3450:221 Calculus I, Final Sample Problems

These problems provide a sample of typical problems you are expected to be able to solve.

1. Limits
   (a) Graphical
      i. Section 2.2, problems 7, 9, 17
      ii. Section 2.6, problem 5
   (b) Computational
      i. $\lim_{x\to 0} \dfrac{|x|}{x}$
      ii. Section 2.3, problems 11, 17, 21, 23
      iii. $\lim_{x\to 1} \ln\left(\dfrac{5-x^2}{1+x}\right)$
      iv. $\lim_{x\to 0} \ln(1+e^{-x})$
      v. $\lim_{x\to 0} \ln(1-e^{-x})$
      vi. $\lim_{x\to \infty} \ln(1+e^{-x})$
      vii. $\lim_{x\to -\infty} \ln(1+e^{-x})$
      viii. Section 2.6, problems 15-33 odd, 32
      ix. Section 3.3, problems 39, 40, 42, 43
      x. $\lim_{x\to 0}(1+x)^{1/x}$
      xi. $\lim_{x\to 0}(1+ax)^{1/x}$
      xii. Section 4.4, problems 15, 25, 27, 35, 45, 50, 51, 57
   (c) Continuity
      i. Section 2.5, problem 20
      ii. Is $f(x)$ below continuous at $x=1$? Why or why not?
      $$f(x)=\begin{cases} x^2, & \text{if } x\le 1;\\ 1+x, & \text{if } x>1 \end{cases}$$
      iii. Find the value of k that makes $f(x)$ below be continuous at $x=3$.
      $$f(x)=\begin{cases} 1+x^2, & \text{if } x\le 3;\\ 2+kx, & \text{if } x>3 \end{cases}$$
2. Derivatives
   (a) Graphical
      i. Section 2.8, problem 3
      ii. Section 4.3, problems 9, 13, 15, 17, 33, 45, 50
      iii. Section 4.5, problems 1, 11, 25
   (b) Definition
      i. Section 2.8, problems 23, 27
      ii. Use the definition of the derivative to find $f'(x)$ for $f(x)=\dfrac{1}{\sqrt{1+x}}$.

(c) Computational. Simplify all derivatives as much as possible.

i. Find $f'(x)$ and $f''(x)$ for $f(x) = \frac{5}{8}x^{8/3} - \frac{5}{8}x^{-3/5} + \pi^2$.

ii. Find $f'(x)$ for $f(x) = e^x + x^e$.

iii. Find $f'(y)$ for $f(y) = y^{1/3}(y-2)^{2/3}$.

iv. Find $f'(x)$ for $f(x) = \dfrac{1+e^{x^2}}{1-e^{-x^2}}$.

v. Find $f'(r)$ for $f(r) = \dfrac{r^2(r+1)^{1/3}}{(r+2)^{2/3}}$.

vi. Find $g'(\theta)$ for $g(\theta) = \dfrac{\sin 3\theta}{\sin 2\theta}$.

vii. Find $h'(x)$ for $dsh(x) = \ln(1+e^{-x})$.

viii. Find $f'(x)$ for $f(x) = 3^{x^2}$.

ix. Find $r'(p)$ for $r(p) = \sec(\ln p + 1)$.

x. Find $f'(x)$ for $f(x) = \ln(\tan(e^{x^2} + 2x))$.

xi. Find $c'(x)$ for $c(x) = \displaystyle\int_1^{e^x} t \ln t^2 \, dt$.

xii. Find $f'(t)$ for $f(t) = e^{3t}\cos 5t$.

xiii. Find $f'(x)$ for $f(x) = (x^2+1)^4(x + \sin(\ln x))^{1/3}$.

xiv. Find $\alpha'(\theta)$ for $\alpha(\theta) = \sin^2(\theta^2)$.

xv. Find $f'(x)$ for $f(x) = \arctan(e^x)$.

xvi. Find $f'(x)$ for $f(x) = \cot^{-1}(e^x)$.

xvii. Find $f'(x)$ for $f(x) = \arctan(3x)$.

xviii. Find $f'(x)$ for $f(x) = \arctan\left(\dfrac{x}{2}\right)$.

xix. Find $g'(x)$ for $g(x) = \dfrac{\sinh x}{\cosh x + 1}$.

(d) Implicit Differentiation.

i. Section 3.5, problems 7, 9, 15, 25

(e) Logarithmic Differentiation.

i. Find y' for $y = \dfrac{e^{-3x}\sqrt{x^2+4}}{(x+2)^2(x+3)^3}$.

ii. Find y' for $y = x^{\sin x}$.

(f) Applications

i. Related Rates: section 3.9, problems 3-6, 13, 15, 17

ii. Use a linear approximation for $f(x) = (8+x)^{1/3}$ to estimate $8.07^{1/3}$.

(g) Extreme Values and Critical Numbers

i. Find the Critical Numbers of $f(x) = x^{2/3}(x+1)^3$.

ii. Find the Critical Numbers of $f(t) = t^{6/7} - 3t^{3/7}$.

iii. Find the Critical Numbers of $g(x) = x^3 + 6x^2 - 15x + 4$.

iv. Extreme Value Theorem: section 4.1, problems 47, 52, 55, 59

v. Mean Value Theorem: section 4.2, problem 11

vi. Optimization: section 4.7, problems 2, 7, 11, 14, 35, 51

3. Integrals

(a) Find $f(x)$ if $f''(x) = \dfrac{15}{16}x^{1/4} - \dfrac{6}{125x^{11/15}}$.

(b) Find $f(x)$ if $f'(x) = \dfrac{1}{3}x^{9/4} - \dfrac{1}{5}x^{-1/5}$ and $f(1) = 3$.

(c) Evaluate these integrals.

i. $I = \displaystyle\int_1^2 \frac{1}{3}x^{3/2} + \frac{1}{2}x^{1/2}\,dx$

ii. $I = \displaystyle\int_1^4 \sqrt{2x+1}\,dx$

iii. $I = \displaystyle\int_0^1 (2r+1)^{17}\,dr$

iv. $I = \displaystyle\int_0^4 |x-1|\,dx$

v. $I = \displaystyle\int_0^{18} \sqrt{\frac{3}{z}}\,dz$

vi. $I = \displaystyle\int \frac{1}{4+x^2}\,dx$

vii. $I = \displaystyle\int \frac{1}{1+9x^2}\,dx$

viii. $I = \displaystyle\int \frac{e^x}{1+e^x}\,dx$

ix. $I = \displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

x. $I = \displaystyle\int \frac{x}{4+x}\,dx$

xi. $I = \displaystyle\int \frac{x}{7+x^2}\,dx$

xii. $I = \displaystyle\int \tan 3x\,dx$

xiii. $I = \displaystyle\int \frac{1}{x}(\ln x + 1)\,dx$

xiv. $I = \displaystyle\int_8^\infty \frac{e^{1/w}}{w^2}\,dw$

xv. $I = \displaystyle\int_0^2 \frac{1}{(4-2x)^{5/2}}\,dx$

xvi. $I = \displaystyle\int \frac{1}{\sqrt{1-x^2}\arcsin x}\,dx$

xvii. $I = \displaystyle\int x^5\sqrt{x^3+2}\,dx$