Calculus I. Summer '17 Test 3 Review.
Make sure you also study all the quizzes, then notes and homework examples!

Overview of Derivatives

Power Rule: $y = x^2$, $7x^{-3}$, $\sqrt[5]{x^7}$, $x^{\sqrt{3}}$.

Trig: $y = \sin x$, $\cos x$, $\tan x$, $\sec x$, $\csc x$, $\cot x$, $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$.

Hyperbolic Trig: $y = \sinh x$, $\cosh x$, $\tanh x$.

Exponential: $y = e^x$, 3^x, $(\ln 2)^x$.

Logs: $y = \ln x$, $\log_5 x$, $\log_{2\pi} x$.

1. Find critical numbers.
a) $f(x) = x^{(4/5)}(x-4)^2$

b) $f(x) = x^2 e^{-3x}$

c) $f(x) = x^{-2} \ln x$

d) (Given the derivative.) $f'(x) = \dfrac{x^3+8}{e^x - 1}$

2. Find local min and/or max. Use the second derivative test.
a) $f(x) = x^2 \ln x$

b) $f(x) = e^{2x} + e^{-x}$

c) $f(x) = x^4 e^{-x}$

d) $f(x) = x + 2\sin x, \quad 0 \le x \le 2\pi$

3. Find absolute min and max on the given interval.

a) $f(x) = x - \ln x, \quad [\frac{1}{2}, 2]$

b) $f(x) = x + \frac{1}{x}, \quad [\frac{1}{5}, 4]$

4. Find all inflection points.

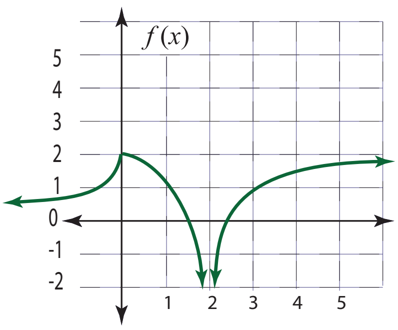
a) $f(x) = x^4$.

b) $f(x) = x^5 - x^4$.

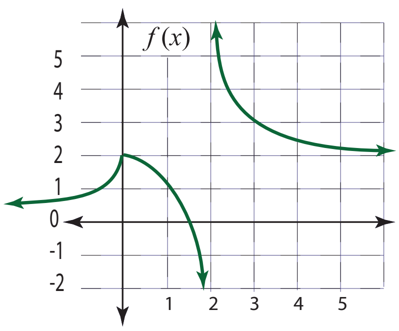
c) $f(x) = x^4 - 4x^3$.

5. Describe using graph. May be matching.

For each of these 6 graphs: use the graph to describe: local min and max, intervals where f is increasing or decreasing, where f is concave up or down, where f' is positive or negative, where f'' is positive or negative, and the points of inflection.



A.



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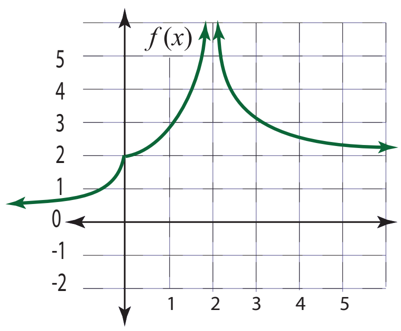

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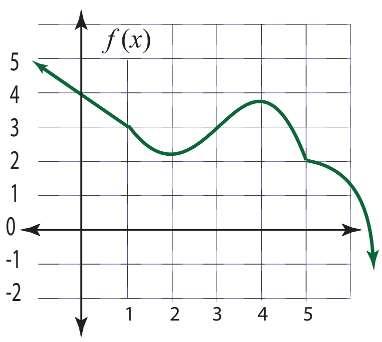

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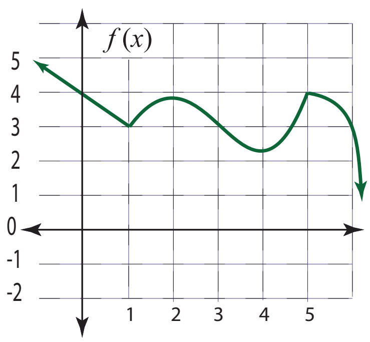

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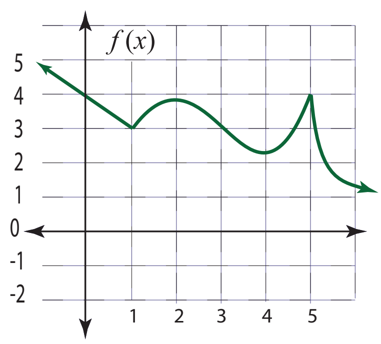

C.

6. Find limits.

a) $\lim_{x\to 2} \frac{e^{(x^3)} - e^{(4x)}}{x-2}$

b) $\lim_{x\to 2} \frac{\sin x}{(x-2)^4}$

7. Find the area of the largest rectangle made by using 50 ft of fencing but leaving an opening on one side that is exactly one third of the width of the rectangle on that side.

8. Find the minimum length of fencing needed to make a rectangle that has only three sides fenced (the last side is left open, along a river) if the area must be 10 square feet.

9. Find the maximum volume of an open-top cylindrical can that uses 5 square feet of tin.

10. Antiderivatives.

a) $\int (x^4 + \sin x) dx$

b) $\int \left(\frac{\sqrt{x}+1}{x}\right) dx$

c) $\int \left(\frac{2e^x+1}{2}\right) dx$

d) Find the definite integral: $\int_0^\pi \sin x \cos^3 x dx$.

e) Find the area under the curve $y = x\sqrt{9+x^2}$ from $x = 0$ to $x = 4$.

f) Find the area under the curve $y = 2^x\sqrt{8+2^x}$ from $x = 0$ to $x = 3$.