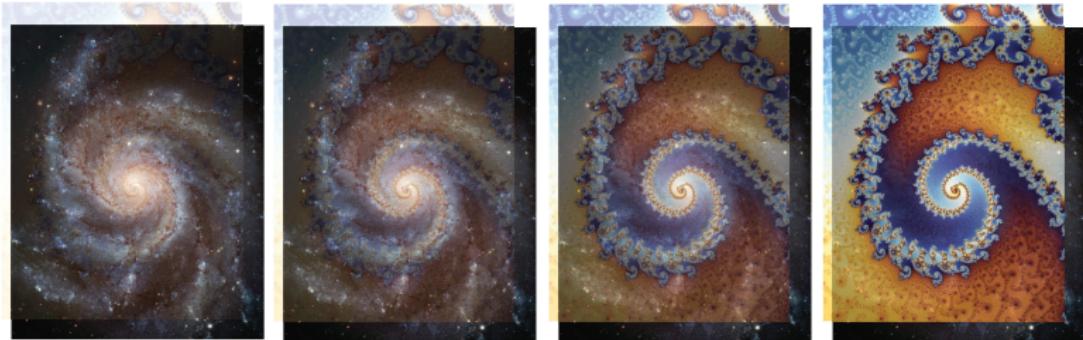


Polytopes, phylogenetics, puddles and fractals



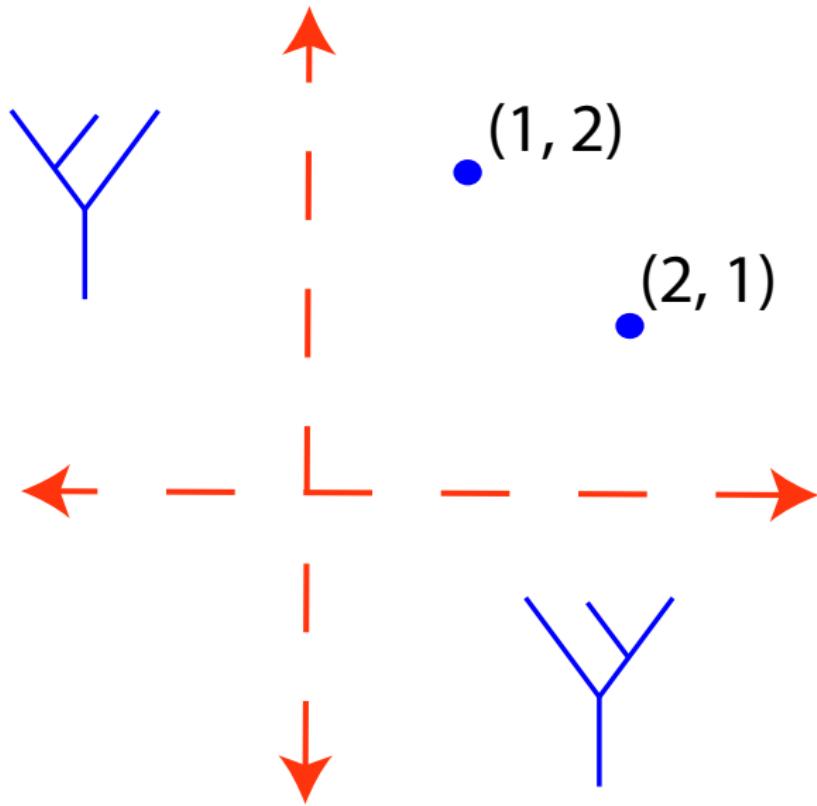
Stefan Forcey, University of Akron

November 8, 2011

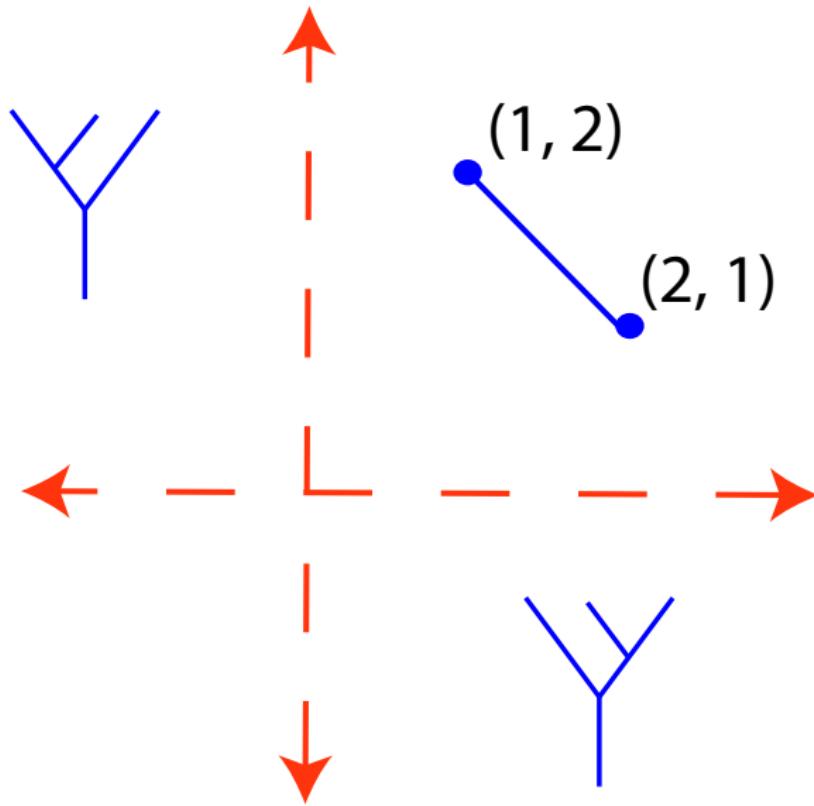


*Nature image credits: NASA, Wikimedia and Google Earth. * "Nature" includes the Mandelbrot set.

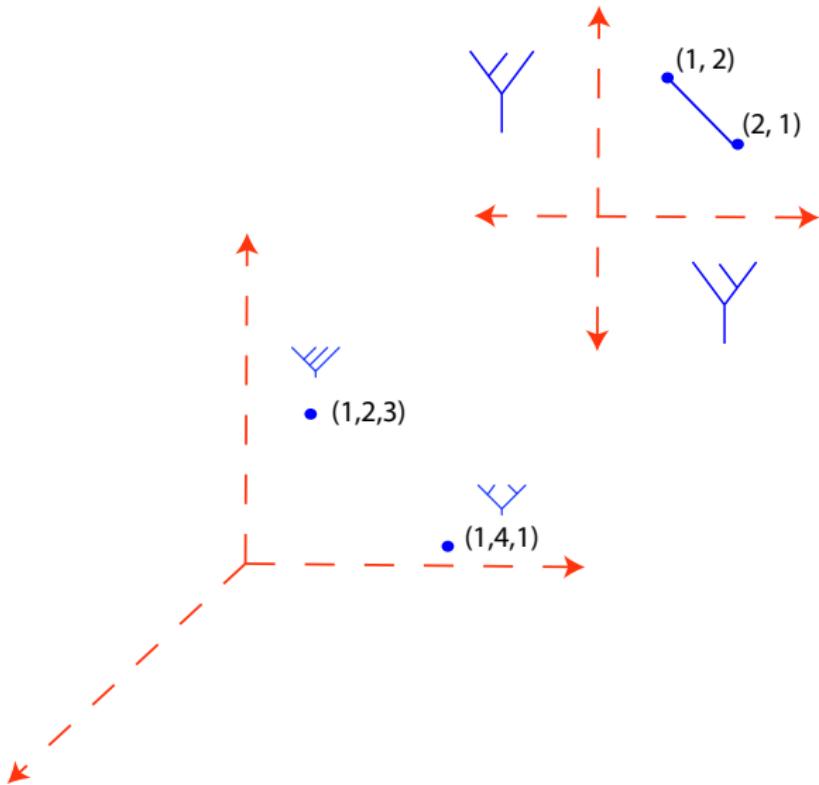
A convex hull of binary trees.



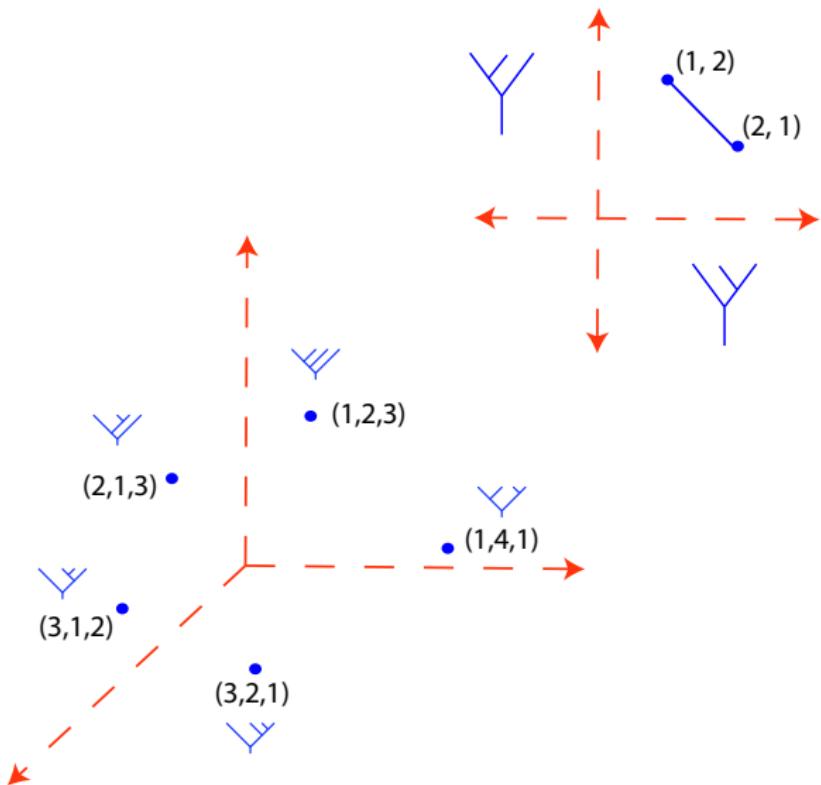
A convex hull of binary trees.



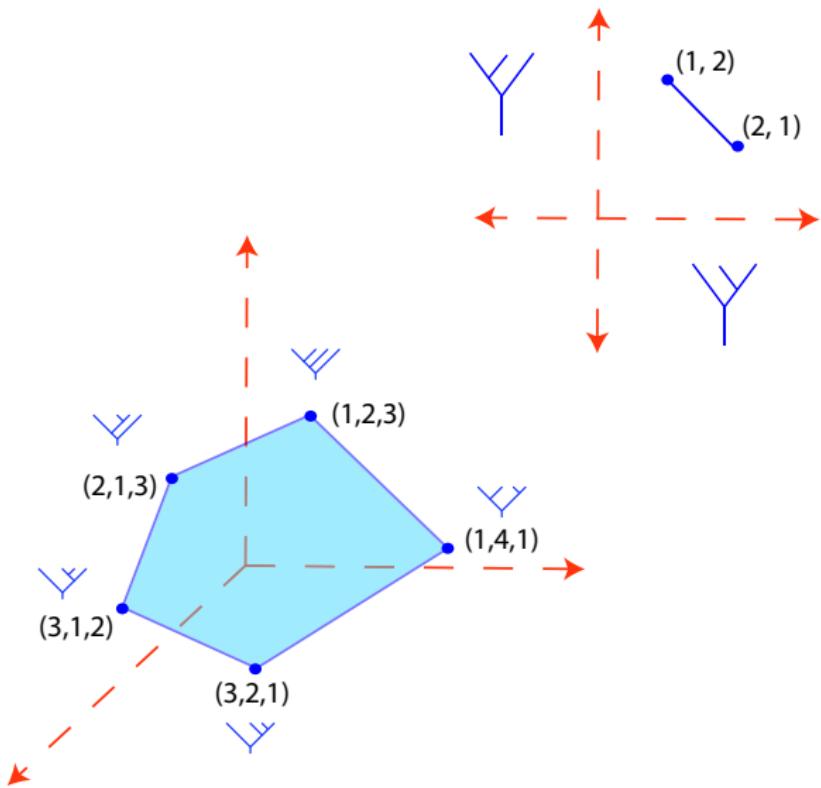
A convex hull of binary trees.



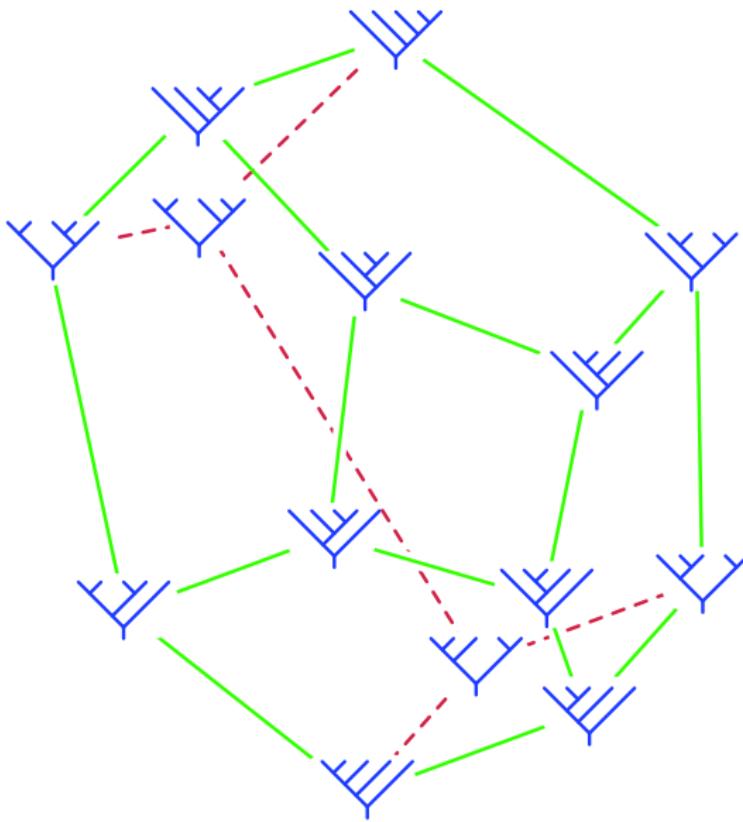
A convex hull of binary trees.



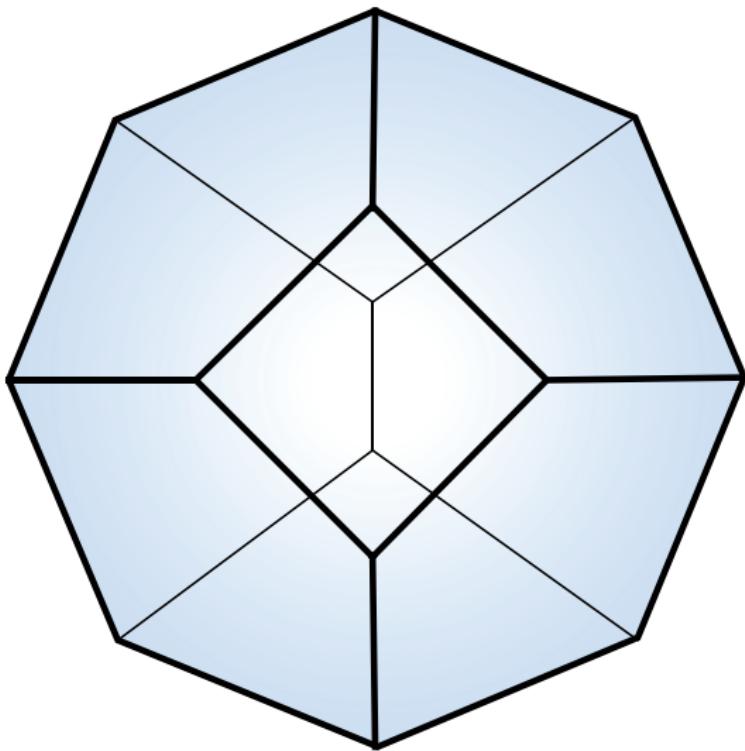
A convex hull of binary trees.



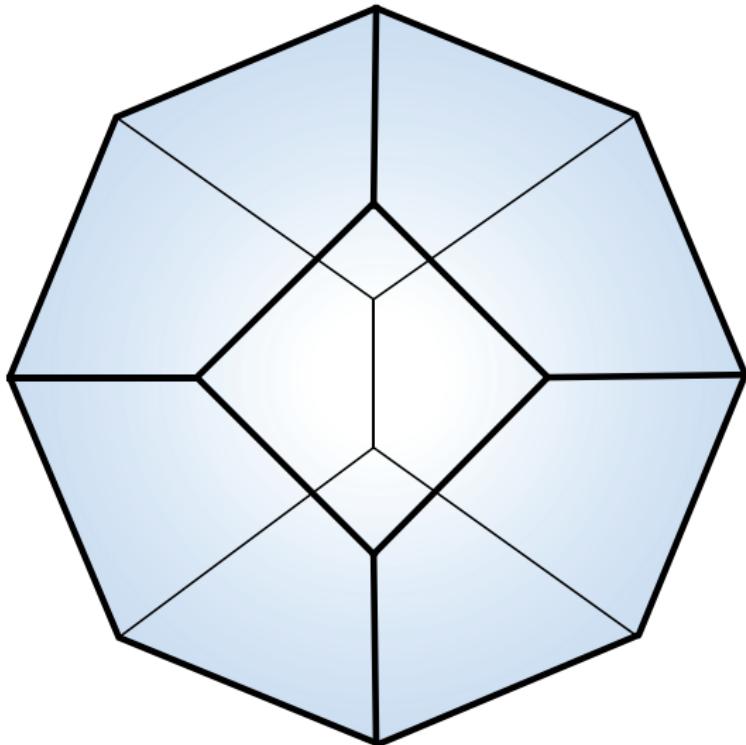
A convex hull of binary trees.



The associahedron.



The associahedra, and the Catalan numbers.



1 , 1 , 2 , 5 , 14 , 42 , ...

The Catalan numbers and the Mandelbrot sequences.

Definition

Mandelbrot sequences: Take a complex number z and add it to its square. Repeat, by squaring your result and adding the same original z .

$$\begin{aligned} z \\ \mapsto z + z^2 \end{aligned}$$

The Catalan numbers and the Mandelbrot sequences.

Definition

Mandelbrot sequences: Take a complex number z and add it to its square. Repeat, by squaring your result and adding the same original z .

z

$$\mapsto z + z^2$$

$$\mapsto z + (z + z^2)^2$$

The Catalan numbers and the Mandelbrot sequences.

Definition

Mandelbrot sequences: Take a complex number z and add it to its square. Repeat, by squaring your result and adding the same original z .

z

$$\mapsto z + z^2$$

$$\mapsto z + (z + z^2)^2$$

$$= z + z^2 + 2z^3 + z^4$$

The Catalan numbers and the Mandelbrot sequences.

Definition

Mandelbrot sequences: Take a complex number z and add it to its square. Repeat, by squaring your result and adding the same original z .

z

$$\mapsto z + z^2$$

$$\mapsto z + (z + z^2)^2$$

$$= \mathbf{1}z + \mathbf{1}z^2 + \mathbf{2}z^3 + z^4$$

The Catalan numbers and the Mandelbrot sequences.

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Mandelbrot sequences: Take a complex number z and add it to its square. Repeat, by squaring your result and adding the same original z .

z

$$\mapsto z + z^2$$

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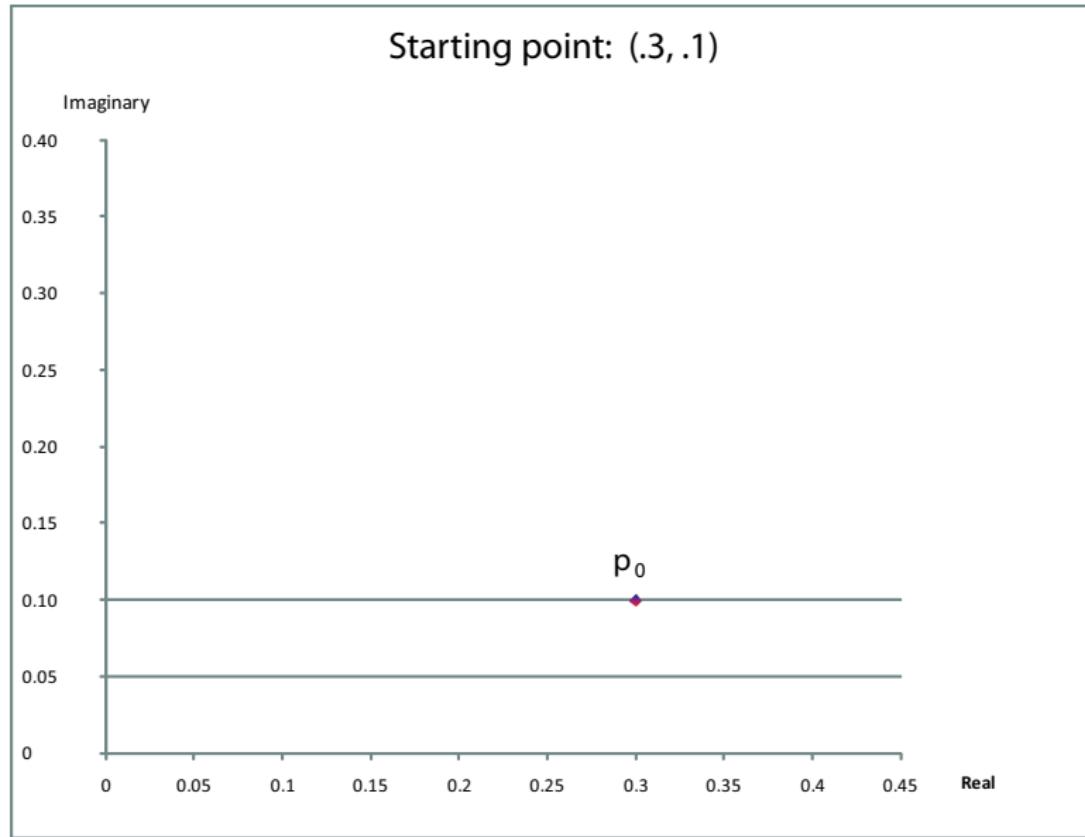
$$= 1z + 1z^2 + 2z^3 + z^4$$

$$\mapsto 1z + 1z^2 + 2z^3 + 5z^4 + 6z^5 + 6z^6 + 4z^7 + z^8$$

\vdots

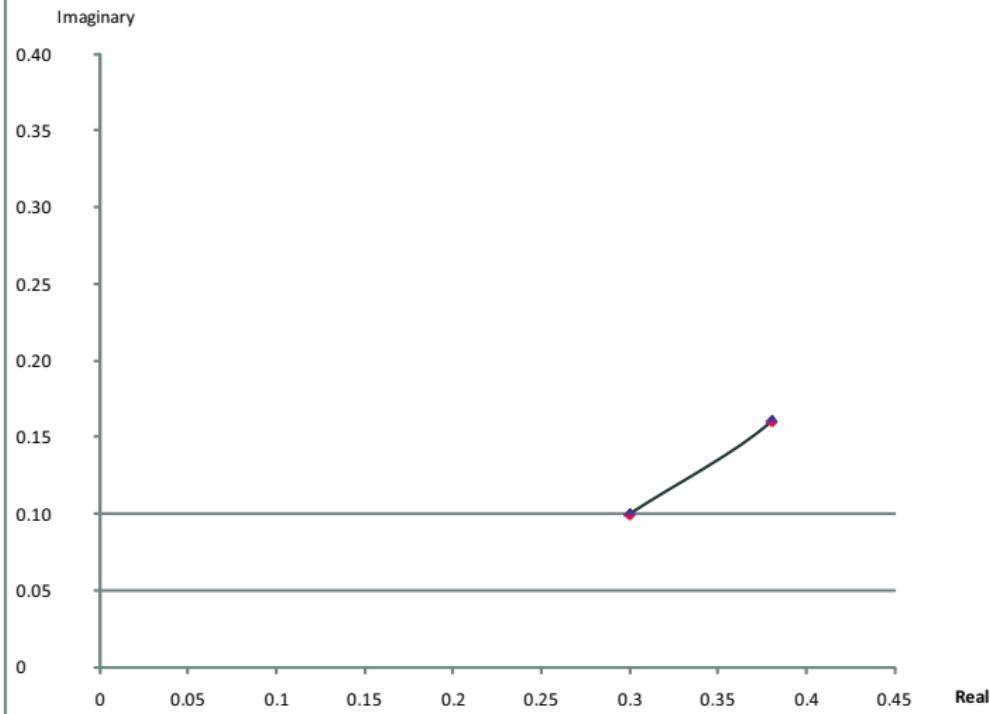
$$\mapsto 1z + 1z^2 + 2z^3 + 5z^4 + 14z^5 + 42z^6 + \dots$$

Example Mandelbrot sequence: $z \mapsto z + z^2$



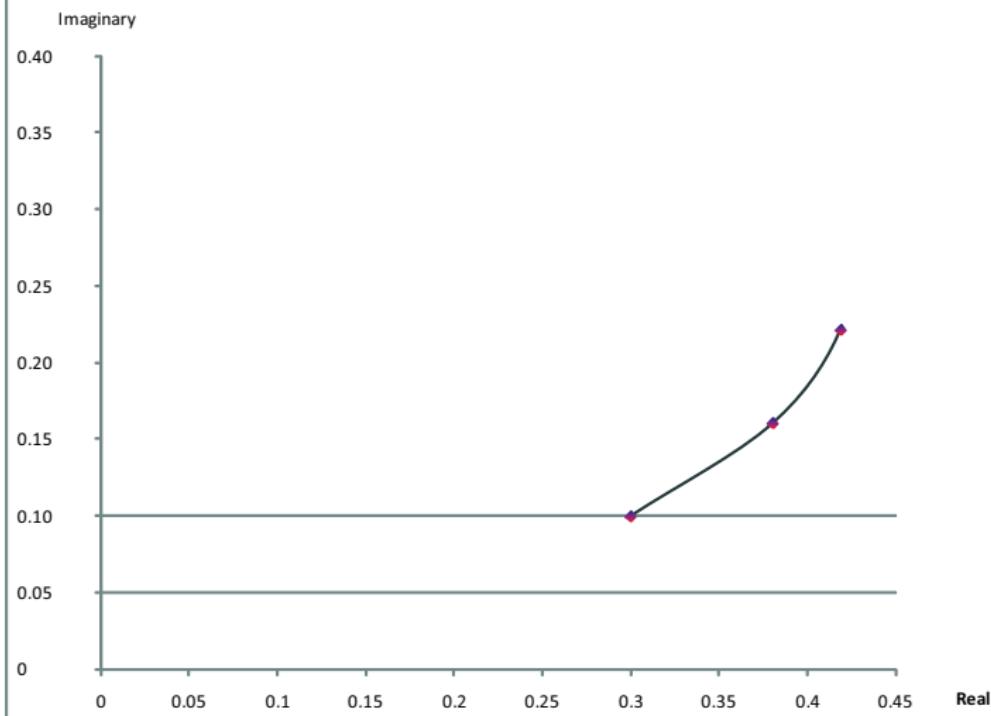
Mandelbrot $z \mapsto z + z^2$

Starting point: (.3,.1)



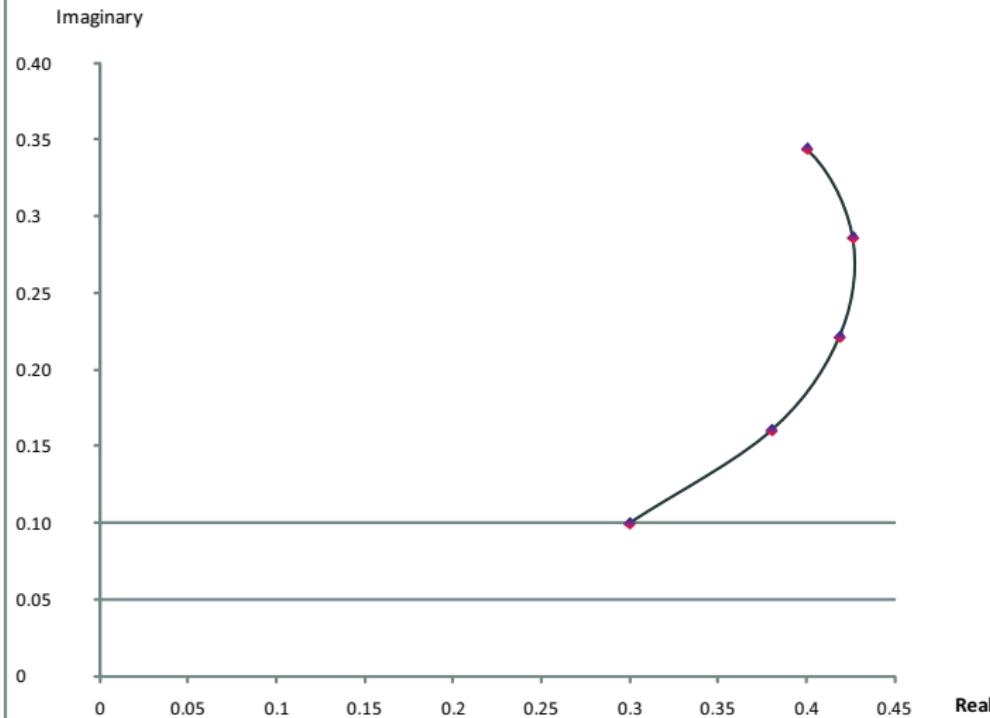
Mandelbrot $z \mapsto z + z^2$

Starting point: (.3,.1)



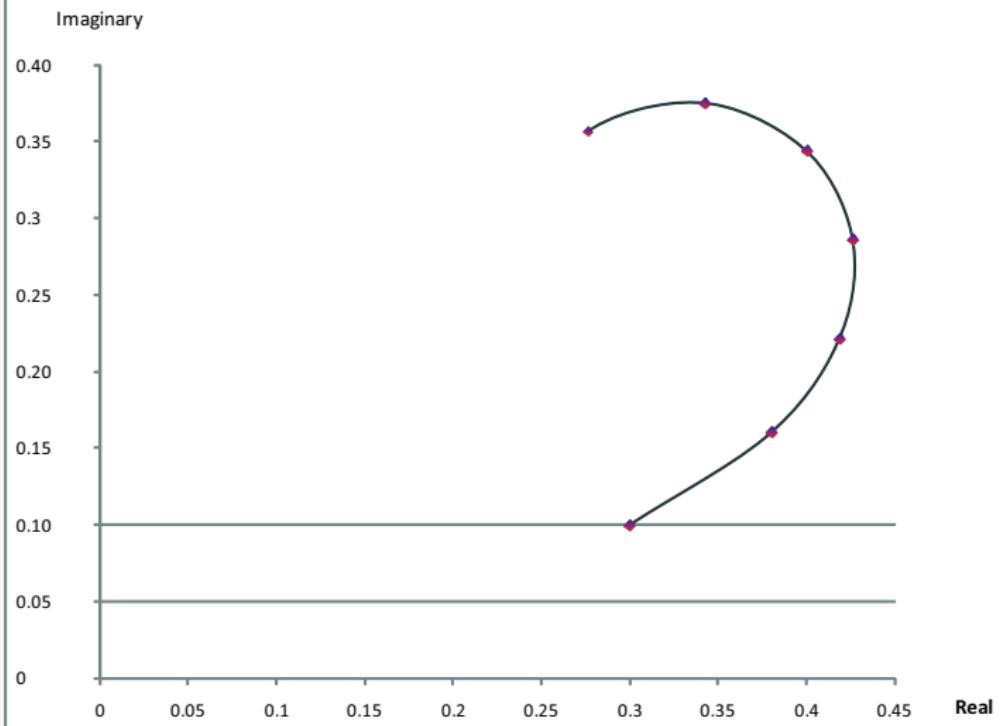
Mandelbrot $z \mapsto z + z^2$

Starting point: (.3,.1)

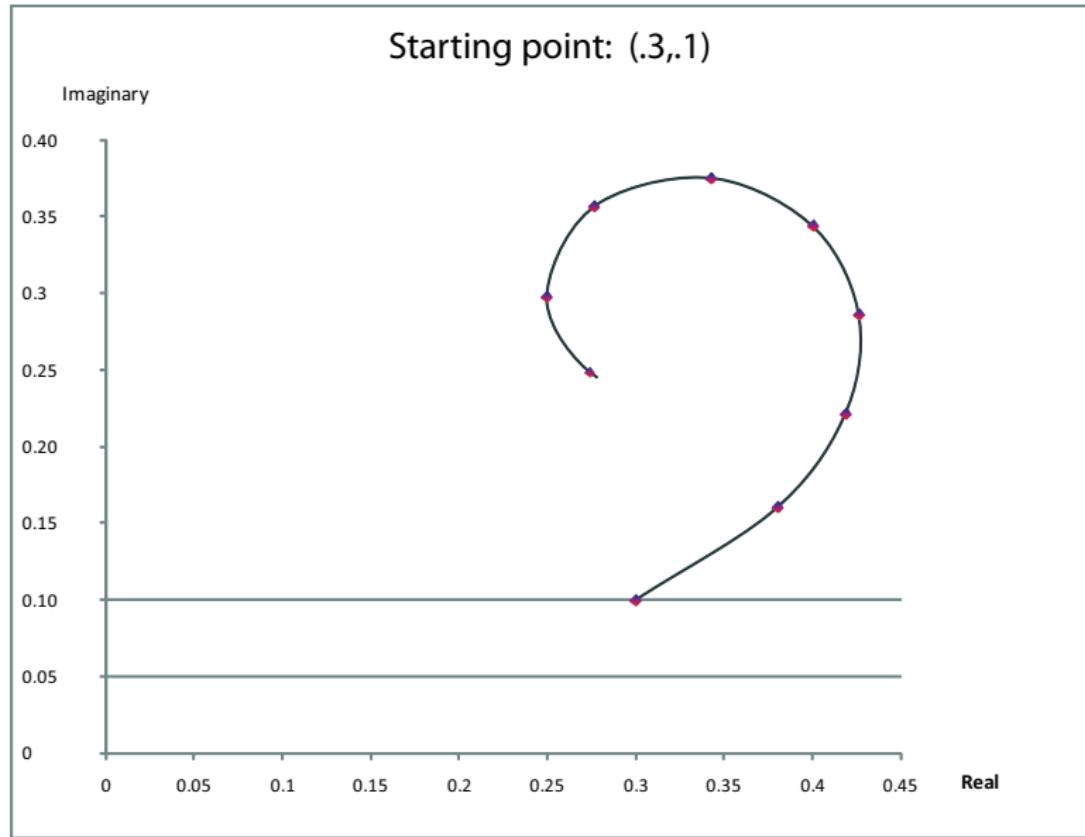


Mandelbrot $z \mapsto z + z^2$

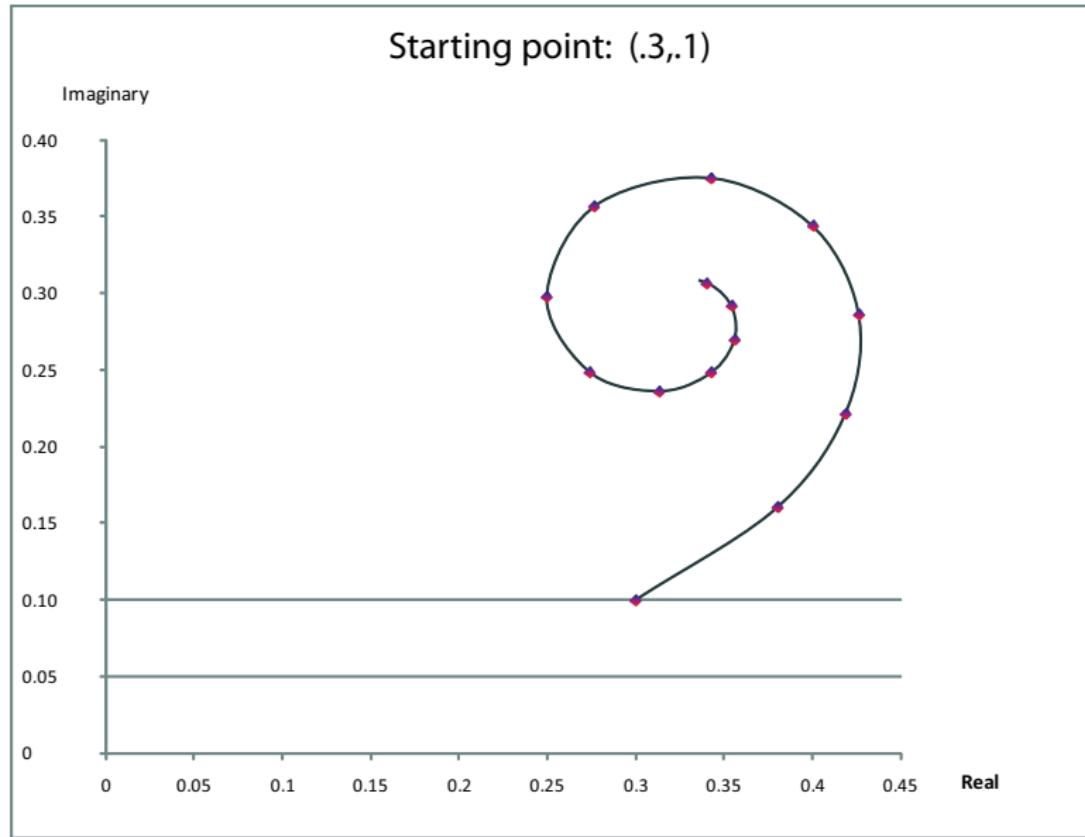
Starting point: (.3,.1)



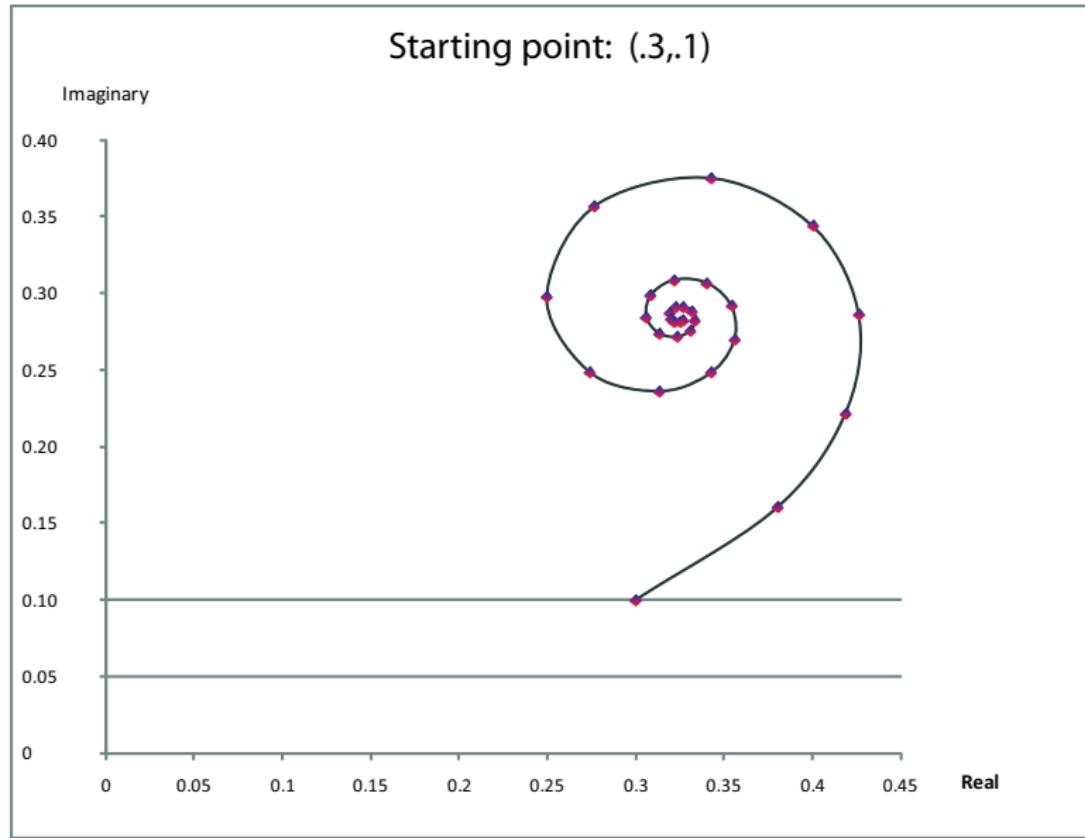
Mandelbrot $z \mapsto z + z^2$



Mandelbrot $z \mapsto z + z^2$

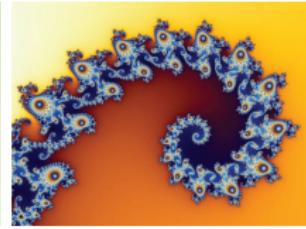
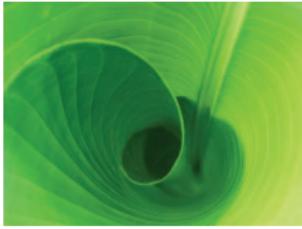


Mandelbrot $z \mapsto z + z^2$



Logarithmic spiral.

$$r = e^{\theta}.$$

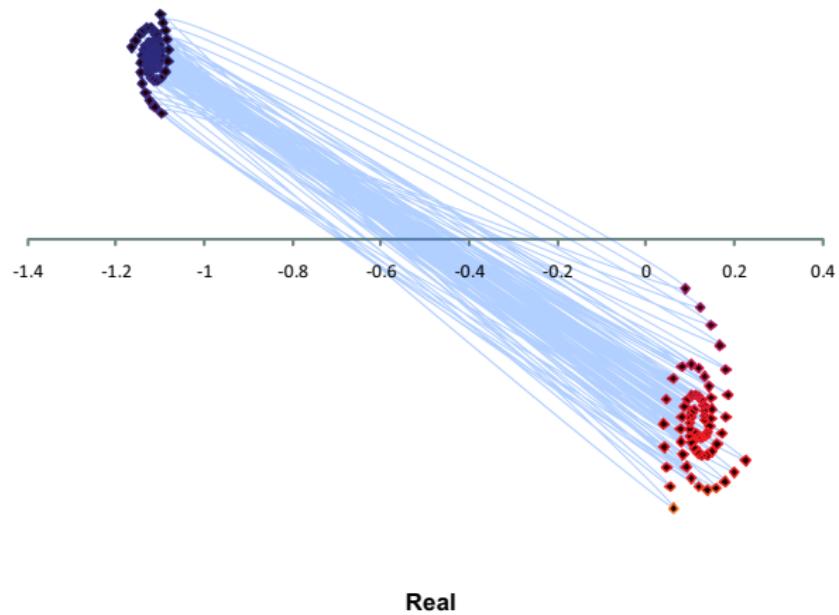


Image

credits: McSweeney's (first two) Wikimedia (last two).

Mandelbrot $z \mapsto z + z^2$

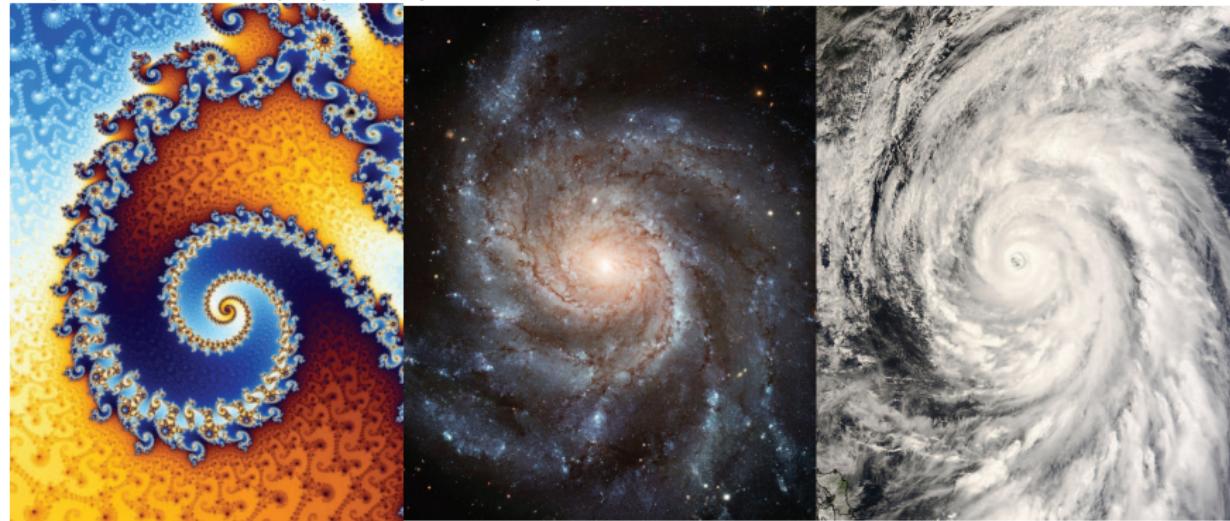
Starting point: (-1.1, .22)



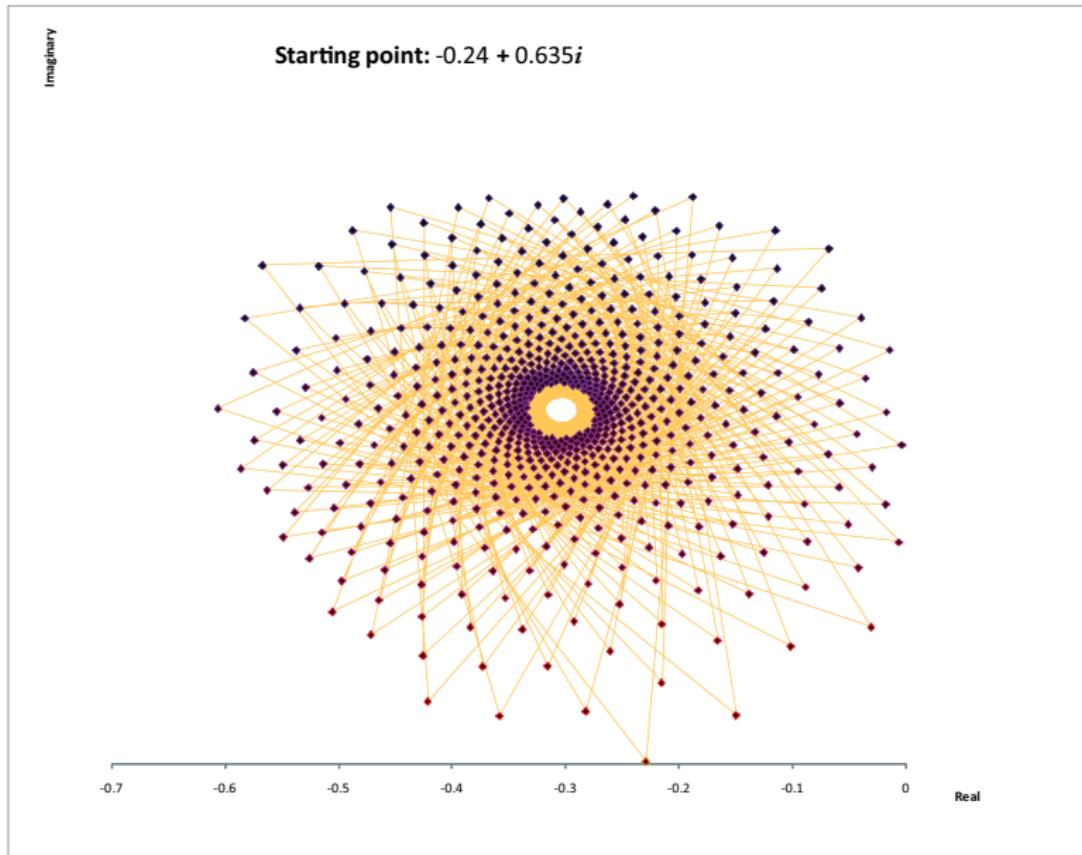
Logarithmic spiral.

$$\ln r = \theta.$$

Image credits: Wikimedia (first), NASA (second two).

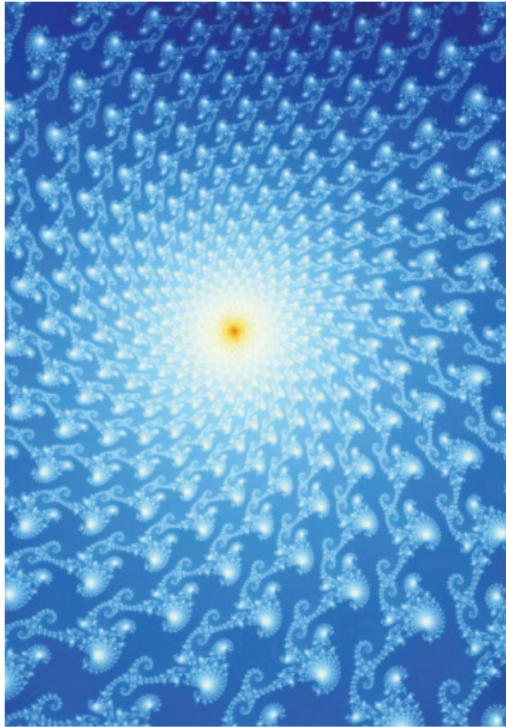


Mandelbrot $z \mapsto z + z^2$



Logarithmic spiral.

$$\ln r = \theta.$$



Mandelbrot $z \mapsto z + z^2$

Starting point: $-0.26 + 0.635i$

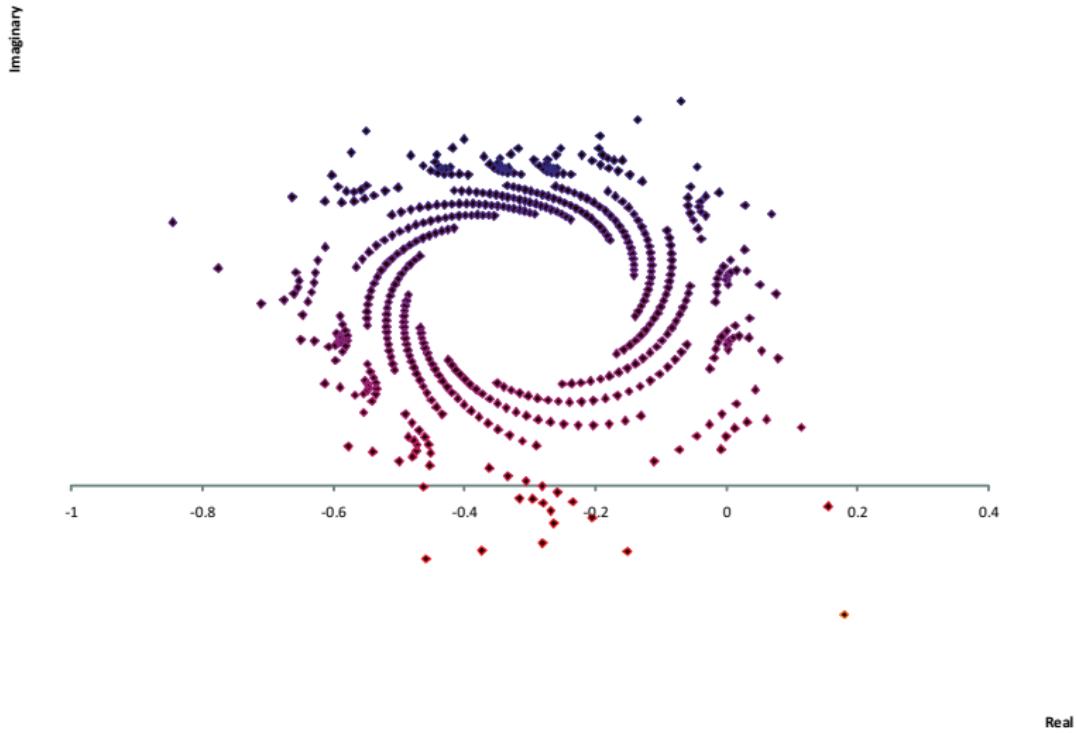
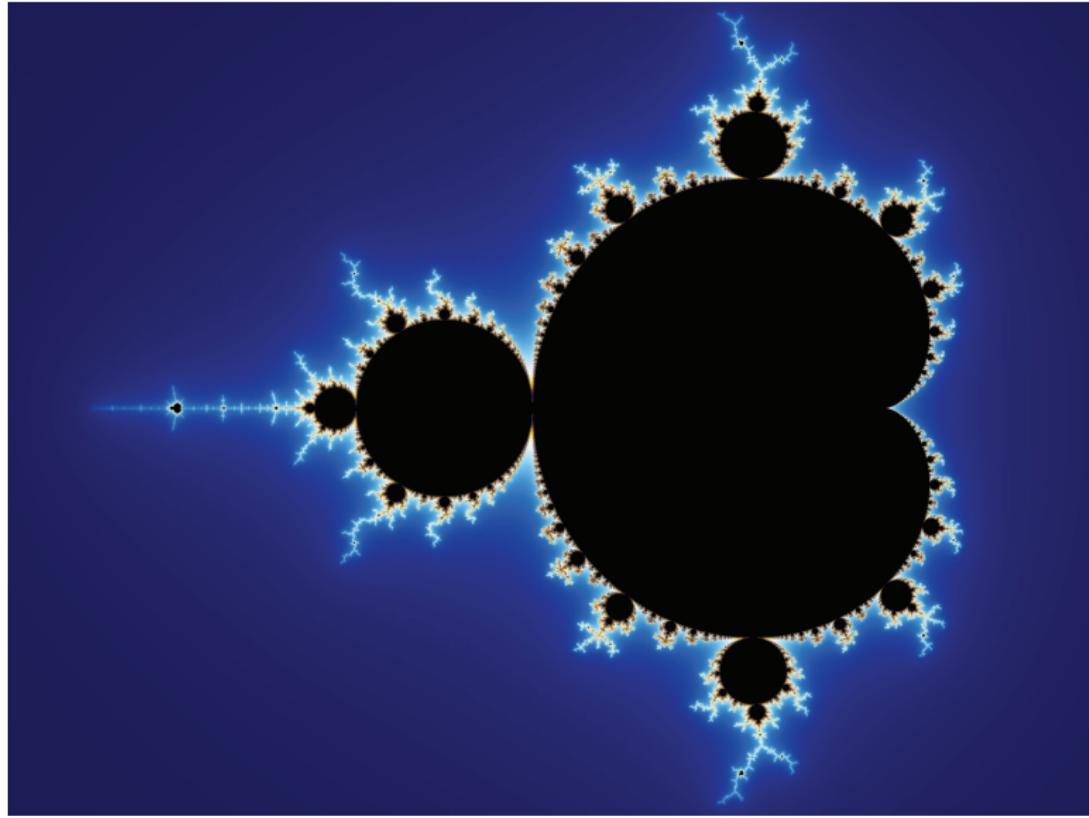


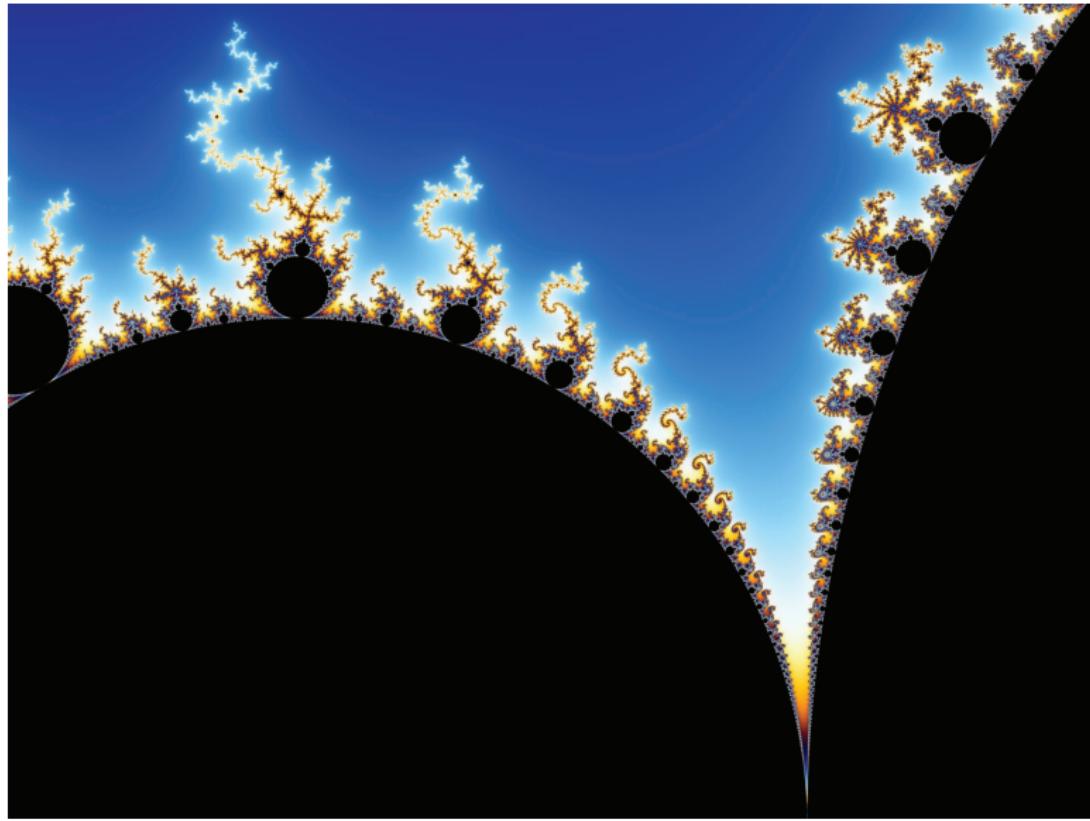
Image credits (all following Mandelbrot slides):
Wikimedia, Wolfgang Beyer.

<http://commons.wikimedia.org/wiki/User:Wolfgangbeyer>

Mandelbrot set.



Mandelbrot set.



Fractal

Victoria crater.

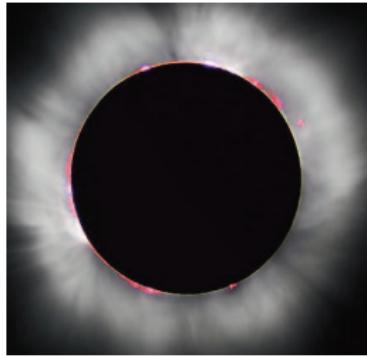
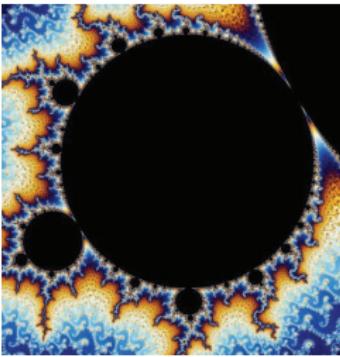
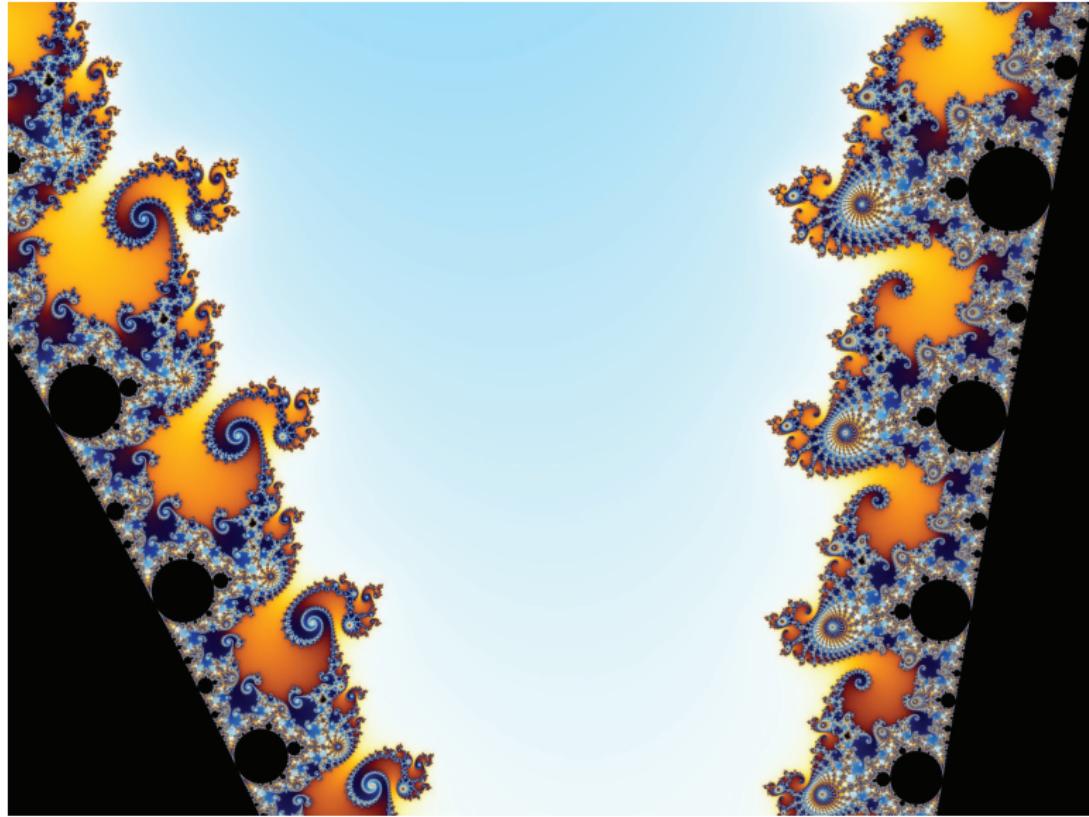
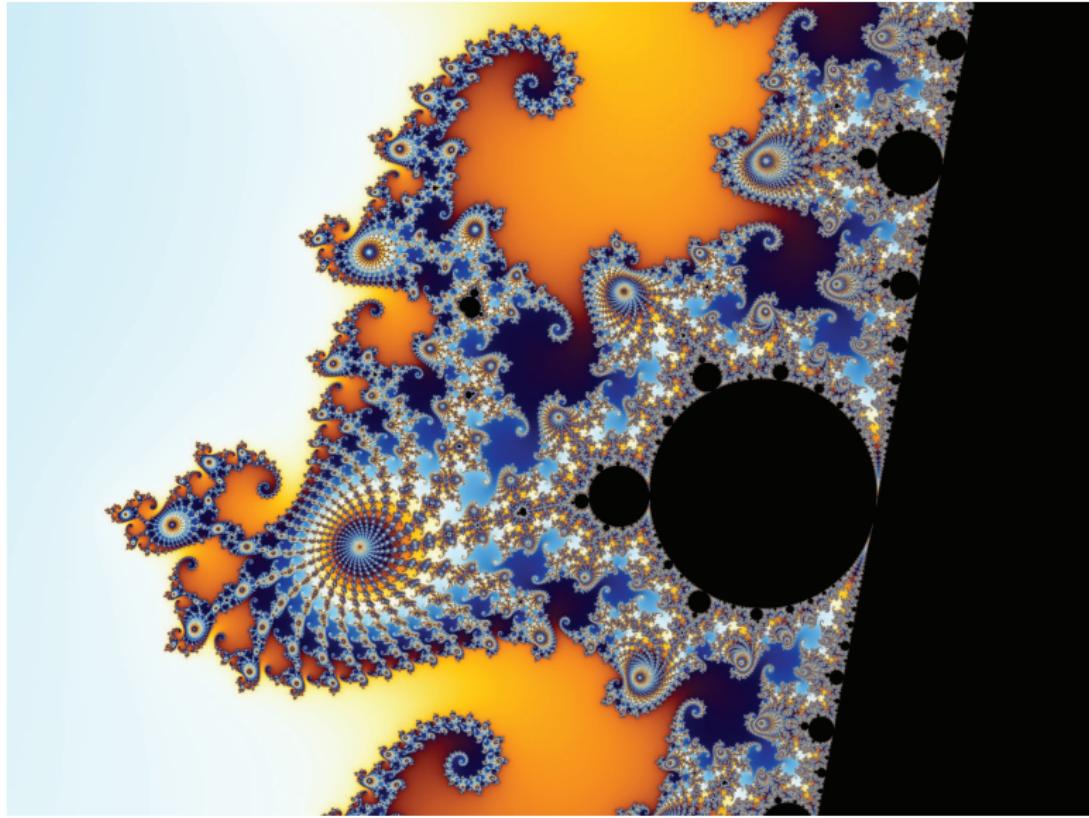


Image credit for Victoria crater: NASA.

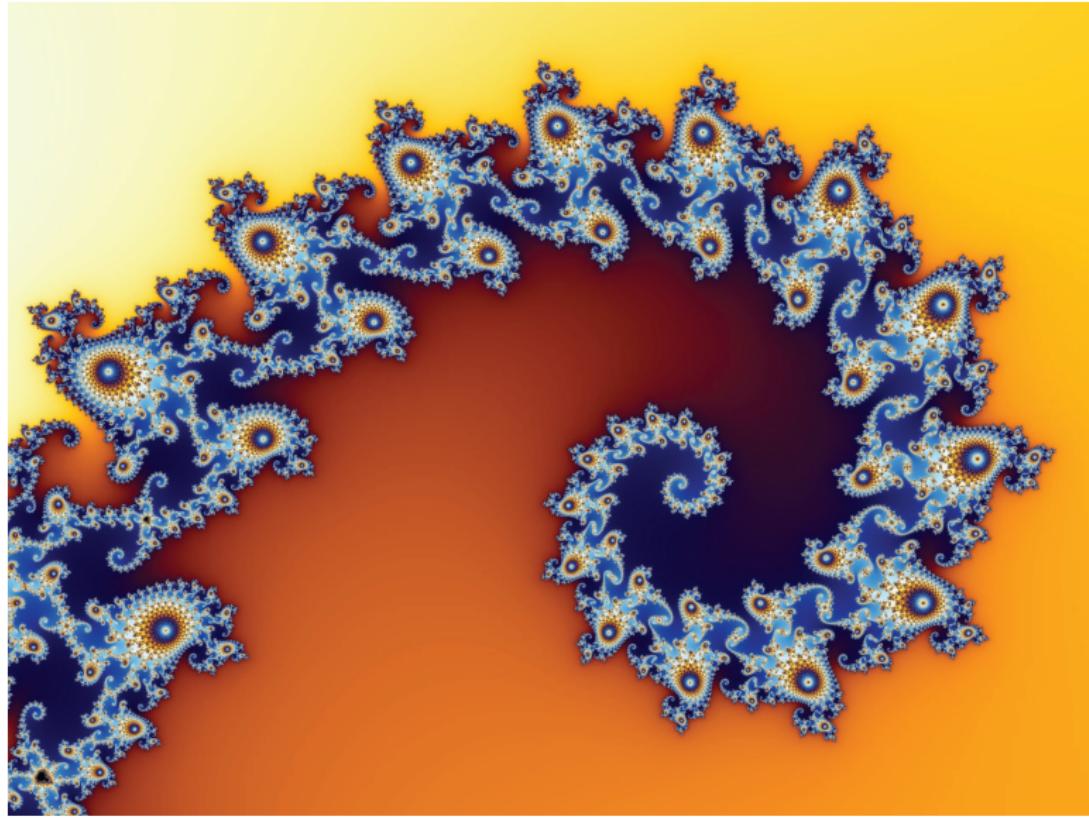
Mandelbrot set.



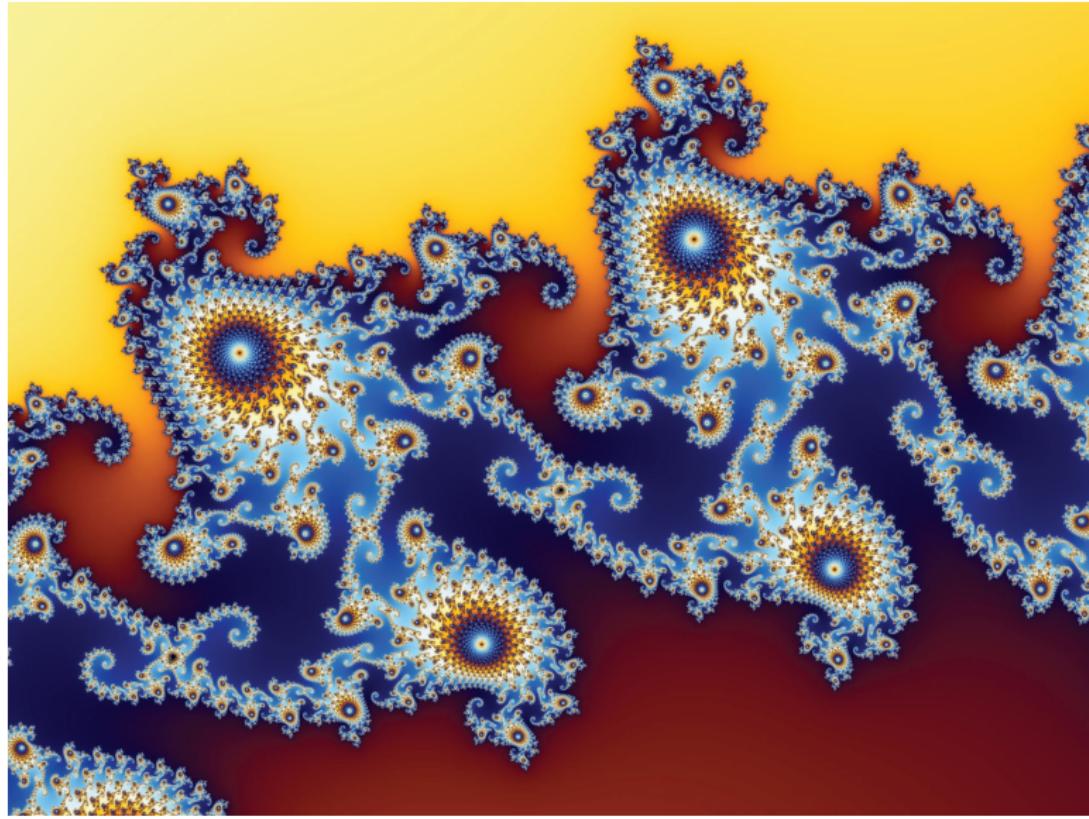
Mandelbrot set.



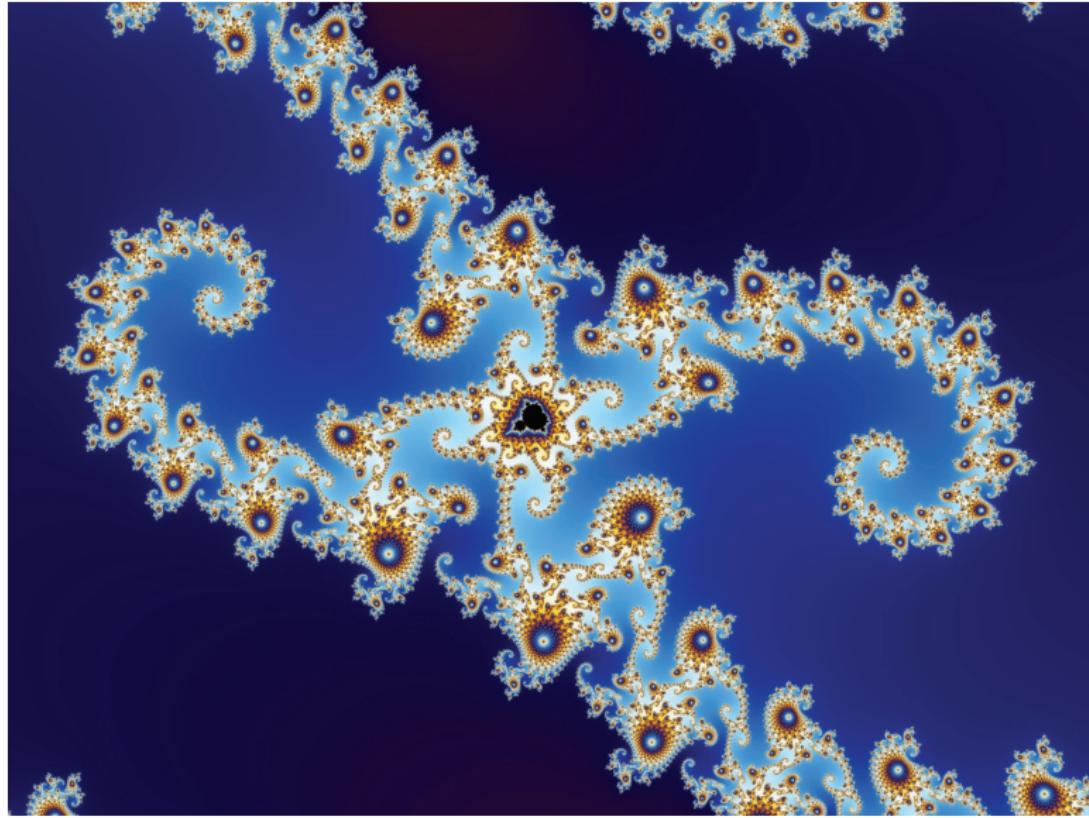
Mandelbrot set.



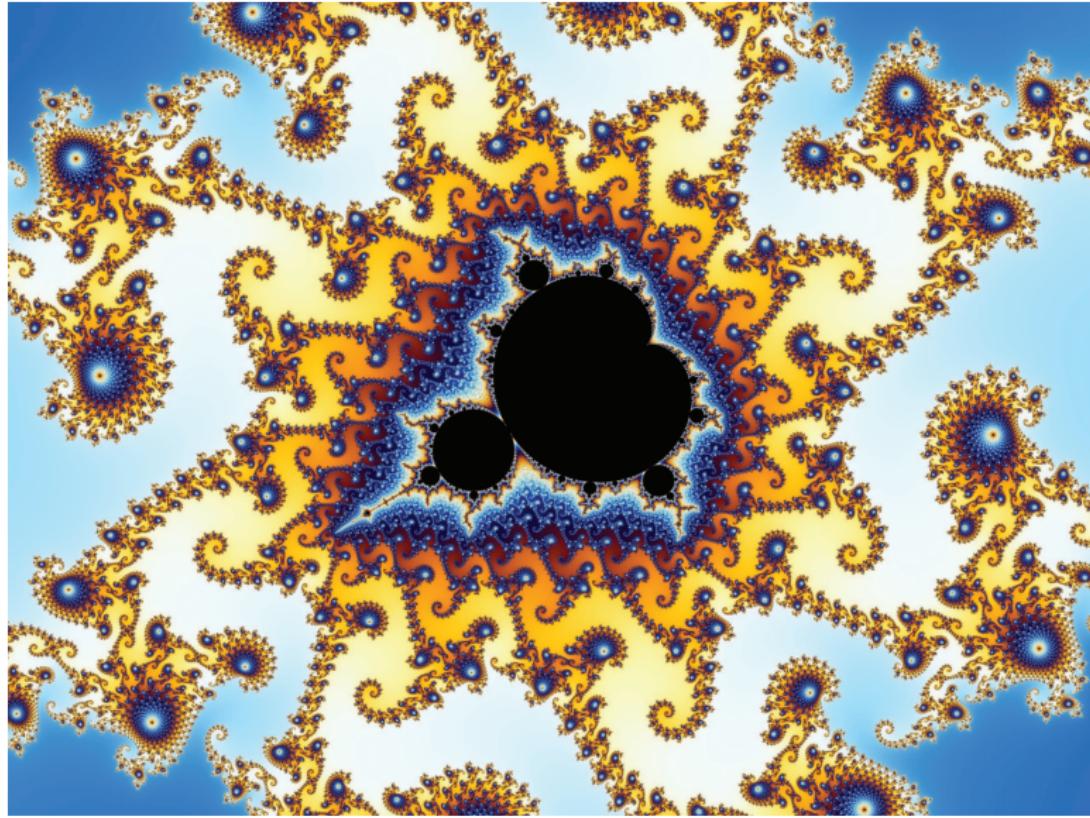
Mandelbrot set.



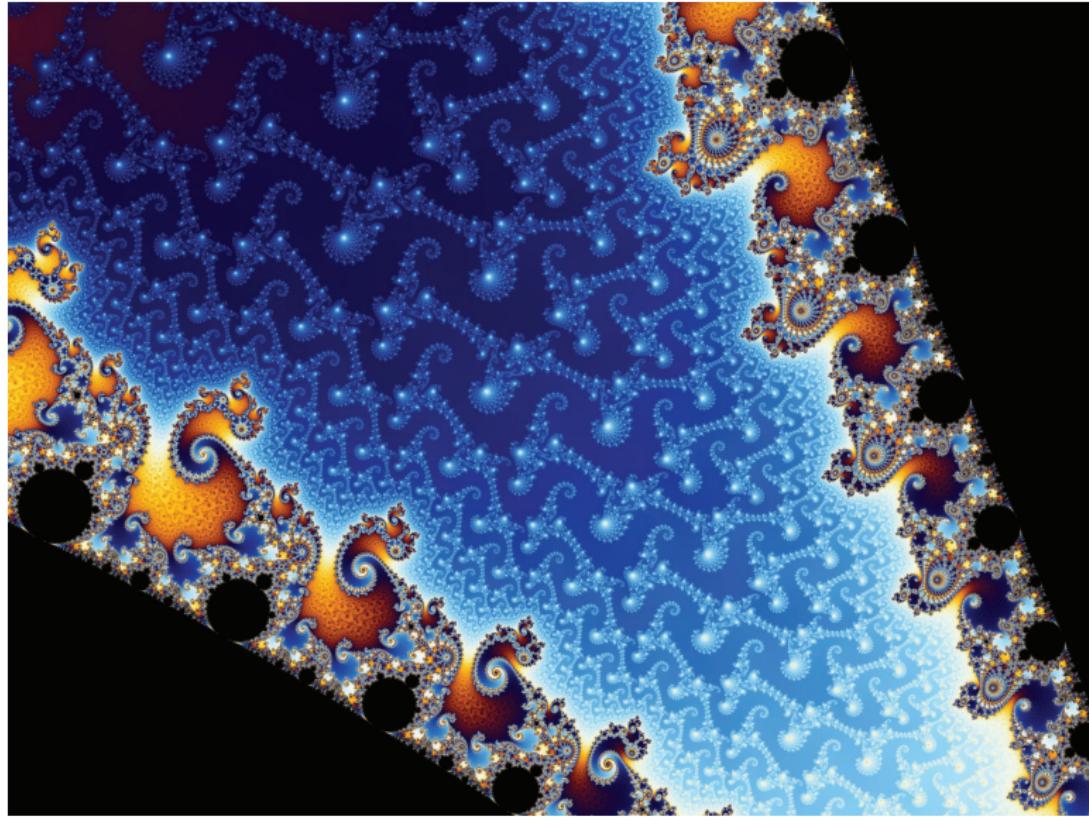
Mandelbrot set.



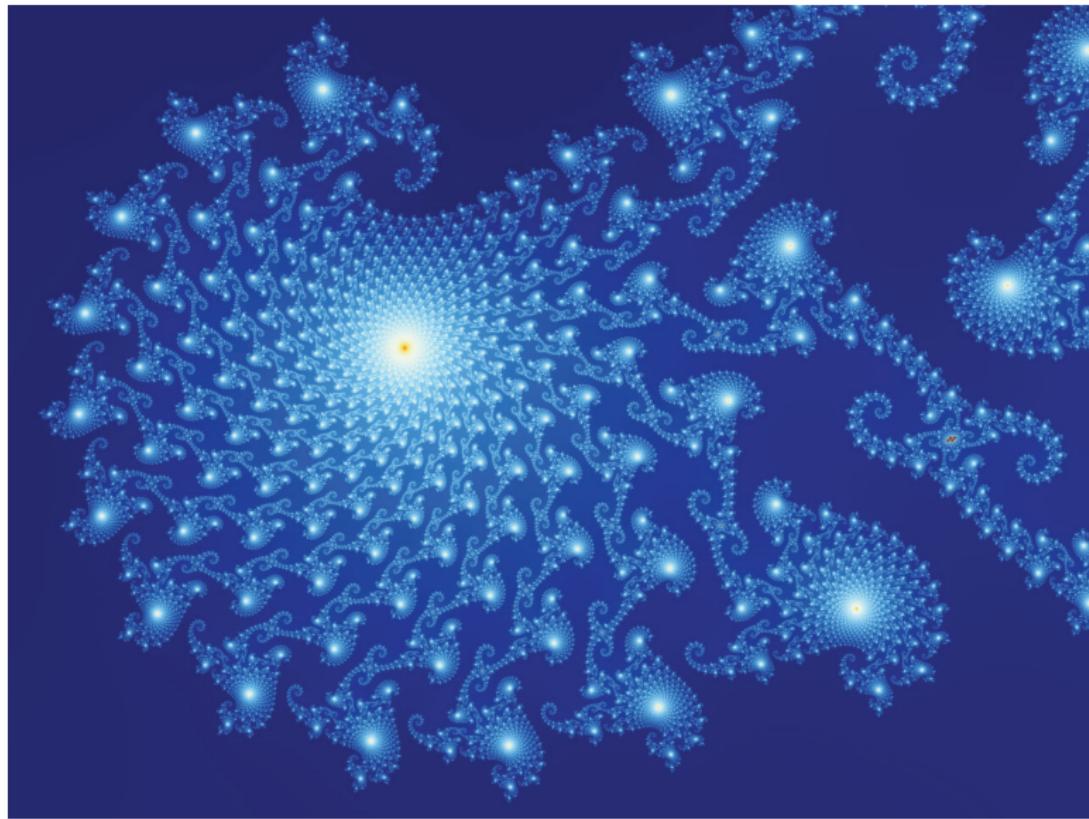
Mandelbrot set.



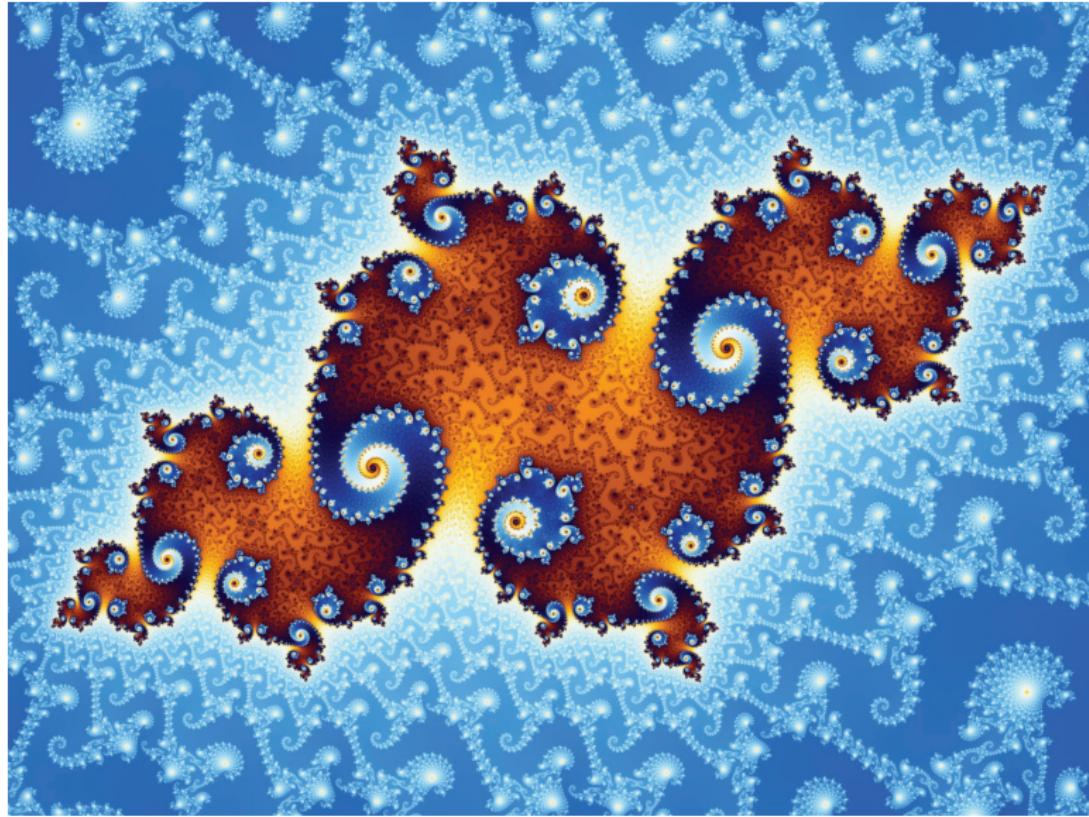
Mandelbrot set.



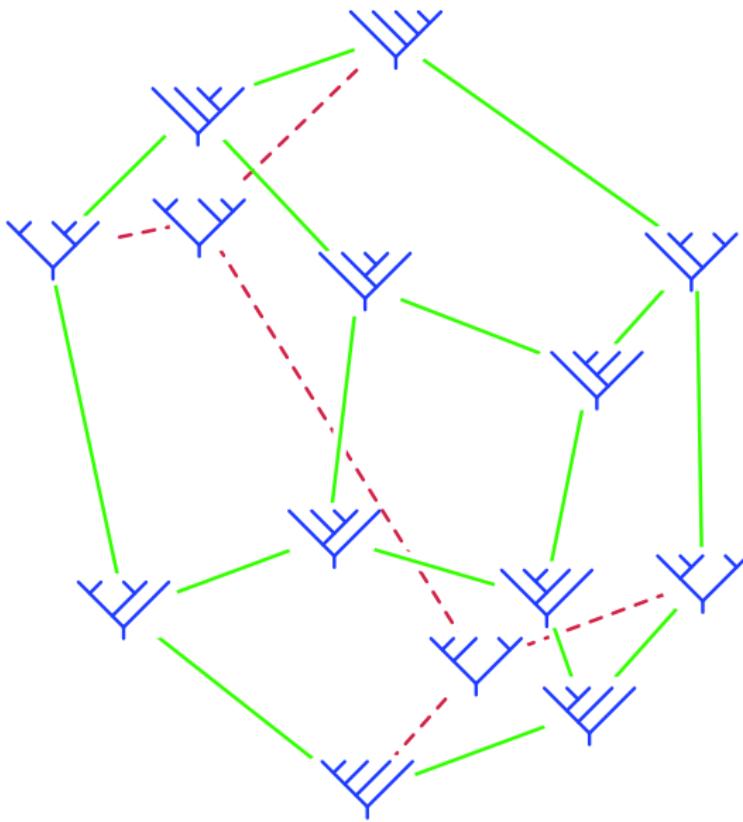
Mandelbrot set.



Mandelbrot set.



Back to binary trees.



Trees

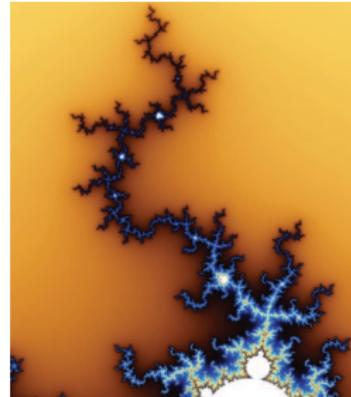
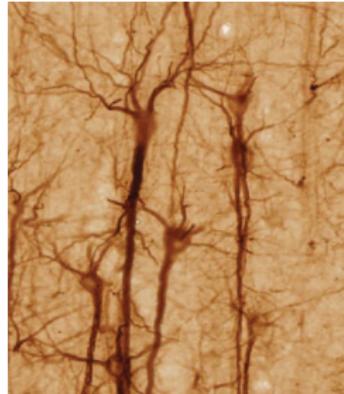


Image credits: Wikimedia.

Trees



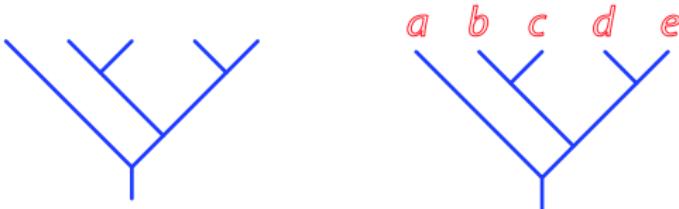
Image credits: Google Earth, Wikimedia.

Trees

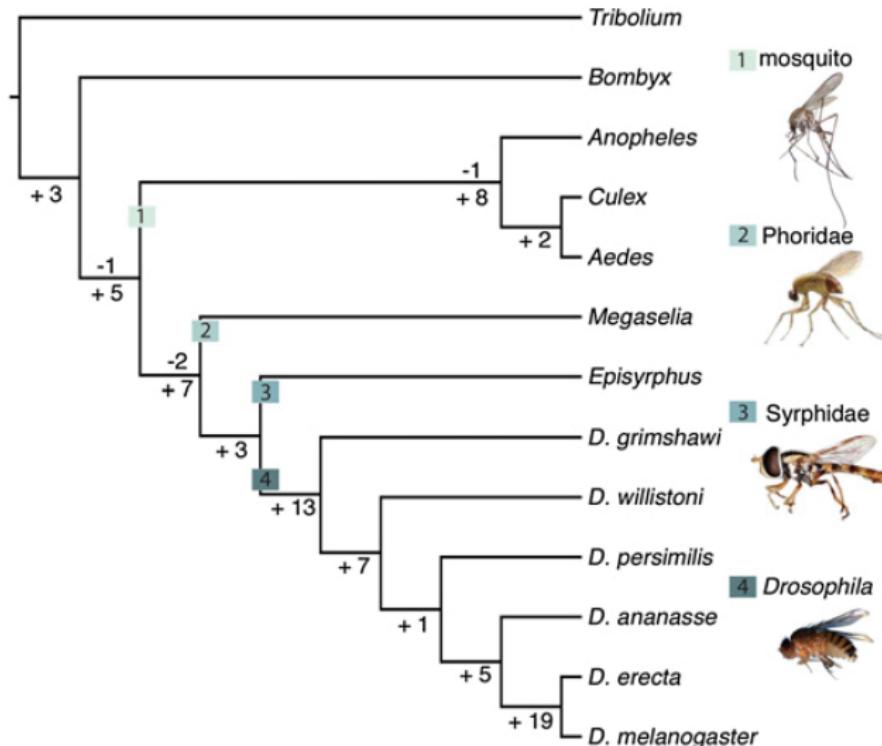


Image credits: Google Earth, Wikimedia (you may guess which!).

Labeled tree.

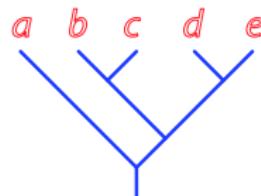


Trees



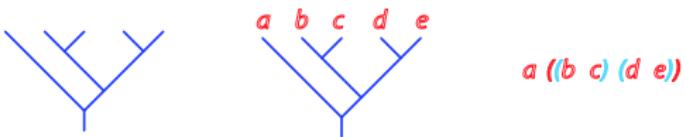
Episodic radiations in the fly tree of life, Wiegmann et.al. PNAS 2011

Bracketed word.

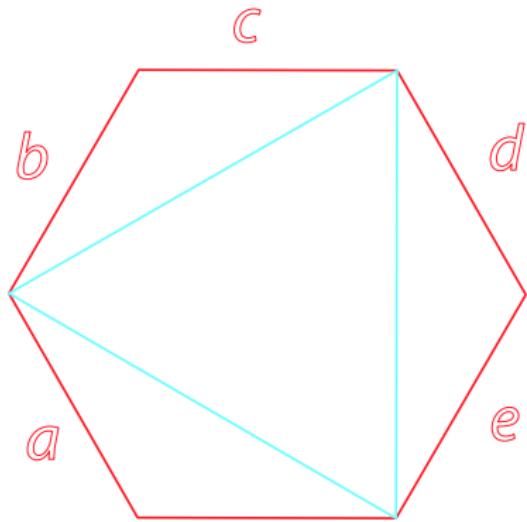


$a ((b \ c) (d \ e))$

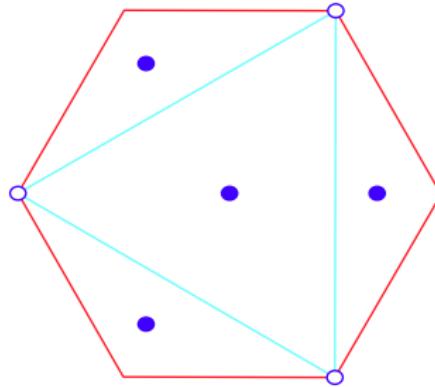
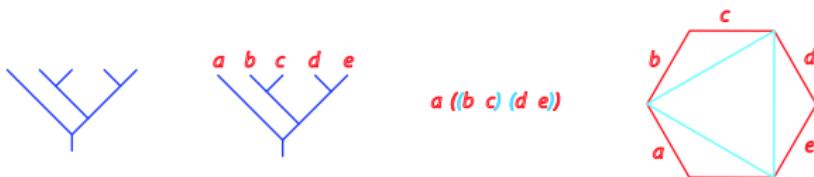
Triangulated polygon.



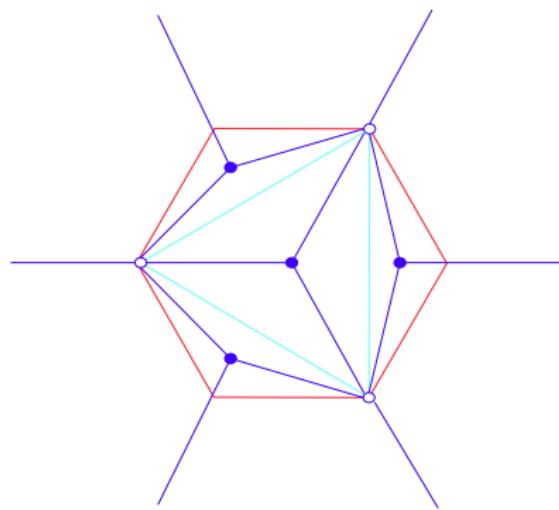
$a ((b\ c)\ (d\ e))$



Triangulated polygon.

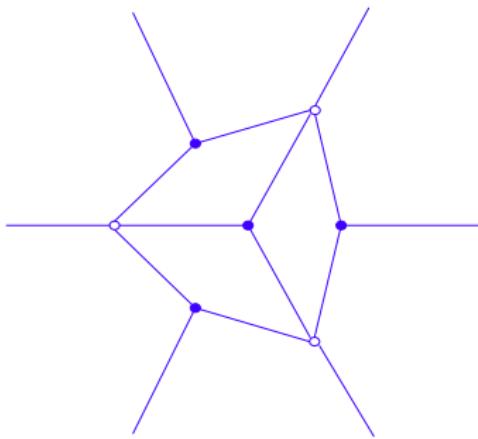
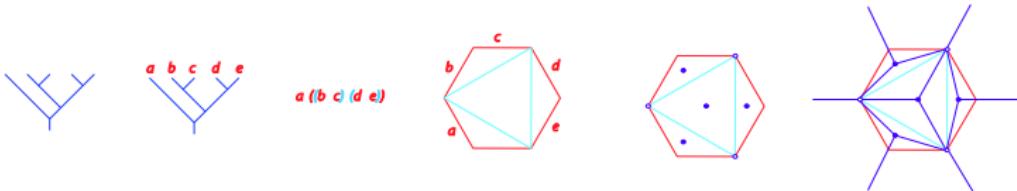


Graph.

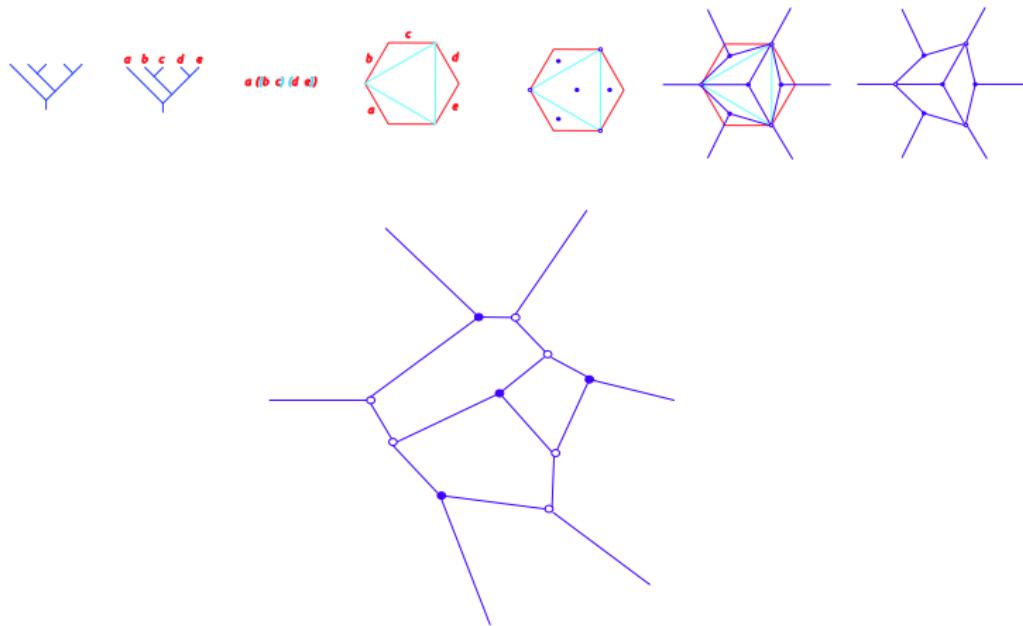


Kodamaa and Williams, KP solitons, total positivity, and cluster algebras,
PNAS 2011

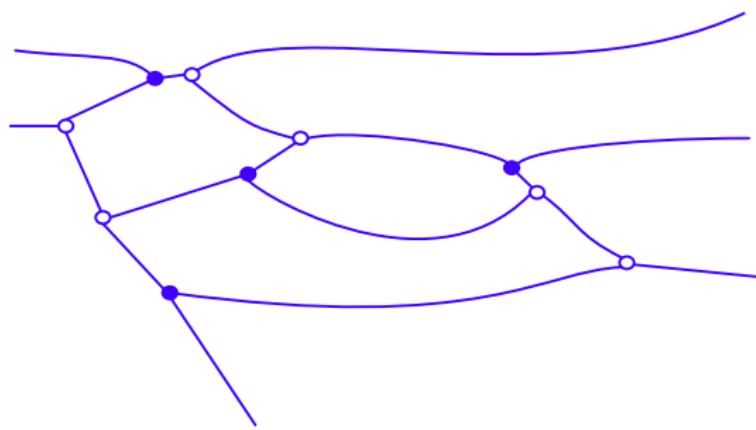
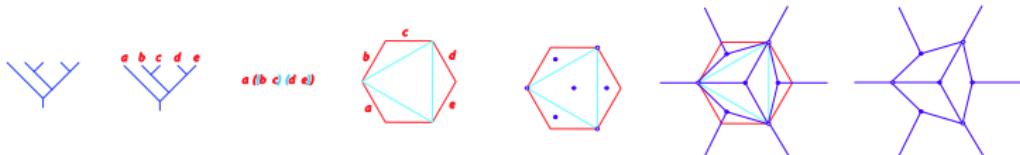
Graph.



Colliding waves.



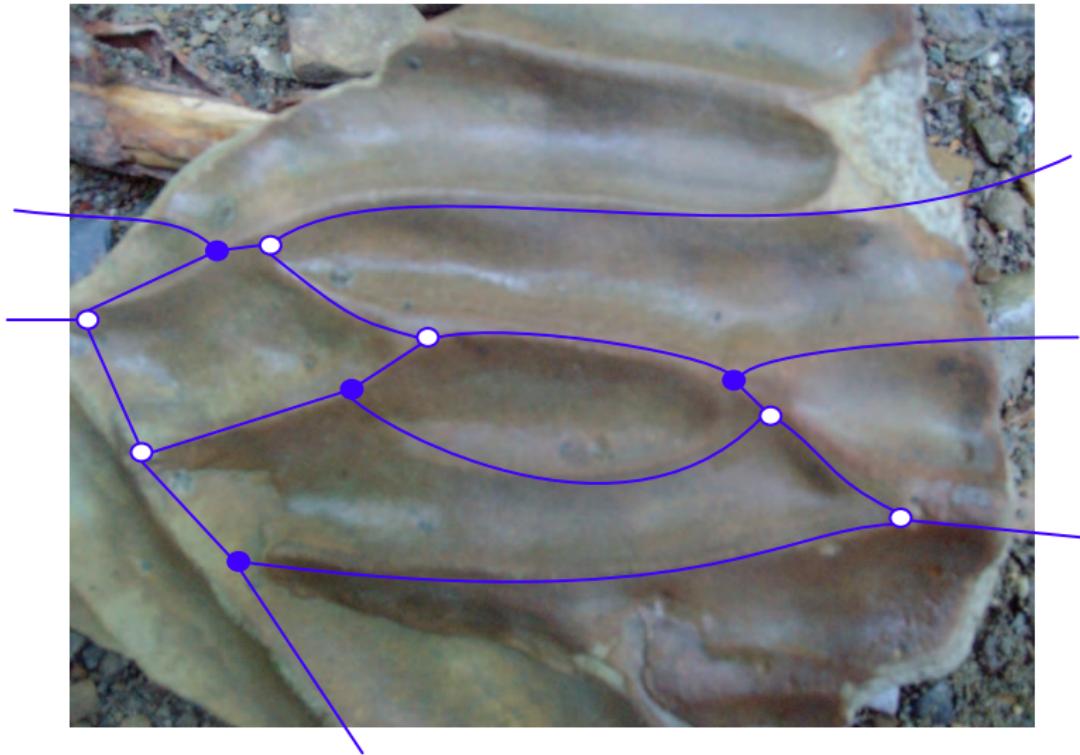
Colliding waves.



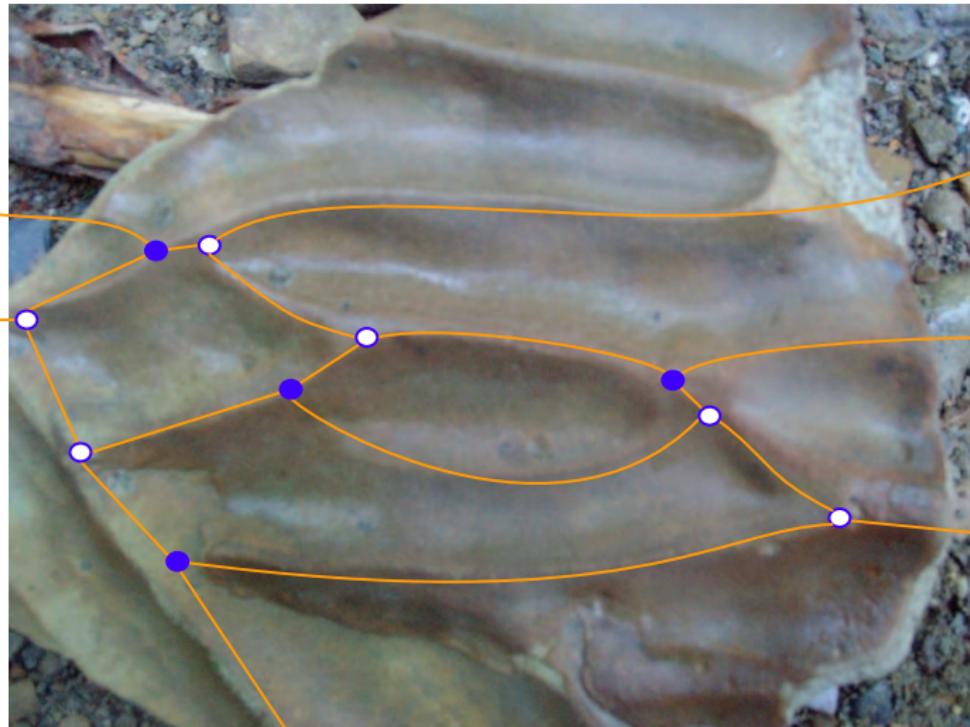
Colliding waves.



Colliding waves.



Colliding waves.



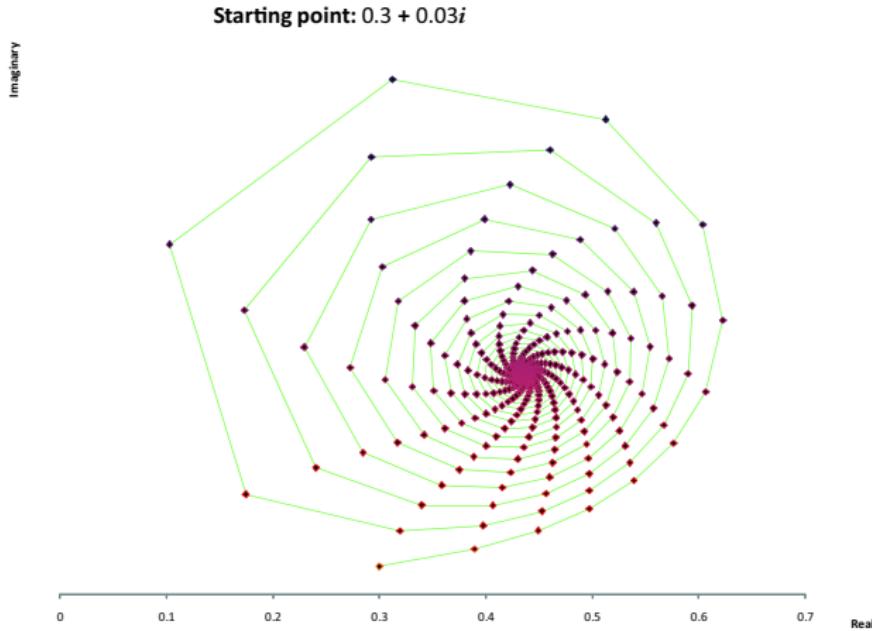
Colliding waves.



Colliding waves.



Thanks!



There are many more patterns to be discovered than there are already known...