Discrete final review.

1. Given universe $\mathcal{U} = \{3, 4, 5, 7, 9, 10, 11, 23\}$; $A = \{5, 7, 9, 10, 11, 23\}$; and $B = \{5, 3, 7\}$. Find the following:

   - $\overline{\overline{(B - A)} \cap B}$

   - $\overline{A \cup \overline{B}}$

   - $|B \times \mathcal{P}(A \times B)|$

   - $|A \cup B|$

2. Find the number of PINs using $\{0, ..., 9\}$, with 7 digits, no repeated numbers, where the first digit cannot be 3 and the fourth digit cannot be 5.

3. Find the number of DNA sequences using $\{A, G, T, C\}$, of length 5, where the first and second location cannot repeat, and the first location cannot be A.

4. Use contradiction to prove: $\forall z \in \mathbb{Z},\ z^2 \equiv 7 \pmod 6 \Rightarrow z$ is odd.
   a) Negate the statement.

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   b) Write the assumptions, translated to algebraic equations.

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   c) We will use the assumptions to show the falsehood $2|7$, which is translated $7 = 2w$ for some integer w. Show the proof steps, from assumptions to $2|7$.

5. Use a direct proof to prove: $\forall z \in \mathbb{Z},\ z \mod 3 = 2 \Rightarrow 9|(3z^2 + 6)$.
   a) Write the assumption, translated to an algebraic equation.

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   b) Write what we want to show, translated to an algebraic equation.

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   c) Proof steps:

6. Use induction to prove: $\forall n \in \mathbb{Z}$, if $n \geq 4$ then $3|(2^{2n-5} + 1)$.
   a) Show the base case.

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   b) State the induction assumption, translate to algebraic equation.

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   c) State what we need to show, translate to algebraic equation.

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   d) Do the proof steps.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

7. Consider the sequence $a_n = (n^2 + 10) \mod 12$; starting at $n = 1$. Use it to encrypt the word SAT. Your answer will be the new word.

n	*letter*	std. num.	find a_n	encrypt	letter
1	S				
2	A				
3	T				

8. Consider the one-time-pad sequence $a_n = (2, 8, 11)$; starting at $n = 1$. It has been used to encrypt a message, and the encrypted message is DDF. Use the same sequence to decrypt and find the original word.

n	*letter*	std. num.	a_n	decrypt	letter
1	D				
2	D				
3	F				

9. Consider the BBS (Blum Blum Shub) sequence $a_n = (a_{n-1})^2 \mod pq$; with $a_0 = 2$ and with $p = 5, q = 5$. Starting at $n = 1$, use this sequence to encrypt the binary number 1010. Your answer will be the new binary number. You may use either method from class.

n	*bit*	find a_n	encrypt	bit
1	1			
2	0			
3	1			
4	0			

10. From 7 library books, how many subsets of exactly 3 books are there? Answer as a whole number.

11. For 4 books and 9 shelves of a bookcase, find the number of ways to distribute the books on the shelves (just in piles, not in order.)

12. For 9 books on 4 shelves of a bookcase, find the number of ways to place the books on the shelves in ordered rows.

13. For 8 books and 6 shelves of a bookcase, find the number of plans for shelving, where at least 3 books are planned for the top shelf (a plan only tells how many books on each shelf.)

14. For 4 books and 9 shelves of a bookcase, find the number of ways to distribute the books on the shelves, where the bottom shelf has at most one book (just in piles, not in order.)

15. For 7 books and 9 shelves of a bookcase, find the number of ways to place the books on the shelves in ordered rows, where the bottom shelf has no more than 2 books.

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2.

Version B

1. Given the original statement of implication: $((x < 2y) \wedge (x \geq 5)) \Rightarrow ((y > 8) \vee (3x \text{ is even}))$.

- Find the contrapositive of the original; write it without "not" and without "∼."

- Find the negation of the original; write it without "not" and without "∼."

2. Given the statement:

$\forall x \in \mathbb{Z}, \exists y \in \mathbb{N}$ s.t. $(yx \geq y + 7) \Rightarrow ((x \text{ is even }) \wedge \ (y + x \text{ is odd }))$.

- Find its negation; write it without "not" and without "∼."

3. Given the original statement "If you have salt then you have sodium." Answer the following without "not" and without "∼.'

- Write the original statement using the word sufficient.

- Write the converse of the original using the words "only if".

- Write the contrapositive of the original using the word necessary.

4 . Translate the following numbers.

- binary: 1101011 hexidecimal:____________________
- binary: 1111010 hexidecimal:____________________
- hexidecimal: FA3 binary:____________________
- binary: 1010 decimal:____________________

5. Suppose that $P = F$ (false) and $Q = T$ (true). Find whether each of these statements is true (T) or false (F). Put a box around each final answer of T or F.

- $Q \wedge \sim (P \Rightarrow Q)$.
- $((\sim P) \wedge Q) \iff ((\sim P) \vee \sim Q)$.
- $(\sim (P \vee Q)) \Rightarrow P$.
- $(P \iff (\sim Q)) \Rightarrow ((\sim P) \Rightarrow P)$.

6. For $S = \{1, -3, -4, -12\}$, find an example making the following true:

$\exists x \in S$ *s.t.* $((x \mid 7) \vee (|x| > 3)) \Rightarrow ((x \text{ is odd }) \wedge (x \leq -1))$

7. Given the inputs of each circuit, fill in the outputs.