

Calculus I. Fall '19 Test 3 Review (selected answers).

Make sure you also study all the quizzes, then notes and homework examples!

1. Find critical numbers.

a) $f(x) = x^{(4/5)}(x - 4)^2$

$$x = 0, 4, 8/7$$

b) $f(x) = x^2 e^{-3x}$

$$x = 0, 2/3$$

c) $f(x) = x^{-2} \ln x$

$$x = \sqrt{e}$$

d) $f(x) = \frac{1}{x} + 3x$

$$x = \sqrt{1/3}, -\sqrt{1/3}$$

e) (Here you are just given the derivative; assume $f(x)$ exists.) $f'(x) = 2 \cos(3x) + 1; 0 < x < \pi$.

$$x = 2\pi/9, 4\pi/9, 8\pi/9$$

2. Find local min and/or max. Use the first derivative test.

a) $f(x) = x^4 e^{-x}$

local min at $x = 0$ local max at $x = 4$

b) $f(x) = x^5 - x^4$

local max at $x = 0$ local min at $x = 4/5$

c) $f(x) = x^4 - 4x^3$

local min at $x = 3$ neither at $x = 0$

3. Find local min and/or max (just x -values). Use the second derivative test.

a) $f(x) = \frac{1}{3}x^3 - x$

local min at $x = 1$ local max at $x = -1$

b) (Here you are just given the derivative; assume $f(x)$ exists.) $f'(x) = 2 \cos(3x) + 1; 0 < x < \pi$.

local max at $x = 2\pi/9, 8\pi/9$ local min at $x = 4\pi/9$

4. Find absolute min and max on the given interval.

a) $f(x) = x - \ln x, \quad \left[\frac{1}{e}, e\right]$

abs. min $y = 1$ at $x = 1$; abs. max $y = e - 1$ at $x = e$.

b) $f(x) = x + \frac{1}{x}, \quad [\frac{1}{5}, 4]$

abs. min $y = 2$ at $x = 1$; abs. max $y = 26/5$ at $x = 1/5$.

5. Find all inflection points.

a) $f(x) = x^4$.

none

b) $f(x) = x^5 - x^4$.

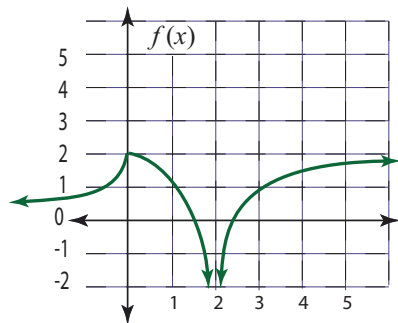
only $x = 3/5$

c) $f(x) = x^4 - 4x^3$.

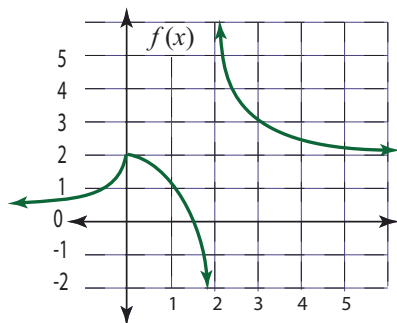
both $x = 0, 2$

6. Describe using graph. May be matching.

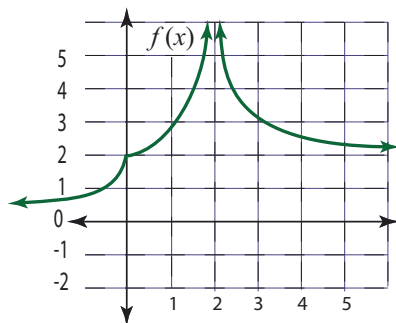
For each of these 6 graphs: use the graph to describe: local min and max, intervals where f is increasing or decreasing, where f is concave up or down, where f' is positive or negative, where f'' is positive or negative, and the points of inflection.



A.



B.



C.

A: $x = 0$ is a local and abs max, and an i.p. , where $f'(0) = DNE$

$f'(x) > 0$ on $(-\infty, 0)$ and $(2, \infty)$

$f'(x) < 0$ on $(0, 2)$.

$f''(x) > 0$ on $(-\infty, 0)$

$f''(x) < 0$ on $(0, 2)$ and $(2, \infty)$

(more in class!)

7. Find limits.

a) $\lim_{x \rightarrow 2} \frac{e^{(x^3)} - e^{(4x)}}{x - 2}$

$8e^8$

b) $\lim_{x \rightarrow 2} \frac{3x - 12}{2x - 1}$

-2

c) $\lim_{x \rightarrow 2} \frac{\sin x}{(x - 2)^4}$

∞

d) $\lim_{x \rightarrow 0} \frac{e^x - e^{3x}}{1 - e^{2x}}$

1

8. Find the length of one side of the largest area rectangle made by using 50 ft of fencing but leaving an opening on one side that is exactly one third of the width of the rectangle on that side.

one side is 15 ft, the other is 12.5 ft
 $(A = 187.5 ft^2)$

9. Find the minimum length of fencing needed to make a rectangle that has only three sides fenced (the last side is left open, along a river) if the area must be 10 square feet.

$$L = 2\sqrt{20} = 4\sqrt{5}.$$

10. Find the radius of the maximum volume open-top cylindrical can that uses 5 square feet of tin.

radius is $\sqrt{\frac{5}{3\pi}}$
 $(V = \frac{5}{3}\sqrt{\frac{5}{3\pi}})$

11. Antiderivatives.

a) $\int (x^4 + \sin x) dx$

$$\frac{x^5}{5} - \cos x + c$$

b) $\int \left(\frac{\sqrt{x} + 1}{x} \right) dx$

$$2\sqrt{x} + \ln x + c$$

$$\text{c) } \int \left(\frac{2e^x + 1}{2} \right) dx$$

$$e^x + \frac{x}{2} + c$$

$$\text{d) } \int (3^x + 1) dx$$

$$x + \frac{3^x}{\ln 3} + c$$

$$\text{e) } \int (\cos x - \sec^2 x) dx$$

$$\sin x - \tan x + c$$

12. Find points demonstrating the MVT on the given interval.

$$\text{a) } f(x) = x - 2 \ln x, \quad [1, 2]$$

$$x = 1/\ln 2$$

$$\text{b) } f(x) = x^4, \quad [0, 2]$$

$$x = \sqrt[3]{2}$$