(1) From the following matrices: decide which one could not represent the derivative $\mathcal{P}^3 \to \mathcal{P}^3$. Explain your answer.

$$B = \begin{bmatrix} 0 & 0 & 1 & 3 \\ 0 & 4 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 3 & -3 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 0 & 3 & 0 \\ 0 & 0 & 4 & 3 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

(2) From the following matrices: decide whether either, neither or both represent an onto transformation. Explain your answer.

$$D = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 3 & 0 & 0 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

(3) Solve $2x\mathbf{y''} + 3x^2\mathbf{y'} - 3x\mathbf{y} = 18x^4 + 36x^2 + 6x^3 + 5x$ for $\mathbf{y} = f(x) \in \mathcal{P}^3$. Use a matrix representative of $T: \mathcal{P}^3 \to \mathcal{P}^4$, with standard bases, where $T(\mathbf{y}) = 2x\mathbf{y''} + 3x^2\mathbf{y'} - 3x\mathbf{y}$. Write your answer as a polynomial with the homogeneous and the particular solution.