Problem 1

```
clear all; close all;
D = load('icevelocity.txt') %load in ice velocity data
```

```
D = 91×2
         0   80.0867
    2.0000   79.4700
    4.0000   80.0735
    6.0000   80.5074
    8.0000   79.5507
   10.0000   80.0381
   12.0000   82.3864
   14.0000   80.1030
   16.0000   79.4725
   18.0000   80.4856
      ⋮
```

```
z = D(:,1) % depths
```

```
z = 91×1
     0
     2
     4
     6
     8
    10
    12
    14
    16
    18
     ⋮
```

```
v = D(:,2) % velocities
```

```
v = 91×1
   80.0867
   79.4700
   80.0735
   80.5074
   79.5507
   80.0381
   82.3864
   80.1030
   79.4725
   80.4856
      ⋮
```

```
figure(1);

c = [0,1,2,3,4]; %set up degrees for testing
P = {} %create empty cell array
```

```
P =

  0×0 empty cell array
```

```
Y = zeros(length(v),length(c)) %create empty vecotr for speed
```

```
Y = 91×5
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     ⋮
```

```
Rms = zeros(1,5)
```

```
Rms = 1×5
     0     0     0     0     0
```

```
S{1} = 'data'
```

```
S = 1×1 cell array
    {'data'}
```

```
for i = 1:numel(c)
     x = c(i)
     P{i} = polyfit(z,v, x) % call ployfit function for each degree to be tested
     Y(:,i) = polyval(P{i},z) % finds polyval values from P polyfit inputs
     Rms(i) = sqrt(mean((v-Y(:,i)).^2)) %finds rmse of plyval vaules from actual veloci
     S{i+1}= (strcat('Degree =', num2str(x), 'RMSE =', num2str(Rms(i))))
end
```

```
x = 0
P = 1×1 cell array
    {[75.4946]}
Y = 91×5
   75.4946         0         0         0         0
   75.4946         0         0         0         0
   75.4946         0         0         0         0
   75.4946         0         0         0         0
   75.4946         0         0         0         0
   75.4946         0         0         0         0
   75.4946         0         0         0         0
   75.4946         0         0         0         0
   75.4946         0         0         0         0
   75.4946         0         0         0         0
     ⋮

Rms = 1×5
    5.0855         0         0         0         0
S = 1×2 cell
'data'      'Degree =0RMSE =5.0855'
x = 1
P = 1×2 cell
```

	1	2
1	75.4946	[-0.0821,82....

```
Y = 91×5
   75.4946   82.8820         0         0         0
   75.4946   82.7178         0         0         0
   75.4946   82.5537         0         0         0
```

```
   75.4946   82.3895         0         0         0
   75.4946   82.2254         0         0         0
   75.4946   82.0612         0         0         0
   75.4946   81.8970         0         0         0
   75.4946   81.7329         0         0         0
   75.4946   81.5687         0         0         0
   75.4946   81.4045         0         0         0
      ⋮

Rms = 1×5
    5.0855    2.6957         0         0         0
S = 1×3 cell
'data'      'Degree =0RMSE =5.0855''Degree =1RMSE =2.6957'
x = 2
P = 1×3 cell
```

	1	2	3
1	75.4946	[-0.0821,82....	[-0.0008,0.0581,78...

```
Y = 91×5
   75.4946   82.8820   78.7222         0         0
   75.4946   82.7178   78.8354         0         0
   75.4946   82.5537   78.9423         0         0
   75.4946   82.3895   79.0430         0         0
   75.4946   82.2254   79.1374         0         0
   75.4946   82.0612   79.2257         0         0
   75.4946   81.8970   79.3077         0         0
   75.4946   81.7329   79.3834         0         0
   75.4946   81.5687   79.4530         0         0
   75.4946   81.4045   79.5163         0         0
      ⋮

Rms = 1×5
    5.0855    2.6957    1.8895         0         0
S = 1×4 cell
'data'      'Degree =0RMSE =5.0855''Degree =1RMSE =2.6957''Degree =2RMSE =1.8895 ···
x = 3
P = 1×4 cell
```

	1	2	3	4
1	75.4946	[-0.0821,82....	[-0.0008,0.0581,78...	[0,0.001,-0.07,80.59...

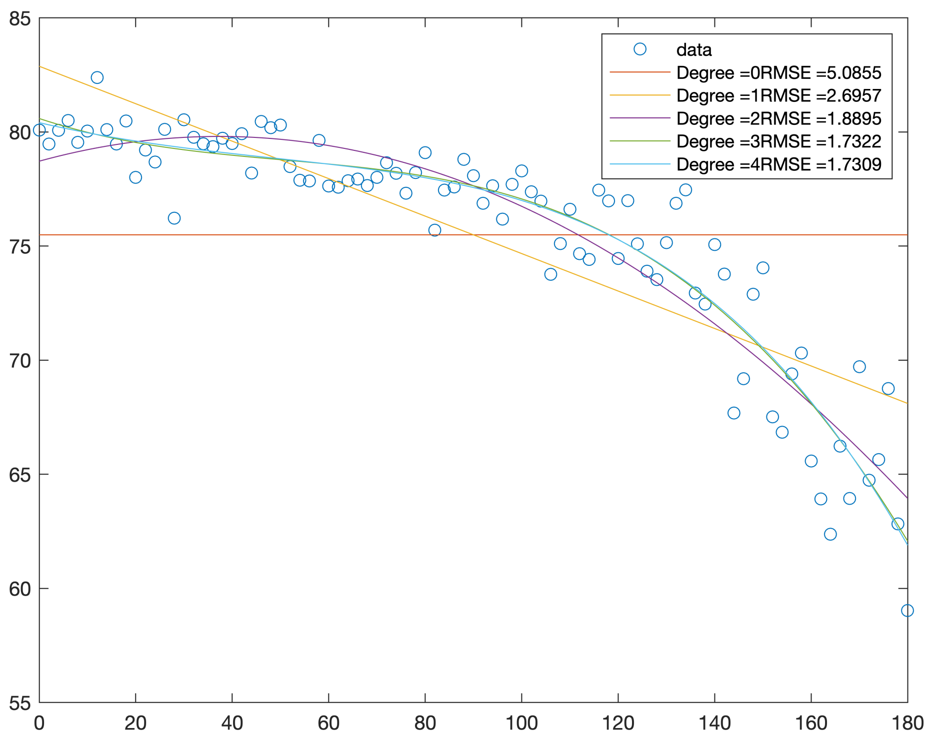
```
Y = 91×5
   75.4946   82.8820   78.7222   80.5917         0
   75.4946   82.7178   78.8354   80.4556         0
   75.4946   82.5537   78.9423   80.3272         0
   75.4946   82.3895   79.0430   80.2064         0
   75.4946   82.2254   79.1374   80.0926         0
   75.4946   82.0612   79.2257   79.9857         0
   75.4946   81.8970   79.3077   79.8852         0
   75.4946   81.7329   79.3834   79.7910         0
   75.4946   81.5687   79.4530   79.7026         0
   75.4946   81.4045   79.5163   79.6198         0
      ⋮

Rms = 1×5
    5.0855    2.6957    1.8895    1.7322         0
S = 1×5 cell
'data'      'Degree =0RMSE =5.0855''Degree =1RMSE =2.6957''Degree =2RMSE =1.8895 ···
x = 4
P = 1×5 cell
```

...

	1	2	3	4
1	75.4946	[-0.0821,82....	[-0.0008,0.0581,78...	[0,0.001,-0.07,80.59...

```
Y = 91×5
   75.4946   82.8820   78.7222   80.5917   80.4099
   75.4946   82.7178   78.8354   80.4556   80.3142
   75.4946   82.5537   78.9423   80.3272   80.2222
   75.4946   82.3895   79.0430   80.2064   80.1337
   75.4946   82.2254   79.1374   80.0926   80.0488
   75.4946   82.0612   79.2257   79.9857   79.9672
   75.4946   81.8970   79.3077   79.8852   79.8888
   75.4946   81.7329   79.3834   79.7910   79.8136
   75.4946   81.5687   79.4530   79.7026   79.7414
   75.4946   81.4045   79.5163   79.6198   79.6720
     ⋮

Rms = 1×5
     5.0855    2.6957    1.8895    1.7322    1.7309
S = 1×6 cell
'data'      'Degree =0RMSE =5.0855''Degree =1RMSE =2.6957''Degree =2RMSE =1.8895⋯
```

```
plot(z,v, 'o', z, Y, '-') %plots the velocities versus depth
legend(S)
```




```
% plot(z,v, 'o', z, Y, '-')
% legend(strcat('RMSE is', num2str(Rms(2))))
% plot(z,v, 'o', z, Y{1,3}, '-')
```

```
% legend(strcat('RMSE is', num2str(Rms(3))))
% plot(z,v, 'o', z, Y{1,4}, '-')
% legend(strcat('RMSE is', num2str(Rms{4})))
% plot(z,v, 'o', z, Y{1,5}, '-')
% legend(strcat('RMSE is', num2str(Rms{5})))
```

Problem 2

```
clear  P S Y Rms

P = {};

Rms = zeros(1,5);
S{1} = 'data'
```

```
S = 1×1 cell array
    {'data'}
```

```
perc = 0.9 % 90 percent of data
```

```
perc = 0.9000
```

```
Ix = 1:length(v);
id = round(length(v)*perc);  %length of samples
Y = zeros(id,length(c));

for i = 1:numel(c)
    for q = 1:1000
        R = randsample(Ix, id); %randomly smaple Ix, and id arrays
        vt = v(R); %test velocities
        zt = z(R);% test depths
        x = c(i);% give degree for model
        P{i}(q,:) = polyfit(zt,vt, x) ;% stores all the columns for qth row in the P c
        Y(:,i) = polyval(P{i}(q,:),zt); %clauclate polyval using polyfit inputs
        Rms(q,i) = sqrt(mean((vt-Y(:,i)).^2)); %caluculates Rmse of test velocities mi
        S{i+1}= (strcat('Degree =', num2str(x), 'RMSE =', num2str(Rms(i)))); %create s
    end
end

Rms_mu = mean(Rms) %find mean of Rms
```

```
Rms_mu = 1×5
    5.0732    2.6937    1.8825    1.7283    1.7250
```

```
Rms_std = std(Rms) %find standard deviation of Rms
```

```
Rms_std = 1×5
    0.1586    0.0766    0.0552    0.0575    0.0582
```

```
% T = [Rms_mu; Rms_std];

T = table([c;Rms_mu; Rms_std,], 'VariableNames', {'Data'}, 'RowNames', {'Degree', 'mea
```

```
T = 3×1 table
```

...

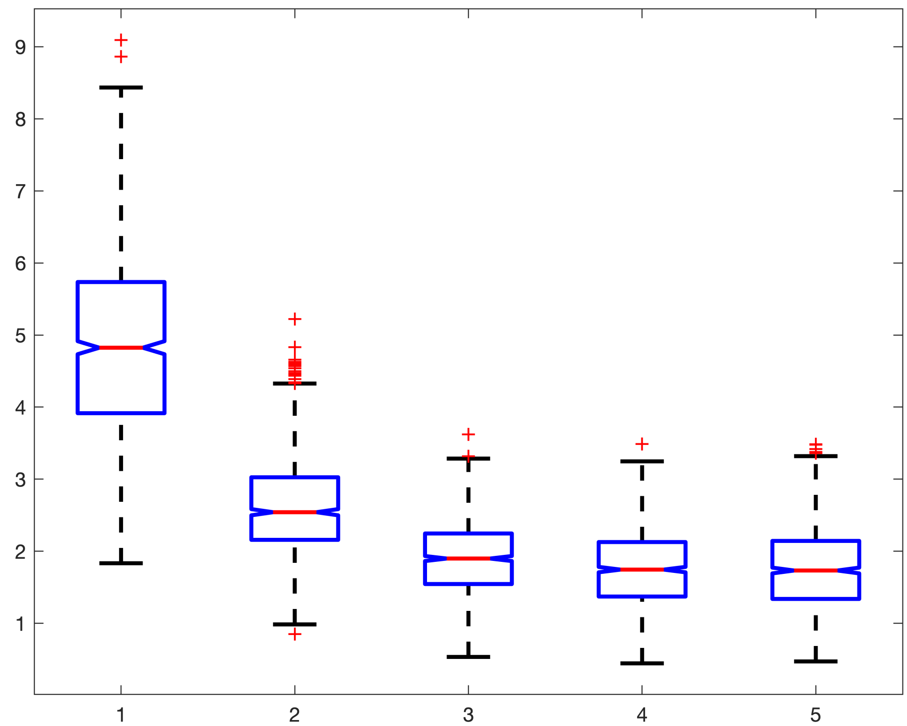
	Data		
1 Degree	0	1	2
2 mean RMS	5.0732	2.6937	1.8825
3 Std RMS	0.1586	0.0766	0.0552

Problem 3

```
clear q P Y S;
nmc= 1000
```

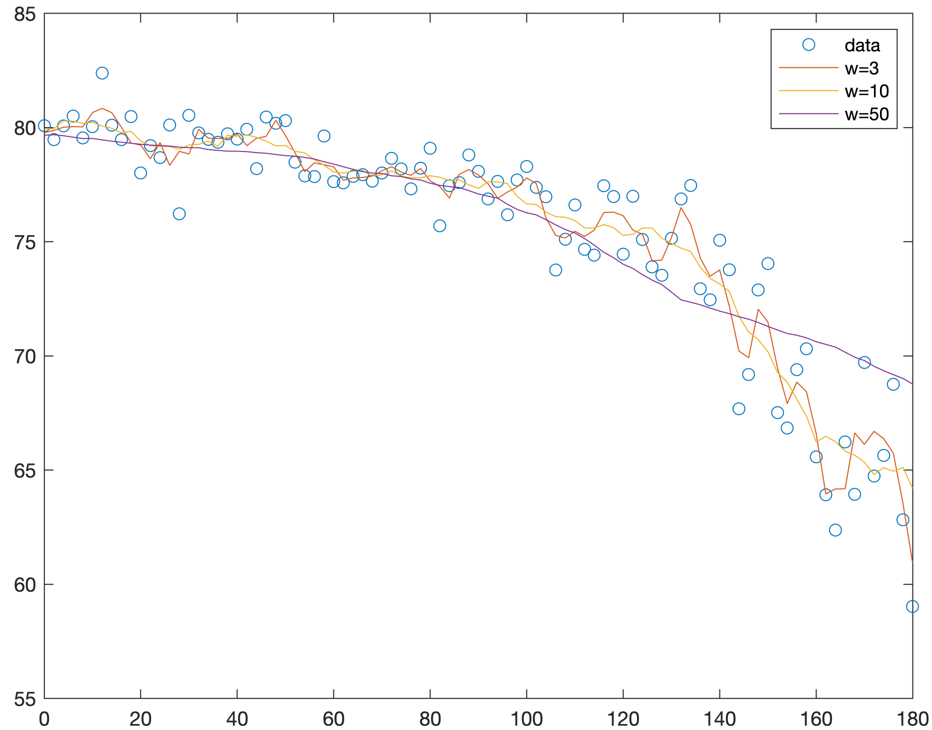
```
nmc = 1000
```

```
pTrain = 0.9;
%still use perc from last question for 90%
rmseCV = zeros(nmc, length(c)); %create empty cell array
for q = 1:length(c)
    for p = 1:nmc
    [trainset, testset] = getTrainTest([z,v], pTrain);
    ztrain = trainset(:,1); % depths for training
    vtrain = trainset(:,2); % velocitites for training
    ztest = testset(:,1); % depths for testing
    vtest = testset(:,2); % velocities for testing
    P = polyfit(ztrain, vtrain, c(q)); %use polyfit function on test sets
    Y = polyval(P, ztest) ; %evaluate at the test depths
    rmseCV(p,q) = sqrt(mean((vtest-Y).^2));
    end
end

figure(2);
h = boxplot(rmseCV, 'notch', 'on', 'labels', {'1','2','3','4','5'});
set(h, 'linewidth', 2);
```



Problem 4

```
clear q p Ix figure(4);
figure(4); clf
plot(z,v, 'o') %plot

winsize =[3 10 50];
z0 = 0:2:180; %sample size
for q = 1:length(winsize)
    for p = 1:length(z0)
        Ix = find(z>(z0(p)-winsize(q)) & z<(z0(p)+winsize(q))); %find values within wi
        um(p,q) = mean(v(Ix)); %take mean of values within window Ix

    end
end
figure(4); hold on
plot(z0,um) % plots z0 values against mean values within window size
legend('data', 'w=3', 'w=10','w=50')
```

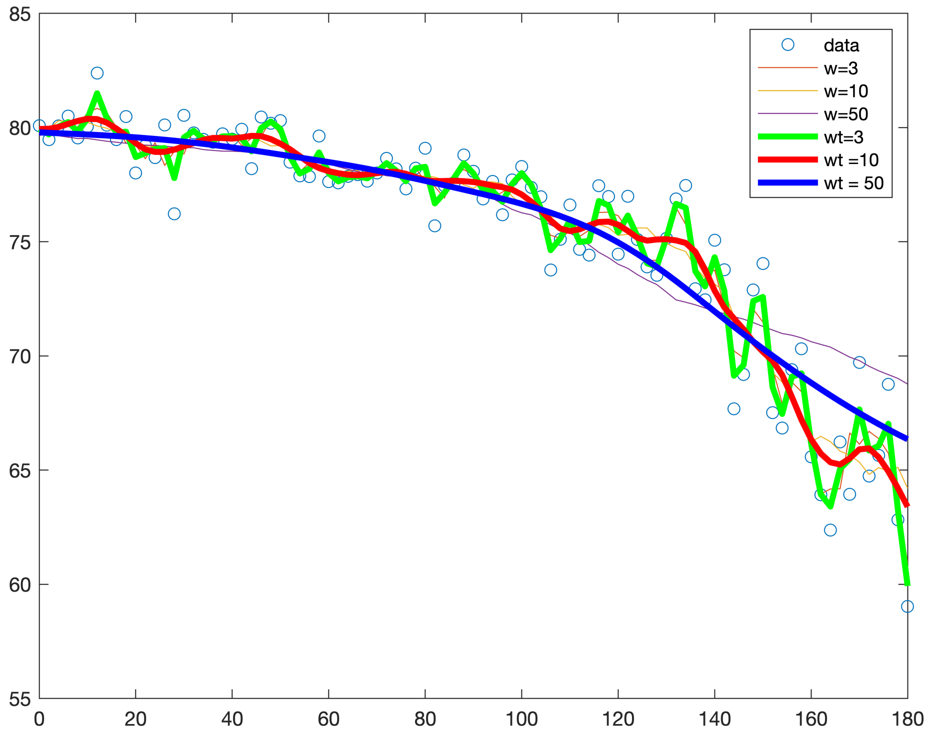


Problem 5

```
%weighted moving window
um2= {} %creat empty cell array
```

```
um2 =

  0×0 empty cell array
```

```
for r = 1:length(winsize)
   um2{r} = nonparametric_smooth(z,v,z0, winsize(r)); %call nonparametric_smooth funct
end
hold on
plot(z0, um2{1}, 'g', 'linewidth', 3) %these all plot the lines with different degrees
plot(z0, um2{2}, 'r', 'linewidth', 3)
plot(z0, um2{3}, 'b', 'linewidth', 3)
legend('data', 'w=3', 'w=10','w=50', 'wt=3', 'wt =10', 'wt = 50')
```



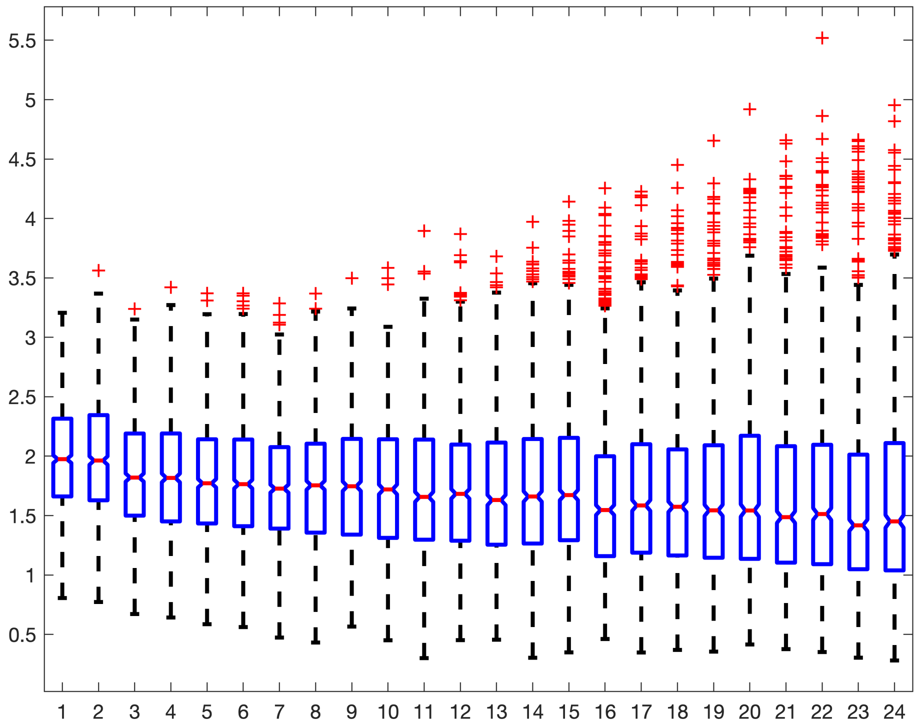
Problem 6

```
% find optimal window
clear q rmseCV ztest vtest ztrain vtrain p ;
nmc= 1000  % values to sample
```

```
nmc = 1000
```

```
pTrain = 0.9; % test 90% of data
P_u = {}; %create empty cell array
Y_u ={}; %create emtpy cell array
%still use perc from question 2 for 90%
winsize= 3:2:50;
rmseCV = zeros(nmc, length(winsize));
for q = 1:length(winsize)
    for p = 1:nmc
    [trainset, testset] = getTrainTest([z,v], pTrain); %call functin getTrainTest
    ztrain = trainset(:,1); % depths for training
    vtrain = trainset(:,2); % velocitites for training
    ztest = testset(:,1); % depths for testing
    vtest = testset(:,2); % velocities for testing
    P_u{q}(p,:) = nonparametric_smooth(ztrain,vtrain,z0, winsize(q)); %use nonparamtri
    Y_u{q}(p,:) = nonparametric_smooth(ztest,vtest,z0, winsize(q)) ; %evaluate at the 
    rmseCV(p,q) = sqrt(nanmean((Y_u{q}(p,:)-P_u{q}(p,:)).^2));  % calculate rmse value
    end
end
%'labels', {'3','10','50'}
figure(5);
```

```
H = boxplot(rmseCV, 'notch', 'on'); %create boxplot
set(H, 'linewidth', 2);
```



Answer: The optimal Window size is 47

Problem 7

```
clear um;
g =9.8 % [m/s^2]
```

```
g = 9.8000
```

```
rho = 917 %[kg/m^3]
```

```
rho = 917
```

```
theta = 10*pi/180 %convert theta to radian
```

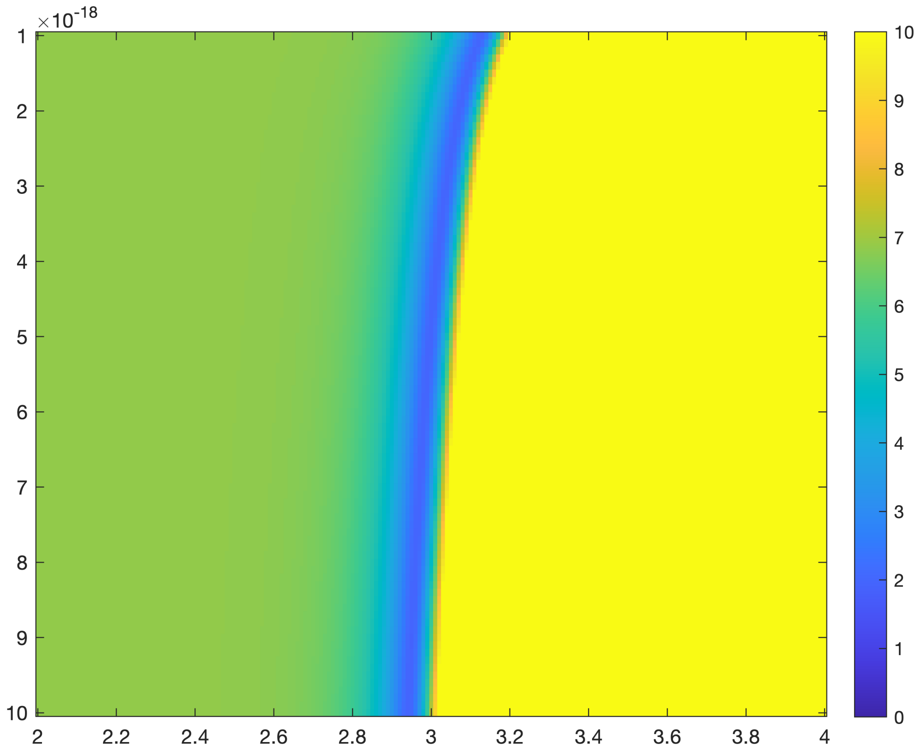
```
theta = 0.1745
```

```
n = 2:0.01:4; %create range of n values to find optimum n value for function
A = 1e-18:0.1e-18:10e-18; % define range of A values to test
rms = zeros(length(A),length(n)); % create empty data array for rmse values
for p =1:length(n) %loop through values of n
    for q=1:length(A) %loop through values of A
        um = v(1)-A(q).*(rho*g*sin(theta)).^n(p).*z.^(n(p)+1); %eq  6 in hw 3
        rms(q,p) = sqrt(mean((um-v).^2)); %RMSE for each combo of n and A
    end
```

```
end
```

Problem 8

```
figure(6); clf
imagesc(n,A,rms, [0 10]); colorbar  %create color bar
```



Answer for 7

```
% using imagesc and the color bar the optimum values for A are any values when
% n = 3
```

Problem 9

```
fh =@(A)icevel(z,v,A) % function handle, A can be tuned to data in v,and z
```

```
fh = function_handle with value:
    @(A)icevel(z,v,A)
```

```
A0 = 1e-18
```

```
A0 = 1.0000e-18
```

```
[Abest, nval]= fminsearch(fh, A0) %create fminsearch handle to fine tune A values from
```

```
Abest = 4.8375e-18
```

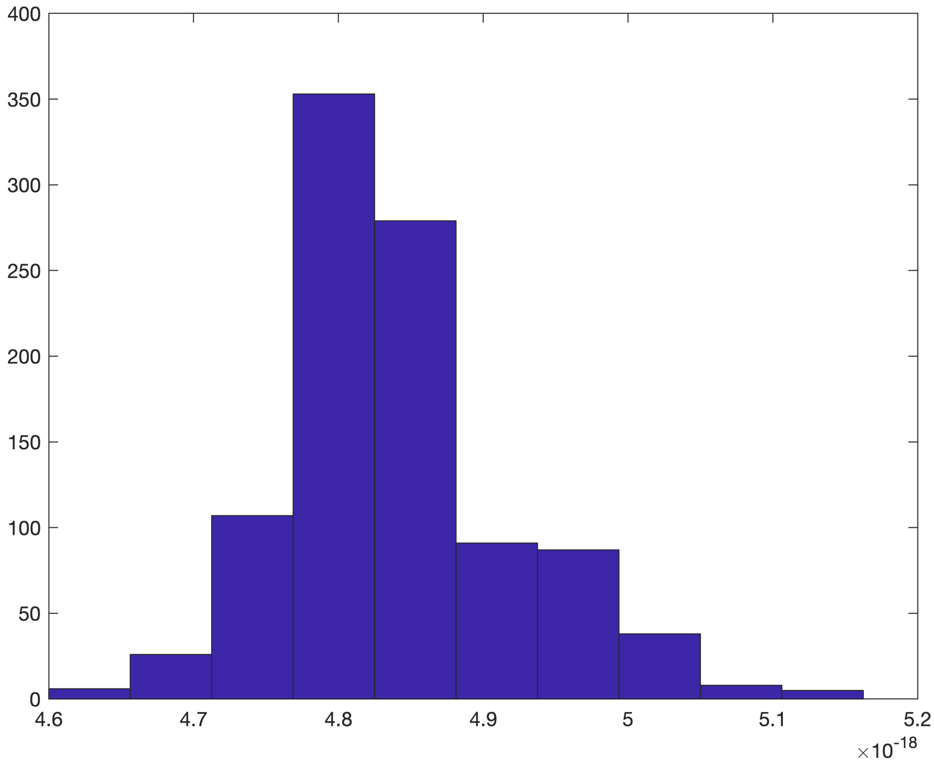
```
nval = 1.9649
```

Problem 10

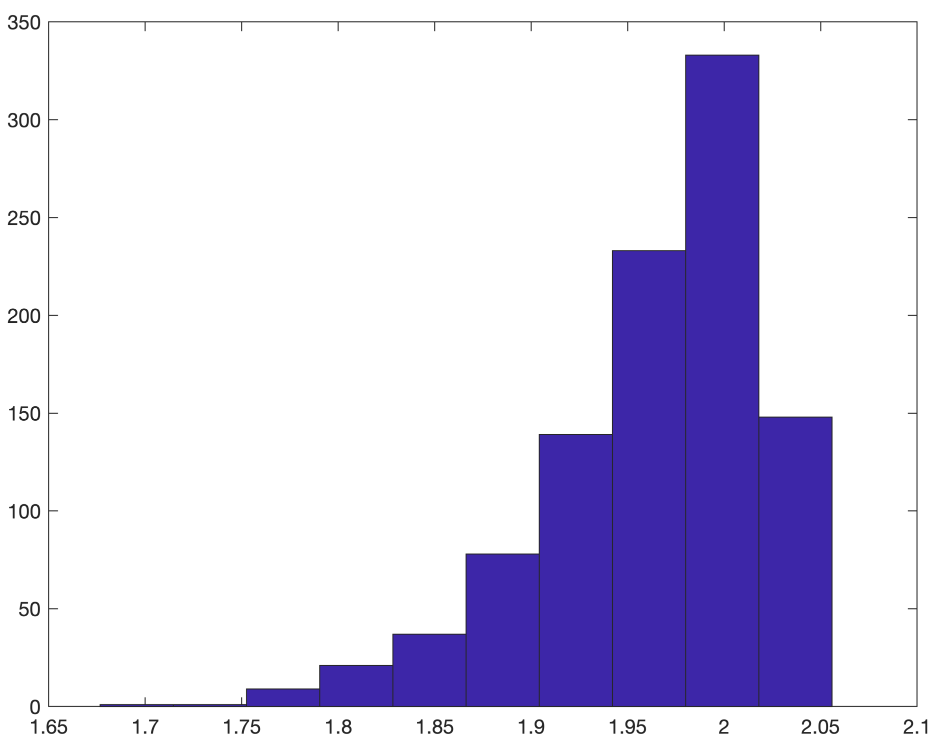
```
clear  P S Y Rms ICE

P = {}; %create empty cell array
% ICE = {};
% Rms = zeros(1,5);
% S{1} = 'data'
perc = 0.9 % 90 percent of data
```

```
perc = 0.9000
```

```
Ix = 1:length(v); % create index  array to keep track of positions
id = round(length(v)*perc);  %length of samples
% Y = zeros(id,length(c));
A0 = 1e-18 ; %define intial A value
% for i = 1:numel()
for q = 1:1000 %for loop for monte carlo evaluations
        R = randsample(Ix, id);
        vt = v(R); %test velocities
        zt = z(R); %test depths
%         ICE(q) = icevel(zt,vt,A);
        P =@(A)icevel(zt,vt,A); % function handle, A can be tuned to data in v,and z
        [Abest(q), nval(q)]= fminsearch(P, A0); % Nval is the value of the function tha
%         P{i}(q,:) = fminsearch(zt,vt, x) ;% stores all the columns for qth row in the
%         Y(:,i) = fminsearch(P{i}(q,:),zt);

%         S{i+1}= (strcat('Degree =', num2str(x), 'RMSE =', num2str(Rms(i))));
    end

figure(7); clf
hist(Abest); %hist of abest
```




```
figure(8);clf
hist(nval); %histogram of rmse values
```



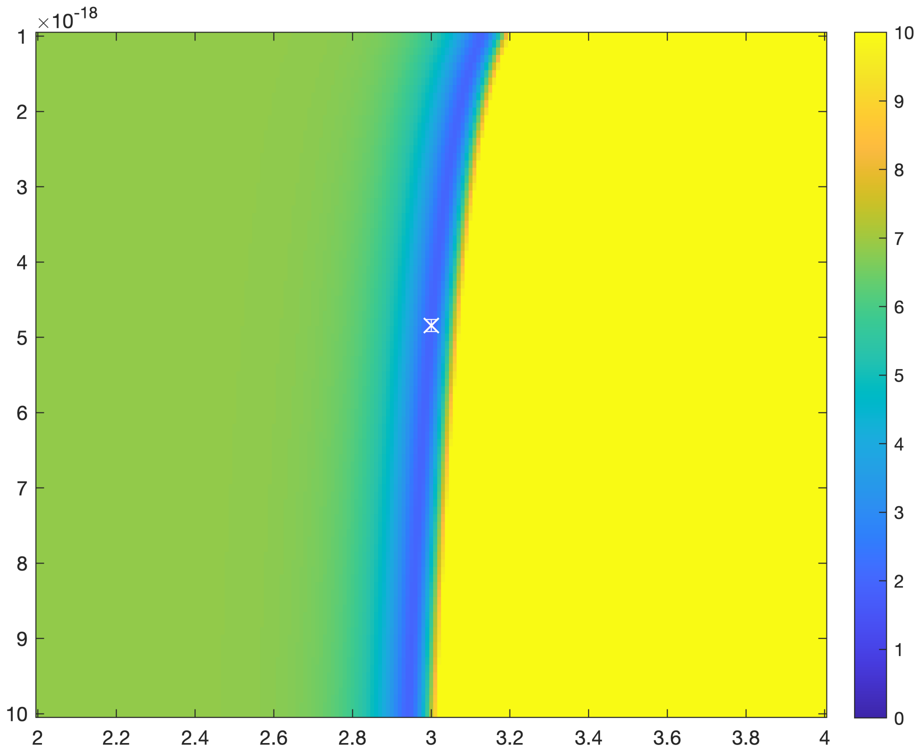
```
% xlim([-0.15, 0.15])
```

Problem 11

```
muAbest= mean(Abest); %find mean of Abest array
sigAbest = std(Abest); %fidn standard deviation of Abest array
n0=3 %initialize n value
```

```
n0 = 3
```

```
figure(9); %reacreate figure from probelm 8
imagesc(n,A,rms, [0 10]); colorbar; hold on
plot(n0, muAbest, 'wx', 'Markersize', 10, 'linewidth', 2); hold on  %plot the mean opt
errorbar(n0, muAbest, sigAbest, 'w-') %create errorbar
```



Problem 12

```
rmse_mu = mean(nval); %find the mean of rmse_mu array
Asim= randn(1000,1).*sigAbest+muAbest; %find simulated model estimation for A values f
Rsim = randn(1000,1).*std(nval)+rmse_mu; %find the RMSE suimulated values from icevel
```

Problem 13

```
[H, P] = kstest2(Asim, Abest); %find probablity of asim and Abest being same
[H1,P1] = kstest2(Rsim, nval); %find the probaility of RMSE values being the same
```

Fmin search function

```
function rmse = icevel(x,y,A) %define rmse function
%Inputs: x = independent variable
%        y = dependent variable
%         A = parameter in the physics based model
g =9.8; % [m/s^2]
rho = 917; %[kg/m^3]
theta = 10*pi/180; %convert theta to radian
% n = 2:0.01:4;
n = 3; %preallocate the n value for the icevelocity model
% A = 1e-18:0.1e-18:21e-18;
ymod = 80.086027-A.*(rho*g*sin(theta)).^n.*x.^(n+1); %define model equation
rmse = sqrt(mean((y-ymod).^2)); %calculate rmse
end
```

Wieghted Moving Window Function

```
function ymod = nonparametric_smooth(x,y, xmod, winsize) %define the function
%this function smooths a 20d dataset using a bisquare kernal
%input: x = indep vairable [n,1]
%       y = dependent variable  [n,1]
%       xmod = locations of estimates [*,1]
%       winsize = size of window (same uits as x)
% Output: ymod = nonparametric smoothed estimate
x = x(:); y=y(:);  %define x and y vectors
xmod = xmod(:); %give xmod data
ymod = zeros(size(xmod)); % create emtpy ymod vector for speed
for i = 1:length(xmod) %loop through all positions in xmod
    dist = sqrt((x-xmod(i)).^2); %distance from each data point to the estimate locati
    Ix = find(dist<winsize); %find values within window size and save index
    Ix = Ix(isfinite(y(Ix))); %removing NaNs from Ix
    if isempty(Ix) %removes empty values
        ymod(i)=NaN; %use NaN if no data in windown
    else

        w = 15/16*(1-(dist(Ix)/winsize).^2).^2; %bisquare kernal
        ymod(i) = sum(w.*y(Ix))./sum(w); %unbiased estimate
    end
end
end
```

RMSE Function

```
function rmse=poly1(x,y,A)
%INPUTS:    x = independent variable
%           y = dependent variable
%           A = parameters [slope,intercept]
```

```
ymod=A(1)*x+A(2); %define ymod equation
rmse=sqrt(mean((y-ymod).^2)); %calculate rmse value
end
```

TRAIN TEST FUNCTION

```
function [trainset, testset] =  getTrainTest(data, pTrain)
%input = data = [ns, nd] ns = number of samples nd = number of dimension
% outputs = trainset = data to use to train model ..
% testset = data for testin model
[ns,nd] = size(data); %create data array
nsTrain = round(pTrain *ns); %number of samples to train

Ix = 1:ns; %vector of indices
IxTrain = randsample(Ix, nsTrain); % randomly grab data
Ix3 = ones(size(Ix)); %vector of ones
Ix3(IxTrain) = 0 ;%set rows from training data to zero
IxTest = find(Ix3); %indices to the test observations

trainset = data(IxTrain,:); %datat for training
testset = data(IxTest,:); %data for testing
end
```