Calculus III. Test 2 Review

I.

$$\text{Let } f(x,y) = \frac{\ln x}{y} + x.$$

Let $g(x,y) = x^3 + y^3 - 3xy + 4$. Let $h(x,y) = x\sin(\sin y)$.

1. __________ Consider the curve $\mathbf{r}(t) = \langle 3t+1, \sin t\rangle$ under $z = g(x,y)$. Find the value of $\frac{dz}{dt}$ at $t = 0$.

   What is the partial derivative $g_x(1,0)$? ________________________________.

2. __________ Find the directional derivative of $g(x,y)$ over the point $(\sqrt{3}, 0)$ in the direction of $\theta = \frac{\pi}{3}$.

   The vector $\nabla g(\sqrt{3}, 0)$= ________________________________.

3. __________ Find the z-value of the local minimum of $g(x,y)$.

   The value of D at this point is ________________________________.

4. __________ Find the tangent plane to the point $(1, \pi, 0)$ on $h(x,y)$.

   The normal vector of this tangent plane is ________________________________.

5. __________ Find the maximum rate of increase in $f(x,y)$ over the point $(x,y) = (1,2)$.

   The 2d vector showing the direction of that greatest increase is ________________________________.

6. __________ Approximate $f(\frac{2}{3}, 2.01)$ using the linearization of f near (1,2).

   The normal vector of the tangent plane to $f(x,y)$ at $(1,2)$ = ________________________________.

7. __________ Find the z-value of the point on the surface $z = f(x,y)$ which has a horizontal tangent plane (find the critical point).

   Is this point locally a min, max, saddle, or inconclusive? ________________________________.

II. Given

$z = f(x, y)$ is a surface	$g(x, y)$ is a surface	$\mathbf{r}(t) = \langle t^2 - 1, t \rangle$
$f(0, 1) = 0$	$g(1, 1) = 2$	
$f_x(0, 1) = 5$	$g_x(1, 1) = 0 = g_y(1, 1)$	
$f_y(0, 1) = -2$	$g_{xx}(1, 1) = 7$ and $g_{yy} = 2$	
	$g_{xy} = -3$	

1. Find the 2d direction vector of max increase for $z = f(x, y)$ over $(x, y) = (0, 1)$.

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2. Find the directional derivative of f over $(0, 1)$ in the direction of $\langle 4, 6 \rangle$.

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3. Find the largest rate of decrease for f over $(0, 1)$.

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4. Find the tangent plane equation for $g(x, y)$ over $(x, y) = (1, 1)$.

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5. Find whether the point on g over $(1, 1)$ is a local max, local min, saddle or inconclusive.
6. Find the instant rate of change in z with respect to x at (0,1) where y is held constant.
7. Find the instant rate of change in z with respect to t at $t = 1$ where (x, y) is constrained to the curve $\mathbf{r}(t)$.

III.

1.

$$\text{Let } f(x,y) = e^y(y^2 - x^2), \text{ so that } f_x = -2xe^y \text{ and } f_y = e^y(y^2 + 2y - x^2).$$

Find the critical points and classify them using the Second Derivative Test.

2.

$$\text{Given} \qquad f(x,y) = x^2 3^y + y^2 - 2y$$

The point on f over $(0,1)$ has a horizontal tangent plane. Find D and decide: is this point a local min, max, saddle or inconclusive?

3.

$$\text{Given} \qquad f(x,y) = \ln(\cos x + y) + x$$

Use linearization over $(x,y) = (\frac{\pi}{2}, 1)$ to find $L(1.5, \frac{\pi}{2})$, which is the approximation of $f(1.5, \frac{\pi}{2})$.

4. Use Lagrange Multipliers to find the local min and max of e^{xy} on the curve $x^2 + y = 12$. You may assume that every solution you find is a local extremum.

5. Use Lagrange Multipliers to find the absolute min and max of $3^{(x+y)}$ on the curve $x^2 + 3y^2 = 12$.

6. Integrate the function $z = 4xy$ over the triangle with vertices $(0,0)$, $(2,0)$, and $(2,1)$.
You may use any set-up you like. Put a box around your set-up and final answer.

7. Find the volume under the surface $z = 4 - y^2$, above the $z = 0$ plane, and between the planes $x = 0$ and $x = 1$.
You may use any set-up you like. Put a box around your set-up and final answer.

8. Find the integral $\displaystyle\int_0^1 \int_{2y}^2 2e^x \, dxdy$.

9. Find the integral $\displaystyle\int_0^1 \int_{2x}^2 e^{y^2} \, dydx$.

10. Given that a surface $z = f(x,y)$ has a tangent plane $6x + 2z - 2y = 4$ at the point $(1,2)$, find the gradient of f and the directional derivative in the direction $< 4, -3 >$.

11. Study the quizzes and the homework problems! These are good test questions too.