Name: Shawn
Class: Real Analysis

1. **Show that if M is the open interval (a,b), and p is in M, then p is a limit point of M.**

> **Solution:**
> Let M be the open interval (a,b) and p∈M, where a,b∈$\mathbb{R}$.
> Let S = $(p-\epsilon, p+\epsilon)$, where ϵ is an arbitrary positive real number and $|\epsilon|$ is such that $S \subseteq M$.
> $(p+\epsilon/2)$, p∈S for any ϵ∈$\mathbb{R}^+$, and so S is an arbitrary interval that contains p and contains another point of M.
> Therefore p is a limit point of M.

2. **Show that if M is the closed interval [a,b], and p is not in M, then p is not a limit point of M.**

> **Solution:**
> Let M be the closed interval [a,b], and p∉M. Assume, without loss of generality, that p > b.
> Let $\epsilon = \frac{p-b}{2}$, and let S = $(p-\epsilon, p+\epsilon)$. Hence $S \cap M = \emptyset$.
> Since S is an open interval that contains p but doesn't contain any element of M, p cannot be a limit point of M.

3. **Show that if M is a point set having a limit point, then M contains 2 points**

> **Solution:**
> Let M be a point set that contains a limit point p. Let $S_\epsilon = (p-\epsilon, p+\epsilon)$, where $\epsilon \in \mathbb{R}^+$. Since p is a limit point there must be exist point in S_ϵ (besides p) for every ϵ. Let k be another point in S_ϵ, where k≠p. Now let $S_{\epsilon_2} = (p-\epsilon_2, p+\epsilon_2)$ where $\epsilon_2 < |p-k|$. So then $k \notin S_{\epsilon_2}$, but there must exist another point in S_{ϵ_2} that is not p.
> Using the Archemedian property and the above argument shows that there are infinite points in M.