UMass Boston CS 310 Homework 8
Due in class on Tuesday, May 2, 2017

1. Let d be the maximum degree of the vertices in a graph G. Prove that we can color G with $d+1$ colors. Hint: by induction.
2. Consider this statement: In any directed graph $G = (V, E)$, when DFS visits a vertex $u \in V$, then every undiscovered vertex v such that u has a path to v must be discovered before DFS returns from u. Is this statement true or false? If true, give a proof; if false, give a counterexample.
3. Consider this statement: In any undirected graph $G = (V, E)$, there must be an even number of vertices whose degree is odd. Is this statement true or false? If true, give a proof; if false, give a counterexample.
4. Consider the problem of determining whether an undirected graph $G = (V, E)$ contains a *triangle* (cycle of length three).
   (a) Give an $O(|V|^3)$ to find a triangle if one exists.
   (b) Improve your algorithm to run in time $O(|V| \cdot |E|)$. You may assume $|V| \leq |E|$.
5. You are given a weighted graph and its minimum spanning tree. There are n vertices and m edges in the graph.
   (a) Design a polynomial-time algorithm that finds the smallest change in the weight of a non-MST edge that would cause a change in the MST.
   (b) Analyze the runtime of your algorithm in terms of n and m.