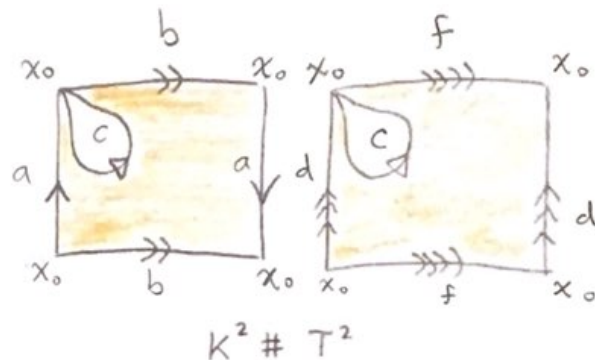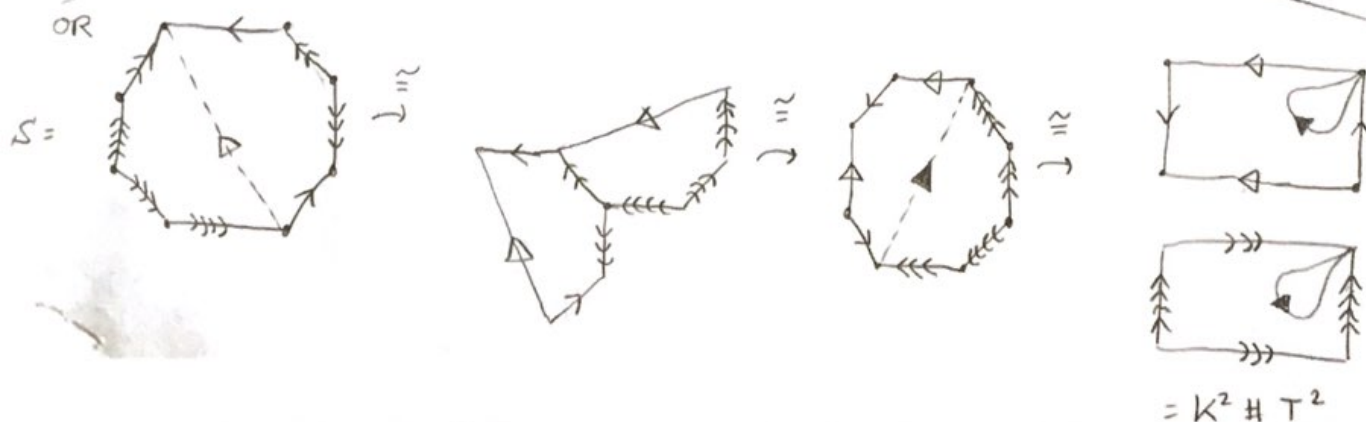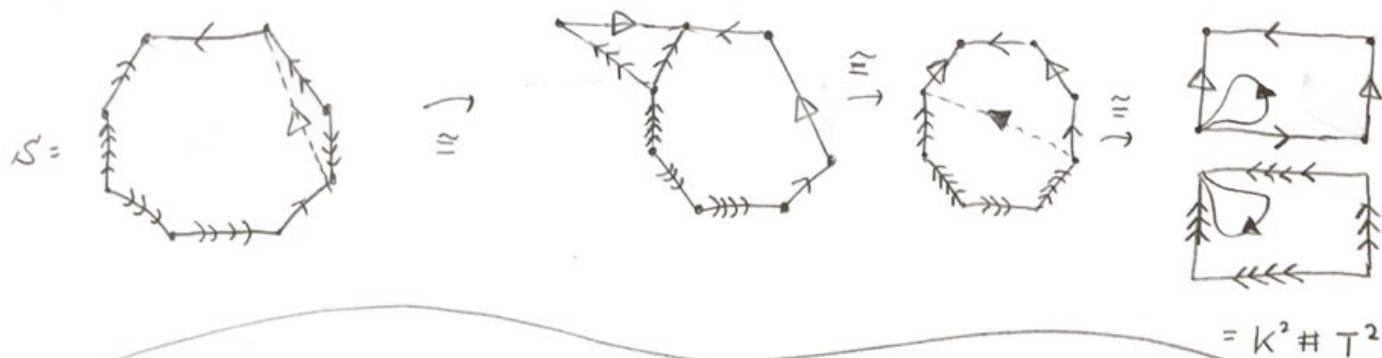Topology Quiz 9   Name ___Key___

$K^2 \# T^2$

$\chi(S) = 1 - 4 + 1 = \boxed{-2}$

$\chi(K^2 \# T^2) = 1 - 5 + 2 = \boxed{-2}$

$\pi_1(S) = \langle a, b, c, d \mid ba^{-1}b^{-1}c = d^{-1}cda \rangle$

$\pi_1(K^2 \# T^2) = \langle a, b, c, d, f \mid acb = ba^{-1}, \ dcf = fd \rangle$

Find a sequence of cuts that shows $S \cong K^2 \# T^2$.

$= K^2 \# T^2$

OR

$= K^2 \# T^2$

Topology Quiz 10   Name ___Key___

Here is an unknot $K = $.

1) Find a presentation for $\pi_1(\mathbb{R}^3 - k)$ as shown.

$$\langle x_1, x_2 \mid x_1 x_1 x_1^{-1} = x_2, \quad x_2 x_2 x_2^{-1} = x_1 \rangle$$

2) Show that your group is $\cong$ to $\mathbb{Z}$.

$$= \langle x_1, x_2 \mid x_1 = x_2, \quad x_2 = x_1 \rangle$$

$$= \boxed{\langle x_1 \mid \emptyset \rangle} \underset{\sim}{=} \mathbb{Z}.$$

Here is the figure-8 knot:

$K = $

3) Find a presentation for $\pi_1(\mathbb{R}^3 - k)$.

$$\langle x_1, x_2, x_3, x_4 \mid x_2 x_1 x_2^{-1} = x_3, \quad x_3 x_4 x_3^{-1} = x_2, \quad x_4 x_2 x_4^{-1} = x_1,$$

$$x_1 x_3 x_1^{-1} = x_4 \rangle$$

$$= \langle x_1, x_2, x_3, x_4 \mid x_2 x_1 = x_3 x_2, \quad x_3 x_4 = x_2 x_3, \quad x_4 x_2 = x_1 x_4,$$

$$x_1 x_3 = x_4 x_1 \rangle$$