ISTANBUL TECHNICAL UNIVERSITY

BLG354E - Recitation 1

25.04.2022

Question 1

Impulse response of a DT system is given as $h[n]=[0.2, 0.1, 0.5]$

$x(t) = t(u(t) - u(t-30)) + (60-t)(u(t-30) - u(t-60))$ is given.

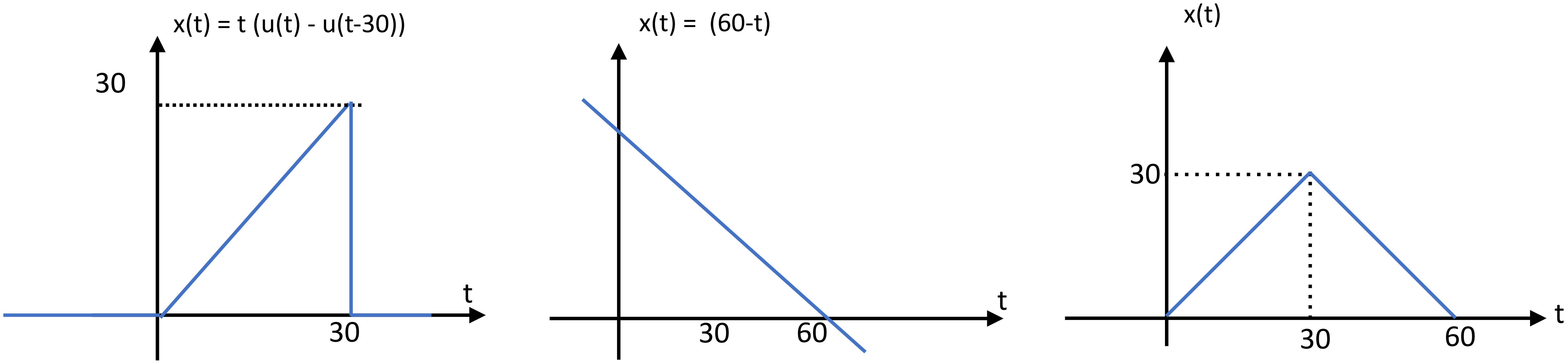
A. Plot the $x(t)$

B. Convert $x(t)$ to digital $x[n]$ with $f_s = 0.1$ Hz and plot

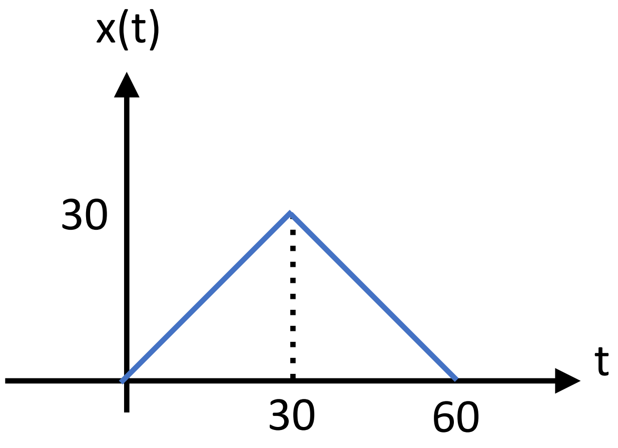
C. Find and sketch the output of the system for the obtained input signal $x[n]$

Question 1

$$x(t) = t\,(u(t) - u(t-30)) + (60 - t)(u(t - 30) - u(t - 60))$$



Question 1



x[n] = x(t = nTs) is used

fs = 0.1Hz => Ts = 10 sn

x[0] = x(0) = 0

x[1] = x(10) = 10

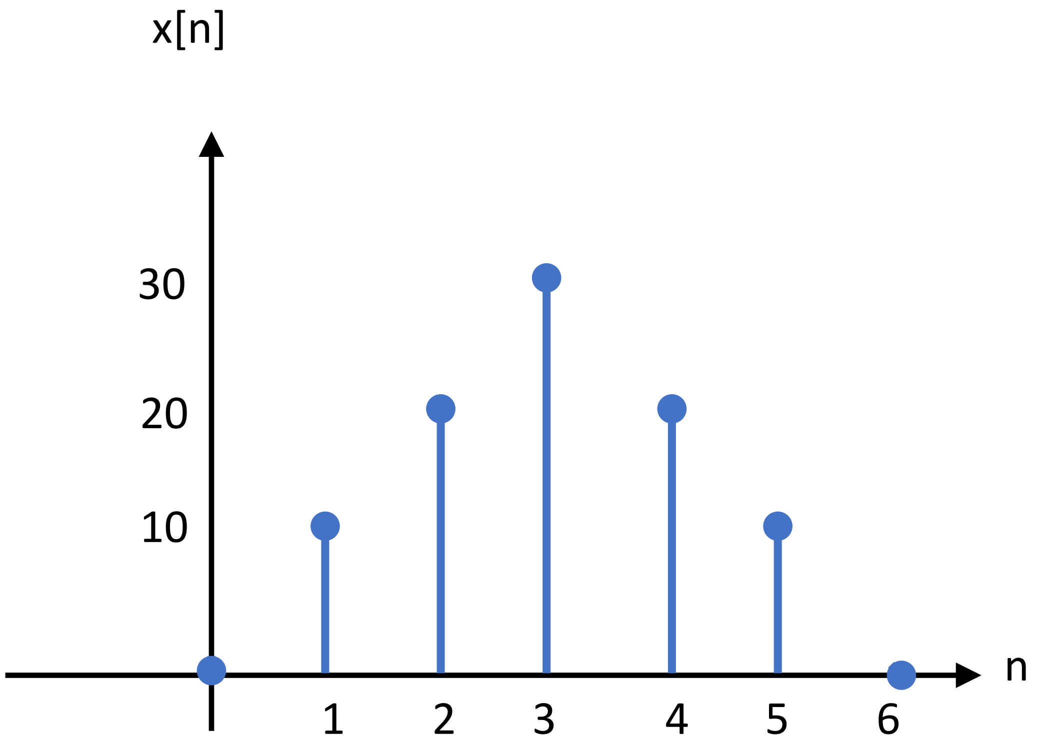
x[2] = x(20) = 20

x[3] = x(30) = 30

x[4] = x(40) = 20

x[5] = x(50) = 10

x[6] = x(60) = 0



x[n] = [0, 10, 20, 30, 20, 10]

Question 1

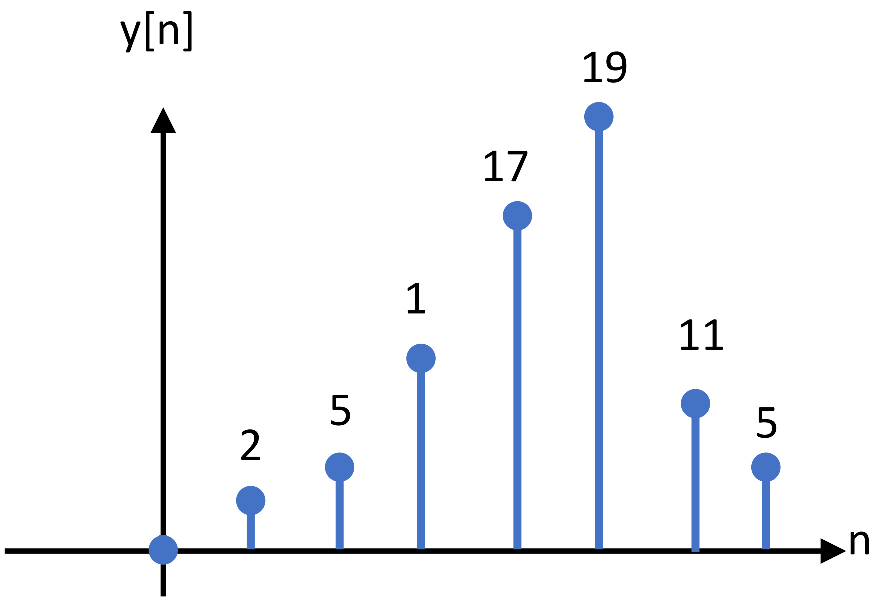
$$y[n] = x[n] * \mathrm{h}[n] = \sum_{k=-\infty}^{k=\infty} x[k] \cdot \mathrm{h}[n-k]$$

x[n] = [0, 10, 20, 30, 20, 10]

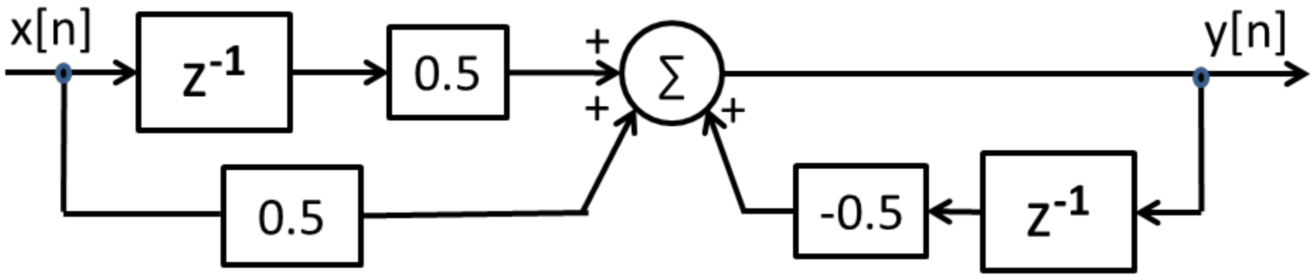
h[n] = [0.2, 0.1, 0.5]

y[n] = {0, 2, 5, 13, 17, 19, 11, 5}

n	-2	-1	0	1	2	3	4	5	6	7
h[n]			0.2	0.1	0.5					
x[n]			0	10	20	30	20	10		
h[-n]	0.5	0.1	0.2							
h[1-n]		0.5	0.1	0.2						
h[2-n]			0.5	0.1	0.2					
h[3-n]				0.5	0.1	0.2				
h[4-n]					0.5	0.1	0.2			
h[5-n]						0.5	0.1	0.2		
h[6-n]							0.5	0.1	0.2	
h[7-n]								0.5	0.1	0.2
y[n]			0	2	5	13	17	19	11	5



Question 2



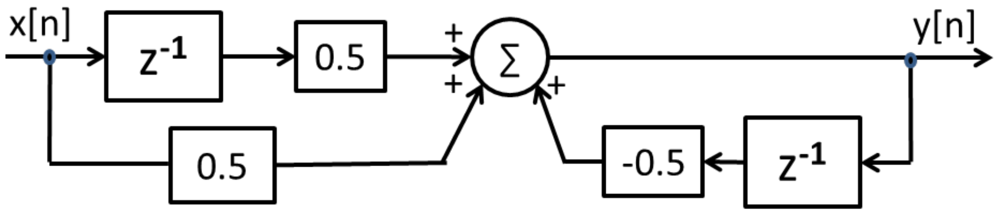
Consider the discrete system shown in the figure below where x[n] is the input, y[n] is the output and z^{-1} represents the unit delay.

a) Write the difference equation that relates the output y[n] and the input x[n]

b) Find and draw the system output values for n=0 to 5 if input signal x[n] is defined as:

$$x[n]=n\cdot(u[n-1]-u[n-3])-2n\cdot\delta[n-2]$$

Question 2



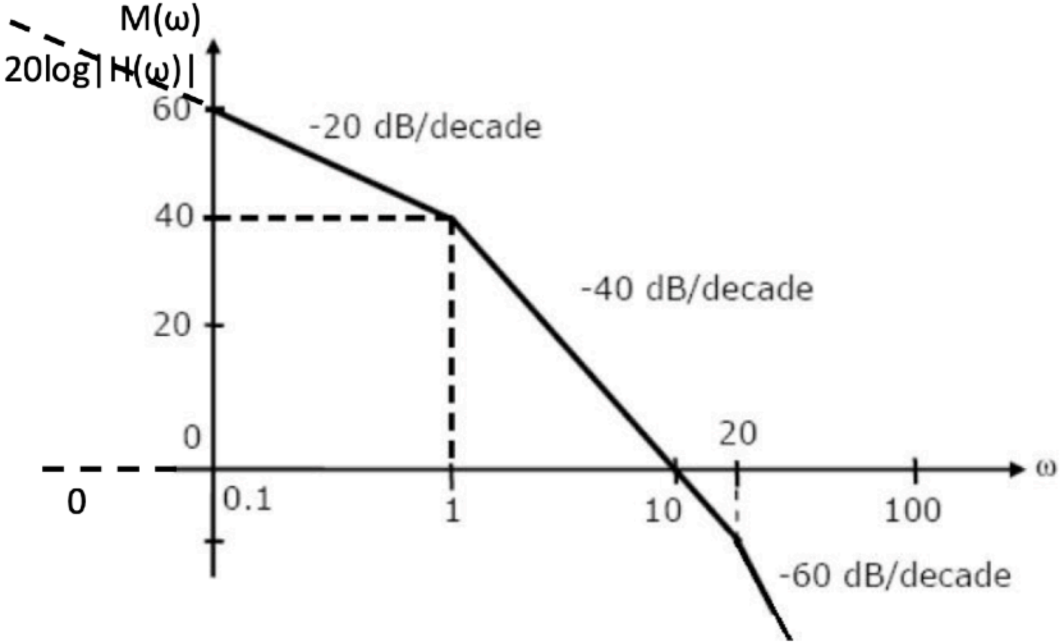
a) $y[n] = 0.5\,x[n-1] + 0.5\,x[n] - 0.5\,y[n-1]$

b) $x[n] = n\cdot(u[n-1]-u[n-3])-2n\cdot\delta[n-2]$

n	x[n]	x[n-1]	y[n]	y[n-1]
0	0	0	0	0
1	1	0	1/2	0
2	-2	1	-3/4	1/2
3	0	-2	-5/8	-3/4
4	0	0	5/16	-5/8
5	0	0	-5/32	5/16

Question 3

Example: Magnitude response of H(s) is given in the below Bode plot. Find the transfer function H(s)



$$H(s)=\frac{K(1+\frac{s}{z_1})(1+\frac{s}{z_2})(1+\frac{s}{z_3})\cdots}{(1+\frac{s}{p_1})(1+\frac{s}{p_2})(1+\frac{s}{p_3})\cdots}$$

$$H(s)=\frac{K}{s(1+\frac{s}{1})(1+\frac{s}{20})}$$

$$H(j\omega)=\left.\frac{K}{s(1+j\omega)(1+\frac{j\omega}{20})}\right|_{\omega=0.1}=60dB$$

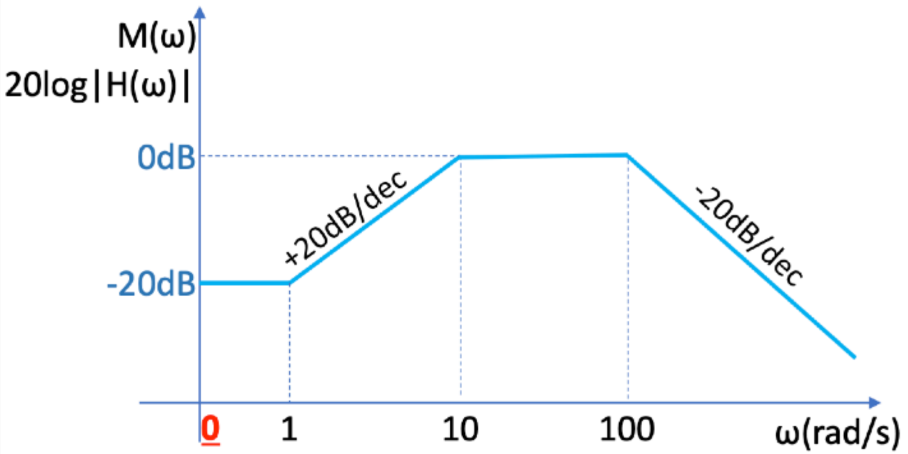
$$|H(j\omega)|=\left.\frac{K}{\omega\sqrt{1+\omega^2}\left(\sqrt{1+\frac{\omega}{20}^2}\right)}\right|_{\omega=0.1}=10^{\frac{60}{20}}=1000$$

For ω=0.1 → $\sqrt{1+\omega^2}\cong 1$ $\sqrt{1+\frac{\omega}{20}^2}\cong 1$ $\frac{K}{\omega}=\frac{K}{0.1}\cong 1000$ → K=100

$$H(s)=\frac{100}{s(1+s)(1+0.05s)}$$

Question 4

Example: Frequency response (magnitude) of H(s) is given in the below Bode plot. Find the transfer function H(s)



$$H(s) = \frac{K(1+\frac{s}{z_1})}{(1+\frac{s}{p_1})(1+\frac{s}{p_2})}$$

$$H(s) = \frac{K(1+\frac{s}{1})}{(1+\frac{s}{10})(1+\frac{s}{100})} = \frac{K(1+s)}{\frac{1}{10}(10+s)\frac{1}{100}(100+s)}$$

$$H(s) = \frac{1000K(1+s)}{(10+s)(100+s)}$$

$$H(j\omega) = \frac{1000K(1+j\omega)}{(10+j\omega)(100+j\omega)}$$

For ω=0 → 20log|H(ω)|)=-20 → $|H(j\omega)|_{\omega=0} = \frac{1000K}{10\cdot 100} = 10^{\frac{-20}{20}} = 0.1$ →K=0.1

$$H(s) = \frac{100(1+s)}{(10+s)(100+s)}$$

Question 5

Consider the periodic square wave x(t) shown in the figure. Determine the first two non-zero harmonic components of its trigonometric Fourier Series.