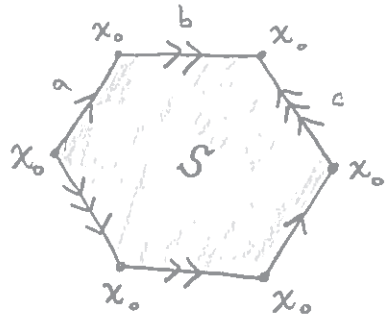


Name: \_\_\_\_\_

Consider the "mystery surface":



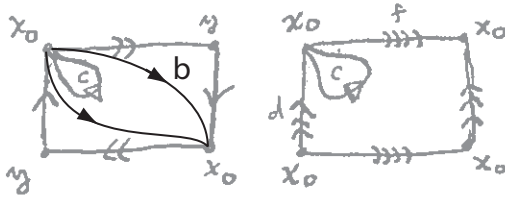
1) Find  $\chi(S)$ .

$$1 - 3 + 1 = -1$$

2) Find a presentation for  $\pi_1(S)$ .

$$\langle a, b, c \mid abc^{-1} = cba \rangle$$

3)  $P^2 \# T^2 =$



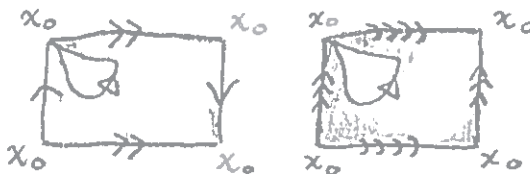
a) Find  $\chi(P^2 \# T^2)$ .

$$2 - 5 + 2 = -1$$

b) Find a presentation for  $\pi_1(P^2 \# T^2)$

$$\langle b, c, d, f \mid cb = b^{-1}, dcf = fd \rangle$$

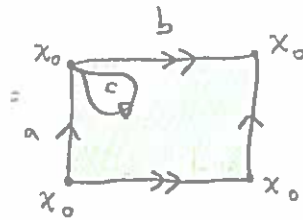
4)  $K^2 \# T^2 =$



Find  $\chi(K^2 \# T^2)$ .

$$1 - 5 + 2 = -2$$

5)  $T^2 - D^2$



a) Find  $\chi(T^2 - D^2)$

$$1 - 3 + 1 = -1$$

b) Find a presentation of  $\pi_1(T^2 - D^2)$

$$\langle a, b, c \mid acb = ba \rangle$$

6) Recall:  $\chi$  and  $\pi_1$  are homotopy invariants, and thus also homeomorphism invariants.

That means, from (4),  $S \neq K^2 \# T^2$ , and  $S \neq K^2 \# T^2$ .

However,  $\chi$  and  $\pi_1$  don't tell us which spaces are homeomorphic. A cut-and-glue sequence allows us to construct a homeomorphism.

