Discrete Test 2 Review: first study quizzes!

(1) Let $a, b \in \mathbb{Z}$. Prove that if $a \mod 6 = 5$ and $b \mod 4 = 3$ then $4a + 6b \mod 8 = 6$.

(2) Suppose we were to prove or find a counterexample to the statement "$\forall x \in S, y \in \mathbb{Z}$, $y \leq 25 \Rightarrow 5|(x+y)$." (Answer without using the word "not" or the symbol "$\sim$.")

a)For a direct proof we assume ______________________ and show ______________________.

b) For proof using the contrapositive we assume ______________________ and show ______________________.

c) For proof by contradiction we assume ______________________________________ and show that we reach a false conclusion.

d) To disprove, using a counterexample, we find: ______________________________________.

(3) Let $a_1 = 2, a_2 = 4$, and $a_n = 5a_{n-1} - 6a_{n-2}, n \geq 3$. Prove that $\forall n \in \mathbb{N}, n \geq 3 \Rightarrow a_n = 2^n$ for all natural numbers n.

(4) Use contradiction to prove: $\forall a, b \in \mathbb{Z}$, if a is even and b is odd then 4 does not divide $(a^2 + 2b^2)$.
a) Negate the statement.

b) Assuming that negation, prove that 4 divides 2.

(5) Prove that $\sqrt{5}$ is irrational. You may assume that if $5|x^2$ then $5|x$, by the F.T. of arithmetic.

(6) Prove $\forall n \in \mathbb{Z}, n \geq 2 \Rightarrow \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$.

(7) Disprove $\forall n \in \mathbb{N}, (n^2 - n + 5)$ is prime.

(8) Prove that: $\forall n \in \mathbb{N}$, if $n \geq 2$ then $3|(2^{(4n-4)} + 2^{(2n-3)})$.

For your use:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

(9) Given the one-time-pad sequence $(2, 6, 13, 1)$ encrypt the word COOL. Your output will be letters.

(10) Use the BBS sequence to encrypt the word ZAP. Use the seed $a_0 = 11$ and the constant $pq = 7 * 13 = 91$.

(11) Use the same BBS sequence to decrypt the word LLJ. Use the seed $a_0 = 11$ and the constant $pq = 7*13 = 91$.

(12) Use the same BBS sequence to encrypt the digits 1101.

(13) Use the same BBS sequence to decrypt the digits 1110.