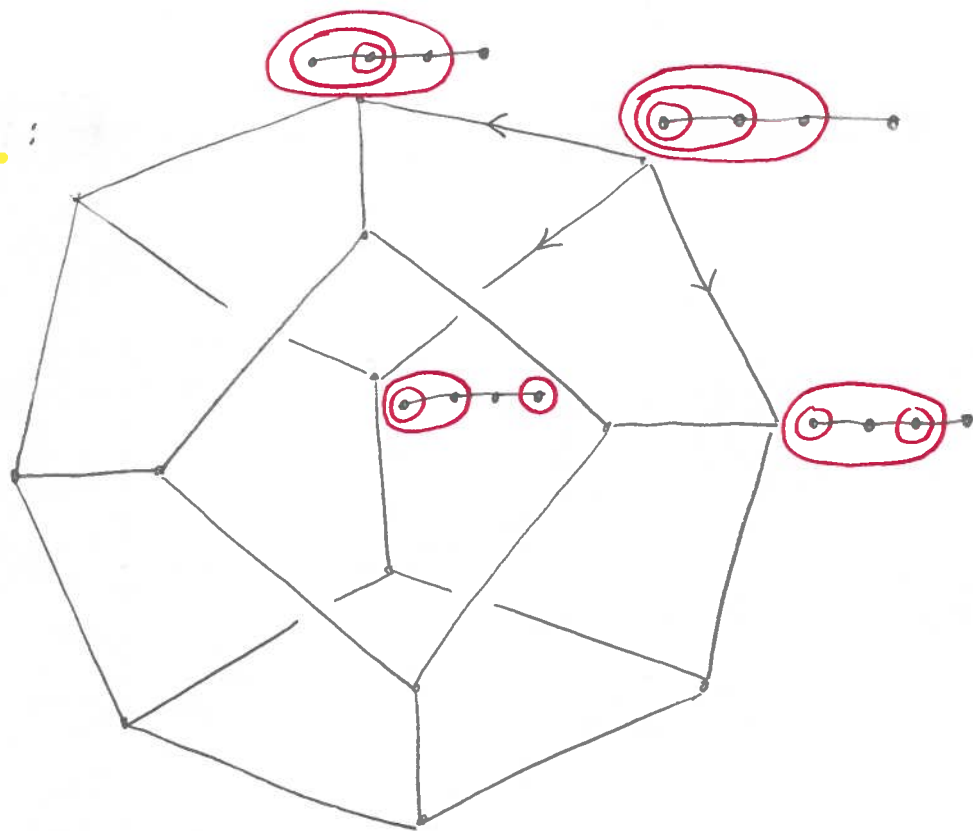**Definitions:** **Tube** t on a **graph**:
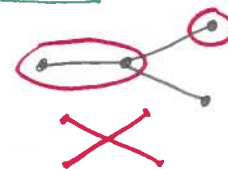induced, connected, subgraph.

Tubing T: a set of tubes,
each pair nested or disjoint,
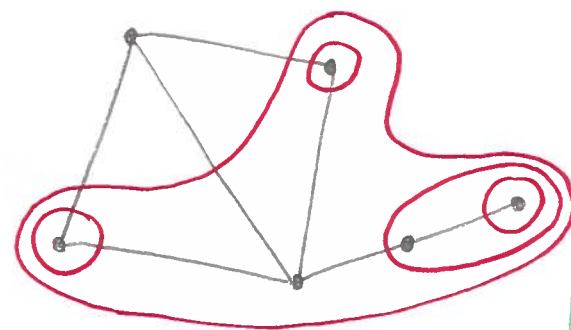and unions of them must be induced subgraphs

Path:

Associahedra

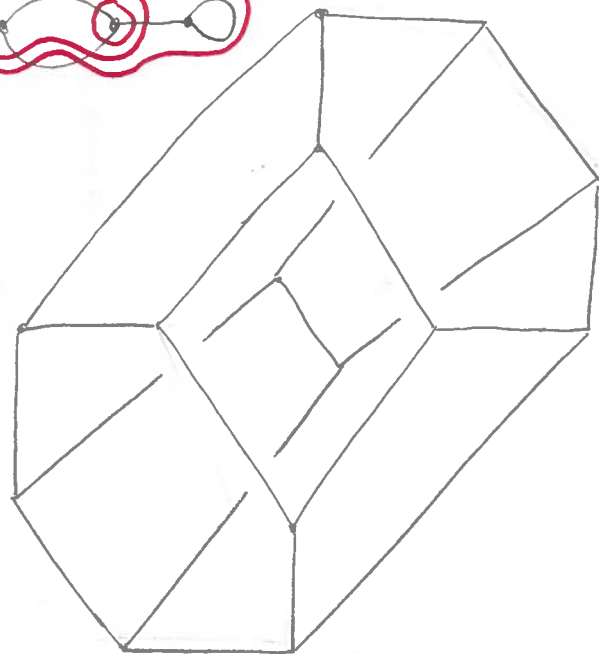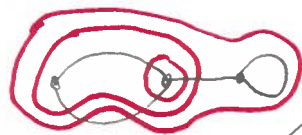[Stasheff]

Any graph:

Graph - Associahedra.

[Carr, Devadoss]
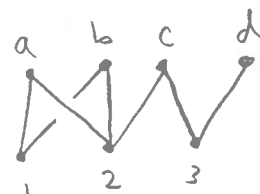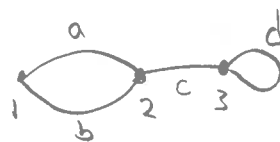
Definitions: **Tube** t on a pseudograph:

Filled, connected, sub-pseudograph.

↳ [if end-nodes of a (multi) edge(s) are in t then at least one of those multi edges is in t]

Tubing T: a set of pairwise nested or disjoint tubes with filled unions.

Pseudograph Associahedra

[Carr, Devadoss, F.]

Notice:

pseudograph ⟷ poset (Hasse diagram)

Definitions:

Tube t on a **poset**:

filled, connected, lower set.

ζ [if t contains the set ∂x of all $y < x$
then t intersects the set b_x of y s.t. $\partial y = \partial x$.]

Tubing T: a set of pairwise nested
or disjoint tubes with filled unions.
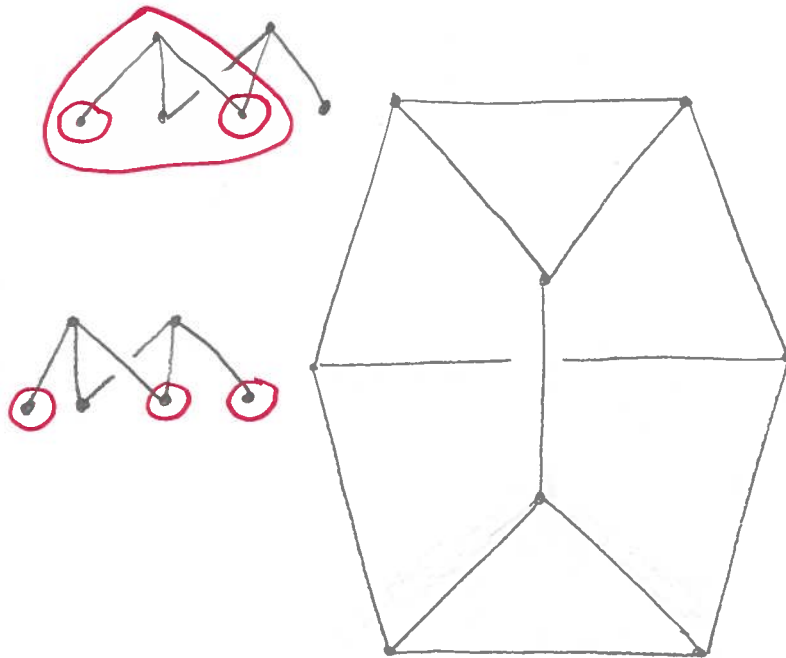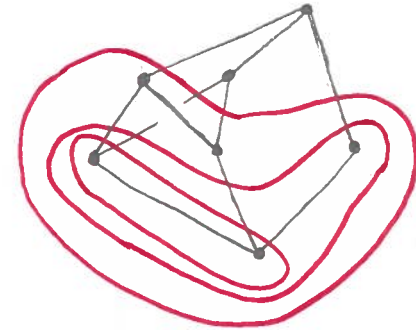
[Devadoss, F., Reisdorf, Showers]

Poset   Associahedra

[Katz, Olsen]

Questions:

- Find a realization of the poset associahedra.
- Find formulas for h-vectors, $h(t,q)$.
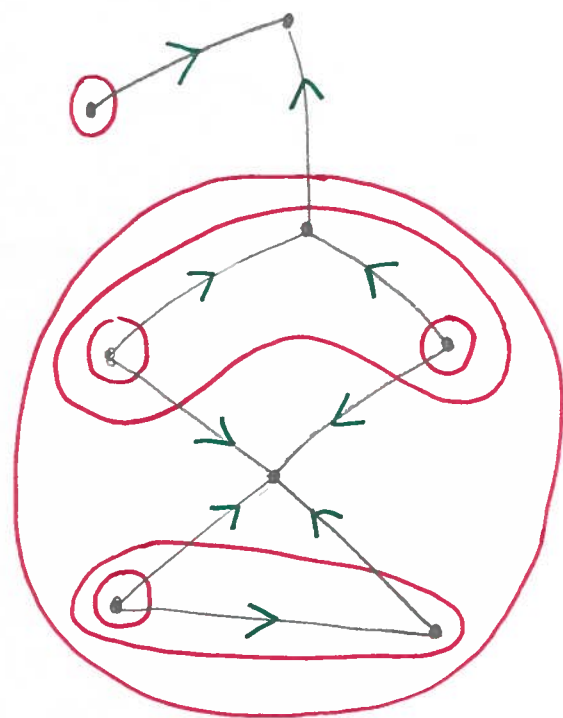- Relate to Galashin's poset associahedra.

Tubings $\longrightarrow$ Orientations
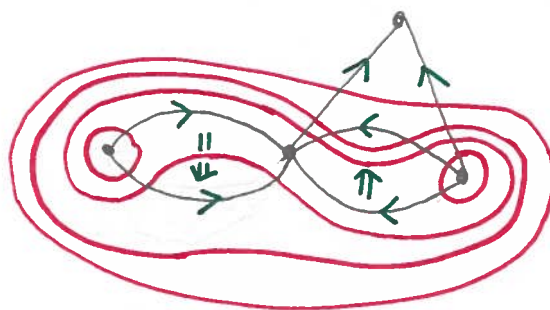
$T \longmapsto \mathcal{O}_T$

where $\mathcal{O}_T(H)$ chooses the element $x \in H$

such that $x \in t \Rightarrow H \subseteq t$.

$[H$ is any $\partial_c y \subseteq b_x$, or any non-minimal b_y,
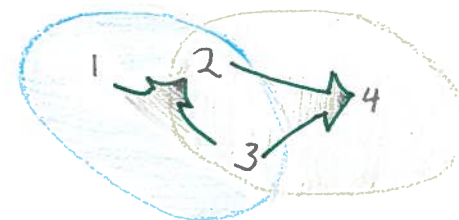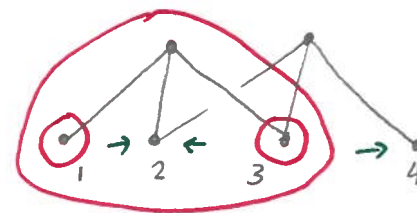
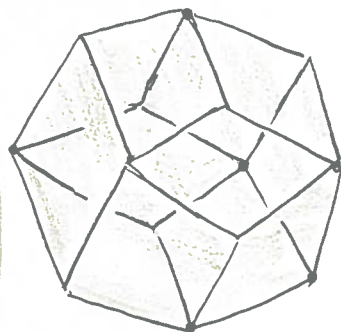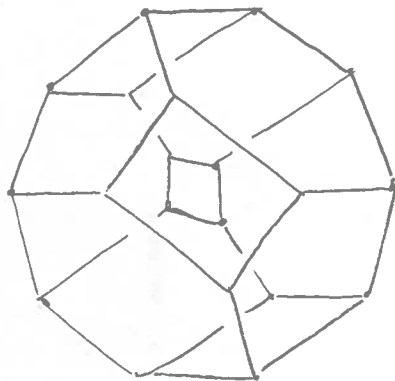and $\partial_c y = $ maximal elements of $\partial y]$

- Directed Acyclic Graph
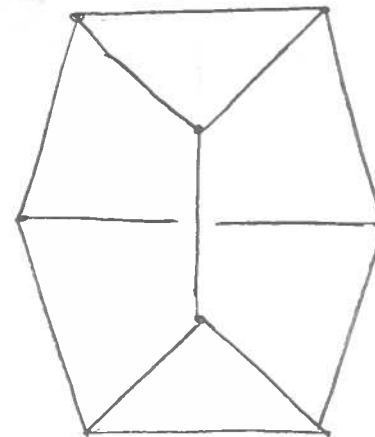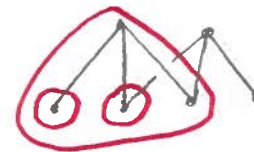
- Directed Acyclic Pseudo graph
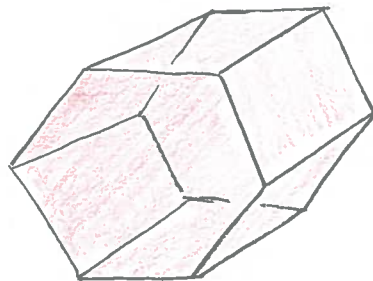
- Pasting diagram

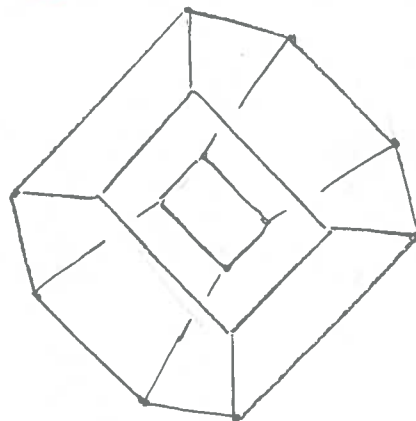- Directed Acyclic Hyper graph
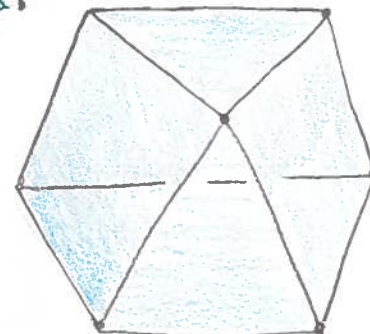
- Poset Gradient

Polytope fan refinements

↓

graphical zonotope

[Féray, Reiner]

Q.
Are all poset gradient polytopes
products of hypergraphic polytopes?

↓

⇓
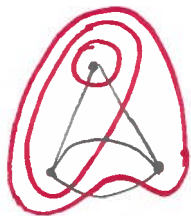
↓ Open Question!

Hypergraphic polytopes

[Benedetti, Bergeron, Machacek]
[Bergeron, Pilaud]