

Fuzzy sets

Uncertainty

- **Vagueness** :- Indefinite or indistinct in nature or character, as ideas or feelings
 - Fuzzy sets
 - The elements in a set will give different values
 - Eg: White color car in a campus.
- **Ambiguity** :- The quality of being open to more than one interpretation; inexactness.
 - Fuzzy measurements
 - The set itself will give different values
 - Eg: Criminal trial

Fuzzy sets: Basic concepts

- The characteristic function of a crisp set assigns a value of **either 1 or 0** to each individual in the universal set.
- This function is can be generalized such that the **values assigned to the elements of the universal sets fall within a specific range** and indicate the **membership grade** of these elements in the set.
- Larger values denote higher degrees of set membership. Such a function is called a **membership function** and the set defined by it a **fuzzy set**.

Fuzzy sets: Basic concepts

Two distinct notations are most commonly employed in the literature to denote membership functions. In one of them, the membership function of a fuzzy set A is denoted by μ_A ; that is,

$$\mu_A : X \rightarrow [0, 1].$$

In the other one, the function is denoted by A and has, of course, the same form:

$$A : X \rightarrow [0, 1].$$

A fuzzy set is a set containing elements that have **varying degrees of membership** in the set.

members of a crisp set would not be members unless their membership is **full, or complete**, in that set (i.e., their membership is assigned a value of 1).

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A notation convention for fuzzy sets when the universe of discourse, X , is discrete and finite, is as follows for a fuzzy set \tilde{A} :

$$\tilde{A} = \left\{ \frac{\mu_{\tilde{A}}(x_1)}{x_1} + \frac{\mu_{\tilde{A}}(x_2)}{x_2} + \cdots \right\} = \left\{ \sum_i \frac{\mu_{\tilde{A}}(x_i)}{x_i} \right\}. \quad (2.20)$$

$$\tilde{A} = \left\{ \frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\} \quad \text{and} \quad \tilde{B} = \left\{ \frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5} \right\}.$$

Fuzzy subset

Let A and B any two fuzzy sets with membership functions μ_A and μ_B then A is called a fuzzy subset of B if $\mu_A(x) \leq \mu_B(x)$ For every $x \in U$ and is denoted by

$$\underline{A} \preceq B$$

The fuzzy set A is called proper fuzzy set B if $\mu_A(x) \leq \mu_B(x)$ For every $x \in U$ and $\mu_A(x) < \mu_B(x)$ For at least one $x \in U$ and denoted by

$$A \prec B$$

Equal Fuzzy sets

A fuzzy set A is said to be equal to fuzzy set B if $\mu_A(x) = \mu_B(x)$

$$\forall x \in U$$

Eg: Let $U = \mathbb{R}$

A = “All integer numbers close to zero” $A = \{(x_i, \eta_A(x_i)) / x_i\}$

$$A = \{(-2, 0.3), (-1, 0.5), (0, 1), (1, 0.5), (2, 0.3)\}$$

Proper fuzzy subset of A

$$B = \{(-2, 0.25), (0, 1), (2, 0.25)\}$$

Equal Fuzzy sets

A fuzzy set A is said to be equal to fuzzy set B if $\mu_A(x) = \mu_B(x)$

$$\forall x \in U$$

$$A = \{(-2, 0.3), (-1, 0.5), (0, 1), (1, 0.5), (2, 0.3)\}$$

$$B = \{(-2, 0.3), (-1, 0.4), (0, 0.9), (1, 0.5), (2, 0.2)\}$$

B is a Proper fuzzy subset of A

$\mu_A(x) \neq \mu_B(x)$ so not equal [Equality of fuzzy set]

Empty fuzzy set

An empty fuzzy set can be defined by the membership function

$$\eta_{\phi} : U \rightarrow [0,1]$$

and

$$\eta_{\phi}(x) = 0; \forall x \in U$$

Support of a fuzzy set

Let A be any fuzzy set with membership function η_A

Support of A is denoted as A^* or $\text{Supp}(A)$

$$\text{Supp}(A) = \{x \in U / \eta_A(x) > 0\}$$

A^* is a crisp set under line set of fuzzy set A

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$$\text{Supp}(A) = \{x \in U / \eta_A(x) > 0\}$$

A^* is a crisp set under line set of fuzzy set A

$$A = \{(-2, 0.3), (-1, 0.5), (0, 1), (1, 0.5), (2, 0.3)\}$$

$$A^* = \{-2, -1, 0, 1, 2\}$$

Support of a fuzzy set

The *height*, $h(A)$, of a fuzzy set A is the largest membership grade obtained by any element in that set. Formally,

$$h(A) = \sup_{x \in X} A(x). \quad (1.12)$$

A fuzzy set A is called *normal* when $h(A) = 1$; it is called *subnormal* when $h(A) < 1$. The height of A may also be viewed as the supremum of α for which ${}^\alpha A \neq \emptyset$.

α - cut

α - cut of a fuzzy set A denoted by

$$A_{\alpha} = \{x \in U / \eta_A(x) \geq \alpha, \alpha \in [0,1]\}$$

A_{α} is a crisp set.

$$A = \{(-2,0.3), (-1,0.5), (0,1), (1,0.5), (2,0.3)\}$$

$$A_{0.5} =$$

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$$A_{0.5} = \{-1,0,1\}$$

Strong α - cut

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$$A_\alpha = \{x \in U / \eta_A(x) > \alpha, \alpha \in [0,1]\}$$

A_α is a crisp set.

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$$A_{0.5} =$$

Strong α - cut

Strong α - cut of a fuzzy set A denoted by

$$A_{\alpha} = \{x \in U / \eta_A(x) > \alpha, \alpha \in [0,1]\}$$

A_{α} is a crisp set.

$$A = \{(-2,0.3), (-1,0.5), (0,1), (1,0.5), (2,0.3)\}$$

$$A_{0.5} = \{0\}$$

$$\alpha_1 \leq \alpha_2 \leq \alpha_3 \leq \alpha_4 \dots \leq \alpha_n$$

then

$$A_{\alpha_1} \geq A_{\alpha_2} \geq A_{\alpha_3} \geq \dots A_{\alpha_n}$$

Level set of a fuzzy set

Level set of a fuzzy set A denoted by

$$\lambda_A = \{\alpha \in [0,1] / \eta_A(x) > 0\}$$

$$\lambda_\alpha = \{\alpha \in [0,1] / \eta_A(x) > \alpha\}$$

for at least one $x \in U$

$$A = \{(-2,0.3), (-1,0.5), (0,1), (1,0.5), (2,0.3)\}$$

$$\lambda_A = \{0.3, 0.5, 1\}$$

Level set of a fuzzy set

Level set of a fuzzy set A denoted by

$$\lambda_A = \{\alpha \in [0,1] / \eta_A(x) > 0\}$$

$$\lambda_\alpha = \{x \in U / \eta_A(x) > \alpha\}$$

for at least one $x \in U$

$$B = \{(-2,0.3), (-1,0.4), (0,0.9), (1,0.5), (2,0.2)\}$$

$$\lambda_B = \{0.3, 0.4, 0.9, 0.5, 0.2\}$$

Fuzzy Set Operations

Define three fuzzy sets A , B , and C on the universe X . For a given element x of *the* universe, the following function-theoretic operations for the set-theoretic operations of union, intersection, and complement are defined for A on X :