Calculus I. Summer 17 Test 2 Review (with answers).
Make sure you also study all the quizzes, then notes and homework examples!

Overview of Derivatives

Power Rule: $y = x^2, \quad 7x^{-3}, \quad \sqrt[5]{x^7}, \quad x^{\sqrt{3}}.$

$y' = 2x, \quad -21x^{-4}, \quad \frac{7}{5}x^{(\frac{2}{5})}, \quad \sqrt{3}x^{(\sqrt{3}-1)}.$

Trig: $y = \sin x, \quad \cos x, \quad \tan x, \quad \sec x, \quad \csc x, \quad \cot x, \quad \sin^{-1} x, \quad \cos^{-1} x, \quad \tan^{-1} x.$

$y' = y = \cos x, \quad -\sin x, \quad \sec^2 x, \quad \sec x \tan x, \quad -\csc x \cot x, \quad -\csc^2 x, \quad \frac{1}{\sqrt{1-x^2}}, \quad \frac{-1}{\sqrt{1-x^2}}, \quad \frac{1}{1+x^2}.$

Hyperbolic Trig: $y = \sinh x, \quad \cosh x, \quad \tanh x.$

$y' = \cosh x, \quad \sinh x, \quad \operatorname{sech}^2 x.$

Exponential: $y = e^x, \quad 3^x, \quad (\ln 2)^x.$

$y' = e^x, \quad 3^x \ln 3, \quad (\ln 2)^x \ln(\ln 2).$

Logs: $y = \ln x, \quad \log_5 x, \quad \log_{2\pi} x.$

$y = \frac{1}{x}, \quad \frac{1}{x \ln 5}, \quad \frac{1}{x \ln(2\pi)}.$

Combining functions: sums, products, quotients, compositions. Find $y'$ using implicit differentiation and logarithmic differentiation.

1. Find $y'$. Don't simplify.

a) $y = \dfrac{x^4 - \sqrt{x}}{\sin 3x}$

$$y' = \frac{\sin 3x(4x^3 - \frac{1}{2}x^{(-1/2)}) - (x^4 - \sqrt{x})3\cos 3x}{\sin^2 3x}$$

b) $y = \dfrac{1}{\sqrt[7]{t^5}}$

$$y' = \frac{-5}{7}x^{(-12/7)}$$

c) $y = e^p \cosh^3(2^p)$

$$y' = e^p \cosh^3(2^p) + e^p 3\cosh^2(2^p)\sinh(2^p)2^p \ln 2$$

d) $y = \sec(\log_2(x))$

$$y' = \sec(\log_2(x))\tan(\log_2(x))\frac{1}{x\ln(2)}$$

e) $y = \dfrac{\tan x}{e^x - \sqrt{x}}$

$$y' = \frac{(e^x - \sqrt{x})\sec^2 x - \tan x(e^x - \frac{1}{2}x^{(-1/2)})}{(e^x - \sqrt{x})^2}$$

f) $x3^y = (x+1)y$ Step 1: $3^y + x3^y(\ln 3)y' = y + (x+1)y'$

$$y' = \frac{y - 3^y}{x3^y\ln 3 - x - 1}$$

g) $xy = \csc y$ Step 1: $y + xy' = -(\csc y\cot y)y'$

$$y' = \frac{-y}{x + \csc y\cot y}$$

h) $y = x^{(\frac{5}{x})}$

$$y' = x^{(\frac{5}{x})}\left(\frac{5}{x^2} - \frac{5}{x^2}\ln x\right)$$

i) $y = \sin(x^{(\frac{5}{x})})$

$$y' = \cos\left(x^{(\frac{5}{x})}\right)x^{(\frac{5}{x})}\left(\frac{5}{x^2} - \frac{5}{x^2}\ln x\right)$$

j) $y = \sin^{-1}(2^r)$

$$y' = \frac{1}{\sqrt{1-(2^r)^2}}2^r\ln 2$$

2. Find the tangent slope to $y = \frac{7^x}{\sin(e^x)}$ at $x = 3$.

$$m = \frac{\sin(e^3)7^3\ln 7 - 7^3\cos(e^3)(e^3)}{\sin^2(e^3)}$$

3. Find the tangent line to the curve given by $xy + y = 7^x$ at $(x, y) = (0, 1)$.

$y = ((\ln 7) - 1)x + 1$

4. Find the linearization $L(x)$ to $f(x) = x^3 + 4x$ at $x_1 = 1$. Use it to approximate $f(1.01)$. Also give the differentials $dx$ and $dy$.

$L(x) = 7(x - 1) + 5$
$f(1.01) \approx L(1.01) = 7(0.01) + 5 = 5.07$
$dx = 0.01;\quad dy = 7(0.01) = 0.07$

5. Estimate $\ln(1.01)$ using linearization.

$L(x) = x - 1$
$\ln(1.01) \approx L(1.01) = 0.01$

6. Let the functions $f(x)$ and $g(x)$ be given such that $f(2) = 1,\ \ f'(2) = 3,\ \ g(2) = -1,\ \ g'(2) = 5$.

a) If $y = f(x)g(x) + g(x) - \dfrac{g(x)}{f(x)}$ find the value of the derivative $y'$ at $x = 2$.

$y' = -1.$

b) If $y = \sin(\pi g(x))$ find the value of the derivative $y'$ at $x = 2$.

$y' = -5\pi$

7. A particle is moving along the curve given by $xy = y^2 e^{(x-1)}$. At the point (1,1) the $x$-coordinate is increasing at the rate 5 m/s. Find the rate of change in the $y$-coordinate.

$y' = 0$

8. A light on a 3 ft pole shines on a 1 inch mouse running away at 2 ft/s. How fast is the tip of the mouse shadow moving when it is 4 ft away?

$y' = \frac{72}{35}$ ft/s

9. A cylindrical tank with radius 5 m is being filled at a rate of 3 $m^3$/min. How fast is the height of the water increasing?

$h' = \frac{3}{25\pi}$ m/min