Graphs

Chapter 10

Chapter Summary

- Graphs and Graph Models
- Graph Terminology and Special Types of Graphs
- Representing Graphs and Graph Isomorphism
- Connectivity
- Euler and Hamiltonian Graphs
- Shortest-Path Problems
- Planar Graphs
- Graph Coloring

Graphs and Graph Models

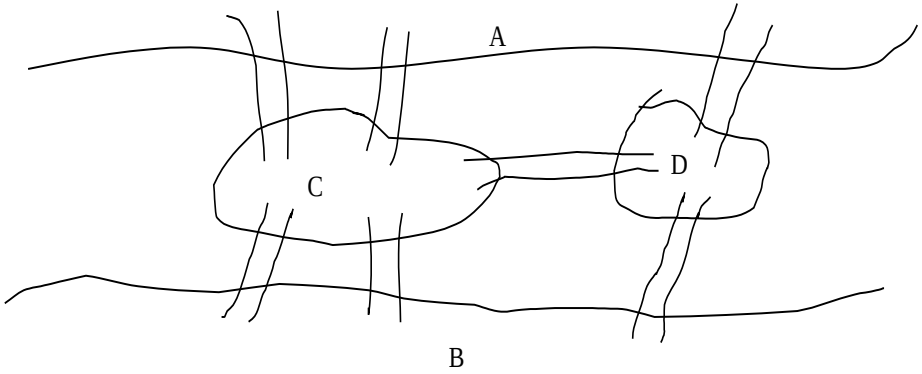
Section 10.1

Section Summary

- A Brief History of Graph Theory
- The Concept of Graph
- Graph Models

The Creation of Graph Theory

- Königsberg Seven Bridge Problem : Euler 1736

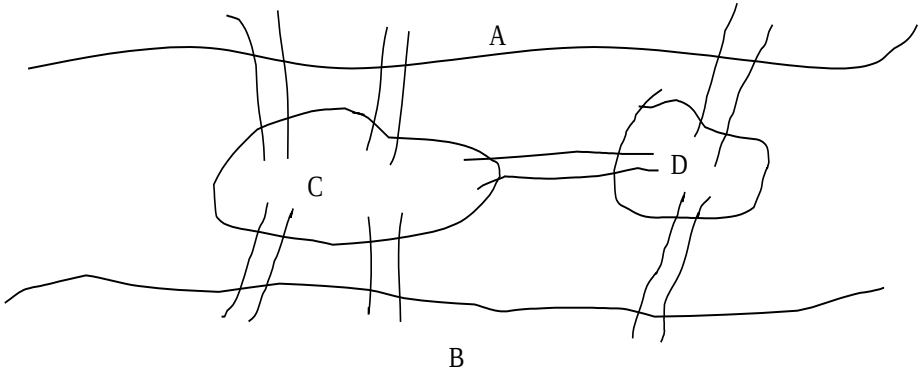


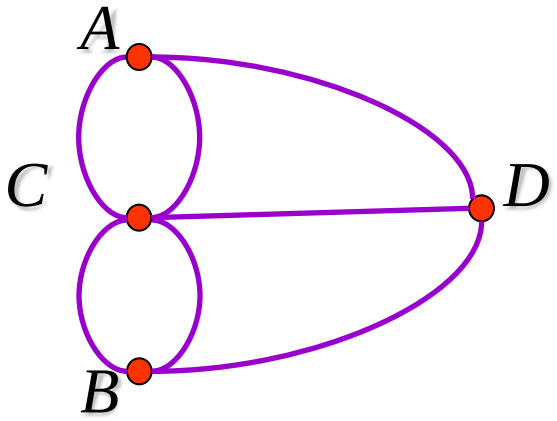
Problem:

Whether it was possible to start at some location in the town, travel across all the bridges without crossing any bridge twice, and return the starting point.

The Creation of Graph Theory

- Konigsberg Seven Bridge Problem : Euler 1736





The equivalent question:

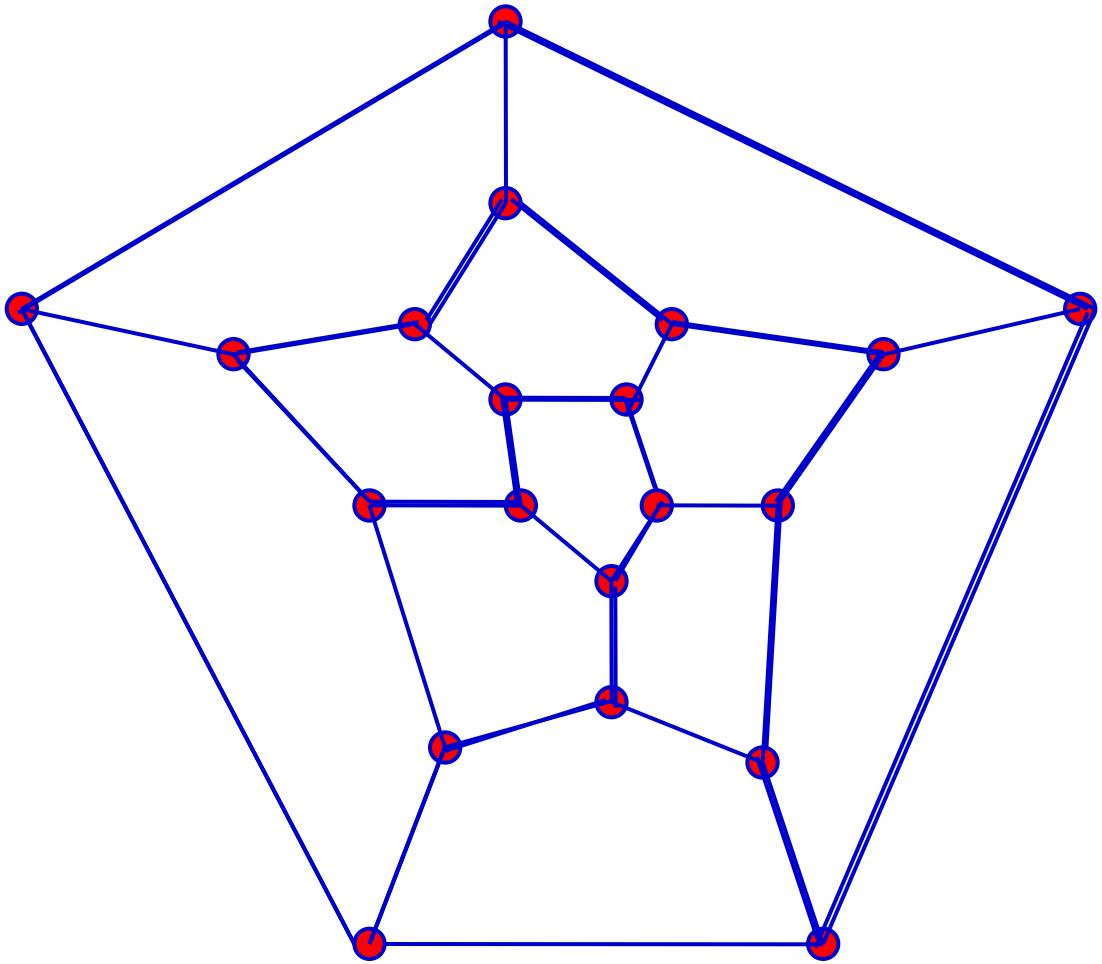
Is there a simple circuit in this multigraph that contains every edge?

Euler proved that a solution for this problem does not exist.

The Development of Graph Theory

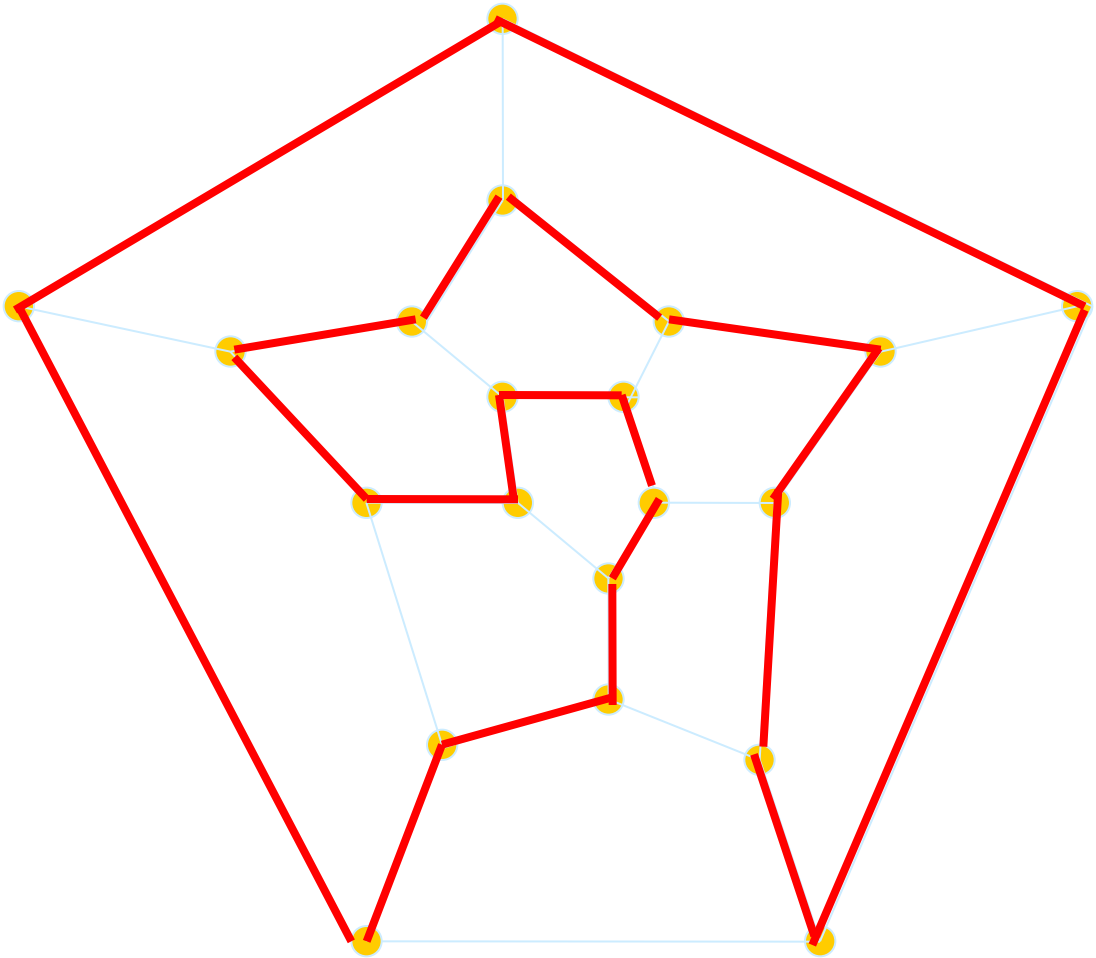
- Kirchhoff developed the theory of trees for their applications in electrical networks (1847)
- Guthire proposed four-color problem (1852)
- Cayley used tree to count the number of the isomers of compounds of the form C_nH_{2n+2} (1857)
- Hamilton's puzzle(1856)

Hamilton's puzzle (1856)



The object in the puzzle was to find a route along the edges of the dodecahedron, passing through each of the 20 cities exactly once?

Hamilton's puzzle (1856)



The equivalent question: Is there a circuit travelling along the edges of the dodecahedron that passes through each vertex exactly once?

The Development of Graph Theory

- Kirchhoff developed the theory of trees for their applications in electrical networks (1847)
- Guthire proposed the problem of Four Colors (1852)
- Cayley used tree to count the number of the isomers of compounds of the form CnH2n+2 (1857)
- Hamilton's puzzle(1856)

The research on graph theory had related to mathematical puzzles early.

- After twenty century, graph theory has already penetrated into a wide variety fields such as Physics ,Chemistry, Informatics, Operational Research ,Computer Network, Sociology etc.

KÖnig worte the first book on graph theory in 1936.

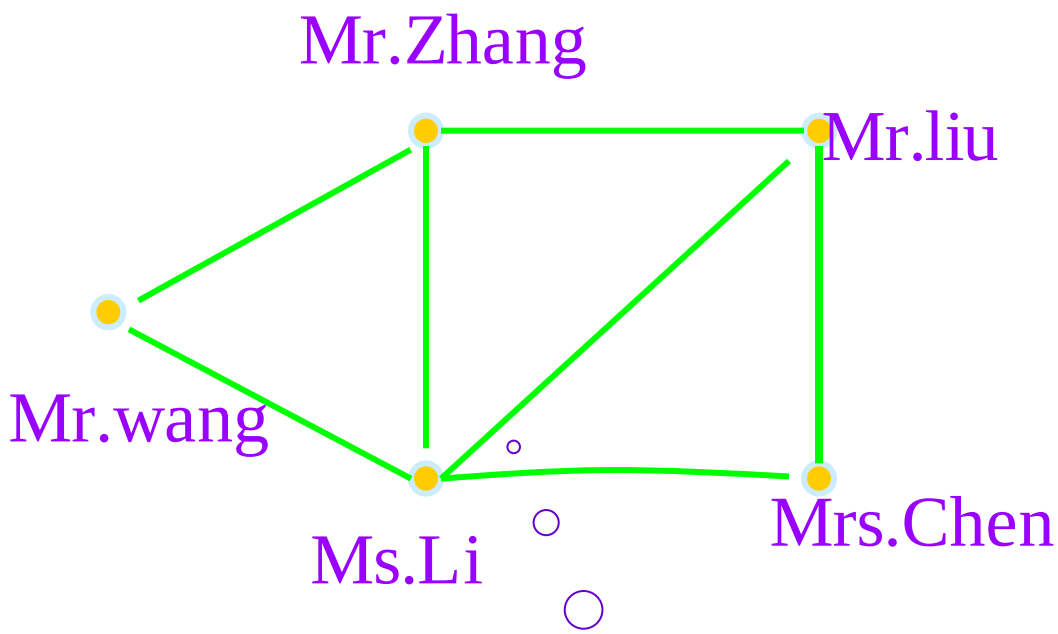
The Concept of Graph

- Things in the world and phenomena in natural world usually exist some positive connection. People need to study in order to reveal these relation.
- Things and phenomena can be represented by vertices (or nodes), and relation can by represented by directed edges or undirected edges. Vertices and edges construct the graph in Graph Theory.

Remark:

Graph is mathematical abstract of relation existing in some concrete things in the objective world.

- **There are 5 representatives in a party. Graph can be used to show their acquaintanceship.**



Using a diagram to represent their acquaintanceship is simple and clear.

【Definition 1】

A graph G=(V,E) consists of V, a nonempty set of vertices (or nodes) and E, a set of edges.

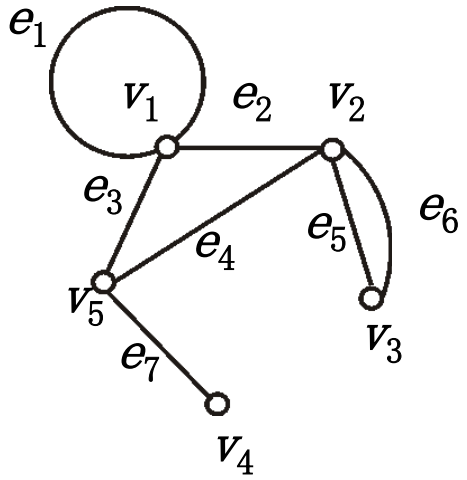
Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to connect its endpoints.

For example,

$G=(V,E)$

Where $V=\{v_1, v_2, \ldots, v_5\}$

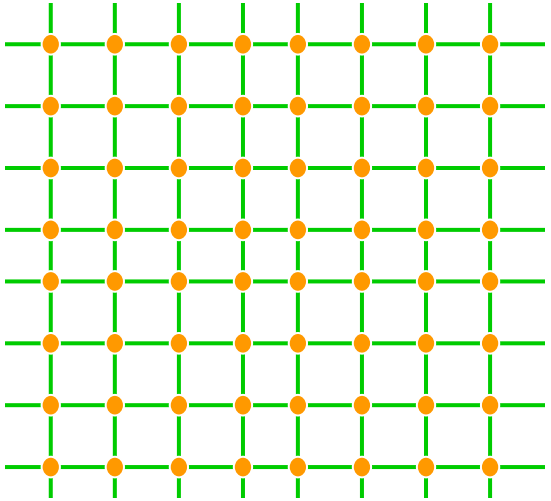
$E=\{\{v_1,v_1\}, \{v_1,v_2\}, \{v_2,v_3\}, \{v_2,v_3\}, \{v_2,v_5\}, \{v_1,v_5\}, \{v_4,v_5\}\}$



Remark:

- **infinite graph:** a graph with an infinite vertex set or an infinite number of edges

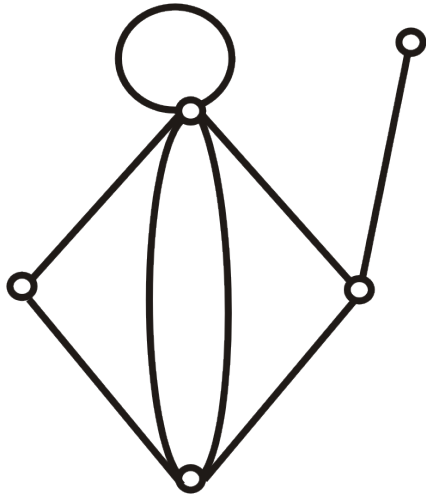
Portion of a infinite graph

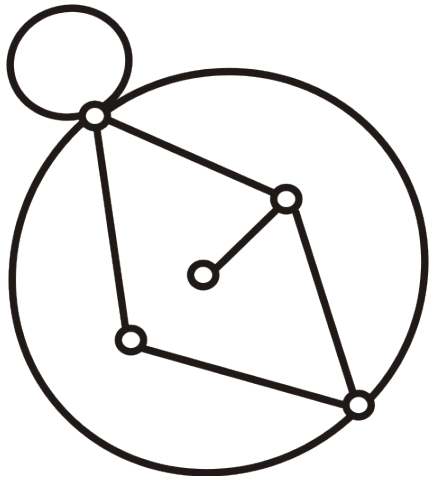


- **finite graph:** a graph with a finite vertex set and a finite number of edges

We shall consider only finite graph.

◆In drawing a graph, it is immaterial whether the lines are drawn straight or curved, long or short; what is important is the incidence between the edges and vertices.



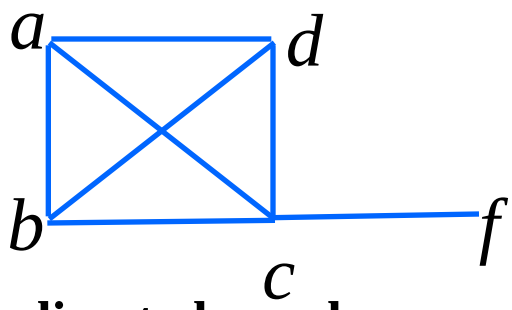


- **Types of graph:**

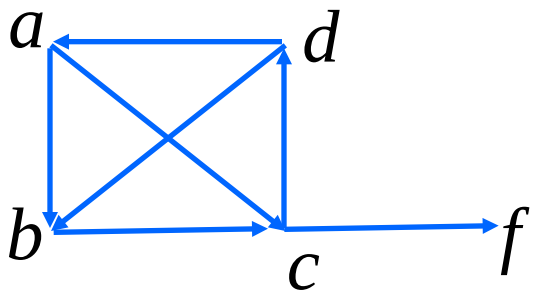
  undirected graph: a graph with undirected edges.

  directed graph : a graph with directed edges.

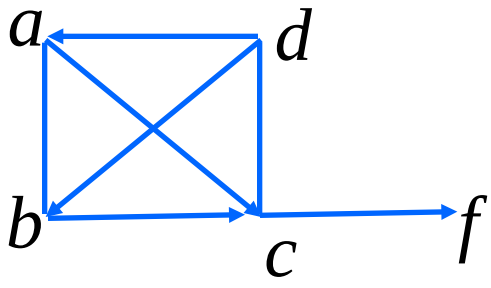
  mixed graph: a graph with both directed and directed edges.



undirected graph



directed graph



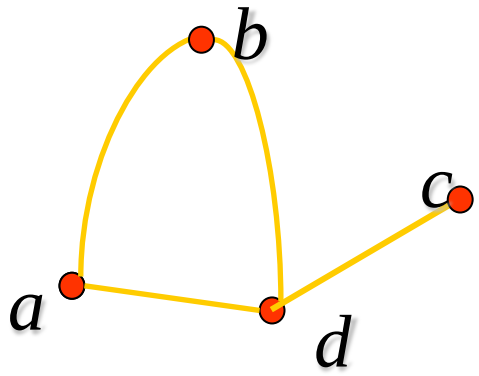
mixed graph

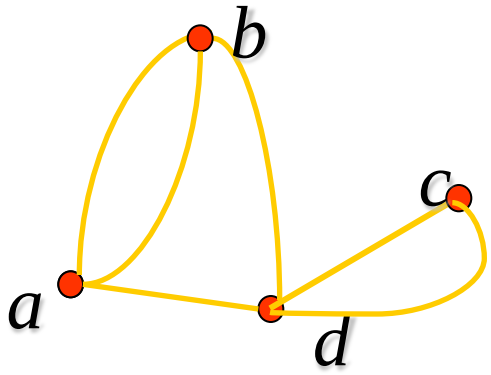
- undirected graph can be classified into:

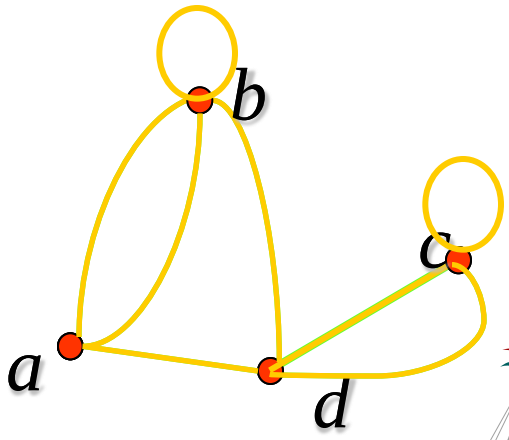
  Simple graph: A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices.

  Multigraph: Graphs that may have multiple edges connecting the same vertices.

  Pseudograph: Graphs that may include loops, and possibly multiple edges connecting the same pair of vertices.







【Definition 2】

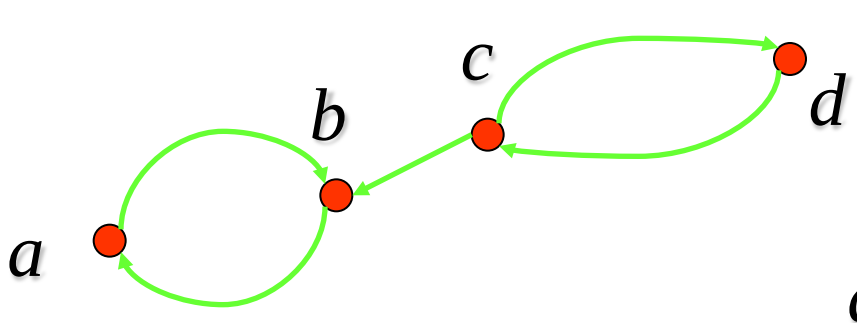
A directed graph (or digraph) (V, E) consists of a nonempty set of vertices V and a set of directed edges (or arcs) E.

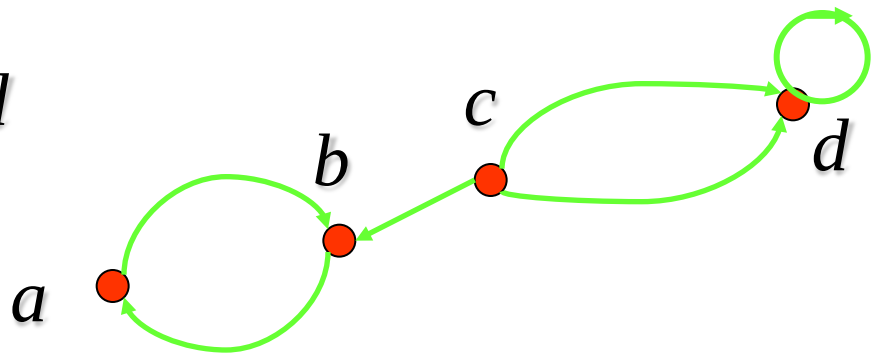
Each directed edge is associated with an ordered pair of vertices. The directed edge associated with the ordered pair (u,v) is said to start at u and end at v.

- **Types of digraphs:**

  simple directed graph: a directed graph has no loops and has no multiple directed edges.

  directed multigraph: a directed graphs that may have multiple directed edges from a vertex to a second (possibly the same) vertex.



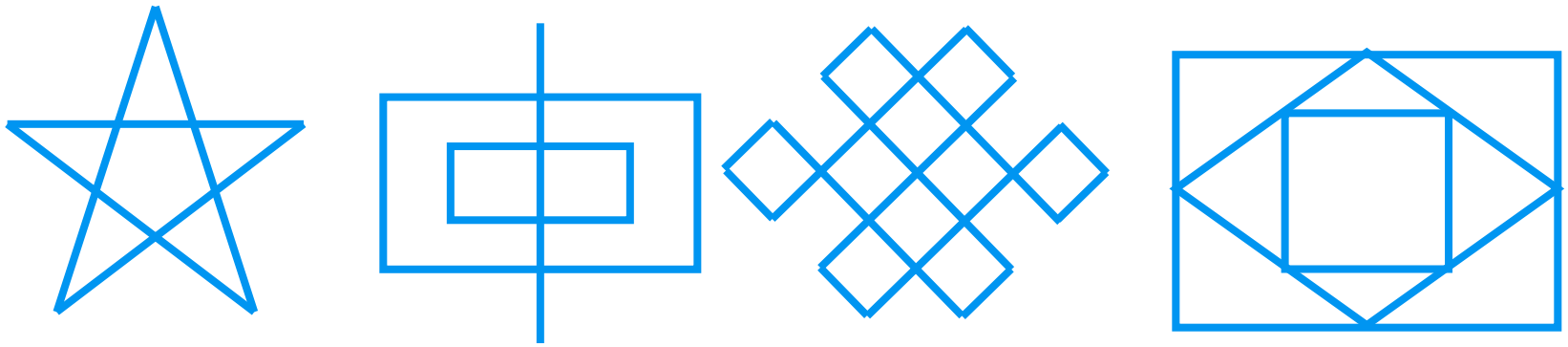


(c, d) is an edge of multiplicity 2.

Graph Models

- Problems in almost every conceivable discipline can be solved using graph models.

- One-stroke drawing problem

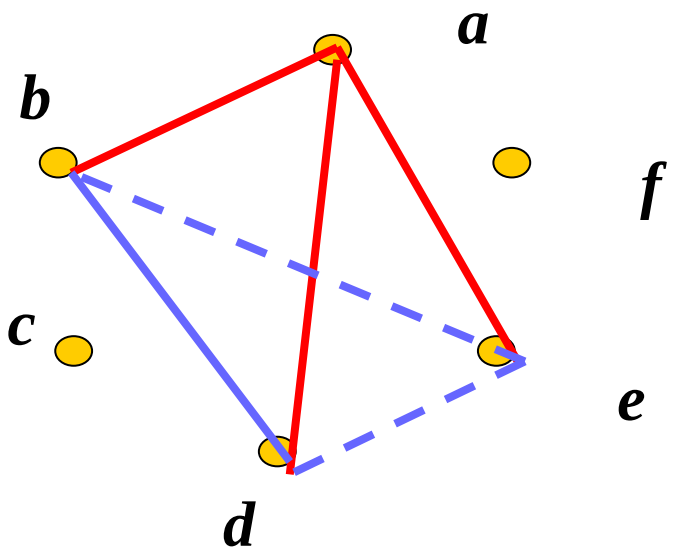


Problem:

Draw a picture in a continuous motion without lifting a pencil so that no part of the picture is retraced.

Ramsey problem

Each pair of individuals consists of two friends or two enemies. There are either three mutual friends or three mutual enemies in the group of six people.



Seating problem

There are seven people denoted by A, B, C, D, E, F, G. Suppose that the following facts are known.

A--English (A can speak English.)

B--English, Chinese

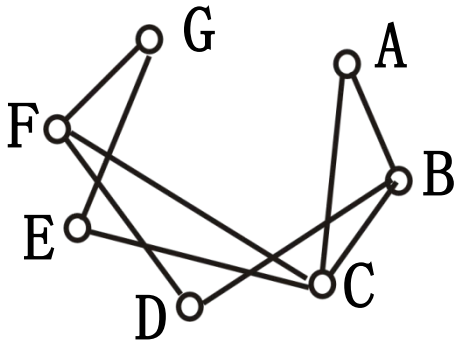
C--English, Italian, Russian

D--Japanese, Chinese

E--German, Italia

F--French, Japanese, Russian

G--French, German

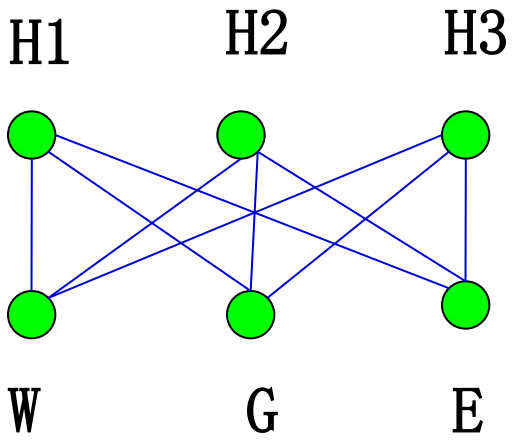


How to arrange seat for the round desk such that the seven people can talk each other?

Utilities problem

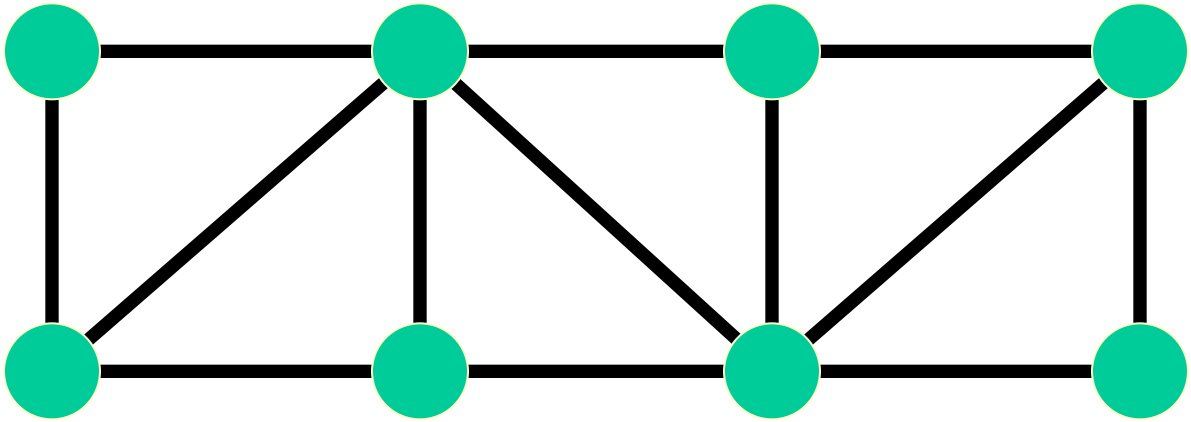
There are three houses H1,H2,H3, each to be connected to each of three utilities- water (W), gas(G), and electricity(E) - by means of conduits.

Is it possible to make such connections without any crossover of the conduits?



The answer to this problem: no

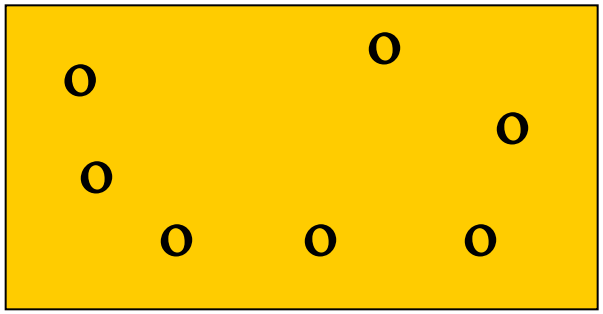
- *A small puzzle*

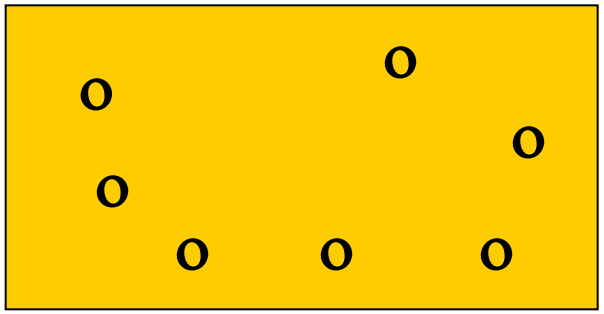


Problem:

There is one person standing at position of every vertex. Persons standing at two endpoints of the same edges have different pose. The least number of different pose?

Part machining



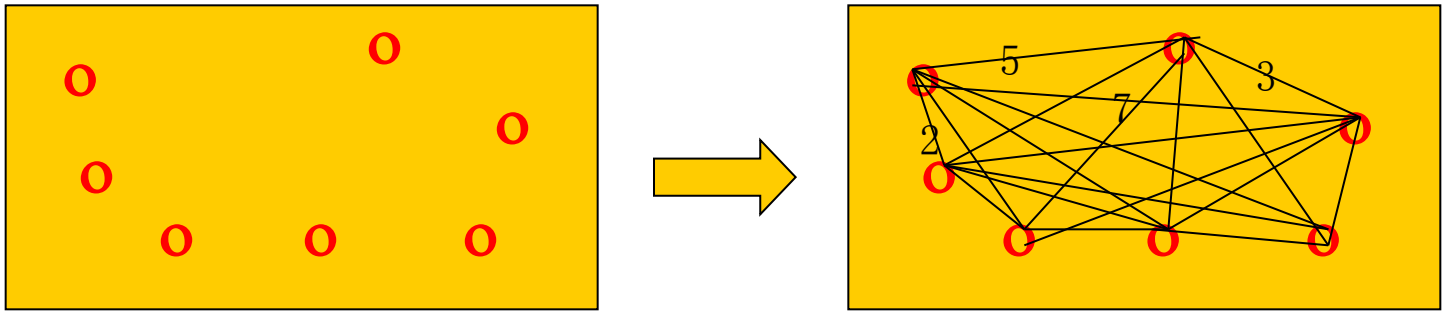


The order given by the above graph is not best obviously.

Problem:

How to determine the order of drilling holes with minimum machining time?

Part machining



- The problem becomes that find a shortest path starting from one vertex and visiting each vertex exactly once.
- The similar problems: the optimal problem for travelling, project, cost etc.

There are many other examples…

- **Social Network**

  Graphs are extensively used to model social structures based on different kinds of relationships between people or groups of people.

- **Acquaintanceship and Friendship Graphs**

  We can use a simple graph to represent whether two people know each other, that is whether they are acquainted, or whether they are friends.

  The Acquaintanceship graph of all people in the word has more than six billion vertices and probably more than one trillion edges.

- **Influence Graphs**

  A directed graph can used to study group behavior that certain people can influence the thinking of others.

Information Network

Graphs can be used to model various networks that link particular types of information.

The Web Graph

The World Wide Web can be modeled as a directed graph .Each Web page is represented by a vertex. The link between Web page is represented by a directed edge.

A snapshot of the Web in 1999 produce a Web graph with over 200 million vertices and over 1.5 billion edges. Many Web Search engines have used the Web Graph.

Citation Graphs

A directed graph can used to represent citations in different types of documents, including academia papers, patents, and legal opinions.

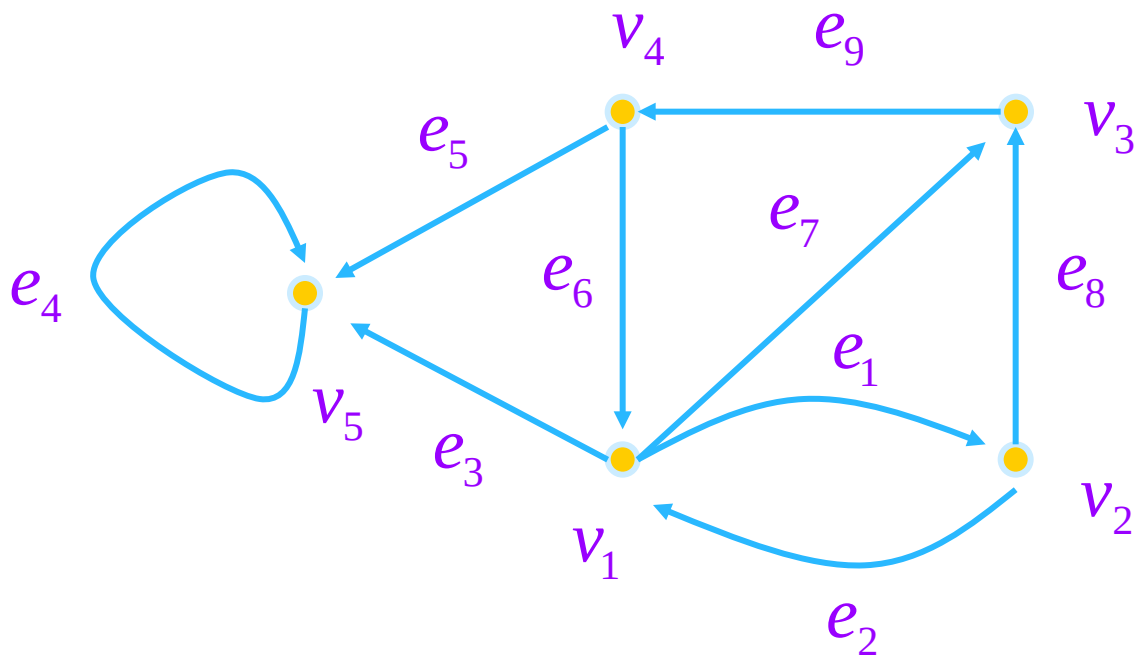
Software Design Application

Graphs models are useful tools in the design of software.

Module Dependency Graph

Each module is represented by a vertex. There is a directed edge from a module to a second module if the second module depends on the first.

A module dependency graph provides a useful tool for understanding how different modules of a program interact.

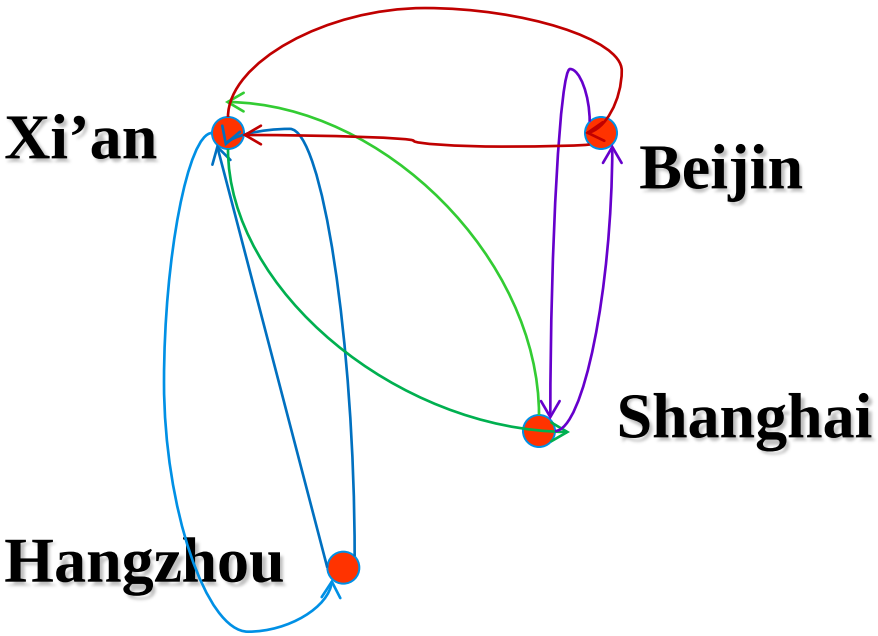


Transportation Networks

Graphs can be used to model many different types of transportation networks, including road, air, and rail networks , as well shipping networks.

Airline Routes

A directed multigraph is used to model airline network. Each airport is represented by a vertex. The flights by a particular airline each day is represented by a directed edge.



More…

- Communication Networks
- Biological Networks
- Tournaments
- …

Homework:

Seventh Edition:

P. 649 1,3-9,33