Calculus II. Quiz 6.5. Name Key Time

Show all work for full or partial credit. Put a box around your final answer in each part.

1. For each series, what does the $p$-series test tell us? [not applicable, converge, diverge, or inconclusive] Show your work.

a) $\sum_{n=1}^{\infty} \frac{7^n}{(\sqrt{2})^n}$ $\boxed{\text{not applicable}}$

b) $\sum_{n=1}^{\infty} \frac{55}{n^{(0.27)}}$ $\boxed{\text{diverge}}$ $\boxed{0.27 < 1}$ p

c) $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^{1.3}$ $\boxed{\text{converge}}$ $\boxed{1.3 > 1}$ p

2. For each series, what does the comparison test tell us? [not applicable, converge, or diverge] Show your work.

a) $\sum_{n=1}^{\infty} \frac{5}{3n^2+1}$ $\boxed{\text{converge}}$

$$\frac{5}{3n^2+1} < \frac{5}{3n^2}$$

OR $\frac{1}{3n^2+1} < \frac{1}{n^2}$

[And $\sum \frac{1}{n^2}$ converges by p-series]

b) $\sum_{n=1}^{\infty} \frac{7^n}{3+8^n}$ $\boxed{\text{converge}}$

$$\frac{7^n}{3+8^n} < \left(\frac{7}{8}\right)^n$$

[and $\sum \left(\frac{7}{8}\right)^n$ converges by geo. series]

3. For each series, what does the limit comparison test tell us? [not applicable, converge, or diverge] Show your work.

a) $\sum_{n=1}^{\infty} \frac{n+3}{(n+1)^2}$ $\boxed{\text{diverge}}$

[lim compare to $\sum_{n=1}^{\infty} \frac{1}{n}$]

$$\lim_{n\to\infty} \frac{\frac{n+3}{(n+1)^2}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n+3}{(n+1)^2}\cdot\frac{n}{1} = \lim_{n\to\infty} \frac{n^2+3n}{n^2+2n+1} \boxed{=1}$$

(using L'Hospital's)

(since $0 < L < \infty$) $\boxed{L=1}$ AND $\sum \frac{1}{n}$ diverges by p-series.

b) $\sum_{n=1}^{\infty} \frac{3^n}{7^n - n}$ $\boxed{\text{converge}}$

[lim compare to $\sum_{n=1}^{\infty} \left(\frac{3}{7}\right)^n$]

$$\lim_{n\to\infty} \frac{\frac{3^n}{7^n-n}}{\frac{3^n}{7^n}} = \lim_{n\to\infty} \frac{3^n}{7^n-n}\cdot\frac{7^n}{3^n} = \lim_{n\to\infty} \frac{7^n}{7^n-n} = \boxed{1}$$

$$= \lim_{n\to\infty} \frac{7^n \ln 7 (\ln 7)}{7^n \ln 7 (\ln 7)}$$

(using L'Hospital's twice)

(since $0 < L < \infty$) $\boxed{L=1}$

and $\sum \left(\frac{3}{7}\right)^n$ converges by geo series.