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Homework 3

CS320L

1. Professor P. has an odd habit: He only buys red, green, and blue books, and he keeps them in completely unordered stacks. However, he does keep track of the total number of books he owns. One day he decides to get a bookshelf so that he can put all books of one of the colors onto it. Unfortunately, when he is at the shelf store, he only knows that he has a total of 271 books, but he does not know how many books of each color he owns. What is the minimum number of books that the new shelf must be able to hold so that Professor P. can be sure he can put all his books of one of the colors onto it? He does not care which color it will be.

Well, this is not exactly the pigeonhole principle that applies, but the idea is very similar. The question is: If a set of 271 books is divided into 3 disjoint subsets, what is the maximum size that the smallest subset could possibly have? The smallest subset will be maximal if the three subsets are equal in size. The closest we can get to equal size is having 90, 90, and 91 books in the three sets, which means that the smallest set can never contain more than 90 books. Thus, Professor P. needs a shelf that can hold at least 90 books.

2. Use mathematical induction to show that the equation $1 + 3 + 5 + \ldots + (2n - 1) = n^2$ is true for all positive integers.

Proof:
Step 1: For n = 1, the above equation holds true. $1 = 1^2$
Step 2: Assume the equation is true for n, and prove the equation is true for n + 1.

$1 + 3 + 5 + \ldots + (2n - 1) + (2(n + 1) - 1) = (n + 1)^2$	(simplify left side)
$n^2 + (2n + 1) = (n + 1)^2$	(replace left sum with n^2)
$n^2 + 2n + 1 = (n + 1) * (n + 1)$	(multiply right side)
$n^2 + 2n + 1 = n^2 + 2n + 1$	(QED)

We can see from the above equation that the equation is true for n + 1, therefore it must be true for any value of n.

3. You got five dice for your birthday, and feel obligated to actually use them. So you are playing around with them and starting to do some probability computations about the outcomes of rolling all five dice at the same time.

a) What is the probability that none of the five dice show the number two?

$\left(\frac{5}{6}\right)^5 = \frac{3125}{7776}$

b) What is the probability that all five dice show the same number?

$\frac{6}{7776}$

c) **Bonus:** What is the probability that no two dice show the same number?

$\frac{6}{6} * \frac{5}{6} * \frac{4}{6} * \frac{3}{6} * \frac{2}{6} = \frac{720}{7776}$

d) **Bonus:** What is the probability that two of the dice show the number six and the other three show the number one?

$\frac{10}{7776}$

4. Give a recursive definition of each of the following sequences (n = 1, 2, 3, …):

A. $a_n = n + 3$
$a_0 = 3$
$a_{n+1} = a_n + 1$

B. $a_n = 2n$
$a_0 = 0$
$a_{n+1} = a_n + 2$

C. $a_n = (-1)^n$
$a_0 = 1$
$a_{n+1} = -1a_n$

D. $a_n = 2n!$
$a_0 = 2$
$a_{n+1} = a_n * n$

5. Show every step of your computation for the following conversions:

a) Convert the decimal number 2885 into its hexadecimal expansion.

2885 % 16 = 5
180 % 16 = 4
11 % 16 = 11

Hexadecimal: $(B45)_{16}$

b) Convert the binary number $(1101100)_2$ into its decimal expansion.

$(1*2^6) + (1*2^5) + (0*2^4) + (1*2^3) + (1*2^2) + (0*2^1) + (0*2^0) = 4 + 8 + 32 + 64 = 108$

Decimal: $(108)_{10}$

c) Convert the octal number $(7435)_8$ into its hexadecimal expansion.

$(7435)_8 = (7 * 8^3) + (4 * 8^2) + (3 * 8^1) + (5 * 8^0) = 3869$

3869 % 16 = 13
241 % 16 = 1
15 % 16 = 15

Hexadecimal: $(F1D)_{16}$

d) Convert the hexadecimal number $(AF81)_{16}$ into its binary expansion.
Hex to decimal:
$(AF81) = (10 * 16^3) + (15 * 16^2) + (8 * 16^1) + (1 * 16^0) = 44929$

44929% 2 = 1
22464 % 2 = 0
11232 % 2 = 0
5616 % 2 = 0
2808 % 2 = 0
1404 % 2 = 0
702 % 2 = 0
351 % 2 = 1
175 % 2 = 1
87 % 2 = 1
43 % 2 = 1
21 % 2 = 1
10 % 2 = 0
5 % 2 = 1
2 % 2 = 0
1 % 2 = 1

Binary: $(1010\ 1111\ 1000\ 0001)_2$

6. Count the following passwords and show how you did it. You do not have to compute the explicit numbers but can leave expressions such as $26^6 + 5^6$ as they are.

a) How many passwords of length 6 are there if all English lower- and uppercase letters and digits are allowed?

Letters and digits: 26 + 26 + 10 = 62
Passwords: 62^6

b) How many passwords of at most length 6 are there if all English lower- and uppercase letters and digits are allowed?

Letters and digits: 26 + 26 + 10 = 62
Passwords: $62^6 + 62^5 + 62^4 + 62^3 + 62^2 + 62$

c) How many passwords of length 6 are there if all English lower- and uppercase letters and digits are allowed and the password must contain at least one digit?

Letters and digits: 26 + 26 + 10 = 62
Passwords: $62^5 * 10$

d) How many passwords of length 6 are there if all English lowercase letters and digits are allowed but no letter or digit may occur more than once?

Letters and digits: 26 + 26 + 10 = 62
P(62,6)
Passwords: 62 * 61 * 60 * 59 * 58 * 57

e) **Bonus:** How many passwords of length 6 are there if they have to consist of 4 English uppercase letters and 2 digits in any order?

Letters: 26
Passwords: $26^4 * 10^2$

f) **Bonus:** How many passwords of length 6 are there if they have to consist of 4 English uppercase letters and 2 digits, and the 2 digits cannot be next to each other?

Letters: 26
Passwords: