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Calculus 2 Test 2, Spring '21. Pg. 1

My signature here is to pledge that I have answered each test question from my own knowledge and understanding, without giving or receiving any unauthorized help.

Sign:__________________

Name: key

Time:__________________

Date:__________________

Show all your work clearly on the test paper for full/partial credit! Read directions carefully, and put a box around the final answer in each part.

All angles are in radians. Simplify only the basics: adding, multiplying, etc. for constants

1. Show the correct form for a partial fraction decomposition of this function. Don't actually solve for the variables A, B, C, D, E.

$$\frac{7x^2 + 29 + 2x}{(x+1)(3x^2+1)(x-1)^2}$$

$$\boxed{\frac{A}{x+1} + \frac{Bx+C}{3x^2+1} + \frac{D}{x-1} + \frac{E}{(x-1)^2}}$$

2. Find the partial fraction decomposition of $\frac{2}{x(x-5)}$. $= \frac{A}{x} + \frac{B}{x-5}$

$$2 = A(x-5) + Bx$$

$$x = 5 \Rightarrow 2 = 5B$$

$$\boxed{B = \frac{2}{5}}$$

$$2 = (A+B)x - 5A$$

$$2 = -5A$$

$$\boxed{A = \frac{-2}{5}}$$

$$= \boxed{\frac{-2/5}{x} + \frac{2/5}{x-5}}$$

3. For each series, use the limit test for divergence or the alternating series test to decide: [converge, diverge, or inconclusive] Show your work by performing the test.

a) $\sum_{n=1}^{\infty} \frac{(-1)^n e^n}{n}$

$$\lim_{n\to\infty} \frac{e^n}{n} = \frac{\infty}{\infty} \quad \text{L'Hospitals}$$

$$= \lim_{n\to\infty} \frac{e^n}{1} = \boxed{\infty} \Rightarrow \lim_{n\to\infty} \frac{(-1)^n e^n}{n} \left(\boxed{\text{DNE}}\right) \neq 0$$

$\boxed{\text{diverges}}$

b) $\sum_{n=1}^{\infty} \frac{3n}{n^2 - 5n - 34}$

$$\lim_{n\to\infty} \frac{3n}{n^2-5n-34} = \boxed{0}$$

$\boxed{\text{inconclusive}}$

c) $\sum_{n=1}^{\infty} \frac{4n^2+1}{3+n^2e^2}$

$$\lim_{n\to\infty} \frac{4n^2+1}{3+n^2e^2} = \boxed{\frac{4}{e^2}} \neq 0$$

$\approx .5413$

$\boxed{\text{diverges}}$

4. For each series, what does the geometric series test tell us? [not applicable, converge, diverge, or inconclusive] If applicable, show your work by showing the deciding inequality (with r), deciding converge or diverge, and if it converges find the value it converges to.

a) $\sum_{n=1}^{\infty} \frac{4^n}{2(\pi^n)}$ $= \frac{1}{2} \sum_{n=1}^{\infty} \left(\frac{4}{\pi}\right)^n$, $\boxed{\left|\frac{4}{\pi}\right| > 1}$ $\Rightarrow$ $\boxed{\text{diverges}}$

c) $\sum_{n=1}^{\infty} \left(\frac{-1}{2}\right)^n$ $\boxed{\left|\frac{-1}{2}\right| < 1}$ $\Rightarrow$ $\boxed{\text{converges}}$

$$= \frac{-\frac{1}{2}}{1-\left(-\frac{1}{2}\right)} = \frac{-\frac{1}{2}}{\frac{3}{2}} = \boxed{\frac{-1}{3}}$$

5. For this series, use the limit comparison test to decide: [converge, or diverge or inconclusive]. 1) you must limit-compare to the p-series $\sum_{n=1}^{\infty} \frac{1}{n^3}$. 2) Set up the limit and find it. 3) Decide.

$\sum_{n=1}^{\infty} \frac{12n^2 - 5n}{9n^5 + 1}$ $\boxed{\sum_{n=1}^{\infty} \frac{1}{n^3} \text{ converges}}$

$$\boxed{\lim_{n\to\infty} \frac{\frac{12n^2-5n}{9n^5+1}}{\frac{1}{n^3}}} = \lim_{n\to\infty} \frac{12n^2-5n}{9n^5+1} \cdot \frac{n^3}{1}$$

$$= \lim_{n\to\infty} \frac{12n^5 - 5n^4}{9n^5+1}$$

and $0 < \frac{4}{3} < \infty$

$$= \boxed{\frac{12}{9}} = \left(\frac{4}{3}\right) \Rightarrow \boxed{\text{converges}}$$

$$= \left(\frac{1440}{1080}\right)$$

6. For each series, what does the p-series test tell us? [not applicable, converge, diverge, or inconclusive] If applicable, show your work by showing the deciding inequality (with p), and deciding converge or diverge.

a) $\sum_{n=1}^{\infty}\left(\frac{2}{n}\right)^5 = \sum_{n=1}^{\infty} \frac{2^5}{n^5} = 32 \sum_{n=1}^{\infty} \frac{1}{n^5}$

$\boxed{\text{converges}}$: $\boxed{p = 5} > 1$

b) $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}\right)^7 = \sum_{n=1}^{\infty} \frac{1}{n^{7/2}}$ $\boxed{\text{converges}}$ $\boxed{p = \frac{7}{2}} > 1$

7. Use the ratio test. Decide if the sum converges or diverges and show the test (all steps) to explain why.

a) $\sum_{n=1}^{\infty} \frac{n!(5^n)n!}{(2n)!}$

$$\boxed{\lim_{n\to\infty} \left| \frac{(n+1)!\, 5^{n+1} (n+1)!}{(2(n+1))!} \cdot \frac{(2n)!}{n!\, 5^n n!} \right|}$$

$$= \lim_{n\to\infty} \frac{(n+1)\cancel{n!}\, \cancel{5^{n+1}} (n+1)\cancel{n!}\cancel{(2n)!}}{(2n+2)(2n+1)\cancel{(2n)!}\,\cancel{n!}\,\cancel{5^n}\cancel{n!}}$$

$$= \lim_{n\to\infty} \frac{(n+1)(n+1)5}{(2n+2)(2n+1)}$$

$$= \lim_{n\to\infty} \frac{5n^2 + 10n + 5}{4n^2 + 6n + 2}$$

$$= \boxed{\frac{5}{4}} > 1 \Rightarrow \boxed{\text{diverges}}$$