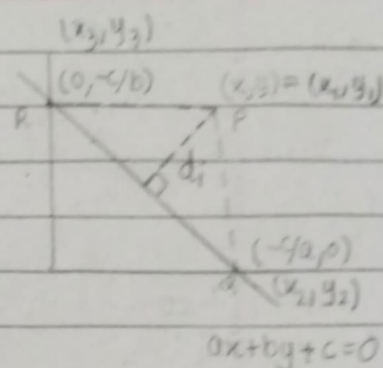


## Perpendicular Distance of point from line

Equation of line  $ax+by+c=0$ for  $x=0$  for  $y=0$ 

$$y = -\frac{c}{b}$$

$$x = -\frac{c}{a}$$



$$\text{Area}(\Delta PQR) = \frac{1}{2} \text{Base} \cdot \text{Height}$$

$$2 \cdot \text{Area}(\Delta PQR) = \text{Height} = d_1$$

Base

$$\text{Area}(\Delta PQR) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} \left[ x_1 \left(0 + \frac{c}{b}\right) + \left(-\frac{c}{a}\right) \left(-\frac{c}{b} - y_1\right) \right]$$

$$= \frac{1}{2} \left[ \frac{x_1 c}{b} + \frac{c^2}{ab} + \frac{c y_1}{a} \right] = \frac{1}{2} \frac{c}{ab} [ax_1 + c + by_1] \dots (1)$$

$$\text{Base} = \sqrt{\left(0 - \frac{c}{b}\right)^2 + \left(-\frac{c}{a} + 0\right)^2}$$

$$= \sqrt{\frac{c^2}{b^2} + \frac{c^2}{a^2}} = \sqrt{\frac{b^2 c^2 + a^2 c^2}{a^2 b^2}} = \sqrt{\frac{c^2 (b^2 + a^2)}{a^2 b^2}}$$

$$= \frac{c}{ab} \sqrt{b^2 + a^2} \dots (2)$$

$$d_1 = \frac{2 \cdot \frac{1}{2} \frac{c}{ab} [ax_1 + by_1 + c]}{\frac{c}{ab} \sqrt{a^2 + b^2}} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \dots (3)$$

Minimization of the distance.

The equation of straight line.

$$y_i = mx_i + c$$

$$\Rightarrow mx_i - y_i + c = 0$$

$$ax + by + c = 0$$

$$\text{here } a = m, b = -1$$

equation (3) becomes

$$d_i = \frac{|mx_i - y_i + c|}{\sqrt{m^2 + 1}} = \hat{e}_i$$

$$D(m, c) = \sum_{i=1}^n d_i(m, c)$$

$$\frac{\partial}{\partial m} \left( \frac{mx_i - y_i + c}{\sqrt{m^2 + 1}} \right)^2$$

$$\frac{\partial (mx_i - y_i + c)^2}{\partial m (m^2 + 1)} =$$

$$= \sum_{i=1}^n \frac{2(m^2 + 1)(mx_i - y_i + c)x_i - m(mx_i - y_i + c)^2}{(m^2 + 1)^2}$$

$$= \sum_{i=1}^n \frac{2x_i(mx_i - y_i + c)}{m^2 + 1} - \frac{2m(mx_i - y_i + c)}{(m^2 + 1)^2}$$

Now

$$\frac{\partial (mx_i - y_i + c)^2}{\partial c (m^2 + 1)}$$

$$= \sum_{i=1}^n \frac{2(mx_i - y_i + c)}{m^2 + 1}$$