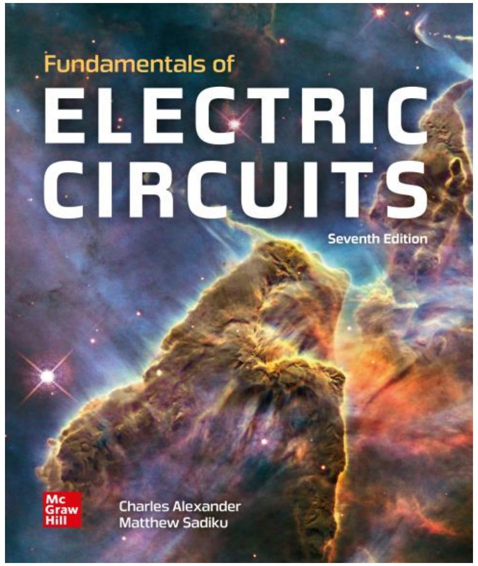
EHB 211E
Basics of Electrical Circuits

Asst. Prof. Onur Kurt

Operational Amplifiers



Fundamentals of Electric Circuits by C. Alexander and M. Sadiku, Seventh Edition (McGraw-Hill Education)

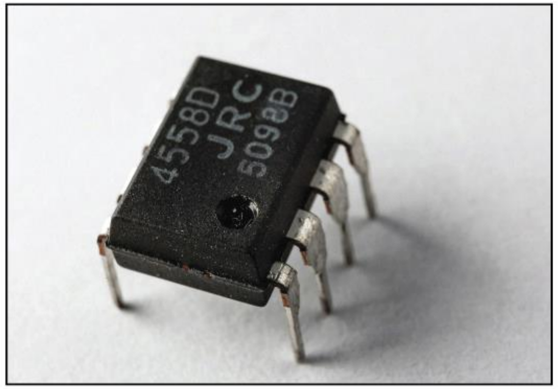
Introduction: Operational Amplifiers

- What is an operational amplifier (op amp)?
  - Active element designed to performed mathematical operations: addition, subtraction, multiplication, division, differentiation, and integration.
  - Sum, amplify, integrate, or differentiate a signal.
  - Versatile circuit building block.
  - Electronic unit that behaves like a voltage-controlled voltage source (VCVS)
- Operational amplifiers (op amps) are popular in practical circuit design:
  - Versatile
  - Inexpensive
  - Easy to use
- Electronic device consisting of a complex arrangement of resistors, transistors, capacitors, and diodes.
- Only consider external characteristics of op amps in this course.

Operational Amplifiers

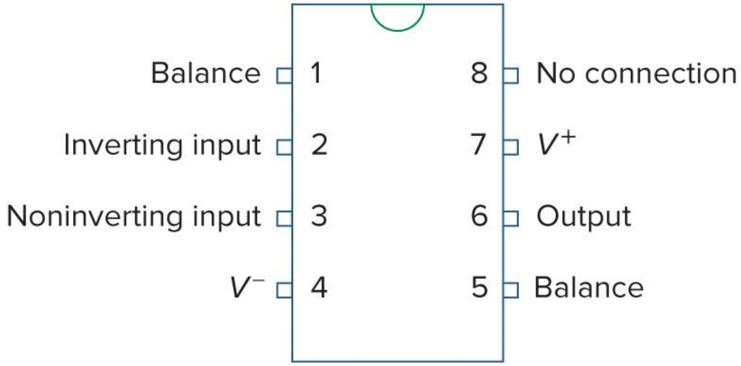
- Consist of two inputs and one output
- Connect to two power supplies, positive (V^+) and negative (V^-).
- Minus(-): inverting input.
- Positive (+): noninverting input
- Inputs applied to noninverting terminal appears with same polarity at the output
- Input applied to inverting terminal appears inverted at the output.

Typical op amp package

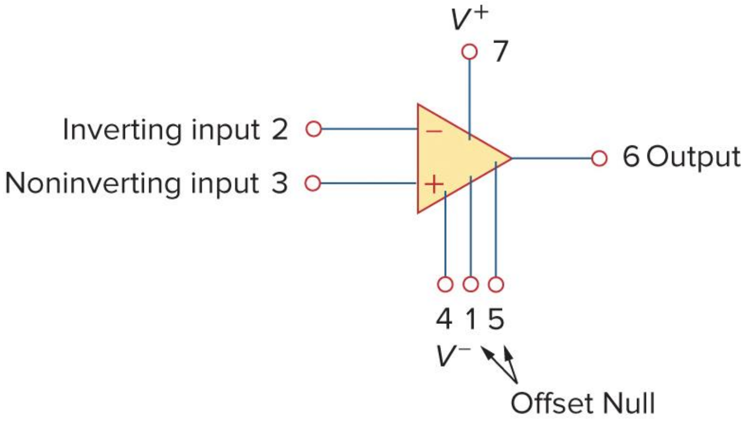


Mark Dierker/McGraw-Hill Education

Top view of op amp



Circuit symbol of op amp



Operational Amplifiers

- The op amp is powered by a voltage supply.
- Apply KCL: $\sum i_{in} = \sum i_{out}$

$$i_0 = i_1 + i_2 + i_+ + i_-$$

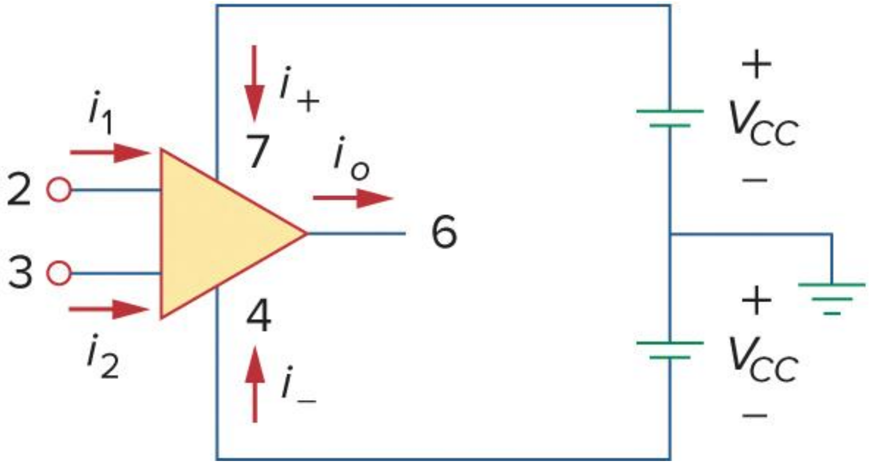
- v_1: inverting terminal
- v_2: noninverting terminal
- R_i: Thevenin equivalent resistance seen at input
- R_0: Thevenin equivalent resistance seen at output
- The differential input voltage v_d is given by

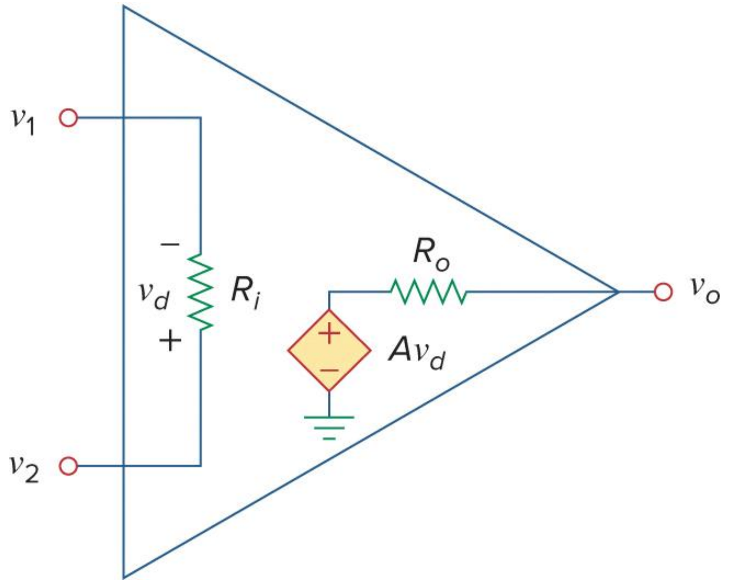
$$v_d = v_2 - v_1$$

- The output of the operational amplifier is given by

$$v_0 = Av_d = A(v_2 - v_1)$$

- where A: open loop voltage gain (no external feedback from output to input)





Operational Amplifiers

TABLE 5.1

Typical ranges for op amp parameters.

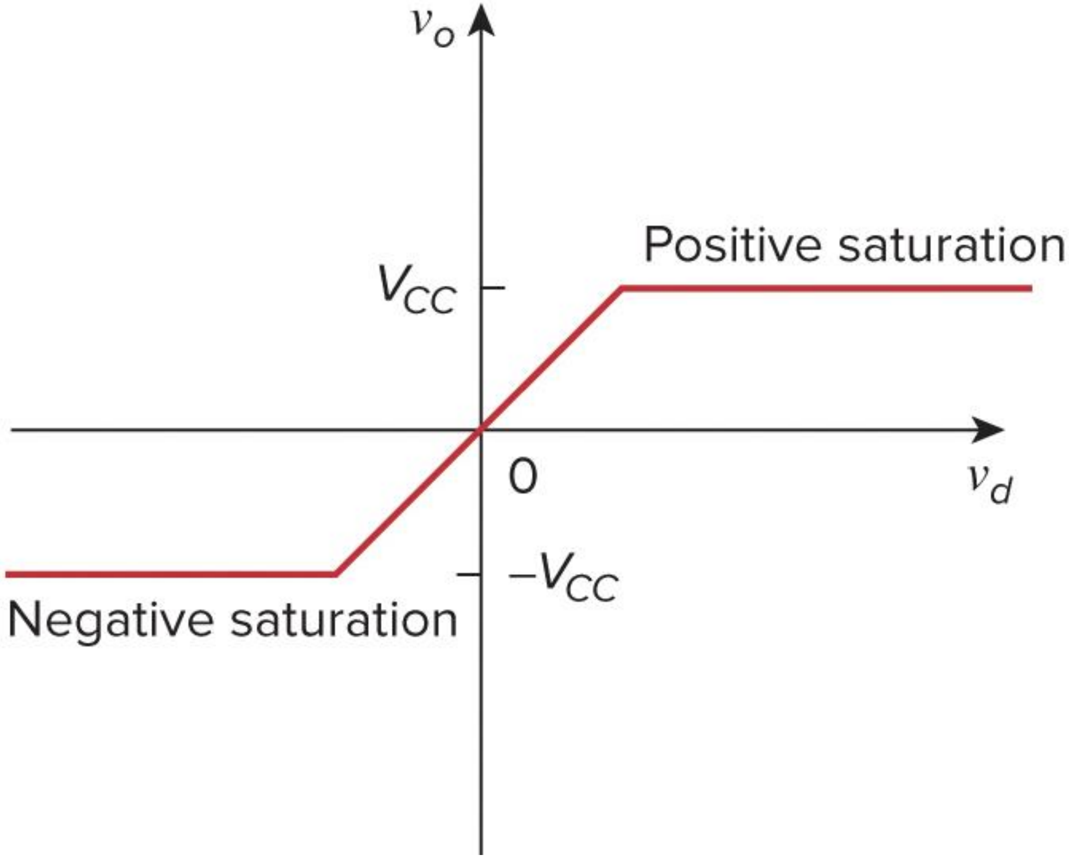
Parameter	Typical range	Ideal values
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_i	10^5 to $10^{13}\ \Omega$	$\infty\,\Omega$
Output resistance, R_o	10 to 100 Ω	$0\,\Omega$
Supply voltage, V_{CC}	5 to 24 V	

Operational Amplifiers

- The magnitude of v_0 cannot exceed power supply voltage.
- The output voltage is dependent on and is limited by the power supply.

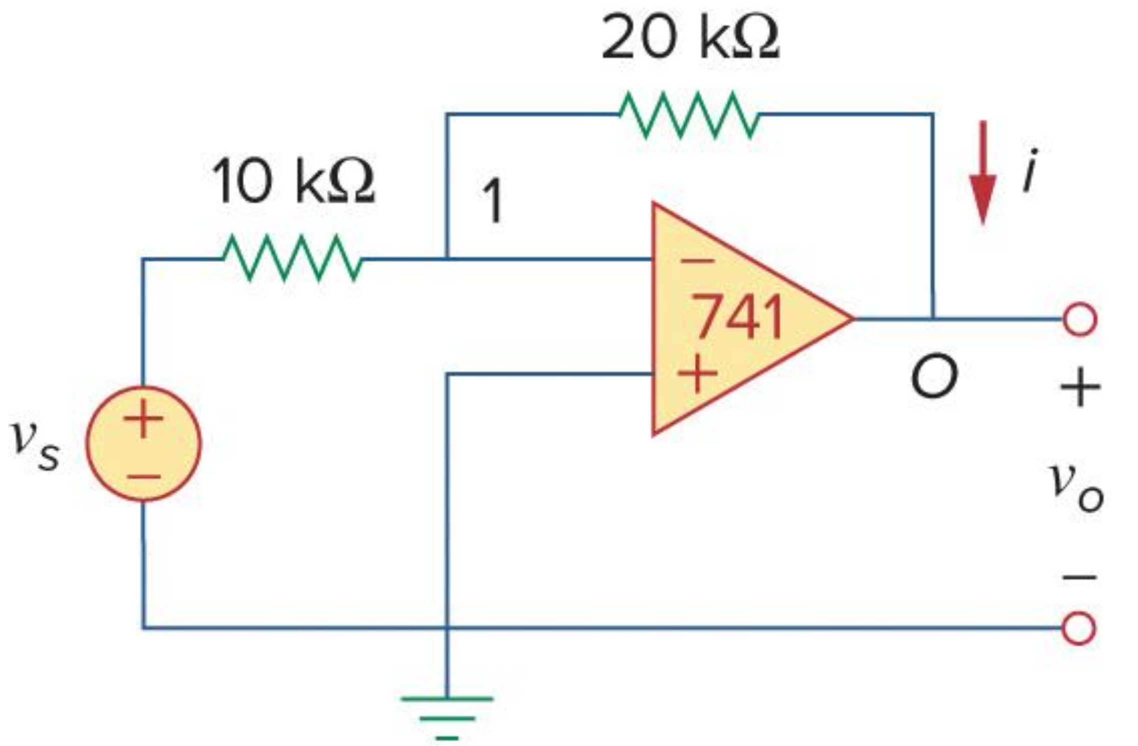
$$-V_{CC} \leq v_0 \leq V_{CC}$$

- Op amp in three modes as follows:
  - Positive saturation: $v_0 = V_{CC}$
  - Linear region: $-V_{CC} \leq v_0 = AV_d \leq V_{CC}$
  - Negative saturation: $v_0 = -V_{CC}$
- Output voltage v_0 is also dependent on differential input v_d



Example 1

A 741-op amp has an open-loop voltage gain of 2×10^5, input resistance of $2\ M\Omega$, and output resistance of $50\ \Omega$. The op amp is used in the circuit shown below. Find the closed-loop gain $\frac{v_0}{v_s}$. Determine current i when $v_s = 2\ V$.



Solution

- Apply nodal analysis to equivalent circuit.
- Apply KCL to node 1: $\sum i_{in} = \sum i_{out}$

$$\frac{v_s - v_1}{10 \times 10^3} = \frac{v_1}{2000 \times 10^3} + \frac{v_1 - v_o}{20 \times 10^3}$$

$$200v_s = 301v_1 - 100v_o$$

$$2v_s \simeq 3v_1 - v_o \quad \Longrightarrow \quad v_1 = \frac{2v_s + v_o}{3}$$

- Apply KCL to node 0: $\dfrac{v_1 - v_o}{20 \times 10^3} = \dfrac{v_o - Av_d}{50}$

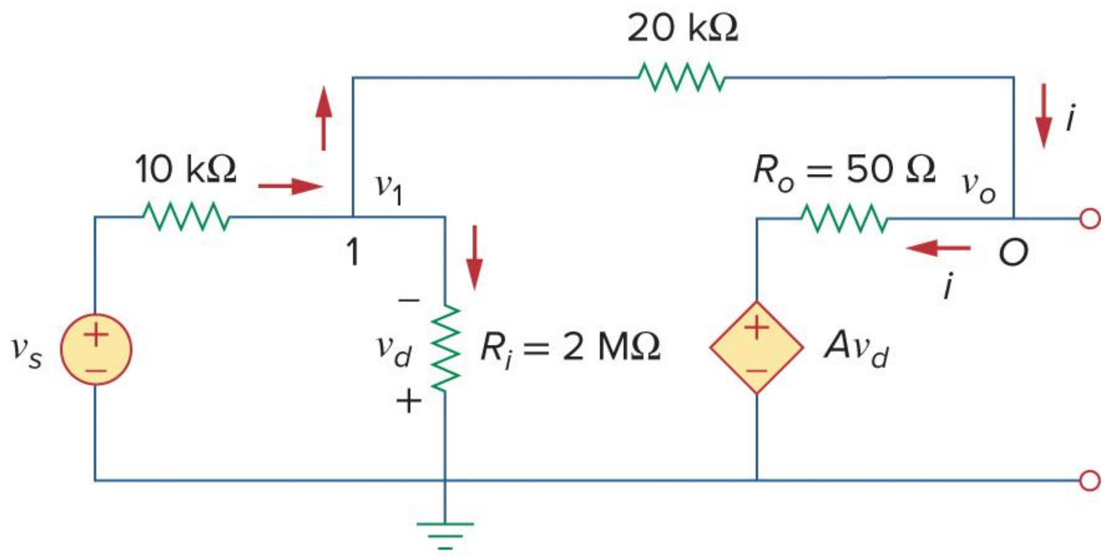
$$v_d = -v_1 \text{ and } A = 200{,}000$$

$$v_1 - v_o = 400(v_o + 200{,}000v_1)$$

$$0 \simeq 26{,}667{,}067v_o + 53{,}333{,}333v_s \Longrightarrow \frac{v_o}{v_s} = -1.9999699$$

$$\text{When } v_s = 2\text{ V},\ i = \frac{v_1 - v_o}{20 \times 10^3} = 0.19999\text{ mA}$$

Equivalent circuit



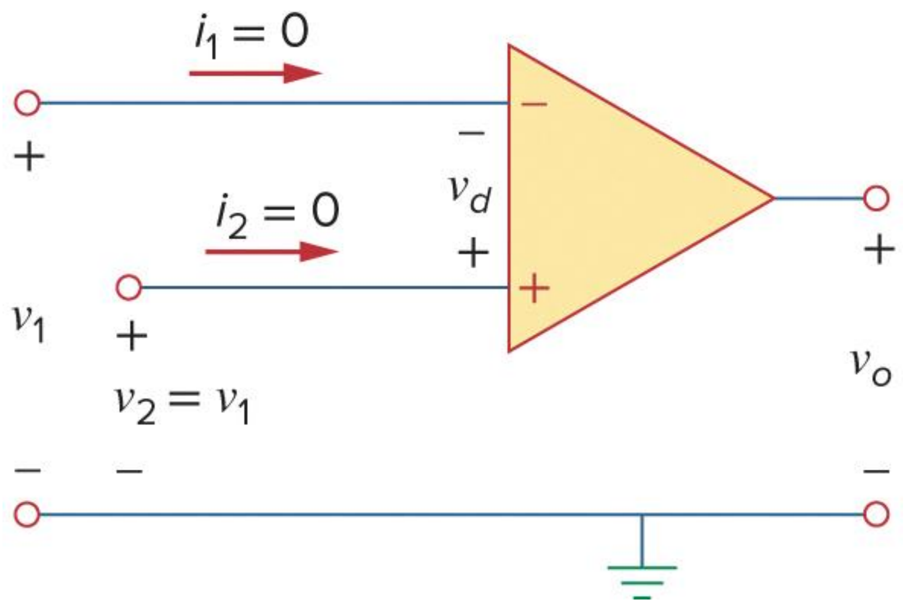
- This is closed-loop gain because the $20\ k\Omega$ feedback resistor closes the loop between the output and input terminals

Recall:

Current flows from higher potential (+) to lower potential (-)

Ideal Op Amp

- An ideal Op amp is ideal if it has the following characteristics:
  - Infinite open-loop gain, A$\simeq \infty$
  - Infinite input resistance, $R_i \simeq \infty$
  - Zero output resistance, $R_0 \simeq 0$
- An ideal op amp is an amplifier with infinite open-loop gain, infinite input resistance, and zero output resistance.
- Two important characteristics of the ideal op amp are:
  - The currents into both input terminals are zero
    $\boxed{i_1 = 0 \text{ , } i_2 = 0}$
  - The voltage across the input terminals is equal to zero
    $v_d = v_2 - v_1 = 0 \Rightarrow \boxed{v_1 = v_2}$



Example 2

Determine the closed-loop gain and current i_0 when $v_s = 1\ V$ in the circuit shown below. Assume the op amp is ideal.

Solution:

- Input currents are zero
- Two inputs of op amp have the same voltages

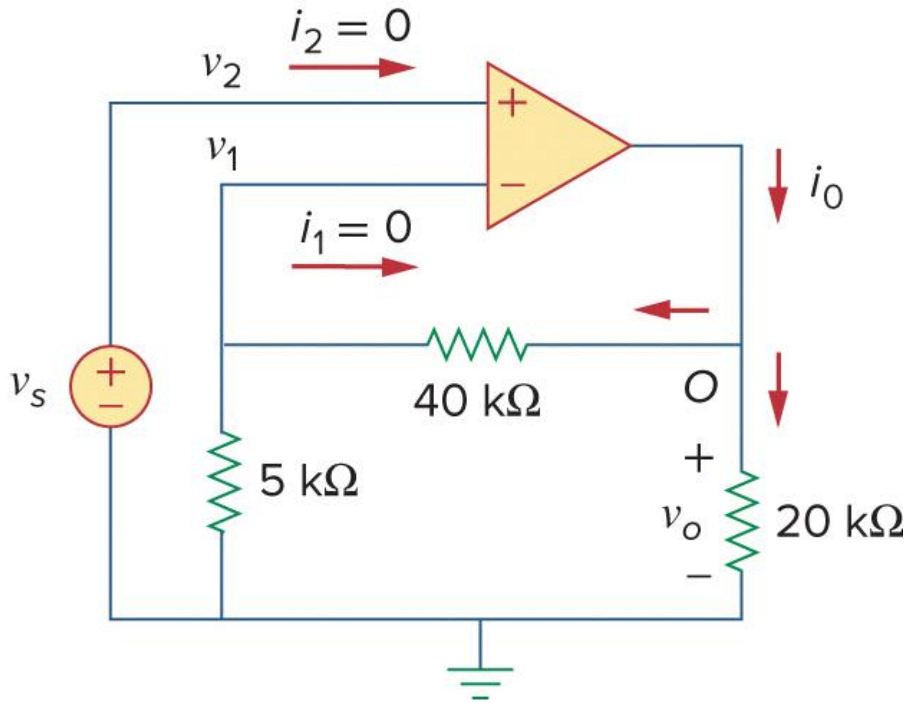
  $v_2 = v_s$

- Since $v_1 = v_2$, $v_1 = v_2 = v_s$
- Since $i_1 = 0$ (no current flow into amp), $40\ \Omega$ series with $5\ \Omega$.
- Using voltage division:

$$v_1 = \frac{5}{5 + 40} v_o = \frac{v_o}{9} \qquad v_s = \frac{v_o}{9} \Rightarrow \frac{v_o}{v_s} = 9$$

- Apply KCL to node O:

$$i_o = \frac{v_o}{40 + 5} + \frac{v_o}{20} \text{mA}$$



when $v_s = 1$ V, $v_o = 9$ V.

$$i_o = 0.2 + 0.45 = 0.65 \text{ mA}$$

Inverting Amplifier

- Inverting amplifier reserves the polarity of the input signal while amplifying it.
- Input v_i is connected to the inverting terminal through R_1, and feedback resistor R_f is connected between the inverting input and output.
- Noninverting input is grounded.
- Apply KCL at node 1:

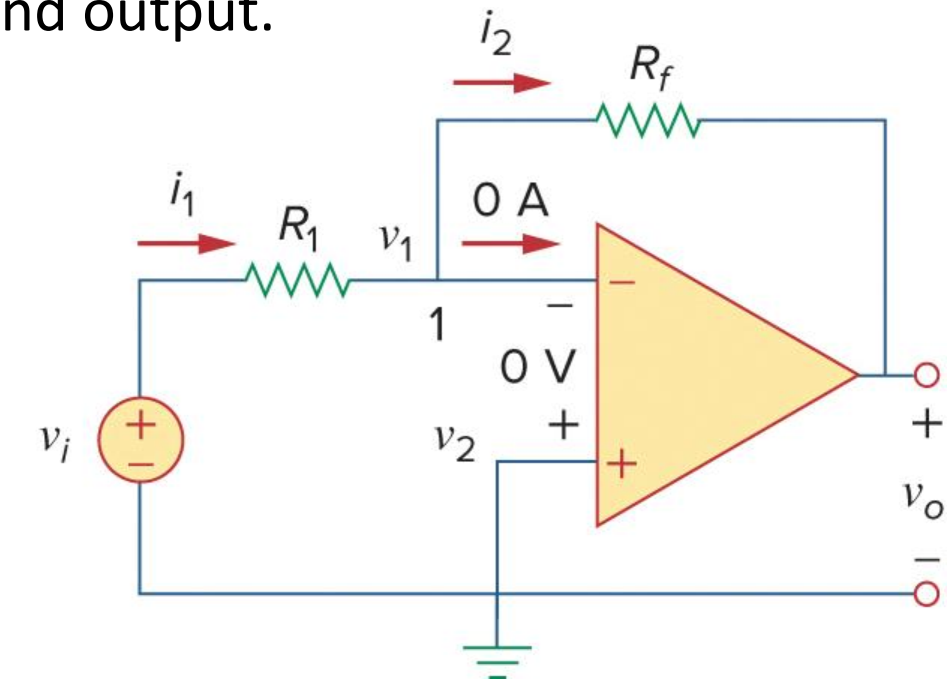
$i_1 = i_2$ (no current flows into amp)

$$i_1 = \frac{v_i - v_1}{R_1} \qquad i_2 = \frac{v_1 - v_0}{R_f}$$

$$\frac{v_i - v_1}{R_1} = \frac{v_1 - v_0}{R_f} \qquad v_1 = v_2 = 0$$

$$\frac{v_i}{R_1} = -\frac{v_0}{R_f} \Rightarrow \boxed{v_0 = -\frac{R_f}{R_1} v_i}$$

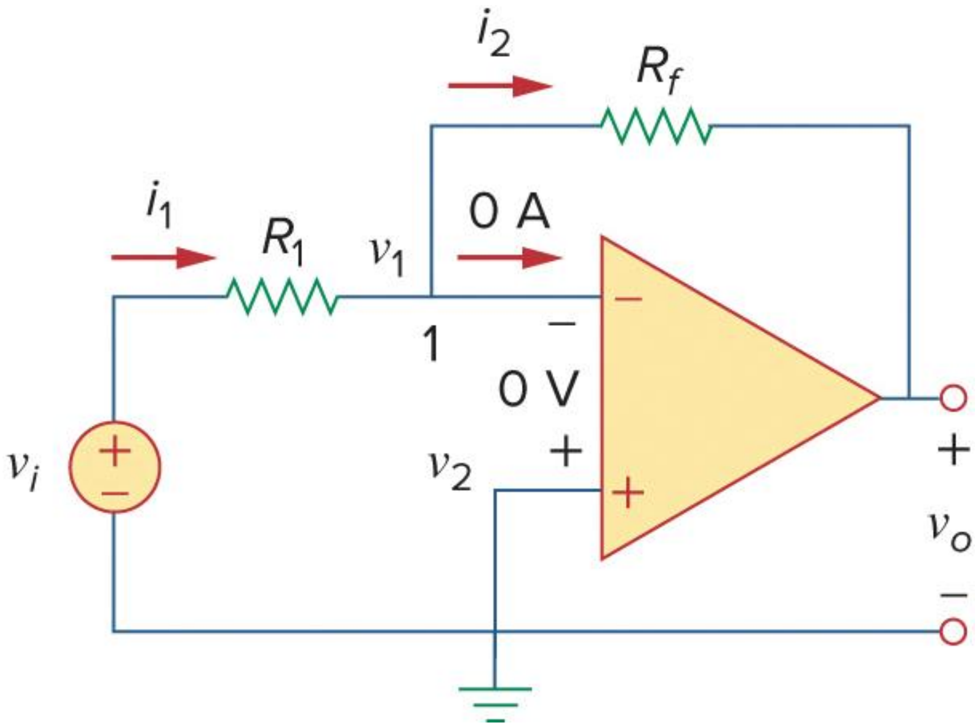
Provide negative output voltage



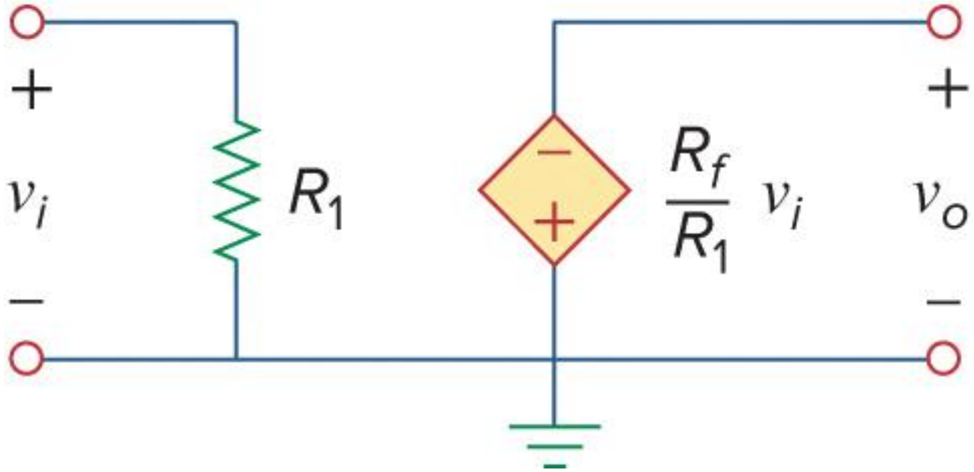
$$A_v = \frac{v_0}{v_i} = -\frac{R_f}{R_1}$$ A_v: voltage gain

Gain depends only on the external elements connected to op amp

Inverting Amplifier

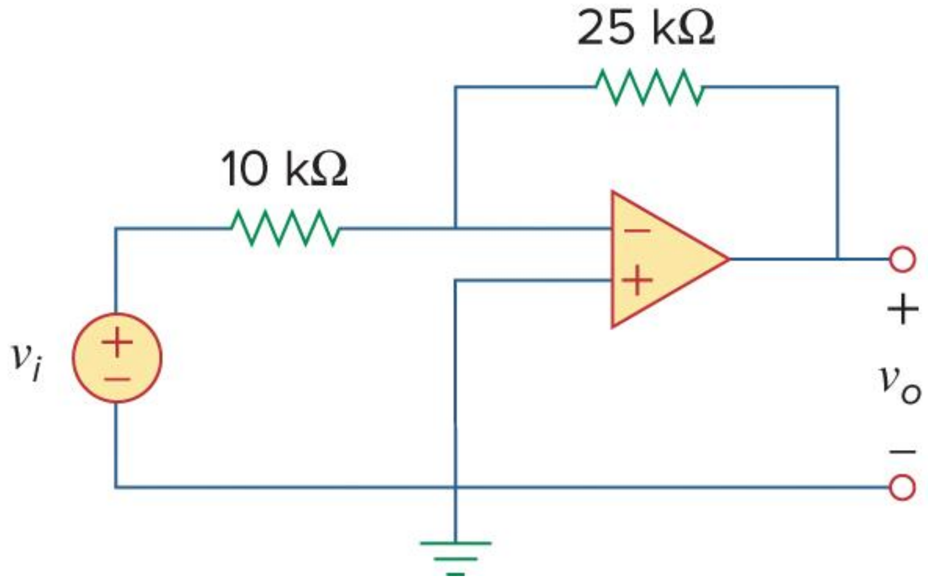


Equivalent circuit of the inverting amplifier



Example 3

For the op amp shown below, if $v_i = 0.5\ V$, calculate a-) the output voltage v_0, and b-) the current in the $10\ k\Omega$ resistor.



Solution:

a-) $$v_0 = -\frac{R_f}{R_1} v_i$$

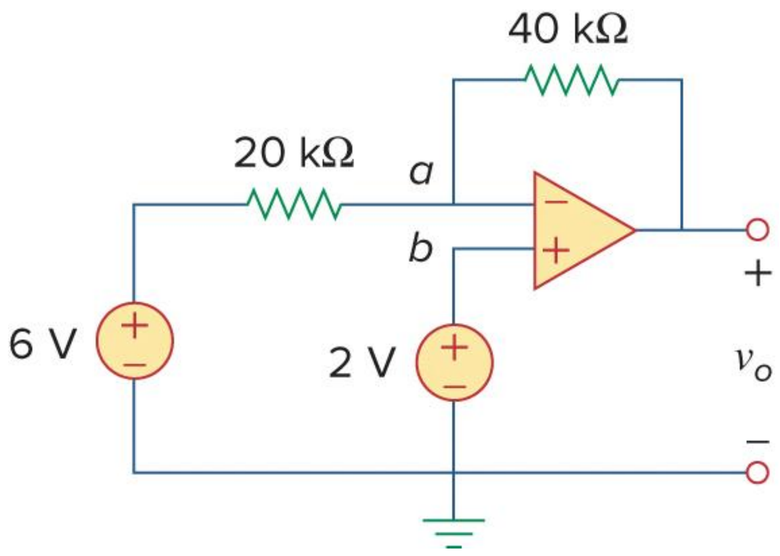
$$\frac{v_o}{v_i} = -\frac{R_f}{R_1} = -\frac{25}{10} = -2.5$$

$$v_o = -2.5 v_i = -2.5(0.5) = -1.25\text{ V}$$

b-) $$i = \frac{v_i - 0}{R_1} = \frac{0.5 - 0}{10 \times 10^3} = 50\ \mu\text{A}$$

Example 4

Determine v_0 in the op amp circuit shown below



Solution:

Applying KCL at node a,

$$\frac{v_a - v_o}{40\ \text{k}\Omega} = \frac{6 - v_a}{20\ \text{k}\Omega}$$

$$v_a - v_o = 12 - 2v_a \Rightarrow v_o = 3v_a - 12$$

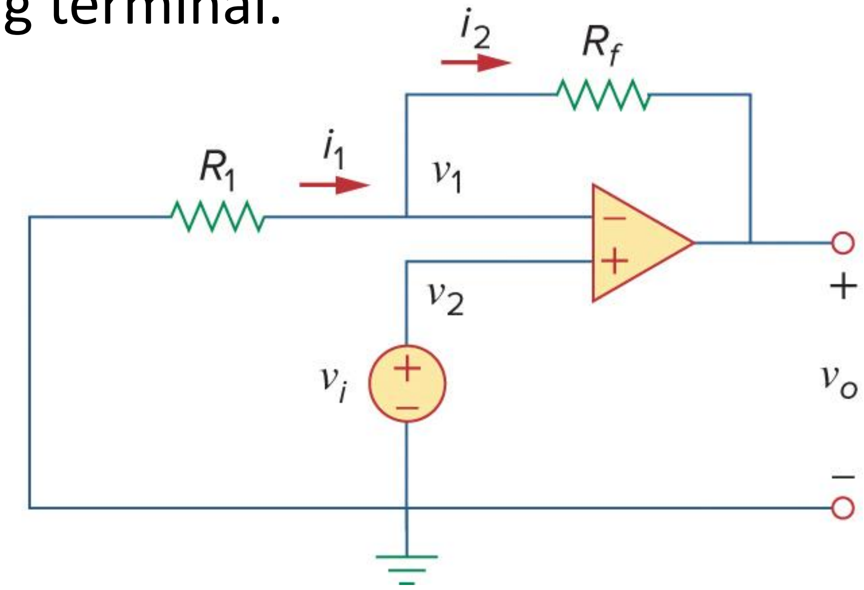
$$v_a = v_b = 2\ \text{V}$$

$$v_o = 6 - 12 = -6\ \text{V}$$

Notice that if $v_b = 0 = v_a$, then $v_o = -12$

Noninverting Amplifier

- Noninverting amplifier is an op amp circuit designed to provide a positive voltage gain.
- Input voltage v_i is directly applied at the noninverting terminal, and the resistor R_1 is connected between the ground and inverting terminal.
- Apply KCL at node 1:



$i_1 = i_2$ (no current flows into amp)

$$i_1 = \frac{0 - v_1}{R_1} \qquad i_2 = \frac{v_1 - v_0}{R_f}$$

$$\frac{0 - v_1}{R_1} = \frac{v_1 - v_0}{R_f} \qquad v_1 = v_2 = v_i$$

$$-\frac{v_i}{R_1} = \frac{v_i - v_0}{R_f} \Rightarrow v_0 = \frac{v_i R_1 + v_i R_f}{R_1} \Rightarrow \boxed{v_0 = \left(1 + \frac{R_f}{R_1}\right) v_i}$$

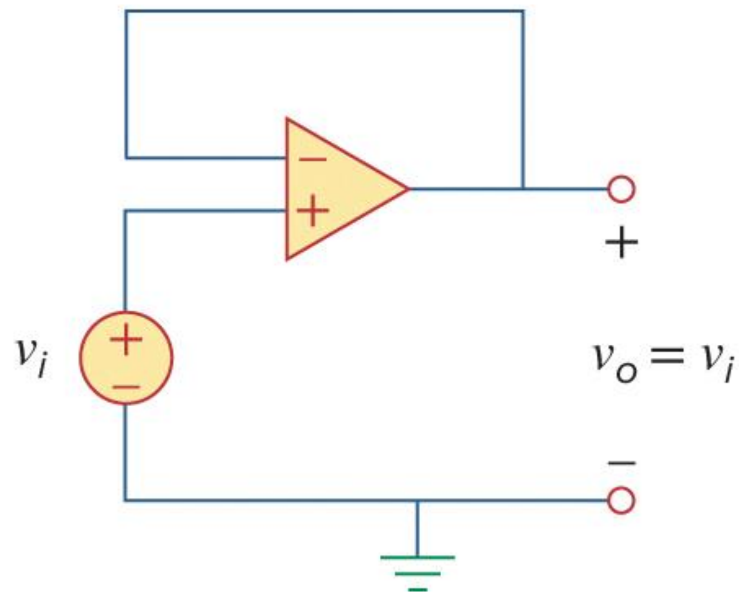
Provide positive output voltage

$$A_v = \frac{v_0}{v_i} = 1 + \frac{R_f}{R_1}$$ A_v: voltage gain

Gain depends only on the external elements connected to op amp

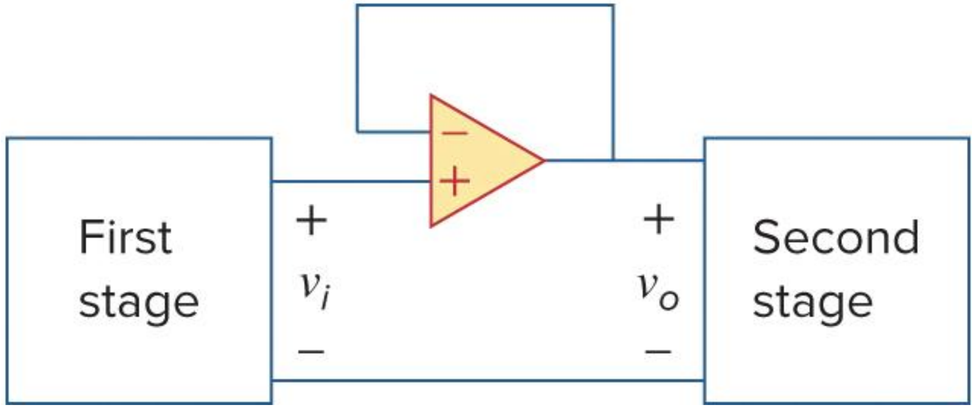
Inverting Amplifier: Special Case

- If feedback resistor $R_f = 0$ (short circuit) and $R_1 = \infty$ (open circuit), we will have the following circuit.
- The gain becomes 1.
- It is called voltage follower (or unity gain amplifier) because output follows input.



$\boxed{v_0 = v_i}$ → Voltage follower does not provide any amplification to the signal

- The voltage follower is used as an intermediate-stage (or buffer) amplifier to isolate one circuit from another. It minimizes the interaction between two stages and eliminates interstage loading.



Example 5

For the op amp circuit shown below, calculate the output voltage v_0.

Solution:

Method 1: Using superposition

let $v_o = v_{o1} + v_{o2}$

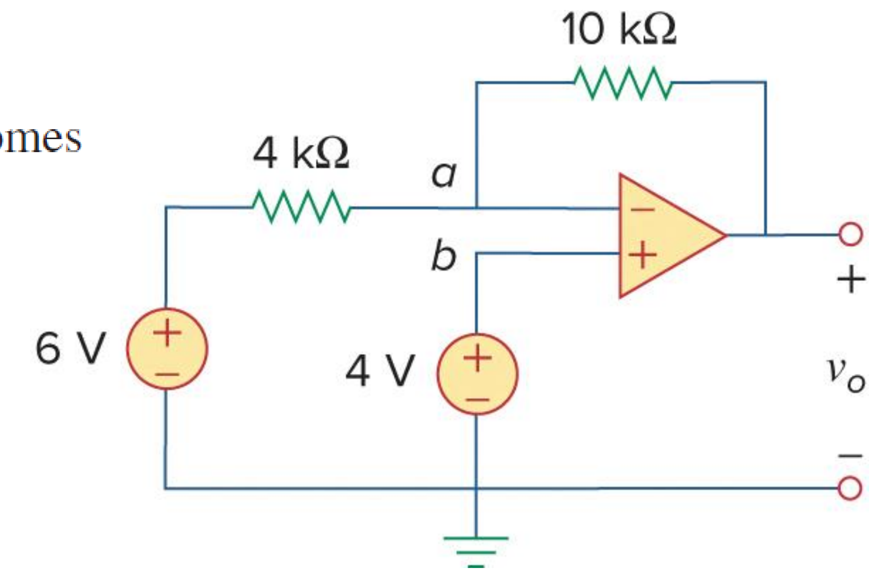
where v_{o1} is due to the 6-V voltage source, and v_{o2} is due to the 4-V input. To get v_{o1}, we set the 4-V source equal to zero. Under this condition, the circuit becomes an inverter.

$$v_{o1} = -\frac{10}{4}(6) = -15 \text{ V}$$

To get v_{o2}, we set the 6-V source equal to zero. The circuit becomes a noninverting amplifier

$$v_{o2} = \left(1 + \frac{10}{4}\right)4 = 14 \text{ V}$$

$$v_o = v_{o1} + v_{o2} = -15 + 14 = -1 \text{ V}$$



Solution

Method 2: Using nodal analysis

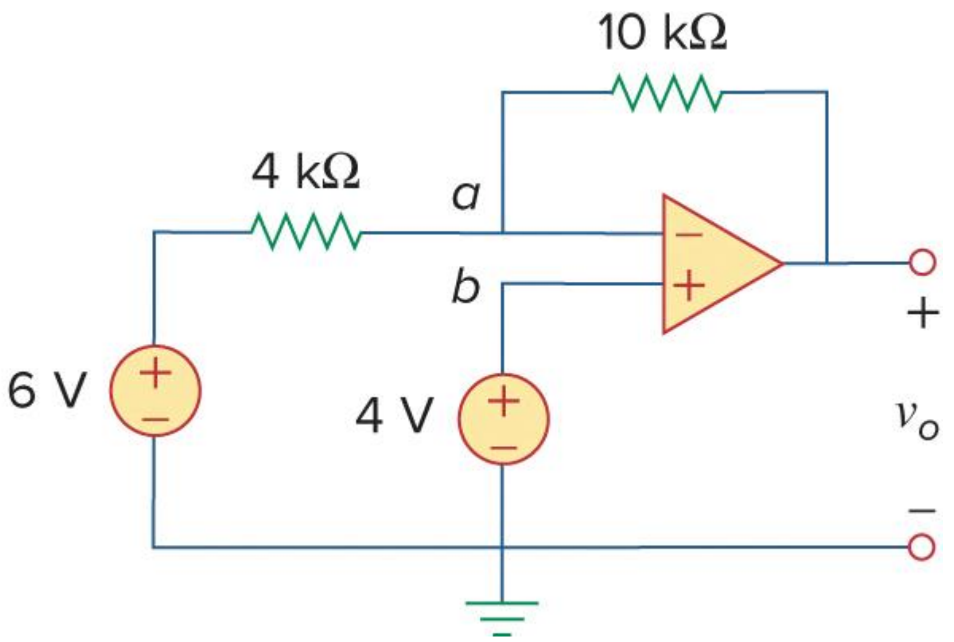
Applying KCL at node a,

$$\frac{6 - v_a}{4} = \frac{v_a - v_o}{10}$$

$$v_a = v_b = 4$$

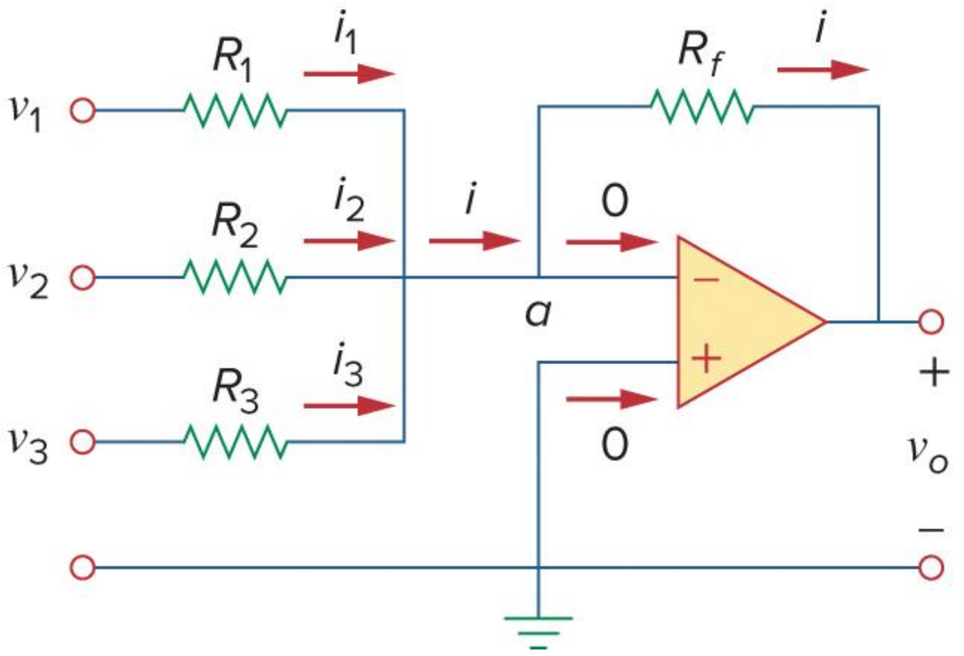
$$\frac{6 - 4}{4} = \frac{4 - v_o}{10} \quad \Rightarrow \quad 5 = 4 - v_o$$

$$v_o = -1 \text{ V}$$



Summing Amplifier

- Op amp can perform addition besides amplification.
- A summing amplifier (aka summer) is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.
- Current entering each op amp input is zero.
- Apply KCL at node a:



$$i = i_1 + i_2 + i_3$$

$$i_1 = \frac{v_1 - v_a}{R_1} \qquad i_2 = \frac{v_2 - v_a}{R_2}$$

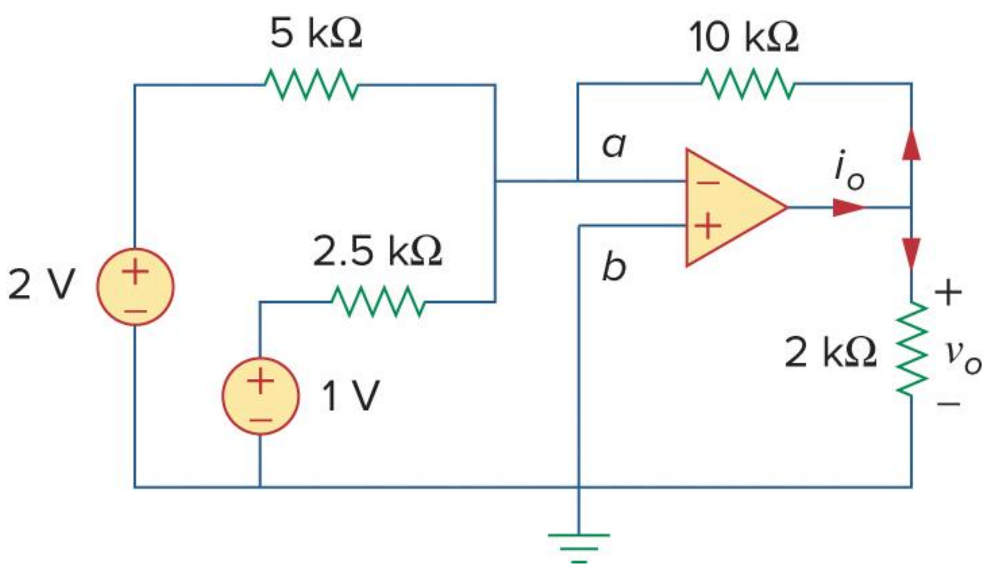
$$i_3 = \frac{v_3 - v_a}{R_3} \qquad i = \frac{v_a - v_0}{R_f} \qquad v_a = 0$$

$$-\frac{v_0}{R_f} = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} \Rightarrow \boxed{v_0 = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)}$$

Example 6

- Calculate v_0 and i_0 in the op amp circuit shown below.

Solution:



This is a summer with two inputs.

$$v_o = -\left[\frac{10}{5}(2) + \frac{10}{2.5}(1)\right] = -(4 + 4) = -8 \text{ V}$$

The current i_o is the sum of the currents through the 10-kΩ and 2-kΩ resistors. Both of these resistors have voltage $v_o = -8$ V across them, since $v_a = v_b = 0$. Hence,

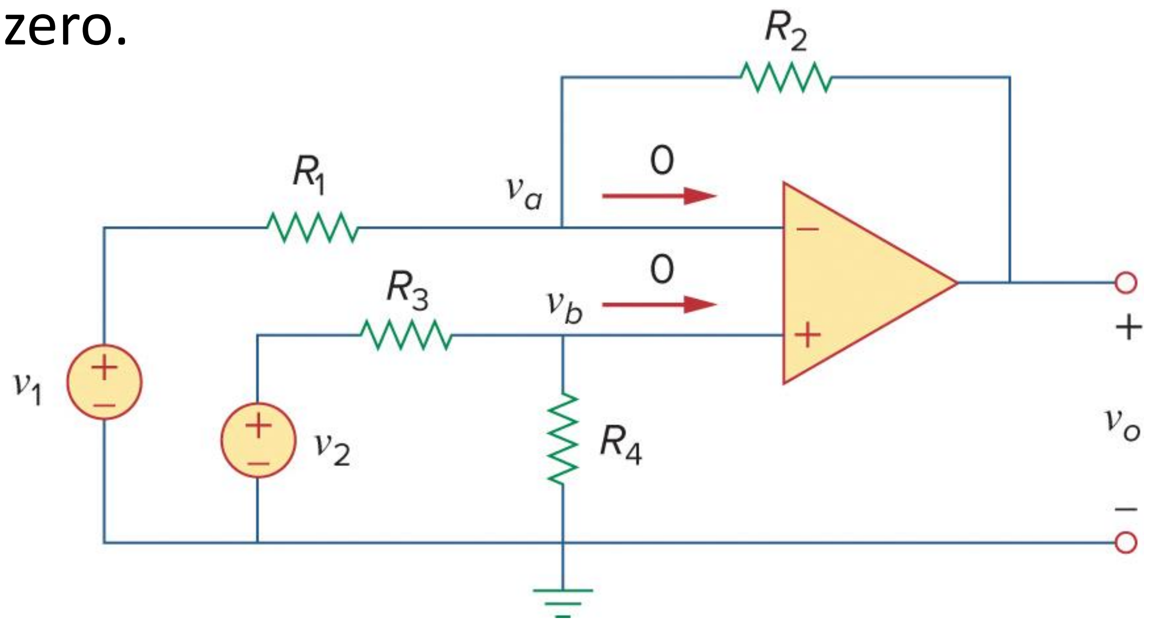
$$i_o = \frac{v_o - 0}{10} + \frac{v_o - 0}{2} \text{ mA} = -0.8 - 4 = -4.8 \text{ mA}$$

Difference Amplifier

- Difference (aka differential) amplifiers are used in various applications where there is a need to amplify the difference between two input signals.
- A difference amplifier is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.
- Current entering each op amp input is zero.
- Apply KCL at node a:

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_0}{R_2}$$

$$v_0 = \left(\frac{R_2}{R_1} + 1\right) v_a - \frac{R_2}{R_1} v_1 \longrightarrow \text{Eq 1}$$



Difference Amplifier

- Apply KCL at node b:

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4} \Rightarrow v_b = \frac{R_4}{R_3 + R_4} v_2$$

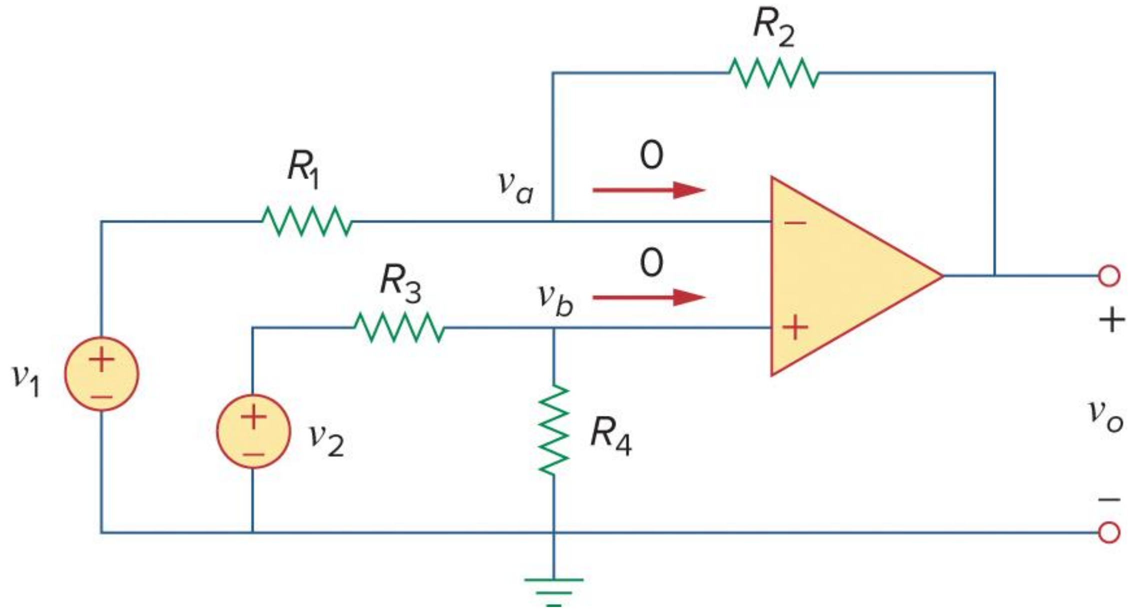
$$v_a = v_b$$

- Substitute v_b into 1st Eq

$$\boxed{v_0 = \frac{R_2\left(1 + \frac{R_1}{R_2}\right)}{R_1\left(1 + \frac{R_3}{R_4}\right)} v_2 - \frac{R_2}{R_1} v_1}$$

- If $R_1 = R_2$ and $R_3 = R_4$, the difference amplifier becomes a subtractor with output

$$v_0 = v_2 - v_1$$



- When $\frac{R_1}{R_2} = \frac{R_3}{R_4}$, $v_0 = \frac{R_2}{R_1}(v_2 - v_1)$

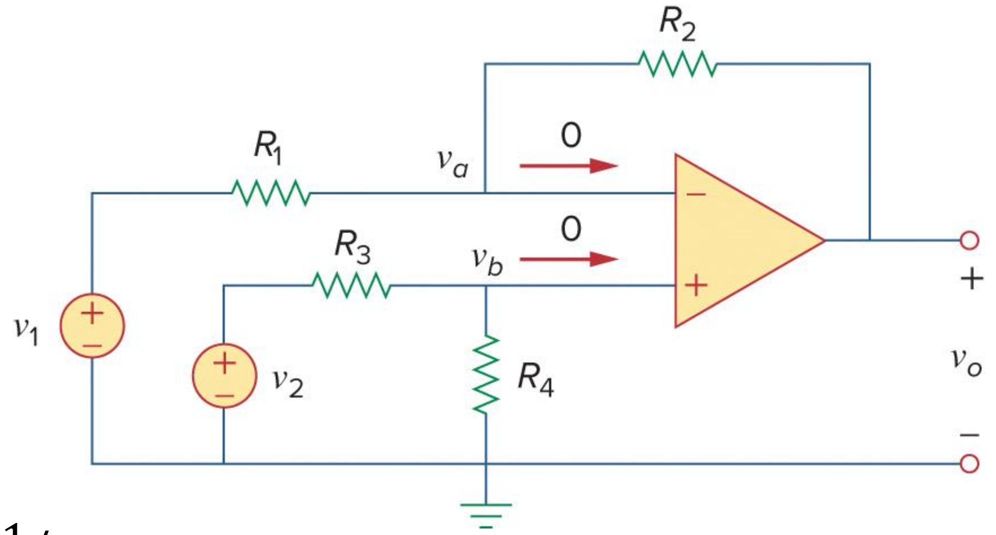
- In this case, the difference amplifier rejects a signal common to the two inputs, i.e., $v_0 = 0$ when $v_1 = v_2$

Example 7

Design an op amp circuit with inputs v_1 and v_2 such that $v_0 = -5v_1 + 3v_2$

Solution:

- This circuit can be designed in two ways.
- **Design 1:** design it using only one op amp.
- Two inputs since $v_0 = 3v_2 - 5v_1$



$$v_0 = \underbrace{\boxed{\frac{R_2\left(1+\frac{R_1}{R_2}\right)}{R_1\left(1+\frac{R_3}{R_4}\right)}}}_{3} v_2 - \underbrace{\boxed{\frac{R_2}{R_1}}}_{5} v_1$$

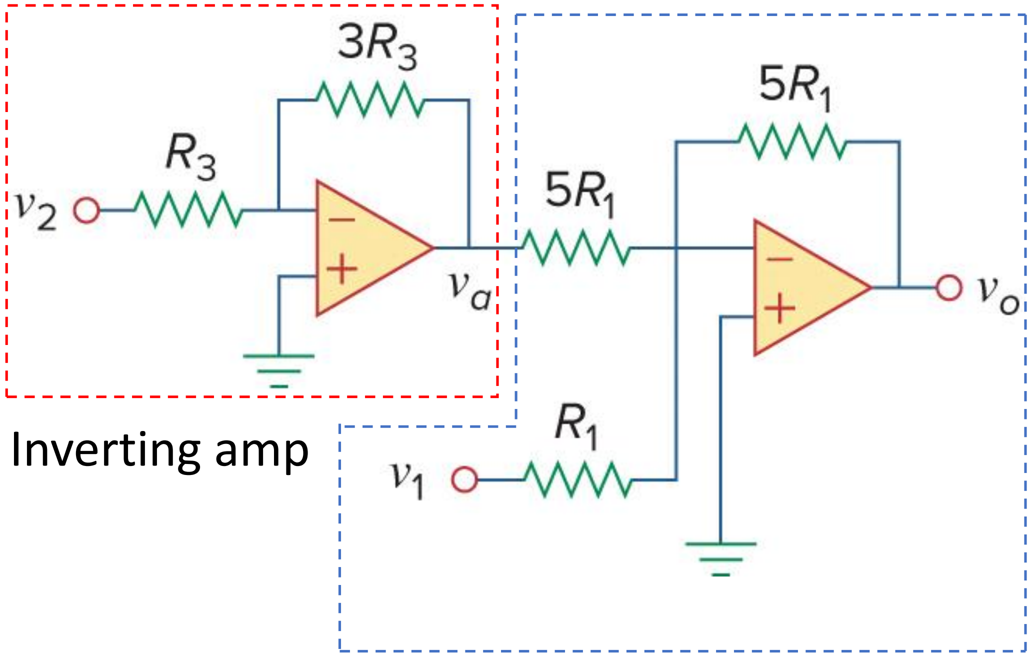
$$5\frac{\left(1+\overbrace{\frac{R_1}{R_2}}^{1/5}\right)}{\left(1+\frac{R_3}{R_4}\right)} = 3 \Rightarrow R_3 = R_4 \qquad \left(\frac{R_1}{R_2} = \frac{1}{5}\right)$$

$$\frac{R_2}{R_1} = 5 \Rightarrow R_2 = 5R_1$$

We may choose: $\begin{cases} R_1 = 10\ k\Omega & R_3 = 20\ k\Omega \\ R_2 = 50\ k\Omega & R_4 = 20\ k\Omega \end{cases}$

Solution

- **Design 2:** design it using two op amps.
- In this case, cascade an inverting amp and two input inverting summer amp as shown in the figure.



- For the summer:

$$v_0 = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

$$v_0 = -\left(\frac{5R_1}{5R_1}v_a + \frac{5R_1}{R_1}v_1\right) \Rightarrow v_0 = -v_a - 5v_1$$

- For the inverter amp:

$$v_0 = -\frac{R_f}{R_1}v_i$$

$$v_a = -\frac{3R_3}{R_3}v_2 \Rightarrow v_a = -3v_2$$

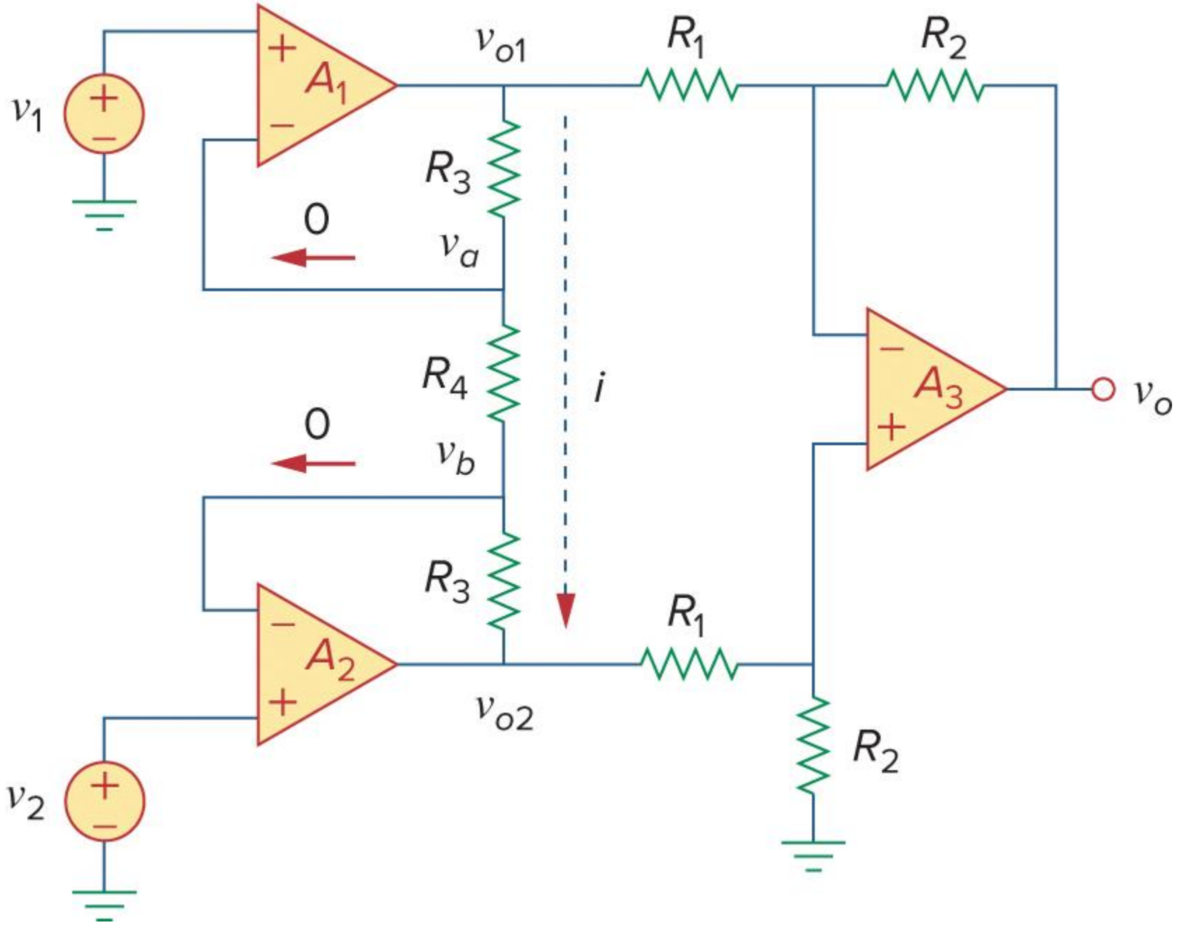
$$v_0 = 3v_2 - 5v_1$$

We may choose: $R_1 = R_3 = 10\ k\Omega$

Example 8

An instrumental amplifier shown below is an amplifier of low-level signals used in process control or measurement applications and commercially available in single-package units. Show that

$$v_0 = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$



Solution

- Amplifier A_3 is a difference amplifier.

$$v_o = \frac{R_2}{R_1}(v_{o2} - v_{o1}) \longrightarrow \text{Eq 1}$$

Since the op amps A_1 and A_2 draw no current, current i flows through the three resistors as though they were in series. Hence,

$$v_{o1} - v_{o2} = i(R_3 + R_4 + R_3) = i(2R_3 + R_4) \longrightarrow \text{Eq 2}$$

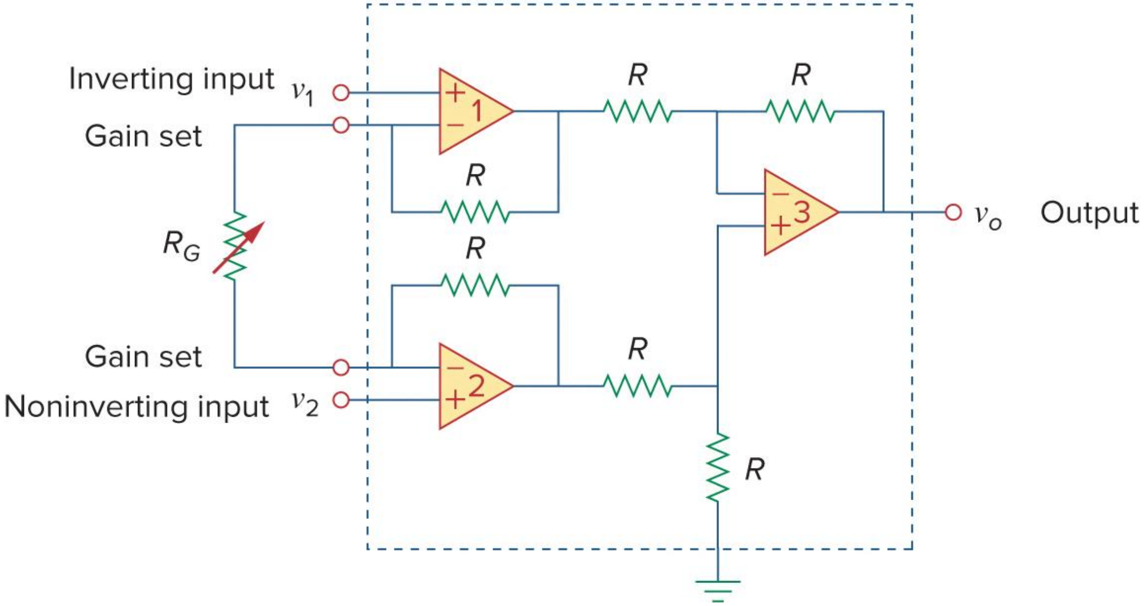
$$\left.\begin{aligned} i &= \frac{v_a - v_b}{R_4} \\ v_a &= v_1,\ v_b = v_2 \end{aligned}\right\} \quad i = \frac{v_1 - v_2}{R_4} \longrightarrow \text{Eq 3}$$

- Substitute Eq 3 into Eq 2 and then substitute into Eq 1

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

Instrumentation Amplifiers

- One of the most useful and versatile op amp circuit which can be used for precision measurement and process control.
- Typical applications of IAs: isolation amplifiers, data acquisition system, etc.
- Extension of the difference amplifier. It amplifies the difference between its input signal.
- Typically consists of three op amps and several resistors.
- R_G: external gain-setting resistor.



$$v_0 = \underbrace{\frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)}_{A_v}(v_2 - v_1)$$

$A_v \longrightarrow$ Voltage gain

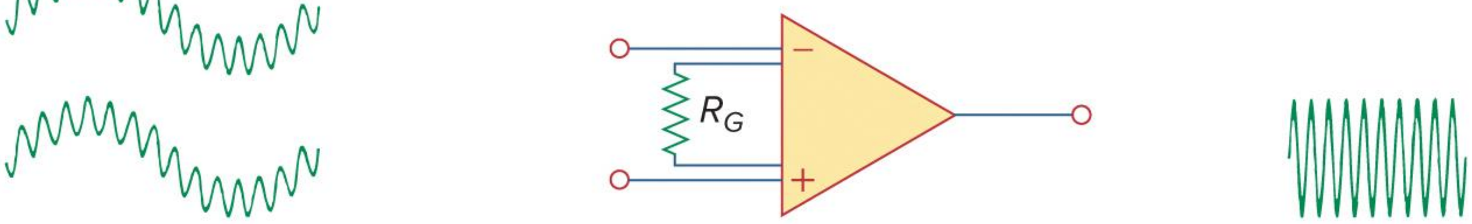
$$A_v = 1 + \frac{2R}{R_G} \qquad v_0 = A_v(v_2 - v_1)$$

Instrumentation Amplifiers

- The instrumentation amplifier has three major characteristic as follows:
  - The voltage gain is adjusted by one external resistor R_G.
  - The input impedance of both inputs is very high and does not vary as the gain is adjusted.
  - The output v_0 depends on the difference between the inputs v_1 and v_2, not the voltage common to them (common-mode voltage).
- For IA, very small changes or differences in the input will result in very large output. How big the output signal also depends on how big its gain is.

$$v_0 = A_v(v_2 - v_1)$$

Schematic symbol of IA



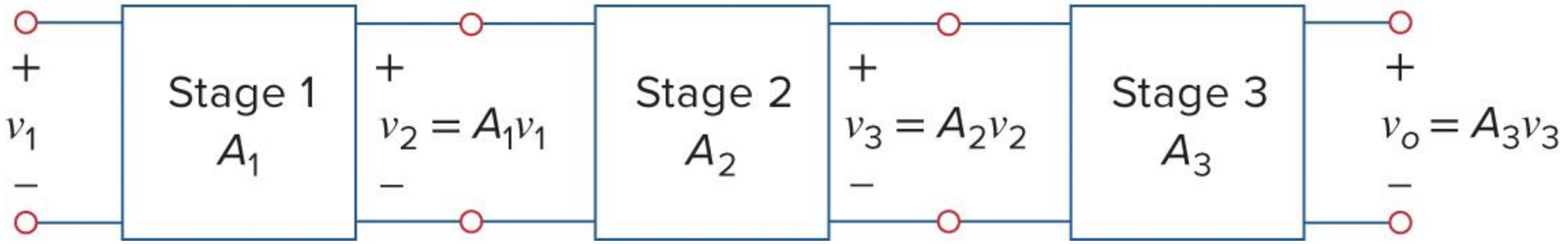
Small differential signals riding on larger common-mode signals

Instrumentation amplifier

Amplified differential signal, no common-mode signal

Cascaded Op Amp Circuit

- To obtain a large overall gain, connect op amp circuits in cascaded way.
- **Cascaded connection**: head-to-tail arrangement of two or more op amp circuits such that the output of one is the input of the next
- Cascaded connection is connecting block in a head-to-tail way.
- In cascaded connection, each circuit in the string is called a "stage".
- Input signal increased by the gain of the individual stage.

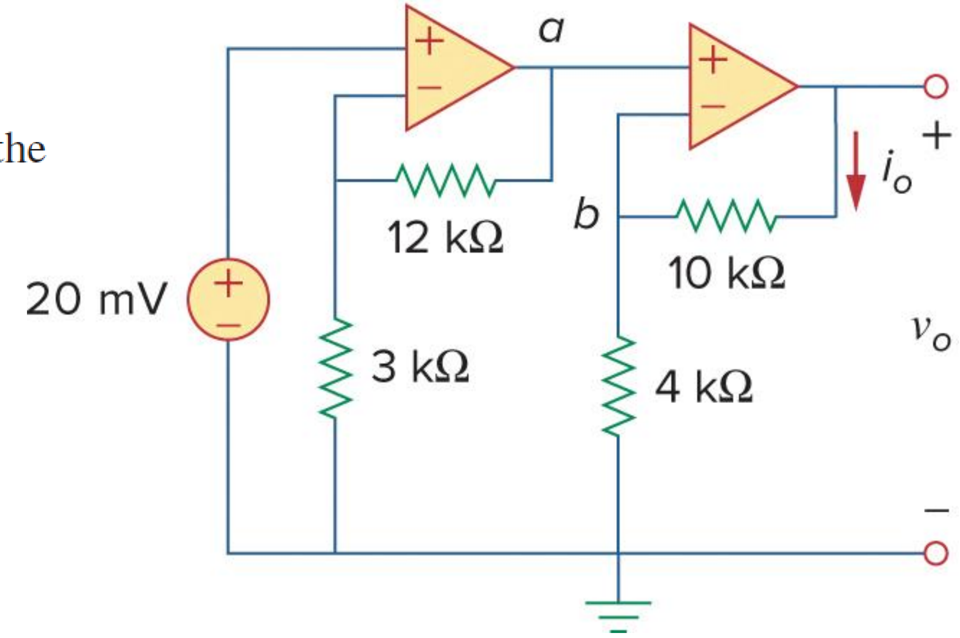


- Overall gain: $A = A_1 A_2 A_3$

Example 9

For the circuit shown below, find v_0 and i_0.

Solution:



This circuit consists of two noninverting amplifiers cascaded. At the output of the first op amp,

$$v_a = \left(1 + \frac{12}{3}\right)(20) = 100 \text{ mV}$$

At the output of the second op amp,

$$v_o = \left(1 + \frac{10}{4}\right)v_a = (1 + 2.5)100 = 350 \text{ mV}$$

The required current i_o is the current through the 10-kΩ resistor.

$$i_o = \frac{v_o - v_b}{10} \text{ mA}$$

$$v_b = v_a = 100 \text{ mV}.$$

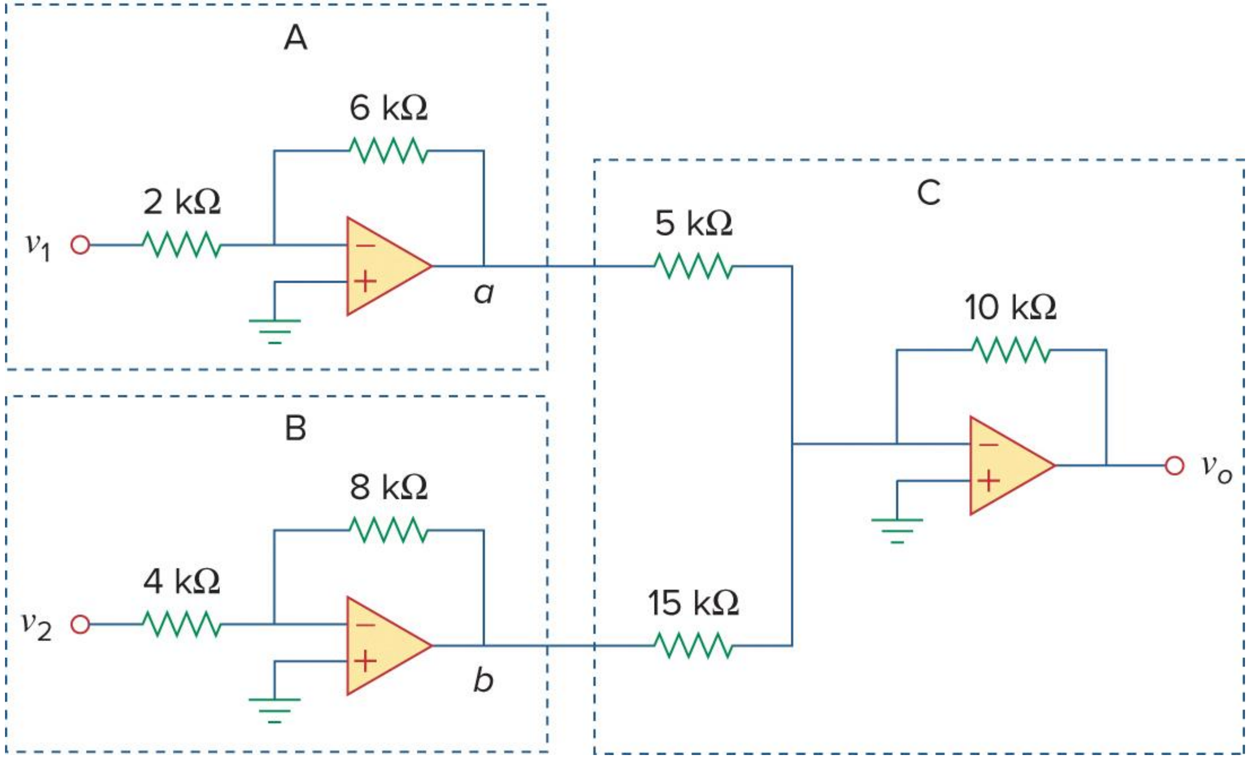
$$i_o = \frac{(350 - 100) \times 10^{-3}}{10 \times 10^3} = 25\ \mu\text{A}$$

Example 10

If $v_1 = 1\ V$ and $v_2 = 2\ V$, find v_0 in the op amp circuit shown below.

Solution:

- The op amp circuit is composed of three circuits
- Two inverting amps and one summing (summer) amp
- A and B: inverting amp
- C: Summing amp



$$v_0 = -\frac{R_f}{R_1} v_i \longrightarrow v_a = -\frac{6}{2} 1 = -3\ V$$

$$v_b = -\frac{8}{4} 2 = -4\ V$$

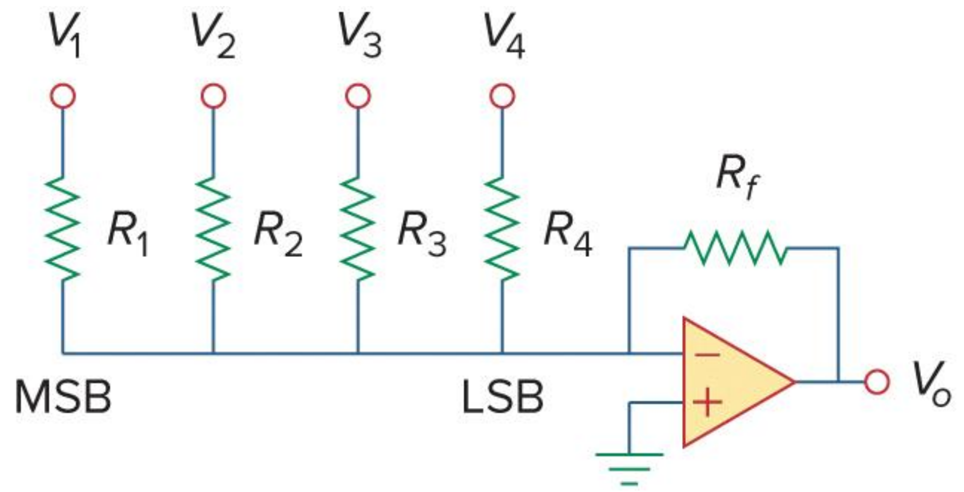
$$v_0 = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right) \longrightarrow v_0 = -\left(\frac{10}{5}(-3) + \frac{10}{15}(-4)\right) \Rightarrow v_0 = 8.667\ V$$

Digital-to-Analog Converter

- Digital-to-analog converter (A/D or DAC): convert digital signal into analog signal.
- The four-bit DAC can be implemented in many ways. A simple implementation is the binary weighted ladder as shown in the figure.

$$-v_0 = \frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3 + \frac{R_f}{R_4}v_4$$

where v_1: Most Significant Bit (MSB)
v_2: Least Significant Bit (LSB)



- Each of the binary inputs v_1, v_2, v_3, v_4 can have only two voltage value: 0 or 1 V

Example 11

In the op amp circuit shown below, let $R_f = 10\ k\Omega$, $R_1 = 10\ k\Omega$, $R_2 = 20\ k\Omega$, $R_3 = 40\ k\Omega$, and $R_4 = 80\ k\Omega$. Obtain the analog output for the binary input [0000], [0001], [0010],…,[1111].

Solution:

DAC provides single output related to inputs

$$-v_0 = \frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3 + \frac{R_f}{R_4}v_4$$

$$-v_0 = \frac{10}{10}v_1 + \frac{10}{20}v_2 + \frac{10}{40}v_3 + \frac{10}{80}v_4$$

$$-v_0 = v_1 + 0.5v_2 + 0.25v_3 + 0.125v_4$$

1st digital input: $[v_1\ v_2\ v_3\ v_4]$=[0000] $\longrightarrow$ $v_0 = 0$

2nd digital input: $[v_1\ v_2\ v_3\ v_4]$=[0001] $\longrightarrow$ $-v_0 = 0.125\ V$

3rd digital input: $[v_1\ v_2\ v_3\ v_4]$=[0010] $\longrightarrow$ $-v_0 = 0.25$ V

TABLE 5.2

Input and output values of the four-bit DAC.

Binary input $[V_1V_2V_3V_4]$	Decimal value	Output $-V_o$
0000	0	0
0001	1	0.125
0010	2	0.25
0011	3	0.375
0100	4	0.5
0101	5	0.625
0110	6	0.75
0111	7	0.875
1000	8	1.0
1001	9	1.125
1010	10	1.25
1011	11	1.375
1100	12	1.5
1101	13	1.625
1110	14	1.75
1111	15	1.875

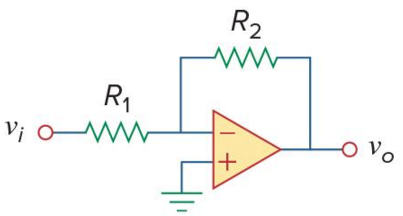
Table summarizes the result of the digital-to-analog conversion

Summary:

TABLE 5.3

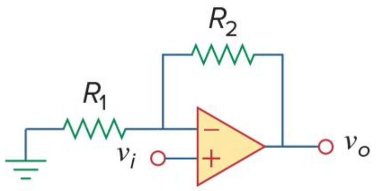
Summary of basic op amp circuits.

Op amp circuit | **Name/output-input relationship**



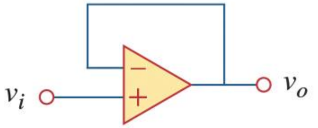
Inverting amplifier

$$v_o = -\frac{R_2}{R_1} v_i$$



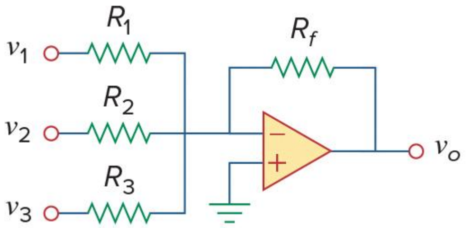
Noninverting amplifier

$$v_o = \left(1 + \frac{R_2}{R_1}\right) v_i$$



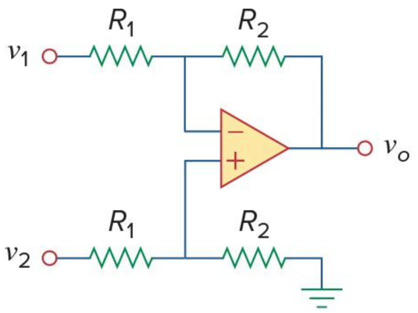
Voltage follower

$$v_o = v_i$$



Summer

$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right)$$



Difference amplifier

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$