Linear. Quiz 6. Name _____ Time_____

(1) Consider the following linear transformation $T : \mathcal{P}^2 \rightarrow \mathcal{P}^3$, given by $T(f) = 4x^2 f'$. That is, take the first derivative and then multiply by $4x^2$.

(a) Find the matrix for $T$ with respect to the standard bases of $\mathcal{P}^n$: that is, find $[T]_{\mathcal{E}}^{\mathcal{E}}$, where $\mathcal{E} = \{1, x, x^2, ..., x^n\}$.

(b) Find $N(T)$ and $R(T)$. You can either work with polynomials or with their coordinate vectors with respect to the standard basis. Write the answers as spans of polynomials.

(c) Find the the matrix for $T$ with respect to the alternate bases: $[T]_{\mathcal{A}}^{\mathcal{B}}$, where $\mathcal{A} = \{x - 1, x, x^2 + 1\}$, $\mathcal{B} = \{x^3, x, x^2, 1\}$.

(2) Consider the following linear transformation $T : \mathcal{P}^2 \rightarrow \mathcal{P}^3$, given by $T(f) = \frac{d}{dx}(4x^2 f)$. That is, multiply by $4x^2$ and then take the first derivative.

(a) Find the matrix for $T$ with respect to the standard bases of $\mathcal{P}^n$: that is, find $[T]_{\mathcal{E}}^{\mathcal{E}}$, where $\mathcal{E} = \{1, x, x^2, ..., x^n\}$.

(b) Find the matrix for $S(f) = 8xf$, again with the standard bases. Check that your answer to 2a equals the matrix from 1a plus the matrix for $S$. Why should that be?