Review Questions: Also study quizzes and homework.

I. Let $\mathbf{a} = \left\langle \frac{1}{2}, -1, 0 \right\rangle$, $\mathbf{b} = \langle 4, 1, -1 \rangle$,

and

$\mathbf{r}(t) = \left\langle e^{2t},\ \ln(t+1),\ t + \sec(t) + 2 \right\rangle$.

1. Find $\text{comp}_{\mathbf{b}}\mathbf{a}$.

2. Find $\cos\theta$ where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

3. Find the tangent vector to $\mathbf{r}(t)$ at $t = 0$.

4. Find $(\mathbf{b} \times \mathbf{a}) \cdot \mathbf{a}$.

5. Find the unit tangent to $\mathbf{r}(t)$ at $t = 0$.

6. Find $\frac{1}{2}\mathbf{b} - 2\mathbf{a}$.

7. Find a vector parallel to $\mathbf{a}$ but twice as long as $\mathbf{a}$.

8. Find the area of the parallelogram with sides the vectors $\mathbf{a}$ and $\mathbf{b}$.

II. Let $P = (0, -1, 2)$, $Q = (2, 1, -1)$,

and

$\mathbf{r}(t) = \left\langle \cos(1 - e^t),\ t\ln(1-t),\ t^2 + 2t \right\rangle$.

1. Find symmetric equations for the line through points P and Q
   The vector $\overrightarrow{PQ}$ =_____.

2. Find parametric equations for the tangent line to $\mathbf{r}(t)$ at $t = 0$.
   The vector $\mathbf{r}'(0)$ = _____.

3. Find parametric equations for the line through P and perpendicular to the plane $7 - 3z = 0$.
   The normal vector of the plane $7 - 3z = 0$ is: _____.

4. Find the plane containing P and perpendicular to $\overrightarrow{QP}$.
   The normal vector $\overrightarrow{QP}$ =_____ .

5. Find the plane through the point Q and perpendicular to $\mathbf{r}(0)$.
   The normal vector $\mathbf{r}(0)$ = _____.

   For the following 3:

$$\text{Let } \mathbf{r}(t) = \left\langle e^{2t},\ 2\tan t,\ \ln(t+1) \right\rangle .$$

6. Find the normal component of acceleration, $a_{_N}(0)$ of $\mathbf{r}(t)$.

   $\mathbf{r}''(0)$ = _____.

7. Find the curvature $\kappa(0)$ of $\mathbf{r}(t)$.

   $\mathbf{r}'(0) \times \mathbf{r}''(0)$ = _____.

8. Find the tangential component of acceleration, $a_{_T}(0)$ of $\mathbf{r}(t)$.

   $\mathbf{r}'(0)$ = _____.

9. $$\text{Given} \qquad \mathbf{r}(t) = \left\langle 5e^{2\tan t},\ 1 + \frac{t^3}{t+1},\ t3^t \right\rangle .$$

   Find the tangent line to the curve $\mathbf{r}(t)$ at $t = 0$. Give parametric equations for the line.

10. $$\text{Given} \qquad P = (1,2,2);\quad Q = (0,1,0);\quad R = (0,2,2).$$

    Find the plane through these three points. Simplify the plane equation so that all constants are combined on the right hand side.

11. $$\text{Given} \qquad \mathbf{a} = \langle 1,1,3 \rangle \text{ and } \mathbf{b} = \langle 1,0,0 \rangle .$$

    Find the area of the triangle with these vectors (arrows) as two of its sides. Give your answer as a real number; you may leave any roots as you found them.

12. Given
$$\mathbf{r}(t) = \langle \ln t,\ 2,\ t^2 + t \rangle \text{ and } \mathbf{r}'(t) = \left\langle \frac{1}{t},\ 0,\ 2t+1 \right\rangle \text{ and } \mathbf{r}''(t) = \left\langle \frac{-1}{t^2}, 0, 2 \right\rangle$$
Find $\mathbf{v}(1),\ \mathbf{a}(1),\ a_T(1),\ a_N(1), \kappa(1)$.

13. Given
$$\mathbf{r}'(2) = \langle 0, 0, 3 \rangle, \mathbf{T}'(2) = \langle 1, 3, 0 \rangle, \text{ and } a_T(2) = 5$$
Find $\mathbf{N}(2),\ \mathbf{a}(2),\ a_N(2),\ \kappa(2)$.

14. Given
$$\mathbf{r}'(2) = \langle 6, 0, 3 \rangle, \mathbf{r}''(2) = \langle 1, 1/3, 0 \rangle.$$
Find $\mathbf{T}(2),\ \mathbf{a}(2),\ a_N(2),\ \ a_T(2),\ \kappa(2),\ \mathbf{N}(2)$.

15. Find the x-value of the maximum curvature of $y = x^6$.

16. Find the x-value of the maximum curvature of $y = 3e^x$.

17. Study all quiz questions!