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CS320L

Homework 5

**Question 1: Family Issues**

a) Write down the relation *R* = {(*a*, *b*) | *a* is a parent of *b*} defined on the set *P* of the seven people, so that it reflects the family structure specified above. Use the set notation and the matrix notation.

R = {(Mary, Peter), (Mary, Robert), (Peter, Christine), (Peter, John), (Robert, Elena), (Christine, Daniel)}

$$M_R = \begin{array}{c} \begin{array}{ccccccc} M & P & R & C & J & E & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

b) Use the matrix notation as your starting point for computing the transitive closure of *R*. Apply the Boolean power method we discussed in class. Once you have derived the matrix representing the transitive closure of *R*, also translate it into set notation.

$$M_R = \begin{array}{c} \begin{array}{ccccccc} M & P & R & C & J & E & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_R^{[2]} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_R^{[3]} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_R^{[4]} = M_R^{[5]} = M_R^{[6]} = M_R^{[7]} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M^R = [M^R \text{ v } M_R^{[2]} \text{ v } M_R^{[3]}] = \begin{array}{c} \begin{array}{ccccccc} M & P & R & C & J & E & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

R = {(Mary, Robert), (Mary, Peter), (Mary, Christine), (Mary, John), (Mary, Elena), (Mary, Daniel), (Peter, Christine), (Peter, John), (Peter, Daniel), (Robert, Elena), (Christine, Daniel)}

c) What does the transitive closure of *R* specify? What name could you give it?

The transitive closure transforms this from a parent relation, to a descendent relation. Where b is a descendent of a.

*R* = {(*a*, *b*) | *b is a descendent of a*}

**Question 2: Count the Relations**

a) How many different equivalence relations can we define on the set A = {x, y, z}?

There are 5 relations:
{(x, x), (y, y), (z, z)},
{(x, x), (x, y), (y, x), (y, y), (z, z)},
{(x, x), (x, z), (y, y), (z, x), (z, z)},
{(x, x), (y, y), (y, z), (z, y), (z, z)},
{(x, x), (x, y), (x, z), (y, x), (y, y), (y, z), (z, x), (z, y), (z, z)}

b) How many different partial orderings can we define on the set A = {a, b}?

There are 3 relations:
{(a, a), (b, b)},
{(a, a), (b, b), (b, a)},
{(a, a), (b, b), (a, b)}

c) How many different total orderings can we define on the set A = {p, q}?

There are 2 relations:
{(p, p), (q, q), (q, p)},
{(p, p), (q, q), (p, q)}

**Question 3: Relations**

a) The empty relation R = ∅ defined on the natural numbers.

Symmetric, Transitive.

b) The complete relation R = **N**×**N** defined on the natural numbers.

Reflexive, Symmetric, transitive.

c) The relation R on the positive integers where aRb means a | b (i.e., a divides b).

Reflexive, transitive, antisymmetric.

d) The relation R on the positive integers where aRb means a < b.

asymmetric.

e) The relation R on the positive integers where aRb means $a \geq b$.

Reflexive, antisymmetric.

f) The relation R on {w, x, y, z} where R = {(w, w), (w, x), (x, w), (x, x), (x, z), (y, y), (z, y), (z, z)}.

Reflexive.

g) The relation R on the integers where aRb means $a^2 = b^2$.

Reflexive, Symmetric, transitive.

**Question 4: Possible and Impossible Graphs**

a) A simple graph with 6 vertices, whose degrees are 2, 2, 2, 3, 4, 4.

No. It is not possible to have graph with one vertex of odd degree

b) A simple graph with 8 vertices, whose degrees are 0, 1, 2, 3, 4, 5, 6, 7.

No. It is not possible to have a vertex of degree 7 and a vertex of degree 0.

c) A simple graph with 4 vertices, whose degrees are 1, 2, 3, 3.

No. It does not satisfy the Handshaking Theorem, since the sum of the degrees is odd.

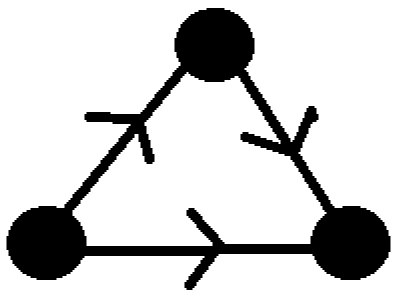
d) A simple graph with 5 vertices, whose degrees are 2, 3, 4, 4, 4.

No. It is not possible to have a graph with one vertex of odd degree.

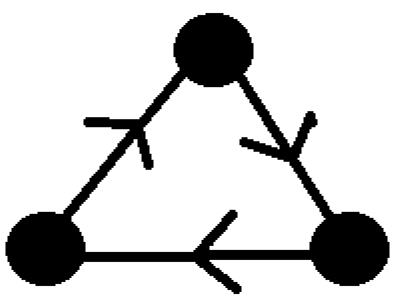
e) A simple graph with 4 vertices, whose degrees are 1, 1, 2, 4.

No. In a simple graph with 4 vertices, the largest degree a vertex can have is 3.

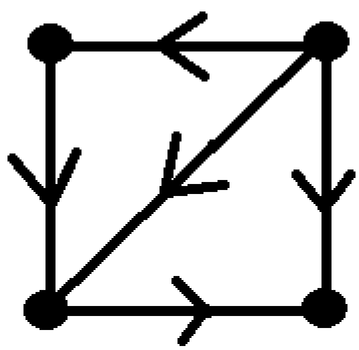
f) A simple digraph with 3 vertices with in-degrees 0, 1, 2 and out-degrees 0, 1, 2.



g) A simple digraph with 3 vertices with in-degrees 1, 1, 1 and out-degrees 1, 1, 1.



h) A simple digraph with 4 vertices with in-degrees 0, 1, 2, 2 and out-degrees 0, 1, 1, 3.



i) A simple digraph with 5 vertices with in-degrees 0, 1, 2, 4, 5 and out-degrees 0, 3, 3, 3, 3.

No. In a simple graph with 5 vertices, there cannot be a vertex with indegree 5.

j) A simple digraph with 4 vertices with in-degrees 0, 1, 1, 2 and out-degrees 0, 1, 1, 1.

No. The sum of the outdegrees must equal the sum of the indegrees.

**Question 5 (Bonus): Equivalent Integers**

a) As you know, any equivalence relation partitions the set on which it is defined into equivalence classes. Write down the partitioning of S by the equivalence relation R.

Equivalent classes of R:
{{2,3,4,5,7,8,9,11,13,16,17,19},{6,10,12,14,15,18,20,21}}

b) Define an equivalence relation Q on the same set S that partitions it into exactly 10 equivalence classes. Write down the definition of Q and the resulting partitioning of S.

Partitioning of Q:
{{2,3},{4,5},{7,8},{9,11},{13,16},{17,19},{6,10},{12,14},{15,18},{20,21}}