Homework 4

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1. INTRODUCTION

This homework involved the use of various numerical integration techniques. One notable method was Gaussian quadrature, which essentially uses a series of orthogonal polynomials to calculate weights of crucial x points of the integrand. These points are then used to calculate the integral. Furthermore, we covered techniques to compute double integrals, gradients, and improper integrals.

2. 5.12

Problem 5.12 involved evaluating the integral $\int_0^\infty \frac{x^3}{e^x-1}dx$ with any numerical method. I chose to use Simpson's rule which takes a linear combination of values of the function to form a quadratic between the points of the data set. I used 1000 points and since Simpson's is third order accurate globally I expected my answer to be accurate to $(\frac{a-b}{1000})^3$. Where a-b is the range of my integral, which in this case is 1 so I expect an accuracy of $1e{-9}$. We can transform $I(\omega) = \frac{h}{4\pi^2 c^2}\frac{\omega^3}{e^{\frac{h\omega}{k_b T}}-1}$ to $I(\omega) = \frac{k_b^4 T^4}{4\pi^2 c^2 h^3}\int_0^\infty \frac{x^3}{e^x-1}dx$ by integrating over all frequencies and setting x to $\frac{h\omega}{k_b T}$ This then gives $dw = \frac{k_b T}{h}dx$ plugging in the value for x and dw gives $I(\omega) = \frac{k_b^4 T^4}{4\pi^2 c^2 h^3}\int_0^\infty \frac{x^3}{e^x-1}dx$ to evaluate this integral we used a change of basis of $x = \frac{z}{1-z}$ and instead of having an integral of the form $I(\omega) = \frac{k_b^4 T^4}{4\pi^2 c^2 h^3}\int_0^\infty \frac{x^3}{e^x-1}dx$ I had an integral of the form $I(\omega) = \frac{k_b^4 T^4}{4\pi^2 c^2 h^3}\int_0^1 \frac{(\frac{z}{1-z})^3}{e^{(\frac{z}{1-z})}-1}dz$.In this way we can make infinity finite by approaching $z = 1$ We can find the Stefan Boltzman constant by evaluating this integral and factoring out T^4. I found the stefan boltzman constant to be $5.670366816083174e-08$ which agrees nicely with the real stefan boltzmann constant of $5.670374419e-08$.

3. 5.14

For 5.14 we must derive the force in the z direction. In this spirit, I labeled the key components of the system.

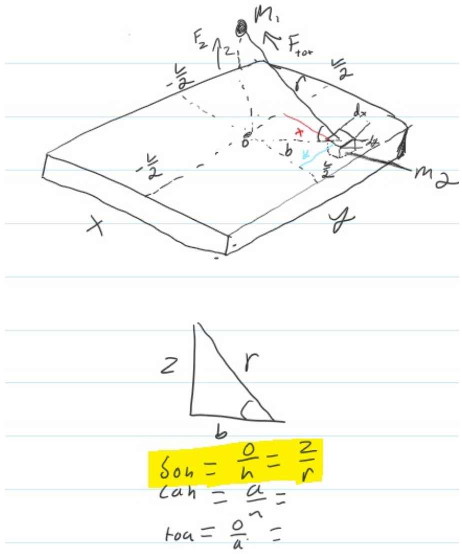


FIG. 1: Diagram of System

we begin with the equation for gravitational force

$$F_G = \frac{m_1 m_2}{r^2} \tag{1}$$

Consider m_2 as an infinitesimal mass a distance r from the 1 kg point mass m_1

$$F_G = \frac{m_1 m_i}{r^2} \tag{2}$$

From the diagram we can see that r is:

$$r = \sqrt{(x^2 + y^2 + z^2)} \tag{3}$$

we also only care about the z direction of the force from the digram and plugging in 3

$$F_{Gz} = \frac{z}{r} F_G \tag{4}$$

$$F_{Gz} = \frac{z}{\sqrt{(x^2 + y^2 + z^2)}} \frac{m_1 m_i}{\sqrt{(x^2 + y^2 + z^2)}^2} \tag{5}$$

$$F_{Gz} = \frac{z}{\sqrt{(x^2 + y^2 + z^2)}} \frac{m_1 m_i}{(x^2 + y^2 + z^2)} \tag{6}$$

$$F_{Gz} = z \frac{m_1 m_i}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \tag{7}$$

integrating over every infinitesimal mass over x length and y length

$$F_{Gz} = m_1 z \int_{\frac{-L}{2}}^{\frac{-L}{2}} \int_{\frac{-L}{2}}^{\frac{-L}{2}} \frac{m_i}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} dxdy \tag{8}$$

We can then integrate over the space and a graph of force z can be seen in 2. Notice that as you approach zero the force abruptly goes to zero. This is not physical because the force between two solid objects radius ≈ 0 will never be zero because Vanderwalls forces and intermolecular forces will come into effect at this small of a distance and the force will be dominated by these processes. Not to mention that gravity has not been resolved for really small scales. As a result, it is clear that this result is not physical instead it is likely a result from our a discrete approach to zero in the code and could be alleviated by a change of basis as we did with integrating to infinity

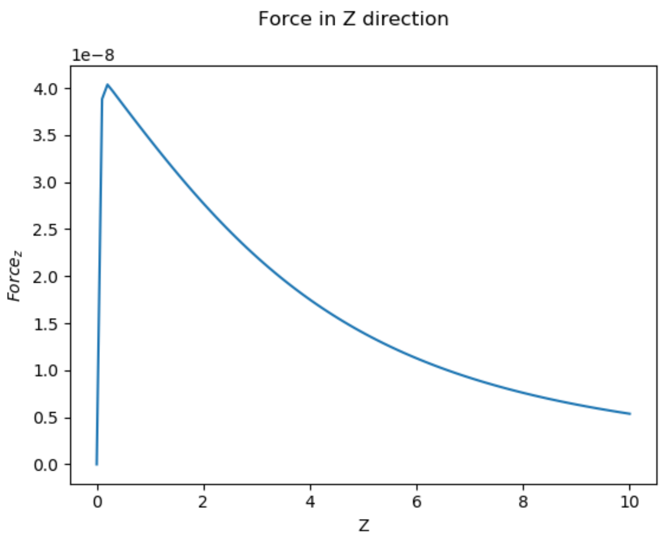
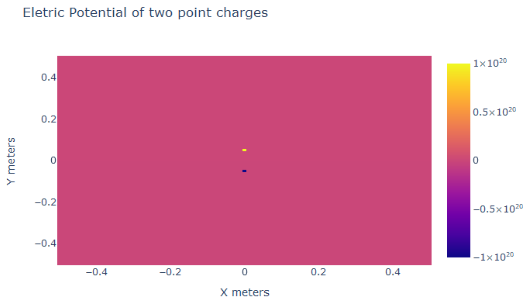


FIG. 2: Force z from the gravity of the plate

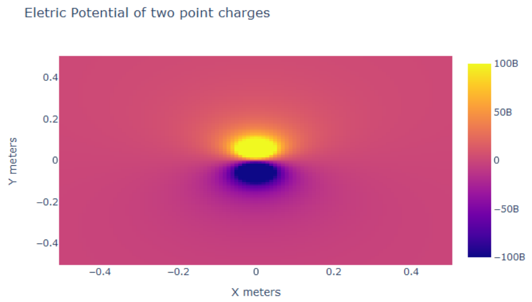
4. 5.21

Problem 5.21 Was a great deal like 5.14 it involved calculating electric potential from a set of charge distributions. For two point charges we can use $\phi = \frac{q}{4\pi\epsilon_0 r}$. Notice though that as r goes to zero the electric potential blows up.

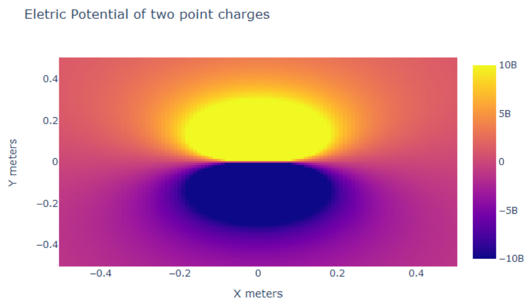
Physically the limit as r approaches 0 is a phenomenon that is left to the realm of quantum mechanics. However, even if r is small and not really small, as it is in quantum mechanics, it makes it hard to visualize the electric potential because the potential around these small r's make negligible any interesting r's that are larger. To deal with this I made several plots that allow different max values and min values to help in the visualization of the electric potential. The same applies for the electric field and these plots can be seen respectively in 3 and 4.



(a) Heat map with max of 10^{20} and min of -10^{20}

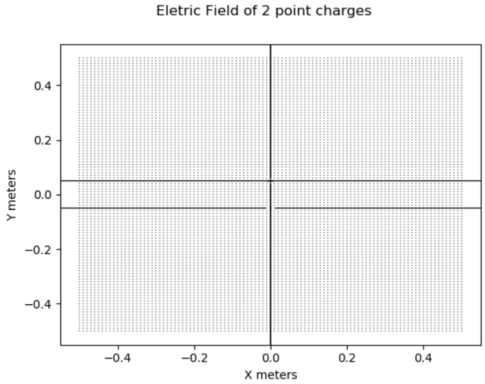


(b) Heat map with max of 10^{11} and min of -10^{11}

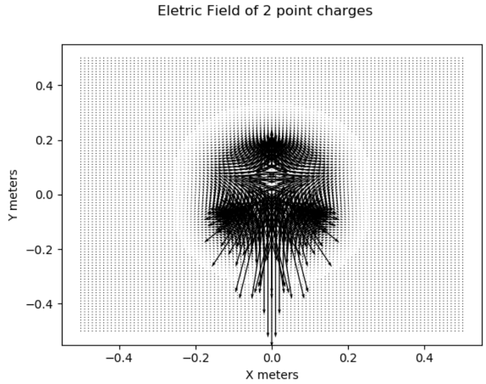


(c) Heat map with max of 10^{10} and min of -10^{10}

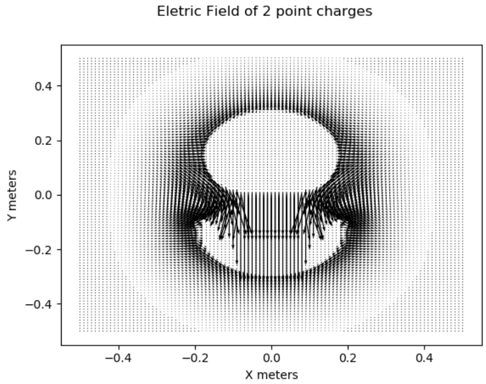
FIG. 3: Part a Potential



(a) Electric Field with max of 10^{20} and min of -10^{20}



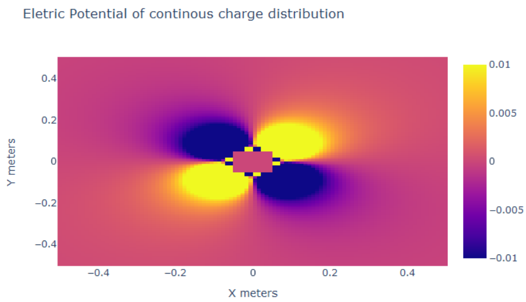
(b) Electric field with max of 10^{11} and min of -10^{11}



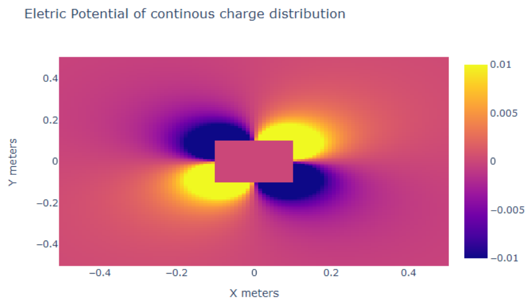
(c) Electric field with max of 10^{10} and min of -10^{10}

FIG. 4: Part a Electric Field

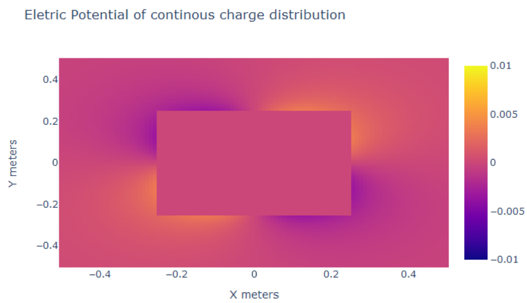
Part c was slightly more complicated, it involved using a continuous charge distribution. This charge distribution was located in a 10cm by 10cm square in the center of our grid. The electric potential is then given by $\int\int_{\frac{-L}{2}}^{\frac{L}{2}} \frac{\sigma}{r} dxdy$ where r is the distance from each infinitesimal $dxdy$ in the charge distribution. To get the electric potential I performed this integral using Gaussian quadrature for each grid of a matrix A where the center of the matrix was where the charge distribution was located, I set the inside of the charge distribution to zero to effectively ignore the case where r=0 for some set of infinitesimal charges . Like in part a, I ran into a problem visualizing the heat map because as one approaches the center, the electric potential blows up. To help with the visualization, I enlarged the square of zero charge distribution so that finer features of the Electric potential and Electric Field can be seen. These graphs can be seen respectively in 5 and 6.



(a) Electric potential with a 10 cm by 10 cm box

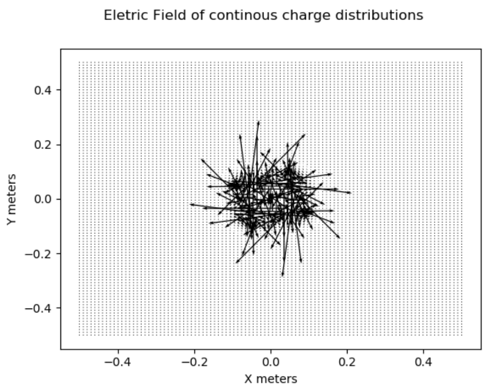


(b) Electric potential with a 10 cm by 10 cm box

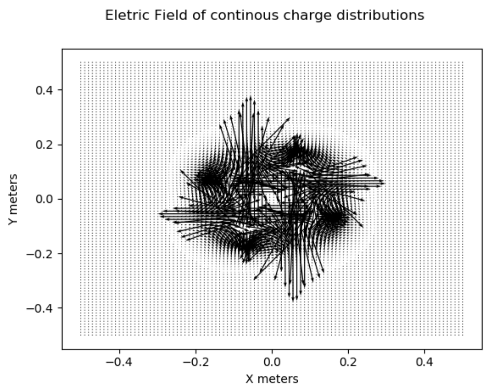


(c) Electric potential with a 10 cm by 10 cm box

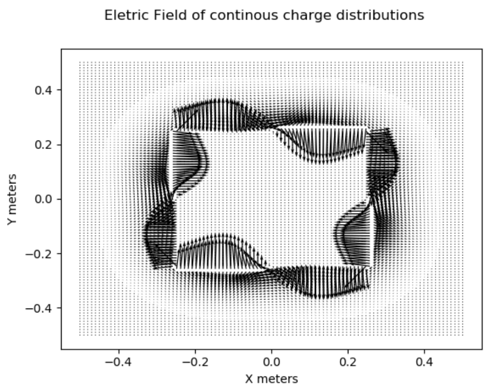
FIG. 5: Part b Potential



(a) Electric field with a 10 cm by 10 cm box



(b) Electric field with a 20 cm by 20 cm box



(c) Electric field with a 50 cm by 50 cm box

FIG. 6: Part b Eletric

5. CONCLUSIONS

I have mixed feelings about this problem set. Although I really loved this problem set and the end results were very interesting, the process of getting quiver and heatmaps to work felt tedious. I would have appreciated more structure in showing how to get quiver and heat maps to work effectively instead of dredging through the trenches of documentation and stack exchange as I did. As far as length goes, I found this homework to be fairly long,and again I wish we had spent more time with the idea of integrating double integrals before using it on the problem set. I enjoyed the derivations having clear end goals, so you could easily check that you were on the right track. I also liked that part c didn't just give you instructions on how to find the potential, and you had to figure it out. It made the problem very fun. Overall I enjoyed this problem set, but it could've used some supplemental materials on the plotting aspects.