

# Biogeography-Based Optimization

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- Biogeography is the study of the **speciation, extinction, and geographical distribution** of biological species.
- Behavior of biological ants - **ant colony optimization**
- Science of genetics - **genetic algorithms**
- Study of animal swarms - **particle swarm optimization**
- Science of biogeography - **biogeography-based optimization**

# Biogeography

- The science of biogeography can be traced to the work of **19th century** naturalists, most notably Alfred Wallace [Wallace, 2006] and Charles Darwin [Keynes, 2001].
- Alfred Wallace - **Father of biogeography**
- Charles Darwin - **Theory of evolution.**

# Biogeography

- Mathematical models of biogeography describe **speciation** (the evolution of new species), the **migration** of species between islands, and the **extinction** of species.
- An **island** is considered any habitat that is geographically isolated from other habitats.
- An island is isolated from other habitats by **water, isolated by stretches of desert, rivers, mountain ranges, predators, man-made artifacts, or other obstacles**.

# Biogeography

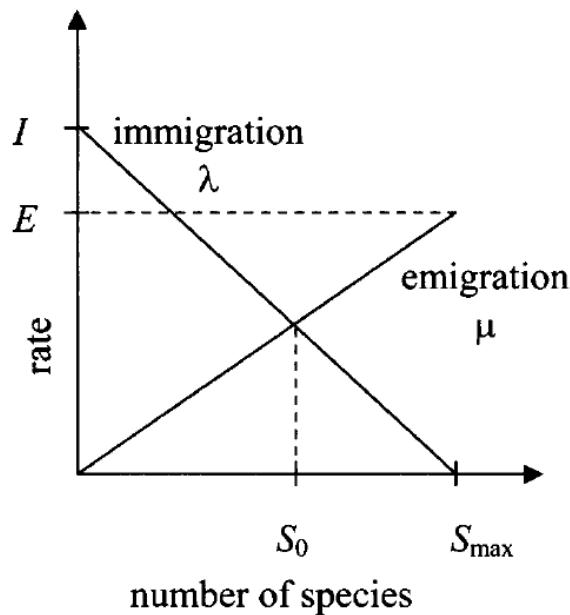
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# Biogeography

- Geographical areas that are **friendly to life** are said to have a **high habitat suitability index (HSI)**.
  - Features that correlate with HSI include such factors as **rainfall, vegetative diversity, topographic diversity, land area, and temperature.**
- The variables which **characterize habitability** are called **suitability index variables (SIVs)**
- **SIVs** are the **independent variables** of the habitat and **HSI** is the **dependent variable.**

# Biogeography

- Islands with a **high HSI** tend to support many species, and islands with a **low HSI** can support only a few species.



Species migration model of an island, based on [MacArthur and Wilson, 1967].  $S_0$  is the equilibrium species count.

# BIOGEOGRAPHY-BASED OPTIMIZATION

- Biogeography is nature's **way of distributing species and optimizing environments** for life, and is analogous to mathematical optimization.
- Suppose that we have an optimization problem and some **candidate solutions**, which we call **individuals**.
- Good individuals perform well on the problem, and poor individuals perform poorly.
- A **good individual** is analogous to an island with a **high HSI**, and a **poor individual** is analogous to an island with a **low HSI**.

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# BIOGEOGRAPHY-BASED OPTIMIZATION

- **Good individuals** resist change more than poor individuals, just like highly habitable islands have lower immigration rates than less habitable islands.
- **Good individuals** tend to share their features (that is, their independent variables) with poor individuals, just like highly habitable islands have high emigration rates.

# BIOGEOGRAPHY-BASED OPTIMIZATION

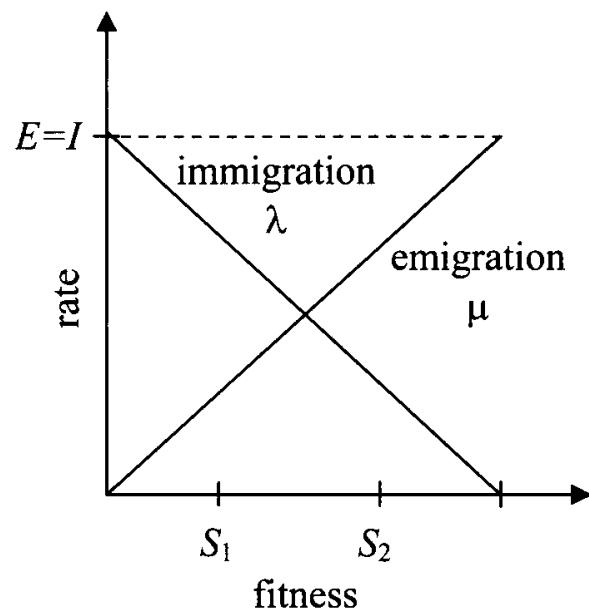
- Poor individuals are likely to accept new features from good individuals, just like less habitable islands are likely to receive many immigrants from highly habitable islands.
- The addition of new features to poor individuals may raise the quality of those individuals.
- The EA that is based on this approach is called biogeography-based optimization (BBO).

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# BIOGEOGRAPHY-BASED OPTIMIZATION

- Assume that each BBO individual is represented by an identical species count curve with  $E = I$  for simplicity.



*$S_1$  represents a poor individual, while  $S_2$  represents a good individual.*

The **immigration** rate for  $S_1$  will be relatively **high**, which means that it will be **likely to receive new features** from other candidate solutions.

The **emigration** rate for  $S_2$  will be relatively **high**, which means that it will be **likely to share its features** with other individuals.

Linear migration model since the I and E values are linear functions of fitness.

# BIOGEOGRAPHY-BASED OPTIMIZATION

- We use the **migration rates** of each individual to probabilistically share information between individuals.
- There are several different ways to implement the details of BBO here we are concentrating method by **D Simon, 2008**.

# BIOGEOGRAPHY-BASED OPTIMIZATION

- Population size of  $N$
- $X_k$  is the  $k$ -th individual in the population
- $X_k(s)$  is the  $s$ -th independent variable in  $X_k$ ,
  - where  $k \in [1, N]$  and  $s \in [1, n]$ .
- At each generation and for each solution feature in the  $k$ -th individual, there is a probability of  $\lambda_k$  (immigration probability) that it will be replaced.

$\lambda_k$  = Probability that  $s$ -th independent variable in  $x_k$  will be replaced

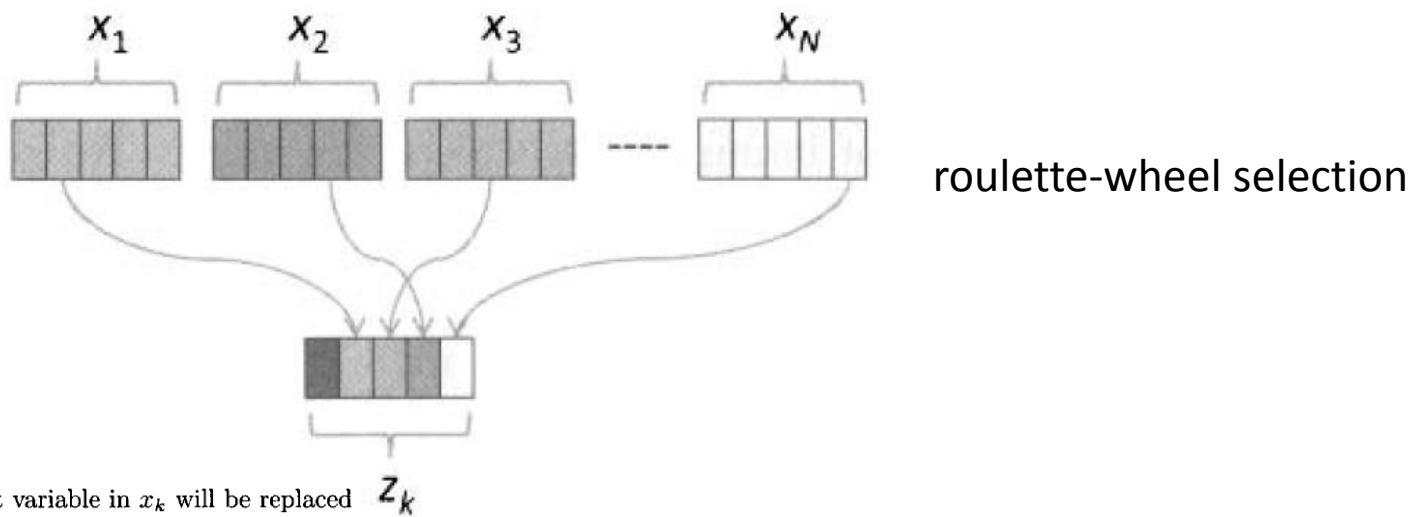
# BIOGEOGRAPHY-BASED OPTIMIZATION

- If a solution feature is selected to be replaced, then we select the emigrating solution with a probability that is proportional to the emigration probabilities  $\{\mu_i\}$

$$\Pr(x_j \text{ is selected for emigration}) = \frac{\mu_j}{\sum_{i=1}^N \mu_j}.$$

# BIOGEOGRAPHY-BASED OPTIMIZATION

- Migration and mutation of each individual in the current generation occurs before any of the individuals are replaced in the population, which requires the use of the temporary population  $Z$



$$\Pr(x_j) \text{ is selected for emigration} = \frac{\mu_j}{\sum_{i=1}^N \mu_j}. \quad (14.14)$$

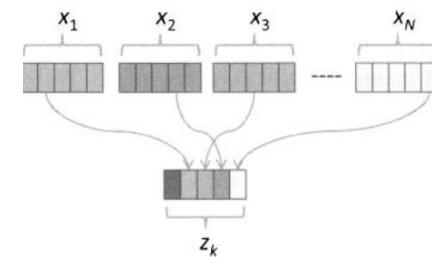
Depicts a *generational BBO algorithm* as opposed to a steady-state algorithm.

# BIOGEOGRAPHY-BASED OPTIMIZATION

1. Immigration is *not* selected for the first feature; that is why the first feature in  $z_k$  remains unchanged.
2. Immigration is selected for the second feature, and Equation (14.14) chooses  $x_1$  as the emigrating individual; that is why the second feature in  $z_k$  is replaced by the second feature from  $x_1$ .
3. Immigration is selected for the third feature, and Equation (14.14) chooses  $x_3$  as the emigrating individual; that is why the third feature in  $z_k$  is replaced by the third feature from  $x_3$ .
4. Immigration is selected for the fourth feature, and Equation (14.14) chooses  $x_2$  as the emigrating individual; that is why the fourth feature in  $z_k$  is replaced by the fourth feature from  $x_2$ .
5. Finally, immigration is selected for the fifth feature, and Equation (14.14) chooses  $x_N$  as the emigrating individual; that is why the fifth feature in  $z_k$  is replaced by the fifth feature from  $x_N$ .

$\lambda_k$  = Probability that  $s$ -th independent variable in  $x_k$  will be replaced

$$\Pr(x_j \text{ is selected for emigration}) = \frac{\mu_j}{\sum_{i=1}^N \mu_j}. \quad (14.14)$$



# BIOGEOGRAPHY-BASED OPTIMIZATION

Initialize a population of candidate solutions  $\{x_k\}$  for  $k \in [1, N]$

While not(termination criterion)

For each  $x_k$ , set emigration probability  $\mu_k \propto$  fitness of  $x_k$ , with  $\mu_k \in [0, 1]$

For each individual  $x_k$ , set immigration probability  $\lambda_k = 1 - \mu_k$

$$\{z_k\} \leftarrow \{x_k\}$$

For each individual  $z_k$

For each solution feature  $s$

Use  $\lambda_k$  to probabilistically decide whether to immigrate to  $z_k$   
 (see Equation (14.13))

If immigrating then

Use  $\{\mu_i\}_{i=1}^N$  to probabilistically select emigrating individual  $x_j$  (see Equation (14.14))

$$z_k(s) \leftarrow x_j(s)$$

End if

Next solution feature

Probabilistically mutate  $\{z_k\}$

Next individual

$$\{x_k\} \leftarrow \{z_k\}$$

$\lambda_k$  = Probability that  $s$ -th independent variable in  $x_k$  will be replaced

$$\Pr(x_j \text{ is selected for emigration}) = \frac{\mu_j}{\sum_{i=1}^N \mu_i}. \quad (14.14)$$

# Simple BBO experiment

String number	$x$ (binary)	$x$ (decimal)	$f(x) = x^2$	$\mu$	$\lambda$
1	01101	13	169	2/5	3/5
2	11000	24	576	4/5	1/5
3	01000	8	64	1/5	4/5
4	10011	19	361	3/5	2/5

$$N = 4$$

use  $1/N$  as the minimum values for  $\lambda$  and  $\mu$ , and  $(N - 1)/N$  as the maximum values

$$z_1(1) = 0, \quad z_1(2) = 1, \quad z_1(3) = 1, \quad z_1(4) = 0, \quad z_1(5) = 1.$$

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Since  $z_1$  is the third most fit individual, immigration rate  $\lambda_1 = 3/5$ , so there is a 60% chance of immigrating to each bit in  $z_1$ . We generate a random number  $r \sim U[0, 1]$  for each bit in  $z_1$  to determine whether or not we should immigrate to that bit.

$$z_1(1) = 0, \quad z_1(2) = 1, \quad z_1(3) = 1, \quad z_1(4) = 0, \quad z_1(5) = 1.$$

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1. Suppose  $r = 0.7$ . Since  $r > \lambda_1$ , we will not immigrate to  $z_1(1)$ , so  $z_1(1)$  remains equal to 0.

# Simple BBO experiment

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2. Suppose the next random number that we generate is  $r = 0.3$ . Since  $r < \lambda_1$ , we will immigrate to  $z_1(2)$ . We use roulette-wheel selection to choose the immigrating bit.  $x_3(2)$  has the greatest probability of immigrating to  $z_1(2)$ ,  $x_1(2)$  has the second greatest probability,  $x_4(2)$  has the third greatest probability, and  $x_2(2)$  has the least probability. We could exclude  $x_1(2)$  from consideration since  $z_1$  is a copy of  $x_1$ , but this is an implementation detail that depends on the preference of the engineer. Suppose that this roulette-wheel selection process results in the choice of  $x_3(2)$  for immigration. Then  $z_1(2) \leftarrow x_3(2) = 1$ . Even though we immigrated to  $z_1(2)$ , it did not change from its original value.

# Simple BBO experiment

String number	$x$ (binary)	$x$ (decimal)	$f(x) = x^2$	$\mu$	$\lambda$
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3. We continue this process for  $z_1(3)$ ,  $z_1(4)$ , and  $z_1(5)$ . Suppose that the random numbers generated result in the following:
- $z_1(3) = 1$  (no immigration);
  - $z_1(4) \leftarrow x_4(4) = 1$  (immigration); and
  - $z_1(5) = 1$  (no immigration).

$$z_1 = 01111$$

# Simple BBO experiment

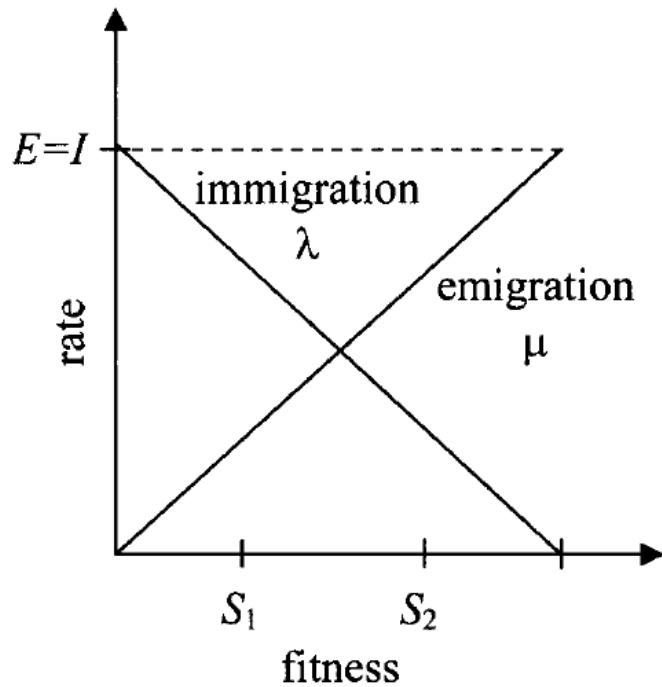
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4. We repeat steps 1–3 for  $z_2$ ,  $z_3$ , and  $z_4$ .
5. We next consider the possibility of mutation for each bit in each temporary individual  $z_1$ ,  $z_2$ ,  $z_3$ , and  $z_4$ .
6. Now that we have a modified population of  $\{z_k\}$  individuals, we copy  $z_k$  to  $x_k$  for  $k \in [1, 4]$ , and the first BBO generation is complete.

# BBO EXTENSIONS

- Migration curve shapes,
- Blended migration
- Alternative approaches

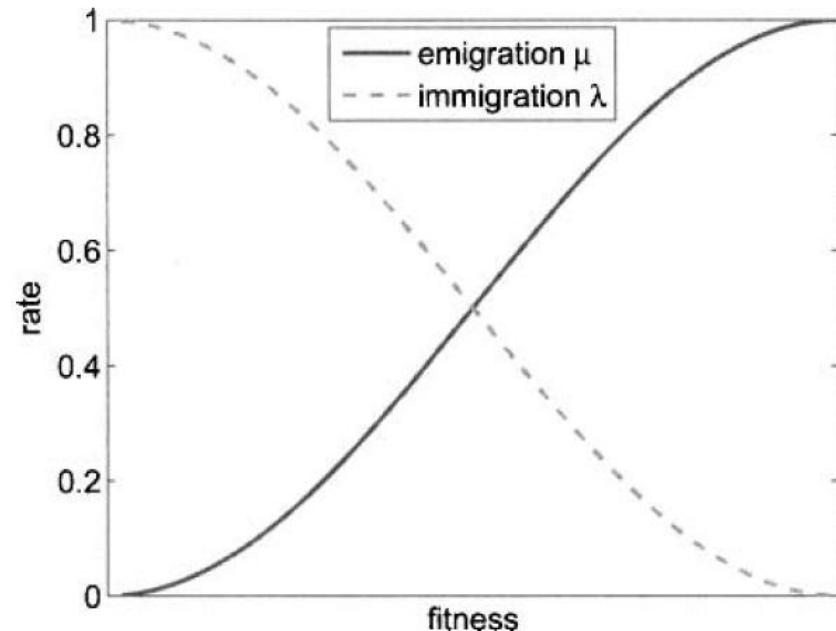
# Migration Curves



$$\mu_k = r_k$$

$$\lambda_k = 1 - r_k$$

linear



$$\mu_k = \frac{1}{2} (1 - \cos(\pi r_k / N))$$

$$\lambda_k = 1 - \mu_k.$$

Sinusoidal BBO migration model

The table shows the normalized minimum found by the two BBO versions, averaged over 50 Monte Carlo simulations.

# Migration Curves

Benchmark	Linear Migration	Sinusoidal Migration
Ackley	1.0373	1
Fletcher	1.2015	1
Griewank	1.2367	1
Penalty #1	1.4249	1
Penalty #2	4.3265	1
Quartic	1.6876	1
Rastrigin	1.0665	1
Rosenbrock	1.0759	1
Schwefel 1.2	1.0980	1
Schwefel 2.21	1.0468	1
Schwefel 2.22	1.0721	1
Schwefel 2.26	1.2471	1
Sphere	1.2582	1
Step	1.2683	1
Average	1.4319	1

set of 20-dimensional benchmark

# Blended Migration

- In blended GA crossover, instead of copying a single parent's gene to a child gene, the child gene is obtained as a convex combination of two parent genes.

$$z_k(s) \leftarrow x_j(s).$$

standard BBO algorithm

$$z_k(s) \leftarrow \alpha z_k(s) + (1 - \alpha)x_j(s)$$

blended migration in BBO

# Blended Migration

Benchmark	Standard BBO ( $\alpha = 0$ )	Blended BBO ( $\alpha = 0.5$ )
Ackley	1.6559	1.0
Fletcher	1.0	2.388
Griewank	3.4536	1.0
Penalty #1	701.47	1.0
Penalty #2	8817.7	1.0
Quartic	49.663	1.0
Rastrigin	1.0	1.6892
Rosenbrock	3.9009	1.0
Schwefel 1.2	12.63	1.0
Schwefel 2.21	4.0846	1.0
Schwefel 2.21	1.3280	1.0
Schwefel 2.26	1.0	4.8213
Sphere	5.4359	1.0
Step	4.5007	1.0
Average	686.34	1.4213

# Other Approaches to BBO

- Partial emigration-based BBO

Initialize a population of candidate solutions  $\{x_k\}$  for  $k \in [1, N]$

While not(termination criterion)

    For each  $x_k$ , set emigration probability  $\mu_k \propto$  fitness of  $x_k$ , with  $\mu_k \in [0, 1]$

    For each individual  $x_k$ , define immigration probability  $\lambda_k = 1 - \mu_k$

$\{z_k\} \leftarrow \{x_k\}$

    For each individual  $x_k$

        For each solution feature  $s$

            Use  $\mu_k$  to probabilistically decide whether to emigrate  $x_k(s)$

            If emigrating then

                Use  $\{\lambda_i\}$  to probabilistically select the immigrating solution  $z_j$

$z_j(s) \leftarrow x_k(s)$

            End if

        Next solution feature

    Next individual

    Probabilistically mutate  $\{z_k\}$

$\{x_k\} \leftarrow \{z_k\}$

Next generation

(there is only one solution feature is considered  
for immigration at a time)

# Total immigration-based BBO

Initialize a population of candidate solutions  $\{x_k\}$  for  $k \in [1, N]$

While not(termination criterion)

    For each  $x_k$ , set emigration probability  $\mu_k \propto$  fitness of  $x_k$ , with  $\mu_k \in [0, 1]$

    For each individual  $x_k$ , define immigration probability  $\lambda_k = 1 - \mu_k$

$\{z_k\} \leftarrow \{x_k\}$

    For each individual  $z_k$

        Use  $\lambda_k$  to probabilistically decide whether to immigrate to  $z_k$

        If immigrating then

            For each solution feature  $s$

                Use  $\{\mu_i\}$  to probabilistically select the emigrating solution  $x_j$

$z_k(s) \leftarrow x_j(s)$

            Next solution feature

        End if

    Next individual

$\{x_k\} \leftarrow \{z_k\}$

Next generation

# Total emigration-based BBO

Initialize a population of candidate solutions  $\{x_k\}$  for  $k \in [1, N]$

While not(termination criterion)

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    For each individual  $x_k$ , define immigration probability  $\lambda_k = 1 - \mu_k$

$\{z_k\} \leftarrow \{x_k\}$

    For each individual  $x_k$

        Use  $\mu_k$  to probabilistically decide whether to emigrate  $x_k$

        If emigrating then

            For each solution feature  $s$

                Use  $\{\lambda_i\}$  to probabilistically select the immigrating solution  $z_j$

$z_j(s) \leftarrow x_k(s)$

            Next solution feature

        End if

    Next individual

$\{x_k\} \leftarrow \{z_k\}$

Next generation

# BBO and Genetic Algorithms

## GA/GUR (BBO is a specific type of GA/GUR)

Initialize a population of candidate solutions  $\{x_k\}$  for  $k \in [1, N]$

While not(termination criterion)

    For  $k = 1$  to  $N$

$\text{Child}_k \leftarrow [ \begin{array}{cccc} 0 & 0 & \dots & 0 \end{array} ] \in R^n$

        For each solution feature  $s = 1$  to  $n$

            Use fitness values to probabilistically select individual  $x_j$

$\text{Child}_k(s) \leftarrow x_j(s)$

        Next solution feature

        Probabilistically mutate  $\text{Child}_k$

    Next child

$\{x_k\} \leftarrow \{\text{Child}_k\}$

Next generation

If we use **global uniform recombination**, and if we also use **fitness-based selection** for each solution feature in each offspring.  
Immigration probability is one

**Thank you**