Review Questions: Also study quizzes and homework.

I. Let $\mathbf{a} = \left\langle \frac{1}{2}, -1, 0 \right\rangle$, $\mathbf{b} = \langle 4, 1, -1 \rangle$,

and

$\mathbf{r}(t) = \left\langle e^{2t},\ \ln(t+1),\ t + \sec(t) + 2 \right\rangle$.

1. __________ Find $\text{comp}_{\mathbf{b}}\mathbf{a}$.
2. __________ Find $\cos\theta$ where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.
3. __________ Find the tangent vector to $\mathbf{r}(t)$ at $t = 0$.
4. __________ Find $(\mathbf{b} \times \mathbf{a}) \cdot \mathbf{a}$.
5. __________ Find the unit tangent to $\mathbf{r}(t)$ at $t = 0$.
6. __________ Find $\frac{1}{2}\mathbf{b} - 2\mathbf{a}$.
7. __________ Find a vector parallel to $\mathbf{a}$ but twice as long as $\mathbf{a}$.
8. __________ Find the area of the parallelogram with sides the vectors $\mathbf{a}$ and $\mathbf{b}$.

II. Let $P = (0, -1, 2)$, $Q = (2, 1, -1)$,

and

$\mathbf{r}(t) = \left\langle \cos(1 - e^t),\ t\ln(1-t),\ t^2 + 2t \right\rangle$.

1. __________ Find symmetric equations for the line through points P and Q
   The vector $\overrightarrow{PQ} =$________________________________.
2. __________ Find parametric equations for the tangent line to $\mathbf{r}(t)$ at $t = 0$.
   The vector $\mathbf{r}'(0) =$ ________________________________.
3. __________ Find parametric equations for the line through P and perpendicular to the plane $7 - 3z = 0$.
   The normal vector of the plane $7 - 3z = 0$ is: ________________________________.
4. __________ Find the plane containing P and perpendicular to $\overrightarrow{QP}$.
   The normal vector $\overrightarrow{QP} =$________________________________ .
5. __________ Find the plane through the point Q and perpendicular to $\mathbf{r}(0)$.
   The normal vector $\mathbf{r}(0) =$ ________________________________.

III.

$$\text{Let } \mathbf{r}(t) = \left\langle e^{2t},\ 2\tan t,\ \ln(t+1) \right\rangle.$$

6. __________ Find the normal component of acceleration, $a_N(0)$ of $\mathbf{r}(t)$.

   $\mathbf{r}''(0) =$ ________________________________.

7. __________ Find the curvature $\kappa(0)$ of $\mathbf{r}(t)$.

   $\mathbf{r}'(0) \times \mathbf{r}''(0) =$ ________________________________.

8. __________ Find the tangential component of acceleration, $a_T(0)$ of $\mathbf{r}(t)$.

   $\mathbf{r}'(0) =$ ________________________________.

9.

$$\text{Given} \qquad \mathbf{r}(t) = \left\langle 5e^{2\tan t},\ 1 + \frac{t^3}{t+1},\ t3^t \right\rangle.$$

Find the tangent line to the curve $\mathbf{r}(t)$ at $t = 0$. Give parametric equations for the line.

10.

$$\text{Given} \qquad P = (1,2,2);\quad Q = (0,1,0);\quad R = (0,2,2).$$

Find the plane through these three points. Simplify the plane equation so that all constants are combined on the right hand side.

11.

$$\text{Given} \qquad \mathbf{a} = \langle 1,1,3 \rangle \text{ and } \mathbf{b} = \langle 1,0,0 \rangle.$$

Find the area of the triangle with these vectors (arrows) as two of its sides. Give your answer as a real number; you may leave any roots as you found them. (4 pts)

12. Given

$$\mathbf{r}(t) = \langle \ln t,\ 2,\ t^2 + t \rangle \text{ and } \mathbf{r}'(t) = \left\langle \frac{1}{t},\ 0,\ 2t+1 \right\rangle \text{ and } \mathbf{r}''(t) = \left\langle \frac{-1}{t^2}, 0, 2 \right\rangle$$

Find $\mathbf{v}(1),\ \mathbf{a}(1),\ a_T(1),\ a_N(1), \kappa(1)$.

13. Given

$$\mathbf{r}'(2) = \langle 0,0,3 \rangle, \mathbf{T}'(2) = \langle 1,3,0 \rangle, \text{ and } a_T(2) = 5$$

Find $\mathbf{N}(2),\ \mathbf{a}(2),\ a_N(2),\ \kappa(2)$.

14. Study all quiz questions!

Answers:
I.

1. $\frac{1}{3\sqrt{2}}$
2. $\frac{2}{3\sqrt{10}}$
3. $\langle 2,1,1\rangle$
4. 0
5. $\frac{1}{\sqrt{6}}\langle 2,1,1\rangle$
6. $\left\langle 1,\frac{5}{2},\frac{-1}{2}\right\rangle$
7. $\langle 1,-2,0\rangle$
8. $\frac{\sqrt{86}}{2}$

II.

1. $\frac{x}{2}=\frac{y+1}{2}=\frac{z-2}{-3}$
2. $\{x=1;\quad y=0;\quad z=2t$
3. $\{x=0;\quad y=-1;\quad z=-3t+2$
4. $-2x-2y+3z=8$
5. $x=2$
6. $\frac{2\sqrt{26}}{3}$
7. $\frac{2\sqrt{26}}{27}$
8. $\frac{7}{3}$
9. $\{x=10t+5;\quad y=1;\quad z=t$
10. $2y-z=2$
11. $\frac{\sqrt{10}}{2}$
12. $\mathbf{v}(1)=\langle 1,0,3\rangle;\qquad \mathbf{a}(1)=\langle -1,0,2\rangle;\qquad a_T(1)=\frac{5}{\sqrt{10}};\quad a_N(1)=\frac{5}{\sqrt{10}};\quad \kappa(1)=\frac{1}{2\sqrt{10}}$
13. $\mathbf{N}(2)=\frac{1}{\sqrt{10}}\langle 1,3,0\rangle;\qquad \mathbf{a}(2)=\langle 3,9,5\rangle;\qquad a_N(2)=\frac{\sqrt{10}}{3};\quad \kappa(2)=3\sqrt{10};$