Defuzzification methods

Defuzzification methods

Defuzzification is the conversion of a fuzzy quantity to a precise quantity, just as fuzzification is the conversion of a precise quantity to a fuzzy quantity.

The output of a fuzzy process can be the logical union of two or more fuzzy membership functions defined on the universe of discourse of the output variable.

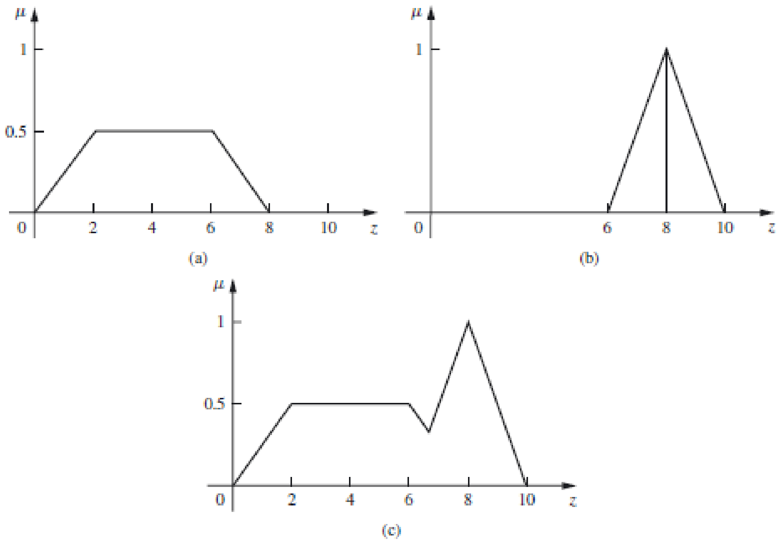


FIGURE 4.11
Typical fuzzy process output: (a) first part of fuzzy output; (b) second part of fuzzy output; and (c) union of both parts.

Defuzzification methods

Among the many methods that have been proposed in the literature in recent years, Four of these methods are first summarized as follows

1. *Max membership principle*

2. *Centroid method*

3. *Weighted average method*

4. *Mean max membership*

Max membership principle

Max membership principle: Also known as the *height method*, this scheme is limited to peaked output functions. This method is given by the algebraic expression

$$\mu_{\underset{\sim}{C}}(z^*) \geq \mu_{\underset{\sim}{C}}(z), \qquad \text{for all } z \in Z, \tag{4.4}$$

where z^* is the defuzzified value, and is shown graphically in Figure 4.12.

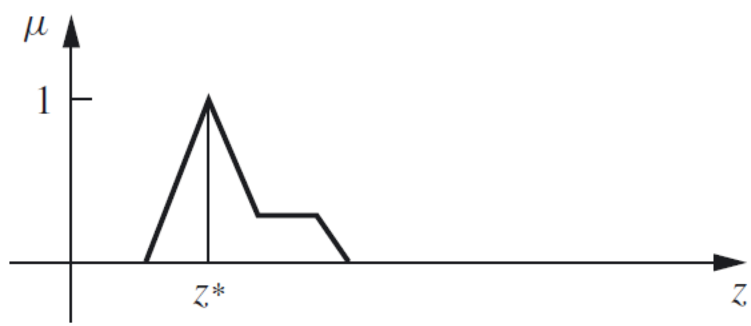


FIGURE 4.12
Max membership defuzzification method.

Mean max membership

Mean max membership: This method (also called *middle-of-maxima*) is closely related to the first method, except that the locations of the maximum membership can be nonunique (i.e., the maximum membership can be a plateau rather than a single point). This method is given by the expression (Sugeno, 1985; Lee, 1990)

$$z^* = \frac{a + b}{2} \tag{4.7}$$

where a and b are as defined in Figure 4.15.

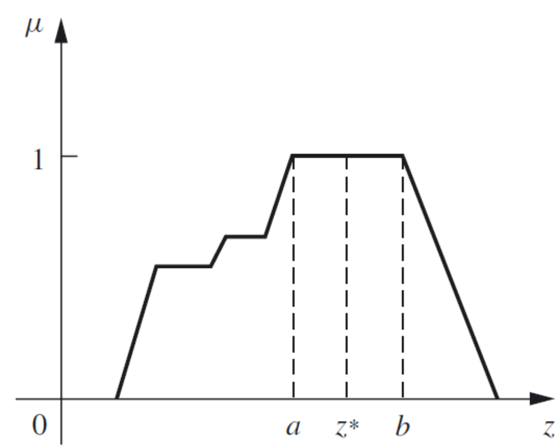


FIGURE 4.15
Mean max membership defuzzification method.

Weighted average method

Weighted average method: The weighted average method is the most frequently used in fuzzy applications since it is one of the more computationally efficient methods. Unfortunately, it is *usually* restricted to symmetrical output membership functions. It is given by the algebraic expression

$$z^* = \frac{\sum \mu_{\underset{\sim}{C}}(\bar{z}) \cdot \bar{z}}{\sum \mu_{\underset{\sim}{C}}(\bar{z})}, \tag{4.6}$$

where $\sum$ denotes the algebraic sum and where $\bar{z}$ is the centroid of each symmetric membership function. This method is shown in Figure 4.14. The weighted average method is formed by weighting each membership function in the output by its

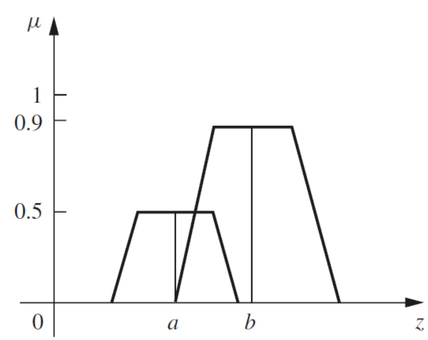


FIGURE 4.14
Weighted average method of defuzzification.

Weighted average method

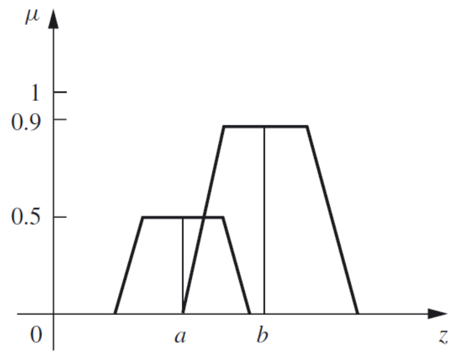


FIGURE 4.14
Weighted average method of defuzzification.

Figure 4.14 would result in the following general form for the defuzzified value:

$$z^* = \frac{a(0.5) + b(0.9)}{0.5 + 0.9}.$$

Since the method can be limited to symmetrical membership functions, the values a and b are the means (centroids) of their respective shapes. This method is sometimes applied to unsymmetrical functions and various scalar outputs (see Sugeno, 1985).

Centroid method

Centroid method: This procedure (also called *center of area* or *center of gravity*) is the most prevalent and physically appealing of all the defuzzification methods (Sugeno, 1985; Lee, 1990); it is given by the algebraic expression

$$z^* = \frac{\int \mu_{\underset{\sim}{C}}(z) \cdot z \, dz}{\int \mu_{\underset{\sim}{C}}(z) \, dz}, \tag{4.5}$$

where $\int$ denotes an algebraic integration. This method is shown in Figure 4.13.

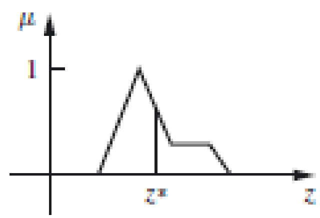


FIGURE 4.13
Centroid defuzzification method.

Example

Example 4.3. A railroad company intends to lay a new rail line in a particular part of a county. The whole area through which the new line is passing must be purchased for right-of-way considerations. It is surveyed in three stretches, and the data are collected for analysis. The surveyed data for the road are given by the sets, $\underset{\sim}{B}_1$, $\underset{\sim}{B}_2$, and $\underset{\sim}{B}_3$, where the sets are defined on the universe of right-of-way widths, in meters. For the railroad to purchase the land, it must have an assessment of the amount of land to be bought. The three surveys on right-of-way width are ambiguous, however, because some of the land along the proposed railway route is already public domain and will not need to be purchased. Additionally, the original surveys are so old (*circa* 1860) that some ambiguity exists on boundaries and public right-of-way for old utility lines and old roads. The three fuzzy sets, $\underset{\sim}{B}_1$, $\underset{\sim}{B}_2$, and $\underset{\sim}{B}_3$, shown in Figures 4.16–4.18, respectively, represent the uncertainty in each survey as to the membership of right-of-way width, in meters, in privately owned land.

We now want to aggregate these three survey results to find the single most nearly representative right-of-way width (z) to allow the railroad to make its initial estimate of the right-of-way purchasing cost. Using Equations (4.5)–(4.7) and the preceding three fuzzy sets, we want to find z^*.

Example

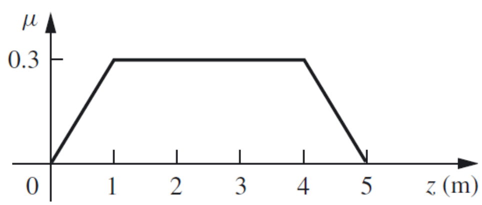


FIGURE 4.16
Fuzzy set $\underset{\sim}{B}_1$: public right-of-way width (z) for survey 1.

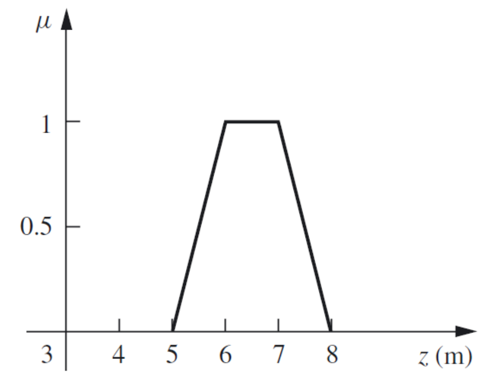


FIGURE 4.18
Fuzzy set $\underset{\sim}{B}_3$: public right-of-way width (z) for survey 3.

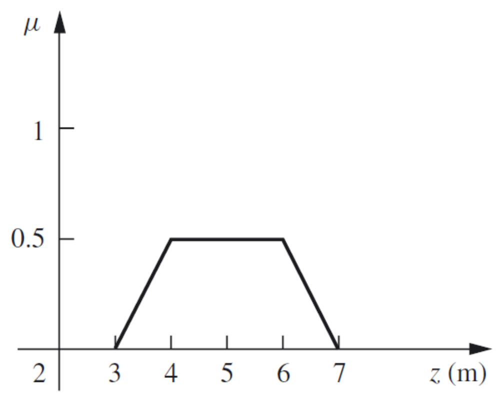


FIGURE 4.17
Fuzzy set $\underset{\sim}{B}_2$: public right-of-way width (z) for survey 2.

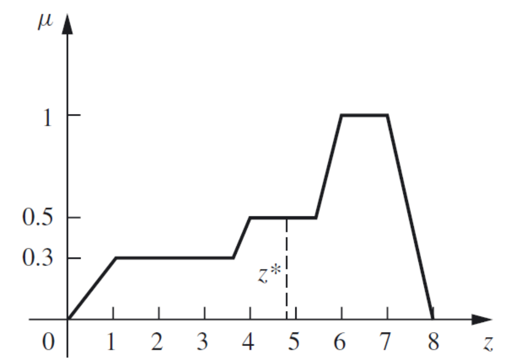


FIGURE 4.19
The centroid method for finding z^*.

Example

$$\int \left(\frac{z - x0}{x1 - x0} \right) mz dz$$

According to the centroid method, Equation (4.5), z^* can be found using

$$z^* = \frac{\int \mu_{\underset{\sim}{B}}(z) \cdot z \, dz}{\int \mu_{\underset{\sim}{B}}(z) \, dz}$$

$$= \left[\int_0^1 (0.3z)z \, dz + \int_1^{3.6} (0.3)z \, dz + \int_{3.6}^4 \left(\frac{z - 3.0}{2} \right) z \, dz + \int_4^{5.5} (0.5)z \, dz \right.$$

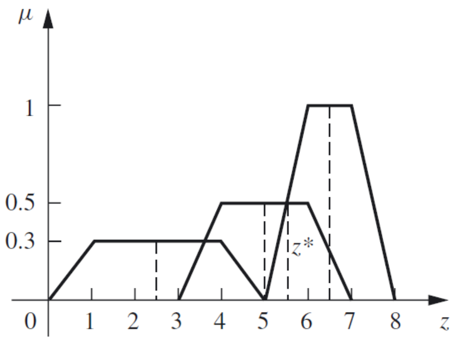
$$\left. + \int_{5.5}^6 (z - 5)z \, dz + \int_6^7 z \, dz + \int_7^8 (8 - z) \, z \, dz \right]$$

$$\div \left[\int_0^1 (0.3z) \, dz + \int_1^{3.6} (0.3) \, dz + \int_{3.6}^4 \left(\frac{z - 3.6}{2} \right) dz + \int_4^{5.5} (0.5) \, dz \right.$$

$$\left. + \int_{5.5}^6 \left(\frac{z - 5.5}{2} \right) dz + \int_6^7 dz + \int_7^8 \left(\frac{7 - z}{2} \right) dz \right]$$

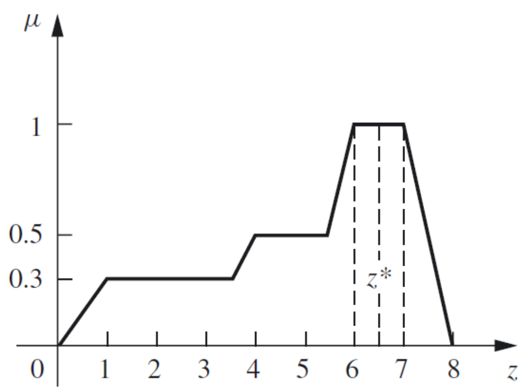
$$= 4.9 \text{ m},$$

Example



$$z^* = \frac{(0.3 \times 2.5) + (0.5 \times 5) + (1 \times 6.5)}{0.3 + 0.5 + 1} = 5.41\,\mathrm{m}$$

FIGURE 4.20
The weighted average method for finding z^*.



z^* is given by $(6 + 7)/2 = 6.5\,\mathrm{m}$

FIGURE 4.21
The mean max membership method for finding z^*.