Why PSBLAS?

Sparse Matrices and Krylov methods

A matrix is sparse when there are so many zeros (nonzeros are typically $\mathcal{O}(n)$) that it pays off to take advantage of them in the computer representation. James Wilkinson

Methods of choice: Search for a solution by projection

$$
\begin{aligned}
\mathbf{x}_m &\in \mathcal{K}_m(A, \mathbf{r}_0) \\
\mathbf{r}_m = \mathbf{b} - A\mathbf{x}_m &\perp \mathcal{K}_m(A, \mathbf{r}_0)
\end{aligned}
$$

$$\mathcal{K}_m(A, \mathbf{r}_0) = Span\{\mathbf{r}_0, A\mathbf{r}_0, A^2\mathbf{r}_0, \dots, A^{m-1}\mathbf{r}_0\}$$

Krylov subspace (growing with iteration until $\mathbf{x}_m$ is good enough)

Conjugate Gradient (CG) for s.p.d. matrices (1952). CG Convergence

$$\frac{\|\mathbf{e}_k\|_A}{\|\mathbf{e}_0\|_A} \leq 2\left(\frac{a-1}{a+1}\right), \quad a = \sqrt{\mu(A) = \lambda_{max}/\lambda_{min}}$$

$\mathbf{e}_k = \mathbf{x} - \mathbf{x}_k$ error at iteration k, λ eigenvalue of A