Chromaticity coordinates in generalized DKL space

Symbols:

l_{ca}, m_{ca}, s_{ca} = cone activations;

l_{mb}, s_{mb} = coordinates in MB space;

l_{dkl}, s_{dkl} = coordinates in DKL space;

(l_0, s_{dkl}) = origin of DKL space;

α_{dkl}, β_{dkl} = scales of LM and S axes in DKL space;

θ_l, θ_s = angles of LM and S axes in DKL space;

Definition of luminance: $L = kl_{ca} + m_{ca}$; *k* a scalar

c_s = 0.371597

c_m = 0.348322

c_l = 0.689903

1) Start in MB space, then change origin, rescale, and rotate axes:

$$l_{dkl} = \alpha_{dkl}(l_{mb} - l_0)cos\theta_l$$

$$s_{dkl} = \beta_{dkl}(s_{mb} - s_0)sin\theta_s \,,$$

where the point (l_0, s_0) is the origin for DKL space; α_{dkl} and β_{dkl} are the separate scaling factors for the LM and S dimensions of DKL space; and θ_l and θ_s are the observer's LM and S color angles.

2) Convert from DKL back to MB space:

$$l_{mb} = \frac{l_{dkl}}{\alpha_{dkl}cos\theta_l} + l_0$$

$$s_{mb} = \frac{s_{dkl}}{\beta_{dkl}sin\theta_s} + s_0 \,.$$

3) Cone activations to MB space:

$$l_{mb} = \frac{c_l l_{ca}}{c_l l_{ca} + c_m m_{ca}}$$

$$s_{mb} = \frac{c_s s_{ca}}{c_l l_{ca} + c_m m_{ca}}$$

4) Define luminance:

$$L = k l_{ca} + m_{ca}$$

$$m_{ca} = L - k l_{ca}$$

5) MB space to cone activations:

$$c_l l_{ca} = l_{mb}(c_l l_{ca} + c_m m_{ca})$$

$$= l_{mb}\big(c_l l_{ca} + c_m(L - k l_{ca})\big) = l_{mb}(c_m L + (c_l - c_m k) l_{ca})$$

$$l_{ca}\big(c_l + l_{mb}(c_m k - c_l)\big) = L c_m l_{mb}$$

$$l_{ca} = \frac{L c_m l_{mb}}{c_l + l_{mb}(c_m k - c_l)}$$

$$= \frac{L c_m l_{mb}\left(\frac{l_{dkl}}{\alpha_{dkl} cos\theta_l} + l_0\right)}{c_l + (c_m k - c_l)\left(\frac{l_{dkl}}{\alpha_{dkl} cos\theta_l} + l_0\right)}$$

$$c_s s_{ca} = s_{mb}(c_l l_{ca} + c_m m_{ca}) = s_{mb}\big(c_l l_{ca} + c_m(L - k l_{ca})\big)$$

$$= s_{mb}\big(c_m L + l_{ca}(c_l - c_m k)\big)$$

$$s_{ca} = c_s^{-1} s_{mb} L\left(c_m - \frac{c_m l_{mb}(c_m k - c_l)}{c_l + l_{mb}(c_m k - c_l)}\right)$$

$$= L\left(\frac{c_m}{c_s}\right)\left(\frac{s_{dkl}}{\beta_{dkl} sin\theta_s} + s_0\right)\left(1 - \frac{(c_m k - c_l)\left(\frac{c_l l_{ca}}{c_l l_{ca} + c_m m_{ca}}\right)}{c_l + (c_m k - c_l)\left(\frac{c_l l_{ca}}{c_l l_{ca} + c_m m_{ca}}\right)}\right)$$