week 2) ↓ Limit Rules & Methods

0) When $f(x)$ is continuous, $\lim_{x \to x_1} f(x) = f(x_1)$ } $\lim_{x \to 7} 4 = 4$

1) When $f(x_1)$ gives $\frac{0}{0}$, try to factor and cancel the common factor before taking limit.

2, 3, 4) Combinations with $+, \times, \div$ can be done a piece at a time. (no division by 0)

→ Piecewise { : } Make sure to use the correct formulas for $x \to x_1^+$, $x \to x_1^-$. > Match for total limit.

→ V.A.) When $f(x_1)$ gives $\frac{\text{non zero}}{0}$, test x_1^- and x_1^+ by factoring (if possible) and compare sides of chart. If match, then $\boxed{\infty}$ or $\boxed{-\infty}$,
week 3) ↓ no match means total limit = $\boxed{\text{DNE}}$

→ V.A.) $b > 1$ $\lim_{x \to 0^+} \log_b x = -\infty$

H.A.) $f(x) = \frac{\text{root}}{\text{root}}$ > divide top and bottom by power of x

→ H.A.)

- For $f(x) = \frac{\text{polynomial}}{\text{polynomial}}$, ↘

  - deg. on top larger: $\lim_{x \to \pm\infty} f(x) = \pm\infty$ (chart!)
  - degrees same: $\lim_{x \to \pm\infty} f(x)$ = ratio of high power coeffs.
  - degree on top smaller: $\lim_{x \to \pm\infty} f(x) = \boxed{0}$

→ H.A.)

$\lim_{x \to \infty} e^x = e^\infty = \boxed{\infty}$	$\lim_{x \to -\infty} e^x = e^{-\infty} = \boxed{0}$
$\lim_{x \to \infty} 2^x = 2^\infty = \boxed{\infty}$	$\lim_{x \to -\infty} 5^x = 5^{-\infty} = \boxed{0}$
$\lim_{x \to \infty} \tan^{-1} x = \tan^{-1}\infty = \boxed{\frac{\pi}{2}}$	$\lim_{x \to -\infty} \tan^{-1} x = \tan^{-1}(-\infty) = \boxed{\frac{-\pi}{2}}$

→ composition

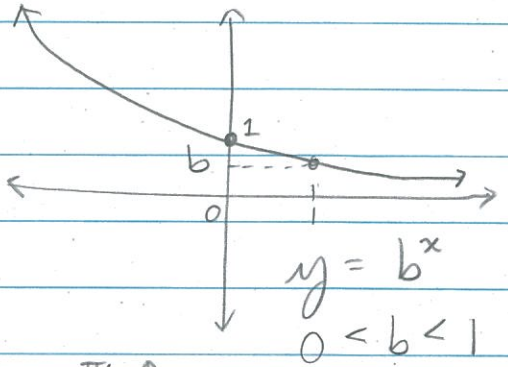
Ex: $\lim_{x \to \infty} \tan^{-1}\left(e^{(x^2/(1-x))}\right) = \tan^{-1}(e^\infty) = \tan^{-1}\infty = \boxed{\frac{\pi}{2}}$

Continuous functions include:

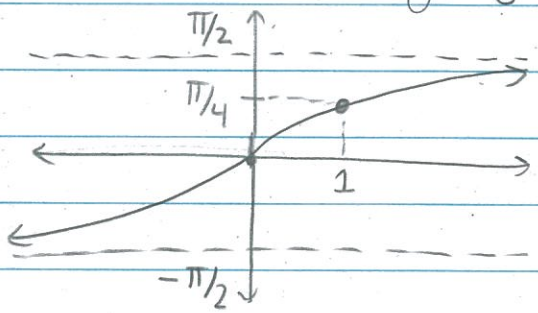
- $y = 3x + 7$; all lines
- $y = x^4 + 2x^3 - \frac{1}{2}x^2 + 5$; all polynomials
- $y = 7$; constants

$$\lim_{x \to 10} 7 = 7$$

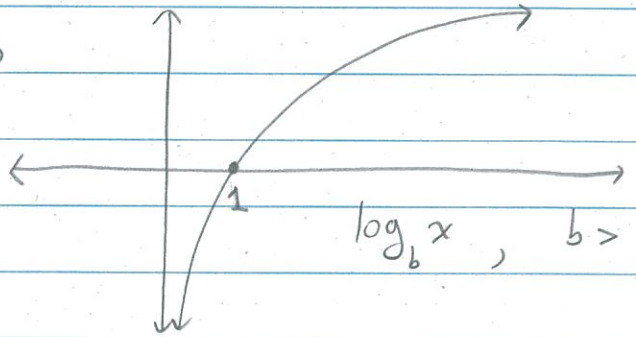
- $y = e^x$
- $y = 5^x$
- $y = 2^{-x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x$



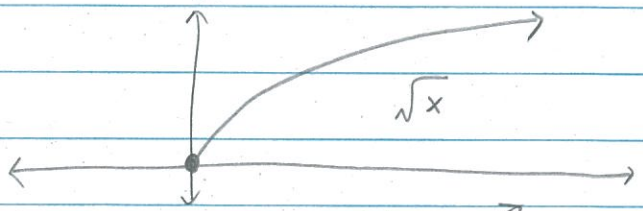
- $y = \sin x$
- $y = \cos x$
- $y = \tan^{-1} x = \arctan x$



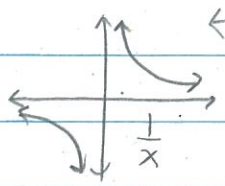
- $y = \ln x$, $x > 0$



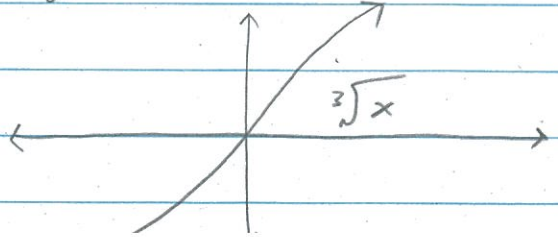
- $y = \sqrt{x}$, $x > 0$



- $y = \frac{1}{x}$, $x \neq 0$



- $y = \sqrt[3]{x} = x^{1/3}$



Rules for combining functions:

2) $$\lim_{x \to x_1} \left(f(x) + g(x) \right) = \left(\lim_{x \to x_1} f(x) \right) + \left(\lim_{x \to x_1} g(x) \right)$$

3) $$\lim_{x \to x_1} f(x)\, g(x) = \left(\lim_{x \to x_1} f(x) \right) \left(\lim_{x \to x_1} g(x) \right)$$

specifically

$$\lim_{x \to x_1} c f(x) = c \lim_{x \to x_1} f(x)$$

for c any real constant

4) $$\lim_{x \to x_1} \frac{f(x)}{g(x)}$$

$$= \frac{\lim_{x \to x_1} f(x)}{\lim_{x \to x_1} g(x)}$$

when $\lim_{x \to x_1} g(x) \neq 0$

Ex: Find limit

$$\lim_{x \to 0} \frac{x^{10}(x-3)e^x}{4x^{10}} \qquad \left(= \frac{0}{0}\right)$$

(cancel)

$$= \lim_{x \to 0} \frac{(x-3)e^x}{4}$$

$$= \frac{(0-3)e^0}{4} = \boxed{\frac{-3}{4}}$$

Ex: Find limit

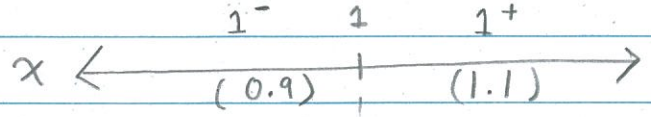
$$\lim_{x \to 0} \frac{5(x-1)}{2(x-1)^3} \qquad \left(= \frac{0}{0}\right)$$

(cance)

$$= \lim_{x \to 0} \frac{5}{2(x-1)^2} \qquad \left(= \frac{5}{0}\right)$$

$\frac{\text{nonzero}}{0} \Rightarrow$ V.A., so limit is ∞, $-\infty$, or DNE

test near $x_1 = 1$



$f(x) = \frac{5}{2(x-1)^2}$	$\frac{\oplus}{\oplus}$	$\frac{\oplus}{\oplus}$
	$= \oplus$	$= \oplus$

matching positives, so answer = $\boxed{\infty}$