Notation : for whole number n

$n! = n(n-1)(n-2)\cdots 2\cdot 1$

$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$

$1! = 1$

by def: $0! = 1$

$6! = 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 720$

Notice: $n! = n\,(n-1)!$

$n! = {}_nP_n$

$7! =$ number of 7-digit PIN's with no repeating digits using only digits $1, 2, \ldots, 7$.

Division principle.

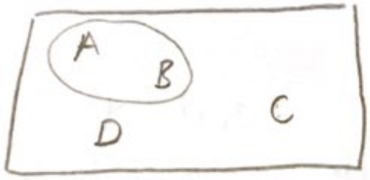
Idea: [count the number of structures (PINs, pizzas, ice cream cones, olympic podiums)]

→ Some times, when using the multiplication principle (decisions, options) we count the number of ways to construct something but we really only want to count the final construction.

→ If each construction has the same number of ways, m, to construct it, then we can count them all, but then divide by m.

Ex: How many ways can you choose a committee of 2 people out of 4?

Construct: $\underset{4}{\_\_}\ \underset{3}{\_\_}\ \Big\}\ 4\cdot 3 = 12$ | But each construction can be done 2 ways: Answer $\frac{12}{2} = \boxed{6}$

Picture 

People $\{A, B, C, D\}$

Committees:

$\{A, B\}$ same as $\{B, A\}$
$\{A, C\}$ same as $\{C, A\}$
$\{A, D\}$ same as $\{D, A\}$
$\{B, C\}$ same as $\{C, B\}$
$\{B, D\}$ same as $\{D, B\}$
$\{C, D\}$ same as $\{D, C\}$

Ex: How many subsets of size 2 are there of $U = \{1, \dots, 10\}$?

Ans: $\dfrac{10 \cdot 9}{2} = \boxed{45}$

Ex: How many subsets of size 3 are there of $U = \{1, \dots, 10\}$?

Ans $\dfrac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}$

$\boxed{= 120}$

For a given subset $\{3, 9, 2\}$
We can build it $3 \cdot 2 \cdot 1$ ways → those are the orders of picking the three numbers

3 9 2
3 2 9
9 2 3
9 3 2
2 3 9
2 9 3

Notation: the number of subsets of size m from a set of size n (like $\{1, \dots, n\}$)

is $$\boxed{\frac{n(n-1)(n-2)\cdots(n-m+1)}{m(m-1)(m-2)\cdots 1} = \frac{{}_nP_m}{m!}}$$

Def: The subsets of size m are called $\boxed{\text{m-combinations.}}$

Shorthand:
Def: $$\boxed{\frac{{}_nP_m}{n!} = \binom{n}{m} = {}_nC_m = \text{"n choose m"}}$$

Handy: multiply on top and bottom by $(n-m)!$

to get $$\frac{{}_nP_m\,(n-m)!}{m!\,(n-m)!} = \frac{n!}{m!\,(n-m)!}$$

Summary: For $U = \{1, 2, \dots, n\}$; Then:

The number of subsets of size m

= the number of m-combinations of n

= n choose m

$= {}_nC_m$

$= \binom{n}{m}$

$= \dfrac{n(n-1)\cdots(n-m+1)}{m(m-1)\cdots 1}$

$= \dfrac{{}_nP_m}{m!}$

$= \dfrac{n!}{m!\,(n-m)!}$