3450:222 Calculus II, Final Sample Problems

These problems provide a sample of typical problems you are expected to be able to solve.

1. Integration Techniques

   (a) $\int \sin^2 x \cos^5 x \, dx$

   (b) $\int \arctan x \, dx$

   (c) $\int \frac{2x+1}{x^2+4x+9} \, dx$

   (d) $\int \frac{x^3+8x^2+21x+13}{x^2+7x+12} \, dx$

   (e) $\int_1^\infty \frac{\ln x}{x^2} \, dx$

   (f) $\int e^{-x} \sin 2x \, dx$

   (g) $\int \tan^5 x \sec x \, dx$

   (h) $\int \frac{2x+1}{x^2+7x+12} \, dx$

   (i) $\int \frac{t^3}{\sqrt{t^2+25}} \, dt$

   (j) $\int \frac{1}{t^3\sqrt{t^2+1}} \, dt$

   (k) $\int x^{99} \ln 2x \, dx$

   (l) $\int \frac{1}{t}\sqrt{t^2-1} \, dt$

   (m) $\int_2^{10} \frac{6}{(w-2)^{4/17}} \, dw$

   (n) $\int_{-\infty}^0 xe^x \, dx$

2. Applications of the Integral

   (a) Find the area between $y = 2 - (x-1)^2$ and $y = 3 - x$.

   (b) Find the arc length of the curve $y = 4x^{3/2} + 7$ for $1 \leq x \leq 7$.

   (c) Rotate the region bounded by $y = 1 - x^2$, $x = 0$, $y = 0$ about the line $x = 3$. Set up the integral for the volume of the region using (a) disks/washers and (b) shells.

   (d) Rotate the region bounded by $y = 1 - x^2$, $x = 0$, $y = 0$ about the x-axis. Set up the integral for the surface area of the solid.

3. Infinite Series

   (a) Evaluate $S = \sum_{n=1}^{\infty} \frac{2^{2n+1}}{3^{3n-2}}$.

(b) Determine whether the series converges absolutely, converges conditionally, or diverges

i. $S = \sum_{n=1}^{\infty} \frac{(n^2+1)^{5/2}}{(n^3+1)^2}$.

ii. $S = \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n(\ln n)^2}$.

iii. $S = \sum_{n=1}^{\infty} \frac{9^n}{6+11^n}$.

iv. $S = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\arctan n}{1+n^2}$

v. $S = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^n}{n! 5^n}$.

(c) Find the interval of convergence and radius of convergence for

i. $f(x) = \sum_{n=1}^{\infty} \frac{2^n (2-3x)^n}{n^{91}}$.

ii. $f(x) = \sum_{n=1}^{\infty} n(x+2)^n$.

(d) Use the geometric series to write the power series expansion for

i. $f(x) = \frac{x}{2-4x}$, centered at $a = 0$.

ii. $f(x) = \frac{1}{2-4x}$, centered at $a = 1$.

(e) Write the first 4 nonzero terms of the Maclaurin expansion for

i. $f(x) = x^2 \left(e^{4x} - 1\right)$.

ii. $f(x) = \cos(3x) - 2\sin(2x)$.

(f) Use the Taylor Series definition to write the expansion for $f(x) = \frac{1}{1-3x}$ centered at $x = 1$.

4. Parametric and Polar Forms

(a) Convert these parametric equations to Cartesian form and sketch the curve:

i. $x(t) = t + 1$, $y(t) = 2t - 1$ for $1 \le t \le 2$.

ii. $x(t) = 5\cos(2t)$, $y(t) = 2\sin(2t)$, $\pi/4 \le t \le 3\pi/4$.

(b) Find the equation of the tangent line to the parametric curve $x(t) = t^2 + 2t$, $y(t) = t^2 + t + 1$ at the point $(3, 3)$.

(c) Find the arc length of the parametric curve $x(t) = 3\cos(2t)$, $y(t) = 3\sin(2t)$ for $0 \le t \le 3\pi/4$.

(d) Sketch these polar curves:

i. $r = 1 - \cos(\theta)$

ii. $r = 1 + \sin^2(\theta)$

(e) Find the area enclosed above by the polar curve $r = e^{-\theta}$ and below by the x-axis.