A Review and Discussion of Processing Algorithms for FSSP Concentration Measurements

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ABSTRACT

The forward-scattering spectrometer probe (FSSP) is an optical particle counter widely used for the measurement of cloud droplet size distributions and concentration. Previous studies have identified operational limitations of these probes and a number of techniques have been developed to minimize the impact of these limitations on the measurements. The majority of effort has been focused on accounting for droplets missed by the FSSP as a result of droplet coincidence and electronic dead time. This note reviews the algorithms that have been developed to account for these losses, describes how and when to apply them to previously acquired measurements, and recommends methods to improve the quality of future measurements.

1. Introduction

A great deal of information about cloud structure has been acquired with the forward-scattering spectrometer probe (FSSP) manufactured by Particle Measuring Systems (Boulder, Colorado). Numerous papers have been published that evaluate the limitations and accuracies of this instrument. Authors have proposed corrections to total concentration (e.g., Baumgardner 1983; Dye and Baumgardner 1984; Baumgardner et al. 1985; Brenguier and Amodei 1989; Brenguier 1989) and to size distribution measurements (e.g., Cooper 1988; Baumgardner and Spowart 1990; Hovenac and Lock 1993). In general the FSSP provides relatively accurate measurements of the size distribution of cloud droplets but only if appropriate corrections are applied to account for the recognized limitations. Failure to make corrections to FSSP measurements will lead to significant errors in sizing and concentration. Baumgardner et al. (1990) estimate errors as large as 30% and 60% in measured sizes and concentration, respectively, that can be reduced significantly when appropriate corrections are applied. The published information on these correction techniques, while useful to those already familiar with the operational characteristics of the FSSP, do not necessarily lend themselves to easy implementation and the variety of techniques leaves the investigator wondering which technique is most appropriate for a specific set of data. The other problem is that most of the techniques require auxiliary information from the FSSP for optimum implementation of the processing algorithms, but many of the earlier model FSSPs do not provide this information. The processing algorithms can still be used on measurements from these probes but additional assumptions must be made. This note presents a simplified explanation of how droplet concentration measurements are made, describes how droplets are missed as a result of coincidence and dead time, reviews the algorithms that account for these losses, and discusses how and when to implement these processing methods.

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2. Fundamentals

a. *Derivation of droplet concentration*

The FSSP detects light scattered in the forward direction from droplets that pass through its laser beam. The concentration N is simply the ratio of the droplet rate n (s^{-1}) through the sensing section of the probe to the corresponding volumetric flow rate of air. The volumetric flow rate is determined from the product of the airspeed v and sensitive beam cross section S. Hence,

$$N = \frac{n}{Sv}. \quad (1)$$

However, not all droplet detections can be used for sizing because of the nonuniform intensity of the laser in both cross section and along its length. Only droplets crossing close to the center of focus are selected by an electro-optical discrimination in the probe and the corresponding acceptance length is defined as the depth of field (DOF). Thus, one of the basic parameters provided by most FSSPs is a measure of the number of particles passing through the DOF, referred to as the "total strobes," or n_T (the appendix lists a glossary of all the variables discussed in this paper). To apply (1) the droplet velocity through the sample volume, the beam diameter (≈ 0.2 mm) and DOF (≈ 3 mm) must also be known (S = beam diameter × DOF). These parameters provide a primary evaluation of the droplet concentration, but the correction algorithms discussed below require a measure of additional parameters; that is,

1) the fast resets n_f (droplets detected outside the DOF) or total resets n_r (all detected droplets, also the sum of n_T and n_f);
2) the activity A that measures the fraction of time the FSSP is active detecting and processing droplets;
3) the electronic delay period τ_d following each droplet detection (typically between 2 and 10 μs);
4) the time response of the FSSP amplifiers that determine the effective droplet transit time.

If these parameters are not available they can be estimated as will be discussed later, but the subsequent corrections will be less accurate.

b. *Errors in derived concentration*

The concentration derived from (1) is correct under the assumption that the FSSP measures n correctly and that S and v are accurately known. There is some uncertainty in v because of possible airflow distortions and turbulence but the primary sources of error are in S and n.

The sensitive beam cross section is defined by the beam diameter and the electro-optically defined DOF. Uncertainties in these dimensions have been discussed by Dye and Baumgardner (1984), and Baumgardner et al. (1990) estimate that the dimensions of the cross section cannot be measured to an accuracy better than 15%.

The measured droplet rate n_d is never identical to n since all droplets in the sample volume are not detected and the derived concentration will always be an underestimate of the true concentration. Two circumstances cause undetected droplets: coincidence and dead time. Droplets that are coincident in the FSSP sample area are measured as a single event. This happens each time a droplet enters the beam before the previous droplet has left the beam. This causes an extended pulse width measured by the system. In this event the FSSP operates like a retriggerable counter.

Each pulse is followed by a fixed electronic delay τ_d needed for accurate droplet sizing. During this delay, however, droplets will not be detected if they pass through the beam and the length of the delay remains fixed. In this, the dead-time loss case, the FSSP behaves as a nonretriggerable counter. These two distinctions are important only because some of the correction algorithms that have been derived to correct for the two types of losses rely upon the mathematical description of these two types of counters.

The errors in concentration measurements that arise from uncertainties in the sample volume cannot be easily eliminated without a new optical design. Algorithms have been developed, however, to account for the droplet detection losses.

c. *Processing algorithms*

The processing algorithms that account for droplet losses are statistically derived since coincidence and dead-time events are random events. These methods are based on the hypothesis that the droplet-counting process is Poisson; that is, that droplet arrivals in the beam are random and independent. In addition, if the droplet concentration is uniform and the sampled volume remains constant during the accumulation period, the counting process is a stationary Poisson process and statistical procedures allow a calculation of the expected value of the actual droplet rate. If the process is Poisson but not stationary (concentration inhomogeneities), the techniques discussed below still provide useful information on the average rates and on the scales of inhomogeneities.

There are two approaches to estimating actual droplet rate through the counter:

1) from measurements of counted rate and/or activity;
2) from measurements of the interarrival times between droplets, using either the slope of their frequency distribution or the compensation method.

1) ACTUAL RATE FROM COUNTED RATE AND/OR ACTIVITY

A particle is in the FSSP laser beam for a period τ determined by its velocity v and the chord l of the beam through which it passes; that is, $\tau = l/v$. If a second particle enters the beam during τ, the pulse duration T is lengthened and only one particle is counted. It follows that the pulse duration is either equal to τ, for single particles, or greater than τ, for series of coincident particles, and its mean value $\bar{T}$

during the accumulation period is always greater than τ. The value of τ, or single particle transit time, is a crucial parameter for the evaluation of coincidence losses, since the probability of a coincidence is the probability for a particle to enter the beam during τ.

The activity of the probe is measured by a clock that is started as a particle enters the beam and remains on until the electronics are reset after the dead time; that is, it is equal to the sum of the pulse durations and electronic delays during the accumulation period:

$$A = n_d(\bar{T} + \tau_d) > n_d(\tau + \tau_d). \qquad (2)$$

The probability of coincidence and dead-time events has been formalized by Baumgardner et al. (1985) and Brenguier and Amodei (1989). The latter investigators took a more theoretical approach, but their final solution is in practice the same as found by Baumgardner et al. The algorithms derived by Brenguier and Amodei (1989) will be presented here. These authors have shown that expected values of counted rate and activity are both functions of the actual droplet rate and τ; that is,

$$n_d = f(n, \tau) \quad \text{and} \quad A = g(n, \tau). \qquad (3)$$

Thus, the expectation of the actual droplet rate can be derived if either n_d or A are measured and τ is known. Further in the text, the term "expectation of" will be omitted. The solution depends on the respective values of the τ and of the electronic delay:

for $\tau_d \leqslant \tau$

$$\bar{T} = \frac{1 - XX_d}{nX} - \tau_d$$
$$n_d = nX$$
$$A = 1 - XX_d, \qquad (4)$$

for $\tau_d > \tau$

$$\bar{T} = \frac{1 - X^2}{nX} - \tau$$
$$n_d = \frac{nX}{1 + nX(\tau_d - \tau)}$$
$$A = 1 - \frac{X^2}{1 + nX(\tau_d - \tau)}, \qquad (5)$$

where $X = e^{-n\tau}$ and $X_d = e^{-n\tau_d}$.

It is also possible to combine these equations to evaluate the homogeneity of the samples (Brenguier 1990) and to check probe function (Brenguier et al. 1993). These equations are strictly valid for a constant τ, but Brenguier (1989) showed that they are still approximately valid for the FSSP when using the average value of τ. Two circumstances cause variations of the τ: the location of the droplet trajectory through the FSSP's approximately cylindrical beam and the amplitude of the detected pulse. The actual duration of a particle in the beam is determined by its location but the electronic response time of the probe leads to a lengthening of the measured pulse duration that varies as a function of pulse amplitude.

Since the beam section is approximately circular and particles will pass randomly through different chords of the beam, it is easy to show that the average chord length l is equal to $\pi/4$ of the beam diameter and the average τ is $0.785 d/v$. The average τ is thus obtained with Eq. (3.3) in Brenguier (1989):

$$\bar{\tau} = \pi\frac{d}{4v} + \frac{\theta}{n_m}\sum_{j=1}^{15} n^j_m \ln\left(\frac{V_j}{V_0} - 1\right), \qquad (6)$$

where n^j is the number of droplets in the size class j, n_m is the total number of droplets that have been sized, V_j is the voltage corresponding to the class j, V_0 is the detection threshold, and θ is the time response of the amplifiers (typically between 0.4 and 0.6 μs for the FSSP). If these values have not been measured on the probe, a value of 0.5 μs for the time response and the voltage scale provided in the manufacturer's documentation are good estimates for the calculation of the electronic lengthening.

For a beam diameter of 0.2 mm, at an airspeed of 100 m s^{-1}, the maximum τ is 2 μs, the average τ is 1.6 μs, and the electronic lengthening can change from 0 to 2 μs, according to the proportion of big droplets in the spectrum. The resulting τ can range from 1.6 μs to more than 3 μs. The coincidence equations are sensitive to the τ value; hence, it is crucial to correctly evaluate the average τ for an accurate correction.

The distinction between (4) and (5) is an important one to understand since it depends upon the conditions under which the FSSP is operated and the electronic setup of the probe. The decision to select (4) or (5) will depend upon the particle velocity, the electronic lengthening—that is, the droplet spectrum—and the electronic delay time. The newer-model FSSPs use two delay cycles. If a droplet is detected in the DOF, its transit is followed by a long delay (typically 6–10 μs), needed for size processing, while if it is detected outside of the DOF, the electronics are more rapidly reset (typically 1–2 μs). At a typical aircraft speed of 100 m s^{-1}, the average τ, as discussed above, will always be shorter than the long delay but can be either longer or shorter than the fast delay.

The fraction of particles that pass through the DOF, f (or DOF fraction), and thus have been followed by a long delay, can be calculated directly if either the fast reset or the total resets are measured:

$$f = \frac{n_T}{n_T + n_f} = \frac{n_T}{n_r}. \qquad (7)$$

The coincidence equations must be combined according to this proportion:

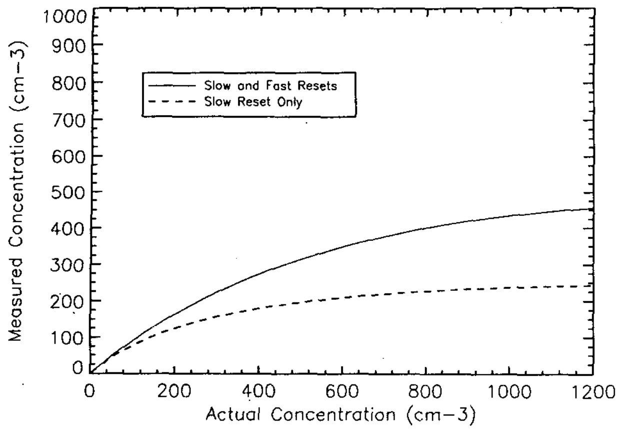

FIG. 1. The measured and actual concentrations are compared in this figure as a function of whether an FSSP has the electronics for a slow delay reset only or has both a slow and a fast reset delay installed.

$$\bar{T} = \bar{T}_1(1 - f) + \bar{T}_2 f$$

$$\frac{1}{n_d} = \frac{1 - f}{n_{d1}} + \frac{f}{n_{d2}}$$

$$A = n_d(\bar{T} + \overline{\tau_d})$$

$$\tau_d = \tau_{d1}(1 - f) + \tau_{d2} f, \qquad (8)$$

where $\bar{T}_1$, $\bar{T}_2$, n_{d1}, and n_{d2} are the average pulse durations and counted rates given by (4) or (5), for τ_d equal to τ_{d1} and τ_{d2}.

The choice of either (4) or (5) depends on the average τ compared to τ_{d1} and τ_{d2}. The previous discussion showed that for τ_{d2} (long delay), (5) always applies ($\tau_d > \bar{\tau}$), while for τ_{d2} (fast reset), the choice of (4) or (5) can be made only after calculation of $\bar{\tau}$. Examination of (4), (5), and (8) shows that they cannot be solved analytically for n, but must be solved numerically. The effect of having the fast reset function of the probe implemented is illustrated in Fig. 1, where (5) for the counted rate is plotted for a τ_d of 10 μs and a $\overline{\tau_d}$ of 3.6. This clearly illustrates the necessity of decreasing the amount of electronic delay to the minimum possible and of compensating for the losses.

The question arises, What is the best evaluation of the actual rate, from counted rate or from activity? The activity in typical FSSPs is measured with a 1-MHz clock, but the subsequent clock counts are divided by 1000 before transmission to the data system. The resolution of activity measurements is thus limited to 1 ms in a standard probe (1024 counts at 100% activity) but can be improved by reducing the divider, for example, 65 000 counts at 100% activity provides a resolution of 15 μs. At very low concentrations it is obvious that the direct measurement of counted rate provides the best evaluation, since counted rate is almost identical to actual rate, while the value of the actual rate derived from activity is directly affected by the error in the evaluation of the τ and the limited accuracy of the activity measurement itself. As the concentration increases, the situation reverses because counted rate reaches a maximum while activity is a continuously increasing function of the actual rate. The simplified set of equations for a pure retriggerable counter is sufficient to illustrate this transition:

$$n_d = nX \quad \text{and} \quad A = 1 - X. \qquad (9)$$

Partial derivatives provide the sensitivity of the actual rate calculation to uncertainties in τ, n_d, and A:

$$\frac{dn}{n} = \frac{1}{1 - n\tau}\left(\frac{dn_d}{n_d} + n\tau\frac{d\tau}{\tau}\right) \qquad (10)$$

when n is derived from n_d,

$$\frac{dn}{n} = \frac{-d\tau}{\tau} + \frac{1 - e^{-n\tau}}{n\tau e^{-n\tau}}\frac{dA}{A} \qquad (11)$$

when n is derived from the activity.

Equation (10) shows that when n_d approaches its maximum (at $n\tau = 1$), the sensitivity of the calculation to errors in n_d and τ tends to infinity. If n is derived from activity, the sensitivity of the calculation to errors in τ is constant, equal to τ^{-1}, and its sensitivity to errors in A at $n\tau = 1$ is equal to $e - 1$, which is less than 2. This is illustrated in Fig. 2, where the relative error in n is plotted as a function of n for a typical $dn_d/n_d = 1\%$, if n is derived from n_d (dashed line) and a typical value of $dA/A = 5\%$ if n is derived from the activity (dotted line). In both cases, $d\tau/\tau = 10\%$. The continuous line represents the contribution of the ab-

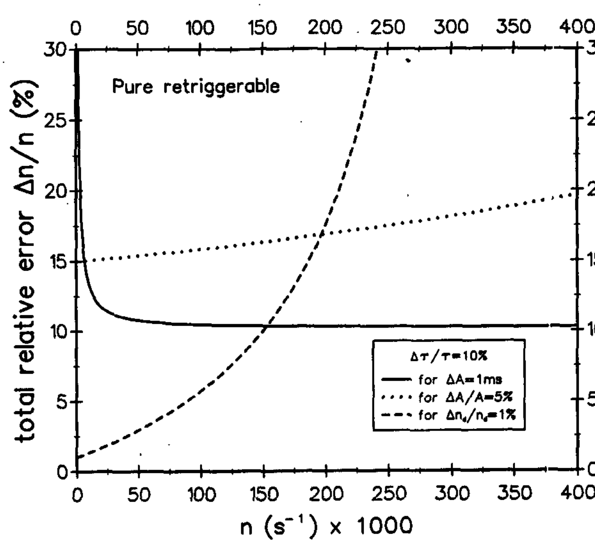

FIG. 2. The relative error in determining n is plotted as a function of n for $dn_d/n_d = 1\%$ if n is derived from n_d (dashed line) and for $dA/A = 5\%$ when n is derived from the activity A (dotted line). In both cases, $d\tau/\tau = 10\%$. The continuous line represents the contribution of the absolute error in A when n is derived from the activity.

solute error in A that results from the limited resolution of the clock and that is significant only at low n values. These values are typical of an FSSP with an activity clock set at 65 000 full range.

Figure 2 shows that at rates below 150 000 s^{-1} (concentration below 750 cm^{-3}), the evaluation of the actual rate from counted rate is the more accurate. At rates above this the solution using activity is the more accurate.

The question arises, what if the total strobes, fast resets, or activity were not recorded? In this case, the concentration correction must be derived from an estimate of n_d, since the activity correction shown above assumes a measured activity that already has the effects of coincidence taken into account in its value. If the FSSP in question was an older model that used only the single reset delay period, then nothing else is required to use the measured n_d to solve for n. If the FSSP used both the slow and fast reset delays, but n_T and n_f were not recorded, an approximation to f in (7) is 0.20. This has been verified experimentally in laboratory studies and by airborne measurements (Dye and Baumgardner 1984) from FSSPs that measure both the total strobes and fast resets; however, this value is dependent on the size spectra and can vary from 0.1 to 0.3.

2) ALTERNATIVE DERIVATIONS OF CONCENTRATION

The algorithms discussed in the previous section pertain to measurements made with conventional FSSPs and data systems. A number of investigators have been recording individual droplet sizes and the time between particle detections in the FSSP. This is different than the conventional systems that only record accumulated size distributions over fixed sampling intervals. The single particle measurements lead to new methods of calculating the droplet rate and minimize any effect of coincidence and dead time.

Baumgardner (1986) suggested deriving n from measurements of the distribution of the interarrival times between droplet detections. The Poisson statistics imply that the probability distribution of an interarrival time Δt being greater than any given time t is exponential; that is,

$$P(\Delta t > t) = e^{-nt}. \quad (12)$$

When an interarrival time frequency distribution is constructed the rate n is just the slope of the distribution (on a log scale). Under the Poisson hypothesis, this exponential law characterizes any frequency distribution of time intervals measured between any time origin and the arrival of the next droplet in the beam, as long as the time origin and the arrival of the next droplet are independent events. This is the case with the fast FSSP (Brenguier 1993) where time intervals are counted from the end of a pulse to the beginning of the next one. When the interarrival time distribution (IATD) is obtained from a standard FSSP, the time interval, measured between detections, consists of the interarrival time plus the pulse duration and the electronic dead time. The electronic dead time, which is constant, can be simply subtracted from the measured intervals but variations of the pulse duration due to coincidence effects are random and cannot be exactly subtracted from the measured values. However, Baumgardner et al. (1993) have shown that rates derived by this method will be negligibly affected if derived only from time intervals greater than t_m, where t_m is roughly the maximum pulse duration, plus dead time, likely to occur. The value of t_m will increase with n. A value of 7 μs is adequate for typical values of n.

Coincidence and dead-time errors can also be eliminated from the droplet rate measurement using the individual interarrival times (Baumgardner et al. 1993). A fixed time, t_m, is subtracted from each interarrival time and any negative times and a fraction (about 1/2) of the zero times are ignored. The actual rate n is calculated from this new sequence of interarrival times by dividing the number of arrivals by the sum of their interarrival times. The first approach we call the slope method and the second the compensation method.

d. Examination of the Poisson assumption

All of the above methods for deriving the actual rate are based on the assumption that the droplets are randomly and independently distributed in space. If this assumption is not valid then solutions of the coincidence equations for counted rate and activity will not be identical and the IATD will not be exponentially distributed. To varying degrees of sensitivity, the hypothesis of Poisson statistics can be tested. The coincidence equations may be combined (Brenguier 1989, 1990), the IATD may be compared with an exponential (Paluch and Baumgardner 1989), or the Fishing statistic (Baker 1992) may be applied to the sequence of interarrival times. The latter method is a well-defined, sensitive, statistical test that also yields information on the scales of the inhomogeneities in the droplet arrivals. Observations of non-Poisson statistics are generally interpreted as an inhomogeneity of the droplet spatial distribution. When stationarity does not apply, for example, when droplet concentrations fluctuate during a sample period, the rate derived by any of the methods described above is not equal to the average concentration of the sample. The rate derived from the coincidence equations (Brenguier 1989) and the rate derived from the slope method (Paluch and Baumgardner 1989), both termed "local rates," are somewhere between the maximum rate and the average rate of the sample. The rate derived from the IATD using the compensation method is less than the average rate (Baumgardner et al. 1993). Thus, from the IATD, some idea of the range of rates involved in the inhomogeneous sample can be obtained as well as upper and lower bounds of the average rate.

3. Summary

Measurements of the concentration from the FSSP underestimate actual droplet concentrations because of coincidence and electronic dead-time losses. The difference between measured and actual concentrations increases with increasing concentration and can easily exceed factor of 2 differences. A significant fraction of these losses can be recovered with appropriate processing algorithms that use the assumption of Poisson distributed particles to apply statistical corrections to the measurements. The FSSPs used by the scientific community are not identical and the processing algorithms are highly sensitive to the operating characteristics and environment of the probe. First-order corrections can be applied if the only measured parameter is the particle rate, which is the most fundamental output of this instrument. Further improvement is gained as additional information is recorded from the probe. The most comprehensive corrections are possible if the activity, total strobes, and fast resets are recorded from the FSSP.

FSSPs whose individual droplet counts can be recorded, along with the arrival time, provide the most robust measure of concentration since the distribution of arrival times provides a measure of the concentration that is virtually unaffected by coincidence and dead time. Many of the newer data systems being built by several organizations are now routinely recording these arrival time distributions without the necessity of recording every droplet event.

New developments are currently under way to improve the FSSP; for example, the fast FSSP (Brenguier 1993) eliminates electronic dead time and provides pulse amplitude, duration, and interarrival time for each detected particle such that coincidence corrections can be made more accurately.

APPENDIX

Definition of Annotation Used in Text

Term	Definition
A	probe activity: the amount of time the probe spends processing droplets (s)
DOF	depth of field (mm)
n	actual rate of droplets in sample volume (s^{-1})
f	DOF fraction: ratio of total strobes to total resets
n_d	measured droplet rate (s^{-1})
n_T	total strobe rate: rate of particles passing through the DOF (s^{-1})
n_f	fast reset rate: rate of particles passing outside the DOF (s^{-1})
n_r	total reset rate: rate of all particles passing through the beam (s^{-1})
N	actual droplet concentration (cm^{-3})
S	cross-sectional area of laser beam where droplets can be detected (cm^2)
v	velocity of droplets through the laser beam ($m\ s^{-1}$)
τ	pulse duration (transit time) of a droplet in the laser beam (s)
τ_d	a fixed electronic delay time after the passage of a droplet (s)
T	pulse duration including coincident droplets

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