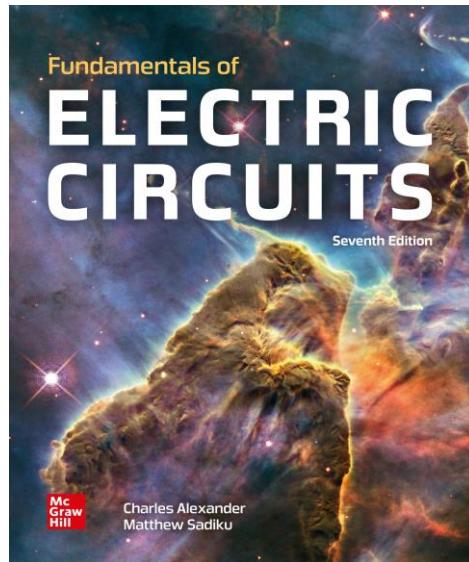


# **EHB 211E**

## **Basics of Electrical Circuits**

*Asst. Prof. Onur Kurt*

### **Basic Laws**



- Fundamental laws:
  - Ohm's law
  - Kirchhoff's law: Kirchhoff's voltage law & Kirchhoff's current law
- Techniques applied in circuit design and analysis:
  - Combining resistors in series or parallel
  - Voltage division & current division
  - Delta-to-wye and wye-to-delta transformation

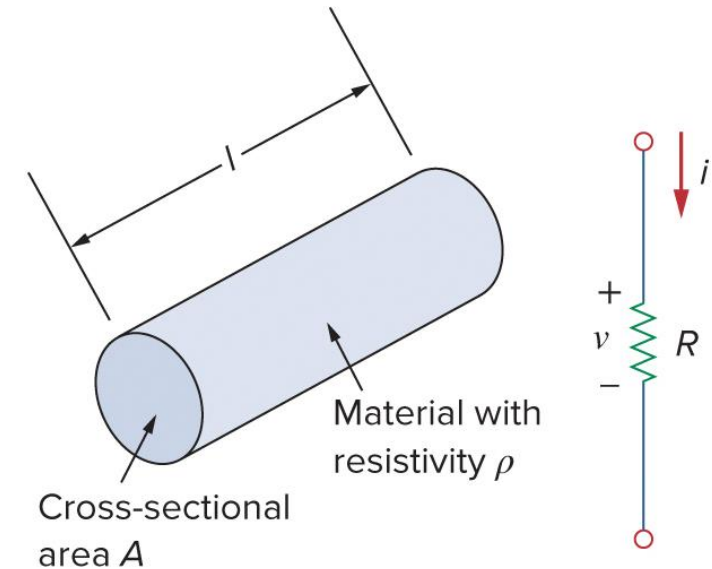
# Ohm's law

- What is resistance?
- Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or **ability to resist current**, is known as resistance.
- Resistance is represented by  $R$ , measure in ohms ( $\Omega$ )
- Resistance in mathematical form:

$$R = \rho \frac{\ell}{A}$$

$\rho$  is resistivity ( $\Omega\cdot\text{m}$ ),  $\ell$  is the length (m), and  $A$  is cross-sectional area ( $\text{m}^2$ )

- Terminology:
  - Resistance: ability to resist current
  - Resistor: passive circuit element that create resistance



# Ohm's law

- Resistivity,  $\rho$ , is material dependent properties.
- Conductor: low resistivity
- Insulator: high resistivity

Resistivities of common materials.		
Material	Resistivity ( $\Omega \cdot m$ )	Usage
Silver	$1.64 \times 10^{-8}$	Conductor
Copper	$1.72 \times 10^{-8}$	Conductor
Aluminum	$2.8 \times 10^{-8}$	Conductor
Gold	$2.45 \times 10^{-8}$	Conductor
Carbon	$4 \times 10^{-5}$	Semiconductor
Germanium	$47 \times 10^{-2}$	Semiconductor
Silicon	$6.4 \times 10^2$	Semiconductor
Paper	$10^{10}$	Insulator
Mica	$5 \times 10^{11}$	Insulator
Glass	$10^{12}$	Insulator
Teflon	$3 \times 10^{12}$	Insulator

# Ohm's law

- What is Ohm's law?

- Ohm's law states that the voltage  $v$  across a resistor is directly proportional to the current  $i$  flowing through the resistor, i.e.,  $v \propto i$

- By definition:

$$v = i R$$

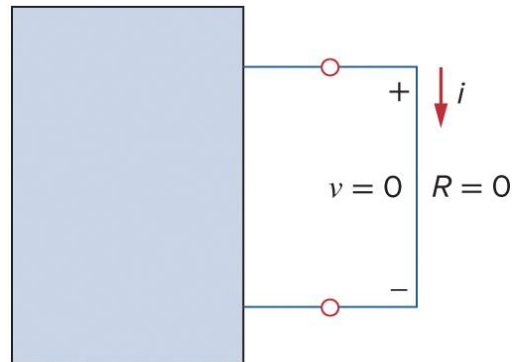


Relation between voltage  
and current for a resistor

$$R = \frac{v}{i} \longrightarrow \Omega = \frac{V}{A}$$

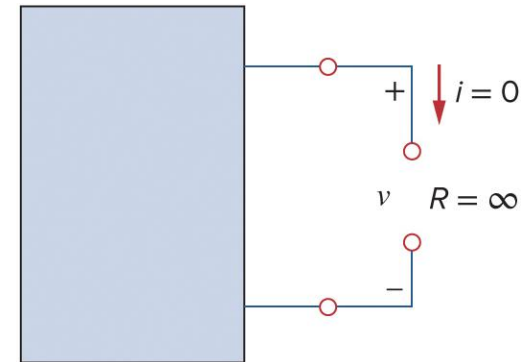
- Two extreme possible values of resistance  $R$ :

Short circuit:  $R$  approaches zero



$$v = i R = 0$$

Open circuit:  $R$  approaches infinity

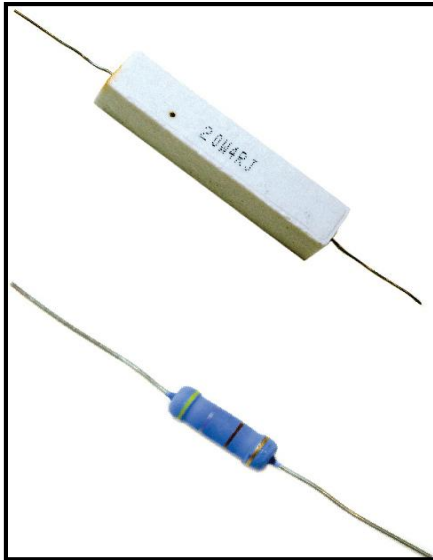


$$i = \lim_{R \rightarrow \infty} \frac{V}{R} = 0$$

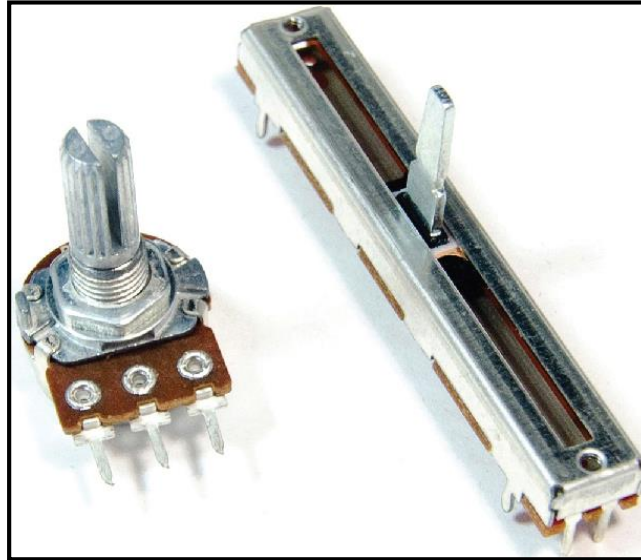
# Resistors

- Two kinds of resistors: Fixed or variable

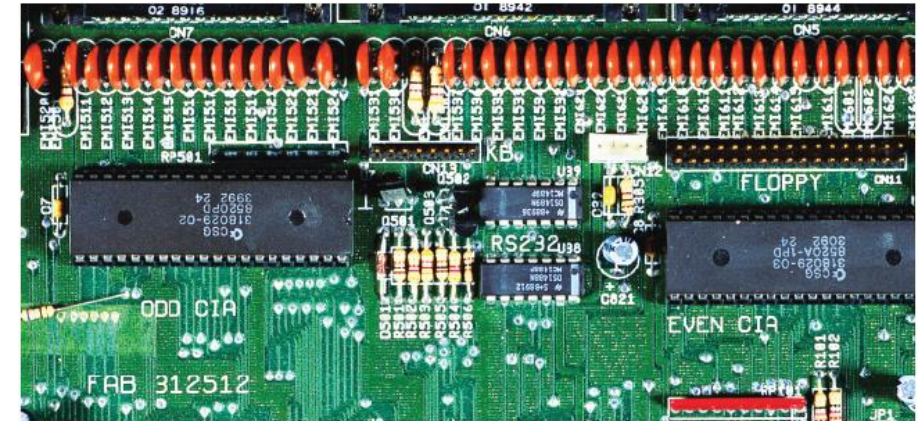
Fixed resistors



Variable resistors



Resistors in an integrated circuit board



Eric Tormey/Alamy

Circuit symbol of fixed resistor



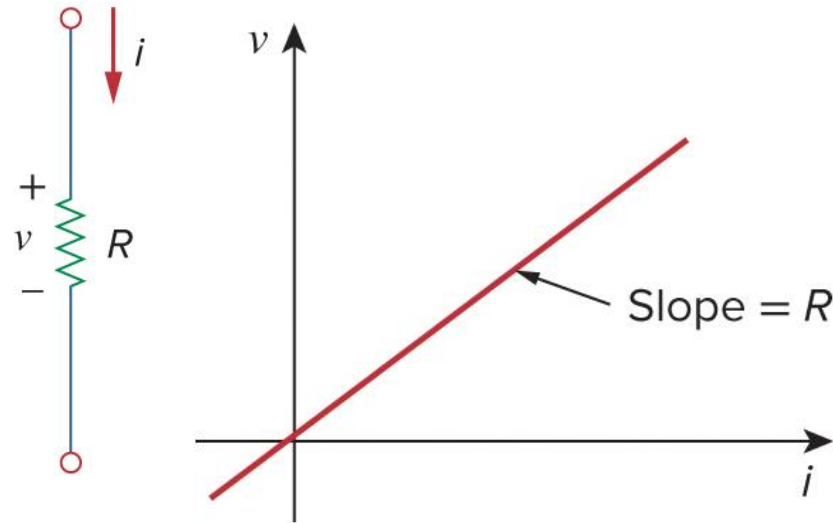
Circuit symbol of variable resistor



# Ohm's law

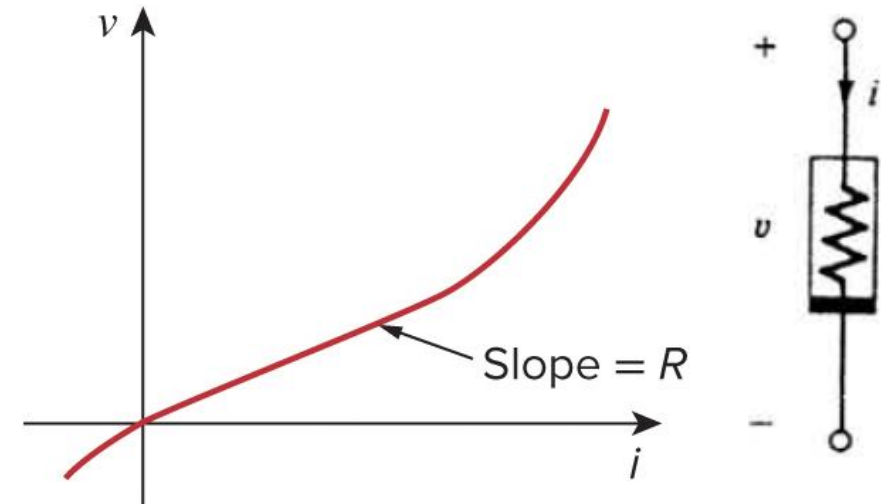
- Not all resistors obey Ohm's law

Linear resistance: Obey Ohm's law



- It has a constant resistance (slope)
- Its  $i$ - $v$  graph is a straight line passing through the origin

Nonlinear resistance: Does not Obey Ohm's law



- Its resistance (slope) varies with current
- Example: Light bulb, diode



# Conductance



- What is conductance?

- The ability of an element to conduct electric current or reciprocal of resistance  $R$
- It is denoted by  $G$ , and measured in mho( $\mathcal{U}$ ) or siemens ( $S$ )

$$\boxed{G = \frac{1}{R} = \frac{i}{v}} \longrightarrow \boxed{i = Gv}$$

$$1\,S = 1\mathcal{U} = 1\,\frac{A}{V}$$

Power dissipated by the resistor (in terms of  $R$ ):

$$\boxed{p = vi = i^2 R = \frac{v^2}{R}}$$

Power dissipated by the resistor (in terms of  $G$ ):

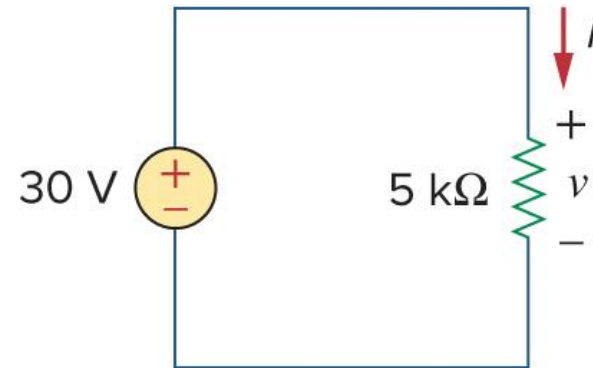
$$\boxed{p = vi = v^2 G = \frac{i^2}{G}}$$

- The power dissipated in a resistor is a nonlinear function of either current or voltage
- Since  $R$  and  $G$  are both positive, the power dissipated in a resistor is positive. Resistor always absorbs power from the circuit. Resistor is passive element (incapable of generating power)



# Example 1:

- In the circuit shown below, calculate the current  $i$ , the conductance  $G$ , and the power  $p$ .



## Solution:

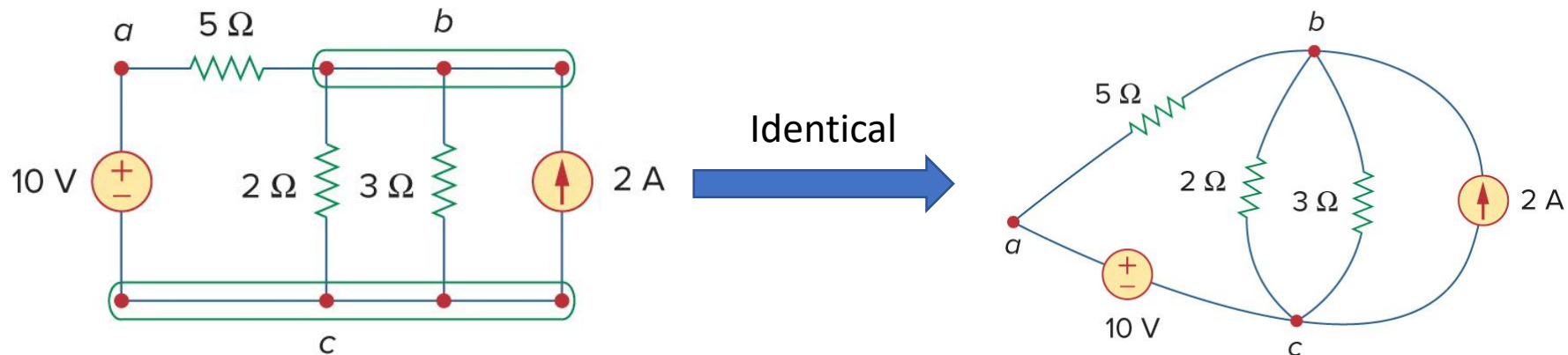
- To calculate the current  $i$ , we need to use Ohm's law, i.e.,  $v = iR$
- Voltage across the resistor is same as the source voltage (30 V) since the source and resistor are connected to the same pair of terminals. Therefore, the current is:

$$v = iR \quad \longrightarrow \quad i = \frac{v}{R} \quad \longrightarrow \quad i = \frac{30}{5k} = 6mA$$

- The conductance  $G$  is:  $G = \frac{1}{R} = \frac{1}{5k} = 0.2 \text{ mS}$
- The power  $p$  is:  $p = vi = i^2R = \frac{v^2}{R}$  or  $p = vi = v^2G = \frac{i^2}{G}$   
 $p = vi = 30 \times 6 \times 10^{-3} = 180 \text{ mW}$

# Nodes-Branches-Loops

- Basic concepts of network topology: Nodes, Branches, and Loops.
- **What is a branch?**
  - A single element such as voltage source or a resistor (any two-terminal element)
- **What is a node?**
  - The point of connection between two or more branches and usually indicated by a dot.
  - If a short circuit (connecting wire) connects two nodes, the nodes constitute a single node.



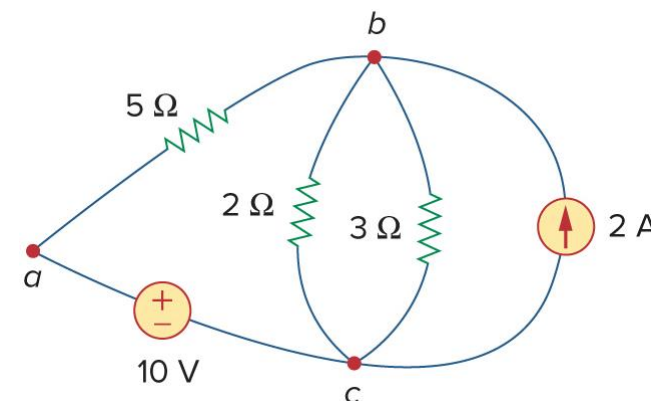
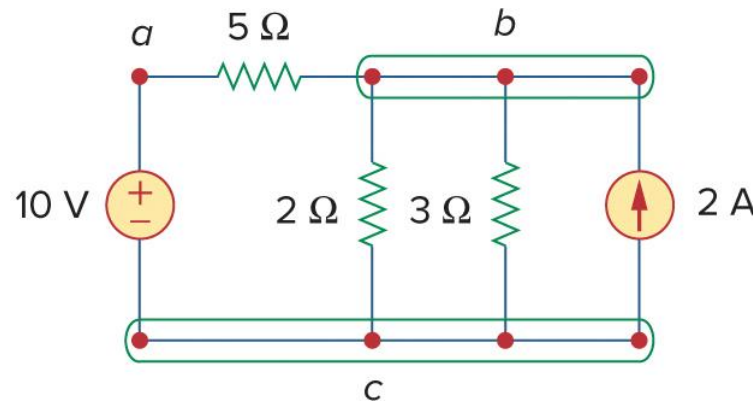
▪ 5 branches: 10 V voltage source, 2 A current source, and three resistors

▪ 3 nodes: node a, node b, and node c

# Nodes-Branches-Loops

- What is a loop?

- Any closed path in a circuit.
- A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any nodes more than once.
- A loop said to be independent if it contains at least one branch which is not a part of any other independent loop
- Independent loops or paths result in independent set of equations.



- Three independent loops:

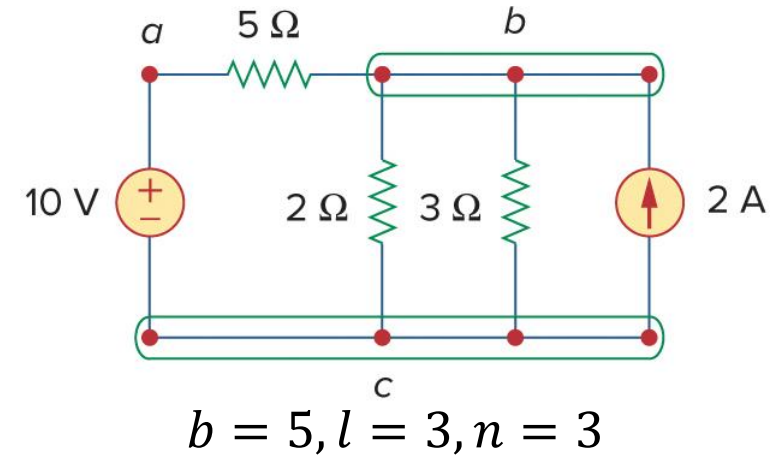
- 1<sup>st</sup> loop: voltage source and two resistors (5 Ω & 2 Ω)
- 2<sup>nd</sup> loop: voltage source and two resistors (5 Ω & 3 Ω)
- 3<sup>rd</sup> loop: voltage source, current source, and a resistor (5 Ω)

# Fundamental Theorem of Network Topology

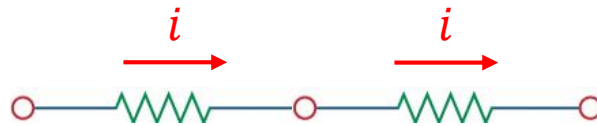
- A network with  $b$  branches,  $n$  nodes, and  $l$  independent loops will always satisfy the fundamental theorem of network topology.

$$b = l + n - 1$$

$b$ : # of branches,  
 $l$ : # of independent loop  
 $n$ : # of nodes

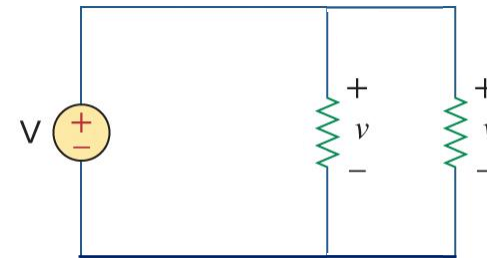


## Series Connection



Two or more elements carry same current if they are in series

## Parallel Connection



Two or more elements have same voltage if they are in parallel

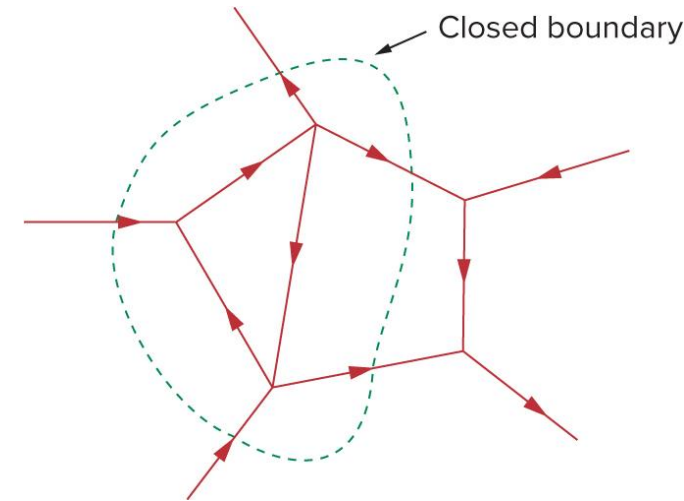
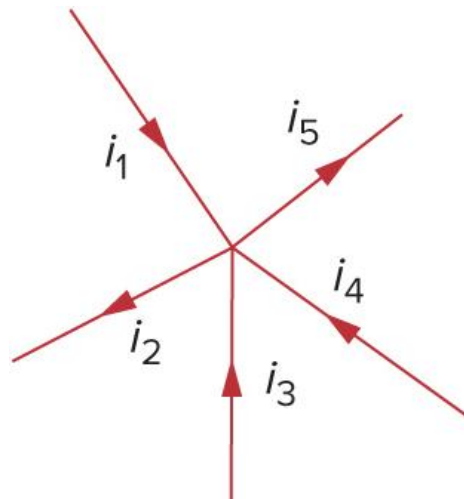
- Ohm's law: not sufficient to analyze circuits by itself
- Ohm's law coupled with Kirchhoff's law: very powerful set of tools for analyzing a large variety of electric circuits.
- Two Kirchhoff's law:
  - Kirchhoff's Current Law (KCL)
  - Kirchhoff's Voltage Law (KVL)

# Kirchhoff's Current Law (KCL)

- What is Kirchhoff's Current Law (KCL)?
  - Algebraic sum of currents entering a node (or a closed boundary) is zero.
  - Sum of currents entering a node is equal to the sum of currents leaving from that node.
- Mathematically, KCL is given by:

$$\sum_{n=1}^N i_n = 0$$

N: # of branches connected to the nodes  
 $i_n$  :  $n$ th current entering (or leaving) the node



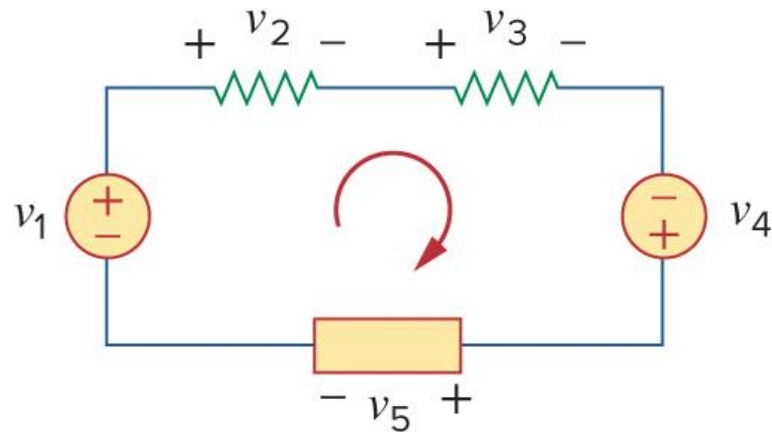
$$i_1 + i_3 + i_4 = i_2 + i_5 \quad \text{or} \quad i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

# Kirchhoff's Voltage Law (KVL)

- What is Kirchhoff's Voltage Law (KVL)?
  - Algebraic sum of all voltages around a closed path (or loop) is zero.
- Mathematically, KVL is given by:

$$\sum_{m=1}^M V_m = 0$$

M: # of voltages in the loop  
 $V_m$  : mth voltage

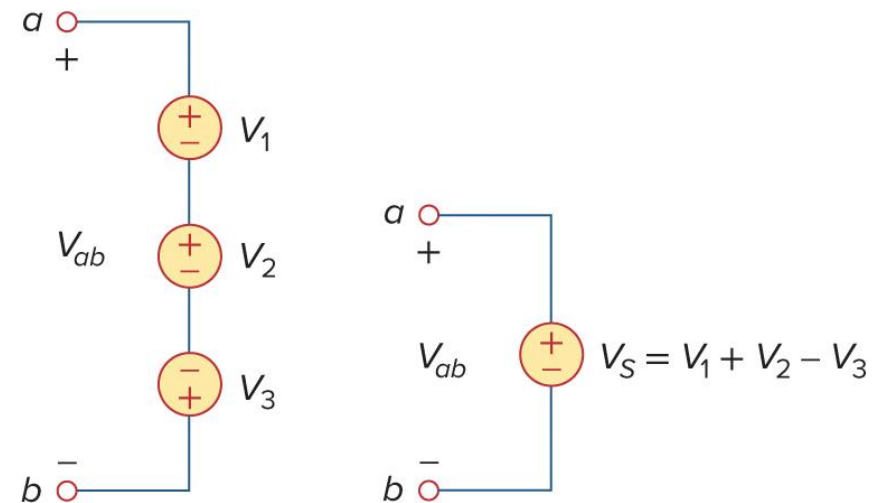


$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

or

$$v_2 + v_3 + v_5 = v_1 + v_4$$

Sum of voltage drop = sum of voltage rise



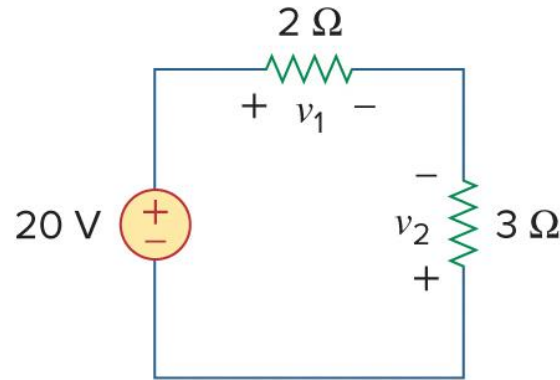
$$-v_{ab} + v_1 + v_2 - v_3 = 0$$

$$v_{ab} = v_1 + v_2 - v_3$$



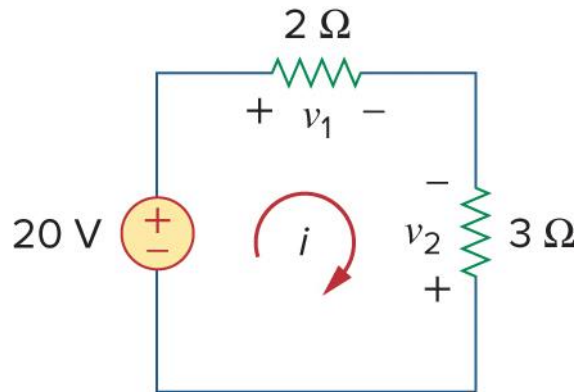
## Example 2:

- For the circuit shown below, find  $v_1$  &  $v_2$



Solution:

- In order to find  $v_1$  &  $v_2$ , we apply Ohm's law and Kirchhoff's voltage law.



Assume that current  $i$  flows through the loop (clockwise direction)

Ohm's law:  $v = iR$

$$v_1 = 2i \quad v_2 = -3i$$

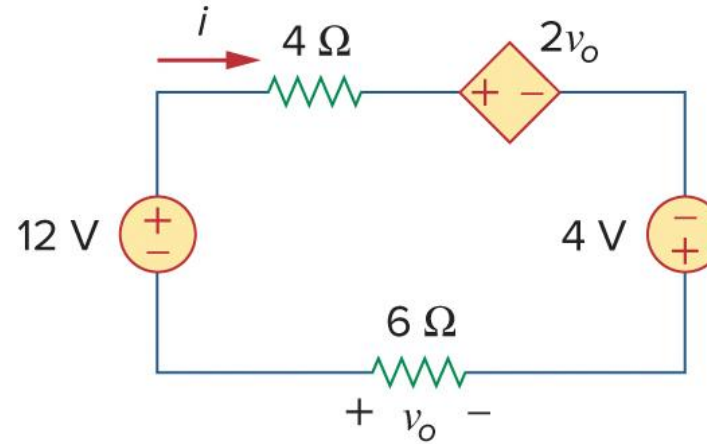
Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0$$

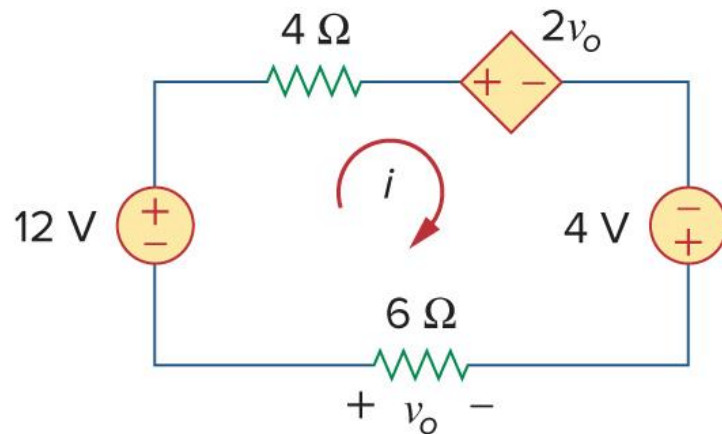
$$-20 + 2i - (-3i) = 0 \Rightarrow i = 4 \text{ A}$$

## Example 3:

- Determine  $v_o$  &  $i$  in the circuit shown below



Solution:



Applying KVL around the loop gives

$$-12 + 4i + 2v_o - 4 - v_o = 0$$

$$-12 + 4i + 2(-6i) - 4 - (-6i) = 0$$

$$i = -8 \text{ A}$$

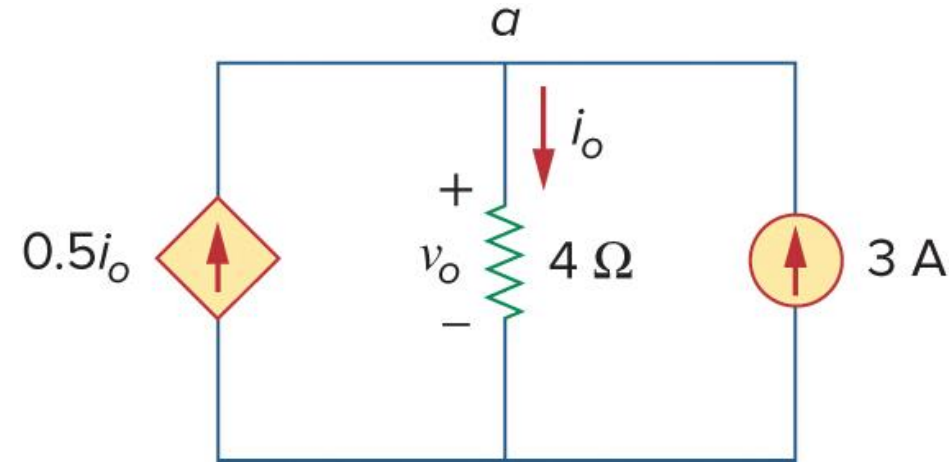
$$v_o = -6i \Rightarrow v_o = 64 \text{ V}$$

Ohm's law:  $v = iR$

$$v_o = -6i$$

## Example 4:

Find current  $i_o$  and voltage  $v_o$  in the circuit



### Solution:

Applying KCL to node  $a$ , we obtain

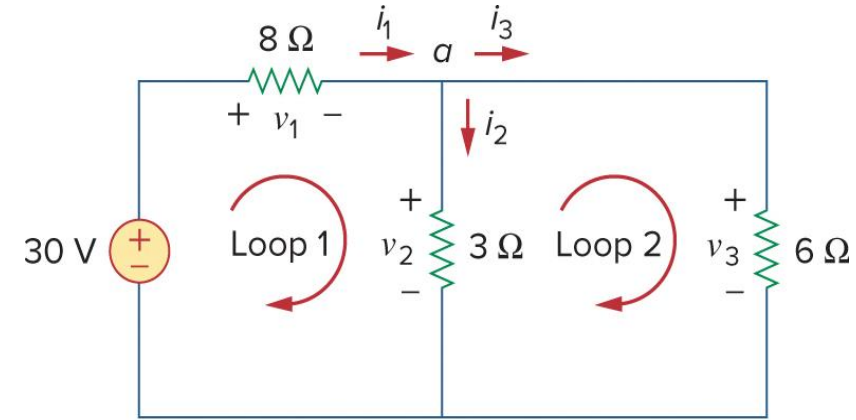
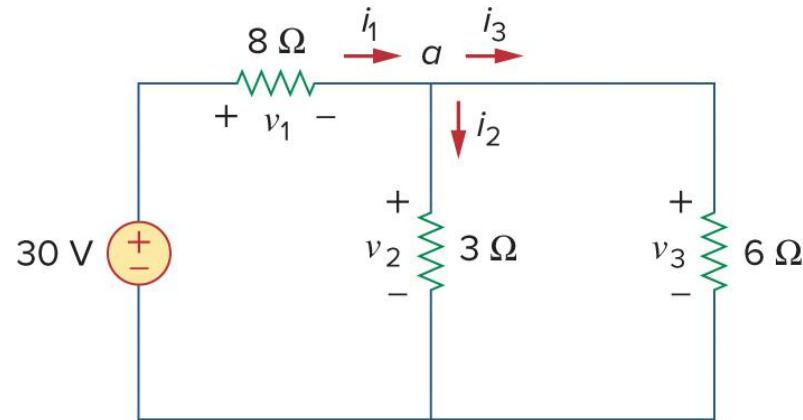
$$3 + 0.5i_o = i_o \quad \Rightarrow \quad i_o = 6 \text{ A}$$

For the  $4\text{-}\Omega$  resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

# Example 5:

Find currents and voltages in the circuit



## Solution:

We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$$v_1 = 8i_1, \quad v_2 = 3i_2, \quad v_3 = 6i_3$$

At node  $a$ , KCL gives

$$i_1 - i_2 - i_3 = 0$$

Applying KVL to loop 1

$$-30 + v_1 + v_2 = 0$$

$$-30 + 8i_1 + 3i_2 = 0$$

$$i_1 = \frac{(30 - 3i_2)}{8}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \Rightarrow v_3 = v_2$$

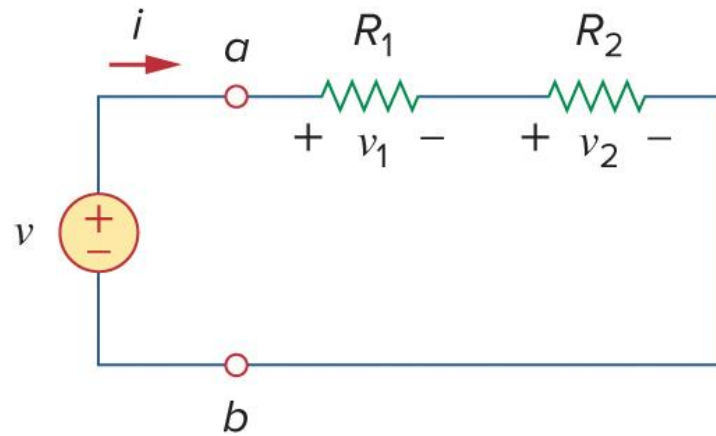
$$6i_3 = 3i_2 \Rightarrow i_3 = \frac{i_2}{2}$$

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0 \Rightarrow i_2 = 2 \text{ A}$$

$$i_1 = 3 \text{ A}, \quad i_3 = 1 \text{ A}, \quad v_1 = 24 \text{ V},$$

$$v_2 = 6 \text{ V}, \quad v_3 = 6 \text{ V}$$

# Series Resistors and Voltage Division



$R_1$  &  $R_2$  are in series since same current  $i$  flows through them

Apply Ohm's law:  $v_1 = iR_1$  &  $v_2 = iR_2$

Apply KVL to the loop (Clockwise):  $-v + v_1 + v_2 = 0$

Combining equations, we get:

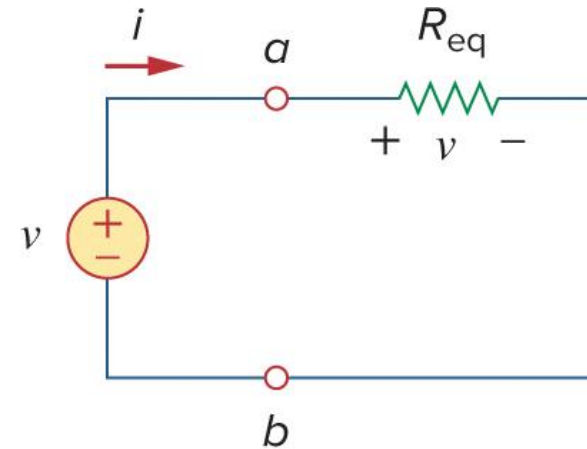
$$v = v_1 + v_2 \Rightarrow iR_1 + iR_2 \Rightarrow i(R_1 + R_2)$$

$$v = i(R_1 + R_2) \text{ or } i = \frac{v}{(R_1 + R_2)}$$

This equation can be written as follows:

$$v = i(R_{eq}) \text{ where } R_{eq} = R_1 + R_2$$

Equivalent Circuit



Equivalent resistance of any number of resistors connected in series is the sum of the individual resistance

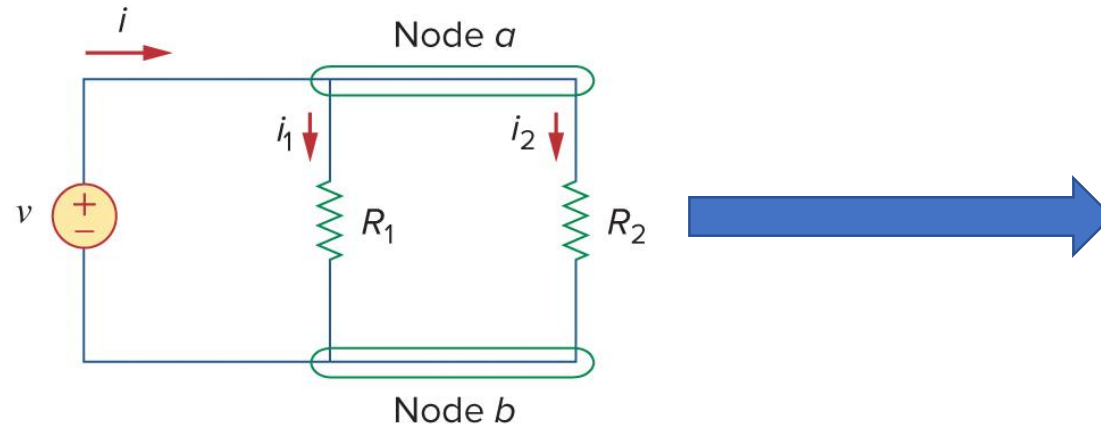
In general, for  $N$  resistors in series:

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_N = \sum_{n=1}^N R_N$$

Voltage division:

$$v_1 = \frac{R_1}{(R_1 + R_2)} v \text{ and } v_2 = \frac{R_2}{(R_1 + R_2)} v$$

# Parallel Resistors and Current Division



$R_1$  &  $R_2$  are connected in parallel and therefore they have same voltage across them

Apply Ohm's law:  $v = i_1 R_1 = i_2 R_2$

Apply KCL at node a:  $i = i_1 + i_2$

Combining equations, we get:

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \Rightarrow \frac{v}{R_{eq}}$$

where  $R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 \times R_2}{R_1 + R_2}$

Equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum

In general, for N resistors in parallel:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}}$$

If  $R_1 = R_2 = R_3 = \dots = R_N = R$ ,

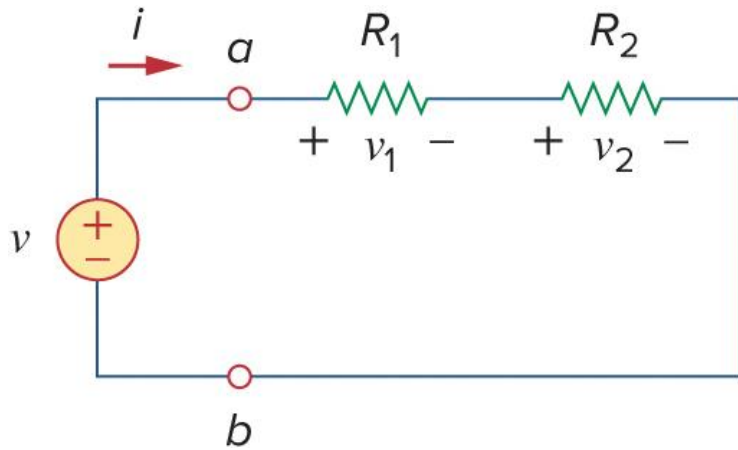
$$R_{eq} = \frac{R}{N}$$

Current division:

$$i_1 = \frac{R_2}{R_1 + R_2} i \quad \text{and} \quad i_2 = \frac{R_1}{R_1 + R_2} i$$

# Equivalent Conductance

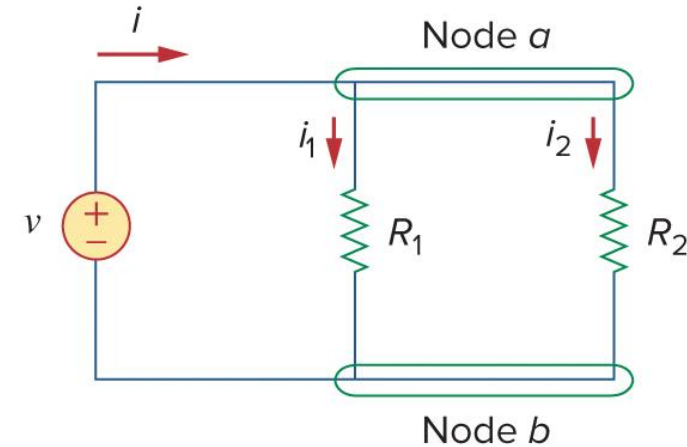
- By definition:  $G = \frac{1}{R}$  (Reciprocal of resistance)



$$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2}$$

Equivalent conductance for N resistors in series:

$$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \cdots + \frac{1}{G_N}$$



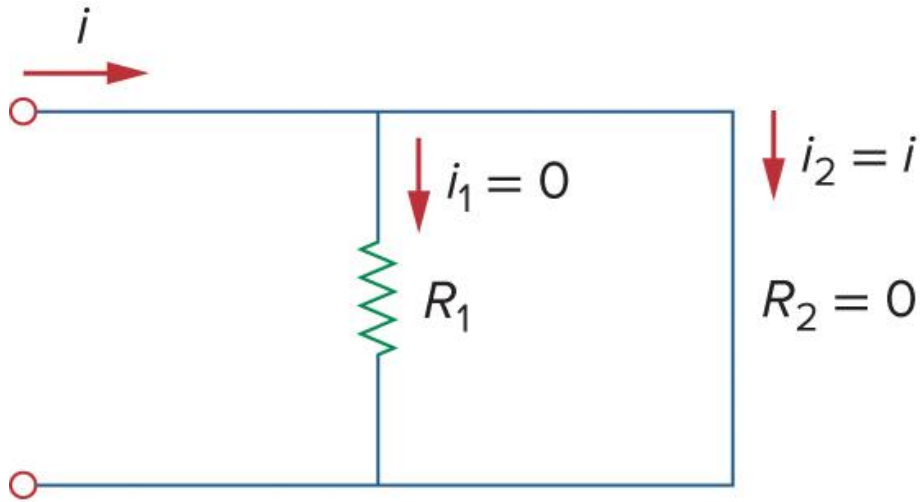
$$G_{eq} = G_1 + G_2$$

Equivalent conductance for N resistors in parallel:

$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

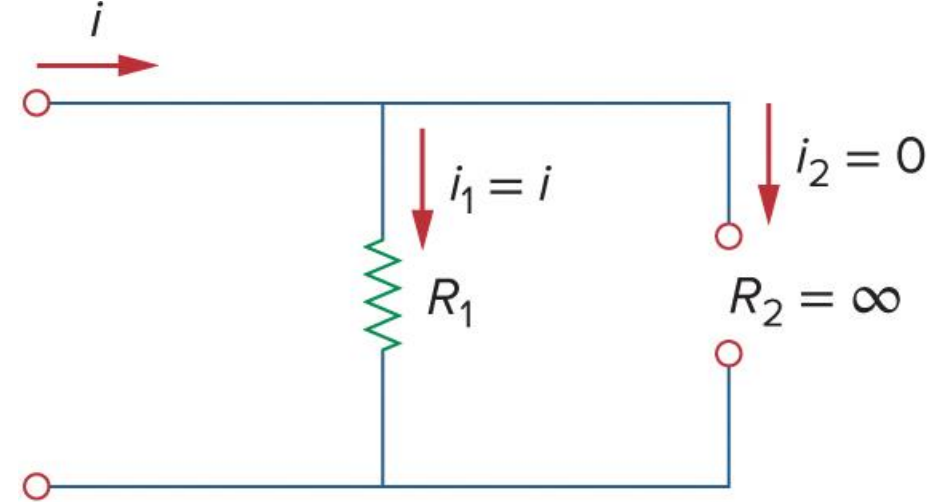


# Short Circuit and Open Circuit Case



Entire current  $i$  bypasses  $R_1$  and flows through the short circuit

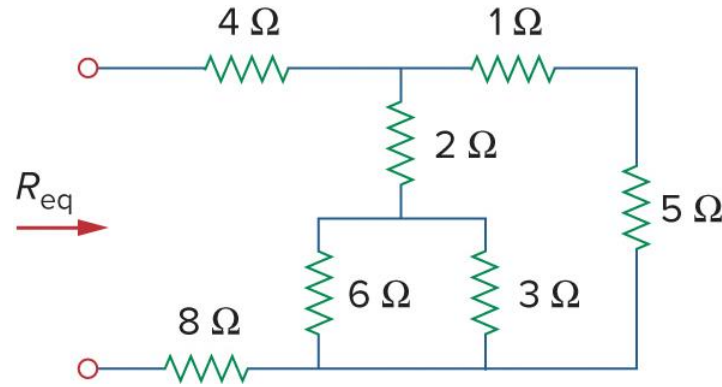
Current always prefers to use path without resistance if possible



Entire current  $i$  flows through  $R_1$  since there is an open circuit on the other side.

# Example 6:

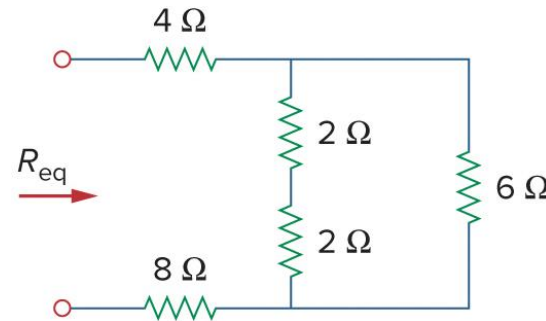
Find  $R_{eq}$  for the circuit



**Solution:**

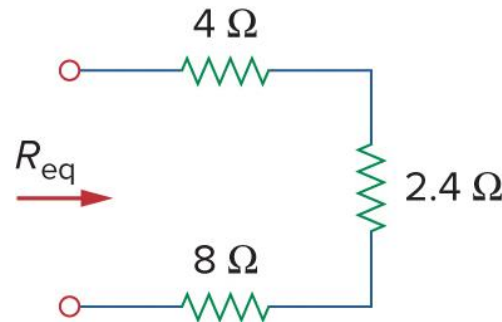
$$6\Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\Omega$$

$$1\Omega + 5\Omega = 6\Omega$$



$$2\Omega + 2\Omega = 4\Omega$$

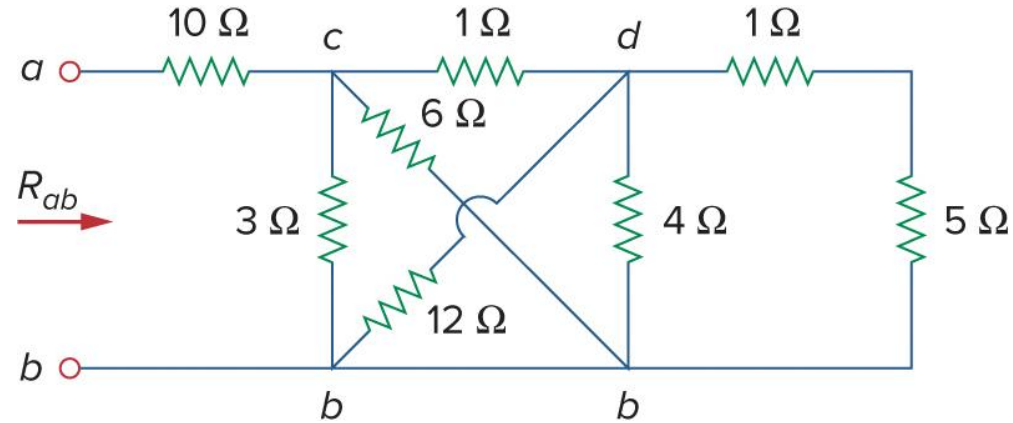
$$4\Omega \parallel 6\Omega = \frac{4 \times 6}{4 + 6} = 2.4\Omega$$



$$R_{eq} = 4\Omega + 2.4\Omega + 8\Omega = 14.4\Omega$$

# Example 7:

Calculate the equivalent resistance  $R_{ab}$

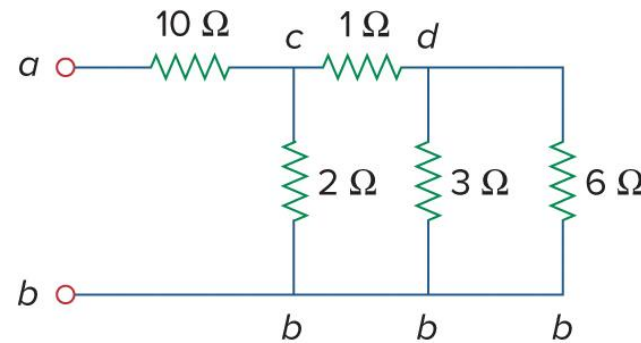


**Solution:**

$$3 \Omega \parallel 6 \Omega = \frac{3 \times 6}{3 + 6} = 2 \Omega$$

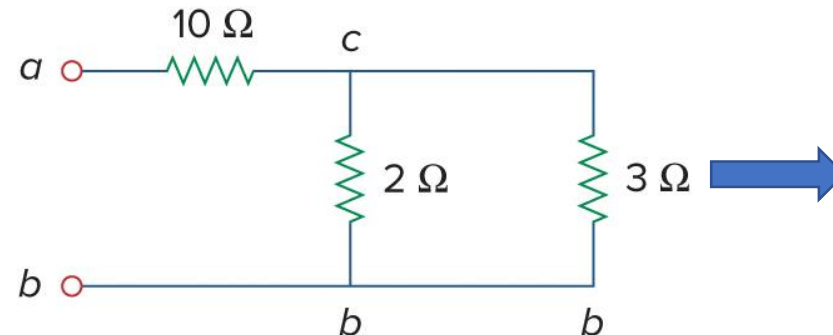
$$12 \Omega \parallel 4 \Omega = \frac{12 \times 4}{12 + 4} = 3 \Omega$$

$$1 \Omega + 5 \Omega = 6 \Omega$$



$$3 \Omega \parallel 6 \Omega = \frac{3 \times 6}{3 + 6} = 2 \Omega$$

$$1 \Omega + 2 \Omega = 3 \Omega$$

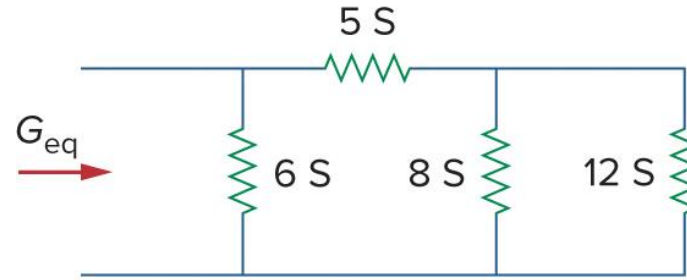


$$2 \Omega \parallel 3 \Omega = \frac{2 \times 3}{2 + 3} = 1.2 \Omega$$

$$R_{ab} = 10 + 1.2 = 11.2 \Omega$$

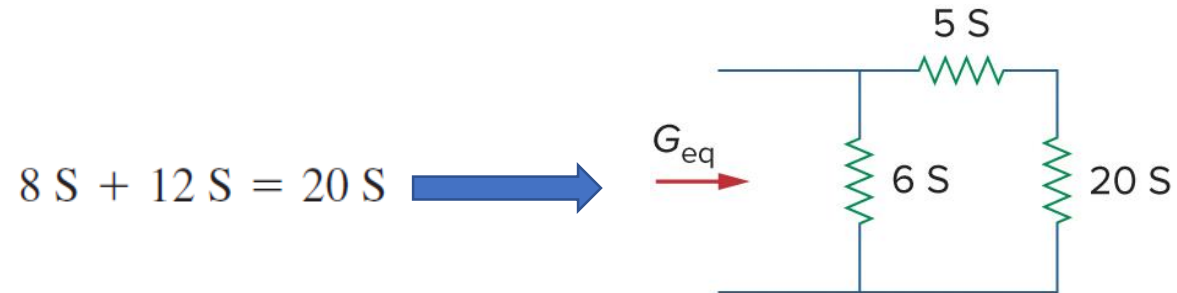
## Example 8:

Find the equivalent conductance  $G_{eq}$



### Solution:

The 8-S and 12-S resistors are in parallel, so their conductance is



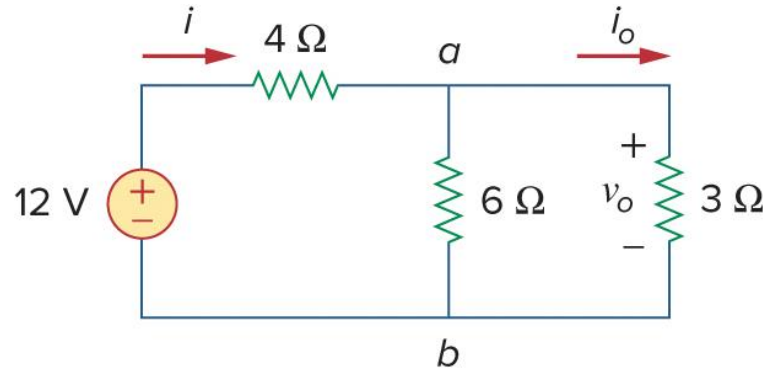
This 20-S resistor is now in series with 5 S

$$\frac{20 \times 5}{20 + 5} = 4\text{ S} \quad \text{This is in parallel with the 6-S resistor.}$$

$$G_{eq} = 6 + 4 = 10\text{ S}$$

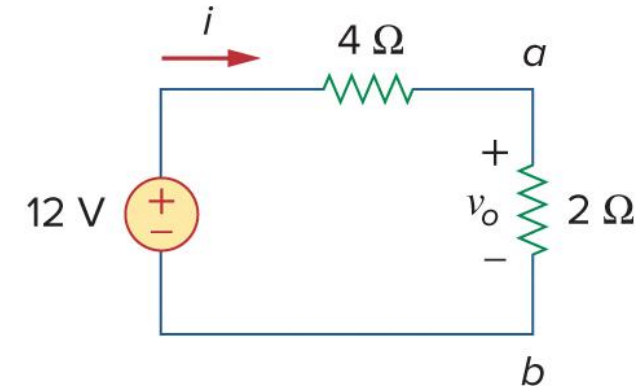
## Example 9:

Find  $i_o$  and  $v_o$  in the circuit shown below. Calculate the power dissipated in 3- $\Omega$  resistor.



Solution:

$$6\Omega || 3\Omega = \frac{6 \times 3}{6 + 3} = 2\Omega$$



Apply voltage division:  $v_o = \frac{R_1}{(R_1 + R_2)} v \Rightarrow v_o = \frac{2}{(2 + 4)} 12 = 4 V$

Apply Ohm's law:  $v = iR \Rightarrow v_o = i_o 3 \Rightarrow i_o = \frac{4}{3} A$

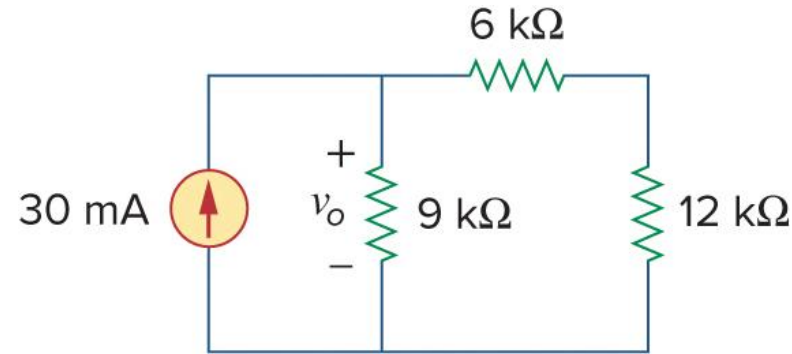
Power dissipation:  $p_o = v_o i_o \Rightarrow p_o = 4 \times \frac{4}{3} = \frac{16}{3} = 5.333 W$

Recall:

Supply power: Negative (-)  
Absorb power: Positive (+)  
Resistor: passive element

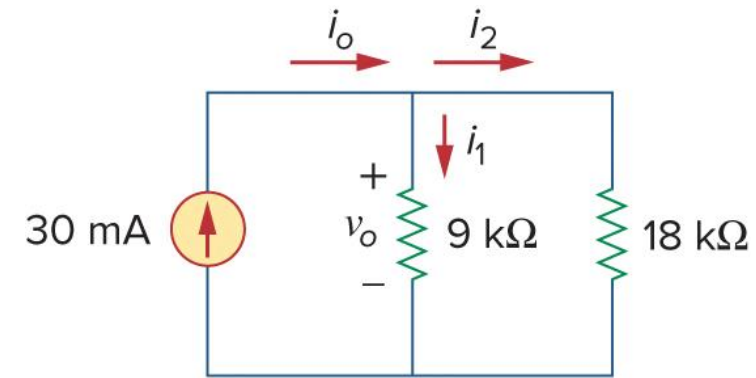
# Example 10:

For the circuit shown below, determine: a-) voltage  $v_o$ , b-) the power supplied by the current source, and c-) the power absorbed by each resistor.



Solution:

6-kΩ and 12-kΩ resistors in series. Thus,  $6+12=18$ -kΩ



Apply current division:

$$i_1 = \frac{R_2}{(R_1 + R_2)} i_0 \Rightarrow i_1 = \frac{18}{(9 + 18)} 30 \text{ (mA)} = 20 \text{ mA}$$

$$i_2 = \frac{R_1}{(R_1 + R_2)} i_0 \Rightarrow i_2 = \frac{9}{(9 + 18)} 30 \text{ (mA)} = 10 \text{ mA}$$

Power supplied = power absorbed

Apply Ohm's law:  $v_o = i_1 9 = 20 \times 9 = 180 \text{ V}$

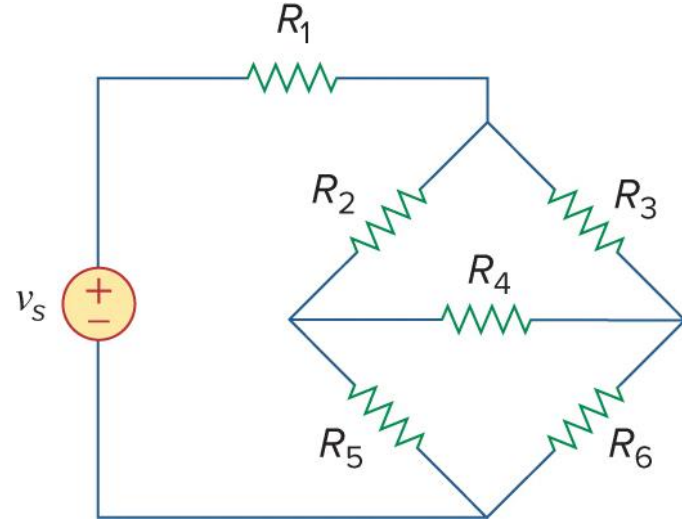
Power dissipation:  $p = vi$

$$p_9 = 180 \times 20 = 3600 \text{ mW} = 3.6 \text{ W}$$

$$p_{12} = vi = (i_2 R) i_2 = i_2^2 R = 10^2 \times 12 = 1.2 \text{ W}$$

$$p_6 = 10^2 \times 6 = 0.6 \text{ W}, p_{cur} = 180 \times 30 = 5.4 \text{ W}$$

# Wye-Delta Transformations

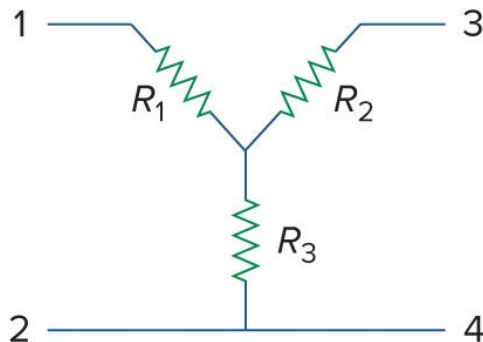


Resistors are neither in parallel nor in series

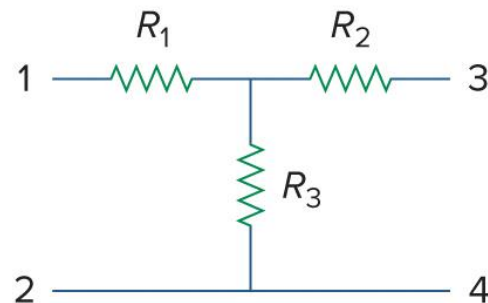
How do we combine resistors  $R_1$  through  $R_6$ ?

- These types of circuits can be simplified by using three-terminal equivalent networks:

Wye (Y) network

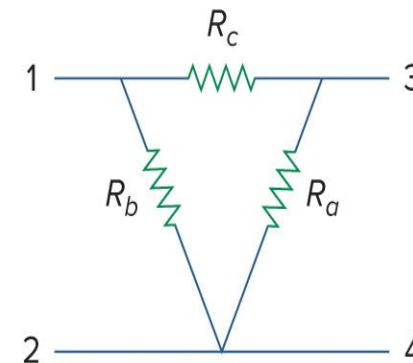


Tee (T) network

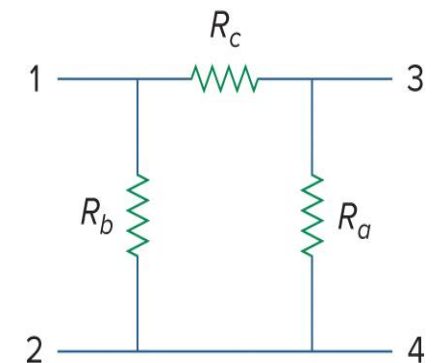


Two forms of same network

Delta ( $\Delta$ ) network



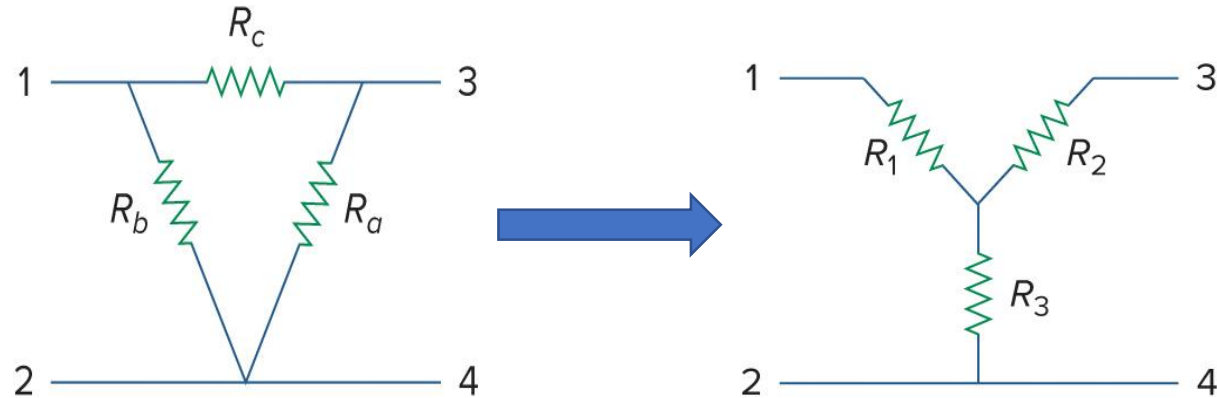
Pi ( $\Pi$ ) network



Two forms of same network



# Delta to Wye Conversion



For terminal 1 & 2:  $R_{12}(Y) = R_1 + R_3$   
 $R_{12}(\Delta) = R_b \parallel (R_a + R_c)$

Setting  $R_{12}(Y) = R_{12}(\Delta)$ :  $R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$  Eq 1

Similarly,  $R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$  Eq 2

$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$  Eq 3

Subtracting eq 1 & eq 3:  $R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c}$  Eq 4

Adding eq 4 & eq 2:

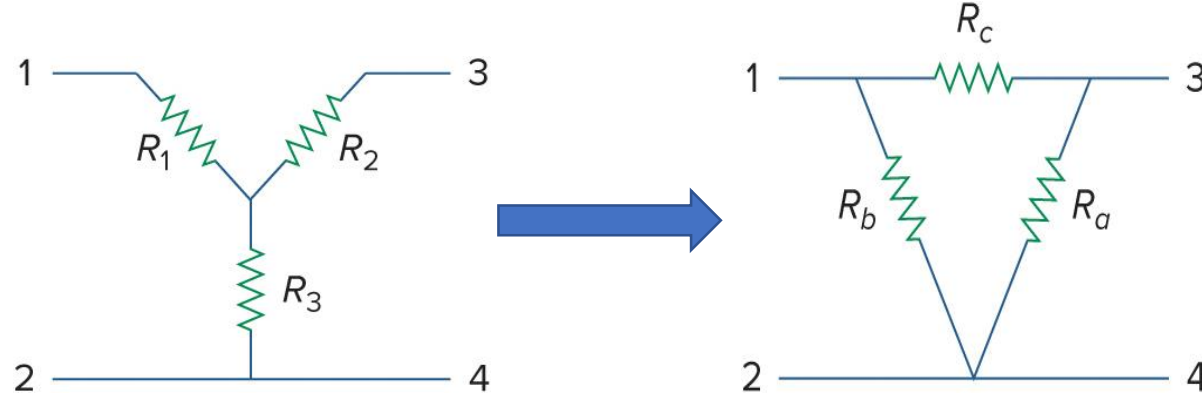
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Each resistor in the Y network is the product of the resistors in the two adjacent  $\Delta$  branches, divided by the sum of the three  $\Delta$  resistors.

# Wye to Delta Conversion



Combining eqs 1, 2, and 3 obtained in the previous step, we get:

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_a R_b R_c (R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} = \frac{R_a R_b R_c}{R_a + R_b + R_c}$$

Dividing the eq by each eq (1, 2, and 3) obtained in the previous step, we get:

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

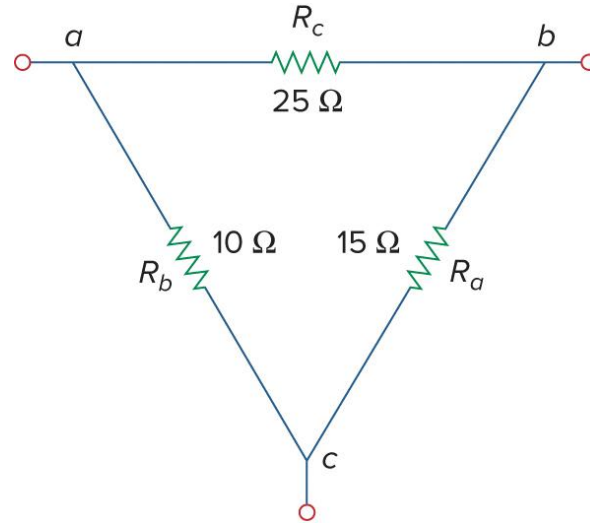
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Each resistor in the  $\Delta$  network is the sum of all possible products of the Y resistor taken two at a time, divided by the opposite Y resistor.

# Example 11:

Convert the  $\Delta$  network in the figure below to an equivalent Y network

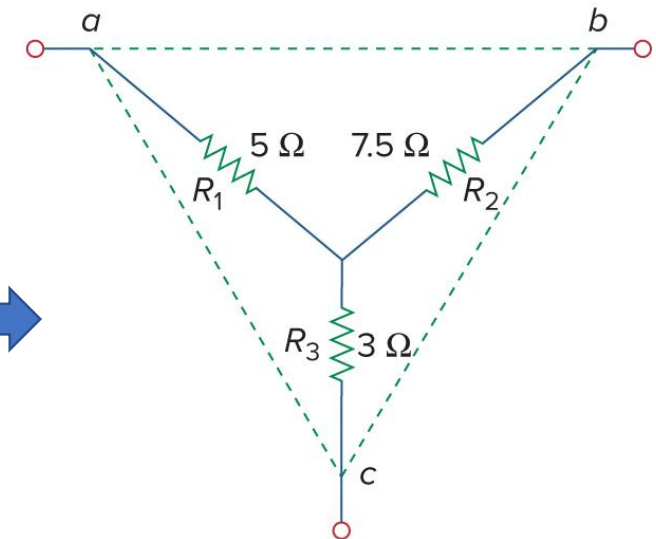


Solution:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5 \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5 \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3 \Omega$$



# Example 12:

Obtain the equivalent resistance  $R_{ab}$  for the circuit shown on the right and use it to find  $i$ .

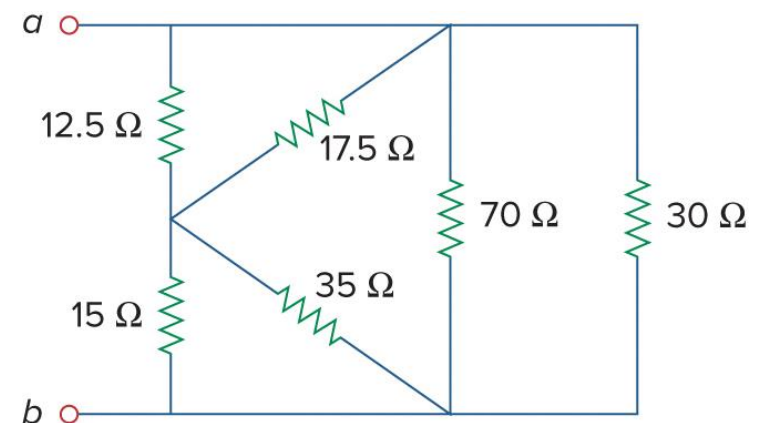
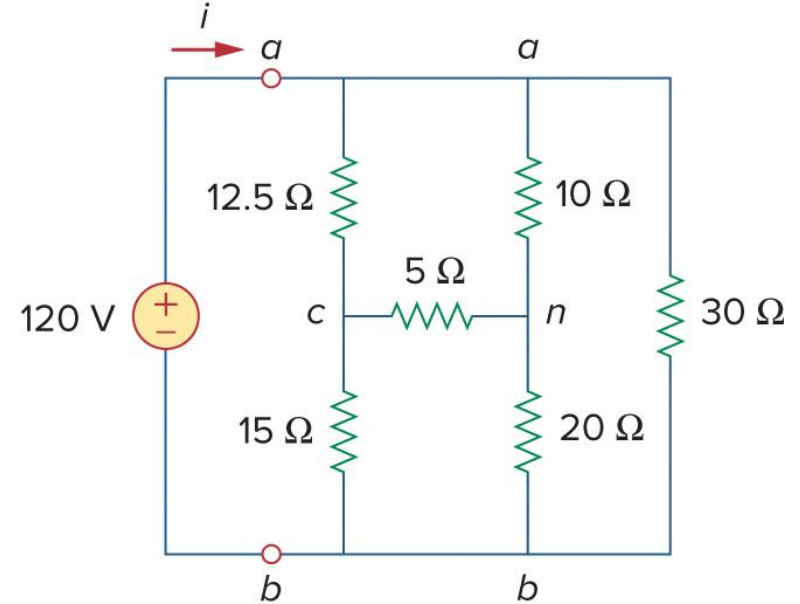
## Solution:

In the circuit, there are two Y networks (one at node  $n$  and the other at node  $c$ ) and three  $\Delta$  networks (can, abn, cnb).

If we convert the Y network comprising the 5- $\Omega$ , 10- $\Omega$ , and 20- $\Omega$  resistors, we may select

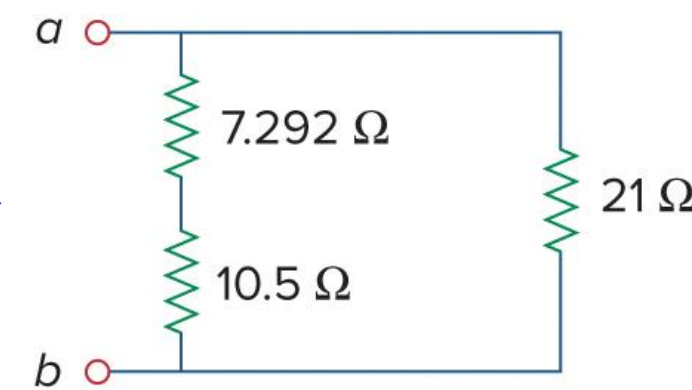
$$R_1 = 10 \, \Omega, \quad R_2 = 20 \, \Omega, \quad R_3 = 5 \, \Omega$$

$$\left. \begin{aligned} R_a &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10} \\ &= \frac{350}{10} = 35 \, \Omega \\ R_b &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = \frac{350}{20} = 17.5 \, \Omega \\ R_c &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{350}{5} = 70 \, \Omega \end{aligned} \right\} \Rightarrow$$



# Example 12 Continuing:

Combining the three pairs of resistors in parallel, we obtain

$$\left. \begin{aligned} 70 \parallel 30 &= \frac{70 \times 30}{70 + 30} = 21 \, \Omega \\ 12.5 \parallel 17.5 &= \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292 \, \Omega \\ 15 \parallel 35 &= \frac{15 \times 35}{15 + 35} = 10.5 \, \Omega \end{aligned} \right\} \rightarrow$$


$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632 \, \Omega}$$

$$\text{Ohm's law: } v = iR \Rightarrow i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458 \, A}$$

# Example 12: Second Way

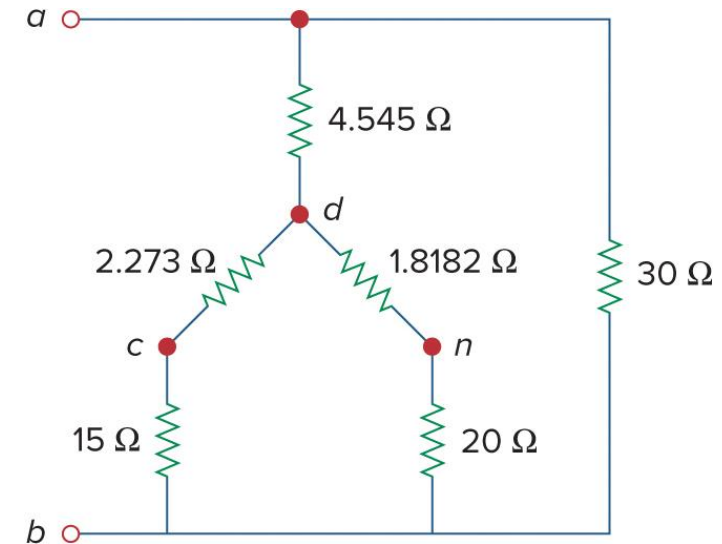
Solving with  $\Delta$  to Y conversion. Transform  $\Delta$  (can) to Y

Let  $R_c = 10\ \Omega$ ,  $R_a = 5\ \Omega$ , and  $R_n = 12.5\ \Omega$ . This will lead

$$R_{ad} = \frac{R_c R_n}{R_a + R_c + R_n} = \frac{10 \times 12.5}{5 + 10 + 12.5} = 4.545\ \Omega$$

$$R_{cd} = \frac{R_a R_n}{27.5} = \frac{5 \times 12.5}{27.5} = 2.273\ \Omega$$

$$R_{nd} = \frac{R_a R_c}{27.5} = \frac{5 \times 10}{27.5} = 1.8182\ \Omega$$



Resistance between d and b, we have two series combination in parallel, i.e.,

$$R_{db} = \frac{(2.273 + 15)(1.8182 + 20)}{2.273 + 15 + 1.8182 + 20} = \frac{376.9}{39.09} = 9.642\ \Omega$$

This is in series with the 4.545- $\Omega$  and the resultant resistance parallel with 30-  $\Omega$  resistor.

$$R_{ab} = \frac{(9.642 + 4.545)30}{9.642 + 4.545 + 30} = \frac{425.6}{44.19} = 9.631\ \Omega$$

$$\text{Ohm's law: } v = iR \Rightarrow i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = 12.458\ \text{A}$$