Linear. Test 1, Review.

Also study the quizzes, and homework problems!

1. Solve this system of equations, any way you like. Write the answer as a set of equations with free variable(s). Then write the answer as a linear combination of constant vectors using the free variable(s) as coefficients, and then as a parameterized line with parameter t.

$$\left\{ \begin{array}{c} x_1 - 2x_2 - 4x_3 = 3 \\ 2x_1 - x_2 + x_3 = 0 \end{array} \right\}$$

2. Solve this system of equations, any way you like. Write the answer as a set of equations with free variable(s). Write the answer as a linear combination of constant vectors using the free variable(s) as coefficients, then as a parameterized line with parameter t.

$$\left\{ \begin{array}{c} x - 3z = 3 \\ y + z = 0 \end{array} \right\}$$

3. Solve this system of equations, any way you like. Write the answer as a linear combination of constant vectors using the free variable(s) as coefficients.

$$\left\{ \begin{array}{c} x - 3z = 3 \\ y = 0 \end{array} \right\}$$

(1)

$$\left[\begin{array}{ccc|c} 1 & -2 & -4 & 3 \\ 2 & -1 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -2 & -4 & 3 \\ 0 & 3 & 9 & -6 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & 3 & -2 \end{array}\right]$$

$$\Rightarrow \begin{array}{l} x_1 + 2x_3 = -1 \\ x_2 + 3x_3 = -2 \\ x_3 = \text{free} \end{array}$$

$$\Rightarrow \boxed{\left\{ \begin{array}{l} x_1 = -1 - 2x_3 \\ x_2 = -2 - 3x_3 \\ x_3 = x_3 \end{array} \right.} \Rightarrow \boxed{L = \left\{ \begin{array}{l} x_1 = -1 - 2t \\ x_2 = -2 - 3t \\ x_3 = t \end{array} \right.}$$

$$\Rightarrow \boxed{\vec{x} = x_3 \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}}$$

(2)

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 3 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\Rightarrow \boxed{\left\{ \begin{array}{l} x = 3z + 3 \\ y = -z \\ z = z \end{array} \right.} \Rightarrow \boxed{L = \left\{ \begin{array}{l} x = 3t + 3 \\ y = -t \\ z = t \end{array} \right.}$$

$$\boxed{\vec{x} = z \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}}$$

(3)

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 3 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \boxed{\left\{ \begin{array}{l} x = 3 + 3z \\ y = 0 \\ z = z \end{array} \right.}$$

$$\Rightarrow \boxed{\vec{x} = z \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}}$$

$$A = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix}$$

4. Find $\det(A)$. $-1(-6) = \boxed{6}$

5. Are the columns of A lin. dep. or lin. indep.? $\boxed{\text{indep.}}$

6. Are the rows of A lin. dep. or lin. indep.? $\boxed{\text{indep}}$

7. Does A have an inverse? If so, find A^{-1}.

$\boxed{\text{Yes}}$ $\left[\begin{array}{ccc|ccc} 0 & 0 & 2 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1/3 \\ 0 & 0 & 1 & 1/2 & 0 & 0 \end{array}\right] \Rightarrow \boxed{A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1/3 \\ 1/2 & 0 & 0 \end{bmatrix}}$

8. How many solutions are there to the equation $A\mathbf{x} = \mathbf{0}$? Find the solution if it exists. $\boxed{\text{One}}$ $\boxed{\vec{x} = \vec{0}}$

9. How many solutions can there be to the equation $A\mathbf{x} = \mathbf{b}$, for $\mathbf{b} \neq \mathbf{0}$? $\boxed{\text{One}}$

10. Solve the equation $A\mathbf{x} = \mathbf{b}$, for $\mathbf{b} = (1,\ 1,\ 1)$. You can always write $\mathbf{b}$ as a column for the sake of setting up the problem.

$\vec{x} = A^{-1}\vec{b} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1/3 \\ 1/2 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ 1/3 \\ 1/2 \end{bmatrix}}$ or $(1, \frac{1}{3}, \frac{1}{2})$

11. Is $\mathbf{b} = (2,\ 1,\ 0)$ in the span of the columns of A? $\boxed{\text{yes}}$

12. Is there a way to make the third column of A as a lin. comb. of the first two columns? $\boxed{\text{no}}$

13. Find $\det(A^{-1})$. $\boxed{1/6}$

14. Find A^t. $\boxed{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 3 \\ 2 & 0 & 0 \end{bmatrix}}$

15. Find $\det(A^t)$.

$\boxed{6}$

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 0 \\ 3 & 0 & 6 \end{bmatrix}$$

16. Find $\det(B)$. $\boxed{0}$

17. Are the columns of B lin. dep. or lin. indep.? $\boxed{\text{dep.}}$

18. Are the rows of B lin. dep. or lin. indep.? $\boxed{\text{dep.}}$

19. Does B have an inverse? If so, find B^{-1}. $\boxed{\text{no}}$

20. How many solutions are there to the equation $B\mathbf{x} = \mathbf{0}$? Find the solution if it exists. $\boxed{\infty}$ (below)

21. How many solutions can there be to the equation $B\mathbf{x} = \mathbf{b}$, for $\mathbf{b} \neq \mathbf{0}$? $\boxed{0 \text{ or } \infty}$

22. Solve the equation $B\mathbf{x} = \mathbf{b}$, for $\mathbf{b} = (1,\ 1,\ 1)$. (below)

23. Find AB.

24. Find BA.

$$\boxed{AB = \begin{bmatrix} 6 & 0 & 12 \\ 1 & 0 & 2 \\ 3 & 0 & 0 \end{bmatrix} \qquad BA = \begin{bmatrix} 0 & 6 & 2 \\ 0 & 0 & 2 \\ 0 & 18 & 6 \end{bmatrix}}$$

25. Find $\det(BA)$. $\boxed{0}$

(20)

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 3 & 0 & 6 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \begin{cases} x_1 = 0 \\ x_2 = x_2 \text{ (free)} \\ x_3 = 0 \end{cases} \Rightarrow \boxed{\vec{x} = x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}$$

(22)

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 1 & 0 & 0 & 1 \\ 3 & 0 & 6 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 \end{array}\right] \Rightarrow \boxed{\text{no solution}}$$

$(0 = -2)$

$$C = \begin{bmatrix} 1 & 0 & 2 & 4 \\ -1 & 0 & -2 & 3 \end{bmatrix}$$

26. Are the columns of C lin. dep. or lin. indep.? $\boxed{\text{dep.}}$

27. Are the rows of C lin. dep. or lin. indep.? $\boxed{\text{indep}}$

28. How many solutions are there to the equation $C\mathbf{x} = \mathbf{0}$? Find the solution if it exists. $\boxed{\infty}$ (below)

29. How many solutions can there be to the equation $C\mathbf{x} = \mathbf{b}$, for $\mathbf{b} \neq \mathbf{0}$? $\boxed{\infty}$

30. Solve the equation $C\mathbf{x} = \mathbf{b}$, for $\mathbf{b} = (7,\ 7)$. (below)

31. How many solutions can there be to the equation $C'\mathbf{x} = \mathbf{b}$, for $\mathbf{b} \neq \mathbf{0}$? $\boxed{0 \text{ or } 1}$

(28) $$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 4 & 0 \\ -1 & 0 & -2 & 3 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & 2 & 4 & 0 \\ 0 & 0 & 0 & 7 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\Rightarrow \begin{cases} x_1 + 2x_3 = 0 \\ x_2 = x_2 \\ x_3 = x_3 \\ x_4 = 0 \end{cases} \Rightarrow \boxed{\vec{x} = x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}}$$

(30) $$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 4 & 7 \\ -1 & 0 & -2 & 3 & 7 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & 2 & 4 & 7 \\ 0 & 0 & 0 & 7 & 14 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$$\Rightarrow \begin{cases} x_1 + 2x_3 = -1 \\ x_2 = x_2 \\ x_3 = x_3 \\ x_4 = 2 \end{cases} \Rightarrow \boxed{\vec{x} = x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 2 \end{bmatrix}}$$