Which are the following matrices in row echelon form? If not, explain why.

a) $\begin{pmatrix} 1 & 5 & 12 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 3 & 2 & 7 \\ 0 & 0 & 1 & 7 \\ 0 & 1 & 9 & -1 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 3 & -5 & 17 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Find the reduced echelon form of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 1 \\ 2 & 1 & 9 \end{pmatrix}$. $\left(\begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \right)$

Find the reduced echelon form of each of the matrices.

$$\begin{pmatrix} 1 & -4 & -2 \\ 3 & -12 & 1 \\ 2 & -8 & 5 \end{pmatrix} \qquad \left(\begin{pmatrix} 1 & -4 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \right)$$

$$\begin{pmatrix} 1 & 3 & 2 & 5 \\ 2 & 5 & 2 & 3 \\ 2 & 7 & 7 & 22 \end{pmatrix} \qquad \left(\begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{pmatrix} \right)$$

Use elementary row operations to transform each augmented coefficient matrix to echelon form.

Then solve the system by back substitution.

$$\begin{array}{rcrcrcr} 2x & - & 4y & + & z & = & 3 \\ x & - & 3y & + & z & = & 5 \\ 3x & - & 7y & + & 2z & = & 12 \end{array}$$

($inconsistent$, $no\, solution$)

$$\begin{array}{rcrcrcr} -3x & - & 2y & + & 4z & = & 9 \\ & & 3y & - & 2z & = & 5 \\ 4x & - & 3y & + & 2z & = & 7 \end{array}$$

$(x = 3\,,\ y = 7\,,\ z = 8)$

Use elementary row operations to transform each augmented coefficient matrix to echelon form.

Then solve the system by back substitution.

$$\begin{array}{rcrcrcr} 2x_1 & + & 8x_2 & + & 3x_3 & = & 2 \\ x_1 & + & 3x_2 & + & 2x_3 & = & 5 \\ 2x_1 & + & 7x_2 & + & 4x_3 & = & 8 \end{array} \qquad (x_1 = 3\,,\ x_2 = -2\,,\ x_3 = 4)$$

$$\begin{array}{rcrcrcr} 2x_1 & + & 5x_2 & + & 12x_3 & = & 6 \\ 3x_1 & + & x_2 & + & 5x_3 & = & 12 \\ 5x_1 & + & 8x_2 & + & 21x_3 & = & 17 \end{array} \qquad (\textit{inconsistent}\,,\ \textit{no solution})$$

Use Gaussian elimination to solve the system.

$$\begin{array}{rcrcrcr} x_1 & + & 3x_2 & + & 3x_3 & = & 13 \\ 2x_1 & + & 5x_2 & + & 4x_3 & = & 23 \\ 2x_1 & + & 7x_2 & + & 8x_3 & = & 29 \end{array}$$

$(x_1 = 4+3t\,,\ x_2 = 3-2t\,,\ x_3 = t)$

$$\begin{array}{rcrcrcrcr} 3x_1 & - & 6x_2 & + & x_3 & + & 13x_4 & = & 15 \\ 3x_1 & - & 6x_2 & + & 3x_3 & + & 21x_4 & = & 21 \\ 2x_1 & - & 4x_2 & + & 5x_3 & + & 26x_4 & = & 23 \end{array}$$

$(x_1 = 4+2s-3t\,,\ x_2 = s\,,\ x_3 = 3-4t\,,\ x_4 = t)$

Use Gaussian elimination to solve the system.

$$\begin{array}{rcrcrcrcr} x_1 & + & x_2 & + & x_3 & & & = & 6 \\ 2x_1 & - & 2x_2 & - & 5x_3 & & & = & -13 \\ 3x_1 & & & + & x_3 & + & x_4 & = & 13 \\ 4x_1 & - & 2x_2 & - & 3x_3 & + & x_4 & = & 1 \end{array}$$

$(x_1 = 2\,,\ x_2 = 1\,,\ x_3 = 3\,,\ x_4 = 4)$

$$\begin{array}{rcrcrcrcr} x_1 & - & 4x_2 & - & 3x_3 & - & 3x_4 & = & 4 \\ 2x_1 & - & 6x_2 & - & 5x_3 & - & 5x_4 & = & 5 \\ 3x_1 & - & x_2 & - & 4x_3 & - & 5x_4 & = & -7 \end{array}$$

$(x_1 = 3 - 2t\,,\ x_2 = -4 + t\,,\ x_3 = 5 - 3t\,,\ x_4 = t)$

Use Gaussian elimination to solve the system.

$$\begin{array}{rcrcrcrcrcr} 2x_1 & + & 4x_2 & - & x_3 & - & 2x_4 & + & 2x_5 & = & 6 \\ x_1 & + & 3x_2 & + & 2x_3 & - & 7x_4 & + & 3x_5 & = & 9 \\ 5x_1 & + & 8x_2 & - & 7x_3 & + & 6x_4 & + & x_5 & = & 4 \end{array}$$

$(x_1 = 2+3t\,,\ x_2 = 1+s-2t\,,\ x_3 = 2+2s\,,\ x_4 = s\,,\ x_5 = t)$

$$\begin{array}{rcrcrcr} 3x_1 & + & x_2 & - & 3x_3 & = & -4 \\ x_1 & + & x_2 & + & x_3 & = & 1 \\ 5x_1 & + & 6x_2 & + & 8x_3 & = & 8 \end{array}$$

$(inconsistent\,,\ no\,solution)$

Use elementary row operations to transform each augmented coefficient matrix to reduced echelon form. Then solve the system by back substitution.

$$\begin{array}{rcrcrcr} x & + & 2y & - & z & = & 1 \\ 2x & + & y & + & 4z & = & 2 \\ 3x & + & 3y & + & 4z & = & 1 \end{array}$$

$(x = 7\,,\ y = -4\,,\ z = -2)$

$$\begin{array}{rcrcrcrcr} x_1 & - & x_2 & + & x_3 & - & x_4 & = & 2 \\ x_1 & - & x_2 & + & x_3 & + & x_4 & = & 0 \\ 4x_1 & - & 4x_2 & + & 4x_3 & & & = & 4 \\ -2x_1 & + & 2x_2 & - & 2x_3 & + & x_4 & = & -3 \end{array}$$

$(x_1 = 1 + t - s\,,\ x_2 = t\,,\ x_3 = s\,,\ x_4 = -1)$

Use elementary row operations to transform each augmented coefficient matrix to reduced echelon form. Then solve the system by back substitution.

$$\begin{array}{rcrcrcr} 3x & + & 9y & + & z & = & 16 \\ 2x & + & 6y & + & 7z & = & 17 \\ x & + & 3y & - & 6z & = & -1 \end{array} \qquad (x = 5 - 3t\,,\ y = t\,,\ z = 1)$$

$$\begin{array}{rcrcrcrcr} x_1 & + & 3x_2 & + & 15x_3 & & 7x_4 & = & 1 \\ 2x_1 & + & 4x_2 & + & 22x_3 & + & 8x_4 & = & 2 \\ 2x_1 & + & 7x_2 & + & 34x_3 & + & 17x_4 & = & 3 \end{array} \qquad (\textit{inconsistent})$$

Determine for what values of k each system has a unique solution, no solution or infinitely many solutions.

$$\begin{array}{rcrcl} 3x & + & 2y & = & 11 \\ 6x & + & ky & = & 21 \end{array}$$

$(k \neq 4\,,\ k = 4\,,\ none)$

$$\begin{array}{rcrcl} 3x & + & 2y & = & 1 \\ 7x & + & 5y & = & k \end{array}$$

$(all\, k\,,\ none\,,\ none)$

Determine for what values of k each system has a unique solution, no solution or infinitely many solutions.

$$\begin{array}{rcrcrcl} x & + & 2y & + & z & = & 3 \\ 2x & - & y & - & 3z & = & 5 \\ 4x & + & 3y & - & z & = & k \end{array} \qquad (none\,,\ k \neq 11\,,\ k = 11)$$

$$\begin{array}{rcrcrcl} x & + & 2y & + & 2z & = & 4 \\ 2x & 4 & ky & + & 4z & = & 8 \\ & & & & kz & = & 4 \end{array} \qquad (k \neq 0 \ and\ k \neq 4\,,\ k = 0\,,\ k = 4)$$

Determine for what values of α each system has a unique solution, no solution or infinitely many solutions. In the case of infinitely many solutions, find the solution set.

$$\begin{array}{rcrcrcl} x & + & y & - & z & = & 1 \\ 2x & + & 3y & + & \alpha z & = & 3 \\ x & + & \alpha y & + & 3z & = & 2 \end{array} \qquad (\alpha \neq 2 \, and \, \alpha \neq -3 \, , \, \alpha = -3 \, , \, \alpha = 2 \, , \, (5t, 1-4t, t))$$

$$\begin{array}{rcrcrcr} x & + & 4y & - & 7z & = & 8 \\ -x & - & 3y & + & 5z & = & -6 \\ 2x & + & 5y & + & (\alpha^2 - 17)z & = & \alpha + 7 \end{array} \qquad (\alpha \neq 3 \, and \, \alpha \neq -3 \, , \, \alpha = -3 \, , \, \alpha = 3 \, , \, (-t, 2+2t, t))$$

Determine for what values of α each system has a unique solution, no solution or infinitely many solutions. In the case of infinitely many solutions, find the solution set.

$$\begin{array}{rcrcrcr} x & - & 2y & + & 3z & = & -2 \\ -x & + & 3y & - & (\alpha+4)z & = & \alpha+2 \\ 2x & - & 3y & + & (\alpha^2-\alpha-11)z & = & 3\alpha+4 \end{array} \quad (\alpha \neq \mp 4\,,\ \alpha = 4\,,\ \alpha = -4\,,\ (-10-9t, -4-3t, t))$$

$$\begin{array}{rcrcrcr} x & + & \alpha y & + & 3z & = & 5 \\ 2x & + & (\alpha+2)y & + & 5z & = & 7 \\ -x & + & (\alpha-4)y & + & (\alpha^2-5)z & = & \alpha-1 \end{array} \qquad (\alpha \neq \mp 2\,,\ \alpha = 2\,,\ \alpha = -2\,,\ (-14-2t, t, 3))$$

Determine for what values of λ and β each system has a unique solution, no solution or infinitely many solutions. In the case of unique and infinitely many solutions, find the solution sets.

$$\begin{array}{rcrcrcr} x & & & - & \beta z & = & 1 \\ -2x & + & \lambda y & - & \beta z & = & -1 \\ -x & + & \lambda y & & \beta z & = & 0 \end{array}$$

$(\lambda \neq 0 \, and \, \beta \neq 0\,,\ \lambda = 0\,,\ \lambda \neq 0 \, and \, \beta = 0\,,\ (1, 1/\lambda, 0)\,,\ (1, 1/\lambda, t))$

$$\begin{array}{rcrcrcr} x & + & 2y & + & 5z & = & 6 \\ & & y & + & 2z & = & 2 \\ -2x & + & 2y & + & (\beta-1)z & = & 0 \\ 2x & + & 2y & + & 6z & = & \lambda+2 \end{array}$$

$(\lambda = 6 \, and \, \beta \neq 3\,,\ \lambda \neq 6\,,\ \lambda = 6 \, and \, \beta = 3\,,\ (2, 2, 0)\,,\ (2-t, 2-2t, t))$

Determine for what values of λ and β each system has a unique solution, no solution or infinitely many solutions. In the case of infinitely many solutions, find the solution set.

$$\begin{pmatrix} 2 & -1 & 2\lambda & 1 \\ 2 & 0 & 2\lambda & 1 \\ 2 & -1 & (2\lambda+1) & (\lambda+1) \\ -2 & 1 & (1-2\lambda) & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \beta \\ 1 \\ 0 \\ -\beta-2 \end{pmatrix}$$

$(\lambda \neq -1\,,\ \lambda = -1\, and\, \beta \neq 2\,,\ \lambda = -1\, and\, \beta = 2\,,\ ((t-3)/2, -1, t-2, t))$

$$\begin{pmatrix} 1 & 0 & 1 & -1 \\ -2 & -1 & -4 & 2 \\ -1 & -1 & -3 & \lambda-1 \\ -1 & 1 & 2\lambda-3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 1 \\ \beta-2 \end{pmatrix}$$

$(\lambda \neq 2\,,\ \lambda = 2\, and\, \beta \neq 5\,,\ \lambda = 2\, and\, \beta = 5\,,\ (-2-r+s, 1-2r, r, s))$

Determine for what values of λ and β each system has a unique solution, no solution or infinitely many solutions. In the case of infinitely many solutions, find the solution set.

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & 2x_3 & - & x_4 & = & 1 \\ -3x_1 & + & 6x_2 & - & x_3 & - & 7x_4 & = & 2 \\ 2x_1 & - & 4x_2 & + & \beta x_3 & - & 2x_4 & = & 2 \\ -x_1 & + & 2x_2 & + & 3x_3 & + & (\lambda-7)x_4 & = & 2\beta-4 \end{array}$$

$(none\,,\ \lambda=-2\,and\,\beta\neq 4\,or\,4\beta+\lambda\neq 14\,for\,\lambda\neq -2\,and\,\beta\neq 4\,,\ \lambda=-2\,and\,\beta=4$
$(2+s-t,-3t,s,1,t))$

$$\begin{array}{rcrcrcrcrcr} x_1 & & & - & x_3 & & & + & x_5 & = & 2 \\ x_1 & + & x_2 & - & x_3 & & & + & 4x_5 & = & \beta+4 \\ & & x_2 & & & + & (2\lambda-2)x_4 & + & 3x_5 & = & 2\lambda-2 \\ -x_1 & & & + & x_3 & + & (\lambda-1)x_4 & - & x_5 & = & \lambda-3 \end{array}$$

$(none\,,\ \beta\neq -2\,,\ \beta=-2\,,\ (2+s-t,-3t,s,1,t))$

Compute the matrix $cA + dB$.

$$A = \begin{pmatrix} 2 & 0 & -3 \\ -1 & 5 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 3 & 1 \\ 7 & 1 & 5 \end{pmatrix}, \quad c = 5, \quad d = -3 \qquad \left(\begin{pmatrix} 16 & -9 & -18 \\ -26 & 22 & 15 \end{pmatrix}\right)$$

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 4 & 0 & -3 \\ 5 & -2 & 7 \end{pmatrix}, \quad B = \begin{pmatrix} 6 & -3 & -4 \\ 5 & 2 & -1 \\ 0 & 7 & 9 \end{pmatrix}, \quad c = 7, \quad d = 5 \qquad \left(\begin{pmatrix} 44 & -22 & -20 \\ 53 & 10 & -26 \\ 35 & 21 & 94 \end{pmatrix}\right)$$

Compute the matries AB and BA if defined.

$A = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -4 & 2 \\ 1 & 3 \end{pmatrix}$ $\left(\begin{pmatrix} -9 & 1 \\ -10 & 12 \end{pmatrix}, \begin{pmatrix} -2 & 8 \\ 11 & 5 \end{pmatrix} \right)$

$\begin{bmatrix} -9 & 1 \\ -10 & 12 \end{bmatrix}, \begin{bmatrix} -2 & 8 \\ 11 & 5 \end{bmatrix}$

$A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 4 \\ 3 & -2 & 5 \end{pmatrix}$ $\left(\begin{pmatrix} 1 & -2 & 13 \\ 5 & -6 & 31 \end{pmatrix}, \text{not defined} \right)$

$\begin{bmatrix} 1 & -2 & 13 \\ 5 & -6 & 31 \end{bmatrix}$, Not defined

Compute the matries AB and BA if defined.

$A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$ $\left(\begin{pmatrix} 26 \end{pmatrix}, \begin{pmatrix} 3 & 6 & 9 \\ 4 & 8 & 12 \\ 5 & 10 & 15 \end{pmatrix} \right)$

$A = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & -2 \\ 3 & 1 \\ -4 & 5 \end{pmatrix}$ $\left(\text{not defined}, \begin{pmatrix} 4 \\ 7 \\ -22 \end{pmatrix} \right)$

Verify both sides of the associative law.

$$A = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 0 \\ 0 & 3 \\ 1 & 4 \end{pmatrix} \qquad \left(\begin{pmatrix} 12 & 15 \\ 8 & 10 \end{pmatrix}\right)$$

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \\ 1 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 \\ 3 & -2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & -1 & 2 \\ 3 & 2 & 0 & 1 \end{pmatrix} \qquad \left(\begin{pmatrix} -4 & -4 & -2 & 2 \\ -9 & -12 & -9 & 12 \\ -14 & -18 & -13 & 17 \end{pmatrix}\right)$$

$$B.C = \begin{bmatrix} -2 & -2 & -1 & 1 \\ -3 & -4 & -3 & 4 \end{bmatrix}$$

$$A.(B.C) = \begin{bmatrix} -4 & -4 & -2 & 2 \\ -9 & -12 & -9 & 12 \\ -14 & -18 & -13 & 17 \end{bmatrix}$$

Write each given homogeneous system in the matrix form $Ax = 0$. Then find the solution in vector form.

$$\begin{array}{rcrcrcl} x_1 & & & - & 5x_3 & + & 4x_4 & = & 0 \\ & & x_2 & + & 2x_3 & - & 7x_4 & = & 0 \end{array}$$

$(x = s(5, -2, 1, 0) + t(-4, 7, 0, 1))$

$$A = \begin{bmatrix} 1 & 0 & -5 & 4 \\ 0 & 1 & 2 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 5s-4t \\ -2s+7t \\ s \\ t \end{bmatrix} = \begin{bmatrix} 5s \\ -2s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} -4t \\ 7t \\ 0 \\ t \end{bmatrix}$$

$$= s(5,-2,1,0) + t(-4,7,0,1)$$

$$\begin{array}{rcrcrcrcl} x_1 & & & & & + & 3x_4 & - & x_5 & = & 0 \\ & & x_2 & & & - & 2x_4 & + & 6x_5 & = & 0 \\ & & & & x_3 & + & x_4 & - & 8x_5 & = & 0 \end{array}$$

$(x = s(-3, 2, -1, 1, 0) + t(1, -6, 8, 0, 1))$

Write each given homogeneous system in the matrix form $Ax = 0$. Then find the solution in vector form.

$$\begin{array}{rcrcrcrcl} x_1 & - & 3x_2 & & & + & 6x_4 & = & 0 \\ & & & & x_3 & + & 9x_4 & = & 0 \end{array}$$

$(x = s(3, 1, 0, 0) + t(-6, 0, -9, 1))$

$$\begin{array}{rcrcrcrcrcl} x_1 & - & x_2 & & & + & 7x_4 & + & 3x_5 & = & 0 \\ & & & & x_3 & - & x_4 & - & 2x_5 & = & 0 \end{array}$$

$(x = r(1, 1, 0, 0, 0) + s(-7, 0, 1, 1, 0) + t(-3, 0, 2, 0, 1))$

Write all 2×2 elementary matrices.

Let $B = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 3 \\ 5 & 9 & -6 \end{pmatrix}$. Find elementary matrix E so that $EB = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 3 \\ 3 & 5 & 0 \end{pmatrix}$.

$$\left(\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}\right)$$

Let $A=\begin{pmatrix} 2 & 4 & -2 \\ 4 & 5 & 1 \\ 0 & 3 & 7 \end{pmatrix}$, $B_1=\begin{pmatrix} 0 & 3 & 7 \\ 4 & 5 & 1 \\ 2 & 4 & -2 \end{pmatrix}$, $B_2=\begin{pmatrix} 1 & 2 & -1 \\ 4 & 5 & 1 \\ 0 & 3 & 7 \end{pmatrix}$, $B_3=\begin{pmatrix} 2 & 4 & -2 \\ 0 & -3 & 5 \\ 0 & 3 & 7 \end{pmatrix}$.

Find elementary matrices E_1, E_2 and E_3 such that $E_1A=B_1$, $E_2A=B_2$ and $E_3A=B_3$.

Determine the inverses of E_1, E_2 and E_3.

$$\left(\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right)$$

Find a 2×2 matrix A with each main diagonal element zero such that $A^2 = I$.

Find a 2×2 matrix A with each main diagonal element zero such that $A^2 = -I$.

Show that there is no 3×3 matrix A such that $A^3 = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ and $A^7 = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

Find A^{-1} of each given matrix A.

$$\begin{pmatrix} 5 & 7 \\ 4 & 6 \end{pmatrix}$$

$$\left(\frac{1}{2}\begin{pmatrix} 6 & -7 \\ -4 & 5 \end{pmatrix}\right)$$

$$\begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}$$

$$\left(\frac{1}{14}\begin{pmatrix} 4 & 2 \\ -1 & 3 \end{pmatrix}\right)$$

$$\begin{pmatrix} 1 & 5 & 1 \\ 2 & 5 & 0 \\ 2 & 7 & 1 \end{pmatrix}$$

$$\left(\begin{pmatrix} -5 & -2 & 5 \\ 2 & 1 & -2 \\ -4 & -3 & 5 \end{pmatrix}\right)$$

Find A^{-1} of each given matrix A.

$$\begin{pmatrix} 1 & -3 & 0 \\ -1 & 2 & -1 \\ 0 & -2 & 2 \end{pmatrix}$$

$$\left(\frac{1}{4}\begin{pmatrix} -2 & -6 & -3 \\ -2 & -2 & -1 \\ -2 & -2 & 1 \end{pmatrix}\right)$$

$$\begin{pmatrix} 2 & 0 & -1 \\ 1 & 0 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\left(\frac{1}{7}\begin{pmatrix} 3 & 1 & 0 \\ -2 & -3 & 7 \\ -1 & 2 & 0 \end{pmatrix}\right)$$

Calculate A^{-1} and then find a matrix X such that $AX = B$.

$A = \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 & -5 \\ -1 & -2 & 5 \end{pmatrix}$ $\quad \left(A^{-1} = \begin{pmatrix} 4 & -3 \\ -5 & 4 \end{pmatrix}, X = \begin{pmatrix} 7 & 18 & -35 \\ -9 & -23 & 45 \end{pmatrix}\right)$

$A = \begin{pmatrix} 1 & 4 & 1 \\ 2 & 8 & 3 \\ 2 & 7 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 2 & 2 \\ -1 & 1 & 0 \end{pmatrix}$ $\quad \left(A^{-1} = \begin{pmatrix} 11 & -9 & 4 \\ -2 & 2 & -1 \\ -2 & 1 & 0 \end{pmatrix}, X = \begin{pmatrix} 7 & -14 & 15 \\ -1 & 3 & -2 \\ -2 & 2 & -4 \end{pmatrix}\right)$

Calculate A^{-1} and then find a matrix X such that $AX = B$.

$A = \begin{pmatrix} 7 & 6 \\ 8 & 7 \end{pmatrix}, B = \begin{pmatrix} 2 & 0 & 4 \\ 0 & 5 & -3 \end{pmatrix}$ $\left(A^{-1} = \begin{pmatrix} 7 & -6 \\ -8 & 7 \end{pmatrix}, X = \begin{pmatrix} 14 & -30 & 46 \\ -16 & 35 & -53 \end{pmatrix}\right)$

$A = \begin{pmatrix} 1 & 5 & 1 \\ 2 & 1 & -2 \\ 1 & 7 & 2 \end{pmatrix}, B = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ $\left(A^{-1} = \begin{pmatrix} -16 & 3 & 11 \\ 6 & -1 & -4 \\ -13 & 2 & 9 \end{pmatrix}, X = \begin{pmatrix} -21 & 9 & 6 \\ 8 & -3 & -2 \\ -17 & 6 & 5 \end{pmatrix}\right)$

Find 2×2 matrix A such that $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} A \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. $\left(\begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix}\right)$

Let $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 0 & -2 \end{pmatrix}$. Find the matrix X such that $XB = (AB)^{-1}$.

$\left(\begin{pmatrix} 2 & 1 \\ 4 & 10 \end{pmatrix}\right)$

Let $A = \begin{pmatrix} 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$. Solve the matrix equation $AB^T X = I$ for the 2x2 matrix X. $\left(\begin{pmatrix} 1/10 & 1/5 \\ -1/10 & 3/10 \end{pmatrix}\right)$

Let $AC^{-1} = \begin{pmatrix} 4 & 1 \\ 7 & 2 \end{pmatrix}$, $B^T = \begin{pmatrix} 3 & 7 \\ 1 & 2 \end{pmatrix}$ and A is a nonsingular matrix. Find the matrix X if $XA + BC = 0$. $\left(\begin{pmatrix} 1 & -1 \\ 0/5 & -1 \end{pmatrix} \right)$

Let $A = \begin{pmatrix} 1 & 1 \\ 3 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$. Find the matrix $X = (A^T - I_2)B^{-1}$. $\left(\begin{pmatrix} -3/10 & 6/5 \\ 3/5 & -7/5 \end{pmatrix} \right)$

Let $A^{-1} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$, $C^T = \begin{pmatrix} 0 & 1 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Find $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ if $(A^T C^{-1})^T X = B$. $\left(\begin{pmatrix} -6 \\ 7 \end{pmatrix} \right)$

Let $A = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 1 & 3 \\ 1 & 0 & 1 \end{bmatrix}$. Use elementary row operations to find A^{-1}.

$\left(\begin{bmatrix} 1/9 & -1/9 & 5/9 \\ 4/9 & 5/9 & -7/9 \\ -1/9 & 1/9 & 4/9 \end{bmatrix} \right)$

Let $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. Use elementary row operations to find A^{-1}. $\left(\begin{bmatrix} -7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \right)$

Let $A^{-1} = \begin{bmatrix} 11 & 4 & -9 \\ -2 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$. Use elementary row operations to find A. $\left(\begin{bmatrix} 1 & 4 & 1 \\ 2 & 7 & 4 \\ 2 & 8 & 3 \end{bmatrix}\right)$

Let $A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$. Use elementary row operations to find A. Use A to solve the linear system $A^{-1}x = b$ where $b = \begin{bmatrix} 4 \\ 7 \\ -2 \end{bmatrix}$. $\left(\begin{bmatrix} 3/2 & -1 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 3 \end{bmatrix}\right)$

Let $A = \begin{bmatrix} 3 & 4 & 1 \\ 1 & -2 & 0 \\ 5 & 3 & 6 \end{bmatrix}$. Find the determinant of A by cofactor expansion (-47)

along the second column.

along the second row.

along the first column.

along the first row.

along the third column.

along the third row.

Use cofactor expansions to evaluate the following determinants. Expand along the row or column that minimizes the amount of computation that is required.

$$\begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} \qquad (4)$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 5 & 0 \\ 3 & 6 & 9 & 8 \\ 4 & 0 & 10 & 7 \end{vmatrix} \qquad (-210)$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 4 \\ 3 & 1 & -1 & 0 & 2 \\ 0 & 1 & 3 & 0 & 1 \\ 2 & -1 & 1 & 0 & 5 \end{vmatrix} \qquad (240)$$

Evaluate the determinants after first simplifying the computation by adding an appropriate multiple of some row or column to another.

$$\begin{vmatrix} 2 & 3 & 4 \\ -2 & -3 & 1 \\ 3 & 2 & 7 \end{vmatrix} \tag{25}$$

$$\begin{vmatrix} 3 & -2 & 5 \\ 0 & 5 & 17 \\ 6 & -4 & 12 \end{vmatrix} \tag{30}$$

$$\begin{vmatrix} 1 & 0 & -1 & 3 \\ 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{vmatrix} \tag{11}$$

Use the method of elimination to evaluate the following determinants.

$$\begin{vmatrix} 2 & 3 & 3 & 1 \\ 0 & 4 & 3 & -3 \\ 2 & -1 & -1 & -3 \\ 0 & -4 & -3 & 2 \end{vmatrix} \qquad (8)$$

$$\begin{vmatrix} 1 & 4 & 4 & 1 \\ 0 & 1 & -2 & 2 \\ 3 & 3 & 1 & 4 \\ 0 & 1 & -3 & -2 \end{vmatrix} \qquad (135)$$

Evaluate the following determinants.

$$\begin{vmatrix} 0 & 0 & a_1 & b_1 \\ 0 & 0 & a_2 & b_2 \\ a_3 & b_3 & 0 & 0 \\ a_4 & b_4 & 0 & 0 \end{vmatrix}$$

$((a_2b_1 - a_1b_2)(a_4b_3 - a_3b_4))$

$$\begin{vmatrix} a_1 & 0 & 0 & b_1 \\ 0 & a_2 & b_2 & 0 \\ 0 & b_3 & a_3 & 0 \\ b_4 & 0 & 0 & a_4 \end{vmatrix}$$

$((a_1a_4 - b_1b_4)(a_2a_3 - b_2b_3))$

Use properties of determinants to show the following. Do not use the cofactor expansion.

$$\begin{vmatrix} -x^2 & xy & xz \\ xy & -y^2 & yz \\ xz & yz & -z^2 \end{vmatrix} = 4x^2y^2z^2.$$

$$\begin{vmatrix} a+d & a-d & g \\ b+e & b-e & h \\ c+f & c-f & k \end{vmatrix} = -2\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & k \end{vmatrix}.$$

Use properties of determinants to show the following. Do not use the cofactor expansion.

$$\begin{vmatrix} x & y+z & y^2+z^2 \\ y & x+z & x^2+z^2 \\ z & x+y & x^2+y^2 \end{vmatrix} = (x+y+z)(x-y)(x-z)(z-y).$$

$$\begin{vmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{vmatrix} = (a-b)^3(a+3b).$$

Let $A = \begin{bmatrix} 2 & 5 & 3 & 4 \\ -1 & -2 & -2 & -3 \\ 2 & 6 & 4 & 4 \\ 1 & 3 & 8 & 9 \end{bmatrix}$.

Evaluate $|A|$. (2)

Find $|A^T|$, $|A^{-1}|$, $|A^4|$ and $|2A|$. (2 , 1/2 , 16 , 32)

Let $A = \begin{bmatrix} 2 & 5 & 3 & 4 \\ -1 & -2 & -2 & -3 \\ 2 & 6 & 4 & 4 \\ 1 & 3 & 8 & 9 \end{bmatrix}$.

Evaluate $|A|$. (2)

Find $|A^T|$, $|A^{-1}|$, $|A^4|$ and $|2A|$. (2 , 1/2 , 16 , 32)

Let $A = \begin{bmatrix} 2 & 2 & 2 \\ 1 & x & x^2 \\ 1 & y & y^2 \end{bmatrix}$. Find $|A|$ and the values of x and y for which A is invertible.

$(2(x-1)(y-1)(y-x)\,,\ x \neq 1\,,\ y \neq 1\,,\ x \neq y)$

Show that the homogeneous system $Ax = 0$ has only the trivial solution for $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$.

$(|A| = 2 \neq 0)$

Use Cramer's rule to solve the following systems.

$$\begin{array}{rcrcrcr} 5x_1 & + & 2x_2 & - & 2x_3 & = & 1 \\ x_1 & + & 5x_2 & - & 3x_3 & = & -2 \\ 5x_1 & - & 3x_2 & + & 5x_3 & = & 2 \end{array}$$

$(x_1 = 1/3\,,\ x_2 = -2/3\,,\ x_3 = -1/3)$

$$\begin{array}{rcrcrcr} 3x_1 & - & x_2 & - & 5x_3 & = & 3 \\ 4x_1 & - & 4x_2 & - & 3x_3 & = & -4 \\ x_1 & & & - & 5x_3 & = & 2 \end{array}$$

$(x_1 = 2\,,\ x_2 = 3\,,\ x_3 = 0)$

Use Cramer's rule to solve the following systems.

$$\begin{array}{rcrcrcr} 2x_1 & + & x_2 & - & 3x_3 & = & 0 \\ 4x_1 & + & 5x_2 & + & x_3 & = & 8 \\ -2x_1 & - & x_2 & + & 4x_3 & = & 2 \end{array} \qquad (x_1 = 4\,,\ x_2 = -2\,,\ x_3 = 2)$$

$$\begin{array}{rcrcrcr} 3x_1 & + & 2x_2 & - & x_3 & = & 1 \\ x_1 & - & 5x_2 & + & 5x_3 & = & -2 \\ 2x_1 & + & x_2 & & & = & 3 \end{array} \qquad (x_1 = -2\,,\ x_2 = 7\,,\ x_3 = 7)$$

Use Cramer's rule to solve the following systems.

$$\begin{array}{rcrcrcr} 2x_1 & & & - & 5x_3 & = & -3 \\ 4x_1 & - & 5x_2 & + & 3x_3 & = & 3 \\ -2x_1 & + & x_2 & + & x_3 & = & 1 \end{array}$$

$(x_1 = -8/7\,,\ x_2 = -10/7\,,\ x_3 = 1/7)$

$$\begin{array}{rcrcrcr} 3x_1 & + & 4x_2 & - & 3x_3 & = & 5 \\ 3x_1 & - & 2x_2 & + & 4x_3 & = & 7 \\ 3x_1 & + & 2x_2 & - & x_3 & = & 3 \end{array}$$

$(x_1 = -7/3\,,\ x_2 = 9\,,\ x_3 = 8)$

Use Cramer's rule to solve the following systems.

$$\begin{array}{rcrcrcr} x_1 & & & + & x_3 & = & 2 \\ -x_1 & + & x_2 & + & x_3 & = & 1 \\ x_1 & + & 2x_2 & + & 3x_3 & = & 8 \end{array}$$

$(x_1 = 2\,,\ x_2 = 3\,,\ x_3 = 0)$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 2x_3 & = & 0 \\ 2x_1 & - & x_2 & + & x_3 & = & -3 \\ -x_1 & + & 3x_2 & - & 2x_3 & = & 3 \end{array}$$

$(x_1 = -2\,,\ x_2 = 3\,,\ x_3 = 4)$

Use Cramer's rule to find x_2.

$$\begin{array}{rcrcrcr} 2x_1 & & & + & x_3 & = & 0 \\ -x_1 & - & x_2 & + & x_3 & = & -7 \\ x_1 & - & 2x_2 & + & 3x_3 & = & 1 \end{array}$$

$(-19/2)$

Use Cramer's rule to find x_1.

$$\begin{array}{rcrcrcrcr} 2x_1 & + & x_2 & + & x_3 & + & x_4 & = & 1 \\ -2x_1 & + & x_2 & & & + & 3x_4 & = & 0 \\ x_1 & + & 3x_2 & & & & & = & 0 \end{array}$$

$(9/40)$

Use Cramer's rule to find y.

$$\begin{array}{rcrcrcrcr} x & + & 2y & & & + & w & = & 2 \\ 2x & - & 3y & & & + & 3w & = & 1 \\ -x & + & 4y & + & 4z & - & w & = & -1 \\ & & y & - & z & & & = & 0 \end{array} \qquad (y = 1/10)$$

$$\begin{array}{rcrcrcrcr} x & + & 2y & + & z & + & w & = & 8 \\ -x & - & y & + & 2z & & & = & 3 \\ 2x & + & 3y & & & & & = & 0 \\ -2x & + & y & - & 2z & - & w & = & 0 \end{array} \qquad (y = 2)$$

Let $A = \begin{bmatrix} 1 & 0 & 2 & -4 \\ 2 & 0 & 4 & -7 \\ 0 & 0 & 3 & -6 \\ 2 & -1 & -5 & -10 \end{bmatrix}$. Express A in the upper triangular form and then use this form to find $\det(A)$.

$\left(\begin{bmatrix} 1 & 0 & 2 & -4 \\ 0 & -1 & -9 & -2 \\ 0 & 0 & 3 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}, 3 \right)$

Find $|A^T|$, $|A^{-1}|$, $|A^3|$, $2|A|$ and $|2A|$. $(3\,,\,1/3\,,\,27\,,\,6\,,\,48)$

Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 4 \\ 7 \\ 10 \end{bmatrix}$ and $AX = B$. Use Cramer's rule to find x_3. (-5/3)

Let $A = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 7 & 4 \\ 2 & 8 & 3 \end{bmatrix}$, $b = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}^T$, $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$.

Show that A is an invertible matrix. $(|A| = -1 \neq 0)$

Find A^{-1} by using elementary row operations. $\left(\begin{bmatrix} 11 & 4 & -9 \\ -2 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}\right)$

For the linear equation system $Ax = b$, find x_2 using Cramer's rule. $(x_2 = 1)$

Let $A = \begin{bmatrix} 3 & 0 & 1 \\ -2 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix}$.

Find the adjoint matrix of A. $\left(\begin{bmatrix} -2 & 1 & -1 \\ -4 & -6 & -2 \\ -2 & -3 & 3 \end{bmatrix} \right)$

Find A^{-1}. $\begin{bmatrix} 1/4 & -1/8 & 1/8 \\ 1/2 & 3/4 & 1/4 \\ 1/4 & 3/8 & -3/8 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & -3 & 3 \\ 2 & 4 & 3 \\ 5 & -1 & 4 \end{bmatrix}$.

Find the adjoint matrix of A. $\left(\begin{bmatrix} 19 & 9 & -21 \\ 7 & -11 & 3 \\ -22 & -14 & 10 \end{bmatrix}\right)$

Show that $(\text{adj}A)A = \det(A)I_3$ for the given matrix A.

Find A^{-1} for the following matrices A by using the adjoint matrix.

$$A = \begin{bmatrix} -5 & -2 & 2 \\ 1 & 5 & -3 \\ 5 & -3 & 1 \end{bmatrix}$$ $$\left(\frac{1}{4}\begin{bmatrix} 4 & 4 & 4 \\ 16 & 15 & 13 \\ 28 & 25 & 23 \end{bmatrix}\right)$$

$$A = \begin{bmatrix} 2 & 4 & -3 \\ 2 & -3 & -1 \\ -5 & 0 & -3 \end{bmatrix}$$ $$\left(\frac{1}{107}\begin{bmatrix} 9 & 12 & -13 \\ 11 & -21 & -4 \\ -15 & -20 & -14 \end{bmatrix}\right)$$

Let $A = \begin{bmatrix} -1 & 1 & 1 \\ -1 & -1 & 0 \\ -2 & 0 & 0 \end{bmatrix}$.

Evaluate $\det(A)$ and $\det(A^2A^T)$. $(-2\,,-8)$

Use elementary row operations to find the inverse of A. $\left(\begin{bmatrix} 0 & 0 & -1/2 \\ 0 & -1 & 1/2 \\ 1 & 1 & 0 \end{bmatrix}\right)$

Use the inverse of A to find the adjoint matrix of A. $\left(\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & -1 \\ -2 & -2 & 0 \end{bmatrix}\right)$

Suppose that A, B, and C are invertible matrices of the same size. Show that the product ABC is invertible and that $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.

Let A be n $n \times n$ matrix such that $Ax = x$ for every n-vector x. Show that $A = I$.

Prove that if A is invertible and $AB = AC$, then $B = C$.

Prove that if C is a symmetric matrix, then C^2 is also a symmetric matrix.

Show that the 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is row equivalent to the 2×2 identity matrix provided that $ad - bc \neq 0$.

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then show that $A^2 = (traceA)A - (ad - bc)I$ where I is the 2×2 identity matrix.

Prove that if A and B are symmetric nxn matrices, then $AB + BA$ is also a symmetric matrix.

The square matrix A is called orthogonal provided that $A^T = A^{-1}$. Show that the determinant of such a matrix must be either 1 or -1.

The matrices A and B are said to be similar provided that $A = P^{-1}BP$ for some invertible matrix P. Show that if A and B are similar, then $|A| = |B|$.

Suppose that $AA^T = A^{-1}B$. Express $|B|$ in terms of $|A|$.

Let $A = [a_{ij}]$ be a 3×3 matrix. Show that $|A^T| = |A|$ by expanding $|A|$ along its first row and $|A^T|$ along its first column.

Suppose that $A^2 = A$. Prove that $|A| = 0$ or $|A| = 1$.

Suppose that $A^n = 0$ for some positive integer n. Prove that $|A| = 0$.

Let B be an nxn invertible matrix. If $\det(B) = b$, find $\det(B^{-1})$, $\det(B^T)$, $\det(B^3)$, $\det(2B)$ and $\det(B^T B)$.

$(1/b\,, b\,, b^3\,, 2^n b\,, b^2)$

Prove the following properties.

$$\begin{vmatrix} ka_{11} & a_{12} & a_{13} \\ ka_{21} & a_{22} & a_{23} \\ ka_{31} & a_{32} & a_{33} \end{vmatrix} = k \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{vmatrix} a_{11} + ka_{12} & a_{12} & a_{13} \\ a_{21} + ka_{22} & a_{22} & a_{23} \\ a_{31} + ka_{32} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Use matrix multiplication to show that if x_1 and x_2 are two solutions of the homogeneous system $Ax = 0$ and c_1 and c_2 are real numbers, then $c_1x_1 + c_2x_2$ is also a solution.

Use matrix multiplication to show that if x_0 is a solution of the homogeneous system $Ax = 0$ and x_1 is a solution of the nonhomogeneous system $Ax = b$, then $x_0 + x_1$ is also a solution of the nonhomogeneous system.

Use matrix multiplication to show that if x_1 and x_2 are solutions of the nonhomogeneous system $Ax = b$, then $x_1 - x_2$ is a solution of the homogeneous system $Ax = 0$.