Assignment 3

Group members' name:

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Question 1

(a) Pigeonholes : score (0-100) = 101
Pigeon : students : ?

- Pigeonhole state that pigeon > pigeonholes which is there will always be at least two pigeons in one hole.
- if each students have each one of the score ~~which if~~ from 0 to 100 and at least two students received the same score, then the students in the class need to be at least 102.

(b) pigeons - students (n = ?).
pigeonhole - grade (m = 5)

$k = 6$

$k = \left\lceil \frac{n}{m} \right\rceil$

$6 = \left\lceil \frac{n}{5} \right\rceil$ , $m = 30$.

∴ minimum students = 30.

Question 2.

2(a) $P(A) = 0.7$

~~(a) P(A|C)~~

(b) $P(B) = 0.3$

(c) $P(C|A) = 0.2$

(d) $P(A \cap C) = ?$

$$P(C|A) = \frac{P(C \cap A)}{P(A)}$$

$$0.2 = \frac{P(C \cap A)}{0.7}$$

$$P(C \cap A) = 0.14.$$

(e) $P(B \cap C) = ?$

$P(C|B) = 0.4$

$$P(C|B) = \frac{P(C \cap B)}{P(B)}$$

$$0.4 = \frac{P(C \cap B)}{0.3}$$

$$P(C \cap B) = 0.12.$$

(f) $P(C) = ?$

$$P(B|C) = \frac{P(C|B)P(B)}{P(C|B)P(B) + P(C|A)P(A)}$$

$$= \frac{(0.4)(0.3)}{(0.4)(0.3) + (0.2)(0.7)}$$

$$= 0.462$$

$$P(B|C) = \frac{P(B \cap C)}{P(C)} \quad \therefore P(C) = 0.26.$$

(g) $$P(A|C) = \frac{P(C|A)P(A)}{P(C|A)P(A) + P(C|B)P(B)}$$

$$= \frac{(0.2)(0.7)}{(0.2)(0.7) + (0.4)(0.3)}$$

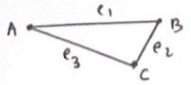
$$= 0.538.$$

Question 3.

(a) vertices - objects that are connected together.

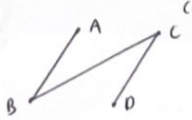

$V = \{A, B, C\}$

(b) edges - connection between the vertices.




$E = \{e_1, e_2, e_3\}$

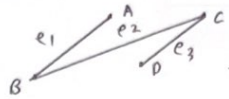
(c) adjacent vertices - two vertices that are connected by an edge are called adjacent.




∴ B and C are adjacent.

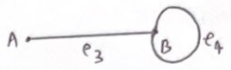
Question 3

(a) incident edge - edge that connects a particular vertex.



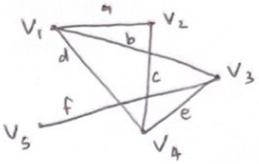

∴ $e_1$ and $e_2$ are incident edge.

(f) loop - an edge with just one endpoint.




∴ $e_4$ is a loop.

Question 4.




deg $(V_1) = 3$

deg $(V_2) = 2$

deg $(V_3) = 3$

deg $(V_4) = 3$

deg $(V_5) = 1$

Question 5.

i. incidence matrix

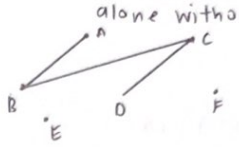
$V = \{1,2,3,4,5,6\}$ $E = \{a,b,c,d,e,f,g,h,i,k\}$

| | a | b | c | d | e | f | g | h | i | k |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 3 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 4 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |

(ii) adjancency matrix

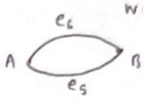
| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 2 | 1 | 0 | 0 |
| 2 | 0 | 0 | 0 | 1 | 0 | 0 |
| 3 | 2 | 0 | 0 | 1 | 1 | 1 |
| 4 | 1 | 1 | 1 | 0 | 0 | 1 |
| 5 | 0 | 0 | 1 | 0 | 0 | 1 |
| 6 | 0 | 0 | 1 | 1 | 1 | 0 |

(e) isolated vertex - vertex that is not connected to any other vertex by an edge. It stands alone without any direct connections.




∴ E and F are isolated vertex.

(g) parallel edge - two or more distinct edges with the same set of endpoints.




∴ $e_5$ and $e_6$ are parallel.

Question 6

* both graph have the same number of vertices and edges.
* $f = Y \to Z$, where $Y = \{A,B,C,D,E,F\}$ and $Z = \{1,2,3,4,5,6\}$;

$f(A) = 6$; $f(B) = 5$; $f(C) = 4$; $f(D) = 3$;

$f(E) = 2$; $f(F) = 1$.

$A_Y =$

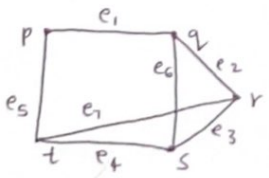
| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 0 | 1 | 0 | 1 | 0 | 0 |
| B | 1 | 0 | 0 | 1 | 1 | 1 |
| C | 0 | 0 | 0 | 1 | 1 | 1 |
| D | 1 | 1 | 1 | 0 | 0 | 1 |
| E | 0 | 1 | 1 | 0 | 0 | 0 |
| F | 0 | 1 | 1 | 1 | 0 | 0 |

$A_Z =$

| | 6 | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|---|
| 6 | 0 | 1 | 0 | 1 | 0 | 0 |
| 5 | 1 | 0 | 0 | 1 | 1 | 1 |
| 4 | 0 | 0 | 0 | 1 | 1 | 1 |
| 3 | 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 |

∴ both graphs are isomorphic.

Question 7




(i) $(p, e_5, t)$, $(p, e_1, q, e_6, s, e_4, t)$, $(p, e_1, q, e_2, r, e_7, t)$, $(p, e_1, q, e_2, r, e_3, s, e_4, t)$, $(p, e_1, q, e_6, s, e_3, r, e_7, t)$.

(ii) shortest path : $(p, e_5, t)$

longest path : $(p, e_1, q, e_2, r, e_3, s, e_4, t)$ / $(p, e_1, q, e_6, s, e_6, r, e_7, t)$

(iii) shortest trail : $(p, e_5, t)$

longest trail : $(p, e_1, q, e_2, r, e_3, s, e_4, t)$ / $(p, e_1, q, e_6, s, e_3, r, e_7, t)$.