Calculus 2 Test 1, Sum '23. Pg. 1

My signature here is to pledge that I have answered each test question from my own knowledge and understanding, without giving or receiving any unauthorized help.

Sign:________________

Name: Key

Time:________________

Date:________________

In each case you must use the type of region that allows a single integral.

1. Find the indefinite integral. No calculator, use the method we learned. $\int \cos^3(x)\sin^{10}(x)dx$

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x\, dx \end{aligned}}$$

$$= \int \cos^2 x \sin^{10} x \cos x \, dx$$

$$= \int (1 - \sin^2 x)\sin^{10} x \cos x \, dx$$

$$= \int (1 - u^2)u^{10}\, du$$

$$= \int u^{10} - u^{12} \quad du$$

$$= \frac{u^{11}}{11} - \frac{u^{13}}{13} + C = \boxed{\frac{\sin^{11} x}{11} - \frac{\sin^{13} x}{13} + C}$$

2. Find antiderivative of $x^4 \ln x$. No calculator: show the steps of the method we used in the notes.

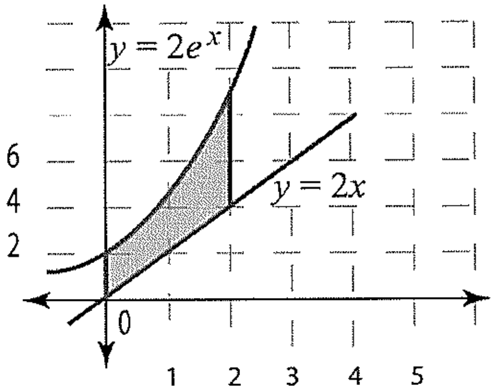
$$\boxed{\begin{aligned} u &= \ln x & dv &= x^4 \\ du &= \frac{1}{x}\,dx & v &= \frac{x^5}{5} \end{aligned}}$$

$$\int x^4 \ln x \, dx$$

$$= \frac{x^5}{5}\ln x - \int \frac{x^5}{5}\cdot\frac{1}{x}\, dx$$

$$= \boxed{\frac{x^5}{5}\ln x - \frac{x^5}{25} + C}$$

3. Set up a single integral for the volume inside the following region rotated around the x-axis. Just set up, don't actually integrate!!

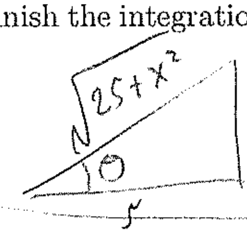


$$V = \boxed{\int_0^2 \pi \left((2e^x)^2 - (2x)^2 \right) dx}$$

$$= \boxed{\int_0^2 \pi \left(4e^{2x} - 4x^2 \right) dx}$$

4. Rewrite the indefinite integral in terms of θ using trig. sub. Make sure to draw the appropriate triangle! You don't need to finish the integration, just the substitution: stop after the integral is all in terms of θ.

$$\int \frac{7 - x^4}{\sqrt{25 + x^2}} dx$$



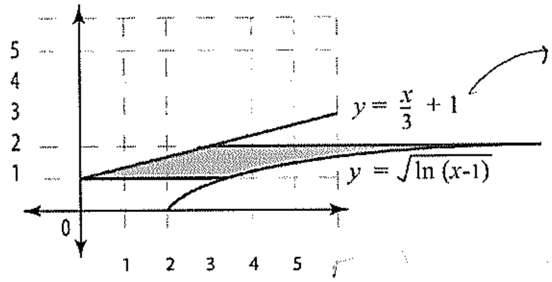
$x = 5 \tan \theta$

$dx = 5 \sec^2 \theta \, d\theta$

$\cos \theta = \frac{5}{\sqrt{25+x^2}}$

$$= \int \frac{7 - 5^4 \tan^4 \theta}{5 \sec \theta} 5 \sec^2 \theta \, d\theta$$

5. Set up an integral for the area inside this shaded region. Just set it up, don't actually integrate. Read the y-axis carefully, since it is not to scale!



$x = 3(y-1) = 3y-3$

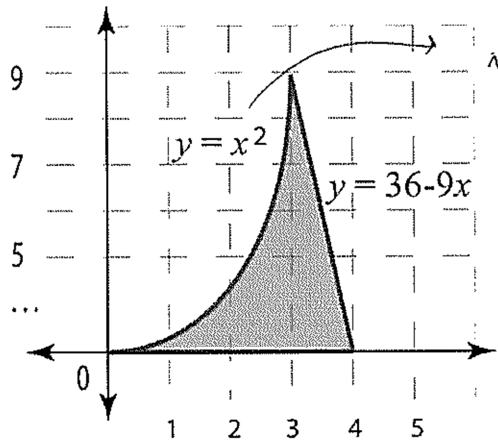
$y^2 = \ln(x-1)$

$e^{y^2} = x-1$

$x = e^{y^2}+1$

$$A = \boxed{\int_1^2 \left(e^{y^2}+1 - (3y-3)\right) dy}$$

6. Set up a single integral for the volume inside the following region rotated around the line $y = 9$. Just set up, don't actually integrate!!



$\sqrt{y} = x$

$\frac{y}{-9} \quad \frac{-36}{-9} = x$

$$V = \boxed{\int_0^9 2\pi (9-y)\left(4 - \frac{y}{9} - \sqrt{y}\right) dy}$$

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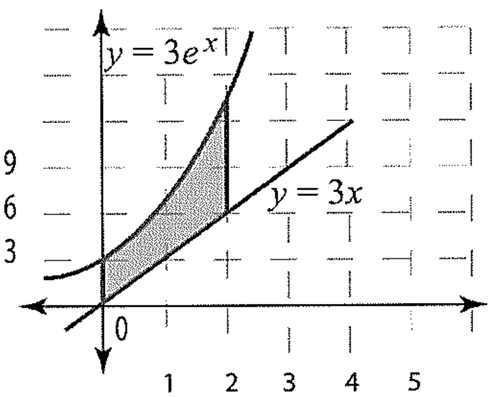
In each case you must use the type of region that allows a single integral.

1. Set up a single integral for the volume inside the following region rotated around the x-axis. Just set up, don't actually integrate!!

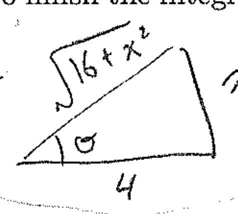


$$\boxed{V = \int_0^2 \pi\left((3e^x)^2 - (3x)^2\right)dx}$$

$$= \boxed{\int_0^2 \pi(9e^{2x} - 9x^2)\, dx}$$

2. Rewrite the indefinite integral in terms of θ using trig. sub. Make sure to draw the appropriate triangle! You don't need to finish the integration, just the substitution: stop after the integral is all in terms of θ.

$$\int \frac{11 - x^5}{\sqrt{16 + x^2}}dx$$



$x = 4\tan\theta$

$dx = 4\sec^2\theta\, d\theta$

$\cos\theta = \dfrac{4}{\sqrt{16+x^2}}$

$4\sec\theta = \sqrt{16+x^2}$

$$= \int \frac{11 - 4^5\tan^5\theta}{4\sec\theta} 4\sec^2\theta\, d\theta$$

$$= \boxed{\int (11 - 4^5\tan^5\theta)\sec\theta\, d\theta}$$

Pg.

3. Find antiderivative of $x^5 \ln x$. No calculator: show the steps of the method we used in the notes.

$$\int x^5 \ln x \, dx$$

$$\boxed{\begin{array}{ll} u = \ln x & dv = x^5 \\ du = \frac{1}{x} dx & v = \frac{x^6}{6} \end{array}}$$

$$\frac{x^6}{6} \ln x - \int \frac{x^6}{6} \cdot \frac{1}{x} \, dx$$

$$= \boxed{\frac{x^6}{6} \ln x - \frac{x^6}{36} + C}$$

4. Find the indefinite integral. No calculator, use the method we learned. $\int \cos^3(x) \sin^6(x) dx$

$$= \int \cos^2 x \sin^6 x \cos x \, dx$$

$$\boxed{\begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array}}$$

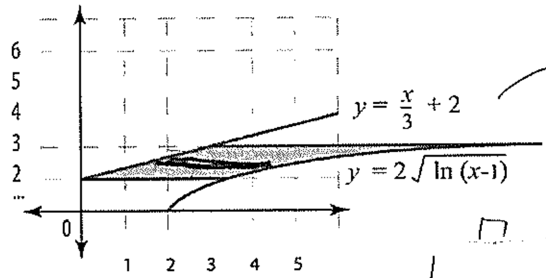
$$= \int (1 - \sin^2 x) \sin^6 x \cos x \, dx$$

$$= \int (1 - u^2) u^6 \, du$$

$$= \int u^6 - u^8 \, du$$

$$= \frac{u^7}{7} - \frac{u^9}{9} = \boxed{\frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C}$$

5. Set up an integral for the area inside this shaded region. Just set it up, don't actually integrate. Read the y-axis carefully, since it is not to scale!



$x = 3(y-2)$

$= 3y - 6$

$\frac{y}{2} = \sqrt{\ln(x-1)}$

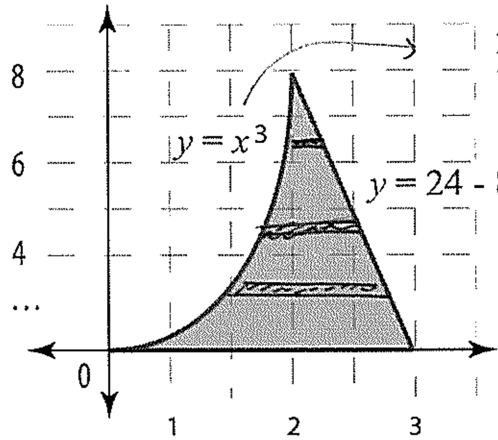
$y^2/4 = \ln x - 1$

$e^{y^2/4} = x - 1$

$e^{y^2/4} + 1 = x$

$$A = \boxed{\int_2^3 \left(e^{y^2/4} + 1 - (3y-6)\right) dy}$$

6. Set up a single integral for the volume inside the following region rotated around the line $y = 9$. Just set up, don't actually integrate!!



$\sqrt[3]{y} = x$

$y - 24 = -8x$

$-\frac{y}{8} + 3 = x$

$$\boxed{\int_0^8 2\pi(9-y)\left(\left(3 - \frac{y}{8}\right) - \sqrt[3]{y}\right) dy}$$

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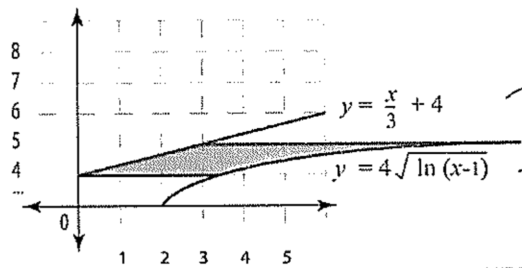
In each case you must use the type of region that allows a single integral.

1. Set up an integral for the area inside this shaded region. Just set it up, don't actually integrate. Read the y-axis carefully, since it is not to scale!



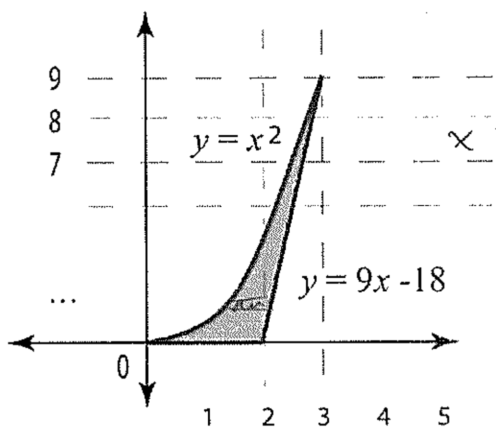
$\to \quad y - 4 = \frac{x}{3} \to x = 3y - 12$

$\to \quad \frac{y^2}{16} = \ln(x-1) \to e^{y^2/4} = x - 1$

$\to x = e^{y^2/4} + 1$

$$\boxed{\int_4^5 \left(e^{y^2/16} + 1 - (3y - 12)\right) dy} = \int_4^5 \left(e^{y^2/16} - 3y + 13\right) dy$$

2. Set up a single integral for the volume inside the following region rotated around the line $y = 9$. Just set up, don't actually integrate!!



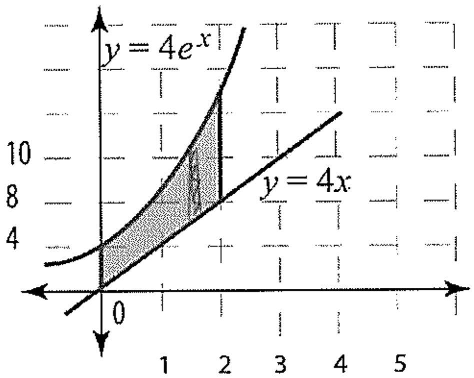
$x = \sqrt{y}$

$y + 18 = 9x$

$x = \frac{y}{9} + 2$

$$\boxed{\int_0^9 2\pi (9 - y)\left(\frac{y}{9} + 2 - \sqrt{y}\right) dy}$$

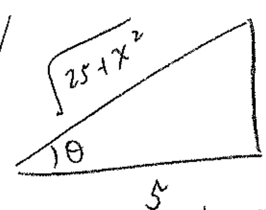
3. Set up a single integral for the volume inside the following region rotated around the x-axis
Just set up, don't actually integrate!!



$$\boxed{\int_0^2 \pi \left((4e^x)^2 - (4x)^2 \right) dx} = \boxed{\int_0^2 \pi \left(16e^{2x} - 16x^2\right) dx}$$

4. Rewrite the indefinite integral in terms of θ using trig. sub. Make sure to draw the appropriate triangle! You don't need to finish the integration, just the substitution.

$$\int \frac{7 - x^3}{\sqrt{25 + x^2}} dx$$



$\cos\theta = \frac{5}{\sqrt{25+x^2}}$

$5\sec\theta = \sqrt{25+x^2}$

$\tan\theta = \frac{x}{5}$

$x = 5\tan\theta$

$dx = 5\sec^2\theta \, d\theta$

$$= \boxed{\int \frac{7 - 125\tan^3\theta}{5\sec\theta} 5\sec^2\theta \, d\theta}$$

$$= \boxed{\int (7 - 125\tan^3\theta)\sec\theta \, d\theta}$$

5. Find antiderivative of $x^3 \ln x$. Show the steps of the method we used in class.

$$\int x^3 \ln x \, dx$$

$$\boxed{\begin{array}{ll} u = \ln x & dv = x^3 dx \\ du = \frac{1}{x} dx & v = \frac{x^4}{4} \end{array}}$$

$$= \underline{x} \ln x - \int \frac{1}{x} \frac{x}{} dx$$

$$= \boxed{\frac{x^4}{4} \ln x - \frac{1}{4}\left(\frac{x^4}{4}\right) + c}$$

$$= \boxed{\frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + c}$$

6. Find the indefinite integral. No calculator, use the method we learned. $\int \cos^3(x) \sin^{12}(x) dx$

$$\boxed{\begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array}}$$

$$= \int \cos^2 x \left(\sin^2 x\right)^6 \cos x \, dx$$

$$= \int (1 - \sin^2 x) \sin^{12} x \cos x \, dx$$

$$= \boxed{\int (1 - u^2) u^{12} du}$$

$$= (u^{12} - u^{14}) du$$

$$= \frac{u^{13}}{13} - \frac{u^{15}}{15} + c \qquad = \boxed{\frac{\sin^{13} x}{13} - \frac{\sin^{15} x}{15} + c}$$

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In each case you must use the type of region that allows a single integral.

1. Find antiderivative of $x^7 \ln x$. No calculator: show the steps of the method we used in the notes.

$$\boxed{\begin{array}{ll} u = \ln x & dv = x^7 \\ du = \frac{1}{x}dx & v = \frac{x^8}{8} \end{array}}$$

$$\int x^7 \ln x \, dx$$

$$= \frac{x^8}{8}\ln x - \int \frac{x^8}{8} \cdot \frac{1}{x} dx$$

$$= \boxed{\frac{x^8}{8}\ln x - \frac{x^8}{64} + c}$$

2. Find the indefinite integral. No calculator, use the method we learned. $\int \cos^3(x)\sin^8(x)dx$

$$\boxed{\begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array}}$$

$$= \int \cos^2 x \sin^8 x \cos x \, dx$$

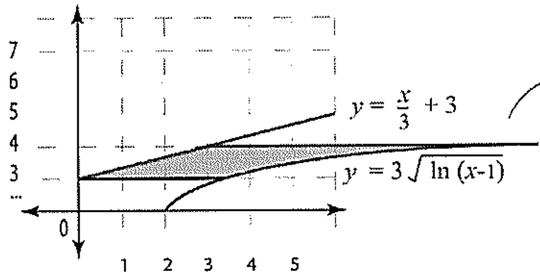
$$= \int (1 - \sin^2 x) \sin^8 x \cos x \, dx$$

$$= \boxed{\int (1 - u^2) u^8 \, du}$$

$$= \int u^8 - u^{10} \, du$$

$$= \frac{u^9}{9} - \frac{u^{11}}{11} + c \quad = \boxed{\frac{\sin^9 x}{9} - \frac{\sin^{11} x}{11} + c}$$

3. Set up an integral for the area inside this shaded region. Just set it up, don't actually integrate. Read the y-axis carefully, since it is not to scale!



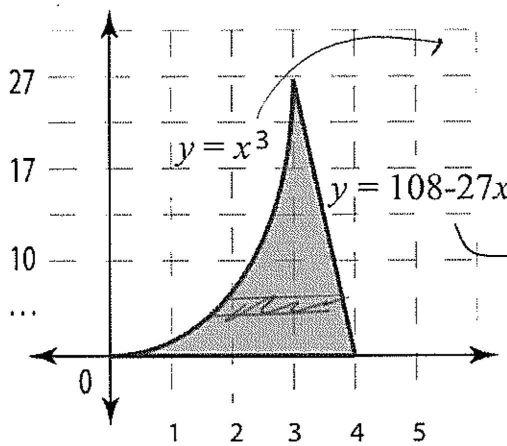
$x = 3(y-3) = 3y - 9$

$\left(\frac{y}{3}\right)^2 = \ln(x-1) \rightarrow e^{y^2/9} = x - 1$

$\rightarrow x = e^{y^2/9} + 1$

$$A = \boxed{\int_3^4 \left(e^{y^2/9} + 1 - (3y-9)\right) dy} = \int_3^4 \left(e^{y^2/9} - 3y + 10\right) d$$

4. Set up a single integral for the volume inside the following region rotated around the line $y = 27$ Just set up, don't actually integrate!!



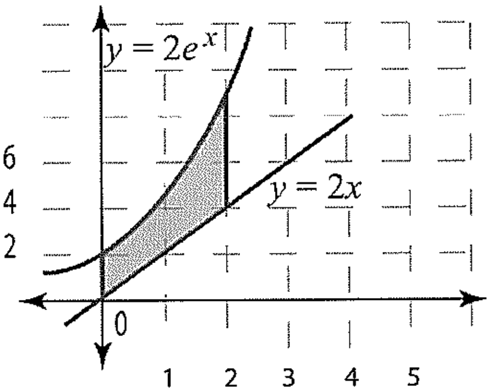
$x = \sqrt[3]{y}$

$x = -\frac{1}{27}(y - 108)$

$= \frac{-y}{27} + 4$

$$V = \boxed{\int_0^{27} 2\pi(27 - y)\left(4 - \frac{y}{27} - \sqrt[3]{y}\right) dy}$$

5. Set up a single integral for the volume inside the following region rotated around the x-axis. Just set up, don't actually integrate!!

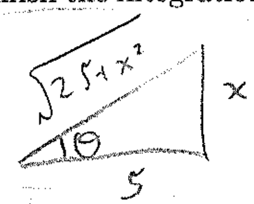


$$V = \int_0^2 \pi \left((2e^x)^2 - (2x)^2 \right) dx$$

$$= \boxed{\int_0^2 \pi \left(4e^{2x} - 4x^2\right) dx}$$

6. Rewrite the indefinite integral in terms of θ using trig. sub. Make sure to draw the appropriate triangle! You don't need to finish the integration, just the substitution: stop after the integral is all in terms of θ.

$$\int \frac{3 - x^7}{\sqrt{25 + x^2}} dx$$



$x = 5\tan\theta$

$dx = 5\sec^2\theta \, d\theta$

$\cos\theta = \dfrac{5}{\sqrt{25+x^2}}$

$$\int \frac{3 - 5^7 \tan^7\theta}{5\sec\theta} 5\sec^2\theta \, d\theta$$

$$= \boxed{\int \left(3 - 5^7 \tan^7\theta\right) \sec\theta \, d\theta}$$