Consider the convex hull of the following seven points in $\mathbb{R}^3$:

$$(1, 2, 3),\ (2, 3, 1),\ (1, 3, 2),\ (2, 1, 3),\ (3, 1, 2),\ (3, 2, 1),\ (1, 4, 1).$$

(1) What is the dimension of the convex hull?

Hint: are they colinear? coplanar? neither? (Prove 'yes' or 'no' for these three questions, and then use your conclusions to demonstrate the dimension.)

(2) What are the vertices of the convex hull?

To make this easier, I'll let you assume that there are exactly two original points which are not vertices...all you have to do is find (prove they are not vertices) and discard them.

Note that non-vertices can be found as weighted averages ($\vec{\mathbf{x}}$ in the definition of convex hull) of the full set of points where $0 \leq \lambda_i < 1$. Vertices can be described as points in the convex hull that are uniquely found using the weights $\lambda_i = 0$ for all $i \neq j$, and $\lambda_j = 1$. (Recall that the weights sum to 1.)