\bar{c} + abc + \bar{a}bc$
b. E(a, b, c, d) = $\bar{a}b\bar{d} + bcd + abc + a\bar{b}d + bc\bar{d} + ad + \bar{a}bc$