

Calculus II. Review for Test 2.

1. Show the correct form for a partial fraction decomposition of these functions. Don't actually solve for the variables.

a)  $\frac{A}{x^2} + \frac{B}{x} + \frac{C}{x+2}$

b)  $\frac{A}{x-2} + \frac{B}{x+2}$

c)  $\frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

2. Decompose the function into its partial fractions. (Actually solve for the variables.)

a)  $\frac{7/4}{x-1} + \frac{(-7/4)x + 21/4}{x^2 + 3}$

b)  $\frac{1/5}{x-2} + \frac{-1/5}{x+3}$

3. Find the indefinite integrals:

a)  $x + 3 \ln|x| - 2 \ln|1+x| + c$  (Hint: first do long division since the degree of the numerator and denominator are both 2. This will turn the integral into:  $\int(1 + \frac{x+3}{x(x+1)}dx$ .)

b)  $\frac{-4}{3(x+1)} + \frac{11}{9} \ln|x-2| - \frac{11}{9} \ln|x+1| + c$

4. Find these definite integrals and classify as "divergent" or "convergent":

a)  $\frac{1}{2e^9}$  (*converges*)

b)  $-\infty$  (*diverges*) (Hint: it's  $-\infty$  because you take  $\lim_{t \rightarrow 0^-}$ .)

c)  $\frac{32}{3}$  (*converges*)

5. For each of these sequences, find the limits, if they exist, and decide "diverges" or "converges."

a) 0 (*converges*) (Hint:  $(2/3)^n$  goes to zero since  $2/3$  is a fraction smaller than 1.)

b) 0 (*converges*)

c)  $1/7$  (*converges*)

d) *DNE* (*diverges*) (Hint:  $\cos(n\pi) = (-1)^n$  just from looking at the graph of  $\cos x$ .)

e)  $\pi/6$  (*converges*)

f)  $0$  (*converges*)

g) *DNE* (*diverges*)

6. For each series, what does the limit test for divergence tell us? [converge, diverge, or inconclusive] Show your work by performing the test.

a) *diverges* (since limit  $= 1/5$ .)

b) *inconclusive* (since limit  $= 0$ .)

c) *inconclusive* (since limit  $= 0$ .)

7. For each series, what does the geometric series test tell us? [not applicable, converge, diverge, or inconclusive] Show your work, and find the value if it converges.

a) *converges* :  $\frac{3/\pi}{1 - 3/\pi}$

b) *diverges* :  $r = \frac{5}{\sqrt{3}} > 1$

c) *diverges* :  $r = 2 > 1$

d) *converges* :  $-1/3$

e) *converges* :  $3 \frac{1/(e^2)}{1 - (1/(e^2))}$

f) *NA* (Hint: does not apply since there is no way to write as  $r^n$ .)

8. For each series, what does the  $p$ -series test tell us? [not applicable, converge, diverge, or inconclusive] Show your work.
- $NA$
  - diverges*  $p = 0.5 \leq 1$
  - diverges*  $p = 1 \leq 1$
  - converges*  $p = 3 > 1$
9. For each series, what does the integral test tell us? [not applicable, converge, diverge, or inconclusive] Show your work.
- converges* :  $\int_{x=1}^{\infty} \frac{\sqrt{x} + 4}{x^2} dx = 6$
  - diverges* :  $\int_{x=1}^{\infty} \frac{1}{\sqrt{x+1}} dx = \infty$
  - converges* :  $\int_{x=1}^{\infty} \left(\frac{1}{x}\right)^3 dx = 1/2$
  - $NA$  (since not positive)
- Note  $\sum_{n=1}^{\infty} \frac{1}{2^n}$  does converge by the integral test since the integral is  $\int_1^{\infty} 1/(2^x) dx = \frac{1}{2 \ln 2}$ .
- $NA$  (since  $1/\cos^2 x$  is not decreasing)
10. For each series, what does the comparison test tell us? [not applicable, converge, or diverge] Show your work.
- converges* :  $\frac{1}{n^3} \geq \frac{1}{2n^3 + 1}$  (Hint: for comparison test and limit comparison we will always be comparing to a  $p$ -series or geometric series!)
  - converges* :  $\frac{9^n}{10^n} \geq \frac{9^n}{3 + 10^n}$
  - diverges* :  $\frac{6^n}{5^n} \leq \frac{6^n}{-4 + 5^n}$
  - $NA$  (since not positive)

11. For each series, what does the limit comparison test tell us? [not applicable, converge, or diverge] Show your work.

a) *diverges* :  $\lim_{n \rightarrow \infty} \frac{\frac{1}{2n+1}}{\frac{1}{n}} = 1/2$

b) *converges* :  $\lim_{n \rightarrow \infty} \frac{\frac{n+2}{(n+1)^3}}{\frac{1}{n^2}} = 1$

c) *converges* :  $\lim_{n \rightarrow \infty} \frac{\frac{2^n}{5^n - n}}{\frac{2^n}{5^n}} = 1$

12. For each series, use the alternating series test or the limit test for divergence to decide: [converge, or diverge]. Show your work.

a) *converges* :  $\lim_{n \rightarrow \infty} \frac{1}{\ln(n+1)} = 0$

b) *converges* :  $\lim_{n \rightarrow \infty} \frac{1}{4n+1} = 0$

c) *diverges* :  $\lim_{n \rightarrow \infty} \frac{(-1)^n}{e^{-n}} = DNE$  since  $\lim_{n \rightarrow \infty} \frac{1}{e^{-n}} = \infty$

13. Decide if the sums converge or diverge, explain why. If there is a formula for the sum, find the value.

a) *converges* :  $\int_{x=1}^{\infty} x^2 e^{-x^3} dx = \frac{1}{3e}$  (by integral test)

b) *diverges* :  $\lim_{n \rightarrow \infty} e^{2n} = \infty$  (by limit test for divergence)

c) *converges* :  $\frac{2/e^3}{1 - 2/e^3}$  (by geometric series test)

14. For each series, what does the ratio test tell us? [converge absolutely, diverge, or inconclusive]. Show your work by finding the limit.

(a)  $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

A: Limit =  $0 < 1$ , so converges absolutely.

$$(b) \sum_{n=1}^{\infty} \frac{2^{(n^2)}}{(2n)!}$$

A: Limit  $= \infty > 1$ , so diverges.

15. For each series, what does the root test tell us? [converge absolutely, diverge, or inconclusive]. Show your work by finding the limit.

$$(a) \sum_{n=1}^{\infty} \left( \frac{-2n}{3n+1} \right)^n$$

A: Limit  $= 2/3 < 1$ , so converges absolutely.

$$(b) \sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n}} \right)^n$$

A: Limit  $= 0 < 1$ , so converges absolutely.

16. Also study the quizzes, and the homework questions. These are good test questions too!