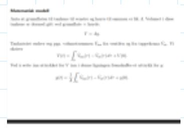
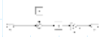
Innledning og modellering

Saturday, 23 August 2025 09:30



(1)

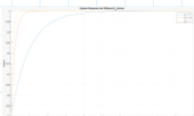
a) · Prosess



· Regulator

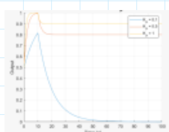
System + Scope

b)



Betraktning: Desto høyere p-verdi, desto mindre tidskonstant og mindre avvik mellom r og y.

c)

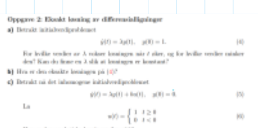


Betraktning: Det stasjonære avviket blir mindre for høyere K_p, men større enn b)

d)

Betraktning: I lukket sløyfe med $K=-1$ vil vil avviket øke proporsjonalt. Systemet vokser derfor ustabilt negativt

(2)



Oppgave 2: Eksakt løsning av differensialligninger

a) Betrakt initialverdiproblemet

$$\dot{y}(t) = \lambda y(t), \quad y(0) = 1. \tag{4}$$

For hvilke verdier av λ vokser løsningen når t øker, og for hvilke verdier minker den? Kan du finne en λ slik at løsningen er konstant?

b) Hva er den eksakte løsningen på (4)?

c) Betrakt nå det inhomogene initialverdiproblemet

$$\dot{y}(t) = \lambda y(t) + bu(t), \quad y(0) = 0. \tag{5}$$

La

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \tag{6}$$

Hva er den analytiske løsningen for $y(t)$?

a) $\dot{y}(t) = \lambda y(t)$, $y(0) = 1$

Løsningen konvergerer når $\lambda < 0$ og ikke når $\lambda > 0$. Når $\lambda = 0$, er $y(t) = 1$

b) $y(t) = e^{\lambda t}$

c) $\dot{y}(t) = \lambda y(t) + bu(t)$, $y(0) = 0$

$$\dot{y}(t) - \lambda y(t) = bu(t) \mid e^{-\lambda t}$$

$$e^{-\lambda t}(\dot{y}(t) - \lambda y(t)) = e^{-\lambda t} bu(t)$$

$$\frac{d}{dt}(e^{-\lambda t} y(t)) = e^{-\lambda t} bu(t)$$

$$\int_0^t \frac{d}{dt}(e^{-\lambda t} y(t))\, dt = \int_0^t e^{-\lambda t} bu(t)\, dt$$

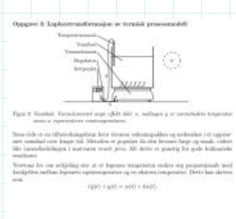
$$e^{-\lambda t} y(t) = y(0) + \int_0^t e^{-\lambda \tau} bu(\tau)\, d\tau$$

$$y(t) = y(0)e^{\lambda t} + \int_0^t b e^{\lambda(t-\tau)}\, d\tau$$

$$y(t) = \int_0^t b e^{\lambda(t-\tau)}\, d\tau$$

(3)

a)



Oppgave 3: Laplacetransformasjon av termisk prosessmodell

$$\tau \dot{y}(t) + y(t) = w(t) + ku(t)$$

$$w(t) = w_0 = 20$$

$$y(0) = 4 \qquad u(t) = 0$$

$$\tau \dot{y}(t) = -y(t) + 20$$

$$\dot{y}(t) = -\frac{1}{\tau} y(t) + \frac{20}{\tau}$$

$$\dot{y}(t) + \frac{1}{\tau} y(t) = \frac{20}{\tau}$$

$$\frac{d}{dt}\left(y(t) e^{t/\tau}\right) = \frac{20}{\tau} e^{t/\tau}$$

$$y(t) e^{t/\tau} = C + \frac{20}{\tau}\int e^{t/\tau}\, dt$$

$$y(t) = e^{-t/\tau}\left(C + \frac{20}{\tau} \cdot \tau e^{t/\tau}\right)$$

$$y(t) = Ce^{-t/\tau} + 20$$

$$y(t) = Ce^{-t/\tau} + 20$$

$$y(0) = C + 20 = 4$$

$$C = -16$$

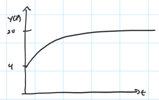
$$y(t) = -16e^{-t/\tau} + 20$$

$y(t) = Ce \quad +20$

$y(0) = C + 20 = 4$

$C = -16$

$\underline{y(t) = -16e^{-t/\tau} + 20}$



b)

$w(t) = w_0 \quad u(t) = 0$

$y_\infty = \lim_{t\to\infty} y(t) = \lim_{t\to\infty}\left(-16e^{-t/\tau} + 20\right) = \underline{20 = w_0}$

c) $\tau\dot{y}(t) + y(t) = w(t) + Ku(t)$

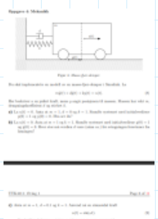
$\tau\left(s\mathcal{L}(y) - y(0)\right) + \mathcal{L}(y) =$

$\tau\mathcal{L}(y) + \mathcal{L}(y) - \tau y(0) = K\mathcal{L}\{u(t)\} + \mathcal{L}\{w(t)\}$

$\mathcal{L}\{y\}(s\tau + 1) = K\mathcal{L}\{u\} + \mathcal{L}\{w\}$

$$y(s) = \frac{K}{s\tau + 1}u(s) + \frac{1}{\tau s + 1}w(s)$$

④



$m\ddot{y}(t) + d\dot{y}(t) + Ky(t) = u(t)$

$\ddot{y}(t) = \frac{1}{m}\left(u(t) - d\dot{y}(t) - Ky(t)\right)$

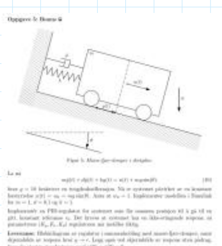
a) Ser stående svingninger

b) For ca. d=2, forsvinner svingningene fra systemet

c) $\underline{w = k}$

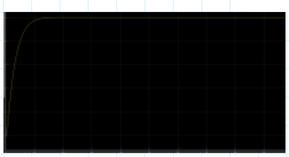
⑤

I: Blokkdiagram av system



II: Response uten pådrag

III: Med pådrag



⑥

a)

1)

$$\det(A_1 - \lambda I) = \det\begin{pmatrix} \frac{3}{2}-\lambda & 1 \\ \frac{1}{4} & \frac{3}{2}-\lambda \end{pmatrix}$$

$\left(\frac{3}{2}-\lambda\right)\left(\frac{3}{2}-\lambda\right) - \frac{1}{4}$

$\frac{9}{4} - 2\cdot\frac{3}{2}\lambda + \lambda^2 - \frac{1}{4}$

$\lambda^2 - 3\lambda + 2 = 0$

$(\lambda-1)(\lambda-2) = 0$

$\underline{\lambda_1 = 1 \quad \lambda_2 = 2}$

2)

Nedre triangulær $\Rightarrow$ $\underline{\lambda_1 = 2 \quad \lambda_2 = 1}$

3)

$$\det(A_3 - \lambda I) = \det\begin{pmatrix} 3-\lambda & -2 \\ 1 & -\lambda \end{pmatrix}$$

$= (3-\lambda)(-\lambda) + 2 = 0$

$= -3\lambda + \lambda^2 + 2 = 0$

$= \lambda^2 - 3\lambda + 2 = 0$

$= (\lambda-1)(\lambda-2) = 0$

$\underline{\lambda_1 = 1 \quad \lambda_2 = 2}$

b)

I) Egenverdier med multiplisitet (s)

B_1:

$\lambda = 1, s = 3$

B_3: Nede triangulær

$\lambda = 1 \quad s = 2$

B_2:

$$\det(B_2 - \lambda I) = \det\begin{pmatrix} -\lambda & -2 \\ \frac{1}{2} & 2-\lambda \end{pmatrix}$$

$$= (-\lambda)(2-\lambda) + 1 = 0$$

$$= -2\lambda + \lambda^2 + 1 = 0$$

$$= \lambda^2 - 2\lambda + 1 = 0$$

$$= (\lambda-1)^2 = 0$$

$\lambda = 1 \quad s = 2$

II) En matrise er diagonaliserbar om den har s-lineært uavh vektorer

B_1: Ja!

B_2: No!

$$\left(\begin{array}{cc|c} -1 & -2 & 0 \\ \frac{1}{2} & 1 & 0 \end{array}\right) \quad 2R_2 + R_1$$

$$\left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$x_1 + 2x_2 = 0$

$x_1 = -2x_2, \; x_2 = a$

$v = a\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

B_3: Nei

$$\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 2 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 2 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)$$

$x_1 = 0$

$v = a\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

c) $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3, \lambda_4 = 4$

for C_1, C_2 og C_3

d)

$A = \begin{bmatrix} 8 & -1 \\ 6 & 3 \end{bmatrix}$

$\det(A - \lambda I) = 0$

$\det\begin{pmatrix} 8-\lambda & -1 \\ 6 & 3-\lambda \end{pmatrix} = 0$

$(8-\lambda)(3-\lambda) + 6 = 0$

$24 - 11\lambda + \lambda^2 + 6 = 0$

$\lambda^2 - 11\lambda + 30 = 0$

$(\lambda-6)(\lambda-5) = 0$

$\lambda_1 = 6 \quad \lambda_2 = 5$

$A = \begin{bmatrix} 8 & -1 \\ 6 & 3 \end{bmatrix}$

$\Lambda = \begin{pmatrix} 5 & 0 \\ 0 & 6 \end{pmatrix}$

$\lambda_1 = 5$

$$\left(\begin{array}{cc|c} 3 & -1 & 0 \\ 6 & -2 & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 3 & -1 & 0 \\ 3 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 3 & -1 & 0 \end{array}\right)$$

$3x_1 - x_2 = 0$

$x_1 = \frac{1}{3}x_2$

$v_2 = s\begin{pmatrix} \frac{1}{3} \\ 1 \end{pmatrix} = \frac{1}{3}s\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\lambda_2 = 6$

$$\left(\begin{array}{cc|c} 2 & -1 & 0 \\ 6 & -3 & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 2 & -1 & 0 \\ 2 & -1 & 0 \end{array}\right) \quad 2\tfrac{1}{3}R_2 - R$$

$$\left(\begin{array}{cc|c} 2 & -1 & 0 \end{array}\right)$$

$2x_1 = x_2$

$x_1 = \frac{x_2}{2}$

$v_1 = s\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = 2s\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$Q = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$

$$\left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right) \quad R_2 - 3R_1$$

$$\left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & -1 & -3 & 1 \end{array}\right) \quad \begin{array}{l} R_1 + R_2 \\ (-1) \end{array}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & 3 & -1 \end{array}\right)$$

$Q^{-1} = \begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix}$

$$A = Q\Lambda Q^{-1} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 5 & 0 \\ 0 & 6 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix}$$

e) Egenverdiene til $\Sigma_1, \Sigma_2, \Sigma_3$ er da $\lambda_1 = 1$ og $\lambda_2 = 2$ med algebraisk multiplisitet 2 for begge

7 Jordan form

Egenvektormatrisen til A kan inverteres om den har n-lineært uavh egenvektor.

V har ikke full rang så ikke invertibar

3 Egenverdianalyse

Oppgave 6: Egenverdier

a) Finn egenverdier i følgende matriser:

$$A_1 = \begin{bmatrix} \frac{3}{2} & 1 \\ \frac{1}{4} & \frac{3}{2} \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}. \tag{11}$$

b) Finn egenverdier (med multiplisitet) i følgende matriser:

$$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & -2 \\ \frac{1}{2} & 2 \end{bmatrix}, \quad B_3 \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}. \tag{12}$$

Hvilke av matrisene kan diagonaliséres?

c) Finn egenverdier i følgende matriser:

$$C_1 = \begin{bmatrix} 1 & a & b & c \\ 0 & 2 & d & e \\ 0 & 0 & 3 & f \\ 0 & 0 & 0 & 4 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 2 & 0 & 0 \\ b & d & 3 & 0 \\ c & e & f & 4 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}. \tag{13}$$

d) La

$$A = \begin{bmatrix} 8 & -1 \\ 6 & 3 \end{bmatrix}. \tag{14}$$

Diagonalisér A (om mulig). Det vil si, finn en diagonal Λ og en invertibel Q slik at

$$A = Q\Lambda Q^{-1}. \tag{15}$$

e) Anta at egenverdiene til matrisen A er $\lambda_1 = 1$ og $\lambda_2 = 2$. Anta videre at egenverdiene til matrisen B er $\mu_1 = 1$ og $\mu_2 = 2$. Hva er da egenverdiene til

$$\Sigma_1 = \begin{bmatrix} A & X \\ 0 & B \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} A & 0 \\ Y & B \end{bmatrix}, \quad \Sigma_3 = \begin{bmatrix} B & 0 \\ 0 & A \end{bmatrix}? \tag{16}$$

Oppgave 7: Jordanform

Ikke alle systemer er diagonalisérbare. Disse kan bringes på Jordanform. La fire matriser være gitt ved:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & -1 & -1 \\ -1 & 0 & 3 & 2 \\ 1 & 0 & -2 & -1 \end{bmatrix}, \; J = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; Q = \begin{bmatrix} 0 & -2 & 0 & -1 \\ -2 & 2 & -2 & 0 \\ -2 & -2 & 1 & -2 \\ 2 & 0 & -1 & 2 \end{bmatrix}. \tag{17}$$

$$A=\begin{pmatrix}0&1&0\\0&0&1\\-a_2&-a_1&-a_0\end{pmatrix}$$

$$c=\begin{pmatrix}b_0\\b_1\\b_2\end{pmatrix} \quad b=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

⑧

a)

S:

$$\dddot{x} = u - a_0\ddot{x} - a_1\dot{x} - a_2 x$$

$$y = b_2\ddot{x} + b_1\dot{x} + b_0 x$$

$x_1 = x$

$x_2 = \dot{x}$

$x_3 = \ddot{x}$

$$\dot{x}_1 = 0 + x_2 + 0$$
$$\dot{x}_2 = 0 + 0 + x_3$$
$$\dot{x}_3 = -a_2x_1 - a_1x_2 - a_0x_3 + u$$
$$y = b_0x_1 + b_1x_2 + b_2x_3$$

$$\frac{d}{dt}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}0&1&0\\0&0&1\\-a_2&-a_1&-a_0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} + \begin{pmatrix}0\\0\\1\end{pmatrix}u$$

$$y = \begin{pmatrix}b_0\\b_1\\b_2\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

O:

$$\dot{x}_1 = -a_2x_1 + x_2 + b_2u$$
$$\dot{x}_2 = -a_1x_1 + x_3 + b_1u$$
$$\dot{x}_3 = -a_0x_1 + b_0u$$
$$y = x_1$$

$$A=\begin{pmatrix}-a_2&1&0\\-a_1&0&1\\-a_0&0&0\end{pmatrix}$$

$$b=\begin{pmatrix}b_2\\b_1\\b_0\end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}-a_2&1&0\\-a_1&0&1\\-a_0&0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} + \begin{pmatrix}b_2\\b_1\\b_0\end{pmatrix}u$$

$$y = \begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

S_*:

$$\dot{x}_1 = -a_2x_1 + x_2$$
$$\dot{x}_2 = -a_1x_1 + x_3$$
$$\dot{x}_3 = -a_0x_1 + u$$
$$y = b_2'x_3 + b_1'x_2 + b_0'x_1$$

$$\frac{d}{dt}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}-a_2&1&0\\-a_1&0&1\\-a_0&0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} + \begin{pmatrix}0\\0\\1\end{pmatrix}u$$

$$y = \begin{pmatrix}b_0'\\b_1'\\b_2'\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

O_*:

$$\dot{x}_1 = 0 + x_2 + 0 + b_2'u$$
$$\dot{x}_2 = 0 + 0 + x_3 + b_1'u$$
$$\dot{x}_3 = -a_2x_1 - a_1x_2 - a_0x_3 - b_0'u$$
$$y = x_1$$

$$\frac{d}{dt}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}0&1&0\\0&0&1\\-a_2&-a_1&-a_0\end{pmatrix} + \begin{pmatrix}b_2'\\b_1'\\b_0\end{pmatrix}$$

$$y = \begin{pmatrix}1\\0\\0\end{pmatrix}x \qquad u$$

b)

D:

$$\dot{x}_1 = b_1u + \lambda_1x_1$$
$$\dot{x}_2 = b_2u + \lambda_2x_2$$
$$\dot{x}_3 = b_3u + \lambda_3x_3$$
$$y = c_1x_1 + c_2x_2 + c_3x_3$$

$$\frac{d}{dt}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}\lambda_1&0&0\\0&\lambda_2&0\\0&0&\lambda_3\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} + \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}u$$

$$y = \begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

J:

$$\dot{x}_1 = \lambda x_1 + u$$
$$\dot{x}_2 = \lambda x_2 + x_1$$
$$\dot{x}_3 = \lambda x_3 + x_2$$
$$y = x_3$$

$$\frac{d}{dt}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}\lambda&0&0\\1&\lambda&0\\0&1&\lambda\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} + \begin{pmatrix}1\\0\\0\end{pmatrix}u$$

$$y = \begin{pmatrix}0\\0\\1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

⑨

a) Teori:

Har funnet:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

Vil finne:

$$Y(s) = H(s)U(s)$$

der $H(s)$ er transfer funksjonen

Anvender laplace transform på originalt system

I: $sX(s) = AX(s) + BU(s)$

II: $Y(s) = CX(s) + DU(s)$

$$sX(s) - AX(s) = BU(s)$$

$$(sI - A)X(s) = BU(s)$$

$$X(s) = (sI-A)^{-1}BU(s)$$

$$Y(s) = (C(sI-A^{-1})B + D)U(s)$$

Endelig form:

$$X(s) = (sI-A)^{-1}BU(s)$$

$$Y(s) = H(s)U(s)$$

$$H(s) = C(sI-A)^{-1}B + D$$

Kode:

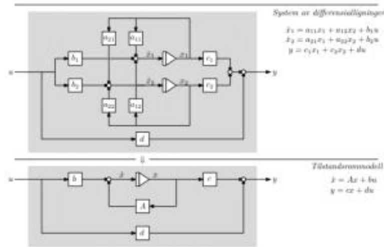
Oppgave:

Her er

$$Q^{-1} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{6} & -\frac{5}{6} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} & 0 \\ -1 & 0 & 1 & 1 \end{bmatrix}. \tag{18}$$

Sjekk at $AQ = QJ$. Bruk gjerne Matlab til å gjøre dette. Anvend deretter Matlabs **eig** kommando for å finne egenvektormatrisen til A. Kan denne inverteres?

4 Generell systemteori



Figur 6: *Korrespondanse mellom systemer av differensialligninger, tilstandsrommodeller og blokkdiagram.*

Oppgave 8: Blokkdiagram $\Longrightarrow$ tilstandsrommodell

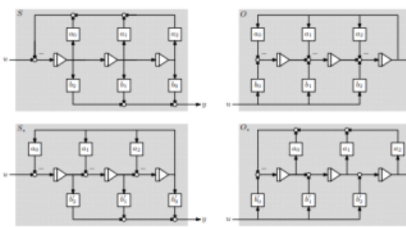
a) Sett opp systemene i fig. 7 som tilstandsrommodeller (finn A, b, c).

b) Sett opp systemene i fig. 8 som tilstandsrommodeller (finn A, b, c).

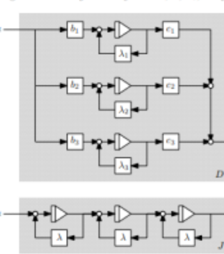
Oppgave 9: Blokkdiagramreduksjon via Laplace (☺)

a) Finn en overføringsfunksjon $H(s)$ til hvert av diagrammene i fig. 7 og fig. 8 slik at $y(s) = H(s)u(s)$.

b) Hva er impulsresponsen til systemet J i fig. 7?



Figur 7: *Blokkdiagram av systemene, S, S_*, O og O_*.*



Figur 8: *Blokkdiagram av systemene D og J.*

c) Hva er impulsresponsen til systemet D i fig. 7?

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(29)

(30)

S:

$$H(s)=\frac{b_2s^2+b_1s+b_0}{s^3+a_0s^2+a_1s+a_2}$$

O:

$$H(s)=\frac{b_2s^2+b_1s+b_0}{s^3+a_2s^2+a_1s+a_0}$$

S_*:

$$H(s)=\frac{b_0+a_1b_2+a_2b_1+b_1s+b_2s^2+a_2b_2s}{s^3+a_2s^2+a_1s+a_0}$$

O_*:

$$H(s)=\frac{(b_0+a_0b_1+a_1b_2+b_1s+b_2s^2+a_0b_2s)}{s^3+a_0s^2+a_1s+a_2}$$

D:

$$H(s)=\frac{b_1c_1}{\lambda_1-s}-\frac{b_2c_2}{\lambda_2-s}-\frac{b_3c_3}{\lambda_3-s}$$

J:

$$H(s)=\frac{-1}{\lambda-s}$$

b) Impulsrespons:

Teori: Impulsrespons i tidsdomenet er laplace transform av H(s)

Oppgaver:

J: $h(t)=e^{\lambda t}$

c) D:

$$h(t)=b_1c_1e^{\lambda_1 t}+b_2c_2e^{\lambda_2 t}+b_3c_3e^{\lambda_3 t}$$

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$x_1=G,\ x_2=I,\ x_3=I_2,\ x_4=M,\ x_5=M_2$

$\dot{x}_1=\theta_1-\theta_2x_2+\theta_4x_4$

$\dot{x}_2=\theta_3^{-1}(x_3-x_2)=\theta_3^{-1}x_3-\theta_3^{-1}x_2$

$\dot{x}_3=-\theta_3^{-1}x_3+u$

$\dot{x}_4=\theta_5^{-1}x_5-\theta_5^{-1}x_4$

$\dot{x}_5=-\theta_5^{-1}x_5+\omega$

$y=x_1$

a)

$$\frac{d}{dt}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{pmatrix}=\begin{pmatrix}0&-\theta_2&0&\theta_4&0\\0&-\theta_3^{-1}&\theta_3^{-1}&0&0\\0&0&-\theta_3^{-1}&0&0\\0&0&0&-\theta_5^{-1}&\theta_5^{-1}\\0&0&0&0&-\theta_5^{-1}\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{pmatrix}+\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}u+\begin{pmatrix}\theta_1\\0\\0\\0\\1\end{pmatrix}\omega$$

$$y=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{pmatrix}$$

b)

$\dot{x}_1=\theta_1-\theta_2x_2+\theta_4x_4$

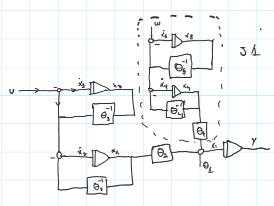
$\dot{x}_2=\theta_3^{-1}(x_3-x_2)=\theta_3^{-1}x_3-\theta_3^{-1}x_2$

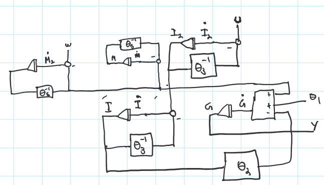
$\dot{x}_3=-\theta_3^{-1}x_3+u$

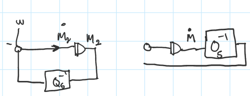
$\dot{x}_4=\theta_5^{-1}x_5-\theta_5^{-1}x_4$

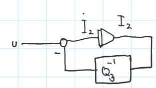
$\dot{x}_5=-\theta_5^{-1}x_5+\omega$

$y=x_1$









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1)

$$e^{At}=\sum_{k=0}^{\infty}\frac{(At)^k}{k!}$$

La A, B være to matriser

$AT=TB$

$(AT)T^{-1}=(TB)T^{-1}$

$A=TBT^{-1}$

$$e^{At}T=\sum_{k=0}^{\infty}\frac{(At)^k}{k!}\cdot T$$

$$=\sum_{k=0}^{\infty}\frac{((TBT^{-1})t)^k}{k!}T$$

$$=\sum_{k=0}^{\infty}\frac{t^k(TBT^{-1})^k}{k!}T$$

$$=\sum_{k=0}^{\infty}\frac{t^kB^k}{k!}T$$

$$=T\sum_{k=0}^{\infty}\frac{(tB)^k}{k!}$$

$$=Te^{Bt}$$

ii)

$(pI-A)^{-1}T=(pI-TBT^{-1})^{-1}T$

$= \dots$

b) $\dddot{y} + \alpha_2 \ddot{y} + \alpha_1 \dot{y} + \alpha_0 y = \beta_2 \ddot{u} + \beta_1 \dot{u} + \beta_0 u$