

Mathematics Done Not Even Wrong

Gently Weird Notes Took In Cambridge

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1 Conventions

1.1 Denotations

Generally I avoid any "bad conventions" that I consider the mathematician use for their narcissistic ego and simple laziness, assuming everyone else suppose them to be quick and convenient, whereas actually they only render vagueness and confusion. Technically, conventions that do not raise confusions, like polymorphisms, are not "bad conventions", and I personally use them a lot. But for the very idea of imposing my (also narcissistic) ideology, I avoid them actively.

Lots of mathematicians also love using $:$ in demonstrating the "type" of a morphism, especially in **1-Set**, while they even don't use type theory related foundations in their works, but rather the conventional set theory. This note is primarily worked in ZF, and actively notes once uses AoC. Thus I avoid using vague notation of $:$.

Personally I also gives the aesthetics of denotations some priority.

I try to include all my notations conventions below.¹ They are done recursively. The arguments enclosed by $[\cdot]$ are in "common conventions", for the convenience of mapping different conventions.

$$[f \text{ is an object in n-category } \mathcal{C}] \implies f \in \mathbf{0}\text{-}\mathcal{C} \quad (1)$$

$$[f \text{ is a morphism in n-category } \mathcal{C}] \implies f \in \mathbf{1}\text{-}\mathcal{C} \quad (2)$$

$$[f \text{ is a k-morphism in n-category } \mathcal{C} (k \leq n)] \implies f \in \mathbf{k}\text{-}\mathcal{C} \quad (3)$$

$$[f \in \text{Hom}_{\mathcal{C}}(A, B)] \implies f \in (A \xrightarrow{\mathbf{1}\text{-}\mathcal{C}} B) \quad (4)$$

$$[f : A \rightarrow B, A, B \in \mathbf{0}\text{-}\mathbf{Set}] \implies f \in (A \xrightarrow{\mathbf{1}\text{-}\mathbf{Set}} B) \quad (5)$$

These are to say, I only recognise morphisms to be legal elements in a category, in case of unnecessary divisions upon $\text{Ob}(\mathcal{C})$ and $\text{Hom}(\mathcal{C})$.

$$[G \text{ and } H \text{ are isomorphic in groups}] \implies G \xlongequal{\mathbf{1}\text{-}\mathbf{Grp}} H \quad (6)$$

$$[G \text{ and } H \text{ are the same group}] \implies G \xlongequal{\mathbf{0}\text{-}\mathbf{Grp}} H \quad (7)$$

$$G = H \quad (8)$$

The equality signs shall read equivalence. I am not serious on (7), as in most cases I only use (8), with assumptions working in $\text{ZF}(\mathcal{C})$.

¹When certain notations later in my note are considered to be vague or not single-typed, I may leave a \cdots to indicate the case, and may fix it later.

$$f \in (A \xrightarrow{\mathbf{1-Set}} B) \implies f^* \in (\mathcal{P}(A) \xrightarrow{\mathbf{1-Set}} \mathcal{P}(B)) \quad (9)$$

$$f^*(S) = \{x \in B \mid (\exists y \in S)(f(y) = x)\} \quad (10)$$

1.2 Assumptions

I hate people assuming too much.

Again, in this note I mainly work on ZF, optionally with AoC. Thus some claims are natural to be made:

- All categories are **strict** categories.
- A

1.3 Lectures

Eventually, this note is called a "note", because I try to use my own way to collate signifiers flying in Cambridge Mathematics Tripos lectures. You may need this to look up the exhaustive list of all tripos lectures.

Sadly, since I decide to take this note from the beginning of my Part IB auditing II lectures, the notes might not include sufficient IA and IB contents. No physics lecture may exist in my note, as I believe physicians should build their own department.

Anywhere in this note, there might spawn out some abbreviations denoting the related lectures, the mappings follow:

Analysis:

Analysis I	\implies	ia-a1	Analysis and Topology	\implies	ib-a2
Linear Analysis	\implies	ii-lan	Complex Analysis	\implies	ib-ca
Optimisation	\implies	ia-op	Probability and Measure	\implies	ii-pm

Algebra:

Groups	\implies	ia-gp	Vector and Matrices	\implies	ia-vm
Linear Algebra	\implies	ib-la	Groups, Rings and Modules	\implies	ib-grm
Algebraic Topology	\implies	ii-at	Galois Theory	\implies	ii-ga
Commutative Algebra	\implies	iii-com			

Foundations:

Automata and Formal Languages	\implies	ii-af	Logic and Set Theory	\implies	ii-ls
Category Theory	\implies	iii-cat	Model Theory	\implies	iii-mod

Probability:

Probability	\implies	ia-pr	Markov Chain	\implies	ib-mc
Probability and Measure	\implies	ii-pm	Stochastic Finance Models	\implies	ii-sfm
Statistics	\implies	ib-st			

Discrete:

Numbers and Sets	\implies	ia-ns	Number Theory	\implies	ii-nt
Graph Theory	\implies	ii-gt	Number Fields	\implies	ii-nf
Coding and Cryptography	\implies	ii-cc			

Geometry:

Geometry	\implies	ib-geo	Riemann Surfaces	\implies	ii-rs
Algebraic Geometry	\implies	ii-ag	Algebraic Geometry(III)	\implies	iii-ag

Semi-physics:

Vector Calculus	\implies	ia-vc	Differential Equations	\implies	ia-de
Methods	\implies	ib-mt	Variational Principles	\implies	ia-vp

1.4 Trivialness

All mathematics are trivial, up to some extent.

2 Prerequisites

3 Uncategorised

3.1 Vector Spaces

Let $V \in \mathbf{0}\text{-Vect}_{\mathbf{F}}$ and $\dim(V) = k < \omega$, essentially $V \xlongequal{\mathbf{1}\text{-Vect}_{\mathbf{F}}} F^k$

When I was in my very young age, I was quite confused with the definition of a group homomorphism. The requirement of $\phi(g)\phi(h) = \phi(gh)$ sounds from nowhere. It is until years later that I've realised the vague rhetorical concept that "Homomorphisms preserve structures". To formalise, one may say $G \in \mathbf{0}\text{-Grp}$, $\phi(G) = \text{Im}(\phi) \in \mathbf{0}\text{-Grp}$.

Let's think about a childish question: Why is morphisms (or for the convenience of thinking, morphisms in $\mathbf{1}\text{-Set}$ denoted as \rightarrow)? Imagine the significance of an arrow, it "shoots into" another object, and thereby