Combinatorics. Quiz 6. Name _____ Time_____
Show all work for full or partial credit. Put a box around your final answer in each part. Try the problem on your own before helping each other understand it.

1. a) Find the ordinary generating function $f(x)$ for the sequence a_n where a_n is the number of integer solutions of $e_1 + e_2 + e_3 + e_4 = n$ where $0 \leq e_1 \leq 5, 2 \leq e_2 \leq 5, 3 \leq e_3 \leq 4$ and $0 \leq e_4$.

   b) Use the ninth derivative of your above answer to find a_9. Use wolframalpha.com, with the command "(d^9/dx^9)[$f(x)$]; $x = 0$" and then divide by 9!. Turn in a screen shot.

   c) Find a_9 using inclusion-exclusion.

2. Find the exponential generating function for the sequence h_n where h_n is the number of permutations of length n using the letters A, C, G, T, with repetition.

3. a) Find the e.g.f. for the sequence b_n where b_n is the number of permutations of length n using the letters A, C, G, T, but there is at least 1 G and an even number of $T's$.

   b) Use the answer above to find a closed formula for b_n.

   c) Check that the formula gives the right answer for $n = 3$ (list by hand all the 3-perms that obey the requirements.)

   d) Check that the third derivative of the e.g.f at zero also gives that answer. (wolframalpha.com, turn in screen shot.)