**Calculus 2 Test 3, Spring '21. Pg. 1**

Name: Key

Time:

My signature here is to pledge that I have answered each test question from my own knowledge and understanding, without giving or receiving any unauthorized help.

Sign:

Date:

Show all your work clearly on the test paper for full/partial credit! Read directions carefully, and put a box around the final answer in each part.

All angles are in radians. Simplify only the basics: adding, multiplying, etc. for constants

1. Find a power series for the following functions, by starting with a fact from the list of known Maclaurin series. Simplify just enough to combine the powers of $x$ into a single expression.

(a) $f(x) = 2xe^{4x^2}$

$$\sum_{n=0}^{\infty} \frac{2x(4x^2)^n}{n!}$$

$$\boxed{= \sum_{n=0}^{\infty} \frac{2(4^n)x^{2n+1}}{n!}}$$

$$= \sum_{n=1}^{\infty} \frac{2(4^{n-1})x^{2(n-1)+1}}{(n-1)!}$$

$$= \sum_{n=1}^{\infty} \frac{2(4^{n-1})x^{2n-1}}{(n-1)!}$$

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(b) $f(x) = 3x^2\cos(5x)$

$$\sum_{n=0}^{\infty} \frac{3x^2(-1)^n(5x)^{2n}}{(2n)!}$$

$$\boxed{= \sum_{n=0}^{\infty} \frac{3(-1)^n(5^{2n})x^{2n+2}}{(2n)!}}$$

$$\boxed{= \sum_{n=0}^{\infty} \frac{3(-1)^n 25^n x^{2n+2}}{(2n)!}}$$

$$= \sum_{n=0}^{\infty} \frac{3(-25)^n x^{2n+2}}{(2n)!}$$

2. For this power series centered at $a = 2$, find the radius of convergence $R$. Then determine the interval of convergence. Make sure to check endpoints.

$$\sum_{n=1}^{\infty} \frac{5^n(x-2)^n}{(-2n)3^n}$$

1) Ratio test

$$\lim_{n\to\infty} \left| \frac{5^{n+1}(x-2)^{n+1}}{(-2(n+1))\,3^{n+1}} \cdot \frac{(-2n)3^n}{5^n(x-2)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{5(x-2)n}{3(n+1)} \right|$$

$$= |x-2| \lim_{n\to\infty} \frac{5n}{3n+3}$$

$$= \frac{5}{3}|x-2| < 1$$

$$\Rightarrow |x-2| < \frac{3}{5} \qquad \boxed{R = \frac{3}{5}}$$

$$\Rightarrow -\frac{3}{5} < x-2 < \frac{3}{5} \qquad \left(\text{add } \frac{10}{5}\right)$$

$$\Rightarrow \frac{7}{5} < x < \frac{13}{5}$$

$x = \frac{7}{5}$: $\sum_{n=1}^{\infty} \frac{5^n}{(-2n)3^n}\left(\frac{-3}{5}\right)^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{-2n} = -\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ converges (alt series)

$x = \frac{13}{5}$: $\sum_{n=1}^{\infty} \frac{5^n}{(-2n)3^n}\left(\frac{3}{5}\right)^n = \sum_{n=1}^{\infty} \frac{1}{-2n} = -\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}$ diverges (p-series)

$$\boxed{\left[\frac{7}{5}, \frac{13}{5}\right)}$$

3. Find the $n = 2$ term of the Taylor series for $f(x) = e^{3x-3} + 2x^2$ centered at $a = 1$.

$$f'(x) = 3e^{3x-3} + 4x$$
$$f^{(2)}(x) = 9e^{3x-3} + 4$$
$$f^{(2)}(1) = 9 + 4 = 13$$

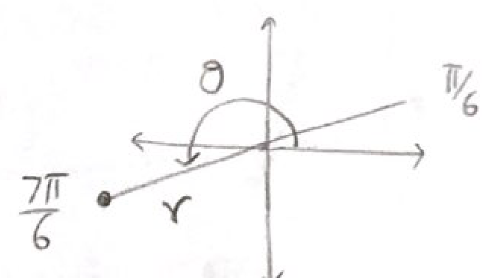
$$\boxed{\frac{13(x-1)^2}{2!}} = \boxed{\frac{13(x-1)^2}{2}}$$

4. For the point $(x, y) = (-5\sqrt{3}, -5)$ find the polar coordinates with positive radius $r$.

$$r = \sqrt{25(3) + 25} = \sqrt{100} = 10$$

$$\tan^{-1}\left(\frac{-5}{-5\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$\theta = \frac{\pi}{6} + \pi = \frac{7\pi}{6}$$




$$\boxed{\left(10, \frac{7\pi}{6}\right)} \quad \text{or} \quad \boxed{\left(10, \frac{-5\pi}{6}\right)}$$

Pg.



5. Given $C = \begin{cases} x = e^t + 4 \\ y = 5e^{(t+3)} - 5t \end{cases}$ $t \in [-5, 11]$.

Also, I already found the first derivative for you: $y' = \dfrac{5e^{(t+3)} - 5}{e^t}$.

(a) Find the $(x, y)$ point(s) with horizontal tangent to the curve. Your answer should be $(x, y)$.

$y' = 0$

$\Rightarrow \dfrac{5e^{t+3} - 5}{e^t} = 0$

$\Rightarrow 5e^{t+3} - 5 = 0$

$\Rightarrow 5e^{t+3} = 5$

$\Rightarrow e^{t+3} = 1$

$\Rightarrow t+3 = \ln 1$

$\Rightarrow t+3 = 0$

$\Rightarrow t = -3$

$x = e^{-3} + 4$

$y = 5e^0 + 15 = 20$

$\boxed{(e^{-3}+4,\ 20)}$

$(4.05, 20)$

(b) Find and use the second derivative to decide min, max or inconclusive at the point in (a).

$y'' = \dfrac{dy'/dt}{dx/dt} = \dfrac{e^t 5e^{t+3} - e^t(5e^{t+3} - 5)}{(e^t)^2} \cdot \dfrac{1}{e^t}$

$= \boxed{\dfrac{e^t 5e^{t+3} - e^t(5e^{t+3} - 5)}{(e^t)^3}}$

$= \dfrac{5e^t}{e^{3t}} = \dfrac{5}{e^{2t}} = \dfrac{5}{(e^t)^2}$

at $t = -3$, $y'' = \dfrac{5}{e^{-6}} = \boxed{5e^6 > 0}$ $= 2017.14$

concave up

$\Rightarrow \boxed{\text{min}}$