Combinatorics Review 2 Answers

6.15a) 1,854

6.25) $6! - 8(5!) + 20(4!) - 20(3!) + 7(2!)$

6.26) $6! - 9(5!) + 26(4!) - 26(3!) + 8(2!) - 0(1!)$

7.13) a) $\dfrac{1}{(1-cx)}$

b) $\dfrac{1}{(1+x)}$

c) $(1-x)^{\alpha}$ for α a positive integer.

d) e^{x}

e) e^{-x}

7.19) $\dfrac{x^2}{(1-x)^3}$

7.20) $\dfrac{x^3}{(1-x)^4}$

7.14) a) $\dfrac{x^4}{(1-x^2)^4}$

b) $\dfrac{1}{(1-x^3)^4}$

c) $\dfrac{1+x}{(1-x)^2}$

d) $\dfrac{(x+x^3+x^{11})(x^2+x^4+x^5)}{(1-x)^2}$

e) $\left(\dfrac{1}{1-x} - (1+x+x^2+x^3+x^4+x^5+x^6+x^7+x^8+x^9)\right)^4$

7.17) $\dfrac{1}{1-x^2}(1+x+x^2)\dfrac{1}{1-x^3}(1+x)$; $h_n = n+1$

7.18) $\dfrac{1}{(1-x^2)(1-x^5)(1-x)(1-x^7)}$

7.24) a) $\left(\dfrac{e^x - e^{-x}}{2}\right)^k$

b) $\left(e^x - 1 - x - \dfrac{x^2}{2} - \dfrac{x^3}{6}\right)^k$

c) skip this one!

d) skip this one too!

7.25) $f(x) = \dfrac{1}{4}(e^{4x} - 1)$; $\begin{cases} 4^{n-1} & n \geq 1 \\ 0 & n = 0 \end{cases}$

7.27) $\dfrac{5^n}{4} - 4^n + \dfrac{3}{2}(3^n) - 2^n + \dfrac{1}{4}$

7.28) $\dfrac{5^n}{2} - 4^n + 3^n - 2^n + \dfrac{1}{2}$

7.48 a) $\begin{cases} 4^{(n-1)/2} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$

b) $\dfrac{1+2x}{1-x-x^2}$ (Stop here; the formula isn't too hard to find, but tedious...you start by factoring the denominator but you have to use the quadratic formula and get lots of $\sqrt{5}$ terms.)

c) $\dfrac{1}{12}(-3+4(3^n)-(-3)^n)$

e) $\dfrac{14}{9}-\dfrac{2}{3}(n+1)+\dfrac{1}{9}(-2)^n$

7.51) $\dfrac{2}{(1-3x)}-\dfrac{4x}{(1-x)^2(1-3x)}$ (The o.g.f is good enough for this one.)

11.11) Left to right the graphs are A, B, C, D. We have $A \cong C$, and no other isomorphism. We prove this by numbering the nodes 1-4 (shown in class and on the review) and give the isomorphism $f : A \to C$ by:

$$f(1) = 1; \quad f(2) = 4; \quad f(3) = 3; \quad f(4) = 2$$

We have $A \ncong B$ by finding their degree sequences: $deg.seq.(A) = (5,3,2,2) \neq deg.seq.(B) = (4,3,3,2)$. We have $A \ncong D$ by finding the number of edges: $|E(A)| = 6 \neq |E(D)| = 7$.

From the drawing on the review of graph G:

$deg.seq.(G) = (3,3,3,3,2,2)$

$diam.(G) = 2$

$1,2,4,2,7,6$ is not a walk, and thus not a path.

$5,4,5,6,5,2,5$ is a walk, but is not a trail.

$1,2,4,5,6,7,1$ is a trail and is a cycle.

$5,4,2,5,6,7,4,5$ is not a cycle, and not a trail.