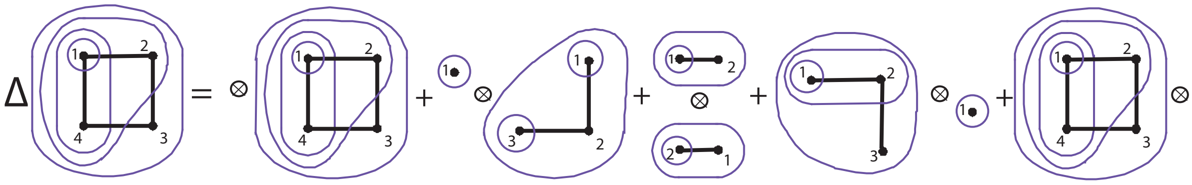

Figure 9. A coproduct in a restrictive sequence coalgebra.

designed to ensure that the coproduct is well-defined. The proof of coassociativity rests upon a straightforward demonstration of the following fact:

Conjecture 3.7. *The generalized Tonks projection from the permutohedra to a sequence K_{G_i} for a restrictive graph sequence is a coalgebra map.*

From the examples in Figure 3.3 we conjecture that there is an infinite filtration of the coalgebra $\mathcal{S}$Sym based on incrementally increasing the connectedness of the early graphs in the restrictive sequence. Again the first coalgebra in the filtration is $\mathcal{Y}$Sym.

3.4. Face algebras. As usual, we conjecture that the algebras and coalgebras based upon the graph associahedra for hereditary and restrictive graph sequences may be enlarged to respective structures on the faces of the polytopes.

3.5. Investigative strategies.

3.5.1. Lattices. One can represent the elements of the symmetric groups in multiple ways. Classically these pictures have allowed lattices such as the Tamari order on binary trees and the Boolean posets to be seen as projections of the weak order on symmetric groups. We have uncovered several new poset structures on the skeletons of the graph associahedra. Given a numbering of the nodes of a graph we define the ordered graph lattice. We can describe the conjectural covering relations as follows: a maximal tubing covers another if the collection of all the tubes of both splits into identical pairs except for one pair of tubes which are unique to their respective tubings.
Compare the two numbered nodes of these which are in no smaller tubes. The smaller node is in the tubing covered by the other. This ordering generalizes both the weak order on permutations and the Tamari ordering of binary trees. Figure 10 shows an example.

Conjecture 3.8. *The 1-skeleton of each graph associahedron is a quotient lattice of the weak order on the symmetric group.*

Of course the projection map we have in mind is the generalized Tonks projection restricted to vertices. The answer to the following question will have important ramifications to finding Möbius inversion.

Question 3.9. *Do the generalized Tonks projections from S_n to our new lattices form lattice congruences or interval retracts?*

3.5.2. Möbius inversion. By performing Möbius inversion on the elements of our basis, with respect to the various lattices, we can find a new basis for the algebra or coalgebra. As mentioned above, this new basis can prove to be perfect for describing primitives or finding structure constants. In [16] we show that the existence of an interval retract between our lattice and the weak order on S_n implies a formula for the Möbius function.