Calculus II. Quiz 5. Name Key Time

Show all work on this page for full and/or partial credit. Put a box around your final answers in each part.

1. Find the improper integral. $\int_4^\infty \frac{3}{e^{2x}}dx$

$$\lim_{t\to\infty}\int_4^t \frac{3}{e^{2x}}\,dx = \lim_{t\to\infty}\int_4^t 3e^{-2x}\,dx$$

$$= \lim_{t\to\infty}\left[\frac{3}{-2}e^{-2x}\right]_4^t$$

$$= \lim_{t\to\infty}\frac{3}{-2}\left(e^{-2t}-e^{-8}\right) = \boxed{\frac{3}{2}e^{-8}}$$

$$= \frac{3}{2e^8}$$

2. Find the improper integral. $\int_4^\infty \frac{3}{x+2}dx$

$$\lim_{t\to\infty}\int_4^t \frac{3}{x+2}\,dx = \lim_{t\to\infty}\left[3\ln(x+2)\right]_4^t$$

$$= \lim_{t\to\infty}\left(3\ln(t+2) - 3\ln 6\right)$$

$$= \boxed{\infty}$$

3. Find the improper integral. $\int_1^\infty \frac{7}{1+x^2}dx$

$$\lim_{t\to\infty}\int_1^t \frac{7}{1+x^2}\,dx = \lim_{t\to\infty}\left[7\tan^{-1}x\right]_1^t$$

$$= \lim_{t\to\infty}\left(7\tan^{-1}t - 7\tan^{-1}1\right)$$

$$= 7\frac{\pi}{2} - 7\frac{\pi}{4} = \boxed{\frac{7\pi}{4}}$$

4. Find the improper integral. $\int_0^2 \frac{1}{x^3}dx$

$$\lim_{t\to 0^+}\int_t^2 x^{-3}\,dx = \lim_{t\to 0^+}\left[\frac{x^{-2}}{-2}\right]_t^2$$

$$= \lim_{t\to 0^+}\left(\frac{2^{-2}}{-2} - \frac{t^{-2}}{-2}\right)$$

$$= \lim_{t\to 0^+}\left(\frac{1}{-8} + \frac{1}{2t^2}\right) = \frac{-1}{8} + \infty = \boxed{\infty}$$

positive infinity