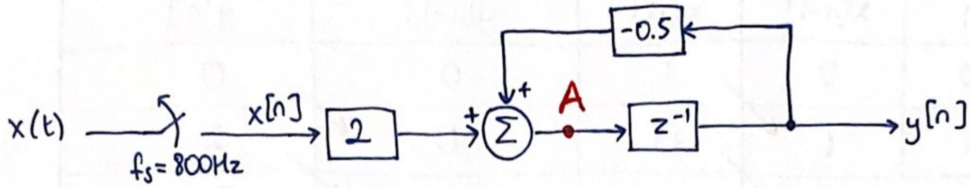
Quiz 3 Solution

$x(t) = 2\sin\left(400\pi t + \frac{\pi}{6}\right) \cdot [u(t) - u(t-0.01)]$



a) $x[n] = x(n.T_s) = 2\sin\left(\cancel{400}\pi . n . \frac{1}{\cancel{800}_{2}} + \frac{\pi}{6}\right) . \left[u\left[\frac{n}{800}\right] - u\left[\frac{n}{800} - \frac{1}{100}\right]\right]$

$\boxed{t \to n.T_s}$

$= 2\sin\left(n.\frac{\pi}{2} + \frac{\pi}{6}\right) . \underbrace{\left[u\left[\frac{n}{800}\right] - u\left[\frac{n-8}{800}\right]\right]}_{\text{it will be 1 for } 0 \leq n < 8}$

$T_s = \frac{1}{f_s} = \frac{1}{800}$

* Therefore, you need to find x[n] for $0 \leq n < 8$.

$x[n] = 2\sin\left(n\frac{\pi}{2} + \frac{\pi}{6}\right)$ for $0 \leq n < 8$

$x[0] = 2\sin\left(\frac{\pi}{6}\right) = 2\sin 30 = 1$

$x[1] = 2\sin(120) = \sqrt{3}$

$x[2] = 2\sin(210) = -1$

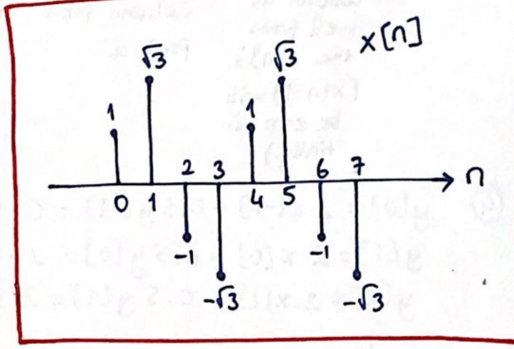
$x[3] = 2\sin(300) = \sqrt{3}$

$x[4] = 1$

$x[5] = \sqrt{3}$

$x[6] = -1$

$x[7] = -\sqrt{3}$



b) Let's call "A" the node after the summation operator.

$A = 2x[n] - 0.5y[n]$

$y[n] = A . z^{-1} = A[n-1]$

$\boxed{y[n] = 2x[n-1] - 0.5y[n-1]}$

c) $y[n] = 2x[n-1] - 0.5y[n-1]$

n	$x[n-1]$	$x[n]$	$y[n-1]$	$y[n]$
0	0	1	0	0
1	1	$\sqrt{3}$	0	2
2	$\sqrt{3}$	-1	2	$2\sqrt{3}-1$
3	-1	$-\sqrt{3}$	$2\sqrt{3}-1$	$-\sqrt{3}-\frac{3}{2}$
4	$-\sqrt{3}$	1	$-\sqrt{3}-\frac{3}{2}$	$-\frac{3\sqrt{3}}{2}+\frac{3}{4}$
5	1	$\sqrt{3}$	$-\frac{3\sqrt{3}}{2}+\frac{3}{4}$	$\frac{3\sqrt{3}}{4}+\frac{13}{8}$
6	$\sqrt{3}$	-1	$\frac{3\sqrt{3}}{4}+\frac{13}{8}$	$\frac{13\sqrt{3}}{8}-\frac{13}{16}$
7	-1	$-\sqrt{3}$	$\frac{13\sqrt{3}}{8}-\frac{13}{16}$	$-\frac{13\sqrt{3}}{16}-\frac{51}{32}$

↓ ② Then, fill this column as well from the $x[n]$'s. ($x[n-1]$ will be zero initially.)

↓ ① First, fill this column from part a.

↓ ③ Later on, $y[n-1]$ will be 0 initially as well and $y[0]$ should be calculated as 0 from the equation above.

④ $y[0] = 2x[-1] - 0.5y[-1] = 0 - 0 = 0$

$y[1] = 2x[0] - 0.5y[0] = 2 - 0 = 2$

$y[2] = 2x[1] - 0.5y[1] = 2\sqrt{3} - (0.5)(2) = 2\sqrt{3} - 1$

⋮

d) $T(z) = \dfrac{Y(z)}{X(z)}$

$$y[n] = 2x[n-1] - 0.5y[n-1]$$

↓ z transform

$$Y(z) = 2 \cdot X(z) \cdot z^{-1} - 0.5\, Y(z) \cdot z^{-1}$$

$$Y(z) + Y(z) \cdot (0.5) \cdot z^{-1} = 2 \cdot z^{-1} \cdot X(z)$$

$$Y(z)\left[1 + 0.5z^{-1}\right] = X(z) \cdot \left[2z^{-1}\right]$$

$$\boxed{T(z) = \frac{Y(z)}{X(z)} = \frac{2z^{-1}}{1 + 0.5z^{-1}}}$$