Analysis of Algorithms II

Recitation 1

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Q1: Do all executions of Gale-Shapley lead to the same stable matching?

A. No, because the algorithm is nondeterministic.
B. No, because an instance can have several stable matchings.
C. Yes, because each instance has a unique stable matching.
D. Yes, even though an instance can have several stable matchings and the algorithm is nondeterministic.

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Q2: Who is the best valid partner for W in the following instance?

6 stable matchings

{ A–W, B–X, C–Y, D–Z }

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	1st	2nd	3rd	4th
A	Y	Z	X	W
B	Z	Y	W	X
C	W	Y	X	Z
D	X	Z	W	Y

	1st	2nd	3rd	4th
W	D	A	B	C
X	C	B	A	D
Y	C	B	A	D
Z	D	A	B	C

Q2: Who is the best valid partner for W in the following instance? A

6 stable matchings

{ A–W, B–X, C–Y, D–Z }

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W	D	A	B	C
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Q3: In every instance of the Stable Matching Problem, there is a stable matching containing a pair (m, w) such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. True or False?

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FALSE

m proposes to w ➡ matched with w

m’ proposes to w’ ➡matched with w’

	1st	2nd
m	w	w’
m’	w’	w

	1st	2nd
w	m’	m
w’	m	m’

Q4: Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair (m, w) belongs to S. True or False ?

Q4: Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair (m, w) belongs to S. True or False ?

TRUE

	1st	2nd
m	w	…
…	…	

	1st	2nd
w	m	..
..	..	

If there is matching pairs such that

m - w'
m' - w

m and w will choose each other

Q5: Another type of stability: competition between TV networks

- Let's assume there are two TV networks A and D.
- There are *n* prime-time show *slots.*
- Each network has *n* TV shows.
- Each show has a fixed rating, which is based on the number of people who watched it last year; we'll assume that no two shows have exactly the same rating.
- Each network wants to devise a schedule –an assignment of each show to a distinct slot– so as to attract as much market share as possible.
- A network wins a given time slot if the show that it schedules for the time slot has a larger rating than the show the other network schedules for that time slot.
- We'll say that the pair of schedules (S , T) is stable if neither network can unilaterally change its own schedule and win more time slots.
- For every set of TV shows and ratings, is there always a stable pair of schedules?

Q5: Another type of stability: competition between TV networks

- There is not always a stable pair of schedules
- Assume n=2

	A	
8pm	a1	20
9pm	a2	40

	D	
8pm	d1	10
9pm	d2	30

- A wins for both slots
- D will want to switch d1&d2

	A	
8pm	a1	20
9pm	a2	40

	D	
8pm	d2	30
9pm	d1	10

- A wins 9pm slot
- D wins 8pm slot
- A will want to switch a1&a2