

3450:221 Calculus I, Final Sample Problems

These problems provide a sample of typical problems you are expected to be able to solve.

1. Limits

(a) Graphical

- i. Section 2.2, problems 7, 9, 17
- ii. Section 2.6, problem 5

(b) Computational

- i. $\lim_{x \rightarrow 0} \frac{|x|}{x} = \boxed{\text{DNE}}$
- ii. Section 2.3, problems 11, 17, 21, 23
- iii. $\lim_{x \rightarrow 1} \ln \left(\frac{5 - x^2}{1 + x} \right) = \boxed{\ln 2}$
- iv. $\lim_{x \rightarrow 0} \ln(1 + e^{-x}) = \boxed{\ln 2}$
- v. $\lim_{x \rightarrow 0} \ln(1 - e^{-x}) = \boxed{\text{DNE}}$
- vi. $\lim_{x \rightarrow \infty} \ln(1 + e^{-x}) = \boxed{0}$
- vii. $\lim_{x \rightarrow -\infty} \ln(1 + e^{-x}) = \boxed{\infty}$
- viii. Section 2.6, problems 15-33 odd, 32
- ix. Section 3.3, problems 39, 40, 42, 43
- x. $\lim_{x \rightarrow 0} (1 + x)^{1/x} = \boxed{e}$
- xi. $\lim_{x \rightarrow 0} (1 + ax)^{1/x} = \boxed{e^a}$
- xii. Section 4.4, problems 15, 25, 27, 35, 45, 50, 51, 57

(c) Continuity

- i. Section 2.5, problem 20
- ii. Is $f(x)$ below continuous at $x = 1$? Why or why not?
 $f(x) = \begin{cases} x^2, & \text{if } x \leq 1; \\ 1 + x, & \text{if } x > 1 \end{cases} \quad \boxed{\text{No}}, \quad \lim_{x \rightarrow 1^-} f(x) = 1, \quad \lim_{x \rightarrow 1^+} f(x) = 2$
- iii. Find the value of k that makes $f(x)$ below be continuous at $x = 3$.
 $f(x) = \begin{cases} 1 + x^2, & \text{if } x \leq 3; \\ 2 + kx, & \text{if } x > 3 \end{cases} \quad 2 + k3 = 1 + 9$

$$k3 = 8$$

$$\boxed{k = 8/3}$$

2. Derivatives

(a) Graphical

- i. Section 2.8, problem 3
- ii. Section 4.3, problems 9, 13, 15, 17, 33, 45, 50
- iii. Section 4.5, problems 1, 11, 25

(b) Definition

- i. Section 2.8, problems 23, 27

- ii. Use the definition of the derivative to find $f'(x)$ for $f(x) = \frac{1}{\sqrt{1+x}}$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \boxed{-\frac{1}{2}(1+x)^{-3/2}}$$

(c) Computational. Simplify all derivatives as much as possible.

- i. Find $f'(x)$ and $f''(x)$ for $f(x) = \frac{5}{8}x^{8/3} - \frac{5}{8}x^{-3/5} + \pi^2$. $\left\{ \begin{array}{l} f' = \frac{5}{3}x^{5/3} + \frac{3}{8}x^{-8/5} \\ f'' = \frac{25}{9}x^{2/3} - \frac{3}{5}x^{-13/5} \end{array} \right.$
- ii. Find $f'(x)$ for $f(x) = e^x + x^e$. $f' = e^x + e x^{e-1}$
- iii. Find $f'(y)$ for $f(y) = y^{1/3}(y-2)^{2/3}$.
- iv. Find $f'(x)$ for $f(x) = \frac{1+e^{x^2}}{1-e^{-x^2}}$.
- v. Find $f'(r)$ for $f(r) = \frac{r^2(r+1)^{1/3}}{(r+2)^{2/3}}$.
- vi. Find $g'(\theta)$ for $g(\theta) = \frac{\sin 3\theta}{\sin 2\theta}$. $h' = \frac{1}{1+e^{-x}} (-e^{-x})$
- vii. Find $h'(x)$ for $h(x) = \ln(1+e^{-x})$.
- viii. Find $f'(x)$ for $f(x) = 3^{x^2}$. $r' = \sec(\ln p + 1) \tan(\ln p + 1) \left(\frac{1}{p}\right)$
- ix. Find $r'(p)$ for $r(p) = \sec(\ln p + 1)$.
- x. Find $f'(x)$ for $f(x) = \ln(\tan(e^{x^2} + 2x))$. $c' = e^x \ln(e^{2x}) (e^x)$
- xi. Find $c'(x)$ for $c(x) = \int_1^{e^x} t \ln t^2 dt$.
- xii. Find $f'(t)$ for $f(t) = e^{3t} \cos 5t$. $\alpha' = 2 \sin(\theta^2) \cos(\theta^2) (2\theta)$
- xiii. Find $f'(x)$ for $f(x) = (x^2 + 1)^4 (x + \sin(\ln x))^{1/3}$.
- xiv. Find $\alpha'(\theta)$ for $\alpha(\theta) = \sin^2(\theta^2)$.
- xv. Find $f'(x)$ for $f(x) = \arctan(e^x)$.
- ~~xvi. Find $f'(x)$ for $f(x) = \cot^{-1}(e^x)$.~~
- xvii. Find $f'(x)$ for $f(x) = \arctan(3x)$.
- xviii. Find $f'(x)$ for $f(x) = \arctan\left(\frac{x}{2}\right)$. $f' = \frac{1}{1 + \left(\frac{x}{2}\right)^2} \left(\frac{1}{2}\right)$
- xix. Find $g'(x)$ for $g(x) = \frac{\sinh x}{\cosh x + 1}$.

(d) Implicit Differentiation.

- i. Section 3.5, problems 7, 9, 15, 25

(e) Logarithmic Differentiation.

- i. Find y' for $y = \frac{e^{-3x} \sqrt{x^2 + 4}}{(x+2)^2 (x+3)^3}$. $y' = x^{\sin x} \left(\cos x \ln x + \frac{\sin x}{x} \right)$
- ii. Find y' for $y = x^{\sin x}$.

(f) Applications

- i. Related Rates: section 3.9, problems 3-6, 13, 15, 17
- ii. Use a linear approximation for $f(x) = (8+x)^{1/3}$ to estimate $8.07^{1/3}$.

(g) Extreme Values and Critical Numbers

- i. Find the Critical Numbers of $f(x) = x^{2/3}(x+1)^3$. $x = 0, x = -1, x = \frac{-2}{11}$
- ii. Find the Critical Numbers of $f(t) = t^{6/7} - 3t^{3/7}$.
- iii. Find the Critical Numbers of $g(x) = x^3 + 6x^2 - 15x + 4$.
- iv. Extreme Value Theorem: section 4.1, problems 47, 52, 55, 59
- v. Mean Value Theorem: section 4.2, problem 11

vi. Optimization: section 4.7, problems 2, 7, 11, 14, 35, 51

3. Integrals

~~(a) Find $f(x)$ if $f''(x) = \frac{15}{16}x^{1/4} - \frac{6}{125x^{11/15}}$.~~

~~(b) Find $f(x)$ if $f'(x) = \frac{1}{3}x^{9/4} - \frac{1}{5}x^{-1/5}$ and $f(1) = 3$.~~

(c) Evaluate these integrals.

i. $I = \int_1^2 \frac{1}{3}x^{3/2} + \frac{1}{2}x^{1/2} dx = \boxed{\frac{6\sqrt{2}}{5} - \frac{7}{15}}$

ii. $I = \int_1^4 \sqrt{2x+1} dx$

iii. $I = \int_0^1 (2r+1)^{17} dr$

iv. $I = \int_0^4 |x-1| dx = \boxed{5}$

v. $I = \int_0^{18} \sqrt{\frac{3}{z}} dz$

vi. $I = \int \frac{1}{4+x^2} dx \longrightarrow \boxed{\frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C}$

vii. $I = \int \frac{1}{1+9x^2} dx$

viii. $I = \int \frac{e^x}{1+e^x} dx \longrightarrow \boxed{\ln(1+e^x) + C}$

ix. $I = \int \frac{e^x}{1+e^{2x}} dx$

x. $I = \int \frac{x}{4+x} dx \longrightarrow \boxed{4+x - 4 \ln|x+4| + C}$

xi. $I = \int \frac{x}{7+x^2} dx$

xii. $I = \int \tan 3x dx \longrightarrow \boxed{-\frac{1}{3} \ln(\cos(3x)) + C}$

xiii. $I = \int \frac{1}{x} (\ln x + 1) dx$

xiv. $I = \int_8^\infty \frac{e^{1/w}}{w^2} dw \xrightarrow{\text{indefinite}} \boxed{-e^{1/w} + C}$

xv. $I = \int_0^2 \frac{1}{(4-2x)^{5/2}} dx$

xvi. $I = \int \frac{1}{\sqrt{1-x^2} \arcsin x} dx$

xvii. $I = \int x^5 \sqrt{x^3+2} dx \longrightarrow \boxed{\frac{2}{45} (x^3+2)^{3/2} (3x^3-4) + C}$