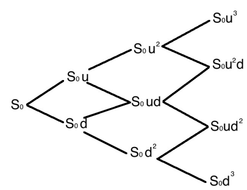
Computer Problem Set 1.1
The Cox-Ross-Rubinstein model

The present problem set is attached to Chapter 2 of the lectures notes. Please read its content in order to proceed to the requested implementation.



1. Consider the $n-$periods binomial model (see the example above with $n = 3$) defined by the coefficients $S_0 > 0, T > 0$,

$$u = u_n = e^{bh_n + \sigma\sqrt{h_n}}, d = d_n = e^{bh_n - \sigma\sqrt{h_n}}, h_n := \frac{T}{n},$$

for some given parameters $\sigma > 0, b \geq 0$. We denote by S_j^n the vector in $\mathbb{R}^{j+1}$ of possible prices at time j defined by $S_j^n(i) := S_0 u^{j-i} d^i$, $i = 0, \ldots, j$.

   (a) Build a function $\mathbf{Sn}(T, n, b, \sigma, j)$ which returns the vector S_j^n.

   (b) Consider a European call option with maturity T and strike $K \in \mathbb{R}_+$. Build a function $\mathbf{Payoffn}(T, n, b, \sigma, K)$ which returns the payoff vector of the option at maturity.

   (c) Let $r \geq 0$ be a constant interest rate. Build a function $\mathbf{Calln}(T, n, r, b, \sigma, K)$ which returns the price at time zero of the European call option.

   (d) At each time $t_j^n := jh_n$, denote by θ_j^n the hedging strategy corresponding to the above European option. Build a function $\mathbf{Deltan}(T, n, r, b, \sigma, K, j)$ which returns the vector of all values of θ_j^n.

   (e) Examine the dependence effect of the strike K on the functions **Calln**, **Deltan**, and comment. You may use various values of $K_i = 80 + i$, $i = 0, 1, 2, \ldots, 40$, a number of periods $n = 50$, and the parameters:

$$\sigma = 0.3, \quad r = b = 5\ \%, \quad S_0 = 100, \quad , \quad T = 2 \text{ (years)}. \tag{1}$$

2. We now consider the limiting Black-Scholes formula.

   (a) Build a function $\mathbf{Call}(T, r, \sigma, K)$ which returns the Black-Scholes price at time zero of the European call option. *The cumulative distribution function of the centered standard gaussian distribution is available in most softwares (Python : scipy.stats.norm.cdf, VBA-excel: NormDist, Scilab: cdfnor, Matlab: cdf).*

   (b) Define $\mathbf{err}(T, n, r, b, \sigma, K) := \frac{\mathbf{Calln}(T,n,r,b,\sigma,K)}{\mathbf{Call}(T,r,\sigma,K)} - 1$, plot the graph of this relative error in terms of the number of periods n, and comment. You may use the parameters (1) togeteher with a strike $K = 105$.