Chp 3 & 4

Vector Spaces & Linear Transformations

$\mathbb{R}^m$, the vectors with m components, is an example of an m-dimensional vector space.

In general: a vector space over the real scalars is any set V with structures of addition and scaling; obeying: For $\vec{x}, \vec{y}, \vec{z} \in V$ and $c, d \in \mathbb{R}$

0) $\vec{x} + \vec{y} \in V$ and $c\vec{x} \in V$ — closure
1) $(\vec{x} + \vec{y}) + z = \vec{x} + (\vec{y} + \vec{z})$ — associative
2) $\vec{x} + \vec{y} = \vec{y} + \vec{x}$ — commutative
3) there exists $\vec{0} \in V$ with $\vec{x} + \vec{0} = \vec{0} + \vec{x} = \vec{x}$ — additive identity
4) there exists $-\vec{x} \in V$ with $\vec{x} + -\vec{x} = \vec{0}$ — additive inverses
5) $c(d\vec{x}) = (cd)\vec{x}$ — compatibility
6) $1\vec{x} = \vec{x}$ — scalar identity
7) $c(\vec{x} + \vec{y}) = c\vec{x} + c\vec{y}$ — distributive
8) $(c+d)\vec{x} = c\vec{x} + d\vec{x}$ — distributive

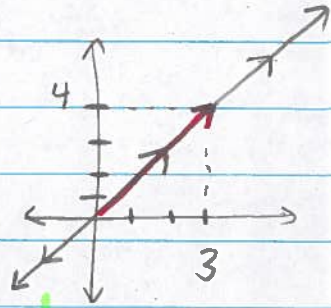
ex) $\mathbb{R}^m$ any m

ex) $M^{m \times n}$ all matrices m rows, n columns

ex) $\left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \middle| \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = c\begin{pmatrix} 3 \\ 4 \end{pmatrix} \right\}$ the set of all scalings of $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

that last one could be written:

$$S = \left\{ \vec{x} \in \mathbb{R}^2 \;\middle|\; \vec{x} = c\begin{pmatrix}3\\4\end{pmatrix},\ c \in \mathbb{R} \right\}$$



this S is a subspace of $\mathbb{R}^2$

[Any subset of a vector space V which is closed under addition and scaling automatically will obey 1-8, so is a subspace.]

* for instance, any subspace contains $\vec{0}$

ex) $$W = \left\{ \vec{x} \in \mathbb{R}^4 \;\middle|\; \vec{x} = c_1\begin{pmatrix}2\\1\\0\\1\end{pmatrix} + c_2\begin{pmatrix}3\\4\\1\\0\end{pmatrix} \right\}$$

check: W is closed, so it is a subspace.

Also, we define the $\boxed{\text{span}}$ of a set of vectors to be the set of all lin. combs of those vectors.

so $$W = \text{Span}\left\{ \begin{pmatrix}2\\1\\0\\1\end{pmatrix}, \begin{pmatrix}3\\4\\1\\0\end{pmatrix} \right\}$$

and $$S = \text{Span}\left\{ \begin{pmatrix}3\\4\end{pmatrix} \right\}$$

In fact, any subspace of a (finite dimensional) vector space can be written as the span of some of its vectors.