Calculus II. S15 Quiz 7. Name Key Time

Show all work for full or partial credit. Put a box around your final answer in each part.

1. For each series, what does the ratio test tell us? [converge absolutely, diverge, or inconclusive]. Show your work by finding the limit.

(a) $\sum_{n=1}^{\infty} \frac{(-2)^n}{(n+2)!}$

$$\lim_{n\to\infty} \left| \frac{2^{n+1}}{(n+1+2)!} \cdot \frac{(n+2)!}{2^n} \right|$$

[absolute value removes $(-1)^n$]

$$= \lim_{n\to\infty} \frac{2^n 2 (n+2)!}{(n+3)(n+2)!\, 2^n}$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{2}{n+3} = \boxed{0 < 1}, \text{ so } \boxed{\text{converges absolutely}}$$

(b) $\sum_{n=1}^{\infty} \frac{2^{(n^2)}}{n!}$

$$\lim_{n\to\infty} \left| \frac{2^{(n+1)^2}}{(n+1)!} \cdot \frac{n!}{2^{n^2}} \right|$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{2^{n^2+2n+1}\, n!}{(n+1)\, n!\, 2^{n^2}} = \lim_{n\to\infty} \frac{2^{2n+1}}{n+1}$$

Use L'Hospitals

$$= \lim_{n\to\infty} \frac{2^{2n+1} \ln 2 \cdot 2}{1} = \boxed{\infty} \boxed{\text{so diverges}}$$