Calculus 2 Test 1, Spring '21. Pg. 1

My signature here is to pledge that I have answered each test question from my own knowledge and understanding, without giving or receiving any unauthorized help.

Sign:____________________

Name: Key

Time:____________________

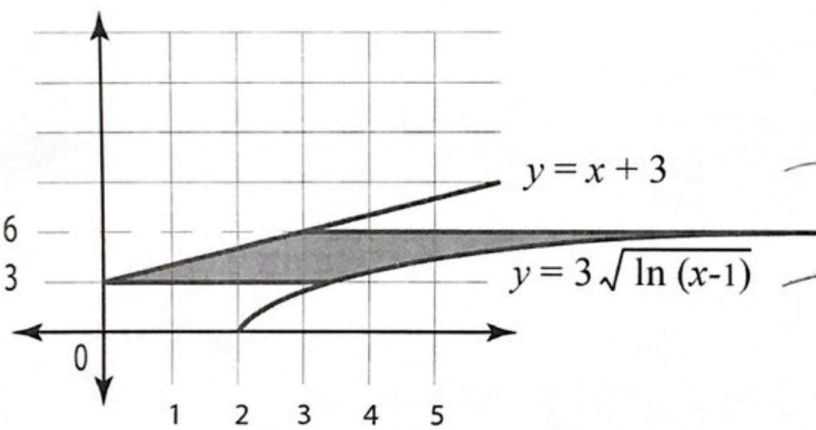
Date:____________________

Show all your work clearly on the test paper for full/partial credit! Read directions carefully, and put a box around the final answer in each part.

All angles are in radians. Simplify only the basics: adding, multiplying, etc. for constants

When the problem says to set up an integration, your answer must be one definite integral. Your answer must accurately represent the area or volume it describes, not a different region or volume.

1. Set up an integral for the area inside this shaded region. Just set it up, don't actually integrate. Read the y-axis carefully, since it is not to scale!



$\rightarrow$ $x = y - 3$

$\rightarrow$ $y^2 = 9 \ln x - 1$

$\frac{y^2}{9} = \ln x - 1$

$e^{y^2/9} = x - 1$

$e^{y^2/9} + 1 = x$

$$\boxed{\int_3^6 \left(e^{y^2/9} + 1 - (y-3)\right) dy}$$

$$= \int_3^6 \left(e^{\left(\frac{y^2}{9}\right)} + 4 - y\right) dy$$

2. Find the average value of the function $x^2 \ln x$ over the interval $[1, 3]$. Set up and integrate.

$$\boxed{\frac{1}{3-1}\int_1^3 x^2 \ln x \, dx}$$

$u = \ln x \quad dv = x^2 dx$

$du = \frac{1}{x}dx \quad v = \frac{x^3}{3}$

WORK MUST be SHOWN

$$= \frac{1}{2}\left(\left[\frac{x^3}{3}\ln x\right]_1^3 - \int_1^3 \underbrace{\frac{x^3}{3}\cdot\frac{1}{x}}_{\frac{x^2}{3}}\,dx\right)$$

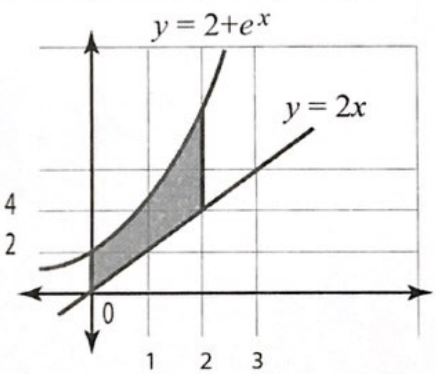
$$= \frac{1}{2}\left(\left[\frac{x^3}{3}\ln x\right]_1^3 - \left[\frac{x^3}{9}\right]_1^3\right) = \frac{1}{2}\left(9\ln 3 - 0 - \left(3 - \frac{1}{9}\right)\right)$$

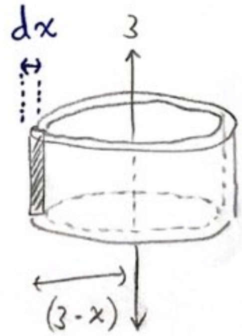
Antiderivatives must be correct.

$$= \boxed{\frac{9}{2}\ln 3 - \frac{3}{2} + \frac{1}{18}} = \frac{9}{2}\ln 3 - \frac{26}{18}$$

$$= 3.4993$$

3. Set up an integral for the volume of the following region rotated around the vertical line $x = 3$. Just set up, don't actually integrate!!





Needs dx!

$$\boxed{\int_0^2 2\pi(3-x)(2+e^x-2x)\,dx}$$

$$= \int_0^2 2\pi\left(3e^x - xe^x + 6 - 8x + 2x^2\right)dx$$

4. Find the indefinite integral. $\int \sin^5(x)\cos^6(x)dx$

$$= \int \sin^4 x \cos^6 x \sin x \, dx$$

$$= \int (\sin^2 x)^2 \cos^6 x \sin x \, dx$$

$$= \int (1-\cos^2 x)^2 \cos^6 x \sin x \, dx$$

$$\left[\begin{array}{l} u = \cos x \\ du = -\sin x \, dx \\ -du = \sin x \, dx \end{array}\right.$$

Careful! The square here covers up a sign error: (u^2-1) is "wrong".

$$= \int (1-u^2)^2 u^6 (-1) du$$

$$= \int (1-2u^2+u^4)(-u^6)\, du$$

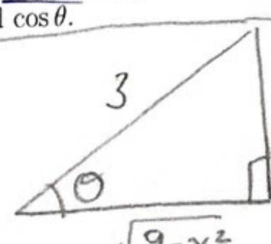
$$= \int -u^6 + 2u^8 - u^{10} \, du$$

$$= -\frac{u^7}{7} + \frac{2u^9}{9} - \frac{u^{11}}{11} + C$$

$$\boxed{= -\frac{\cos^7 x}{7} + \frac{2\cos^9 x}{9} - \frac{\cos^{11} x}{11} + C}$$

5. Rewrite the indefinite integral in terms of θ using trig. sub. Make sure to draw the appropriate triangle! You don't need to finish the integration, just the substitution; your answer should contain only the trig functions $\sin\theta$ and $\cos\theta$.

$$\int \frac{x^2}{2+x\sqrt{9-x^2}}dx$$



$$\left[\begin{array}{l} x = 3\sin\theta \\ dx = 3\cos\theta \, d\theta \end{array}\right.$$

$$\sin\theta = \frac{x}{3}$$

$$\cos\theta = \frac{\sqrt{9-x^2}}{3}$$

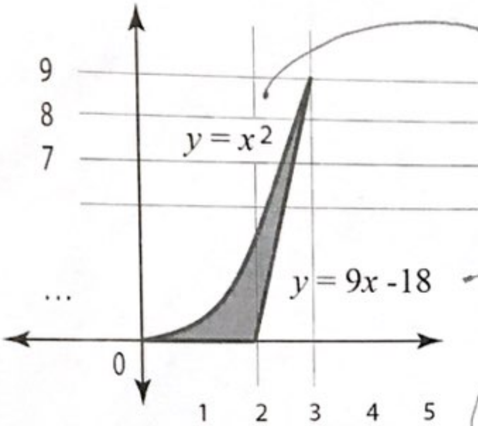
$$3\cos\theta = \sqrt{9-x^2}$$

$$= \int \frac{9\sin^2\theta}{2+3\sin\theta\, 3\cos\theta} 3\cos\theta \, d\theta$$

$$= \boxed{\int \frac{27\sin^2\theta\cos\theta}{2+9\sin\theta\cos\theta} d\theta}$$

can't cancel any further!

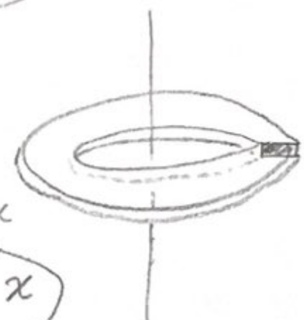
6. Set up an integral for the volume inside the following region rotated around the y-axis. Just set up, don't actually integrate!!



$x = \sqrt{y}$

$y + 18 = 9x$

$\frac{y}{9} + 2 = x$



dy

$$V = \boxed{\int_0^9 \pi \left(\left(\frac{y}{9} + 2 \right)^2 - (\sqrt{y})^2 \right) dy}$$

must be dy

$$= \int_0^9 \pi \left(\left(\frac{y}{9} + 2 \right)^2 - y \right) dy$$

$$\left(\frac{y + 18}{9} \right)^2$$

$$= \int_0^9 \pi \left(\frac{y^2}{81} + \frac{4y}{9} + 4 - y \right) dy$$

$$= \int_0^9 \pi \left(\frac{y^2}{81} - \frac{5y}{9} + 4 \right) dy$$