

## 3450:221 Calculus I, Final Sample Problems

These problems provide a sample of typical problems you are expected to be able to solve.

### 1. Limits

#### (a) Graphical

- i. Section 2.2, problems 7, 9, 17
- ii. Section 2.6, problem 5, 9

#### (b) Computational

- i.  $\lim_{x \rightarrow 0} \frac{|x|}{x}$
- ii. Section 2.3, problems 11, 17, 21, 23, 42
- iii.  $\lim_{x \rightarrow 1} \ln \left( \frac{5 - x^2}{1 + x} \right)$
- iv.  $\lim_{x \rightarrow 0} \ln(1 + e^{-x})$
- v.  $\lim_{x \rightarrow 0} \ln(1 - e^{-x})$
- vi.  $\lim_{x \rightarrow \infty} \ln(1 + e^{-x})$
- vii.  $\lim_{x \rightarrow -\infty} \ln(1 + e^{-x})$
- viii. Section 2.6, problems 15-33 odd, 32
- ix. Section 3.3, problems 39, 40, 42, 43
- x.  $\lim_{x \rightarrow 0} (1 + x)^{1/x}$
- xi.  $\lim_{x \rightarrow 0} (1 + ax)^{1/x}$
- xii. Section 4.4, problems 15, 25, 27, 35, 45, 49, 50, 51, 57

#### (c) Continuity

- i. Section 2.5, problem 20
- ii. Is  $f(x)$  below continuous at  $x = 1$ ? Why or why not?  
$$f(x) = \begin{cases} x^2, & \text{if } x \leq 1; \\ 1 + x, & \text{if } x > 1 \end{cases}$$
- iii. Find the value of  $k$  that makes  $f(x)$  below be continuous at  $x = 3$ .  
$$f(x) = \begin{cases} 1 + x^2, & \text{if } x \leq 3; \\ 2 + kx, & \text{if } x > 3 \end{cases}$$

### 2. Derivatives

#### (a) Graphical

- i. Section 2.8, problem 3
- ii. Section 4.3, problems 9, 13, 15, 17, 27, 29, 33, 45, 50
- iii. Section 4.5, problems 1, 11, 25

#### (b) Definition

- i. Section 2.8, problems 23, 27
- ii. Use the definition of the derivative to find  $f'(x)$  for  $f(x) = \frac{1}{\sqrt{1+x}}$ .

(c) Computational. Simplify all derivatives as much as possible.

- i. Find  $f'(x)$  and  $f''(x)$  for  $f(x) = \frac{5}{8}x^{8/3} - \frac{5}{8}x^{-3/5} + \pi^2$ .
- ii. Find  $f'(x)$  for  $f(x) = e^x + x^e$ .
- iii. Find  $f'(y)$  for  $f(y) = y^{1/3}(y - 2)^{2/3}$ .
- iv. Find  $f'(x)$  for  $f(x) = \frac{1 + e^{x^2}}{1 - e^{-x^2}}$ .
- v. Find  $f'(r)$  for  $f(r) = \frac{r^2(r + 1)^{1/3}}{(r + 2)^{2/3}}$ .
- vi. Find  $g'(\theta)$  for  $g(\theta) = \frac{\sin 3\theta}{\sin 2\theta}$ .
- vii. Find  $h'(x)$  for  $h(x) = \ln(1 + e^{-x})$ .
- viii. Find  $f'(x)$  for  $f(x) = 3^{x^2}$ .
- ix. Find  $r'(p)$  for  $r(p) = \sec(\ln p + 1)$ .
- x. Find  $f'(x)$  for  $f(x) = \ln(\tan(e^{x^2} + 2x))$ .
- xi. Find  $c'(x)$  for  $c(x) = \int_1^{e^x} t \ln t^2 dt$ .
- xii. Find  $f'(t)$  for  $f(t) = e^{3t} \cos 5t$ .
- xiii. Find  $f'(x)$  for  $f(x) = (x^2 + 1)^4(x + \sin(\ln x))^{1/3}$ .
- xiv. Find  $\alpha'(\theta)$  for  $\alpha(\theta) = \sin^2(\theta^2)$ .
- xv. Find  $f'(x)$  for  $f(x) = \arctan(e^x)$ .
- xvi. Find  $f'(x)$  for  $f(x) = \cot^{-1}(e^x)$ .
- xvii. Find  $f'(x)$  for  $f(x) = \arctan(3x)$ .
- xviii. Find  $f'(x)$  for  $f(x) = \arctan\left(\frac{x}{2}\right)$ .
- xix. Find  $g'(x)$  for  $g(x) = \frac{\sinh x}{\cosh x + 1}$ .

(d) Implicit Differentiation.

- i. Section 3.5, problems 7, 9, 15, 25

(e) Logarithmic Differentiation.

- i. Find  $y'$  for  $y = \frac{e^{-3x}\sqrt{x^2 + 4}}{(x + 2)^2(x + 3)^3}$ .
- ii. Find  $y'$  for  $y = x^{\sin x}$ .

(f) Applications

- i. Related Rates: section 3.9, problems 3-6, 13, 15, 17
- ii. Use a linear approximation for  $f(x) = (8 + x)^{1/3}$  to estimate  $8.07^{1/3}$ .

(g) Extreme Values and Critical Numbers

- i. Find the Critical Numbers of  $f(x) = x^{2/3}(x + 1)^3$ .
- ii. Find the Critical Numbers of  $f(t) = t^{6/7} - 3t^{3/7}$ .
- iii. Find the Critical Numbers of  $g(x) = x^3 + 6x^2 - 15x + 4$ .
- iv. Extreme Value Theorem: section 4.1, problems 47, 52, 55, 59

- v. Mean Value Theorem: section 4.2, problem 11
- vi. Optimization: section 4.7, problems 2, 7, 11, 14, 35, 51

### 3. Integrals

- (a) Find  $f(x)$  if  $f''(x) = \frac{15}{16}x^{1/4} - \frac{6}{125x^{11/15}}$ .
- (b) Find  $f(x)$  if  $f'(x) = \frac{1}{3}x^{9/4} - \frac{1}{5}x^{-1/5}$  and  $f(1) = 3$ .
- (c) Evaluate these integrals.

i.  $I = \int_1^2 \frac{1}{3}x^{3/2} + \frac{1}{2}x^{1/2} dx$

ii.  $I = \int_1^4 \sqrt{2x+1} dx$

iii.  $I = \int_0^1 (2r+1)^{17} dr$

iv.  $I = \int_0^4 |x-1| dx$

v.  $I = \int_0^{18} \sqrt{\frac{3}{z}} dz$

vi.  $I = \int \frac{1}{4+x^2} dx$

vii.  $I = \int \frac{1}{1+9x^2} dx$

viii.  $I = \int \frac{e^x}{1+e^x} dx$

ix.  $I = \int \frac{e^x}{1+e^{2x}} dx$

x.  $I = \int \frac{x}{4+x} dx$

xi.  $I = \int \frac{x}{7+x^2} dx$

xii.  $I = \int \tan 3x dx$

xiii.  $I = \int \frac{1}{x}(\ln x + 1) dx$

xiv.  $I = \int_8^{10} \frac{e^{1/w}}{w^2} dw$

xv.  $I = \int_0^2 \frac{1}{(4-2x)^{5/2}} dx$

xvi.  $I = \int \frac{1}{\sqrt{1-x^2} \arcsin x} dx$

xvii.  $I = \int x^5 \sqrt{x^3+2} dx$