Differential Equations. Review for Test 1, Spring '19.

Also study all
the homework and quizzes, as well as examples in class notes.
Note: Some questions on the actual test may state "Set up the differential equation only." Since you don't know which kind, for practice do both the set-up and the solution.

Note: Don't forget that the answer will have an unknown constant or constants, unless it is an IVP.

1. A 100 gallon tank initially contains 60 gallons of water with 20 kg of sugar in solution. An input pipe adds 10 kg of sugar per gallon, at the rate of 5 gallons per second. An output pipe drains 2 gallons of well-stirred mixture per second.
   - Set up the diff. eq. for finding $A(t)$, the amount of sugar in the tank after t seconds.
   - Solve to get the formula for $A(t)$.
   - When will the tank be full, and how much sugar will it contain then?

2. Solve the diff. eq. $y' - y = e^x y^2$. Is it linear, Bernoulli or separable?

3. Solve the diff. eq. $y' = y(xy^3 - 1)$. Is it linear, Bernoulli or separable?

4. Consider the differential equation $x^2y'' - 7xy' + 7y = 0$.
   By using $y = x^m$ find two solutions of the above equation. Write a (family of) solutions that uses the constants c_1, c_2.

5. Solve the differential equation (IVP): $x^3y' = y - xy \;\; ; \;\; y(1) = 7$. Is it Bernoulli or separable?

6. Solve the differential equation generally: $y' = 2^x(1 + y^2)$. Is it linear, Bernoulli or separable?
   Your answer should be solved for y, and will have an unknown constant.

7. Solve the differential equation generally: $y' + \frac{1}{x}y = \sqrt{x^2 + 1}$. Is it linear, Bernoulli or separable?
   Your answer should be solved for y, and will have an unknown constant.

8. Consider the differential equation: $(e^x \cos y + y^2)dx + (2yx - e^x \sin y)dy = 0 \;\; ; \;\; y(2) = 0$.
   - Show whether this diff. eq. is exact or inexact.
   - Solve it (IVP).

9. Solve the differential equation generally: $y' = \dfrac{2e^y + x^3 + 1}{-xe^y}$.
   - Show whether this diff. eq. is exact or inexact.

   Your answer should be an implicit equation with an unknown constant.