In [ ]:
```
import numpy as np
from scipy.interpolate import RegularGridInterpolator
import matplotlib.pyplot as plt

# Define the function F(x, y)
def F(x, y):
    return 1.8 - np.exp(-0.1 * (2.5 * (x + 3)**2 + (y + 3)**2)) - 1.5 * n

x = np.arange(-8, 8, 0.1)
y = np.arange(-8, 8, 0.1)
X, Y = np.meshgrid(x, y)
# Flatten the grid for interpolation
points = np.column_stack((X.flatten(), Y.flatten()))
values = F(X, Y)
# Create RegularGridInterpolator
interpolator = RegularGridInterpolator((x, y), values, method='linear', b

# Stochastic Gradient Descent
learning_rate = [0.001, 0.01, 0.1, 1]
num_iterations = 100

# Initial guess
gradient_points = np.array([-2., -3.])

# Lists to store the trajectory for visualization
grad_x = [gradient_points[0]]
grad_y = [gradient_points[1]]
trajectory_z = [F(*gradient_points)]
initial_points = np.array([[-2, -3], [2, 3], [2, 4]], dtype=float)  # Ens

# Lists to store the trajectory of initial points for visualization
initial_x_traj = [initial_points[:, 0].copy()]
initial_y_traj = [initial_points[:, 1].copy()]
for l_rate in learning_rate:
    for i in range(num_iterations):
        # Generate random indices for the batch
        indices = np.random.choice(len(points), size=10, replace=False)
        batch_points = points[indices]

        # Calculate gradient using RegularGridInterpolator
        gradient = interpolator(np.array([batch_points[:, 0], batch_point
        gradient_points -= l_rate * gradient.mean(axis=0)

        # Store the trajectory for visualization
        grad_x.append(gradient_points[0])
        grad_y.append(gradient_points[1])
        trajectory_z.append(F(*gradient_points))

        # Update the initial points trajectory
        initial_points[:, 0] -= l_rate * gradient[0]  # Update each point
        initial_points[:, 1] -= l_rate * gradient[1]
```

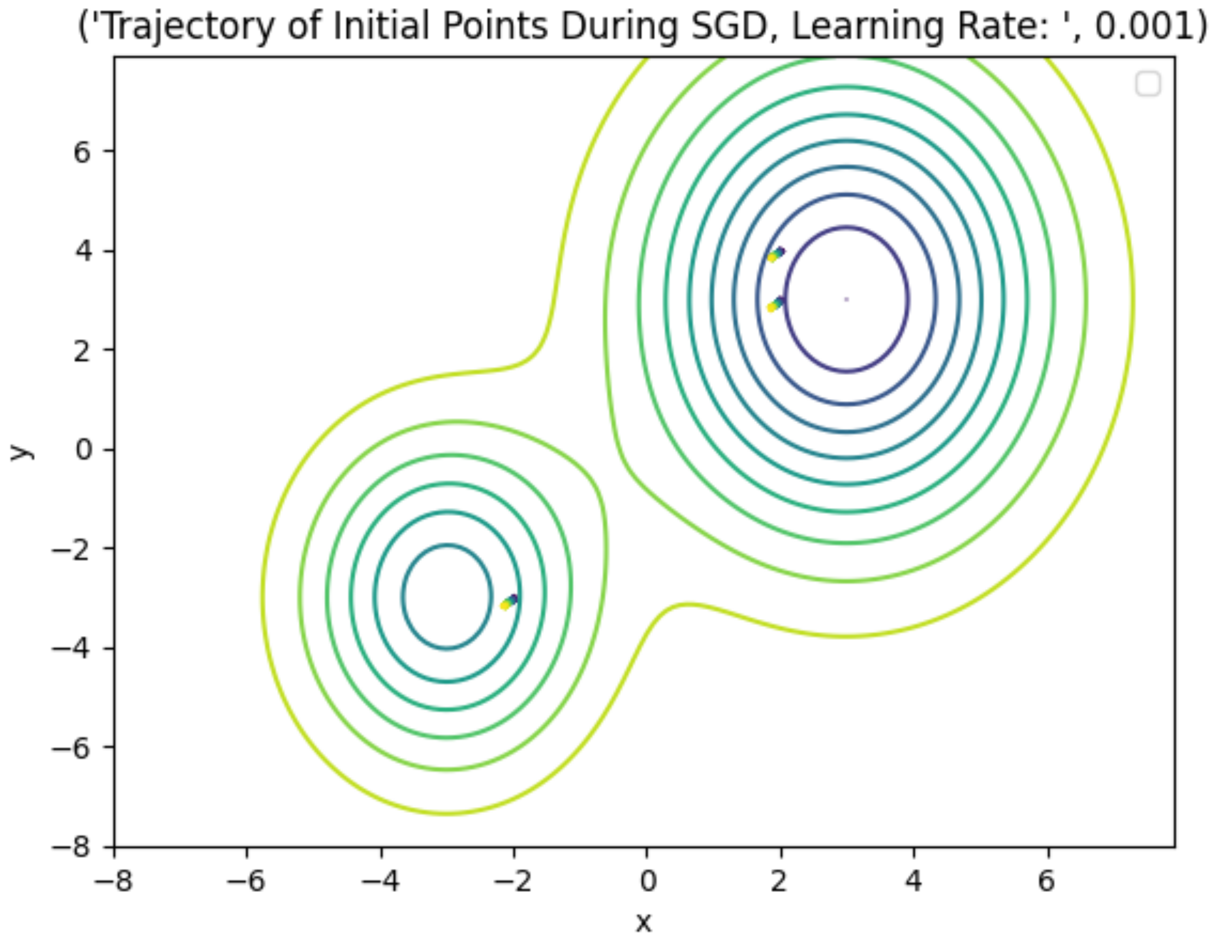
```
        initial_x_traj.append(initial_points[:, 0].copy())
        initial_y_traj.append(initial_points[:, 1].copy())

    # Plot the contour of the function
    plt.contour(X, Y, F(X, Y), levels=10, cmap='viridis')

    # Plot the trajectory of initial points
    x= 0
    for i in range(x,num_iterations):
        plt.scatter(initial_x_traj[i], initial_y_traj[i], marker='x', col
    x+=100
    num_iterations += 100


    plt.title(('Trajectory of Initial Points During SGD, Learning Rate: '
    plt.xlabel('x')
    plt.ylabel('y')
    plt.legend()
    plt.show()
```

```
No artists with labels found to put in legend.  Note that artists whose la
bel start with an underscore are ignored when legend() is called with no a
rgument.
```




```
No artists with labels found to put in legend.  Note that artists whose la
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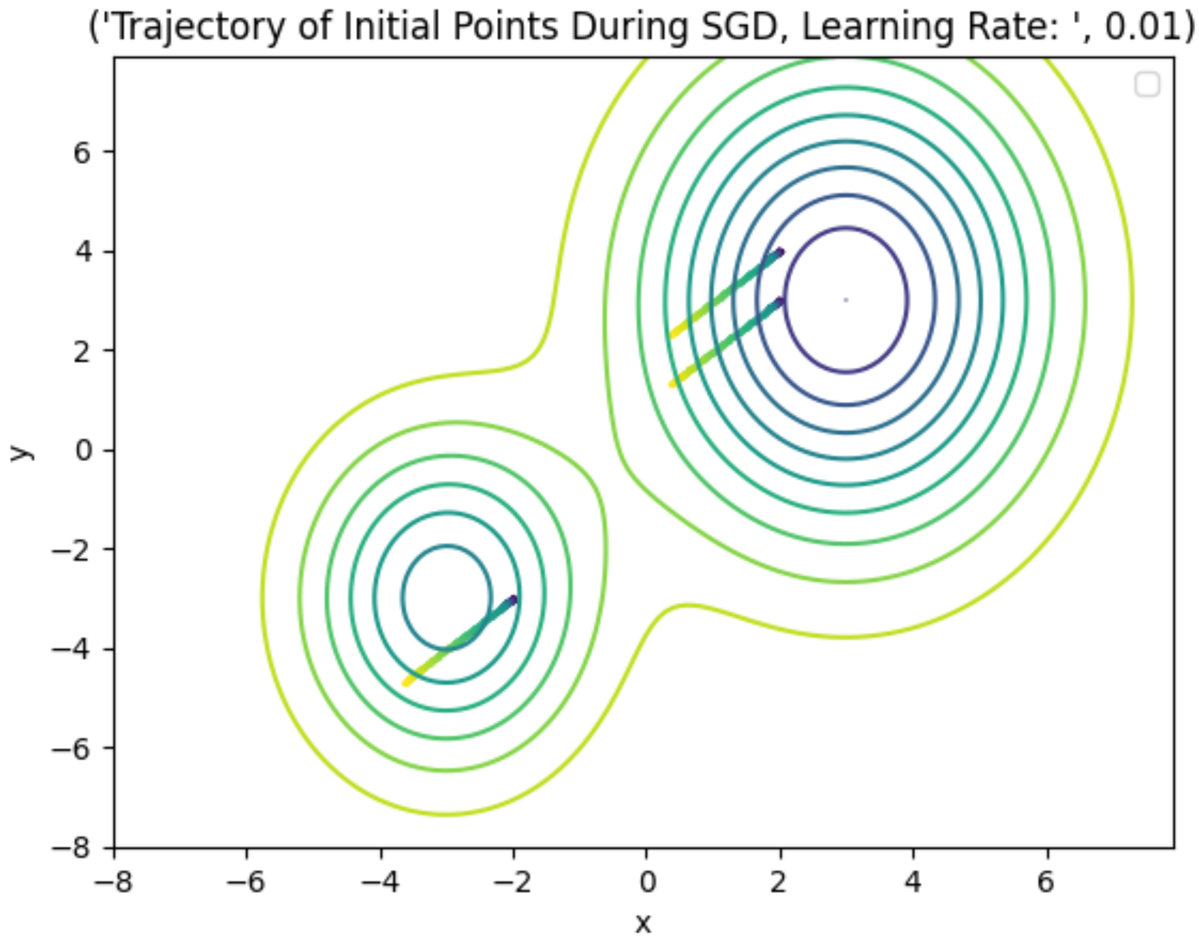


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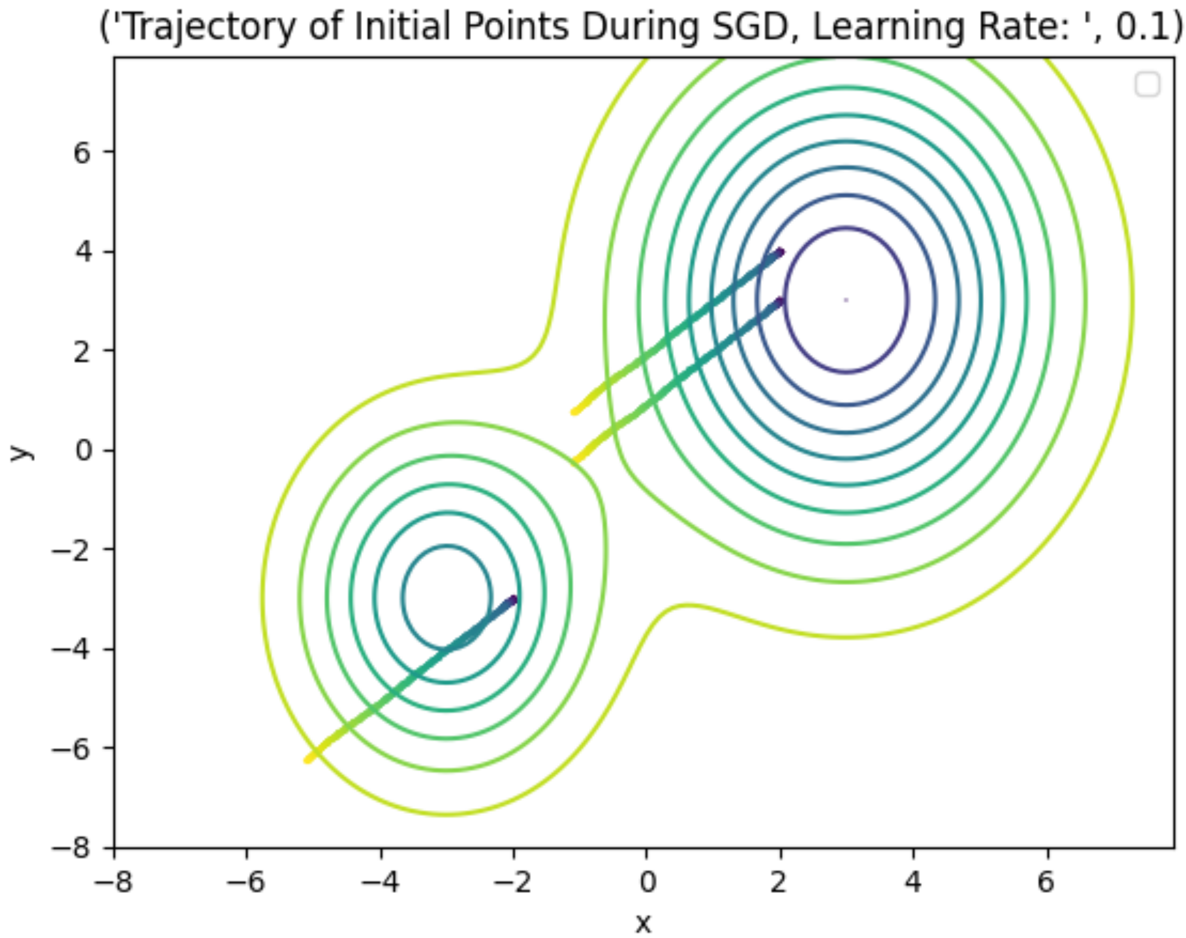


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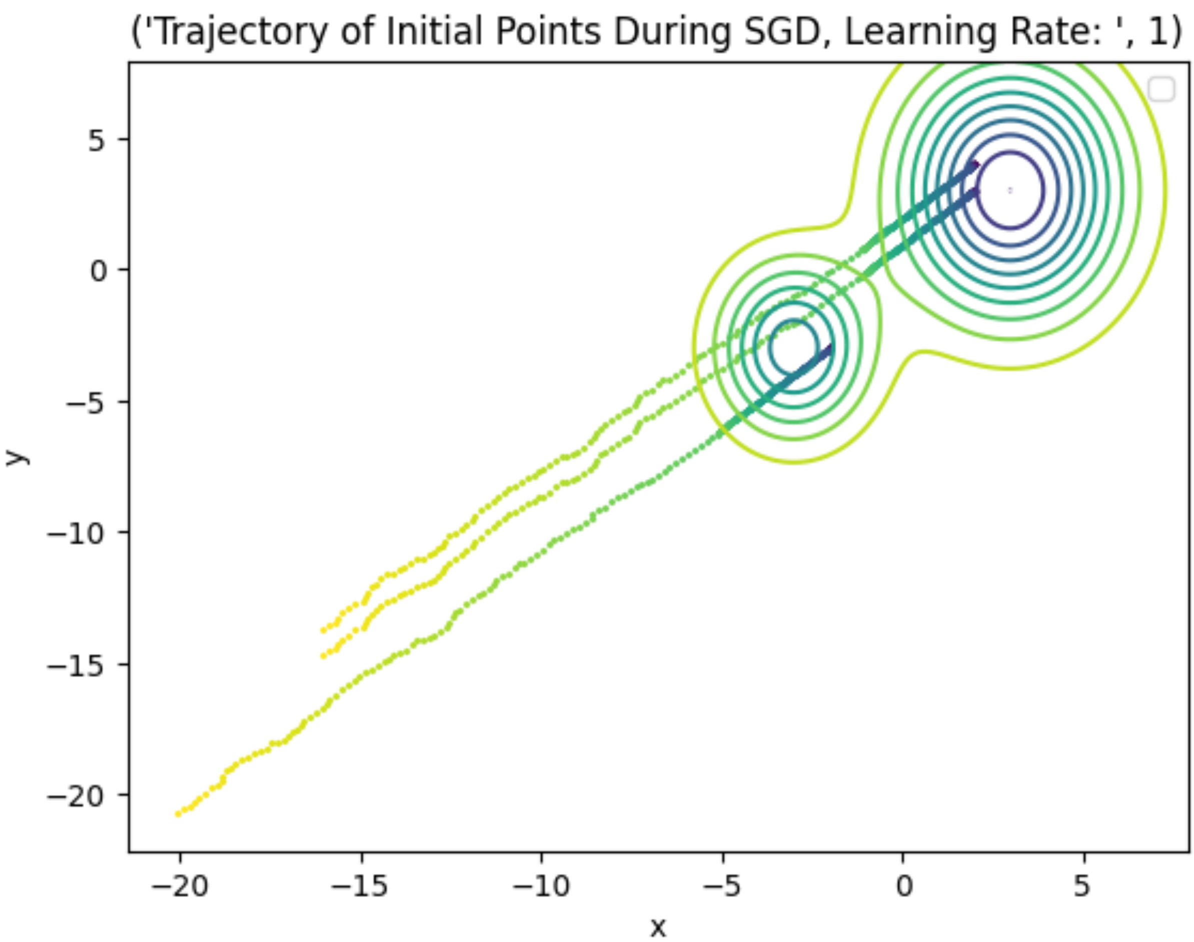
```



In [ ]:
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from scipy.interpolate import RegularGridInterpolator
import matplotlib.pyplot as plt

# Define the function F(x, y)
def F(x, y):
    return 1.8 - np.exp(-0.1 * (2.5 * (x + 3)**2 + (y + 3)**2)) - 1.5 * n

x = np.arange(-8, 8, 0.1)
y = np.arange(-8, 8, 0.1)
X, Y = np.meshgrid(x, y)
# Flatten the grid for interpolation
points = np.column_stack((X.flatten(), Y.flatten()))
values = F(X, Y)
# Create RegularGridInterpolator
interpolator = RegularGridInterpolator((x, y), values, method='linear', b

# Stochastic Gradient Descent
learning_rate = 0.2
num_iterations = 20

# Initial guess
gradient_points = np.array([-2., -3.])

# Lists to store the trajectory for visualization
grad_x = [gradient_points[0]]
```

```
grad_y = [gradient_points[1]]
trajectory_z = [F(*gradient_points)]
initial_points = np.array([[-2, -3], [2, 3], [2, 4]], dtype=float)  # Ens

# Lists to store the trajectory of initial points for visualization
initial_x_traj = [initial_points[:, 0].copy()]
initial_y_traj = [initial_points[:, 1].copy()]

# Plot the contour of the function
plt.contour(X, Y, F(X, Y), levels=10, cmap='viridis')

# Plot the initial points with red stars
plt.scatter(initial_x_traj[0], initial_y_traj[0], marker='*', color='red'

for i in range(num_iterations):
    # Generate random indices for the batch
    indices = np.random.choice(len(points), size=10, replace=False)
    batch_points = points[indices]

    # Calculate gradient using RegularGridInterpolator
    gradient = interpolator(np.array([batch_points[:, 0], batch_points[:,
    gradient_points -= learning_rate * gradient.mean(axis=0)

    # Store the trajectory for visualization
    grad_x.append(gradient_points[0])
    grad_y.append(gradient_points[1])
    trajectory_z.append(F(*gradient_points))

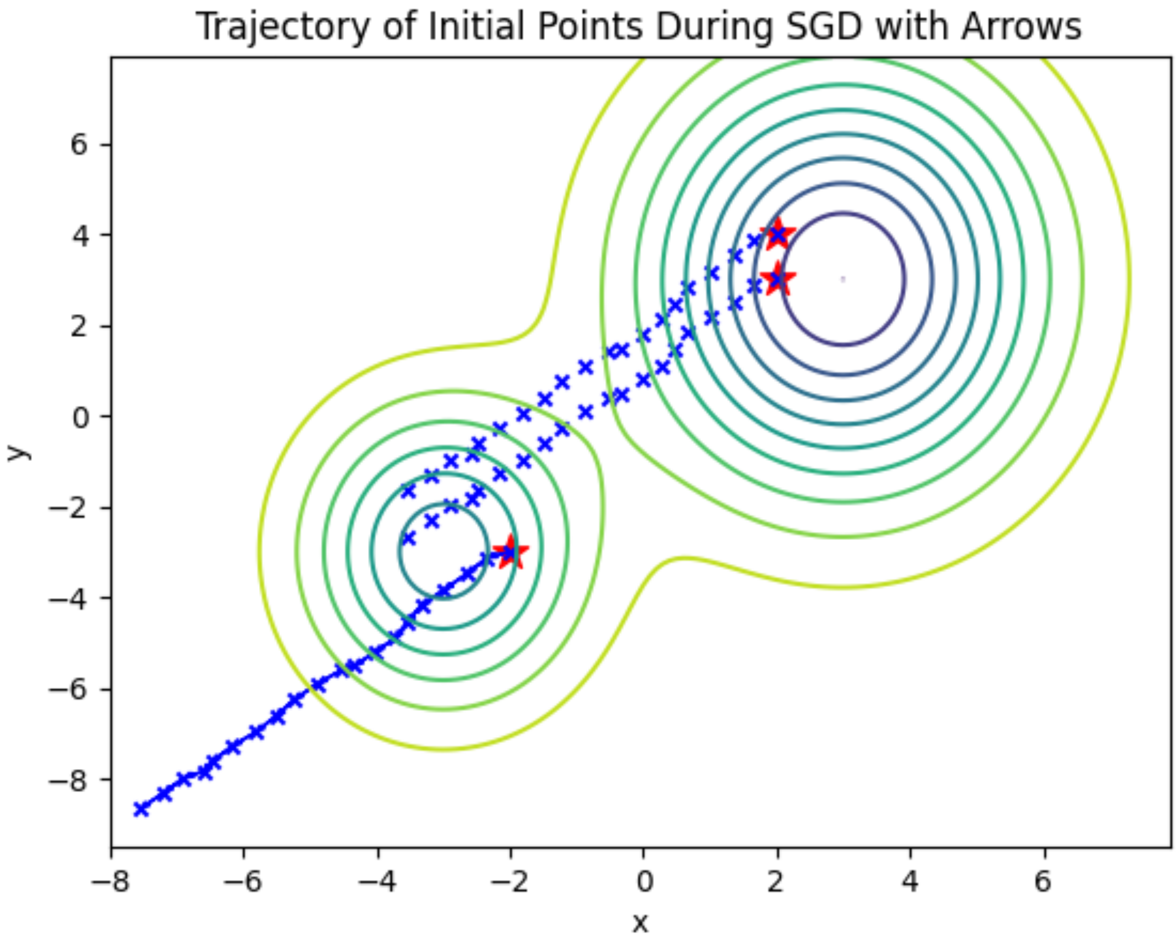
    # Update the initial points trajectory
    initial_points[:, 0] -= learning_rate * gradient[0]  # Update each po
    initial_points[:, 1] -= learning_rate * gradient[1]

    initial_x_traj.append(initial_points[:, 0].copy())
    initial_y_traj.append(initial_points[:, 1].copy())

# Plot the trajectory of initial points with blue "x" markers
for i in range(num_iterations):
    plt.scatter(initial_x_traj[i], initial_y_traj[i], marker='x', color='

# Add arrows to connect consecutive points in the trajectory
for i in range(num_iterations - 1):
    plt.arrow(initial_x_traj[i][0], initial_y_traj[i][0],
              initial_x_traj[i + 1][0] - initial_x_traj[i][0],
              initial_y_traj[i + 1][0] - initial_y_traj[i][0],
              color='blue', width=0.01, head_width=0.1, length_includes_h

plt.title('Trajectory of Initial Points During SGD with Arrows')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```


Trajectory of Initial Points During SGD with Arrows
6
4
2
0
−2
−4
−6
−8
y
−8
−6
−4
−2
0
2
4
6
x