Beyond Games

Gently weird notes took in Cambridge

Yvette Meng

ym401@cam.ac.uk

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0 Preface

0.1 Ontology

Juxtaposition sucks

0.2 Cambridge

I love exonyms like Caergrawnt or Cantabrigia.

0.3 Language

2.034 The structure of a fact consists of the structures of states of affairs.

Ludwig W.

Always believe that, once two ”structures” in mathematics share the very same name, then they are definitely equivalent up to a certain domain of discourse (universe).

1 Category I

1.1 Basic Concepts

2.026 There must be objects, if the world is to have unalterable form.

Ludwig W.

Note that if $\mathscr{C}$ is a **concrete** category, then

Theorem 1.1.

$$
\begin{aligned}
(\, f \in (\, A \xmapsto{\mathbf{1}\text{-}\mathscr{C}} B \,) \subseteq \mathbf{1}\text{-}\mathscr{C} \,) \quad &\iff \quad (\, \exists S \subseteq B \,)(\, f \in (\, A \overset{\mathbf{1}\text{-}\mathscr{C}}{=\!=\!=} S \,)\,) \\
(\, f \in (\, A \xtwoheadrightarrow{\mathbf{1}\text{-}\mathscr{C}} B \,) \subseteq \mathbf{1}\text{-}\mathscr{C} \,) \quad &\iff \quad (\, \exists T \subseteq A \,)(\, f \in (\, T \overset{\mathbf{1}\text{-}\mathscr{C}}{=\!=\!=} B \,)\,)
\end{aligned}
$$

2 Linear Algebra

2.1 Finite vector spaces

For a given field F, finite dimensioned vector spaces are **trivial** up to an equivalence in $\mathbf{1}\text{-}\mathbf{Vect}_{\mathbf{F}}$.
Note that, let $V \in \mathbf{0}\text{-}\mathbf{Vect}_{\mathbf{F}}$ and $\dim(V) = k < \omega$, essentially $V \xlongequal{\mathbf{1}\text{-}\mathbf{Vect}_{\mathbf{F}}} F^k$
Later it may be proved that, for any two normed vector spaces on the same field, they also isomorphic to the extent of $\mathbf{1}\text{-}\mathbf{Vect}_{\mathbf{F}}$?

2.2 Category NVect

Riesz Lemma

2.3 Dual Spaces

A very useful way of thinking what a dual space is in our common finite $V \xlongequal{\mathbf{1}\text{-}\mathbf{Vect}_{\mathbf{F}}} F^n$ is:

Lemma 2.1. $\begin{pmatrix} a_1 & a_2 & \dots & a_n \end{pmatrix} \quad \iff \quad \begin{pmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{pmatrix}; \quad V \quad \iff \quad V^\star$

3 Measure

3.1 Concepts

These are basic definitions sharing the structure adhering to the powerset of a set.

Definition 3.1. *A π-system is*

4 Uncategorised

4.1 Miscellaneous

When I was in my very young age, I was quite confused with the definition of a group homomorphism. The requirement of $\phi(g)\phi(h) = \phi(gh)$ sounds from nowhere. It is until years later that I've realised the vague rhetorical concept that "Homomorphisms preserve structures". To formalise, one may say $G \in \mathbf{0}\text{-}\mathbf{Grp}$, $\phi(G) = \mathrm{Im}(\phi) \in \mathbf{0}\text{-}\mathbf{Grp}$.

Let's think about a childish question: Why is morphisms (or for the convenience of thinking, morphisms in $\mathbf{1}\text{-}\mathbf{Set}$ denoted as $\rightarrow$? Imagine the significance of an arrow, it "shoots into" another object, and thereby

A Conventions

A.0.1 Denotations

Generally I avoid any ”bad conventions” that I consider the mathematician use for their narcissistic ego and simple laziness, assuming everyone else suppose them to be quick and convenient, whereas actually they only render vagueness and confusion. Technically, conventions that do not raise confusions, like polymorphisms, are not ”bad conventions”, and I personally use them a lot. But for the very idea of imposing my (also narcissistic) ideology, I avoid them actively.

Lots of mathematicians also love using : in demonstrating the ”type” of a morphism, especially in **1**-**Set**, while they even don’t use type theory related foundations in their works, but rather the conventional set theory. This note is primarily worked in ZF, and actively notes once uses AoC. Thus I avoid using vague notation of :.

Personally I also gives the aesthetics of denotations some priority.

I try to include all my notations conventions below.[1] They are done recursively. The arguments enclosed by $[\,\cdot\,]$ are in ”common conventions”, for the convenience of mapping different conventions.

[1]When certain notations later in my note are considered to be vague or not single-typed, I may leave a $^{\cdots}$ to indicate the case, and may fix it later.

$$[\text{f is an object in n-category } \mathscr{C}] \iff f \in \mathbf{0}\text{-}\mathscr{C} \tag{A.1}$$

$$[\text{f is a morphism in n-category } \mathscr{C}] \iff f \in \mathbf{1}\text{-}\mathscr{C} \tag{A.2}$$

$$[\text{f is a k-morphism in n-category } \mathscr{C}(k \leq n)] \iff f \in \mathbf{k}\text{-}\mathscr{C} \tag{A.3}$$

$$[f \in \mathrm{Hom}_{\mathscr{C}}(A, B)] \iff f \in (\, A \xrightarrow{\mathbf{1}\text{-}\mathscr{C}} B \,) \tag{A.4}$$

$$[f : A \to B,\ A, B \in \mathbf{0}\text{-}\mathbf{Set}] \iff f \in (\, A \xrightarrow{\mathbf{1}\text{-}\mathbf{Set}} B \,) \tag{A.5}$$

$$[\text{G is homomorphic to H as groups}] \iff G \xrightarrow{\mathbf{1}\text{-}\mathbf{Grp}} H \tag{A.6}$$

$$\iff (\, G \xrightarrow{\mathbf{1}\text{-}\mathbf{Grp}} H \,) \neq \emptyset \tag{A.7}$$

$$[\text{f is a monomorphism as } f \in \mathrm{Hom}_{\mathscr{C}}(A, B)] \iff f \in (\, G \xhookrightarrow{\mathbf{1}\text{-}\mathbf{Grp}} H \,) \tag{A.8}$$

$$[\text{f is a epimorphism as } f \in \mathrm{Hom}_{\mathscr{C}}(A, B)] \iff f \in (\, G \xrightarrow{\mathbf{1}\text{-}\mathbf{Grp}} \!\!\!\!\rightarrow H \,) \tag{A.9}$$

These are to say, I only recognise morphisms to be legal elements in a category, in case of unnecessary divisions upon Ob(C) and Hom(C). More information upon monomorphisms and epimorphisms shall be found in the section 1.1.

$$[\text{G and H are isomorphic in groups}] \iff G \overset{\mathbf{1}\text{-}\mathbf{Grp}}{=\!=\!=} H \tag{A.10}$$

$$[\text{G and H are the same group}] \iff G \overset{\mathbf{0}\text{-}\mathbf{Grp}}{=\!=\!=} H \tag{A.11}$$

$$\iff G = H \tag{A.12}$$

The equality signs shall read equivalence. I am not serious on (12), as in most cases I only use (13), with assumptions working in ZF(C).

$$f \in (\, A \xrightarrow{\mathbf{1}\text{-}\mathbf{Set}} B \,) \iff f^{\star} \in (\, \mathcal{P}(A) \xrightarrow{\mathbf{1}\text{-}\mathbf{Set}} \mathcal{P}(B) \,) \tag{A.13}$$

$$\iff f^{\star}(S) = \{x \in B \mid (\exists y \in S)(f(y) = x)\} \tag{A.14}$$

A.0.2 Assumptions

I hate people assuming too much.

Again, in this note I mainly work on ZF, optionally with AoC. Thus some claims are natural to be made:

- All categories are **strict** categories.
- A

A.0.3 Trivialness

All mathematics are trivial, up to some extent.

B Index

B.0.1 Apparatus

Eventually, this note is called a ”note”, because I try to use my own way to collate signifiers flying in Cambridge Mathematics Tripos lectures. You may need this to look up the exhaustive list of all tripos lectures.

Sadly, since I decide to take this note from the beginning of my Part IB audititting II lectures, the notes might not include sufficient IA and IB contents. No physics lecture may exist in my note, as I believe physicians should build their own department.

Anywhere in this note, there might spawn out some abbreviations denoting the related lectures, the mappings follow:

Analysis:

Analysis I	$\Longleftrightarrow$	ia-a1	Analysis and Topology	$\Longleftrightarrow$	ib-a2
Linear Analysis	$\Longleftrightarrow$	ii-lan	Complex Analysis	$\Longleftrightarrow$	ib-ca
Optimisation	$\Longleftrightarrow$	ia-op	Probability and Measure	$\Longleftrightarrow$	ii-pm

Algebra:

Groups	$\Longleftrightarrow$	ia-gp	Vector and Matrices	$\Longleftrightarrow$	ia-vm
Linear Algebra	$\Longleftrightarrow$	ib-la	Groups, Rings and Modules	$\Longleftrightarrow$	ib-grm
Algebraic Topology	$\Longleftrightarrow$	ii-at	Galois Theory	$\Longleftrightarrow$	ii-ga
Commutative Algebra	$\Longleftrightarrow$	iii-com			

Foundations:

Automata and Formal Languages	$\Longleftrightarrow$	ii-af	Logic and Set Theory	$\Longleftrightarrow$	ii-ls
Category Theory	$\Longleftrightarrow$	iii-cat	Model Theory	$\Longleftrightarrow$	iii-mod

Probability:

Probability	$\Longleftrightarrow$	ia-pr	Markov Chain	$\Longleftrightarrow$	ib-mc
Probability and Measure	$\Longleftrightarrow$	ii-pm	Stochastic Finance Models	$\Longleftrightarrow$	ii-sfm
Statistics	$\Longleftrightarrow$	ib-st			

Discrete:

Numbers and Sets	$\Longleftrightarrow$	ia-ns	Number Theory	$\Longleftrightarrow$	ii-nt
Graph Theory	$\Longleftrightarrow$	ii-gt	Number Fields	$\Longleftrightarrow$	ii-nf
Coding and Cryptography	$\Longleftrightarrow$	ii-cc			

Geometry:

Geometry	$\Longleftrightarrow$	ib-geo	Riemann Surfaces	$\Longleftrightarrow$	ii-rs
Algebraic Geometry	$\Longleftrightarrow$	ii-ag	Algebraic Geometry(III)	$\Longleftrightarrow$	iii-ag

Semi-physics:

Vector Calculus	$\Longleftrightarrow$	ia-vc	Differential Equations	$\Longleftrightarrow$	ia-de
Methods	$\Longleftrightarrow$	ib-mt	Variational Principles	$\Longleftrightarrow$	ia-vp