

Decomposition of Graphs: Representing Graphs

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Graph Algorithms
Data Structures and Algorithms

Learning Objectives

- Provide ways in which a graph can be represented on a computer.
- Understand the distinction between dense and sparse graphs and how it affects algorithm efficiency.

Outline

① Graph Representations

② Density and Runtimes

Last Time

Graphs consist of:

- Vertices (or nodes).
- Edges connecting pairs of vertices.

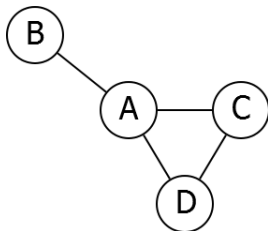
Representing Graphs

To compute things about graphs we first need to **represent** them.

There are many ways to do this.

Edge List

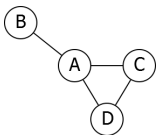
List of all edges:



Edges: (A, B) , (A, C) , (A, D) , (C, D)

Adjacency Matrix

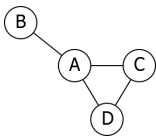
Matrix. Entries 1 if there is an edge, 0 if there is not.



	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
<i>A</i>	0	1	1	1
<i>B</i>	1	0	0	0
<i>C</i>	1	0	0	1
<i>D</i>	1	0	1	0

Adjacency List

For each vertex, a list of adjacent vertices.



A adjacent to *B*, *C*, *D*

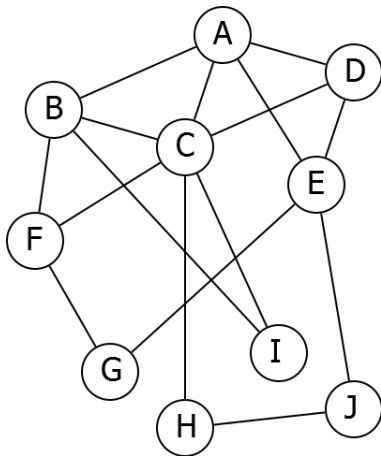
B adjacent to *A*

C adjacent to *A*, *D*

D adjacent to *A*, *C*

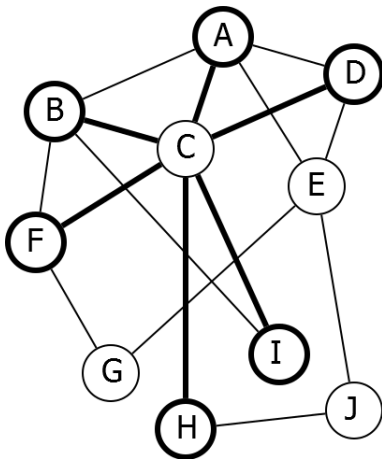
Problem

What are the neighbors of **C**?



Solution

A, B, D, F, H, I.



Summary

Different operations are faster in different representations.

Op.	Is Edge?	List Edge	List Nbrs.
Adj. Matrix	$\Theta(1)$	$\Theta(V ^2)$	$\Theta(V)$
Edge List	$\Theta(E)$	$\Theta(E)$	$\Theta(E)$
Adj. List	$\Theta(\deg)$	$\Theta(E)$	$\Theta(\deg)$

For many problems, want adjacency list.

Outline

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② Density and Runtimes

Algorithm Runtimes

Graph algorithm runtimes depend on $|V|$ and $|E|$.

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For example, $O(|V| + |E|)$ (linear time),

$O(|V||E|)$, $O(|V|^{3/2})$,

$O(|V| \log(|V|) + |E|)$.

Density

Which is faster, $O(|V|^{3/2})$ or $O(|E|)$?

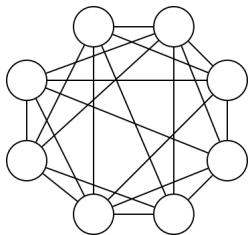
Density

Which is faster, $O(|V|^{3/2})$ or $O(|E|)$?

Depends on graph! Depends on the **density**, namely how many edges you have in terms of the number of vertices.

Dense Graphs

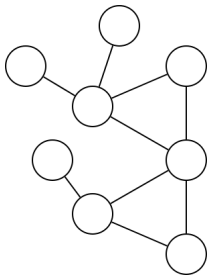
In dense graphs, $|E| \approx |V|^2$.



A large fraction of pairs of vertices are connected by edges.

Sparse Graphs

In **sparse** graphs, $|E| \approx |V|$.



Each vertex has only a few edges.

Next Time

Algorithms for exploring graphs.