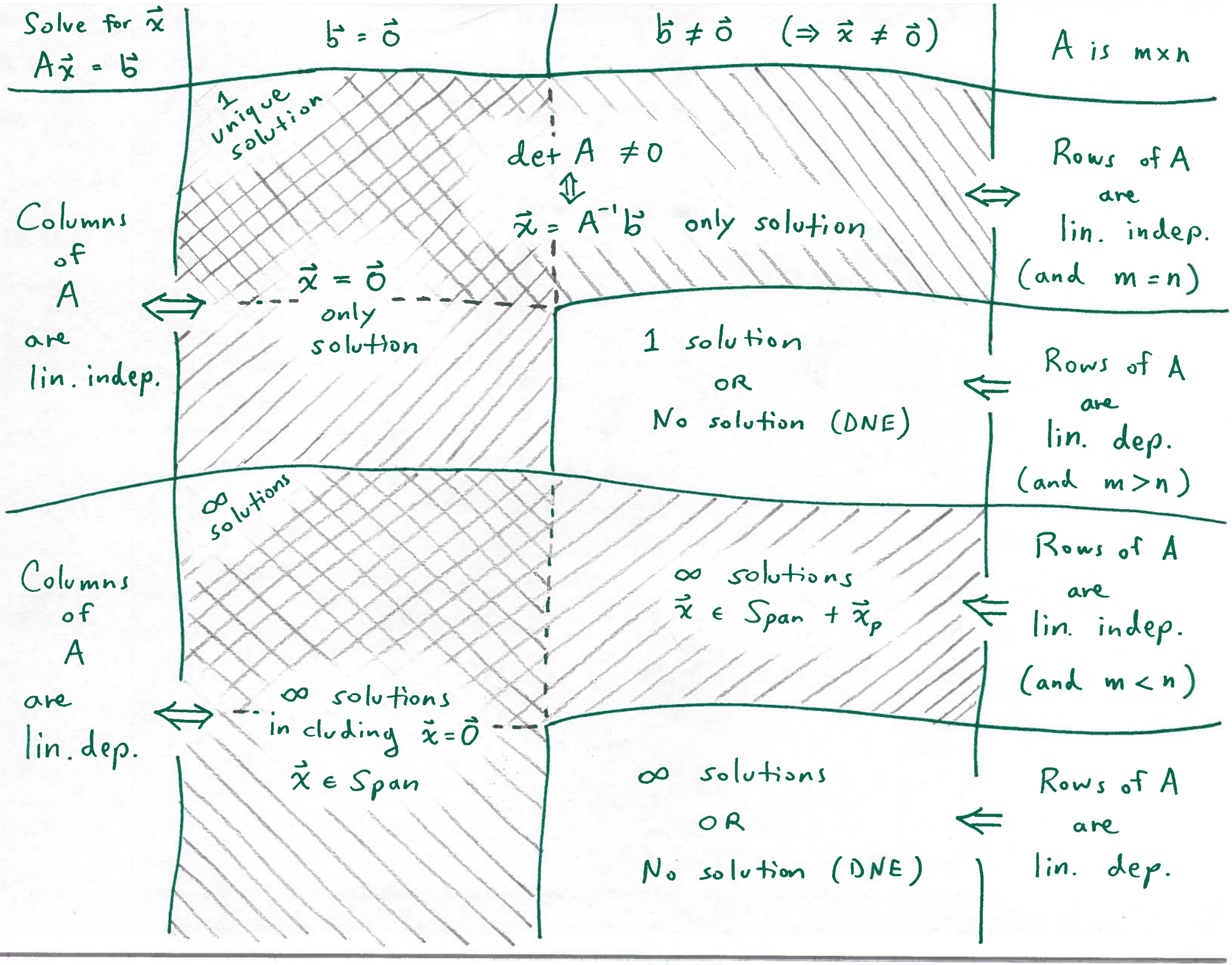
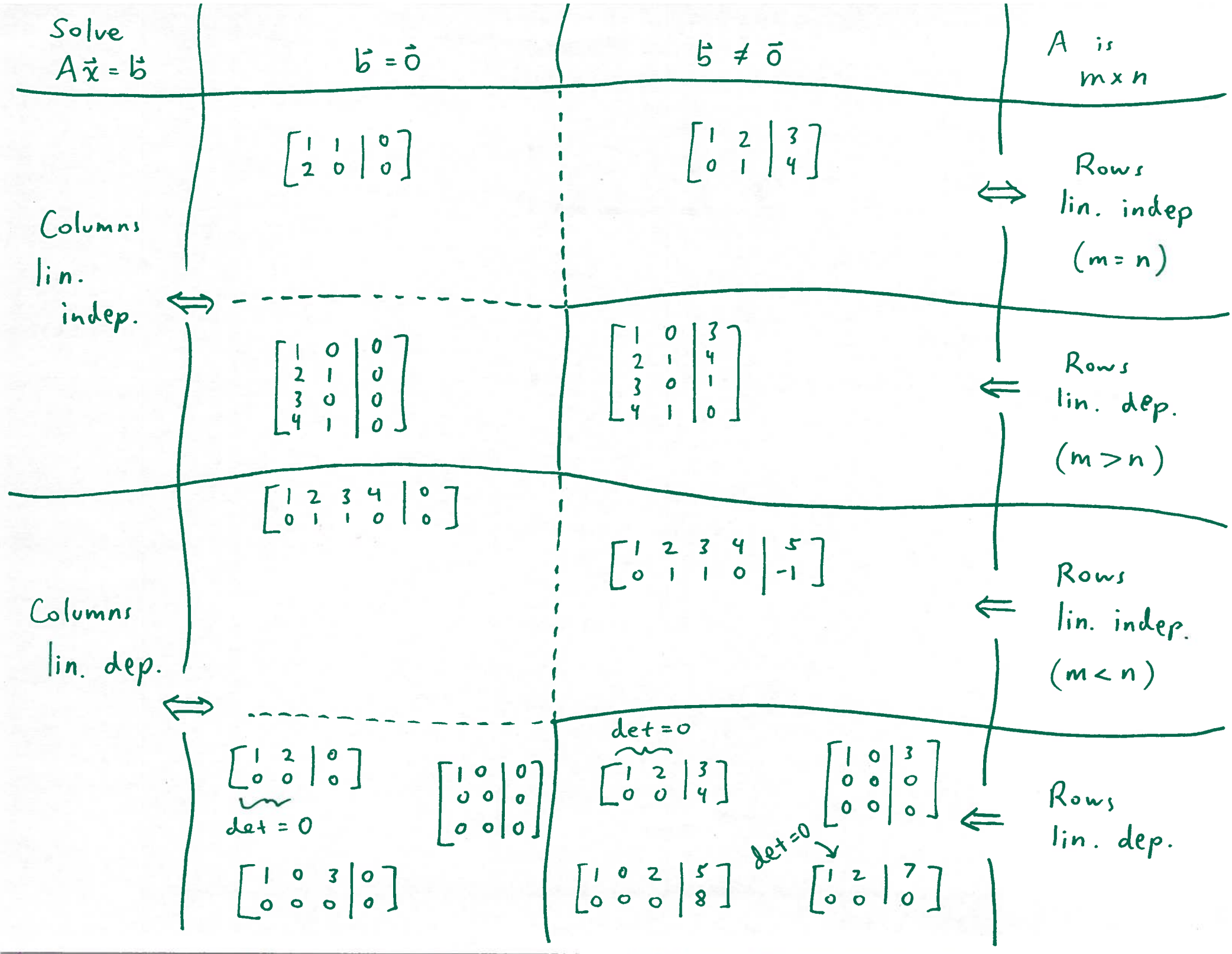

Solve for $\vec{x}$
$A\vec{x} = \vec{b}$
$\vec{b} = \vec{0}$
$\vec{b} \neq \vec{0}$ $(\Rightarrow \vec{x} \neq \vec{0})$
A is m x n
1 unique solution
$\det A \neq 0$
$\vec{x} = A^{-1}\vec{b}$ only solution
Rows of A are lin. indep. (and $m = n$)
Columns of A are lin. indep.
$\vec{x} = \vec{0}$ only solution
1 solution OR No solution (DNE)
Rows of A are lin. dep. (and $m > n$)
∞ solutions
∞ solutions $\vec{x} \in$ Span + $\vec{x}_p$
Rows of A are lin. indep. (and $m < n$)
Columns of A are lin. dep.
∞ solutions including $\vec{x} = \vec{0}$
$\vec{x} \in$ Span
∞ solutions OR No solution (DNE)
Rows of A are lin. dep.


Solve
$A\vec{x} = \vec{b}$
$\vec{b} = \vec{0}$
$\vec{b} \neq \vec{0}$
A is
m x n
$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix}$
$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix}$
Rows
lin. indep
(m = n)
Columns
lin.
indep.
$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 0 \\ 4 & 1 & 0 \end{bmatrix}$
$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 1 \\ 4 & 1 & 0 \end{bmatrix}$
Rows
lin. dep.
(m > n)
$\begin{bmatrix} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$
$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 1 & 0 & -1 \end{bmatrix}$
Rows
lin. indep.
(m < n)
Columns
lin. dep.
$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
det = 0
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
det = 0
$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$
$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
Rows
lin. dep.
$\begin{bmatrix} 1 & 0 & 2 & 5 \\ 0 & 0 & 0 & 8 \end{bmatrix}$
det = 0
$\begin{bmatrix} 1 & 2 & 7 \\ 0 & 0 & 0 \end{bmatrix}$

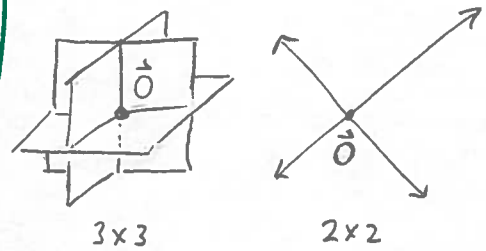
Picture $A\vec{x} = \vec{b}$

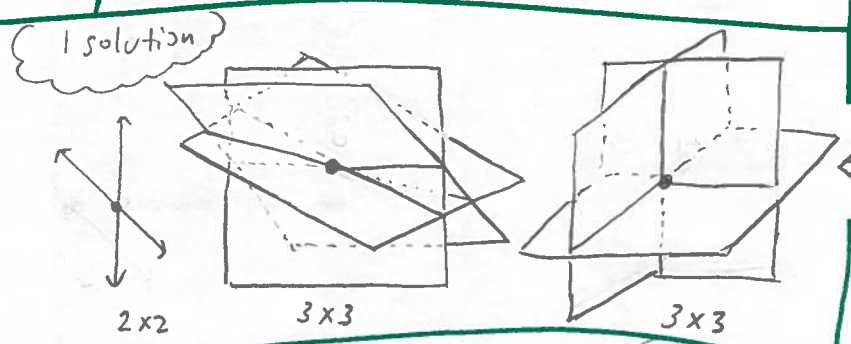
$\vec{b} = \vec{0}$

$\vec{b} \neq \vec{0}$

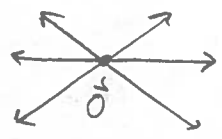
A mxn

Columns A lin. indep. $\Leftrightarrow$

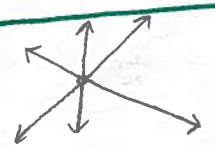




$\Leftrightarrow$ Rows lin. indep. (m=n)

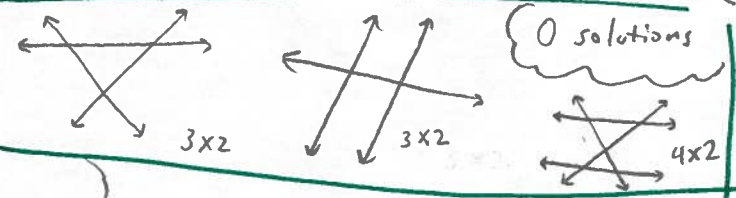


1 solution

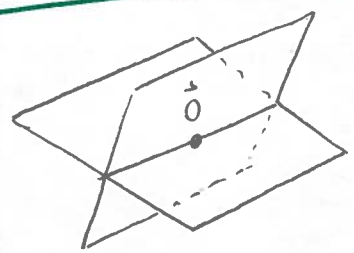


1 solution

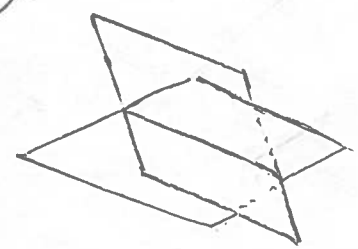
$\Leftarrow$ Rows lin. dep $m > n$



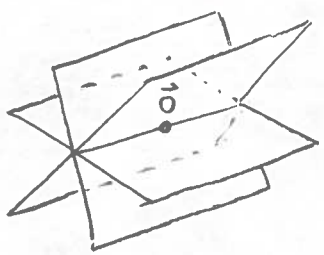
Columns lin dep. $\Leftrightarrow$



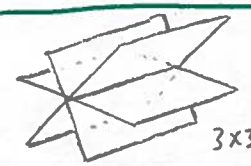
∞ solutions



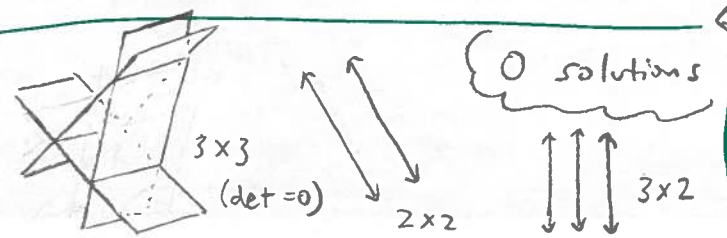
$\Leftarrow$ Rows lin. indep. $m < n$



∞ solutions



∞ solutions



$\Leftarrow$ Rows lin. dep.