... so $\begin{cases} x_1 = 1/3 \\ x_2 = 1 \\ x_3 = 5/3 \end{cases}$

Check: these make all three original equations true

$$3\left(\tfrac{1}{3}\right) - 1 = 0 \quad ✓$$
$$1/3 + 5/3 = 2 \quad ✓$$
$$1/3 + 5/3 - 1 = 1 \quad ✓$$

→ Geometric meaning and 3 types of solution.

For any number of linear equations, with any number of variables, there can only be one of 3 possibilities

→ Zero solutions
→ One solution
→ ∞ solutions

Why?

1) dimension of a space is a counting number 0, 1, 2, 3, 4, 5, 6, ... which describes a collection of points. It tells:

→ the number of independent, free decisions for perpendicular motions

→ the number of real numbers needed to describe a single point location in that space

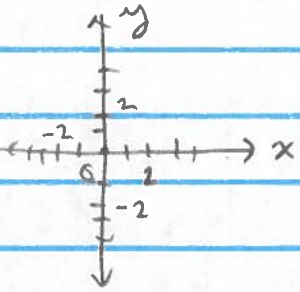
\# degrees of freedom

\# coordinates, # axes R

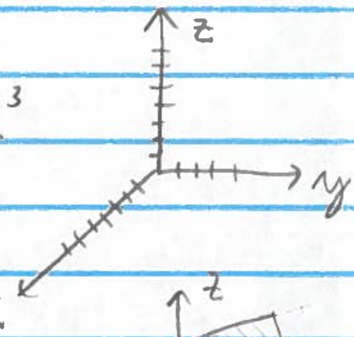
2) Each single (affine) linear equation describes the points in a <u>hyperplane</u> of the space $\mathbb{R}^d$

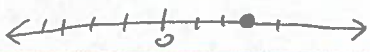
$\mathbb{R}^1 = \mathbb{R}$

$\mathbb{R}^2$

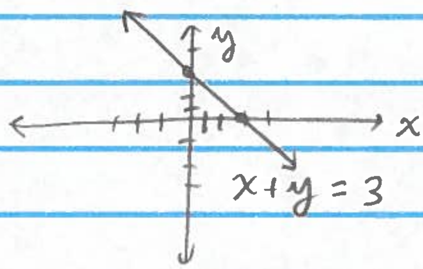


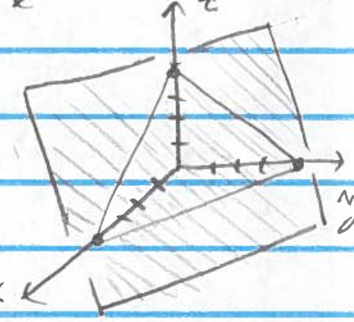
$\mathbb{R}^3$





$x = 3$





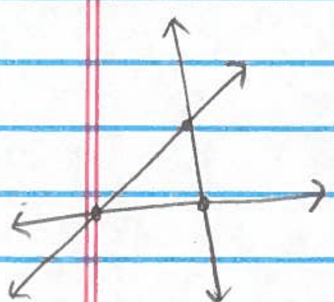
$x+y+z=4$

3) Several hyperplanes in $\mathbb{R}^d$ can either:

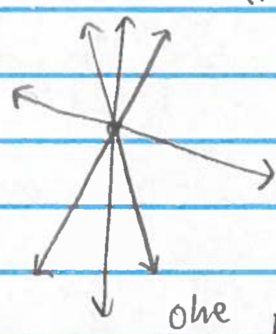
→ not share a common point of intersection (0 solutions)

→ share exactly one common point (needs at least d hyperplanes but no guarantee)

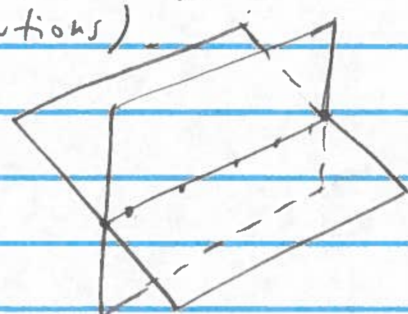
→ or, all intersect in a lower-dimensional plane, line, or hyperplane (∞ solutions)



no point in common - no solution



one point solution

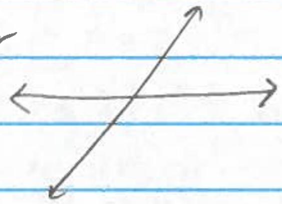


line of solution points → ∞ solutions

Goal # 1: be able to look at equations & know what the picture is, and vice versa: look at the picture and know things about the equations.

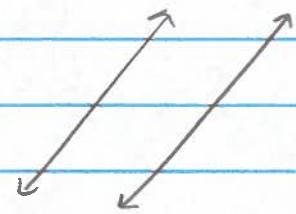
Two lines in $\mathbb{R}^2$:

either



crossing = different slopes

OR



... 2 ways!

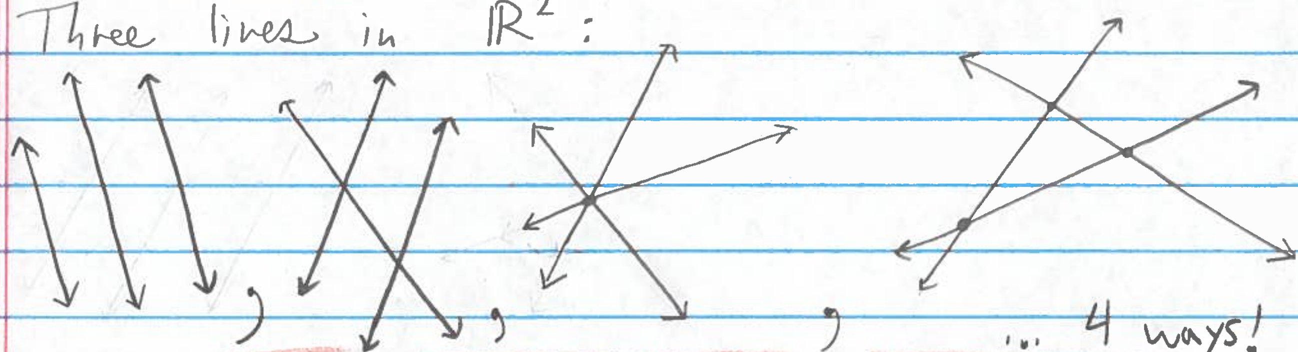
parallel = same slope

ex: $y = \frac{2}{3}x + 1$, $y = \frac{2}{3}x - 5$

$3y - 2x = 3$, $3y - 2x = -15$

slope = coefficients

Three lines in $\mathbb{R}^2$:



... 4 ways!

Four lines: 9 different pictures!

Five lines: 47

Six lines: 791

Seven lines: 37,830

Eight lines: 4,134,940

Nine lines: Unknown

In general n lines? → open research question.