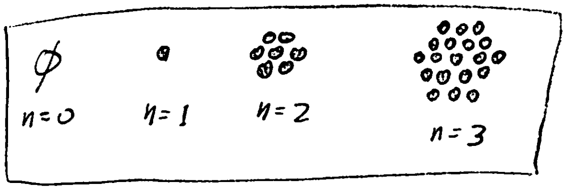

$$\boxed{a_{n+1} = a_n + 6n, \quad n \geq 1} \qquad \boxed{\begin{aligned} a_0 &= 0 \\ a_1 &= 1 \end{aligned}}$$

Since our recurrence is true for $n \geq 1$, we start with:

$$\boxed{\sum_{n=1}^{\infty} a_{n+1} x^n = \sum_{n=1}^{\infty} (a_n + 6n) x^n}$$

$$\Rightarrow \sum_{n=0}^{\infty} a_{n+1} x^n - \underbrace{a_{0+1}}_{1} x^0 = \sum_{n=0}^{\infty} (a_n + 6n) x^n \underbrace{-(a_0 + 6 \cdot 0) x^0}_{0}$$

(mult. by x)

$$\Rightarrow x \sum_{n=0}^{\infty} a_{n+1} x^n - x = x \sum_{n=0}^{\infty} (a_n + 6n) x^n$$

$$\Rightarrow \sum_{n=0}^{\infty} a_{n+1} x^{n+1} - x = x \sum_{n=0}^{\infty} a_n x^n + x \sum_{n=0}^{\infty} 6n x^n$$

$$\Rightarrow \sum_{n=1}^{\infty} a_n x^n - x = xf + x^2 \sum_{n=0}^{\infty} 6n x^{n-1}$$

$$\Rightarrow \sum_{n=0}^{\infty} a_n x^n - \underbrace{a_0 x^0}_{0} - x = xf + x^2 \frac{6}{(1-x)^2}$$

$$\Rightarrow f - x = xf + \frac{6x^2}{(1-x)^2}$$

$$\Rightarrow f - xf = x + \frac{6x^2}{(1-x)^2}$$

$$\Rightarrow f(1-x) = x + \frac{6x^2}{(1-x)^2}$$

$$\Rightarrow f = \boxed{\frac{x}{1-x} + \frac{6x^2}{(1-x)^3}}$$ o.g.f.

Use:

$$\frac{x}{1-x} = \sum_{n=0}^{\infty} x^n - 1$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} n x^{n-1}$$

$$\frac{2}{(1-x)^3} = \sum_{n=0}^{\infty} n(n-1) x^{n-2}$$

$$\boxed{a_n = \begin{cases} 1 + 3n(n-1), & n \geq 1 \\ 0, & n = 0 \end{cases}}$$

closed formula

Now $$f = \sum_{n=0}^{\infty} x^n - 1 + 3x^2 \sum_{n=0}^{\infty} n(n-1) x^{n-2} = \sum_{n=0}^{\infty} (1 + 3n(n-1)) x^n - 1$$