

## Project for Combinatorics, MATH 415 and 515.

### Required steps to completion:

0) Due date: Oct 30. Choose an entry in the OEIS with at least two structures (graphical interpretations) and which allows the following outline to be filled in. Get approval from Dr. Forcey, by emailing the outline, as follows:

- a) The sequence ID, such as [A000108](#)
- b) The first 10 terms: 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862
- c) The two interpretations you plan to illustrate.
  - i) Number of ways to insert  $n$  pairs of parentheses in a word of  $n+1$  letters. E.g., for  $n=2$  there are 2 ways:  $((ab)c)$  or  $(a(bc))$ ; for  $n=3$  there are 5 ways:  $((ab)(cd))$ ,  $((ab)c)d$ ,  $((a(bc))d)$ ,  $(a((bc)d))$ ,  $(a(b(cd)))$ .
  - ii) Consider all the binomial( $2n, n$ ) paths on squared paper that (i) start at  $(0, 0)$ , (ii) end at  $(2n, 0)$  and (iii) at each step, either make a  $(+1, +1)$  step or a  $(+1, -1)$  step. Then the number of such paths that never go below the  $x$ -axis (Dyck paths) is  $C(n)$ .
- d) The four formulas of at least three types: closed, recursive, G.f. and E.g.f.
  - i)  $a(n) = \text{binomial}(2n, n) - \text{binomial}(2n, n-1)$ .
  - ii)  $a(n) = \sum_{k=0}^{n-1} a(k)a(n-1-k)$ .
  - iii)  $a(n) = \prod_{k=2}^n (1 + n/k)$ , if  $n > 1$ .
  - iv) G.f.:  $A(x) = (1 - \sqrt{1 - 4x}) / (2x)$ . G.f.  $A(x)$  satisfies  $A = 1 + xA^2$ .
  - v) E.g.f.:  $\exp(2x) * (I_0(2x) - I_1(2x))$ , where  $I_n$  is Bessel function.

e) (Optional) The team of 2 you plan to work in, and list of responsibilities from steps 1-5 for each partner. Each partner should do some of each step!

The remaining steps are due in two stages: Rough draft by Nov 25, Final draft by Dec. 9.

- 1) Illustrate both structures for the sets counted by  $a_{n1}$ ,  $a_{n2}$  where Dr. Forcey assigns  $n1, n2$ .
- 2) Four demonstrations, subject to the types of the four formulas from step 0.d).
  - Demonstrate a closed formula for  $n1, n2$ , where Dr. Forcey assigns  $n1, n2$ .
  - Demonstrate a recursion for  $j, j+1, j+2$ , where Dr. Forcey assigns  $j$ .
  - Demonstrate the G.f. for  $n1, n2$ , where Dr. Forcey assigns  $n1, n2$ .
  - Demonstrate the E.g.f. for  $n1, n2$ , where Dr. Forcey assigns  $n1, n2$ .
- 3) Experimental bijection: find a rule for one of your structures that maps it to the other, for a set counted by  $a_{n1}$ . Show how the rule works on the next size:  $a_{(n1+1)}$ .
- 4) Experimental superstructure: find a way to organize the set for one of the structures counted by  $a_{n1}$ . Show how the rule works on the next size:  $a_{(n1+1)}$ . Show how the rule works on the other structure, via your bijection from step 3).
- 5) Open question: Describe an open question about your sequence...either by researching or by your own invention.