1. Given universe $\mathcal{U}=\{3,4,5,7,9,10,11,23\}$; $A=\{5,7,9,10,11,23\}$; and $B=\{5,3,7\}$. Find the following:

•
$$\overline{(B-A)} \cap B$$
 = $(B-A) \cup \overline{B}$
= $\{3\} \cup \{4,9,10,11,23\} = \overline{\{3,4,9,10,11,23\}}$

$$\bullet \overline{A \cup \overline{B}} = \widehat{A} \cap \widehat{B} = \widehat{B} - A = \overline{\left\{3\right\}}$$

$$\bullet |B \times \mathcal{P}(A \times B)| = 3(2^{3.6}) = \overline{3(2^{18})}$$

$$\bullet |A \cup B| = 3 + 6 - 2 = \boxed{7}$$

2. Find the number of PINs using $\{0, ..., 9\}$, with 7 digits, no repeated numbers, where the first digit cannot be 3 and the fourth digit cannot be 5.

3. Find the number of DNA sequences using $\{A, G, T, C\}$, of length 5, where the first and second location cannot repeat, and the first location cannot be A.

$$= 4^{5} - 4^{3}(4 + 4 - 1)$$

$$= 4^{5} - 7(4^{3})$$

a) Negate the statement.

b) Write the assumptions, translated to algebraic equations.

$$\overline{z}^2 - 7 = 6k$$
 and $\overline{z} = 2m$

c) We will use the assumptions to show the falsehood 2|7, which is translated 7 = 2w for some integer w. Show the proof steps, from assumptions to 2|7.

$$= (2m)^2 - 7 = 64$$

$$= (2(2m^2 - 3k) = 7)$$

- 5. Use a direct proof to prove: $\forall z \in \mathbb{Z}, z \mod 3 = 2 \Rightarrow 9 | (3z^2 + 6)$.
 - a) Write the assumption, translated to an algebraic equation.

b) Write what we want to show, translated to an algebraic equation.

c) Proof steps:

of steps:

$$3z^2+6=3(3k+2)^2+6$$

$$=3(9k^2+12k+4)+6$$

$$=17k^2+36k+12+6$$

- 6. Use induction to prove: $\forall n \in \mathbb{Z}$, if $n \geq 4$ then $3|(2^{2n-5}+1)$.

a) Show the base case.

$$2^{2(4)-5}$$
 $4/=2+1=9=3(3)$

b) State the induction assumption, translate to algebraic equation.

$$\sqrt{2^{2k-5}+1} = 3m$$

c) State what we need to show, translate to algebraic equation.

that we need to show, translate to
$$\begin{array}{c|c}
2(k+1) - S \\
+ 1 = 3p
\end{array}$$

d) Do the proof steps.

the proof steps.

$$2^{2(k+1)-5} + 1 = 2^{2k-5} + 1$$

$$= 2^{2k-5} + 1$$

$$= 2^{2k-5} + 1$$

$$= 2^{2k-5} + 1$$

$$= 3(2^{2k-5}) + 2^{2k-5} + 1$$

$$= 3(2^{2k-5}) + 3m$$

$$= 3(2^{2k-5}) + 3m$$

$$= 3(4m-1)$$

7. Consider the sequence $a_n = (n^2 + 10) \mod 12$; starting at n = 1. Use it to encrypt the word SAT. Your answer will be the new word.

n	letter	std. num.	find a_n		encrypt	letter
1	S	19	11 mod 12 =	[]	19+11 = 30 mod 26 = 4	D
2	A		14 mod 12 =	2	(1+2) mod 26 = 3	Ċ
3	Т	20	19 mod 12 =	7	(20+7) mod 26 = 1	A

8. Consider the one-time-pad sequence $a_n = (2, 8, 11)$; starting at n = 1. It has been used to encrypt a message, and the encrypted message is DDF. Use the same sequence to decrypt and find the original word.

n	letter	std. num.	a_n	decrypt	letter
1	D	4	2	(4-2) mod 26 = 2	B
2	D	4	8	(4-8) mod 26 = 22	V
3	F	6	11	(6-11) mod 26 = 21	И

9. Consider the BBS (Blum Blum Shub) sequence $a_n = (a_{n-1})^2 \mod pq$; with $a_0 = 2$ and with p = 5, q = 5. Starting at n = 1, use this sequence to encrypt the binary number 1010. Your answer will be the new binary number. You may use either method from class.

n	bit	find a_n	encrypt		bit
1	1	4 mod 25 = 4	1+4 =5	mod 2)
2	0	16 mod 25 = 16	0+16 = 16	mod 2	0
3	1	256 mod 25 = 6	1+6=7	m.d2	1
4	0	36 mod 25 = 11	0+11 = 11	mod 2	1

10, From 7 library books, how many subsets of exactly 3 books are there? Answer as a whole number.

$$\left[\begin{pmatrix} 7 \\ 3 \end{pmatrix} \right] = \frac{7.6.5}{3.2.1} = \left[3.5 \right]$$

For 4 books and 9 shelves of a bookcase, find the number of ways to distribute the books on the shelves (just in piles, not in order.)

= 6561

- For 8 books and 6 shelves of a bookcase, find the number of plans for shelving, where at least 3 books are planned for the top shelf (a plan only tells how many books on each shelf.) $0\ell \left(\frac{13}{r} \right) \left(\frac{12}{q} \right) \left(\frac{11}{q} \right) \left(\frac{10}{q} \right) = 252$
- For 4 books and 9 shelves of a bookcase, find the number of ways to distribute the books on the shelves, where the bottom shelf has at most one book (just in piles, not in order.) $9^4 \binom{4}{2} 8^2 \binom{4}{3} 8^4 1$
- For 7 books and 9 shelves of a bookcase, find the number of ways to place the books on the shelves in ordered rows, where the bottom shelf has no more than 2 books.

$$7! \left(\left(\begin{array}{c} 7+9-1 \\ 9-1 \end{array} \right) - \left(\left(\begin{array}{c} (7-3)+9-1 \\ 9-1 \end{array} \right) \right) = \left[\left(\begin{array}{c} 15 \\ 8 \end{array} \right) - \left(\begin{array}{c} 12 \\ 8 \end{array} \right) \right] 7!$$

$$7! \left(\binom{7+8-1}{8-1} + \binom{(7-1)+8-1}{8-1} + \binom{(7-2)+8-1}{8-1} \right)$$

$$= \left(\binom{14}{7} + \binom{13}{7} + \binom{12}{7} \right) 7!$$

- 1 .. Given the original statement of implication: $((x < 2y) \land (x \ge 5)) \Rightarrow ((y > 8) \lor (3x \text{ is even})).$
 - Find the contrapositive of the original; write it without "not" and without "~."

(2)
$$(y \leq 8 \land 3 \times \text{odd}) \Rightarrow (\chi \geqslant 2 y \lor \chi < 5)$$

• Find the negation of the original; write it without "not" and without "~."

$$(3) \qquad (\chi < 2 \% \land \chi \geqslant 5) \land (\% \leq 8 \land 3 \% \text{ odd})$$

2. Given the statement:

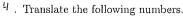
$$\forall x \in \mathbb{Z}, \exists y \in \mathbb{N} \text{ s.t. } (yx \ge y + 7) \Rightarrow ((x \text{ is even }) \land (y + x \text{ is odd })).$$

• Find its negation; write it without "not" and without "~."

- 3. Given the original statement "If you have salt then you have sodium." Answer the following without "not" and without "∼.'
 - Write the original statement using the word sufficient.

• Write the converse of the original using the words "only if".

• Write the contrapositive of the original using the word necessary.



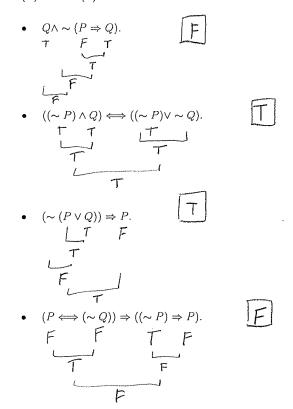
- binary: 1101011, hexidecimal: 6B

 binary: 11111010, hexidecimal: 7A

 hexidecimal: FA3

 hexidecimal: FA3
- binary: 1010 decimal:

5. Suppose that P = F (false) and Q = T (true). Find whether each of these statements is true (T) or false (F). Put a box around each final answer of T or F.



 $\boldsymbol{\mathcal{G}}$. For $\mathcal{S} = \{1, -3, -4, -12\},$ find an example making the following true:

$$\exists x \in S \text{ s.t. } ((x \mid 7) \lor (|x| > 3)) \Rightarrow ((x \text{ is odd }) \land (x \le -1))$$

7. Given the inputs of each circuit, fill in the outputs.

