

## Sparse Matrices and Krylov methods

A matrix is sparse when there are so many zeros (**nonzeros are typically  $\mathcal{O}(n)$** ) that it pays off to take advantage of them in the computer representation.

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**Methods of choice:** Search for a solution by projection

$$\mathbf{x}_m \in \mathcal{K}_m(A, \mathbf{r}_0)$$

$$\mathbf{r}_m = \mathbf{b} - A\mathbf{x}_m \perp \mathcal{K}_m(A, \mathbf{r}_0)$$

$$\mathcal{K}_m(A, \mathbf{r}_0) = \text{Span}\{\mathbf{r}_0, A\mathbf{r}_0, A^2\mathbf{r}_0, \dots, A^{m-1}\mathbf{r}_0\}$$

Krylov subspace (growing with iteration until  $\mathbf{x}_m$  is good enough)