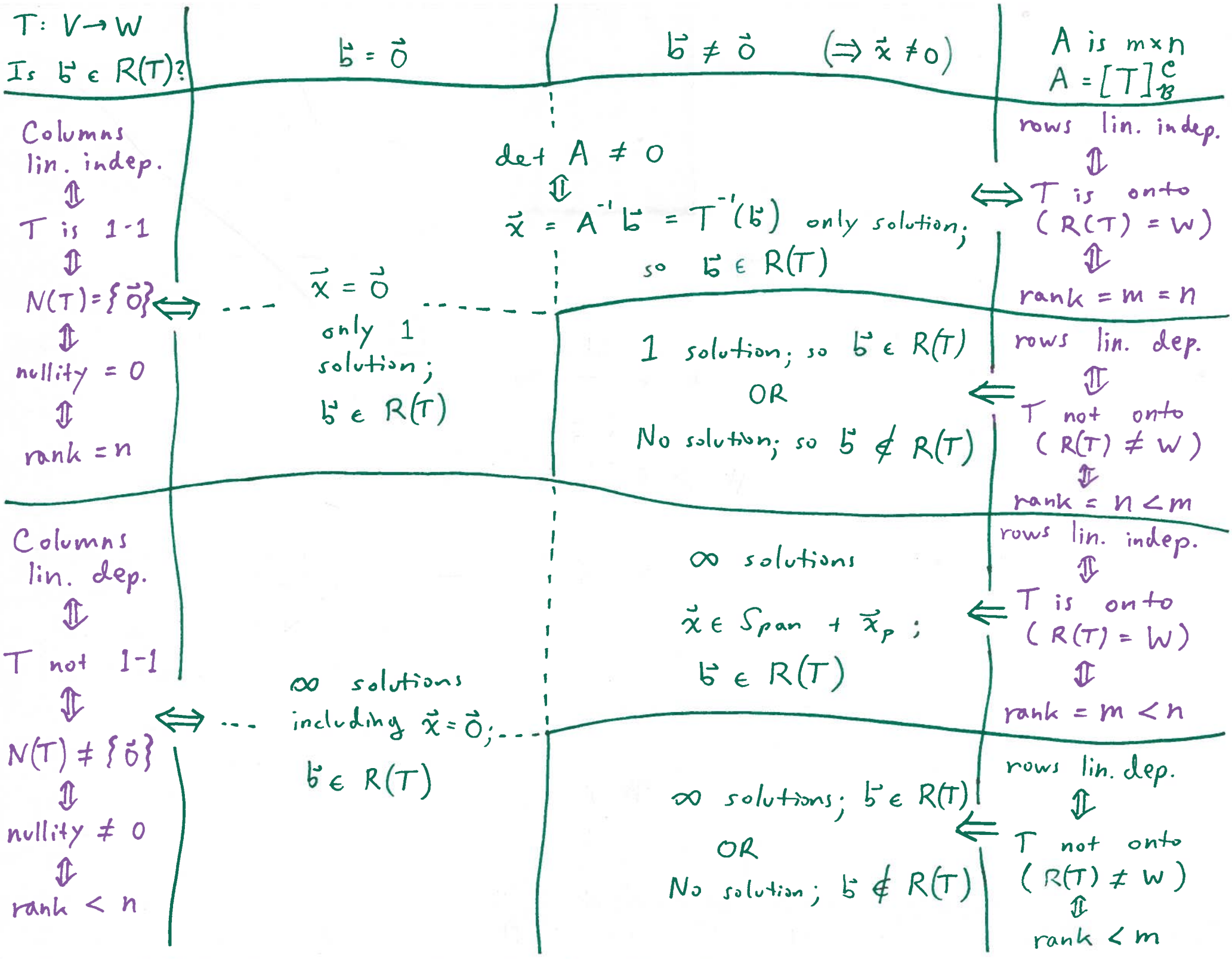
T: V → W
Is $\vec{b} \in R(T)$?
$\vec{b} = \vec{0}$
$\vec{b} \neq \vec{0}$ $(\Rightarrow \vec{x} \neq 0)$
A is m×n
$A = [T]_{\mathcal{B}}^{\mathcal{C}}$
Columns lin. indep.
⇕
T is 1-1
⇕
$N(T) = \{\vec{0}\}$
⇕
nullity = 0
⇕
rank = n
$\vec{x} = \vec{0}$
only 1 solution;
$\vec{b} \in R(T)$
$\det A \neq 0$
⇕
$\vec{x} = A^{-1}\vec{b} = T^{-1}(\vec{b})$ only solution;
so $\vec{b} \in R(T)$
rows lin. indep.
⇕
$\Leftrightarrow$ T is onto
$(R(T) = W)$
⇕
rank = m = n
1 solution; so $\vec{b} \in R(T)$
OR
No solution; so $\vec{b} \notin R(T)$
$\Leftarrow$
rows lin. dep.
⇕
T not onto
$(R(T) \neq W)$
⇕
rank = n < m
Columns lin. dep.
⇕
T not 1-1
⇕
$N(T) \neq \{\vec{0}\}$
⇕
nullity ≠ 0
⇕
rank < n
∞ solutions
including $\vec{x} = \vec{0}$;
$\vec{b} \in R(T)$
∞ solutions
$\vec{x} \in$ Span + $\vec{x}_p$;
$\vec{b} \in R(T)$
rows lin. indep.
⇕
$\Leftarrow$ T is onto
$(R(T) = W)$
⇕
rank = m < n
∞ solutions; $\vec{b} \in R(T)$
OR
No solution; $\vec{b} \notin R(T)$
rows lin. dep.
⇕
$\Leftarrow$ T not onto
$(R(T) \neq W)$
⇕
rank < m