

Fall 2016 Topics in Algebraic Topology 3450:589

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OFFICE HOURS: MTuWF 1:15-2:15pm, and lots more by appointment!

TEXTS:

[PS] Hatcher, Point-Set Topology:

<http://www.math.cornell.edu/~hatcher/Top/Topdownloads.html>

[AT] Hatcher, Algebraic Topology:

<http://www.math.cornell.edu/~hatcher/AT/ATpage.html>

BIBLIOGRAPHY:

Prasolov, V.V., *Intuitive Topology* A.M.S.

Weeks, Jeffrey. *The Shape of Space*.

Website for schedule, homework problems and announcements:

http://www.math.uakron.edu/~sf34/class_home/topo/topos17.htm

GRADING POLICY:

400 pts: Team Moderator (1st time, weeks 2-7)

900 pts. guarantees an A

400 pts: Team Moderator (2nd time, weeks 8-12)

800 pts. guarantees a B

200 pts: Attendance, or turn in HW.

700 pts. guarantees a C

600 pts. guarantees a D

(+,- at my discretion)

Course Outline:

Jan. 18 Category of topological spaces and continuous maps.

Reading [PS] Chapter 1,2 pgs 1-20.

1) _____ Given the spaces

$X = \{1,2,3,4,5,6,7\}$; $\mathcal{C}_X = \{\{\}, X, \{1\}, \{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}, \{1,2,3,7\}\}$ and

$Y = \{y \text{ in } \mathbf{N} \mid 0 < y < 15\}$;

$\mathcal{C}_Y = \{\{\}, Y, \{2\}, \{5\}, \{2,5\}, \{2,4,5\}, \{1,2,5,7\}, \{1,2,4,5,7\}, \{1,2,4,5,6,7\}, \{1,2,4,5,6,7,10\}\}$;

and $Z = \{2,4,6,8\}$; $\mathcal{C}_Z = \{\{\}, Z, \{2\}, \{4,6\}, \{6,8\}\}$.

Which pair A , \mathcal{C}_A is not a topology? Find bases for the two topological spaces. Is the function $f = 2x$ continuous from X to Y ? Is the function g from the unit interval given by $g(t) = 2$ for $t < 1/2$, and $g(t) = 4$ otherwise, a path in X ? Is that same function a path in Y ?

Pick 2 from weeks 2-7, and another 2 topics from weeks 8-12. Email me!

Jan. 25 Reading [PS] Chapter 2 pgs 20-28.

- 2) _____ Connectivity, path connectivity.

Is the space X , \mathcal{O}_X from the previous week path connected? Is it connected? Describe a specific space that is connected but not path connected. Describe a specific space that is connected but for which it is unknown whether it is path connected. For what topologies on the set of vertices of a graph are the edges of the graph connected as subspaces?

Feb 1 Metric spaces. Reading [PS] pgs 9-10. Also see the links below.

- 3) _____ [Mackenzie Jones](#) _____ Draw the unit balls in the taxicab metric (see the wiki: https://en.wikipedia.org/wiki/Taxicab_geometry) and the bus metric (also known as the British Rail metric, on page 94 here: https://www.math.ucdavis.edu/~hunter/m125a/intro_analysis_ch7.pdf). Show that a continuous function from Calculus 1 is a continuous function in the standard topology of the real line.

Feb 8 Gluing and cutting spaces: Quotient Spaces: Reading [PS] pgs 20-21

- 4) _____ [Haley Innes, De Airre Haines](#) _____ ENUMERATE THE CUT POINTS, NON-CUT-POINTS, AND THE HOMEOMORPHISM CLASSES OF THE LETTERS IN THIS SENTENCE.

Feb 15 Reading [PS] pgs 44-51

- 5) _____ [Brandon Pipher, Brandon Martin](#) _____ Surfaces: Figure out what you get by gluing some surfaces by hand--start with polygons and identify edges. [Link to worksheet: http://www.math.uakron.edu/~sf34/class_home/topo/gluing.pdf .]

Feb 22 Reading [AT] pgs 1-10

- 6) _____ [Andrew Mayer](#) _____ Euler characteristic, genus: Find the Euler characteristic of a genus- n surface with k distinct boundary components. Find the Euler characteristic of an n -dimensional polytope.

March 1 Homotopy groups. Reading [AT] pgs 21-39

- 7) _____ [Christopher Benson](#) _____ Find the fundamental groups of the circle, sphere, torus, Klein bottle, two-holed torus, punctured torus and thrice punctured sphere.

March 8 Reading [AT] pgs 40-55

- 8) _____ [Mackenzie Jones](#) _____ Figure out the fundamental groups for some knot complements: the Knot Groups for the trefoil and the figure eight, and one more.

March 15 Reading [AT] pgs 339-340

- 9) ____ [Brandon Pipher, Brandon Martin](#) ____ Higher homotopy: What is the definition of the homotopy group $\pi_2(X, x_0)$. Define $\pi_n(X, x_0)$. Show that $\pi_2(X, x_0)$ is abelian. What is the open problem of homotopy for the spheres?

March 22 Homology groups. Reading [AT] pgs 95-107

- 10) ____ [Andrew Mayer](#) ____ Find the homology groups of the circle, sphere, torus, Klein bottle and projective plane.

April 5 Reading [AT] pgs 137-143, pgs 5-8

- 11) ____ [Haley Innes](#) _____ Use Simplicial and Cellular complexes, simplicial or cellular homology to calculate homology groups. Find the homology groups of the two-holed torus, punctured torus, thrice punctured sphere and 3-torus.

April 12 Reading [AT] pgs 149-159

- 12) _____ Calculate (maybe using Mayer-Vietoris) the real homology sequences of: the solid 3d-ball with a toroidal hole (missing a torus-shaped solid), the solid torus with a missing 3-ball, and the solid torus with a toroidal hole.

April 19 Polytopes from topologies. Reading [Graph Associahedra](#) pgs 2-5

- 13) ____ [De Airre Haines](#) _____ Define graph associahedra in terms of topological bases on nodes in a path graph.

April 26 Spaces of Trees. Reading [Tree spaces](#) pgs 9-24

- 14) _____ Define Billera Holmes Vogt (BHV) space, Balanced Minimal Evolution (BME) polytope

May 3 Spaces of Networks. Reading [Network spaces](#) pgs 2-13

- 15) ____ [Christopher Benson](#) _____ Define the spaces of networks \mathcal{CSN}_n and \mathfrak{S}_n

Phylogenetics. Facets of the BME polytope and cells of \mathcal{S}_n