MITx: 15.071x The Analytics Edge

CLASS ASSIGNMENTS IN AN ELEMENTARY SCHOOL

The Salanter Akiba Riverdale (SAR) Academy is a coeducational, private Modern Orthodox Jewish day school located in New York City. Every summer, the SAR Academy must create class assignments for their elementary school students. Each grade of 80-100 students must be divided into four different classes. Requests for assignments are made by parents, teachers, and school therapists. These requests include pairs of students that should be placed together, pairs of students that should not be placed together, and requests for students to be placed in classes that better suit their academic needs. These requests often conflict with each other, and it falls on the administration to prioritize which requests should be fullfilled over others.

In this exercise, we 'll solve a simplified version of the problem faced by the SAR Academy with 40 students. The full optimization problem is currently being used to assist administrators at the SAR Academy.

PROBLEM 1.1 - SOLVING THE BASIC PROBLEM (3 points possible)

The parents or guardians of each of the 40 students are asked to submit preferences for class 1 or class 2. These preferences often depend on the teaching style of the teachers, the teachers older siblings have had in the past, and characteristics of the class (one class is called an "inclusion class", which is better for students with academic needs). The parents give a ranking of 1 to the class they prefer (their first choice), and a ranking of 2 to their second choice. The data for this problem is in the spreadsheet ClassAssignments.ods for LibreOffice or OpenOffice, and ClassAssignments.xlsx for Microsoft Excel.

Download this file, and then formulate and solve the basic assignment problem. The decision variables are very similar to those in the Pfizer Sales Representatives problem. We want to assign each student to either Class 1, or Class 2. Our objective is to adhere to the preferences of the parents as much as possible (note that since smaller numbers in the preferences are better, we will be minimizing in this problem). We have two types of constraints: (1) each student must be assigned to exactly one class, and (2) there should be exactly 20 students in each class.

What is the optimal objective value?

? **Answer:** 42

EXPLANATION

There are 80 different decision variables (two for each student). For this explanation, let's suppose that you added the decision variables in cells G5:H44.

The objective is the sumproduct of these decision variables with the preferences, SUMPRODUCT(G5:H44, B5:C44), and we are minimizing the objective.

We have 42 constraints - 40 assignment constraints (each student must be assigned to exactly one class), and 2 classroom constraints (each class must have exactly 20 students). An example constraint for student 1 is "G5+H5 = 1" and an example constraint for class 1 is "SUM(G5:G44) = 20".

Additionally, the decision variables must be binary. If we set-up and solve this problem in LibreOffice, the objective value of the solution is 42.

Additional Note: This problem has multiple optimal solutions! So while everyone should get the same optimal objective function value of 42, your values of the decision variables may be different than the values someone else gets. This happens frequently in problems with little structure, like this one.

You have used 0 of 8 submissions

PROBLEM 1.2 - SOLVING THE BASIC PROBLEM (1 point possible)

How many students received their first choice class (according to the parent preferences)?

? **Answer:** 38

EXPLANATION

The objective value is 42, which means that 38 students received their first choice and two students received their second choice (1*38 + 2*2 = 42).

You have used 0 of 3 submissions

PROBLEM 1.3 - SOLVING THE BASIC PROBLEM (1 point possible)

We would like to better balance the boy/girl ratio in the classes. Add the necessary constraint(s) to your model to limit the number of boys in each class to no more than 12, and then resolve the model.

What is the objective value now?

? **Answer:** 46

EXPLANATION

We need to add two constraints to our model. One that constrains the sum of the decision variables for Students 1-23 in class one to be less than or equal to 12, and a second to constrain the sum of the decision variables for Students 1-23 in class two to be less than or equal to 12.

For this explanation, let's suppose that you added the decision variables in cells G5:H44. Then you would need to add the constraints:

Class 1: SUM(G5:G27) <= 12

Class 2: SUM(H5:H27) <= 12

After adding these constraints and resolving the model, the objective value changes to

46.

You have used 0 of 3 submissions

PROBLEM 1.4 - SOLVING THE BASIC PROBLEM (1 point possible)

Now how many students received their first choice class?

? **Answer:** 34

EXPLANATION

The objective value is 46, which means that 34 students received their first choice and 6 students received their second choice (1*34 + 6*2 = 46).

While the boy/girl ratio is now better balanced (a preference of the teachers and staff), fewer parent preferences are met. The administrative staff could adjust the constraints depending on the importance of the teacher preferences versus the parent preferences.

You have used 0 of 3 submissions

PROBLEM 2.1 - ADDING LOGICAL CONSTRAINTS (1 point possible)

In the next few questions, we'll add some logical constraints to our model that capture additional preferences of parents, teachers, and school therapists. A constraint added in one part will be used in all subsequent parts.

Students 10 and 11 are twins, and the school has a policy that twins must be placed in different classes. Add the necessary constraint(s) to implement this policy, and solve the model again.

What is the objective value now?

? **Answer:** 46

EXPLANATION

We need to add two constraints to our model. For this explanation, let's suppose that you added the decision variables in cells G5:H44, so Students 10 and 11 are in rows 14 and 15. Then we need to add the constraints: (1) G14 + G15 <= 1, and (2) H14 + H15 <= 1. This prevents students 10 and 11 from being in the same class. If we add these constraints and resolve the model, the solution changes, but the objective value is still 46.

You have used 0 of 3 submissions

PROBLEM 2.2 - ADDING LOGICAL CONSTRAINTS (1 point possible)

Students 4, 9, 15, 25, 30, and 36 are all from the same neighborhood. The school would like to put at least 2 students from this neighborhood in each class. Add the necessary constraint(s) to implement this policy, and solve the model again.

What is the objective value now?

? **Answer:** 46

EXPLANATION

For this explanation, let's suppose that you added the decision variables in cells G5:H44.

We need to add two constraints to our model: (1) G8 + G13 + G19 + G34 + G40 >= 2, and (2) H8 + H13 + H19 + H34 + H40 >= 2. These force each class to have at least 2 students from this neighborhood. After adding these constraints and resolving the model, the solution changes but the objective value stays at 46.

You have used 0 of 3 submissions

PROBLEM 2.3 - ADDING LOGICAL CONSTRAINTS (1 point possible)

The school therapist strongly recommends that students 20 and 21 are placed in the same classroom, that student 1 is placed in classroom 2, and that student 40 is placed in classroom 2. Add the necessary constraint(s) to implement this policy, and solve the model again.

What is the objective value now?

? **Answer:** 46

EXPLANATION

For this explanation, let's suppose that you added the decision variables in cells G5:H44.

We need to add four constraints: (1) G24 = G25, (2) H24 = H25, (3) H5 = 1, and (4) H44 = 1. If we add these constraints and resolve the model, the objective value remains at 46.

You have used 0 of 3 submissions

PROBLEM 2.4 - ADDING LOGICAL CONSTRAINTS (1 point possible)

How has the objective function value changed in this part, and what does this tell us?

- The objective function value has increased after adding each logical constraint, because adding additional constraints will always make objective function value of the new problem worse than before.
- The objective function value has increased after adding each logical constraint, because we had to put more students in their second choice classes.
- The objective function value has remained the same after adding each logical constraint, because the solution (assignment of students to classrooms) never changed.
- The objective function value has remained the same after adding each logical constraint, because it can't be any larger than the current value.
- The objective function value has remained the same after adding each logical constraint, because the solver was always able to find a solution that satisfies all of the constraints without having to increase the objective value. ✔

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EXPLANATION

The objective value has remained the same after adding each set of logical constraints. This means that there are many solutions that have this objective function value, so the objective value is not very sensitive to adding logical constraints.

You have used 0 of 1 submissions

ACKNOWLEDGEMENTS

This problem is based on the case study "Optimizing the Assignment of Students to Classes in an Elementary School" by Binyamin Krauss, Jon Lee, and Daniel Newman, *INFORMS Transactions on Education* 14(1), p.39-44, September 2013.

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