Calculus II. S15 Quiz 8. Name Key Time

Show all work for full or partial credit. Put a box around your final answer in each part.

1. For each power series, determine the interval of convergence and the radius of convergence.

(a) $\sum_{n=1}^{\infty} \frac{2(x-1)^n}{n3^n}$

$\lim_{n\to\infty} \left| \frac{2(x-1)^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n3^n}{2(x-1)^n} \right|$

$= \lim_{n\to\infty} \frac{n}{3(n+1)} |x-1|$

$= \frac{1}{3}|x-1| < 1$

$\Rightarrow |x-1| < 3$

$\Rightarrow -3 < x-1 < 3$

$\Rightarrow -2 < x < 4$

ENDS:

$x=-2$ $\sum \frac{2(-3)^n}{n(3)^n} = \sum \frac{2(-1)^n}{n}$ $\boxed{\text{converges}}$ by $\boxed{\text{alt series}}$

$x=4$ $\sum \frac{2 \cdot 3^n}{n3^n} = \sum \frac{2}{n}$ $\boxed{\text{diverges}}$ by $\boxed{\text{p-series}}$

$\boxed{\text{i.o.c } [-2,4) \quad \text{r.o.c. } R=3}$

(b) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)^2} \cdot \frac{n^2}{x^n} \right|$

$= \lim_{n\to\infty} \frac{n^2}{n^2+2n+1} |x|$

$= |x| < 1$

$\Rightarrow -1 < x < 1$

ENDS:

$x=-1$ $\sum \frac{(-1)^n}{n^2}$ $\boxed{\text{converges}}$ by alt series

$x=1$ $\sum \frac{1}{n^2}$ $\boxed{\text{converges}}$ by p-series

$\boxed{\text{i.o.c. } [-1,1] \quad \text{r.o.c. } R=1}$

2. Find a power series which converges to the following functions, in the form $\sum_{n=k}^{\infty} a_n x^n$ where $k \geq 0$.

(a) $f(x) = \frac{x^3}{(1-x)^2}$

$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

$\frac{d}{dx}$ $\Rightarrow \frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} nx^{n-1}$

$\cdot x^3$ $\Rightarrow \frac{x^3}{(1-x)^2} = \sum_{n=0}^{\infty} nx^3x^{n-1}$

$= \sum_{n=0}^{\infty} nx^{n+2} = \sum_{n=0}(n+1)x^{n+3}$

$= \boxed{\sum_{n=2}^{\infty} (n-2)x^n}$

OR $\boxed{\sum_{n=3}^{\infty} (n-2)x^n}$

(b) $f(x) = e^{2x}$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\Rightarrow e^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$

$= \boxed{\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}}$