金融经济学导论复习

考试题型

- 填空题4题，10分，
- 选择题5题，10分
- 判断题 4个，10分
- 简答1个，10分
- 计算题4个，40分
- 证明题2题，20分

主要内容

- Chapter 2 Preferences representation and risk aversion
- 2.1 Certainty and Uncertainty
- 2.2 the Expected Utility Theory

2.1.1 Risk, speculation and gambling

What is risk?

What is uncertainty?

- **Definition1**.Risk is the uncertainty of the future earnings and the Probability of occurrence of each state are known.
- **Definition 2** We measure the reward as the difference between the expected HPR(Holding-Period Returns) on the index stock fund and the risk-free rate, that is, the rate you would earn in risk-free assets such as T-bills, money market funds, or the bank.

Definition 3. Speculation is the assumption of considerable investment risk to obtain commensurate gain.

Definition 4. Gambling is to bet or wager on an uncertain outcome.

- **Definition 1** A binary relation $\succcurlyeq$ on X is a collection of pairs of consumption plans (x,y). If (x,y) is in the relation, we write $x \succcurlyeq y$ and say x is preferred to y. If (x,y) is not in the relation, then we write $x \not\succeq y$ and say x is not preferred to y.
- **Definition 2**: A preference relation is binary relation with **transitivity, completeness**.
- **Transitivity:** A binary relation is transitive if $x \succcurlyeq y$ and $y \succcurlyeq z$ imply $x \succcurlyeq z$. That is ,If x is preferred (or indifferent) to y, and y is preferred (or indifferent) to z, then x is preferred (or indifferent) to z.
- **Completeness:** A binary relation is said to be complete if for any two consumption plans x and y, we either have x $\succcurlyeq$ y or y $\succcurlyeq$ x. That is, any two consumption plans can always be compared.

- If the preference relationship is violate consistency of preferences, then the investor is lack of rationality.
- **Theorem 1**: If the portfolio set **X** has limited or countable elements, then the preference relation defined on $\mathbf{X} \times \mathbf{X}$ can be described by the real value utility function.
- **Theorem2**: Assuming that there is a preference relation on P. If it satisfies the following assumptions, **the preference relation** can be described by **von Neumann and Morgenstern's expected utility function**, and it is **unique**.

Chapter 3: the Analysis of Risk Averter's Behavior

- Risk aversion, risk neutral, risk seeking and fair gamble
- Risk-averse individuals 'behavior
- How to estimate the Individual Risk Aversion?
- Absolute Risk Aversion and Relative Risk Aversion

- Definition 1. Fair gamble refers to gambling with a risk premium of 0.

Definition 2:The gamble is fair when its expected payoff is zero, or

$$ph_1 + (1 - p)\,h_2 = 0$$

- **Definition 3:** An individual is said to be risk averse if he is unwilling to accept or is indifferent to any fair gamble. An individual is said to be strictly risk averse if he is unwilling to accept any fair gamble.
- Let u( ) be the utility function of an individual. From the definition of (strict) risk aversion, we have

$$E(u(W_0)) \geq (>) E(u(W_0 + g)) \quad \forall\, E(g) = 0$$

or

$$u(W_0) \geq (>) p\, u(W_0 + h_1) + (1 - p)\, u(W_0 + h_2)$$

where W_0 denotes the individual's initial wealth.

- **Definition4:** An individual is said to be risk neutral if he is indifferent to any actuarially fair gamble.

$$E\big(u(W_0)\big) = E(u(W_0 + g)) \quad \forall\ E(g) = 0$$

or

$$u(W_0) = p\, u(W_0 + h_1) + (1 - p)\, u(W_0 + h_2)$$

- **Definition5:** An individual is said to be risk seeking if he is **happy to** attend all fair gambles.

$$E(u(W_0)) \leq (<) E(u(W_0 + g)) \quad \forall\ E(g) = 0$$

or

$$u(W_0) \leq (<) p\, u(W_0 + h_1) + (1 - p)\, u(W_0 + h_2)$$

- Theorem 1. A (strict) risk averter $\Leftrightarrow$ u is a (strictly) concave function$\Leftrightarrow u'' \leq (<)0$
- Theorem 2. A (strict) risk lover $\Leftrightarrow$ u is a (strictly) convex function$\Leftrightarrow u'' \geq (>)0$
- Theorem 3. Risk neutral $\Leftrightarrow$ u is a linear function $\Leftrightarrow u'' = 0$

- **Proposition 1:**An individual who is risk averse and strictly prefers more to less will invest to risky asset if and only if the rate of return on at least one risky asset greater than the risk-free interest rate.

Proposition 2: In a market where there is only one risky asset whose risk is very low and one risk-free asset, the sufficient and necessary condition for investors to invest all the initial wealth W_0 or even hold a short position in the risk-free asset is the following inequality was satisfied:

$$E\left(r - r_f\right) \geq -\frac{W_0 u''\left(W_0(1 + r_f)\right)}{u'\left(W_0(1 + r_f)\right)} E((r - r_f)^2)$$

- **Proposition 3:** In a market where there is only one risky asset whose risk is very low and one risk-free asset, the sufficient and necessary condition for investors to invest at least λW_0 in the risky asset is the following inequality is satisfied:

$$E\left(r - r_f\right) \geq -\frac{\lambda W_0 u''\left(W_0\left(1 + r_f\right)\right)}{u'\left(W_0\left(1 + r_f\right)\right)} E\left(\left(r - r_f\right)^2\right)$$

- Definition 4: Let $f(W_0, H)$ be the maximum amount of the penalty that investors are willing to pay in order to avoid a gamble (an uncertainty), if it satisfies the following formula:

$$u(W_0 - f(W_0, H)) = p\, u(W_0 + h_1) + (1 - p)\, u(W_0 + h_2)$$

then $f(W_0, H)$ is called the **Markowitz risk premium.** $W_0 - f(W_0, H)$ is called the certainty equivalent wealth.

- Definition 5:Consider a gamble with a level of risk, we can define Pratt-Arrow risk premium as follows,

$$f(W_0, H) = \frac{1}{2}\sigma_H^2\left(-\frac{u''(W_0)}{u'(W_0)}\right)$$

- Definition 6. Absolute risk aversion coefficient is define by $R_A(x) = -\frac{u''(x)}{u'(x)}$.
- Definition7: An individual's utility function displays **decreasing absolute risk aversion** when R_A(z) is a strictly decreasing function , $\text{i.e.}, \frac{dR_A(z)}{dz} < 0, \forall z$. Similarly, $\frac{dR_A(z)}{dz} > 0$, $\forall z$, implies **increasing absolute risk aversion**, and $\frac{dR_A(z)}{dz} = 0$, $\forall z$ implies **constant absolute risk aversion.**

- Theorem 1.
- $\forall z\ , \dfrac{dR_A(z)}{dz} < 0 \Rightarrow \dfrac{da}{dW_0} > 0,\ \forall W_0$
- $\forall z\ , \dfrac{dR_A(z)}{dz} > 0 \Rightarrow \dfrac{da}{dW_0} < 0,\ \forall W_0$
- $\forall z\ , \dfrac{dR_A(z)}{dz} = 0 \Rightarrow \dfrac{da}{dW_0} = 0,\ \forall W_0$

- Definition 8.The Arrow-Pratt measure of relative risk aversion is $R_R(z) \equiv zR_A(z). \textit{ Furtherly, } R_R(z) = -z\frac{u''(z)}{u'(z)}$
- Definition 9. An individual's utility function displays **increasing relative risk aversion**, that is, $\forall z$, $\frac{dR_R(z)}{dz} > 0.$
- An individual's utility function displays **constant relative risk aversion**, that is, when $\forall z$, $\frac{dR_R(z)}{dz} = 0.$
- An individual's utility function displays **decreasing relative risk aversion**, that is, when $\forall z$, $\frac{dR_R(z)}{dz} < 0.$

- Theorem 2
- ① Under **increasing relative risk aversion**, that is, when $\forall z$, $\frac{dR_R(z)}{dz} > 0$, **the wealth elasticity** of the individual's demand for the risky asset is strictly **less than unity**.
- ② Under **constant relative risk aversion**, that is, when $\forall z$, $\frac{dR_R(z)}{dz} = 0$, **the wealth elasticity** of demand for the risky asset would be **unity**.
- ③ Under **decreasing relative risk aversion**, that is,

- **Concave quadratic utility function**

$$u(z) = z - \frac{b}{2}z^2\ , b > 0$$

What kind of absolute risk aversion does the quadratic utility function display?

(decreasing, increasing or constant)

Chapter 4: stochastic dominance

- **Definition 1:**We will say that risky asset A dominates risky asset B in the sense of the First Degree Stochastic Dominance, denoted by $A \geq_{FSD} B$ if all individuals having utility functions in wealth that are increasing and continuous either prefer A to B or are indifferent between A and B. That is,

$$\forall u'(.) \geq 0, E\left(u(1+\widetilde{r_A})\right) \geq E\left(u(1 + \widetilde{r_B})\right)$$

- **Definition 2. Second Degree Stochastic Dominance.**

If we have $\forall u''(.) \leq 0 \ , E(u(\widetilde{r_A})) \geq E(u(\widetilde{r_B}))$

then we will say that risky asset A dominates risky asset B

- **Proposition 1.**

$$A \geq_{FSD} B \Leftrightarrow F_A(z) \leq F_B(z) \quad \forall z \in [0,1]$$

- **Definition 2. Second Degree Stochastic Dominance.**

If we have $\forall u''(.) \leq 0 \ , E(u(\tilde{r}_A)) \geq E(u(\tilde{r}_B))$ then we will say that risky asset A dominates risky asset B in the sense of the **Second Degree Stochastic Dominance**, denoted by $A \geq_{SSD} B$

- Definition 3 If the return of security A is always higher than that of security B in each possible state, that is

$$x_A(w) \geq x_B(w)$$

Chapter 5 Mathematics of the portfolio frontier

- Property 1 : **The portfolio composed of two assets is a hyperbola in the mean-standard deviation coordinate system.**
- Property 2 : The proportion of these two risky assets in minimum variance portfolio is

$$w_1 = \frac{\sigma_2^2 - \operatorname{cov}(r_1, r_2)}{\sigma_1^2 + \sigma_2^2 - 2\operatorname{cov}(r_1, r_2)}$$

- Property 3 : Given the risk aversion level, the proportion of these two risky assets in utility maximization portfolio is

$$w_1 = \frac{E(r_1) - E(r_2) + 0.01A(\sigma_2^2 - \operatorname{cov}(r_1, r_2))}{0.01A(\sigma_1^2 + \sigma_2^2 - 2\operatorname{cov}(r_1, r_2))}$$

$$w_1 = \frac{(E(r_1) - r_f)\sigma_2{}^2 - (E(r_2) - r_f)\operatorname{cov}(r_1, r_2)}{[E(r_1) - r_f]\sigma_2^2 + [E(r_2) - r_f]\sigma_1^2 - [E(r_1) - r_f + E(r_2) - r_f]\operatorname{cov}(r_1, r_2)}$$

- **Nonsystematic risk:** nonmarket or firm-specific risk factors that can be eliminated by diversification, such as legal disputes, strikes, new product development failure, etc. Also called unique risk or diversifiable risk.
- **Systematic risk:** risk factors common to the whole economy, such as the economic prosperity, changes in the overall market interest rates, etc. Also called nondiversifiable risk or market risk.

Assumptions

1.Single-period investment

Single-period investment means that investors invest at the beginning of the period and obtain returns at the end of the period.

2.Investors know the probability distribution of return rate in advance, and the return rate of stocks satisfies normal distribution.

3.The utility function of investors is quadratic.

4.Investors measure the future return rate with the expected rate of return, and measure the uncertainty (risk) with the variance (or standard deviation) of the return rate.

5.Investors are nonsatiable and risk averse, following the principle of stochastic dominance.

- $$\min \frac{1}{2} w^T V w$$
  $$s.t. w^T e = E[\tilde{r}_p]$$
  $$w^T 1 = 1$$

- Where e denotes the N-vector of expected rate of return on the N risky assets, $E[\tilde{r}_p]$ denotes the expected rate of return on portfolio p, and 1 is an N-vector of ones.

$$\lambda = \frac{CE[\tilde{r}_p] - A}{D}$$

$$\gamma = \frac{B - AE[\tilde{r}_p]}{D}$$

where

$$A = \mathbf{1}^\top \mathbf{V}^{-1}\mathbf{e} = \mathbf{e}^\top \mathbf{V}^{-1}\mathbf{1},$$

$$B = \mathbf{e}^\top \mathbf{V}^{-1}\mathbf{e},$$

$$C = \mathbf{1}^\top \mathbf{V}^{-1}\mathbf{1},$$

$$D = BC - A^2.$$

$$\mathbf{w}_p = \mathbf{g} + \mathbf{h}E[\tilde{r}_p] \qquad ($$

where

$$\mathbf{g} = \frac{1}{D}[B(\mathbf{V}^{-1}\mathbf{1}) - A(\mathbf{V}^{-1}\mathbf{e})]$$

and

- **Proposition 1** g is the vector of portfolio weights corresponding to a frontier portfolio having a zero expected rate of return and $g+h$ is the vector of portfolio weights of a frontier portfolio having an expected rate of return equal to 1.
- **Proposition 2** The entire portfolio frontier can be generated by the two frontier portfolios g and $g+h$.
- **Proposition 3** The portfolio frontier can be generated by any two distinct frontier portfolios.

- **Two fund separation theorem:** on the portfolio frontier of all risky assets, any two distinct points represent two distinct frontier portfolios, and any other frontier portfolio can be generated by the linear combination of these two frontier portfolios.

$$\operatorname{cov}(\tilde{r}_p, \tilde{r}_q) = w_p V^T w_q = \frac{C}{D}(E[\tilde{r}_p] - \frac{A}{C})(E[\tilde{r}_q] - \frac{A}{C}) + \frac{1}{C}$$

$$\frac{\sigma^2(\tilde{r}_p)}{1/C} - \frac{(E[\tilde{r}_p] - \frac{A}{C})^2}{D/C^2} = 1$$

- What are efficient portfolios？

Those frontier portfolios which have expected rates of return strictly higher than that of the minimum variance portfolio, A/C, are called efficient portfolios.

- What are inefficient portfolios？

Portfolios that are on the portfolio frontier but are neither efficient nor minimum variance are called inefficient portfolios. For each inefficient portfolio there exists an efficient one having the same variance but a higher expected rate of return.

- **Property 4** The covariance of minimum variance portfolio and any other frontier portfolio is equal to 1/C, which is strictly positive. Therefore, there does not exist a frontier portfolio that has zero covariance with the minimum variance portfolio.

$$E(r_{zcp}) = \frac{A}{C} - \frac{D/C^2}{E(\tilde{r}_p) - A/C}$$

- In the standard deviation-expected rate of return space, the intercept on the E(r)-axis of the line passing through any frontier portfolio p (except the minimum variance portfolio) and tangent to the portfolio frontier is $E[\tilde{r}_{zcp}]$

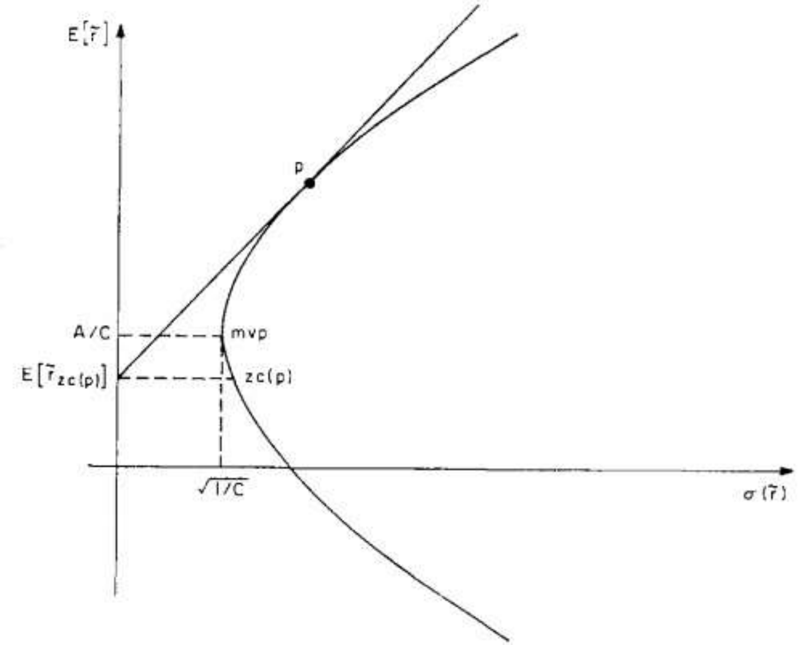

Figure 3.15.1: The Location of a Zero Covariance Portfolio in the $\sigma(\tilde{r})$–$E[\tilde{r}]$ Space

- Alternatively, in the variance-expected rate of return space, the intercept on the E(r)-axis of the line passing through any frontier portfolio p and the mvp is equal to $E[\widetilde{r}_{zcp}]$

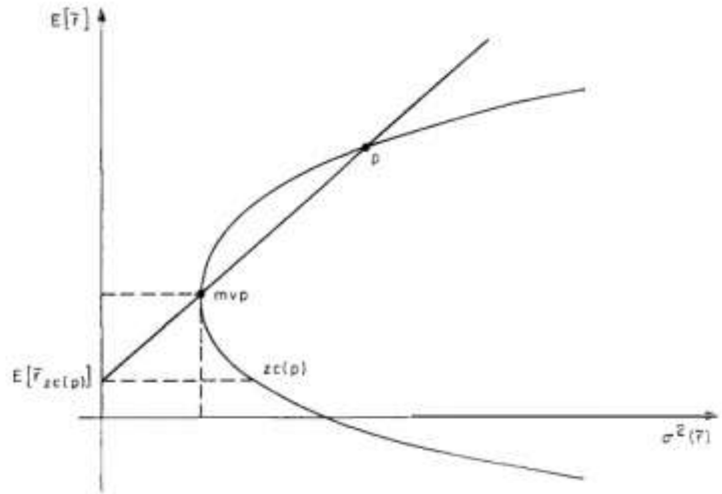

Figure 3.15.2: The Location of a Zero Covariance Portfolio in the $\sigma^2(\tilde{r})$–$E[\tilde{r}]$ Space

Asset pricing model （without riskless asset）

- Theorem 1: The expected rate of return of any portfolio q can be written as a linear combination of the expected rates of return on p and on its zero covariance portfolio $zc(p)$**:**

$$E[\tilde{r}_q] = (1-\beta_{qp})E[\tilde{r}_{zc(p)}] + \beta_{qp}E[\tilde{r}_p]$$

- where

$$\beta_{qp} = \frac{\mathrm{cov}(\tilde{r}_q, \tilde{r}_p)}{\sigma_p^2}$$

The portfolio frontier with riskless asset

$$\min \frac{1}{2} w^T V w$$

$$s.t. w^T e + (1 - w^T 1) r_f = E[\tilde{r}_p]$$

Solving for $\mathbf{w}_p$, we have

$$\mathbf{w}_p = \mathbf{V}^{-1}(\mathbf{e} - r_f \mathbf{1}) \frac{E[\tilde{r}_p] - r_f}{H}, \tag{3.18.1}$$

$$\sigma(\tilde{r}_p) = \begin{cases} \frac{E[\tilde{r}_p] - r_f}{\sqrt{H}} & \text{if } E[\tilde{r}_p] \geq r_f, \\ -\frac{E[\tilde{r}_p] - r_f}{\sqrt{H}} & \text{if } E[\tilde{r}_p] < r_f, \end{cases} \tag{3.18.3}$$

We now consider some special cases.

Case 1. $r_f < A/C$. This case is presented graphically in Figure 3.18.1, where **e** is the tangent point of the half line $r_f + \sqrt{H}\sigma(\tilde{r}_p)$ and the portfolio frontier of all risky assets. To verify this, we only

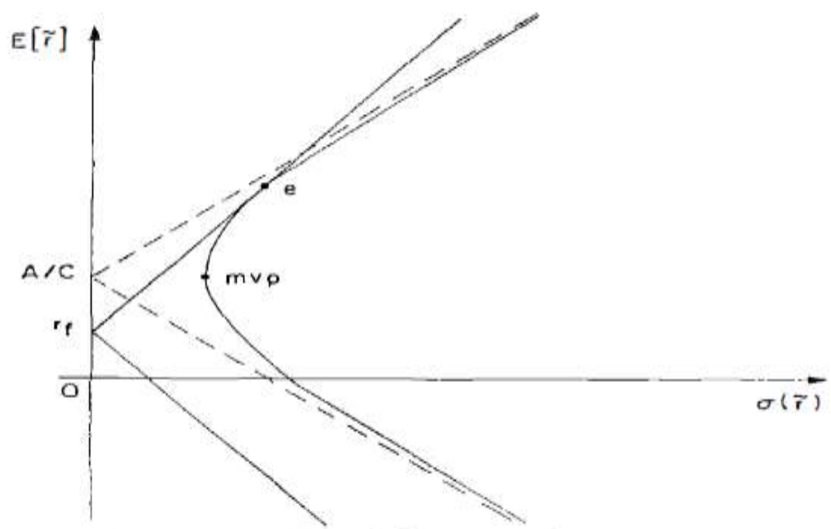

Figure 3.18.1: A Portfolio Frontier when $r_f < A/C$

Case 2. $r_f > A/C$. The story here is a little different. The portfolio frontier of all assets is graphed in Figure 3.18.2. Any portfolio on the half-line $r_f + \sqrt{H}\sigma(\tilde{r}_p)$ involves short-selling portfolio e' and investing the proceeds in the riskless asset. On the other hand, any portfolio on the half-line $r_f - \sqrt{H}\sigma(\tilde{r}_p)$ involves a long position in portfolio e'.

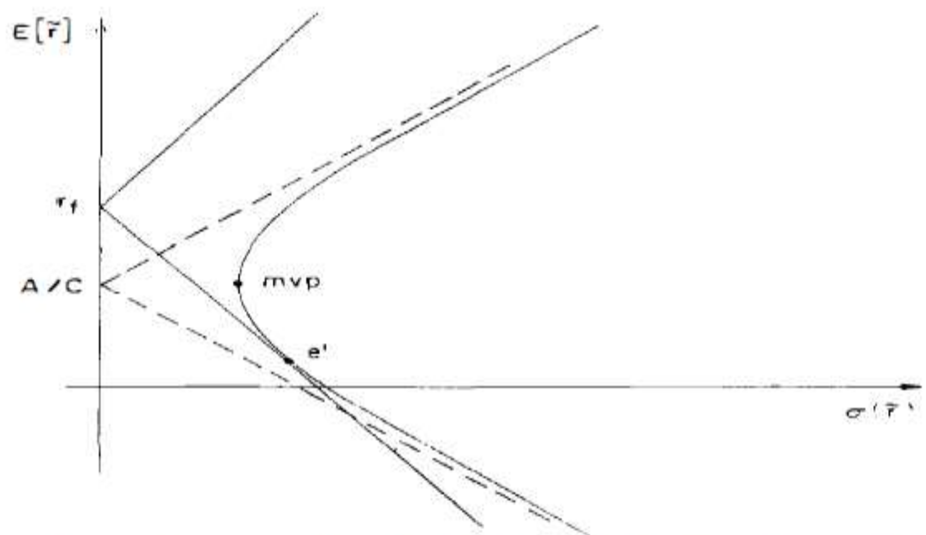

Figure 3.18.2: A Portfolio Frontier when $r_f > A/C$

Case 3. $r_f = A/C$. In this case,

$$\begin{aligned} H &= B - 2Ar_f + Cr_f^2 \\ &= B - 2A\,(A/C) + CA^2/C^2 \\ &= \frac{BC - A^2}{C} = \frac{D}{C} > 0. \end{aligned}$$

Recall that $E[\tilde{r}_p] = A/C \pm \sqrt{D/C}\,\sigma(\tilde{r}_p)$ are the two asymptotes of the portfolio frontier of risky assets. The portfolio frontier of all assets is graphed in Figure 3.18.3.

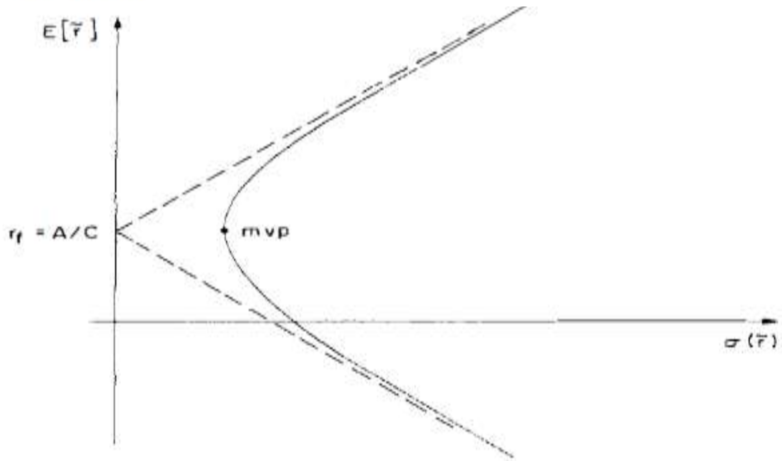

Figure 3.18.3: A Portfolio Frontier when $r_f = A/C$

Asset pricing:

- The relationship between any portfolio q and frontier portfolio p is :

$$E[\tilde{r}_q] - r_f = \beta_{qp}(E[\tilde{r}_p] - r_f)$$

$$\beta_{qp} = \frac{\operatorname{cov}(\tilde{r}_q, \tilde{r}_p)}{\sigma_p^2}$$

Chapter 5 two fund separation and linear valuation

assumptions

1. Individual behavior

   a. Investors are rational, mean-variance optimizers.

   b. Their planning horizon is a single period.

   c. Investors have homogeneous expectations (identical input lists).

2. Market structure

   a. All assets are publicly held and trade on public exchanges, short positions are allowed, and investors can borrow or lend at a common risk-free rate.

   b. All information is publicly available.

   c. No taxes.

   d. No transaction costs.

What is separation property?

What is Market portfolio?

What is market equilibrium?

What is the difference between Capital Market Line and Security Market Line?

Capital Asset Pricing Model

- **Security Market Line:** Graphical representation of the expected return–beta relationship of the CAPM.

$$E(\tilde{r}_i) = r_f + [E(\tilde{r}_M) - r_f]\beta_{iM}$$

$$\beta_{iM} = \frac{\text{cov}(\tilde{r}_i, \tilde{r}_M)}{\sigma_M^2}$$

- where, β_{iM} is the beta.

Extension of CAPM

- ZERO-BETA Capital asset pricing model
- Riskless lending is allowed, but riskless borrowing is not allowed.
- Unequal rates of riskless lending and borrowing
- Property3 When riskless lending is allowed and riskless borrowing is not allowed, the market portfolio m must be located on the hyperbolic frontier on the right side of the tangent portfolio e.
- Property 4 The market portfolio is on curve LB.

Chapter 7
Factor Model and Arbitrage Pricing Model

- Single factor model
- Multi factor model

General Form of Single-Factor-Model

- Generally, in the Single-Factor Model, a single factor F is considered to have a broad impact on security returns.

We can see that the return of any security is composed of three parts: $$\tilde{r}_i = a_i + \beta_i F + \varepsilon_i$$

- α_i : the expected return of the security when the expected growth value of macro factors is zero;
- $\beta_i F$: systemic risk return, that is, the uncertainty (unexpected) return that changes with the entire market movement, and the sensitivity is bi;
- ε_i : factors that have nothing to do with GDP, and it is a non-systematic risk return, that is, the unexpected return formed by unexpected events related only to a single security.

The Relation between CAPM and Factor Model

1. CAPM *VS* Factor Model

- CAPM is actually a special case of Single Factor Model, but the capital asset pricing model is an equilibrium model, while the factor model is not. For example, compare the expected return of CAPM and of the Factor Model:

$$E(R_i) = r_f + (E(r_M) - r_f)\beta_{iM}$$

$$E(R_i) = \alpha_i + \beta_i E(F)$$

2.CAPM *VS* Market Model

- There is a slope ß in both CAPM and the Market Model. These two models include the market more or less, but there are obvious differences between them:

1. CAPM is an equilibrium model, which describes how the price of securities is determined, and the market model is a Factor Model.
2. 2. CAPM is relative to the entire market portfolio, that is, relative to the collection of all securities in the market. The Market Model is relative to a certain market index, that is, based on a sample in the market.
3. 3. In a strict sense, the ß in CAPM is different from the ß value in the Market Model. But in practical terms, since we cannot get exactly the composition of the market portfolio, we generally use the market index instead. In this way, we can use the ß value measured in the Market Model to replace the ß value in CAPM.

Multi-Factor Model

- The equation of the two-factor model in period t:

$$R_{it} = a_i + b_{i1}F_{1t} + b_{i2}F_{2t} + \varepsilon_{it}$$

- F1 and F2 are two factors that have a general impact on the return rate of securities. bi1 and bi2 are respectively the sensitivity of security i to F1 and F2. εit is a random error term, and ai is the expected return rate of security i when both factors are set to 0.

The concept of arbitrage

Arbitrage refers to the behavior of obtaining net income without net investment. It is to get risk-free profits due to the temporary imbalance of prices.

- The first type of arbitrage: the cost is negative, and the future payment is 0
- The second type of arbitrage: The cost is 0, and future payment is positive. In short, the second type of arbitrage gives investors the opportunity to earn positive income without any cost.
- The third type of arbitrage: the cost is negative, and future payments is positive.

Assumptions of APT

- 1 The market is perfectly competitive and frictionless
- 2 Investors cannot be satisfied. When they find arbitrage opportunities, they will definitely construct arbitrage portfolios to make money.
- 3 The expectations of all investors are the same, and the return of any security i satisfies the k-factor model:

$$r_i = a_i + \beta_{i1}F_1 + \beta_{i2}F_2 + \cdots \beta_{ik}F_k + \varepsilon_i$$

- 4. $E[\varepsilon_i] = 0, \mathrm{cov}(\varepsilon_i, F_k) = 0, \mathrm{cov}(\varepsilon_i, \varepsilon_j) = 0, i \neq j, \mathrm{cov}(F_i, F_j) = 0$
- 5. The number of types of securities in the market is greater than k, the number of types of factors.

- Definition of arbitrage portfolio
- Definition 1: an arbitrage portfolio of securities meets the following three descriptions:
- 1. The initial price is zero (no money is spent at the beginning of the period);
- 2. The sensitivity to each factor is zero (that is, the factor risk is zero);
- 3. The expected return is positive.

- The arbitrage portfolios should satisfy the following equations：

$$\begin{cases} w_1 + w_2 + \cdots + w_n = 0 \\ b_{11}w_1 + b_{21}w_2 + \cdots + b_{n1}w_n = 0 \\ b_{12}w_1 + b_{22}w_2 + \cdots + b_{n2}w_n = 0 \\ \qquad \cdots \qquad \cdots \\ b_{1k}w_1 + b_{2k}w_2 + \cdots + b_{nk}w_n = 0 \\ \overline{r_1}w_1 + \overline{r_2}w_2 + \cdots + \overline{r_n}w_3 > 0 \end{cases} \qquad (n > k)$$

The Essence of Arbitrage Pricing

- Property 1: In a fully diversified combination, specific risk of each security can be diversified.
- Property 2: Two fully diversified portfolios with the same β value must have the same expected return when the market is in equilibrium, otherwise there will be risk-free arbitrage opportunities, and the two expected returns will finally be equal through arbitrage.

$$\beta_A = \beta_B \Rightarrow E(r_A) = E(r_B)$$

- Property 3: For two fully diversified portfolios with different β values, the risk compensation must be proportional to the β value. It can be expressed as the following formula:

$$\frac{E(r_p)-r_f}{\beta_p}=\frac{E(r_Q)-r_f}{\beta_Q}$$

- Property 4: For most assets, such as i and j, the risk premium is proportional to the beta value.

$$\frac{E(r_i)-r_f}{\beta_i}=\frac{E(r_j)-r_f}{\beta_j}=K$$

- Property 5: If Property 4 holds for any single asset i in the portfolio, then Property 4 holds for portfolio p.

$$\frac{E(r_p)-r_f}{\beta_p}=K$$

APT of single factor

$$E[\bar{r}_i] = \lambda_0 + \lambda b_1 = r_f + (\delta_1 - r_f) b_1$$

- λ: risk premium or expected return of the factor
  λ
- δ: expected return of a portfolio with unit sensitivity to the factor

$E(r_i)= \lambda 0 +$
$\lambda 1 * bi$在此处键入公式。

APT of multi-factor model

- $$\bar{r}_i = r_f + (\delta_1 - r_f)b_{i1} + \ldots + (\delta_K - r_f)b_{iK}$$

- δ_i: the expected return of a portfolio that only has unit sensitivity to factor i and not sensitive at all to other factors.

Differences between APT and CAPM

- The major difference between APT and CAPM: APT particularly emphasizes the principle of no-arbitrage equilibrium. CAPM is a market equilibrium dominated by the balance of return and risk, and is the result of the joint action of many investors, while the starting point of APT is to eliminate risk-free arbitrage opportunities. A small number of investors will construct large arbitrage positions and generate huge market pressure to restore equilibrium.
- APT does not require the assumption of risk appetite on which CAPM is based.
- APT just requires a fully diversified portfolio, while there must be a market index that can substitute for risky market portfolios in Sing-Factor Model.
- APT does not hold for all securities. APT can be used for a fully diversified portfolio, but for a single asset, APT is not always valid, and it is more practical to use CAPM.

- What is the relationship between the Beta in CAPM and the factor sensitivity in APT

- Single-Factor Model

$$E(r_i) = r_f + \lambda_1 b_i, E(r_i) = r_f + \beta_{iM}(E(r_M) - r_f)$$

$$\lambda_1 = [E(r_M) - r_f]\frac{\text{cov}(F, \tilde{r}_M)}{\sigma_M^2}$$

$$E(r_i) = r_f + \lambda_1 b_{i1} + \lambda_2 b_{i2}, E(r_i) = r_f + \beta_{iM}(E(r_M) - r_f)$$

- Multi-Factor Model

$$\lambda_1 = [E(r_M) - r_f]\frac{\text{cov}(F_1, \tilde{r}_M)}{\sigma_M^2} \qquad \lambda_2 = [E(r_M) - r_f]\frac{\text{cov}(F_2, \tilde{r}_M)}{\sigma_M^2}$$

- What is Efficient market hypothesis?

- **The weak-form hypothesis :** The weak-form hypothesis asserts that stock prices already reflect all information that can be derived by examining market trading data such as the history of past prices, trading volume, or short interest. This version of the hypothesis implies that trend analysis is fruitless. Past stock price data are publicly available and virtually costless to obtain. The weak-form hypothesis holds that if such data ever conveyed reliable signals about future performance, all investors already would have learned to exploit the signals. Ultimately, the signals lose their value as they become widely known.
- Corollary 1: If the weak efficient market hypothesis is true, technical analysis of stock prices will not work, and fundamental analysis may help investors to obtain excess profits.

- **The semi-strong form hypothesis** : The semi-strong form hypothesis states that all publicly available information regarding the prospects of a firm must be reflected already in the stock price. Such information includes, in addition to past prices, fundamental data on the firm's product line, quality of management, balance sheet composition, patents held, earning forecasts, and accounting practices. Again, if investors have access to such information from publicly available sources, one would expect it to be reflected in stock prices.
- Corollary 2: If the semi-strong efficient hypothesis is true, then the use of technical analysis and basic analysis in the market is useless, and the inside information may gain excess profits.

- **The strong form hypothesis :** The strong-form version of the efficient market hypothesis states that stock prices reflect all information relevant to the firm, even including information available only to company insiders. This version of the hypothesis is quite extreme. Few would argue with the proposition that corporate officers have access to pertinent information long enough before public release to enable them to profit from trading on that information. Indeed, much of the activity of the Securities and Exchange Commission is directed toward preventing insiders from profiting by exploiting their privileged situation. Rule 10b-5 of the Security Exchange Act of 1934 sets limits on trading by corporate officers, directors, and substantial owners, requiring them to report trades to the SEC.
- Corollary 3: In a strongly efficient market, there is nothing that helps investors make excess profits, not even funds or insiders.