FUZZY INTERSECTIONS

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The intersection of two fuzzy sets A and B is specified in general by a binary operation on the unit interval; that is, a function of the form

$$i : [0, 1] \times [0, 1] \rightarrow [0, 1].$$

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A *fuzzy intersection/t-norm* i is a binary operation on the unit interval that satisfies at least the following axioms for all $a, b, d \in [0, 1]$:

Axiom i1. $i(a, 1) = a$ (*boundary condition*).

Axiom i2. $b \leq d$ implies $i(a, b) \leq i(a, d)$ (*monotonicity*).

Axiom i3. $i(a, b) = i(b, a)$ (*commutativity*).

Axiom i4. $i(a, i(b, d)) = i(i(a, b), d)$ (*associativity*).

Let us call this set of axioms the *axiomatic skeleton for fuzzy intersections/t-norms*.

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It is often desirable to restrict the class of fuzzy intersections (t-norms) by considering various additional requirements. Three of the most important requirements are expressed by the following axioms:

Axiom i5. i is a continuous function (*continuity*).

Axiom i6. $i(a, a) < a$ (*subidempotency*).

Axiom i7. $a_1 < a_2$ and $b_1 < b_2$ implies $i(a_1, b_1) < i(a_2, b_2)$ (*strict monotonicity*).

A continuous t-norm that satisfies subidempotency is called an *Archimedean t-norm*; if it also satisfies strict monotonicity, it is called a *strict Archimedean t-norm*.

$i(a, b) = \min(a, b)$.

Yager class

$$i_w(a, b) = 1 - \min(1, [(1 - a)^w + (1 - b)^w]^{1/w}) \quad (w > 0).$$

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Standard intersection : $i(a, b) = \min(a, b)$.

Algebraic product : $i(a, b) = ab$.

Bounded difference : $i(a, b) = \max(0, a + b - 1)$.

Drastic intersection : $i(a, b) = \begin{cases} a & \text{when } b = 1 \\ b & \text{when } a = 1 \\ 0 & \text{otherwise.} \end{cases}$

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Theorem 3.9. The standard fuzzy intersection is the only idempotent t-norm.

Proof: Clearly, $\min(a, a) = a$ for all $a \in [0, 1]$. Assume that there exists a t-norm such that $i(a, a) = a$ for all $a \in [0, 1]$. Then, for any $a, b \in [0, 1]$, if $a \leq b$, then

$$a = i(a, a) \leq i(a, b) \leq i(a, 1) = a$$

by monotonicity and the boundary condition. Hence, $i(a, b) = a = \min(a, b)$. Similarly, if $a \geq b$, then

$$b = i(b, b) \leq i(a, b) \leq i(1, b) = b$$

and, consequently, $i(a, b) = b = \min(a, b)$. Hence, $i(a, b) = \min(a, b)$ for all $a, b \in [0, 1]$. ∎

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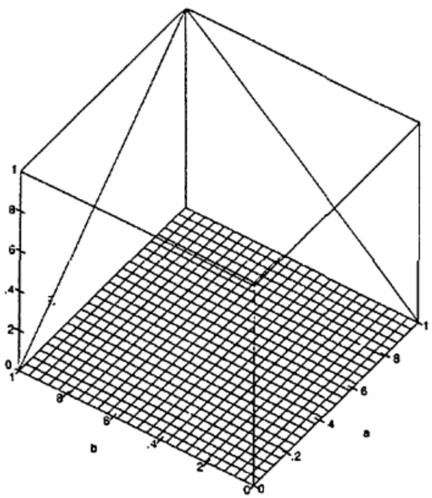
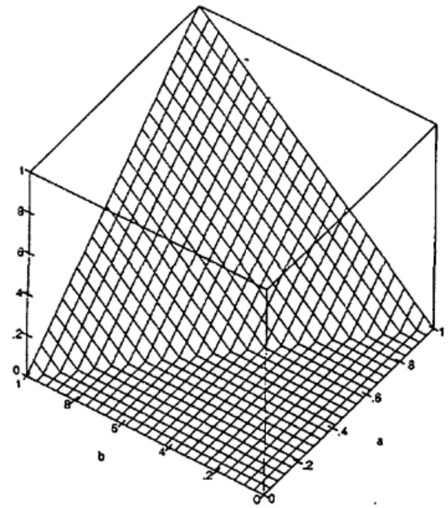
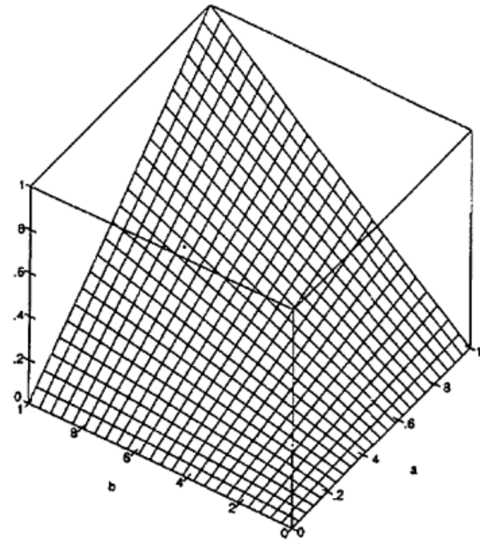


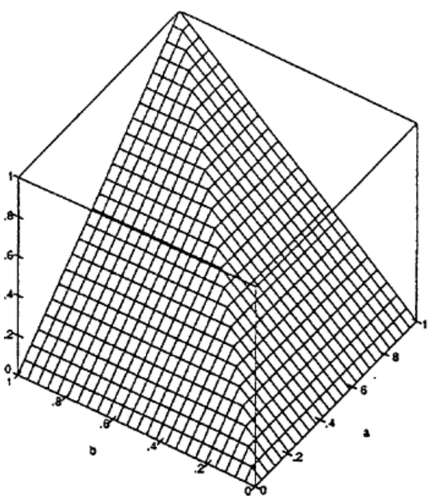
i_{min}



max (o,a+b-1)



ab



min

$$i_{\min}(a, b) \leq \max(0, a + b - 1) \leq ab \leq \min(a, b)$$