FUZZY UNION

FUZZY UNIONS

$$u : [0, 1] \times [0, 1] \to [0, 1].$$

The argument to this function is the pair consisting of the membership grade of some element x in fuzzy set A and the membership grade of that same element in fuzzy set B. The function returns the membership grade of the element in the set $A \cup B$. Thus,

$$(A \cup B)(x) = u[A(x), B(x)]$$

for all $x \in X$.

FUZZY UNIONS

A *fuzzy union/t-conorm* u is a binary operation on the unit interval that satisfies at least the following axioms for all $a, b, d \in [0, 1]$:

Axiom u1. $u(a, 0) = a$ (*boundary condition*).

Axiom u2. $b \leq d$ implies $u(a, b) \leq u(a, d)$ (*monotonicity*).

Axiom u3. $u(a, b) = u(b, a)$ (*commutativity*).

Axiom u4. $u(a, u(b, d)) = u(u(a, b), d)$ (*associativity*).

Since this set of axioms is essential for fuzzy unions, we call it the *axiomatic skeleton for fuzzy unions/t-conorms*.

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The most important additional requirements for fuzzy unions are expressed by the following axioms:

Axiom u5. u is a continuous function (*continuity*).

Axiom u6. $u(a, a) > a$ (*superidempotency*).

Axiom u7. $a_1 < a_2$ and $b_1 < b_2$ implies $u(a_1, b_1) < u(a_2, b_2)$ (*strict monotonicity*).

Any continuous and superidempotent t-conorm is called *Archimedean*; if it is also strictly monotonic, it is called *strictly Archimedean*.

$$u(a, b) = \max(a, b).$$

$$u_w(a, b) = \min(1, (a^w + b^w)^{1/w}) \quad (w > 0).$$

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Standard union: $u(a, b) = \max(a, b)$.

Algebraic sum: $u(a, b) = a + b - ab$.

Bounded sum: $u(a, b) = \min(1, a + b)$.

Drastic union: $u(a, b) = \begin{cases} a & \text{when } b = 0 \\ b & \text{when } a = 0 \\ 1 & \text{otherwise.} \end{cases}$

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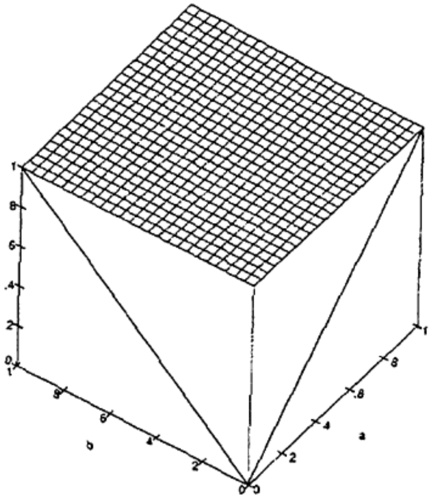
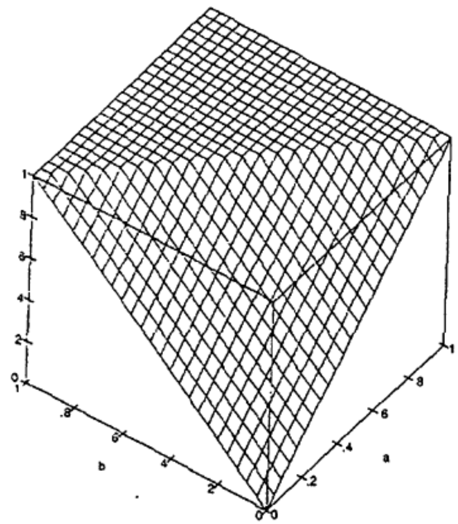
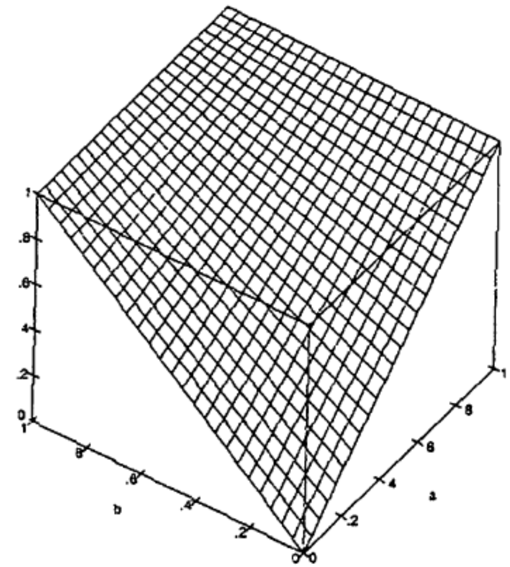


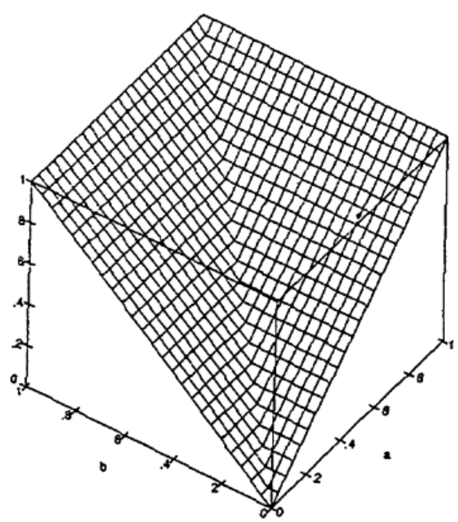
u_{max}



min (1,a+b)



a+b-ab



max

$$\max(a, b) \leq a + b - ab \leq \min(1, a + b) \leq u_{\max}(a, b)$$