Problem Session 1

1. 2012-2013 1. midterm 1.soru

a. A is a 4-bit and B is an 8-bit, **signed**, binary integer, which are given as follows:
$A=(8)_{16}$ and $B=(FA)_{16}$.

Perform necessary operations on **binary numbers** to compare the **absolute values** ($|A|$, $|B|$) of two numbers. Decide which number has the greater absolute value by interpreting the obtained binary result. Explain the operations and interpretation of the result.

b. Assume that the numbers A and B are **unsigned**, and answer the same question.

a.

A= 1000 Sign extension to obtain 8-bit A= 1111 1000

B= 1111 1010 (Because Hex F=1111, Hex A=1010)

Both numbers are negative (sign =1)

To obtain absolute values we apply 2's complement operations.

2's complement of A= 1111 1000 : 0000 0111 + 1 = 0000 1000 = $|A|$

2's complement of B= 1111 1010: 0000 0101 + 1 = 0000 0110 = $|B|$

To compare the absolute values we perform $|A|-|B| = |A|$ + 2's complement of $|B|$

$\|A\|$:	0000 1000
2's complement of $\|B\|$:	+ 1111 1010
	1 000 0010

Interpretation:

Absolute values are unsigned numbers. Therefore we investigate the carry (barrow).
Carry=1, that means no barrow. Consequently $|A| > |B|$

b.

A= 1000 Extension to obtain 8-bit A= 000 1000 (A is unsigned)

B= 1111 1010

As the numbers are unsigned A=$|A|$, and B= $|B|$

To compare the numbers we perform A- B = A+ 2's complement of B

A :	0000 1000
2's complement of B:	+ 0000 0110
	0000 1110

Interpretation:

Absolute values are unsigned numbers. Therefore we investigate the carry (barrow).
Carry=0, that means barrow. Consequently A< B and $|A| < |B|$

2. 2011-2012 1.midterm 1.soru

a. A and B are two 8-bit, **signed**, binary integers. B is given as B=1001 1101. If we perform the operation A-B according to **2's complement** method overflow occurs and the most significant bit of the 8-bit result is 1.

i) What is the sign of A (positive or negative)? Why?

ii) Write the smallest possible integer A that can constitute this situation (result and overflow).

b. A and B are two 8-bit, **unsigned**, binary integers. After the operation A-B according to **2's complement** method the obtained result is a 9-bit number: 1 1001 0110.

Which is true A>B or A<B? Why?

a. B is negative, result is negative, there is an overflow, and operation is subtraction

i) Overflow condition: pos – neg = neg , therefore A must be **positive**. [10 points]

ii) A = 0xxx xxxx   0xxx xxxx
    B= 1001 1101   2's comp. $\underline{+\ 0110\ 0011}$ smallest possible A= 0001 1101
    R= 1xxx xxxx   1xxx xxxx

[10 points]

The same solution by thinking in decimal:

B= $(-99)_{10}$, to generate an overflow result must be at least +128. (Note that result seems to be negative, but due to overflow the real sign of the result is positive.)

A-99=128, smallest possible **A=$(29)_{10}$ = 0001 1101**

b. The carry bit is 1. It means **no borrow**. Therefore A>B.   [10 points]

3. There is a logical function below.

f (a,b,c,d) = Σm (4,5,6,7,9,12,13,15)
Write the second canonical form of *f*, minimize it, and explain.

② $f(a,b,c,d) = \Sigma m(4,5,6,7,9,12,13,15) = \Pi M(0,1,2,3,8,10,11,14)$

a) $f = (a+b+c+d)(a+b+c+\bar{d})(a+b+\bar{c}+d)(a+b+\bar{c}+\bar{d})$
$(\bar{a}+b+c+d)(\bar{a}+b+\bar{c}+d)(\bar{a}+b+\bar{c}+\bar{d})(\bar{a}+\bar{b}+\bar{c}+d)$

$= (a+b+c)(a+b+\bar{c})(\bar{a}+b+d)(\bar{a}+b+\bar{c}+\bar{d})(\bar{a}+\bar{b}+\bar{c}+d)$

$= (a+b)(\bar{a}+b+d)(\bar{a}+b+\bar{c}+\bar{d})(\bar{a}+\bar{b}+\bar{c}+d)(\bar{a}+b+\bar{c})$ → konsensüs

$= (a+b)(\bar{a}+b+d)(\bar{a}+\bar{b}+\bar{c}+d)(\bar{a}+b+\bar{c})$

$= (a+b)(\bar{a}+b+d)(b+d)(\bar{a}+\bar{b}+\bar{c}+d)(\bar{a}+b+\bar{c})(\bar{a}+\bar{c}+d)$

$= (a+b)(b+d)(\bar{a}+b+\bar{c})(\bar{a}+\bar{c}+d)$

$= (a+b)(b+d)(\bar{a}+b+\bar{c})(b+\bar{c})(\bar{a}+\bar{c}+d)$

$= (a+b)(b+d)(b+\bar{c})(\bar{a}+\bar{c}+d)$

Sağlama:

c

	00	01	11	10
00	0	0	0	0
01				
11				0
10	0		0	0

b

$(a+b)(b+d)(b+\bar{c})(\bar{a}+\bar{c}+d)$

4. 2011-2012 Final 1.soru a.

a) Minimize the following function <u>using axioms and theorems</u>.
$f(A,B,C,D)=A'B'CD+AB'CD+AC'D+AC'D'+A'B'CD'+ABCD+ACD'$

$f(A,B,C,D)=A'B'CD+AB'CD+AC'D+AC'D'+A'B'CD'+ABCD+ACD'$

$=(A'+A)B'CD + AC'(D+D') + A'B'CD' + ABCD + ACD'$ (Inverse)

$=(B'+AB)CD + AC' + A'B'CD' + ACD'$ (absorbtion)

$= B'CD + A(CD + C') + (A'B'+A)\ CD'$ (absorbtion)

$= B'CD + AD + AC' + B'CD' + ACD'$

$= B'CD + AD + B'CD' + A(C'+CD')$ (absorbtion)

$= B'CD + AD + B'CD' + AC' + AD'$

$= B'C(D + D') + A(D+C'+D')$ (inverse)

$= B'C + A$