Scott Fenton

Homework 4

CS320L

1. Let *X* be the random variable that is defined as the greater of the two numbers that appear when a pair of dice is rolled. For example, if you roll 2 and 5, then *X* = 5.

a)
P(1) = 1/36
P(2) = 3/36
P(3) = 5/36
P(4) = 7/36
P(5) = 9/36
P(6) = 11/36

E(x) = 1*1/36 + 2*3/36 + 3*5/36 + 4*7/36 + 5*9/36 + 6*11/36
E(x) = 5

Var(x) = (1*1/36)^2 + (2*3/36)^2 + (3*5/36)^2 + 4*7/36 + 5*9/36 + 6*11/36
Var(x) = (1/36 + (36/36) + (

b)
P(1) = 1/12
P(2) = 1/12
P(3) = 1/12
P(4) = 1/12
P(5) = 1/12
P(1) = 7/12

E(x) = 1*1/12 + 2*1/12 + 3*1/12 + 4*1/12 + 5*1/12 + 6*7/12
E(x) = 4.75
E(x) = 5

c)
P(1) = 1/12
P(2) = 1/12
P(3) = 1/12
P(4) = 1/12
P(5) = 1/12
P(1) = 5/12

E(x) = 1*1/12 + 2*1/12 + 3*1/12 + 4*1/12 + 5*1/12 + 6*7/12
E(x) = 3.75
E(x) = 4

d)
In the final state the numbers that appear on the flat die, and the numbers that appear on the normal die are independent. The chance of rolling a number on the flat die is 50% for 1 and 50% for 6. On the normal die, the chance of rolling any number is roughly 16%. However, rolling the flat dice does not affect the chance of rolling a number on the normal dice.

2. Botanists at UMass Boston recently discovered a new local flower species that they named the Boston powerflower. It has beautiful, blue blossoms, and each flower lives for only one summer. During that time, each flower produces 13 seeds. Three of these seeds will turn into flowers in the following year, and the remaining ten seeds will turn into flowers the year after. As the name powerflower suggests, these seeds always turn into flowers; there is no failure ever. During the year of this discovery (let us call it year zero), the scientists found three powerflowers on the UMass campus, and in the following year (year one), there were already eight of them.

a)
Initial Conditions:
$f_0 = 3$
$f_1 = 8$

$f_n = 3f_{n-1} + 10f_{n-2}$

b)
$f_2 = 3f_1 + 10f_0 = 3(8) + 10(3) = 54$
$f_3 = 3f_2 + 10f_1 = 3(54) + 10(8) = 242$
$f_4 = 3f_3 + 10f_2 = 3(242) + 10(54) = 1266$
$f_5 = 3f_4 + 10f_3 = 3(1266) + 10(242) = 6218$

c)
Characteristic equation:
$r^2 - 3r – 10$
$(r + 2)(r – 5)$
$R = -2, 5$

General solution:
$f_n = x5^n + y(-2)^n$
$f_0 = 3 = x + y$
$f_1 = 8 = 5x – 2y$
$x = 2$
$y = 1$

$f_n = 2*5^n + (-2)^n$

3. Let us assume that the probability of the Boston Red Sox to beat the New York Yankees at a baseball game is 70% and that no ties are possible, i.e., the New York Yankees win 30% of the time. Furthermore, the result of each game is independent of the results of any previous games.

a) Easier: Compute probability of complementary event -E, i.e., Red Sox winning 8, 9, or10 games:

p(winning 10 games) = $0.7^{10} = 0.02825$
p(winning 9 games) = $C(10, 9) \cdot 0.7^{9} \cdot 0.3 = 0.12106$
p(-E) = 0.02825 + 0.12106 = 0.14931
p(E) = 1 – p(-E) = 0.85069 or 85.069%

b) p = C(10, 7)·0.37·0.73= 120·0.0000750141 = 0.009 or 0.9%

c) p = $0.7^{4} = 0.2401$ or 24.01%

d) p = $0.3^{4} = 0.0081$ or 0.81%

e)

f)

4. Urns and Probabilities

a) Event A and Event B are independent events if the occurrence of one of them does not affect the probability of the occurrence of the other. When you draw a red ball in event A, the chance of a red ball being pulled are 3 out of 7. In event B, you do not put the ball back from event A, so the chance of pulling another ball is 2 out of 6. In event A, the chance of pulling a red ball out is roughly 43%, and in event B, the chance of pulling a red ball out is 33%. Since event A, effects the probability of picking a red ball in event B, the two events are not independent.

P(A) = 43%
P(B) = 33%

b) Event C and Event D are independent events if the occurrence of one of them does not affect the probability of the occurrence of the other. When you draw the two balls in Event C, the chance of picking a red ball is 3 out of 7. When you draw the red ball in Event D, the chance of picking a red ball is 2 out of 5. The chance is reduced by roughly 3% from Event C to Event D of picking a red ball. Since event C effects the probability of event D, the two events are not independent.

P(C) =
P(D) =