Calculus II. Quiz 4. Name Key Time

Show all work on this page for full and/or partial credit. Put a box around your final answers in each part.

1. Show the correct form for the partial fraction decomposition. (As in example 7 of 7.4) Don't find the numerical values of the variables A,B,etc. Hint, $x = 1$ makes the denominator 0, so $(x-1)$ is a factor of the denominator.

$$\frac{x^2+5x-1}{x^3+x^2-2} = \frac{x^2+5x-1}{(x-1)(x^2+2x+2)} = \boxed{\frac{A}{x-1} + \frac{Bx+C}{x^2+2x+2}}$$

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2. Now solve the partial fraction decomposition: same fraction as in the previous question, but this time find the values of A,B,etc.

$$A(x^2+2x+2) + (Bx+C)(x-1) = x^2+5x-1$$

$$Ax^2+2Ax+2A+Bx^2+Cx-Bx-C = x^2+5x-1$$

$$(A+B)x^2 + (2A+C-B)x + 2A-C = x^2+5x-1$$

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$$\left.\begin{aligned} A+B&=1 \\ 2A-B+C&=5 \\ 2A-C&=-1 \end{aligned}\right\} \quad 5A=5, \ \boxed{A=1} \rightarrow \boxed{B=0}, \ 2-C=-1, \ \boxed{C=3} \quad \boxed{\frac{1}{x-1} + \frac{3}{x^2+2x+2}}$$

3. Find the indefinite integral. You must show the partial fraction method as steps in the solution.

$$\int \frac{x^3+10x^2+8x+11}{(x+3)(2x+1)(x^2+1)}dx = \int \frac{A}{x+3} + \frac{B}{2x+1} + \frac{Cx+D}{x^2+1}\,dx \qquad \underbrace{2x^2+7x+3}$$

$$x^3+10x^2+8x+11 = A(2x+1)(x^2+1) + B(x+3)(x^2+1) + (Cx+D)(x+3)(2x+1)$$

$$= A2x^3 + Ax^2 + 2Ax + A + Bx^3 + 3Bx^2 + Bx + 3B + 2Cx^3 + 7Cx^2 + 3Cx + 2Dx^2 + 7Dx + 3D$$

$$= (2A+B+2C)x^3 + (A+3B+7C+2D)x^2 + (2A+B+3C+7D)x + A+3B+3D$$

$$\begin{aligned} 2A+B+2C&=1 \\ A+3B+7C+2D&=10 \\ 2A+B+3C+7D&=8 \\ A+3B+3D&=11 \end{aligned}$$

$$C+7D=7 \rightarrow 7C+49D=49$$

$$\underbrace{-7C+D=1}$$

$$50D=50$$

$$\boxed{D=1}$$

$$\Rightarrow C+7=7, \ \boxed{C=0} \rightarrow 2A+B=1 \qquad A+3B+3=11, \ 2A+6B=16 \qquad 5B=15 \Rightarrow \boxed{B=3}$$

$$2A+3=1, \ 2A=-2, \ \boxed{A=-1}$$

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$$\int \frac{-1}{x+3} + \frac{3}{2x+1} + \frac{1}{x^2+1}\,dx$$

$$= \boxed{-\ln|x+3| + \frac{3}{2}\ln|2x+1| + \tan^{-1}x + C}$$