Calculus II. Quiz 9. Name Key Time

Show all work for full or partial credit. Put a box around your final answer in each part.

1. Find a Maclaurin series for the following functions, by starting with a known fact from the list of known Maclaurin series. You don't need to simplify.

   (a) $f(x) = e^{3x^2}$

   Use $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$

   $$e^{3x^2} = \boxed{\sum_{n=0}^{\infty} \frac{(3x^2)^n}{n!}} = \boxed{\sum_{n=0}^{\infty} \frac{3^n x^{2n}}{n!}}$$

   (b) $f(x) = x\cos(3x)$

   Use $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

   $$x \cos 3x = \boxed{\sum_{n=0}^{\infty} \frac{x(-1)^n (3x)^{2n}}{(2n)!}} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} x^{2n+1}}{(2n)!}}$$

   $$= \boxed{\sum_{n=0}^{\infty} \frac{(-9)^n x^{2n+1}}{(2n)!}}$$

2. Find the $n = 3$ term of the Taylor series for $f(x) = x^3 + e^{2x}$ centered at $a = 5$.

   Use $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$

   $f^{(0)}(x) = x^3 + e^{2x}$

   $f^{(1)}(x) = 3x^2 + 2e^{2x}$

   $f^{(2)}(x) = 6x + 4e^{2x}$

   $f^{(3)}(x) = 6 + 8e^{2x}$

   $f^{3}(5) = 6 + 8e^{10}$

   term:

   $$\boxed{\frac{(6+8e^{10})(x-5)^3}{6}}$$