Calculus II. S15 Quiz 8. Name Key Time

Show all work for full or partial credit. Put a box around your final answer in each part.

1. For each power series, determine the interval of convergence and the radius of convergence.

(a) $\sum_{n=1}^{\infty} \frac{2(x-1)^n}{n3^n}$

$$\lim_{n\to\infty} \left| \frac{2(x-1)^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n3^n}{2(x-1)^n} \right|$$

$$= \lim_{n\to\infty} \frac{n}{3(n+1)} |x-1|$$

$$= \frac{1}{3}|x-1| < 1$$

$$\Rightarrow |x-1| < 3$$

$$\Rightarrow -3 < x-1 < 3$$

$$\Rightarrow -2 < x < 4$$

ENDS:

$x = -2$: $\sum \frac{2(-3)^n}{n(3)^n} = \sum \frac{2(-1)^n}{n}$ converges by alt series

$x = 4$: $\sum \frac{2 \cdot 3^n}{n3^n} = \sum \frac{2}{n}$ diverges by p-series

i.o.c $[-2, 4)$

r.o.c. $R = 3$

(b) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)^2} \cdot \frac{n^2}{x^n} \right|$$

$$= \lim_{n\to\infty} \frac{n^2}{n^2+2n+1} |x|$$

$$= |x| < 1$$

$$\Rightarrow -1 < x < 1$$

ENDS:

$x = -1$: $\sum \frac{(-1)^n}{n^2}$ converges by alt series

$x = 1$: $\sum \frac{1}{n^2}$ converges by p-series

i.o.c. $[-1, 1]$

r.o.c. $R = 1$