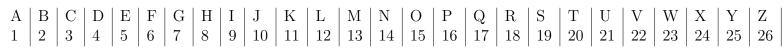
Discrete final review.

- 1. Given universe $\mathcal{U}=\{3,4,5,7,9,10,11,23\}$; $A=\{5,7,9,10,11,23\}$; and $B=\{5,3,7\}$. Find the following:
 - $\bullet \ \overline{(B-A)} \cap B$
 - $\bullet \ \overline{A \cup \overline{B}}$
 - $|B \times \mathcal{P}(A \times B)|$
 - $\bullet |A \cup B|$
- 2. Find the number of PINs using $\{0, ..., 9\}$, with 7 digits, no repeated numbers, where the first digit cannot be 3 and the fourth digit cannot be 5.

3. Find the number of DNA sequences using $\{A, G, T, C\}$, of length 5, where the first and second location cannot repeat, and the first location cannot be A.

- 4. Use contradiction to prove: $\forall z \in \mathbb{Z}, z^2 \equiv 7 \pmod{6} \Rightarrow z \text{ is odd.}$
 - a) Negate the statement.
 - b) Write the assumptions, translated to algebraic equations.
 - c) We will use the assumptions to show the falsehood 2|7, which is translated 7 = 2w for some integer w. Show the proof steps, from assumptions to 2|7.

- 5. Use a direct proof to prove: $\forall z \in \mathbb{Z}, z \mod 3 = 2 \Rightarrow 9 | (3z^2 + 6)$.
 - a) Write the assumption, translated to an algebraic equation.
 - b) Write what we want to show, translated to an algebraic equation.
 - c) Proof steps:
- 6. Use induction to prove: $\forall n \in \mathbb{Z}$, if $n \geq 4$ then $3|(2^{2n-5}+1)$.
 - a) Show the base case.
 - b) State the induction assumption, translate to algebraic equation.
 - c) State what we need to show, translate to algebraic equation.
 - d) Do the proof steps.



7. Consider the sequence $a_n = (n^2 + 10) \mod 12$; starting at n = 1. Use it to encrypt the word SAT. Your answer will be the new word.

n	letter	std. num.	find a_n	encrypt	letter
	a				
	S				
2	A				
3	Т				

8. Consider the one-time-pad sequence $a_n = (2, 8, 11)$; starting at n = 1. It has been used to encrypt a message, and the encrypted message is DDF. Use the same sequence to decrypt and find the original word.

n	letter	std. num.	a_n	$\operatorname{decrypt}$	letter
1	D				
	<i>D</i>				
2	D				
3	F				
_					

9. Consider the BBS (Blum Blum Shub) sequence $a_n = (a_{n-1})^2 \mod pq$; with $a_0 = 2$ and with p = 5, q = 5. Starting at n = 1, use this sequence to encrypt the binary number 1010. Your answer will be the new binary number. You may use either method from class.

n	bit	find a_n	encrypt	bit
	1			
2	0			
3	1			
4	0			

revie	w
10.	From 7 library books, how many subsets of exactly 3 books are there? Answer as a whole number.
11.	For 4 books and 9 shelves of a bookcase, find the number of ways to distribute the books on the shelves (just in piles, not in order.)
12.	For 9 books on 4 shelves of a bookcase, find the number of ways to place the books on the shelves in ordered rows.
13.	For 8 books and 6 shelves of a bookcase, find the number of plans for shelving, where at least 3 books are planned for the top shelf (a plan only tells how many books on each shelf.)
14.	For 4 books and 9 shelves of a bookcase, find the number of ways to distribute the books on the shelves, where the bottom shelf has at most one book (just in piles, not in order.)
15.	For 7 books and 9 shelves of a bookcase, find the number of ways to place the books on the shelves in ordered rows, where the bottom shelf has no more than 2 books.

- Find the contrapositive of the original; write it without "not" and without "~."
- Find the negation of the original; write it without "not" and without "~."

2. Given the statement:

$$\forall x \in \mathbb{Z}, \exists y \in \mathbb{N} \text{ s.t. } (yx \ge y + 7) \Rightarrow ((x \text{ is even }) \land (y + x \text{ is odd })).$$

- Find its negation; write it without "not" and without "~."
- 3. Given the original statement "If you have salt then you have sodium." Answer the following without "not" and without "~.'
 - Write the original statement using the word sufficient.
 - Write the converse of the original using the words "only if".
 - Write the contrapositive of the original using the word necessary.



1101011

• binary:

hexidecimal:

1111010 binary:

hexidecimal:_____

hexidecimal: FA3 binary:__

binary: 1010 decimal:

5. Suppose that P = F (false) and Q = T (true). Find whether each of these statements is true (T) or false (F). Put a box around each final answer of T or F.

•
$$Q \land \sim (P \Rightarrow Q)$$
.



•
$$((\sim P) \land Q) \Longleftrightarrow ((\sim P) \lor \sim Q).$$



•
$$(\sim (P \lor Q)) \Rightarrow P$$
.



•
$$(P \Longleftrightarrow (\sim Q)) \Rightarrow ((\sim P) \Rightarrow P).$$



 \mathcal{S} . For $\mathcal{S} = \{1, -3, -4, -12\}$, find an example making the following true:

$$\exists x \in S \text{ s.t. } ((x \mid 7) \lor (|x| > 3)) \Rightarrow ((x \text{ is odd }) \land (x \le -1))$$

7. Given the inputs of each circuit, fill in the outputs.

