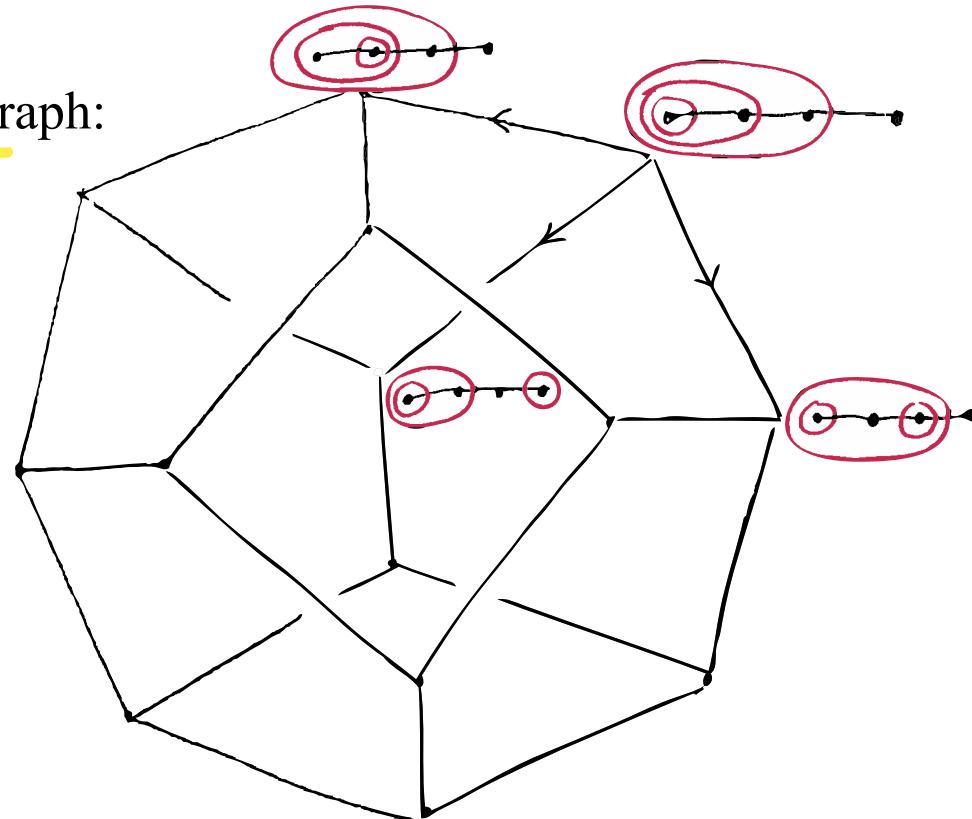


Definitions: **Tube** t on a graph:
is an induced, connected, subgraph.

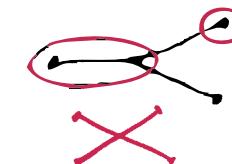
Tubing T :
a set of tubes, each pair nested or
disjoint, and all unions of the tubes
must be induced subgraphs.

Path graph:

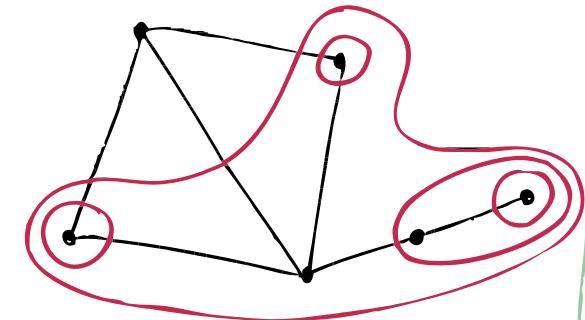


Associahedra

[Stasheff]



Any Graph:



Graph Associahedra

[Carr, Devadoss]

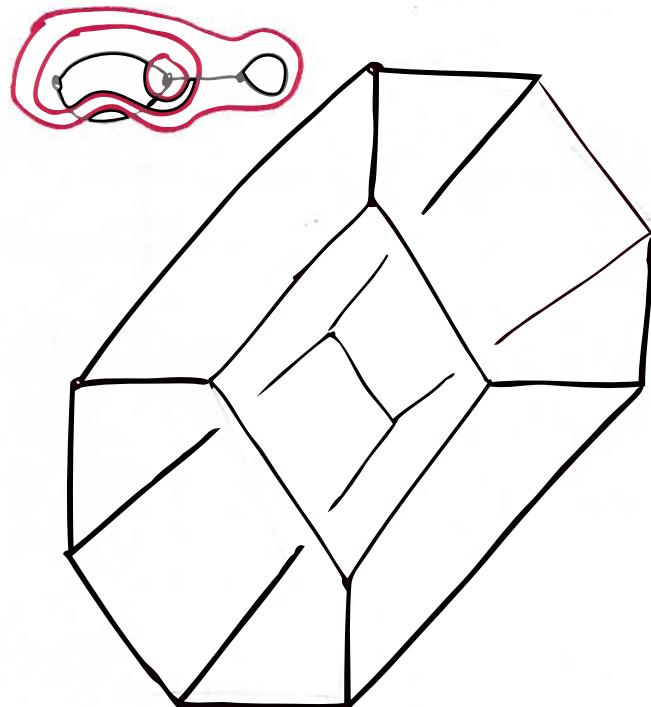
Definitions: **Tube** t on a **pseudograph**:

is a **filled**, connected, sub-pseudograph.

[**filled**: If both vertices of (multi)edge(s) are in t then at least one of those multiedges is in t .]

Tubing T :

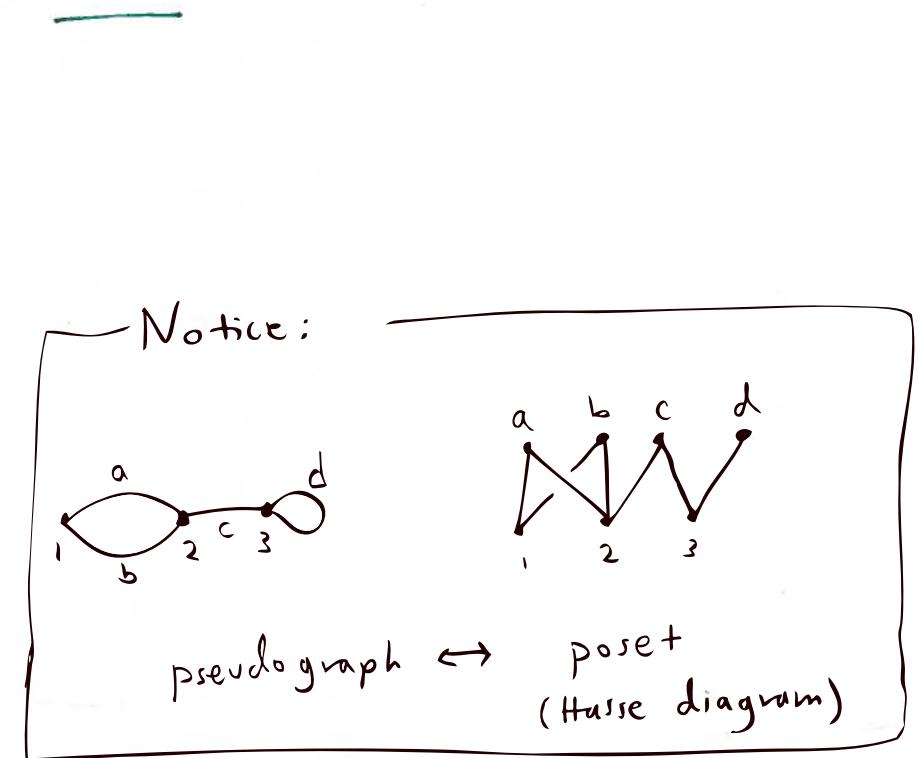
a set of pairwise nested or disjoint tubes with **filled** unions.



Pseudograph

Associahedra

[Carr, Devadoss, F.]



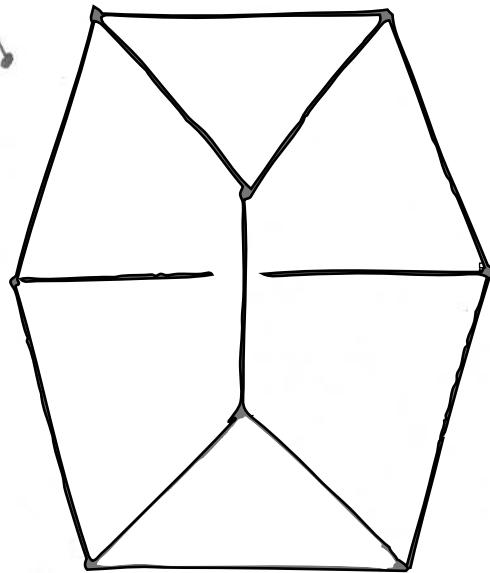
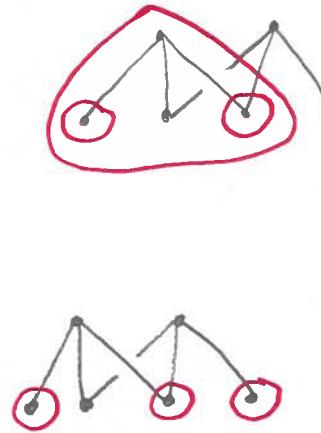
Definitions: **Tube** t on a poset:

is a **filled**, connected, lower set.

[**filled**: If t contains the set ∂x of all $y < x$ then
 t intersects the set $\mathbf{b}_x = \{y \mid \partial x = \partial y\}$.]

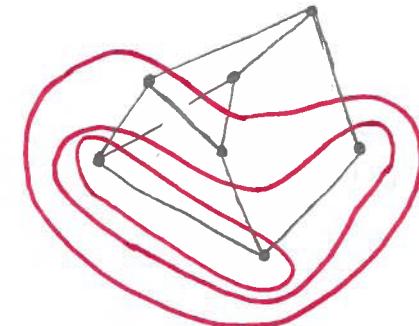
Tubing T :

a set of pairwise nested or disjoint tubes with **filled** unions.



Poset Associahedra

[Devadoss, F., Reisdorf, Showers]



Questions:

Find a geometric realization
of the poset associahedra.

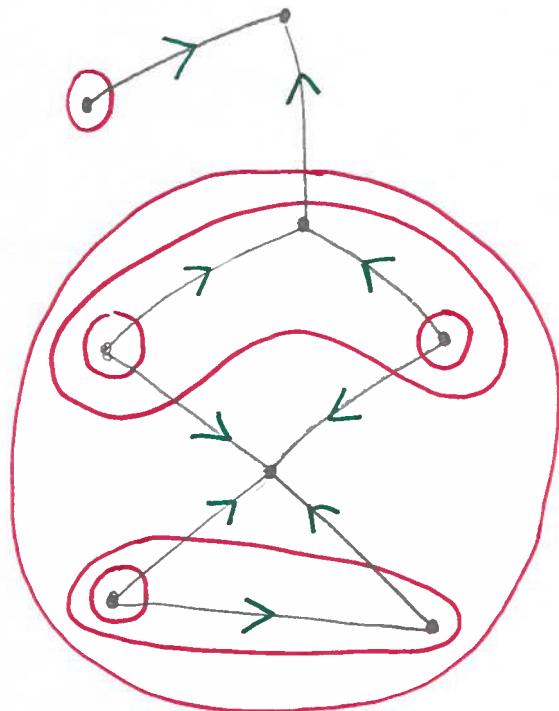
Find relationship to Galashin's
poset associahedra.

Tubings \rightarrow Orientations [Analogy: level curves and gradients]

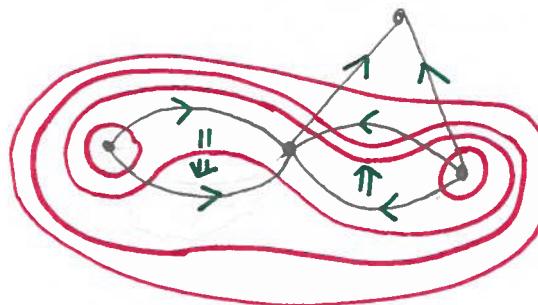
$T \mapsto O_T$

where $O_T(H)$ chooses the element $z \in H$
such that $z \in t \Rightarrow H \subseteq t$.

[H is any $\partial_{\mathcal{D}} \subseteq b_x$, or any non-minimal b_y ,
and $\partial_{\mathcal{D}} =$ maximal elements of $\partial_{\mathcal{D}}$]

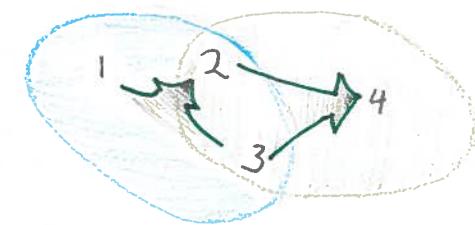
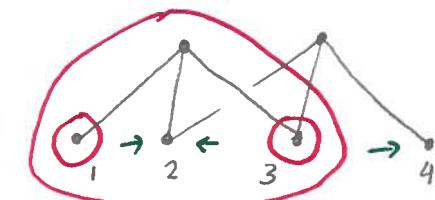


- Directed Acyclic Graph



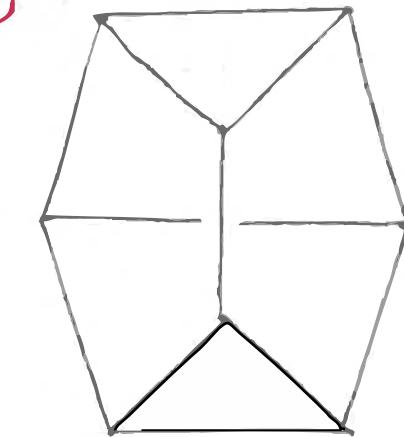
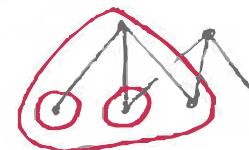
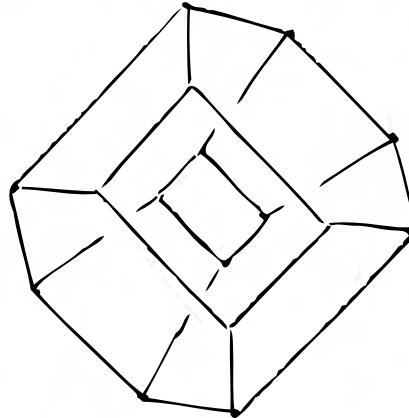
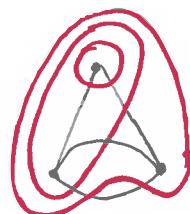
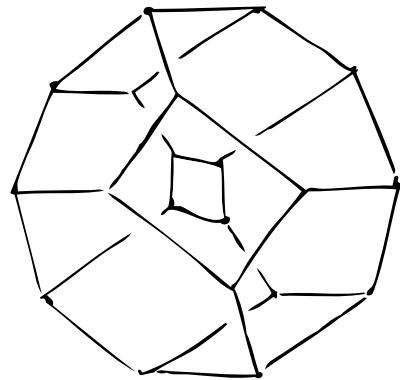
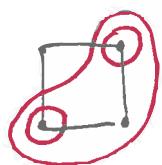
- Directed Acyclic Pseudo graph

- Parting diagram

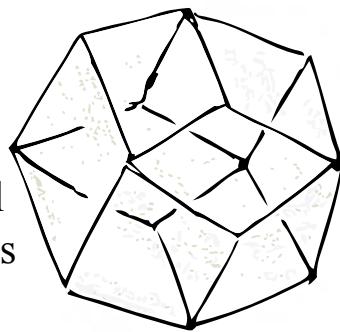


- Directed Acyclic Hypergraph
- Poset Gradient

Tubings to orientations as Polytope fan refinements.



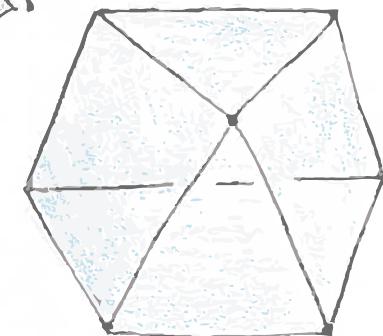
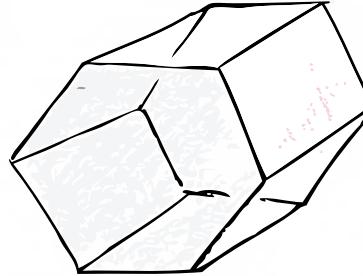
Graphical
Zonotopes



[Féray, Reiner]

Q.

• Are all poset gradient polytopes
products of hypergraphic polytopes?



Open Question!

Hypergraphic Polytopes

[Benedetti, Bergeron, Machacek]

[Bergeron, Pilaud]