Homework 4

Shaun Fedrick
Computational Physics Haverford College
(Dated: March 3, 2020)

1. INTRODUCTION

This homework involved the use of linear algebra, mainly elimination and eigen values,to solve various physics problems like voltages in a circuit or the probability of a particle existing at a given energy state.

2. 6.14

For this problem we had to use Kirchoffs loop law to derive the voltages at various nodes in the circuit. This was done by analyzing each node and recognizing that current flowing into a node must be equal to the current flowing out. Through inspection we are then able to formulate a system of equations describing the voltage at each node. We get the equations

$$\begin{aligned}
&\text{We begin with the first node} \\
&4V_1 - V_2 - V_3 - V_4 = V_+ \quad (1) \\
&\text{By inspecting the voltage difference at node 2 we get a system of:} \\
&-V_1 + 3V_2 - 0V_3 - V_4 = 0 \quad (2) \\
&\text{Similarly we get this for node 3} \\
&-V_1 + 0V_2 + 3V_3 - V_4 = V_+ \quad (3) \\
&\text{The 4th node is isomorphic to the first giving} \\
&-V_1 - V_2 - V_3 + 4V_4 = 0 \quad (4) \\
&\quad (5)
\end{aligned}$$

We then create a matrix using this system of equations and solve the matrix giving a voltage at every node. I found the voltages of V_1, V_2, V_3, V_4 to be $3, 1.66666667, 3.33333333,$ and 2 respectively .

3. 6.9

We must show that from the schrodinger equation $\mathcal{H}\Psi = E\Psi$ we get $\sum_{n=1}^{\infty} \Psi \int_0^L \sin(\frac{\pi m x}{L})\mathcal{H}\sin(\frac{\pi n x}{L})dx = \frac{1}{2}LEE\Psi_m$

we begin with the schrodinger equation

$$\mathcal{H}\Psi = E\Psi \tag{6}$$

We then plug in the fourier expansion of Ψ

$$\mathcal{H}\sum_{n=1}^{\infty} \Psi_n \sin(\frac{\pi n x}{L}) = E\sum_{n=1}^{\infty} \Psi_n \sin(\frac{\pi n x}{L}) \tag{7}$$

we then mutliply both sides by $\sin(\frac{\pi m x}{L})$ giving :

$$\sin(\frac{\pi m x}{L})\mathcal{H}\sum_{n=1}^{\infty} \Psi_n \sin(\frac{\pi n x}{L}) = E\sum_{n=1}^{\infty} \Psi_n \sin(\frac{\pi m x}{L})\sin(\frac{\pi n x}{L}) \tag{8}$$

If we take the integral of both sides from 0 to L we get the following by utilizing the constant value that

$$\int_0^L \sin(\frac{\pi m x}{L})\sin(\frac{\pi n x}{L})dx \text{ becomes when } m = n$$

$$\sum_{n=1}^{\infty} \Psi \int_0^L \sin(\frac{\pi m x}{L})\mathcal{H}\sin(\frac{\pi n x}{L})dx = \frac{1}{2}LE\Psi_m \tag{9}$$

$$\tag{10}$$

It is clear that this can then be written as matrix because Ψ_n on the left hand side will have n elements that will distribute onto the elements of the integral $\int_0^L \sin(\frac{\pi m x}{L})\mathcal{H}\sin(\frac{\pi n x}{L})dx$ over every column m in the matrix $\mathcal{H}$ creating a matrix equation of $\mathcal{H}\Psi_m = E\Psi_m$ and by the definition of this vector $\Psi_m$being equal to a scalar of itself when transformed by$\mathcal{H}$, Ψ_m is an eigen vector of $\mathcal{H}$

analytically we find the matrix components of $\mathcal{H}$ to be $\frac{2}{L}\int_0^L \sin(\frac{\pi m x}{L})\mathcal{H}\sin(\frac{\pi n x}{L})dx$
where it is 0 when both m and n are either even or odd a matrix
it is $\frac{-8amn}{\pi^2 * (m^2-n^2)^2}$ when either but not both of m and n are even
,and it is $\frac{aL^2M+\pi^2n^2h^2}{\pi^2 * (m^2-n^2)^2}$ when m=n
A matrix is symetric when $a_{mn} = anm$ through inspection of the above if statements it is clear that$\mathcal{H}$ is such. This is because the only points that affect symmetry of the matrix is when m doesn't equal n. For the first case where both m and n are even or both are odd the matrix is zero so we can ignore this if statement. This leaves $\frac{-8amn}{\pi^2 * (m^2-n^2)^2}$ when we consider $a_{mn} = anm$ we know the numerator will be the same by the commutative property multiplication and the denominator will be the same because the denominator is simply the absolute value of the distance from m to n which will not change by swapping m and n with each other meaning that $a_{mn} = anm$ is true for the matrix $\mathcal{H}$

I found the first 3 Energy states to be 5.836, 11.18, and 18.66 Electron volts I found that increase the matrix size to 10 caused marginal differences in the eigen values. There was not a noticeable difference of the values until you looked at the 8th eigen value of the matrix. I conclude that one can approximate an infinite matrix very well with only a few elements.

I found that my probability distribution was very small for these states, and because were weren't summing over all states it makes sense that these distribution do not satisfy the condition of the probability distribution equaling one once you integrate over the whole box.

4. CONCLUSIONS

I found this homework to be fair, and interesting even though I do not personally enjoy quantum mechanics. It felt a tad short, but that was much appreciated due to this being midterm season. Overall I enjoyed this problem set.

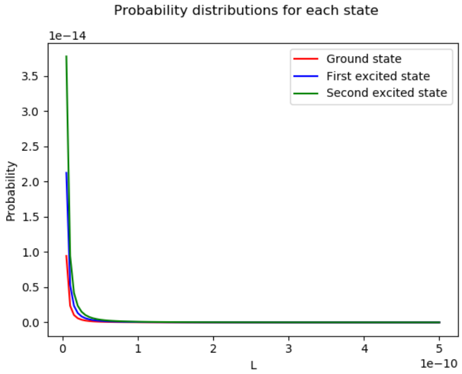

FIG. 1: Probability distribution