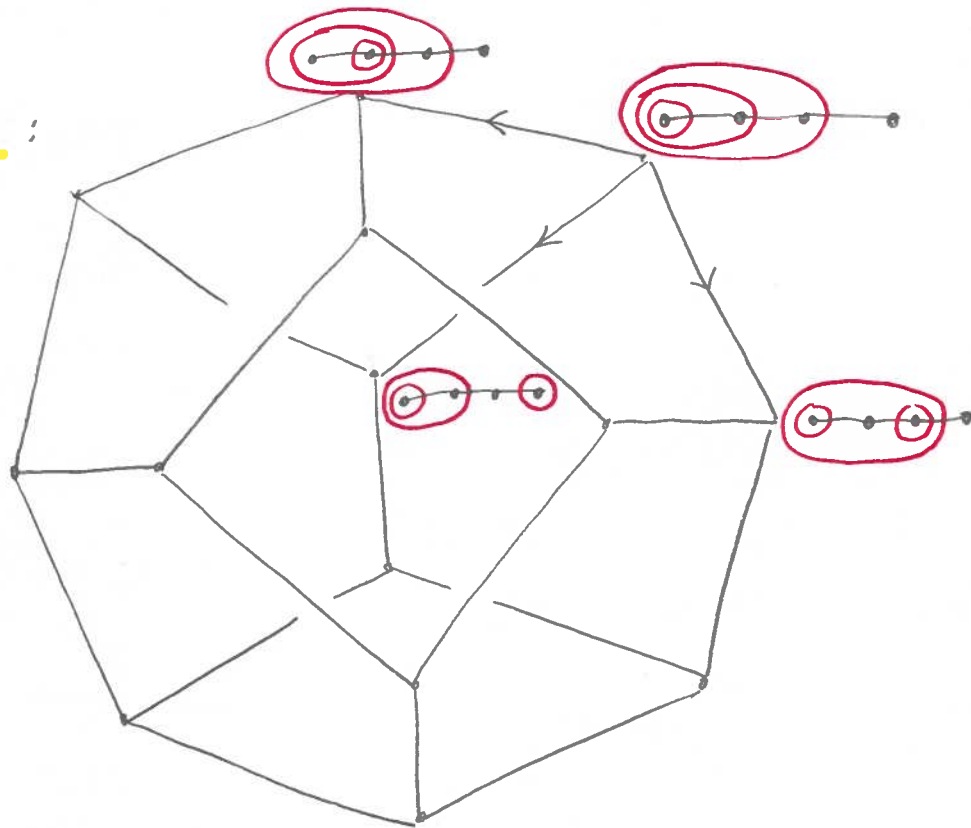


Definitions: Tube  $t$  on a graph:  
induced, connected, subgraph.

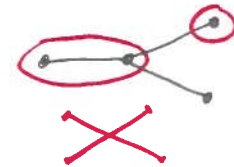
Tubing  $T$ : a set of tubes,  
 each pair nested or disjoint,  
 and unions of them must be induced subgraphs

Path:

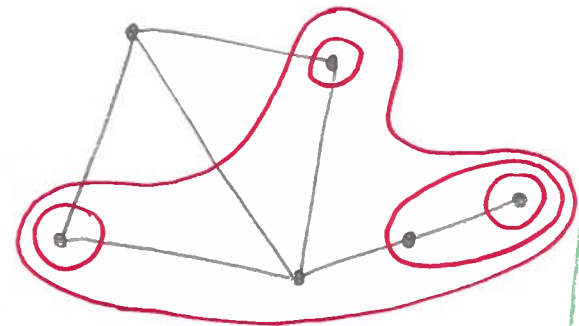


Associahedra

[Stasheff]



Any graph:



Graph - Associahedra.

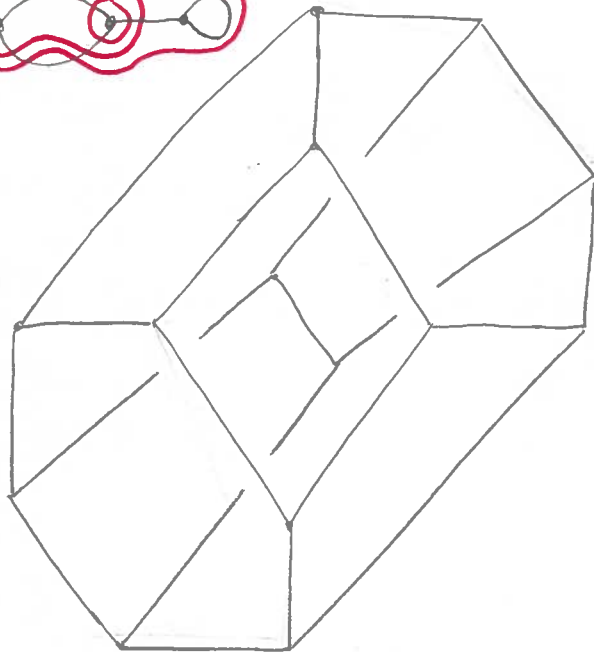
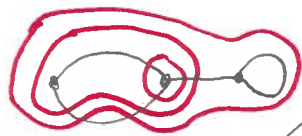
[Carr, Demdoss]

Definitions: **Tube**  $t$  on a pseudograph:

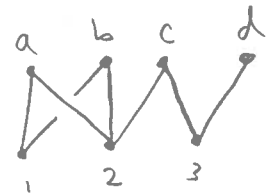
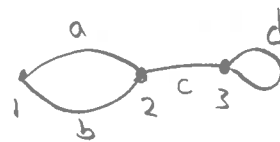
Filled, connected, sub-pseudograph.

↳ [if end-nodes of a (multi) edge(s) are in  $t$  then at least one of those multiedges is in  $t$ ]

**Tubing**  $T$ : a set of pairwise nested or disjoint tubes with filled unions.



Notice:



pseudograph  $\leftrightarrow$  poset  
(Hasse diagram)

Pseudograph Associahedra

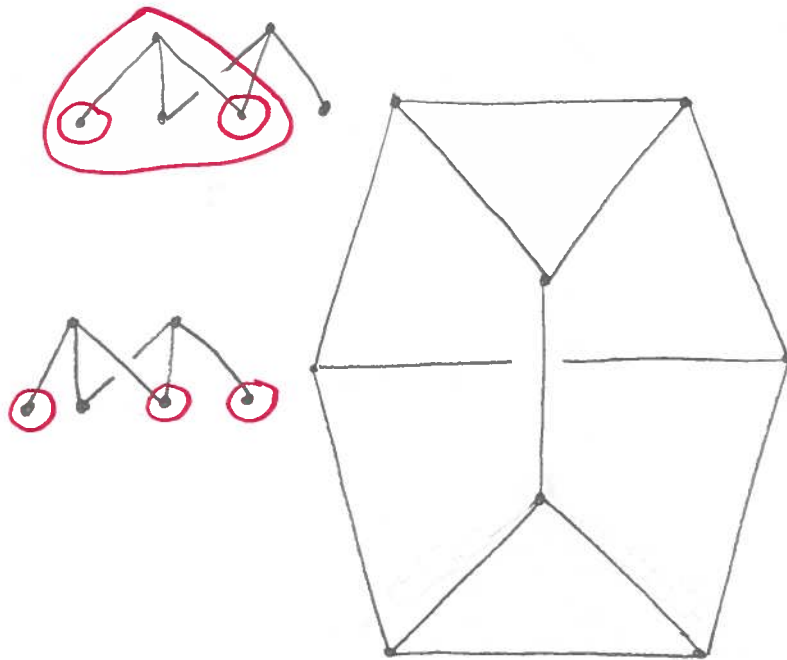
[Carr, Devadoss, F.]

Definitions: Tube  $t$  on a poset:

filled, connected, lower set.

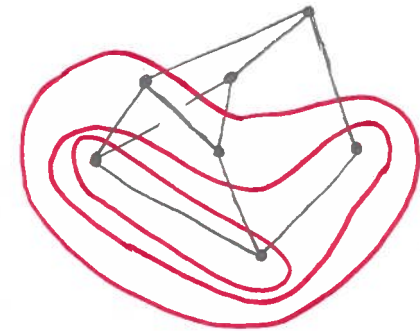
$\hookrightarrow$  [if  $t$  contains the set  $\partial x$  of all  $y < x$   
then  $t$  intersects the set  $b_x$  of  $y$  s.t.  $\partial y = \partial x$ .]

Tubing  $T$ : a set of pairwise nested  
or disjoint tubes with filled unions.



Poset

Associahedra



[Devadoss, F., Reisdorf, Showers]

Questions:

- Find a realization of the poset associahedra.
- Find formulas for  $h$ -vectors,  $h(t, q)$ .
- Relate to Galashin's poset associahedra.

[Katz,  
Olsen]

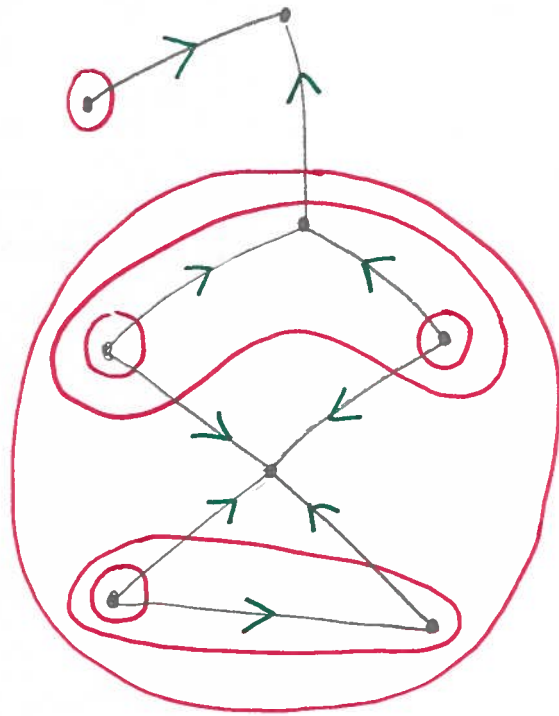


Tubings  $\longrightarrow$  Orientations

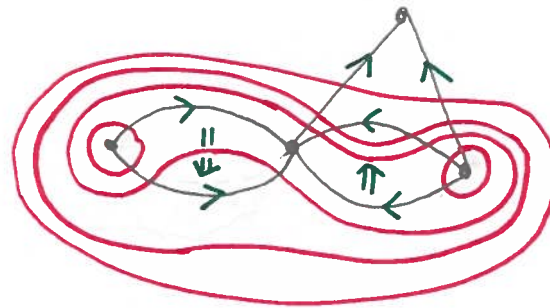
$T \longmapsto \mathcal{O}_T$

where  $\mathcal{O}_T(H)$  chooses the element  $x \in H$   
such that  $x \in t \Rightarrow H \subseteq t$ .

[  $H$  is any bundle  $b_x$ , or any  $\partial y \subseteq b_x$  ]

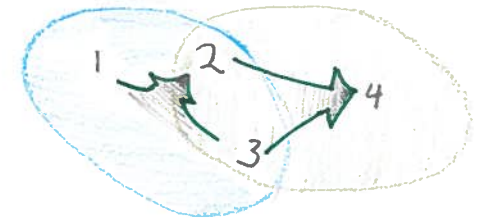
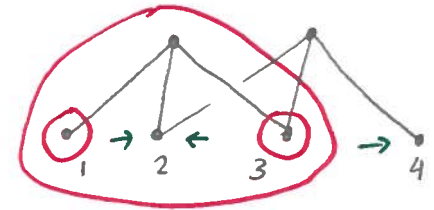


- Directed Acyclic Graph



- Directed Acyclic Pseudo graph

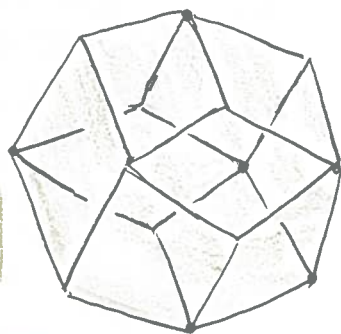
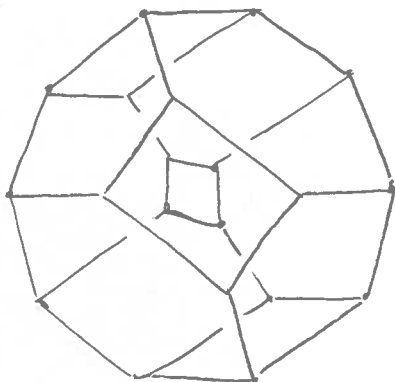
- Pasting diagram



- Directed Acyclic Hypergraph

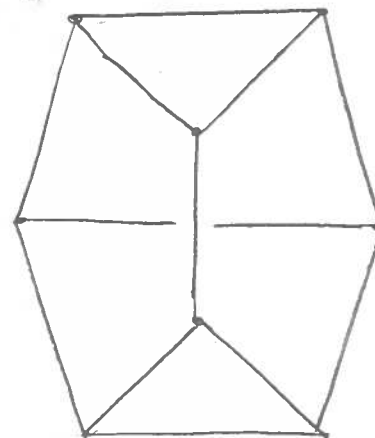
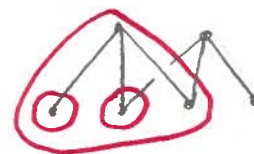
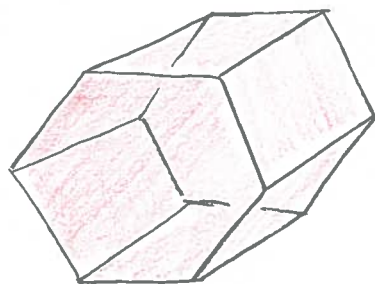
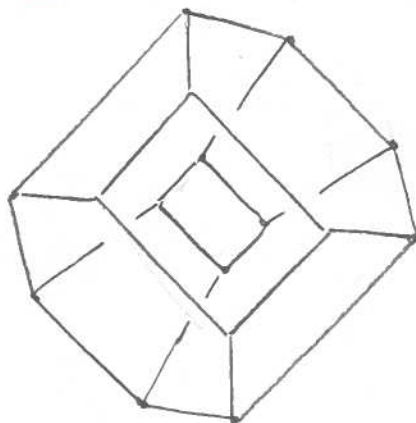
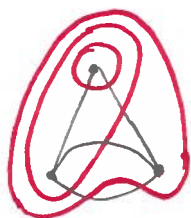
- Poset Gradient

# Polytope fan refinements

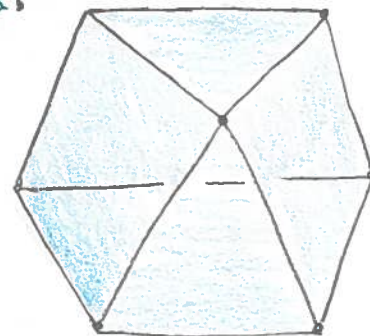


graphical  
zonotope

[Féray, Reiner]



Open Question!



Hypergraphic polytopes

[Benedetti, Bergeron, Machacek]

[Bergeron, Pilaud]

Q.

Are all poset gradient polytopes  
products of hypergraphic polytopes?