**BLG 231E - Digital Circuits**

**Assignment 1**

**Name Surname: Seyfülmülük Kutluk**

**Student ID: 150180073**

**CRN: 11623**

**Part 1 – Computer Arithmetic**

1. ***A*** and ***B*** are two 8-bit binary integers, and ***B*** =**1101 1001**. For the operation ***A - B***, answer the followingquestions:

**a.** If ***A*** and ***B*** are *signed* binary integers, what are the **i)** largest and **ii)** smallest decimal values of **A** that yield valid results (that can be represented using 8 bits) after the operation? Explain your answer briefly.

**Solution: a. i)**The signed integers are in between -128, 127 so the result of the A-B should be in between these values, if the value of **A-B** is not in between these values, the overflow will occur. Also, A should be between the range of -128 to 127. So to find the largest value of A that yield valid results, we have to find the value of A in the case of A-B is equal to 127;

B=11011001\_\_\_\_\_\_\_\_\_\_\_2’s complement\_\_\_\_> 00100111=(2^5\*1)+(2^2\*1)+(2^1\*1)+(2^0\*1)=39

So B= -39

-128 <= A- B < =127 ------------------ -128 <=A+39 <= 127 also -128 <= A <= 127

A-B=+127 --------------------> A = B + 127

**The largest decimal value of A is +88.**

**Solution: 1. a. ii)**To find the smallest value **A** that yield valid results, **A-B** should be smallest, but at the same time the A need to be between the range of -128 to 127.

-128 <= A- B < =127 ------------------ -128 <=A+39 <= 127 also -128 <= A <= 127

**A** can get the smallest value in range of -128:127 which is -128, because it also fits to -128< A- B < 127

**The smallest decimal value of A is -128.**

**b.** Write the binary representation for the largest value of the signed ***A*** you found in **(a.i)**. Carry out the binary operation ***A - B*** using **2’s complement**, and show that the result is valid using binary numbers only.

**Solution: 1. b.)**

A – B = A + B+ 1

B=11011001 B=00100110 B + 1 = 00100111

01011000 : +88 --> A 01011000 : +88 --> A

11011001 : -39 ---> B \_\_\_\_\_\_\_2’s complement 00100111 : 39--> B + 1

- +

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

01111111 : 127

**The result is valid**

**c.** If ***A*** and ***B*** are *unsigned* binary integers, what are the **i)** largest and **ii)** smallest binary values of **A** that yield valid results after the operation? Explain your answer briefly.

**Solution: 1. c.)**The unsigned binary numbers are in the range of 0 to 255, so the A and A-B need to be in range of 0 to 255.

B=11011001\_\_\_\_\_\_\_\_\_\_\_2’s complement\_\_\_\_> 00100111=(2^5\*1)+(2^2\*1)+(2^1\*1)+(2^0\*1)=39

So B= -39

0 <= A- B <= 255 ------------------ 0 <= A+39 <= 255 also 0 < =A < =255

A-B=A+39 A+39=255 A=216 in decimal A= 011011000

**The largest binary value of A is 11011000**

Smallest value of A is “0” since it yields the 0 <= A- B <= 255

A=0 in decimal A= 00000000

**The largest binary value of A is 00000000**

1. ***A*** and ***B*** are two 8-bit,**signed**, binary integers, and ***A* = 1011 1100**. If we perform the operation ***A + B*** ,

**a.** What are the **i)** largest and **ii)** smallest decimal values of ***B*** that yield valid results after the operation? Explain your answer briefly.

**Solution 2. a. i)B** and **A+B** should be in between -128 and 127.

A= 1011 1100\_\_\_\_\_\_\_\_\_\_\_2’s complement\_\_\_\_> 01000100=(2^6\*1)+(2^2\*1)=68

So A is -68

-128<= A+B <= 127 -128<= (-68)+B <= 127 also -128 <= B <= 127

Largest value of B is “127” since it yields the -128 <= (-68)+B <= 127

**The largest decimal value of B is 127**

**Solution 2. a. ii)B** and **A+B** should be in between -128 and 127

A= 1011 1100\_\_\_\_\_\_\_\_\_\_\_2’s complement\_\_\_\_> 01000100=(2^6\*1)+(2^2\*1)=68

So A is -68

-128<= A+B <= 127 -128<= (-68)+B <= 127 also -128 <= B <= 127

-68 + B = -128 B= -60

Smallest value of B is “-60” since it yields the -128<= (-68)+B <= 127

**The smallest decimal value of B is -60**

**b.** Write the binary representation for the smallest value of the signed ***B*** you found in **(a.ii)**. Perform the binary operation ***A + B***, and show that the result is valid using binary numbers only.

**Solution 2. b.)**

60 =00111100 1’s complement 11000011 :::: 2’complement 11000100

-60=11000100

B=11000100

10111100 : -68 --> A

11000100 : -60 ---> B

+

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**1**10000000 -128

the leftmost 1 is ignored the result is 10000000

**The result is valid**

**Part 2 – Boolean Algebra**

**3.** Simplify the following logic expressions using axioms, properties, and theorems of Boolean algebra.a.

**a.** E(a, b, c) = ab̅c + abc̅ + abc + a̅bc

**b.** E(a, b, c, d) = a̅bd̅ + bcd + abc̅ + ab̅d + bc̅d̅ + ad + a̅bc