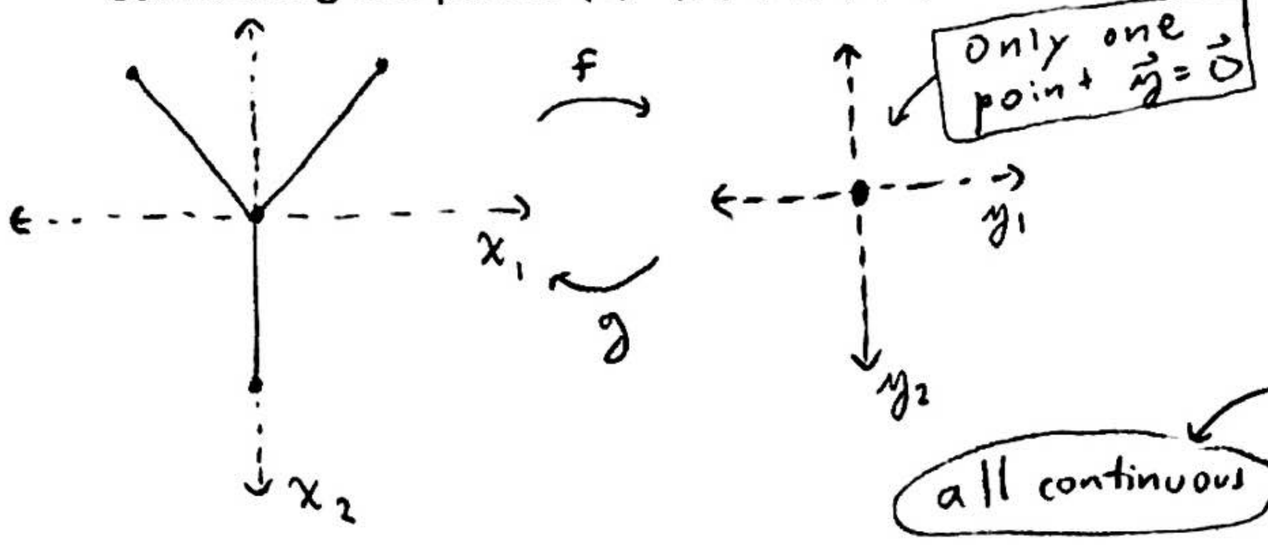
Topology Quiz 7.

Name: Key

1) Show that "Y" is homotopic to "." . That is, find a homotopy equivalence from the "Y" given by connecting the points (0,-1), (0,0), (1,1) and (-1,1) with three line segments; to the single point (0,0).



$f(\vec{x}) = \vec{0}$

$g(\vec{y}) = \vec{y} = \vec{0}$

$F(\vec{x}, t) = t\vec{x}$

$G(\vec{y}, t) = \vec{y}$

Check:

$gf(\vec{x}) = \vec{0}$

$fg(\vec{y}) = \vec{0} = \vec{y}$

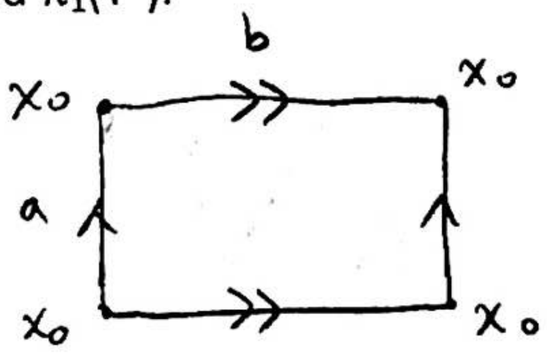
$F(\vec{x}, 0) = \vec{0}$

$G(\vec{y}, 0) = \vec{y} = \vec{0}$

$F(\vec{x}, 1) = \mathbb{1} = G(\vec{y}, 1)$

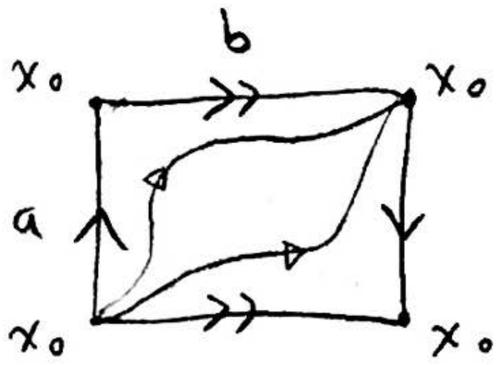
Find the fundamental groups of the torus, Klein bottle, two-holed torus, punctured torus and thrice punctured sphere; each as a group presentation.

2) Find $\pi_1(T^2)$.



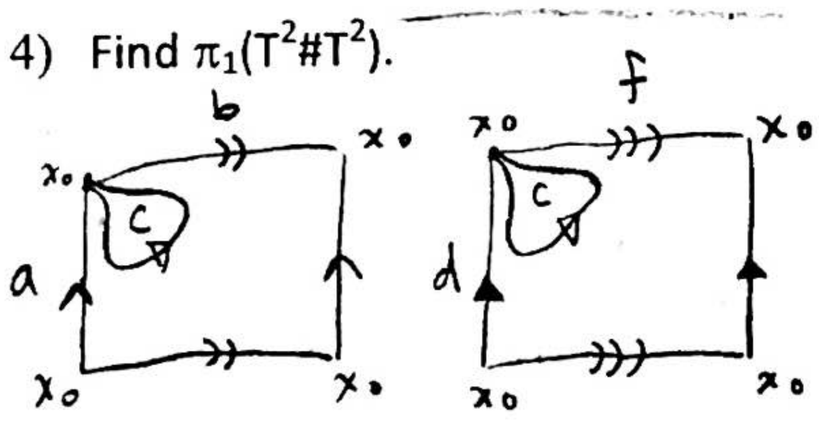
$\pi_1 = \langle a, b \mid ab = ba \rangle$

3) Find $\pi_1(K^2)$.



$\pi_1 = \langle a, b \mid ab = ba^{-1} \rangle$

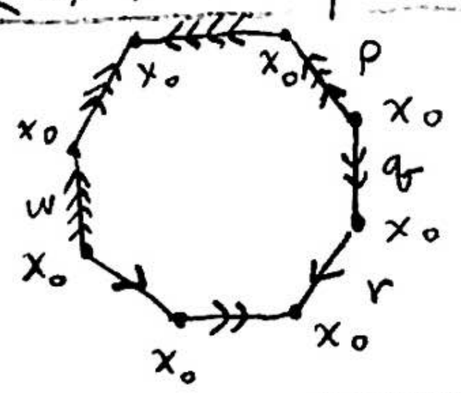
4) Find $\pi_1(T^2 \# T^2)$.



$\pi_1 = \langle a, b, c, d, f \mid acb = ba,\ dcf = fd \rangle$

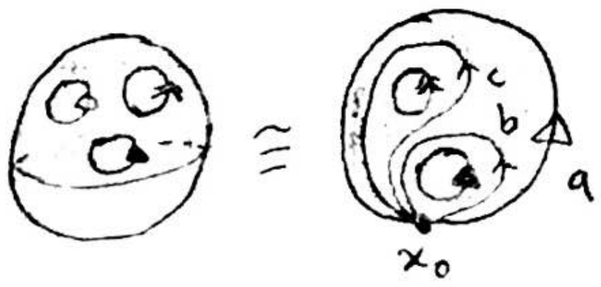
$\cong \langle a, b, d, f \mid ba^{-1}b^{-1}a = fd^{-1}f^{-1}d \rangle$

OR



$\pi_1 = \langle p, q, r, w \mid qrq^{-1}r^{-1} = pwp^{-1}w^{-1} \rangle$

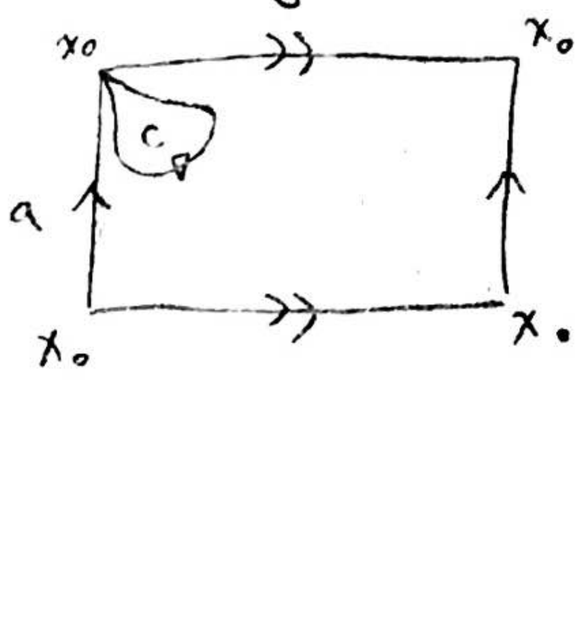
5) Find $\pi_1(S^2\text{-}3D^2)$.



$\pi_1 = \langle a, b, c \mid a = bc \rangle$

$\cong \langle b, c \mid \phi \rangle$

6) Find $\pi_1(T^2\text{-}D^2)$.



$\pi_1 = \langle a, c, b \mid acb = ba \rangle$

$\cong \langle a, b \mid \phi \rangle$

These last two are in fact homotopy equivalent!

(Both retract onto a pair of circles)