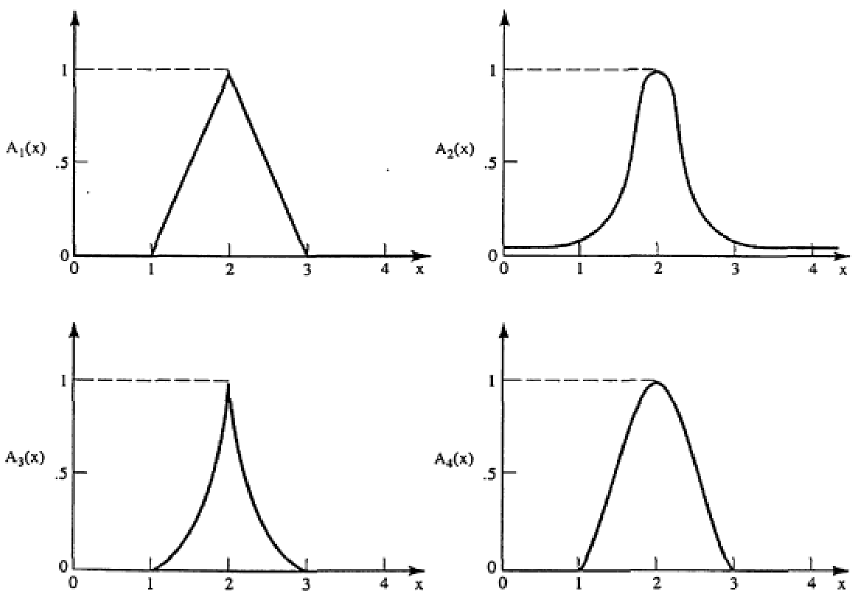
Fuzzy membership function

Fuzzy membership function



$$A_1(x) = \begin{cases} p_1(x-r)+1 & \text{when } x \in [r-1/p_1, r] \\ p_1(r-x)+1 & \text{when } x \in [r, r+1/p_1] \\ 0 & \text{otherwise} \end{cases}$$

$$A_2(x) = \frac{1}{1+p_2(x-r)^2}$$

$$A_3(x) = e^{-|p_3(x-r)|}$$

$$A_4(x) = \begin{cases} (1+\cos(p_4\pi(x-r)))/2 & \text{when } x \in [r-1/p_4, r+1/p_4] \\ 0 & \text{otherwise} \end{cases}$$

Figure 1.2 Examples of membership functions that may be used in different contexts for characterizing fuzzy sets of real numbers close to 2.

(i) $A_i(2) = 1$ and $A_i(x) < 1$ for all $x \neq 2$;
(ii) A_i is symmetric with respect to $x = 2$, that is $A_i(2+x) = A_i(2-x)$ for all $x \in \mathbb{R}$;
(iii) $A_i(x)$ decreases monotonically from 1 to 0 with the increasing difference $|2-x|$.

Fuzzy membership function

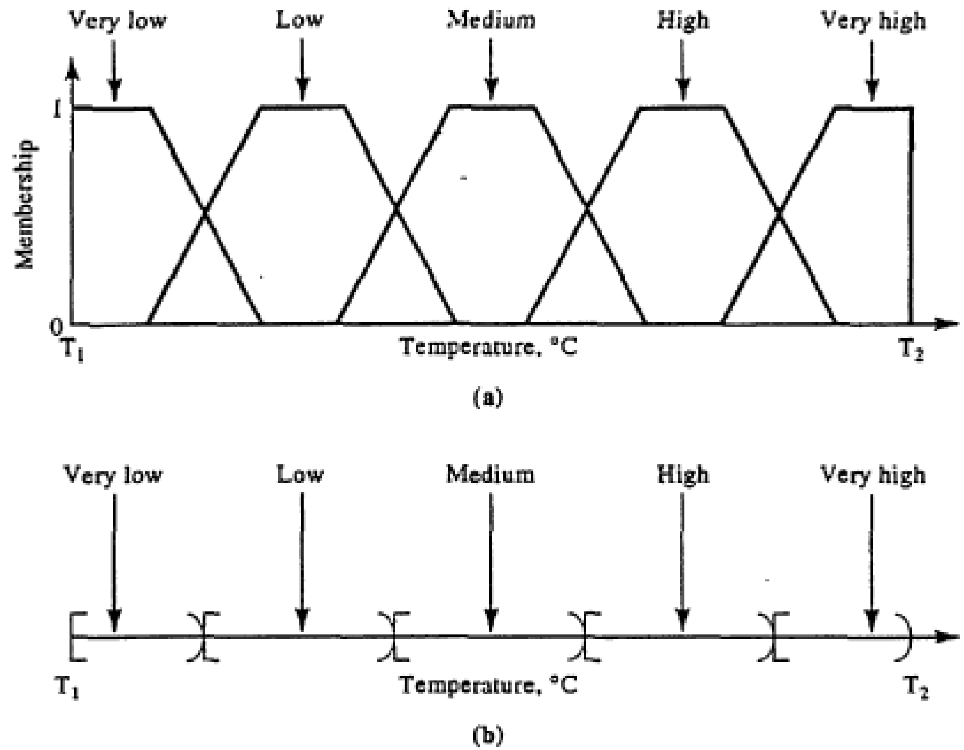


Figure 1.4 Temperature in the range $[T_1, T_2]$ conceived as: (a) a fuzzy variable; (b) a traditional (crisp) variable.

FEATURES OF THE MEMBERSHIP FUNCTION

The *core* of a membership function for some fuzzy set $\underset{\sim}{A}$ is defined as that region of the universe that is characterized by complete and full membership in the set $\underset{\sim}{A}$. That is, the core comprises those elements x of the universe such that $\mu_{\underset{\sim}{A}}(x) = 1$.

The *support* of a membership function for some fuzzy set $\underset{\sim}{A}$ is defined as that region of the universe that is characterized by nonzero membership in the set $\underset{\sim}{A}$. That is, the support comprises those elements x of the universe such that $\mu_{\underset{\sim}{A}}(x) > 0$.

FEATURES OF THE MEMBERSHIP FUNCTION

The *boundaries* of a membership function for some fuzzy set $\underset{\sim}{A}$ are defined as that region of the universe containing elements that have a nonzero membership but not complete membership. That is, the boundaries comprise those elements x of the universe such that $0 < \mu_{\underset{\sim}{A}}(x) < 1$. These elements of the universe are those with some *degree* of fuzziness, or only partial membership in the fuzzy set $\underset{\sim}{A}$. Figure 4.1 illustrates the regions in the universe comprising the core, support, and boundaries of a typical fuzzy set.

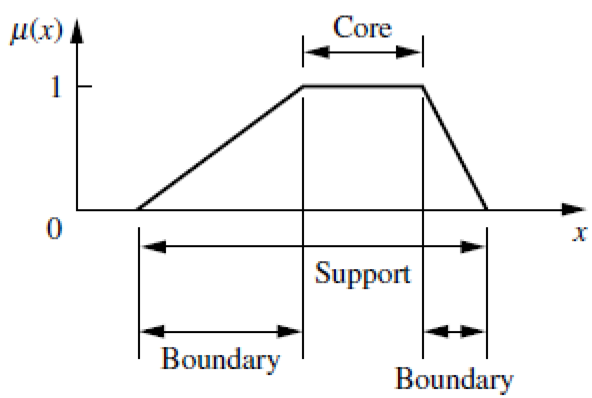


FIGURE 4.1
Core, support, and boundaries of a fuzzy set.

Convex fuzzy set

A *convex* fuzzy set is described by a membership function whose membership values are strictly monotonically increasing, or whose membership values are strictly monotonically decreasing, or whose membership values are strictly monotonically increasing then strictly monotonically decreasing with increasing values for elements in the universe. Said another way, if, for any elements x, y, and z in a fuzzy set $\underset{\sim}{A}$, the relation $x < y < z$ implies that

$$\mu_{\underset{\sim}{A}}(y) \geq \min[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{A}}(z)] \qquad (4.1)$$

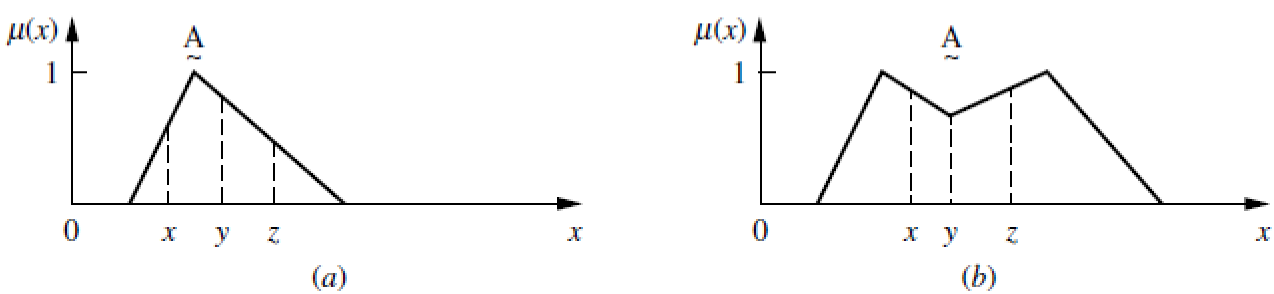


FIGURE 4.3
Convex, normal fuzzy set (*a*) and nonconvex, normal fuzzy set (*b*).

Convex fuzzy set

A special property of two convex fuzzy sets, say $\underset{\sim}{A}$ and $\underset{\sim}{B}$, is that the intersection of these two convex fuzzy sets is also a convex fuzzy set, as shown in Fig. 4.4. That is, for $\underset{\sim}{A}$ and $\underset{\sim}{B}$, which are both convex, $\underset{\sim}{A} \cap \underset{\sim}{B}$ is also convex.

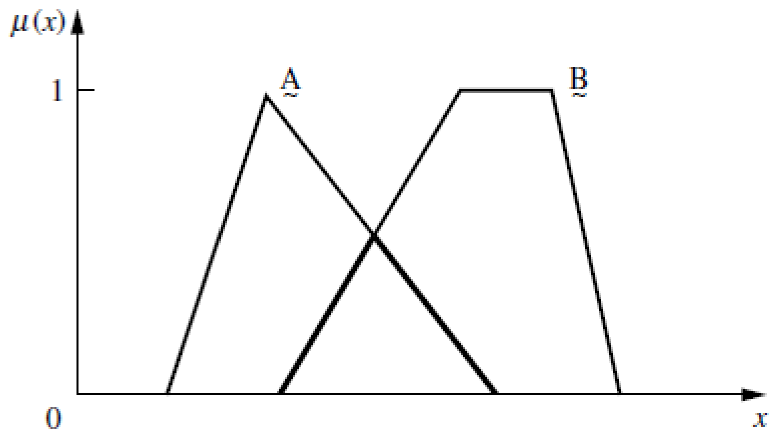


FIGURE 4.4
The intersection of two convex fuzzy sets produces a convex fuzzy set.

Cross over points

The *crossover points* of a membership function are defined as the elements in the universe for which a particular fuzzy set $\underset{\sim}{A}$ has values equal to 0.5, i.e., for which $\mu_{\underset{\sim}{A}}(x) = 0.5$.

The most common forms of membership functions are those that are normal and convex.

Interval-valued membership function

Suppose the level of information is not adequate to specify membership functions with this precision, Such a fuzzy set would be described by an *interval-valued membership function*.

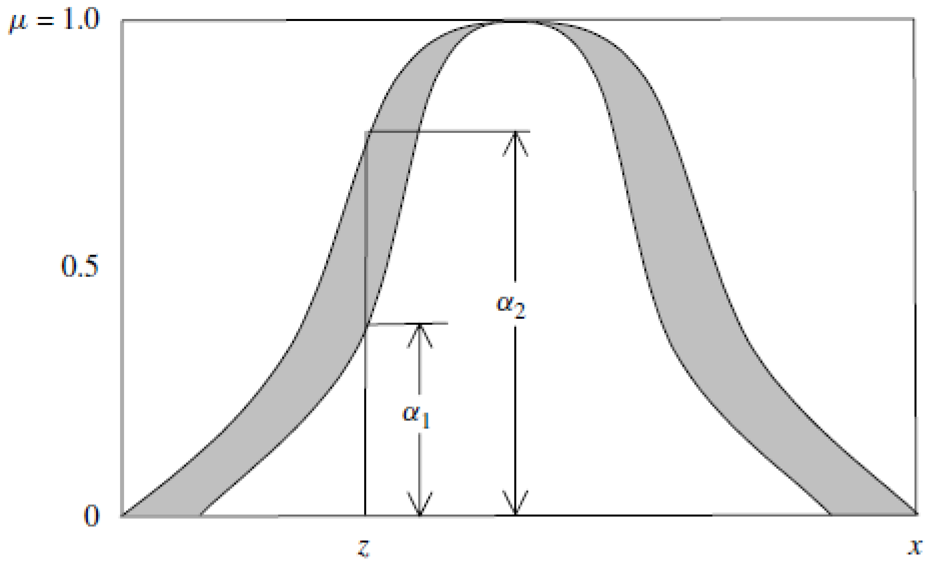
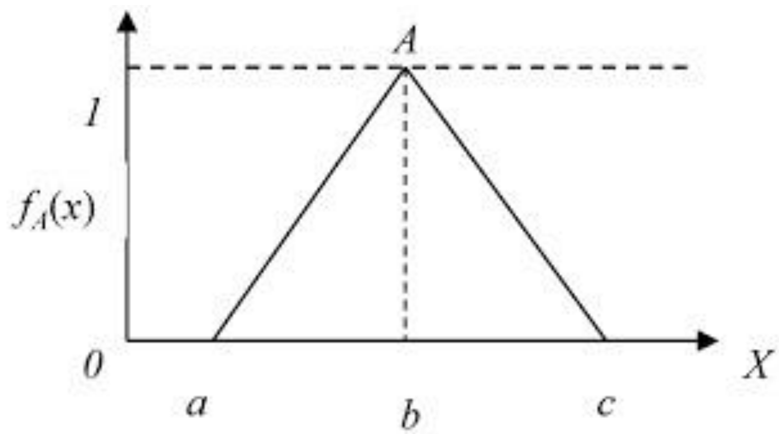


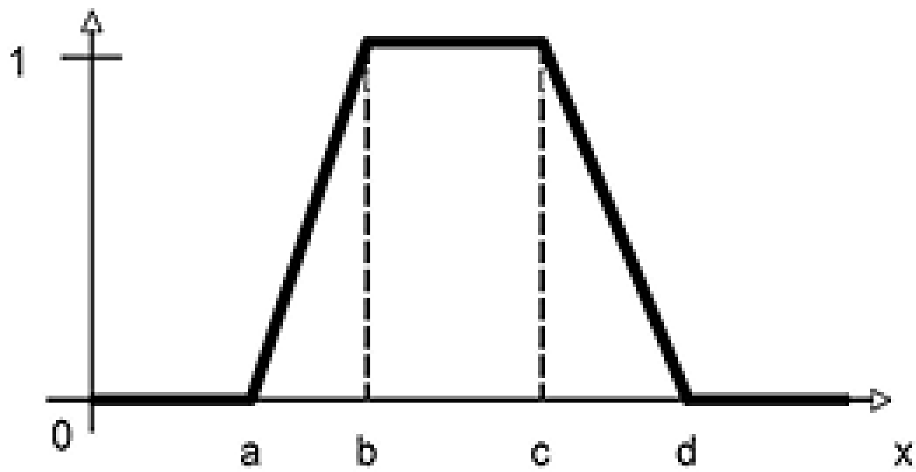
FIGURE 4.5
An interval-valued membership function.

Triangular membership function



$$\text{triangle}(x;a,b,c)=\begin{cases}0, & x\le a.\\ \dfrac{x-a}{b-a}, & a\le x\le b.\\ \dfrac{c-x}{c-b}, & b\le x\le c.\\ 0, & c\le x.\end{cases}$$

Trapezoidal membership function

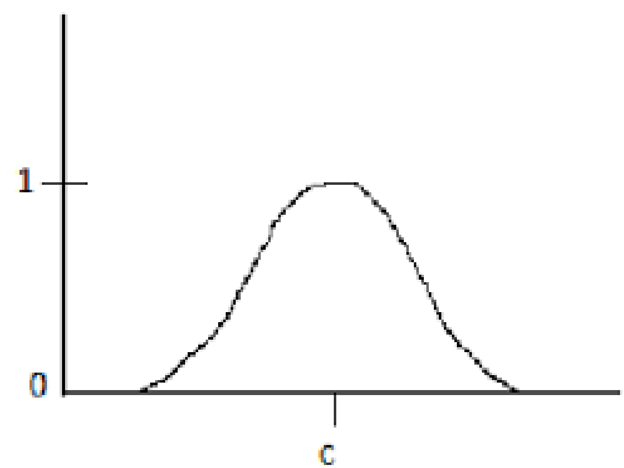


$$\text{trapezoid}(x; a, b, c, d) = \begin{cases} 0, & x \leq a. \\ \dfrac{x-a}{b-a}, & a \leq x \leq b. \\ 1, & b \leq x \leq c. \\ \dfrac{d-x}{d-c}, & c \leq x \leq d. \\ 0, & d \leq x. \end{cases}$$

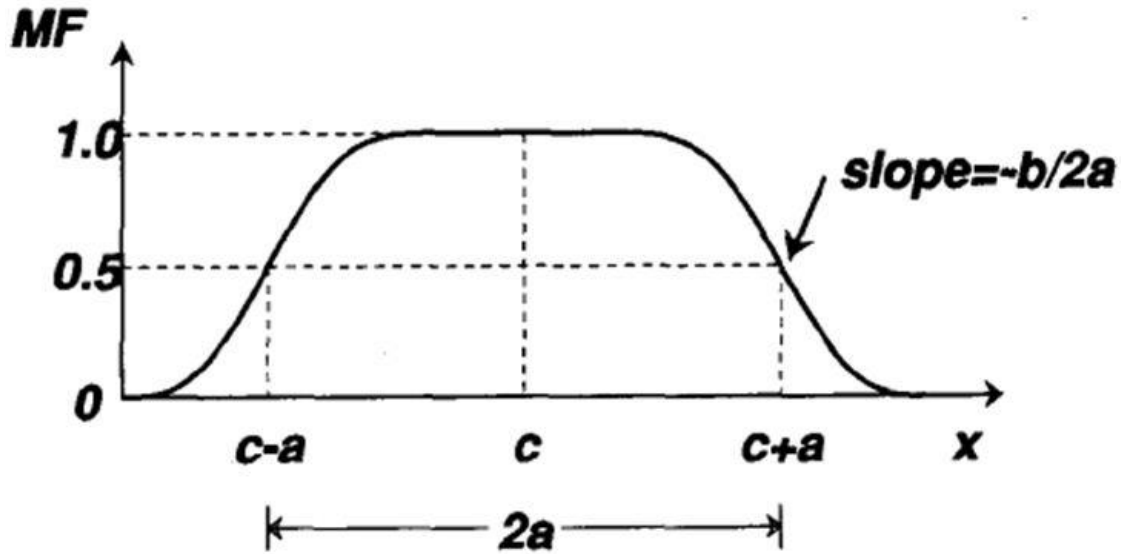
Gaussian membership function

$$\mu_b(x) = exp\left(-\frac{(x-c)^2}{2\sigma^2}\right)$$

- *c* is the centre of the MF
- sigma is the width of the MF
- *exp* is the exponential function

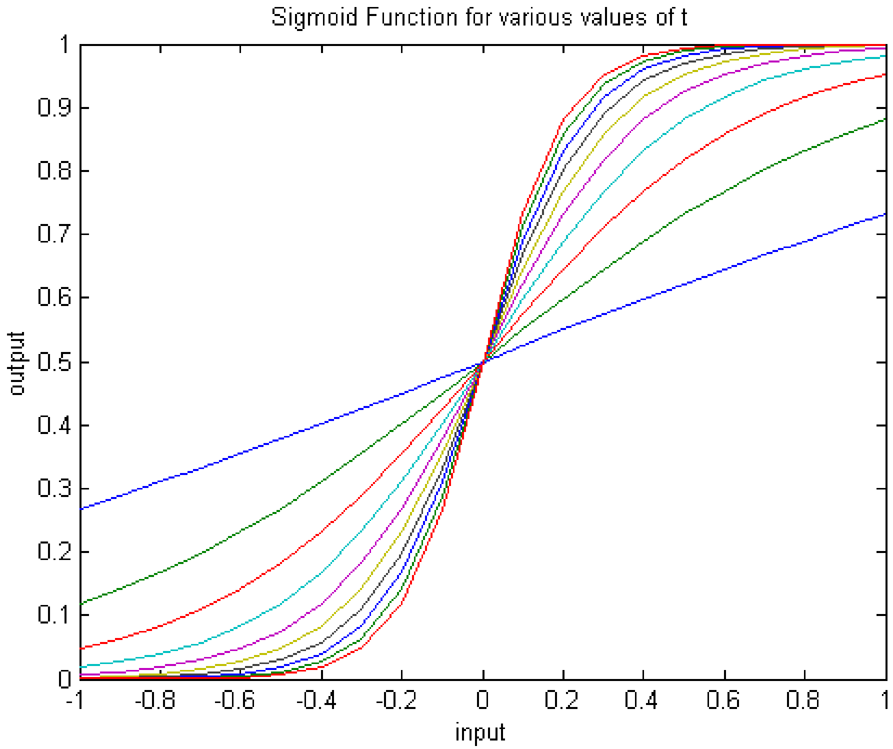


Bell shaped membership function



$$f(x;a,b,c)=\frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}}$$

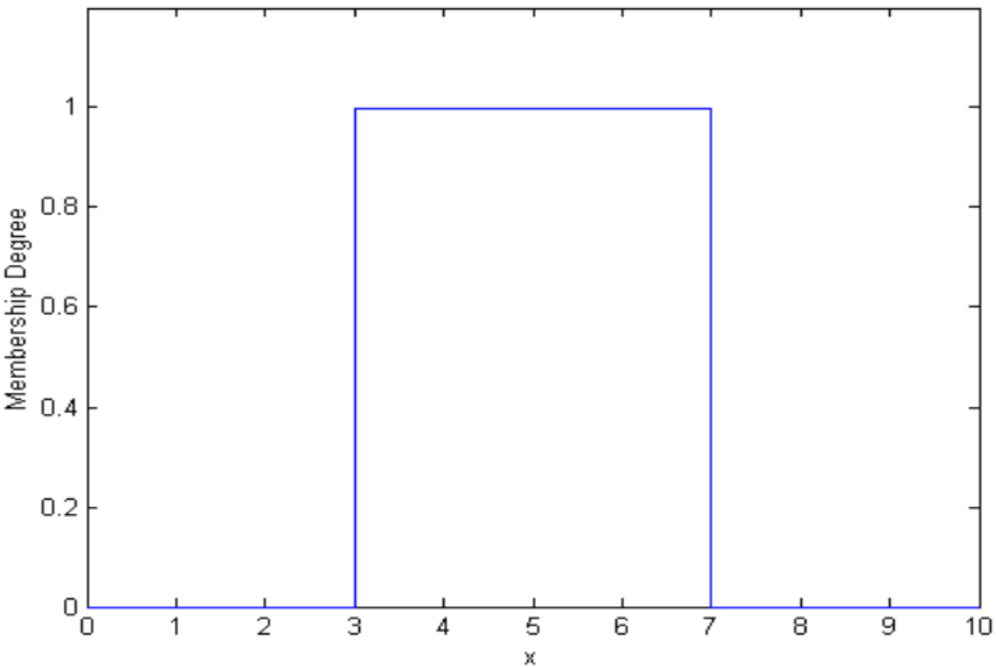
Sigmoid membership function



a = slope
c = center

$$\mathrm{sig}(x; a, c) = \frac{1}{1 + \exp[-a(x - c)]},$$

Rectangular MF

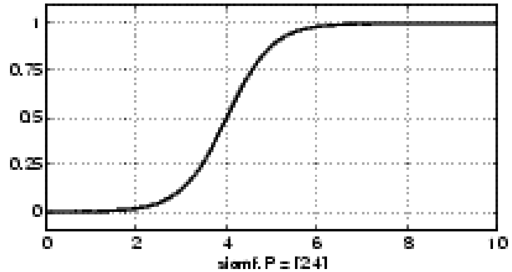


$$\mu(x) = \begin{cases} 1, & l \leq x \leq r \\ 0, & \text{otherwise} \end{cases}$$

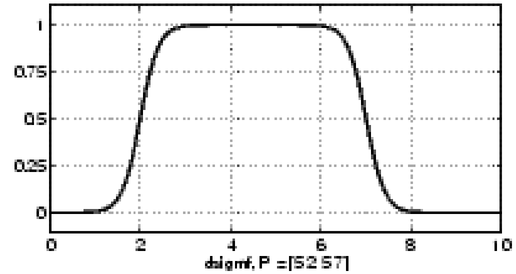
D-Sigmoid and P-Sigmoid membership function

Difference of two sigmoid function is called d sigmoid

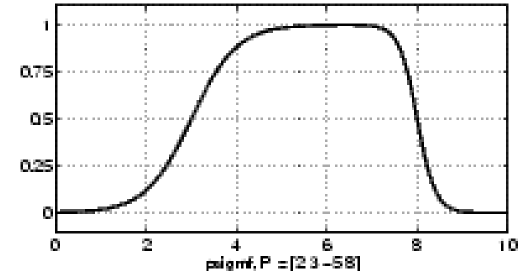
Product of two sigmoid function is called p sigmoid



sigmf

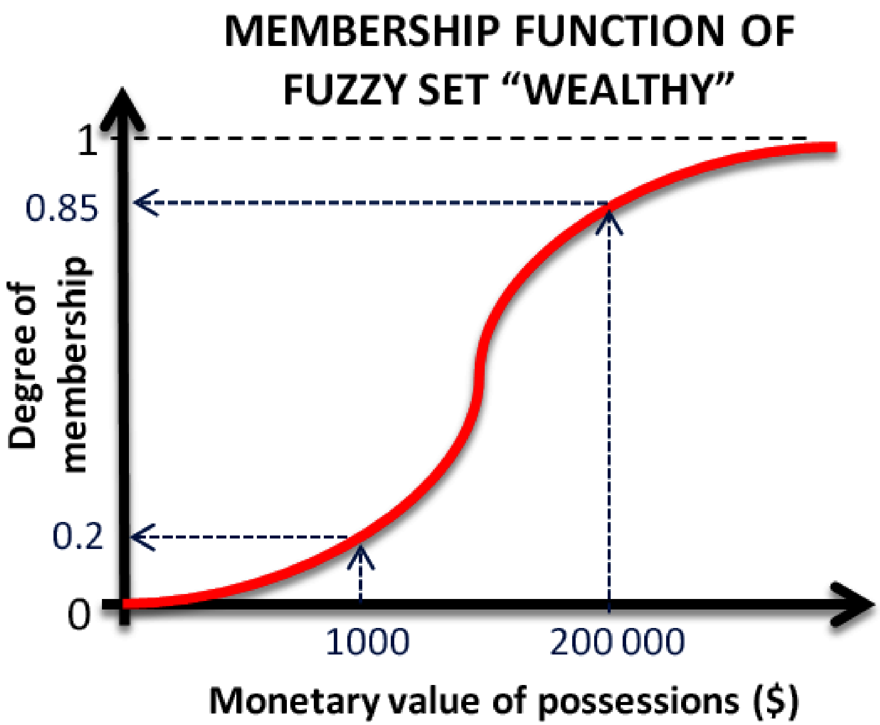


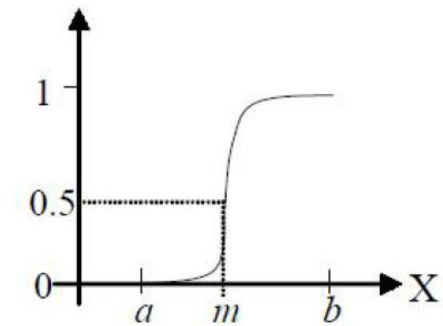
dsigmf



psigmf

S membership function

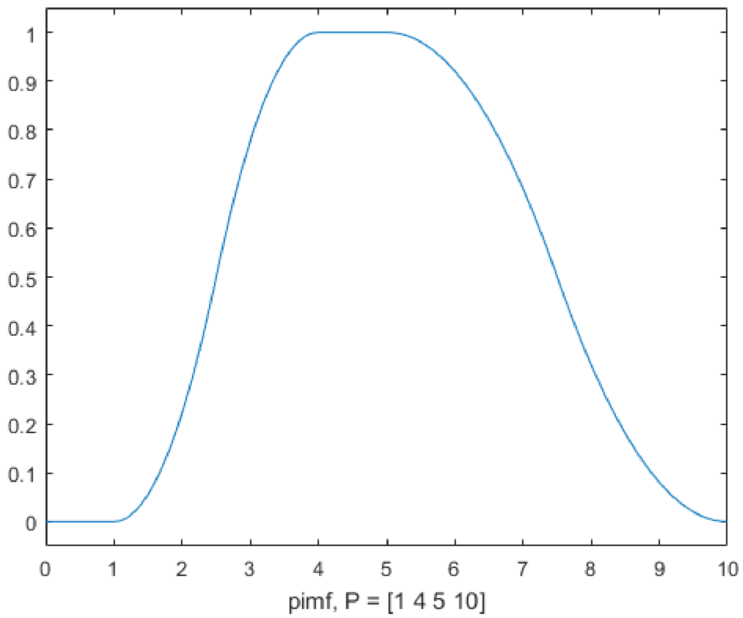




$$m = \frac{a+b}{2}$$

$$Z(x) = \begin{cases} 1, & \text{for } x \le a \\ 1-2\left(\dfrac{x-a}{b-a}\right)^2, & \text{for } a \le x \le \dfrac{a+b}{2} \\ 2\left(\dfrac{x-b}{b-a}\right)^2, & \text{for } \dfrac{a+b}{2} \le x \le b \\ 0, & \text{for } x \ge b \end{cases}$$

Pi membership function



$$\Pi_1 = \frac{1}{1 + \left(\frac{x-a}{b}\right)^2}$$

$$\Pi_2(x,a,b,c,d) = \begin{cases} \dfrac{c}{a+c-x}, x < a \\ 1, a \le x \le b \\ \dfrac{d}{x-b+d} x > b \end{cases}$$