Title: Hopf structures on shuffle algebras and shuffle operads

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Abstract: The set $\mathcal{T}$ of all planar rooted trees, equipped with the grafting operations, determines a monad on the category of graded modules. D. Ginzburg and M. Kapranov described nonsymmetric operads as algebras over this monad. In a more algebraic way, the graded vector space $K[\mathcal{T}]$ spanned by $\mathcal{T}$ may be considered as:

1. the free pre-Lie system spanned by the set of all corollas $\{c_n\}_{n\geq 1}$.
2. the free tridendriform algebra spanned by one element.

The associahedron or Stasheff polytope $\mathfrak{A}_n$ of dimension $n-2$ is the geometric realization of the poset of planar rooted trees with n leaves, equipped with the Tamari order. The boundary map of $\mathfrak{A}_n$ may be described uniquely as a bar construction in the context of pre-Lie systems, and in terms of its relationship with the 0-tridendriform structure. Moreover, there exists a notion of tridendriform bialgebra such that the space of primitive elements of $K[\mathcal{T}]$, equipped with the associahedron boundary map, describes the free $\mathcal{S}_2$- free algebras, where $\mathcal{S}_2$ is the J. McClure and J. Smith's operad, which is homotopically equivalent to the singular homology of the operad of small 2-cubes.

If we replace the planar rooted trees by the surjection maps between finite sets, we get other family of polytopes: the permutohedra, denoted $\mathfrak{P}_n$. The grafting of planar rooted trees is replaced in this case by the action of shuffles. In a joint work with J.-L. Loday, we proved that the set of all surjective maps with substitution gives rise to a monad on arity graded modules, and an algebra over this monad is called a *permutad*. A permutad can be presented with partial operations $\circ_ß$ where ß is a shuffle, as generators or as a suboperad of the *shuffle operad* introduced by V. Dotsenko and A. Khoroshkin. Shuffle algebras or permutads play the rule of pre-Lie systems for the permutohedron, up to now we have no approximation to the space spanned by all surjective maps similar to tridendriform algebras.

The first part of the talk will be devoted to describing all the algebraic structures related to the associahedra. In the second part, we shall explain all the results we were able to extend to the permutohedron, as well as some questions and work in progress arising from them.