Calculus II. Review for Test 2.

1. Set up these approximate integrations, using the method and number of rectangles n that is given. Don't work them out, just set up!

$a)$ $\displaystyle\int_{-3}^{5} \frac{3x}{1+\ln|x|}dx$; Trapezoidal Rule with $n = 5$.

$\frac{4}{5}[\frac{3(-3)}{1+ln|-3|} + 2\frac{3(-7/5)}{1+ln|-7/5|} + 2\frac{3(1/5)}{1+ln|1/5|} + 2\frac{3(9/5)}{1+ln|9/5|} + 2\frac{3(17/5)}{1+ln|17/5|} + \frac{3(5)}{1+ln|5|}]$

$b)$ $\displaystyle\int_{0}^{1} xe^{(\sin x)}dx$; Midpoint Rule with $n = 3$.

$\frac{1}{3}[\frac{1}{6}e^{(\sin(1/6))} + \frac{3}{6}e^{(\sin(3/6))} + \frac{5}{6}e^{(\sin(5/6))}]$

$c)$ $\displaystyle\int_{7}^{13} (x + \sin(\ln x))dx$; Simpson's Rule with $n = 6$.

$\frac{1}{3}[(7+\sin(\ln(7)))+4(8+\sin(\ln(8)))+2(9+\sin(\ln(9)))+4(10+\sin(\ln(10)))+2(11+\sin(\ln(11)))+4(12+\sin(\ln(12)))+(13+\sin(\ln(13)))]$

$d)$ $\displaystyle\int_{-2}^{3} (x^2 - \sin^2 x)dx$; Simpsons Rule with $n = 2$.

$\frac{5}{6}[((-2)^2 - \sin^2(-2)) + 4((1/2)^2 - \sin^2(1/2)) + ((3)^2 - \sin^2(3))]$

2. Show the correct form for a partial fraction decomposition of these functions. Don't actually solve for the variables.

$a)$ $\dfrac{A}{x^2} + \dfrac{B}{x} + \dfrac{C}{x+2}$

$b)$ $\dfrac{A}{x-2} + \dfrac{B}{x+2}$

$c)$ $\dfrac{A}{x-2} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

3. Decompose the function into its partial fractions. (Actually solve for the variables.)

$a)$ $\dfrac{7/4}{x-1} + \dfrac{(-7/4)x + 21/4}{x^2+3}$

$b)$ $\dfrac{1/5}{x-2} + \dfrac{-1/5}{x+3}$

4. Find the indefinite integrals:

   a) $x + 3\ln|x| - 2\ln|1+x| + c$ (Hint: first do long division since the degree of the numerator and denominator are both 2. This will turn the integral into: $\int(1 + \frac{x+3}{x(x+1)}dx.$)

   b) $\dfrac{-4}{3(x+1)} + \dfrac{11}{9}\ln|x-2| - \dfrac{11}{9}\ln|x+1| + c$

5. Find these definite integrals and classify as "divergent" or "convergent":

   a) $\dfrac{1}{2e^9}$ $(converges)$

   b) $-\infty$ $(diverges)$ (Hint: it's $-\infty$ because you take $\lim_{t\to 0^-}$.)

   c) $\dfrac{32}{3}$ $(converges)$

6. For each of these sequences, find the limits, if they exist, and decide "diverges" or "converges."

   a) 0 $(converges)$ (Hint: $(2/3)^n$ goes to zero since $2/3$ is a fraction smaller than 1.)

   b) 0 $(converges)$

   c) $1/7$ $(converges)$

   d) DNE $(diverges)$ (Hint: $\cos(n\pi) = (-1)^n$ just from looking at the graph of $\cos x$.)

   e) $\pi/6$ $(converges)$

   f) 0 $(converges)$

   g) DNE $(diverges)$

7. For each series, what does the limit test for divergence tell us? [converge, diverge, or inconclusive] Show your work by performing the test.

   a) *diverges* (since limit $= 1/5$.)

   b) *inconclusive* (since limit $= 0$.)

c) *inconclusive* (since limit = 0.)

8. For each series, what does the geometric series test tell us? [not applicable, converge, diverge, or inconclusive] Show your work, and find the value if it converges.

   a) *converges* : $\dfrac{3/\pi}{1-3/\pi}$

   b) *diverges* : $r = \dfrac{5}{\sqrt{3}} > 1$

   c) *diverges* : $r = 2 > 1$

   d) *converges* : $-1/3$

   e) *converges* : $3\dfrac{1/(e^2)}{1-(1/(e^2))}$

   f) *NA* (Hint: does not apply since there is no way to write as r^n.)

9. For each series, what does the p-series test tell us? [not applicable, converge, diverge, or inconclusive] Show your work.

   a) *NA*

   b) *diverges* $p = 0.5 \leq 1$)

   c) *diverges* $p = 1 \leq 1$

   d) *converges* $p = 3 > 1$

10. For each series, what does the integral test tell us? [not applicable, converge, diverge, or inconclusive] Show your work.

    a) *converges* : $\displaystyle\int_{x=1}^{\infty} \frac{\sqrt{x}+4}{x^2}dx = 6$

    b) *diverges* : $\displaystyle\int_{x=1}^{\infty} \frac{1}{\sqrt{x+1}}dx = \infty$

$c)$ $converges:$ $\displaystyle\int_{x=1}^{\infty}\left(\frac{1}{x}\right)^3 dx = 1/2$

$d)$ NA (since not positive)

Note $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2^n}$ does converge by the integral test since the integral is $\displaystyle\int_1^{\infty} 1/(2^x)dx = \frac{1}{2\ln 2}$.

$e)$ NA (since $1/\cos^2 x$ is not decreasing)

11. For each series, what does the comparison test tell us? [not applicable, converge, or diverge] Show your work.

$a)$ $converges:$ $\displaystyle\frac{1}{n^3}\geq\frac{1}{2n^3+1}$ (Hint: for comparison test and limit comparison we will always be comparing to a p-series or geometric series!)

$b)$ $converges:$ $\displaystyle\frac{9^n}{10^n}\geq\frac{9^n}{3+10^n}$

$c)$ $diverges:$ $\displaystyle\frac{6^n}{5^n}\leq\frac{6^n}{-4+5^n}$

$d)$ NA (since not positive)

12. For each series, what does the limit comparison test tell us? [not applicable, converge, or diverge] Show your work.

$a)$ $diverges:$ $\displaystyle\lim_{n\to\infty}\frac{\frac{1}{2n+1}}{\frac{1}{n}} = 1/2$

$b)$ $converges:$ $\displaystyle\lim_{n\to\infty}\frac{\frac{n+2}{(n+1)^3}}{\frac{1}{n^2}} = 1$

$c)$ $converges:$ $\displaystyle\lim_{n\to\infty}\frac{\frac{2^n}{5^n-n}}{\frac{2^n}{5^n}} = 1$

13. For each series, use the alternating series test or the limit test for divergence to decide: [converge, or diverge]. Show your work.

$a)$ $converges:$ $\displaystyle\lim_{n\to\infty}\frac{1}{\ln(n+1)} = 0$

$b)$ $converges:$ $\displaystyle\lim_{n\to\infty}\frac{1}{4n+1} = 0$

c) *diverges* : $\lim_{n\to\infty} \frac{(-1)^n}{e^{-n}} = DNE$ since $\lim_{n\to\infty} \frac{1}{e^{-n}} = \infty$

14. Decide if the sums converge or diverge, explain why. If there is a formula for the sum, find the value.

a) *converges* : $\int_{x=1}^{\infty} x^2 e^{-x^3}\, dx = \frac{1}{3e}$ (by integral test)

b) *diverges* : $\lim_{n\to\infty} e^{2n} = \infty$ (by limit test for divergence)

c) *converges* : $\frac{2/e^3}{1-2/e^3}$ (by geometric series test)

15. Also study the quizzes, and the homework questions. These are good test questions too!