Branch and bound

For linear programming: with powers of 2.

a) The problem

- We have a linear programming problem for which one of two things is true:

1) We don't have all the linear inequalities but we have a relaxed version (subset) of them. Ex: TSP. for n>6, with just the basic facets.
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- *But*: we *do* know that the optimal answer (as a vector) has a finite number of allowed values for the coordinates. [Examples: 0 or 1 for STSP, non-negative integers for knapsack problem, powers of 2 for the balanced minimal evolution problem.]

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3) If not, then we create some new LP problems 1A, 1B, etc. by adding *new inequalities one at a time*, just enough to force an offending coordinate away from its disallowed value.
4) We solve each of these (as long as they are still *feasible*) to get answers 1A, 1B, … etc.

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4) We solve each of these (as long as they are still *feasible*) to get answers 1A, 1B, … etc.
5) For each new answer we check whether it is complete, and if not whether it *merits further branching* into more new problems.
6) The process ends when no more branching is indicated; and the final answer is the optimal one from among the complete answers found.

b) Example in 2d

Maximize p = 6.75x + 5y subject to

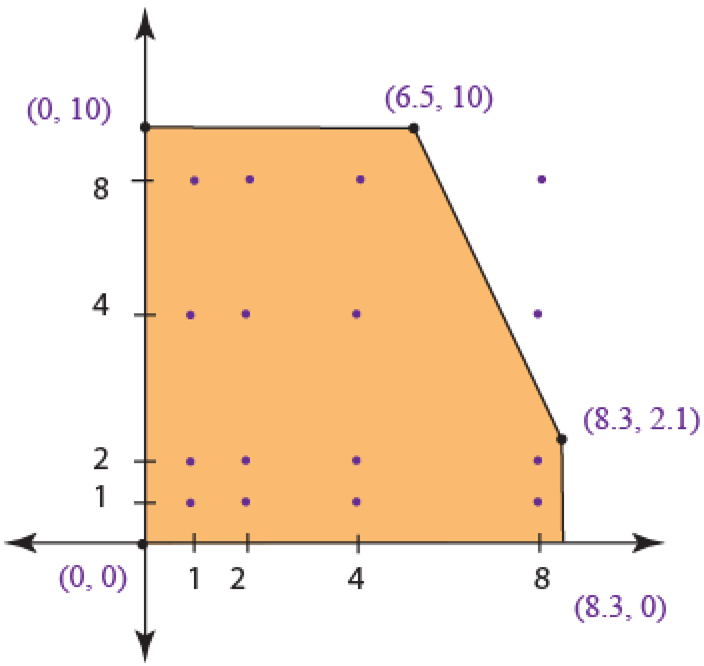
-x <= 0

-y <= 0

y <= 10

x <= 8.3

79x + 18y <= 693.5   Require all coordinates of answer (x,y) to be powers of 2.



Optimal Solution: p = 93.87; x = 6.5, y = 10

Answer zero: p = 93.87; x = 6.5, y = 10

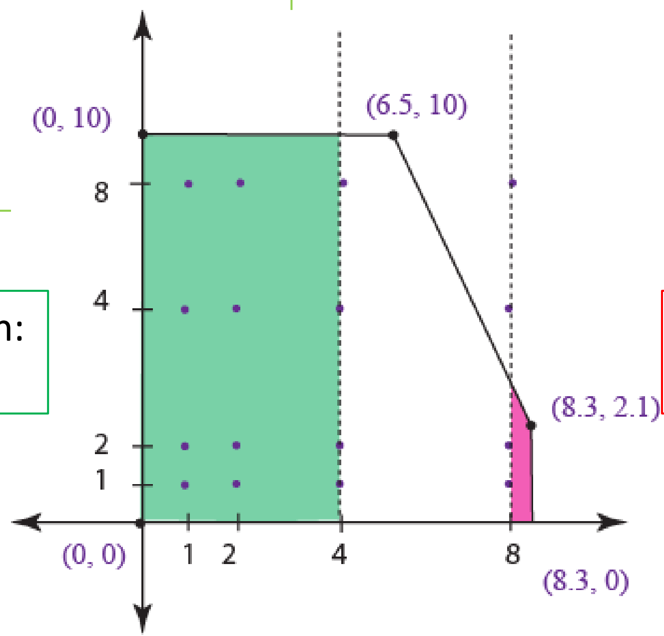
Branches:

x<=4

x>=8

Maximize p = 6.75x + 5y subject to
-x <= 0
-y <= 0
y <= 10
x <= 8.3
79x + 18y <= 693.5
x <= 4

Maximize p = 6.75x + 5y subject to
-x <= 0
-y <= 0
y <= 10
x <= 8.3
79x + 18y <= 693.5
x >= 8



1A: Optimal Solution:
p = 77; x = 4, y = 10

1B: Optimal Solution:
p = 71.08; x = 8, y = 3.417

Answer 1A: p = 77; x = 4, y = 10

Branches: y <= 8 | y >= 16

Maximize p = 6.75x + 5y subject to

-x <= 0

-y <= 0

y <= 10

x <= 8.3

79x + 18y <= 693.5

x <= 4

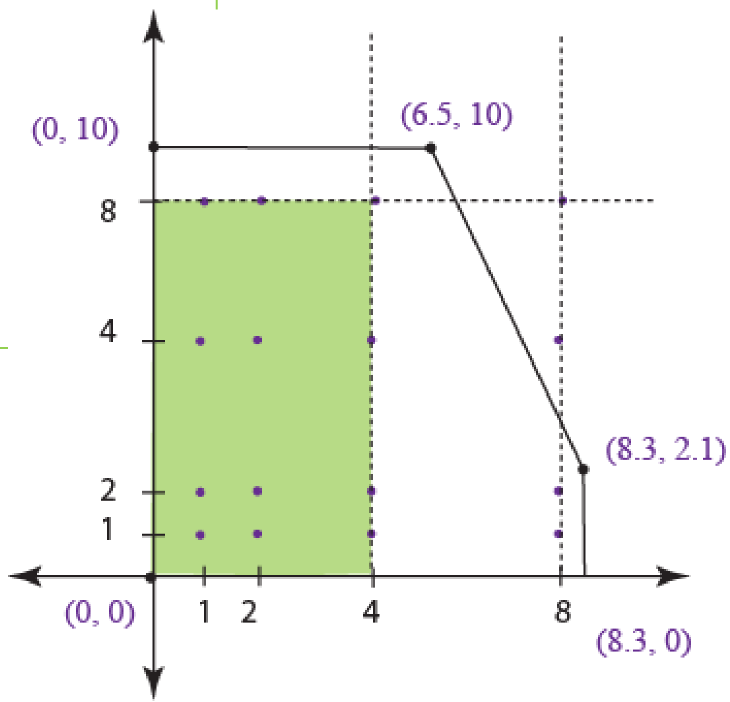
y <= 8

2A: Optimal Solution:

p = 67; x = 4, y = 8

Complete solution.

2B: Not feasible.



Answer 1B: p = 71.08; x = 8, y = 3.417

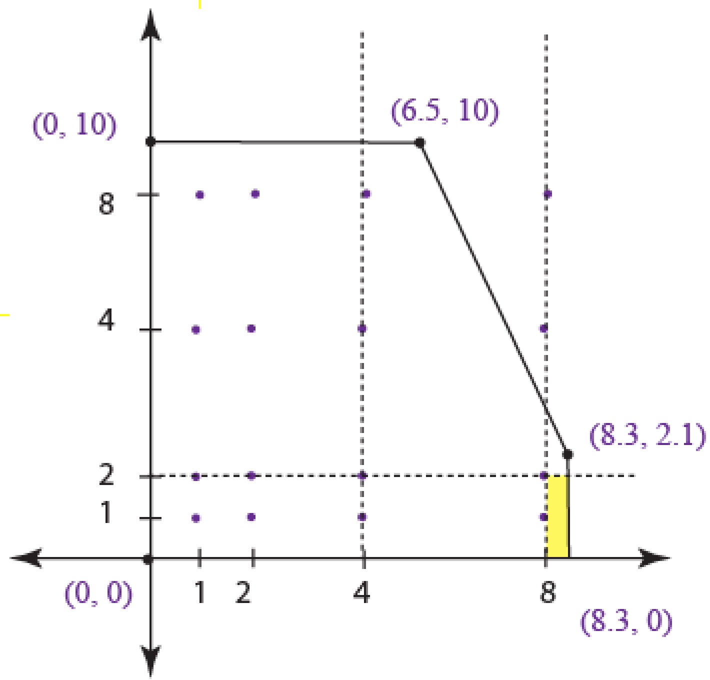
Branches: y <= 2 | y >= 4

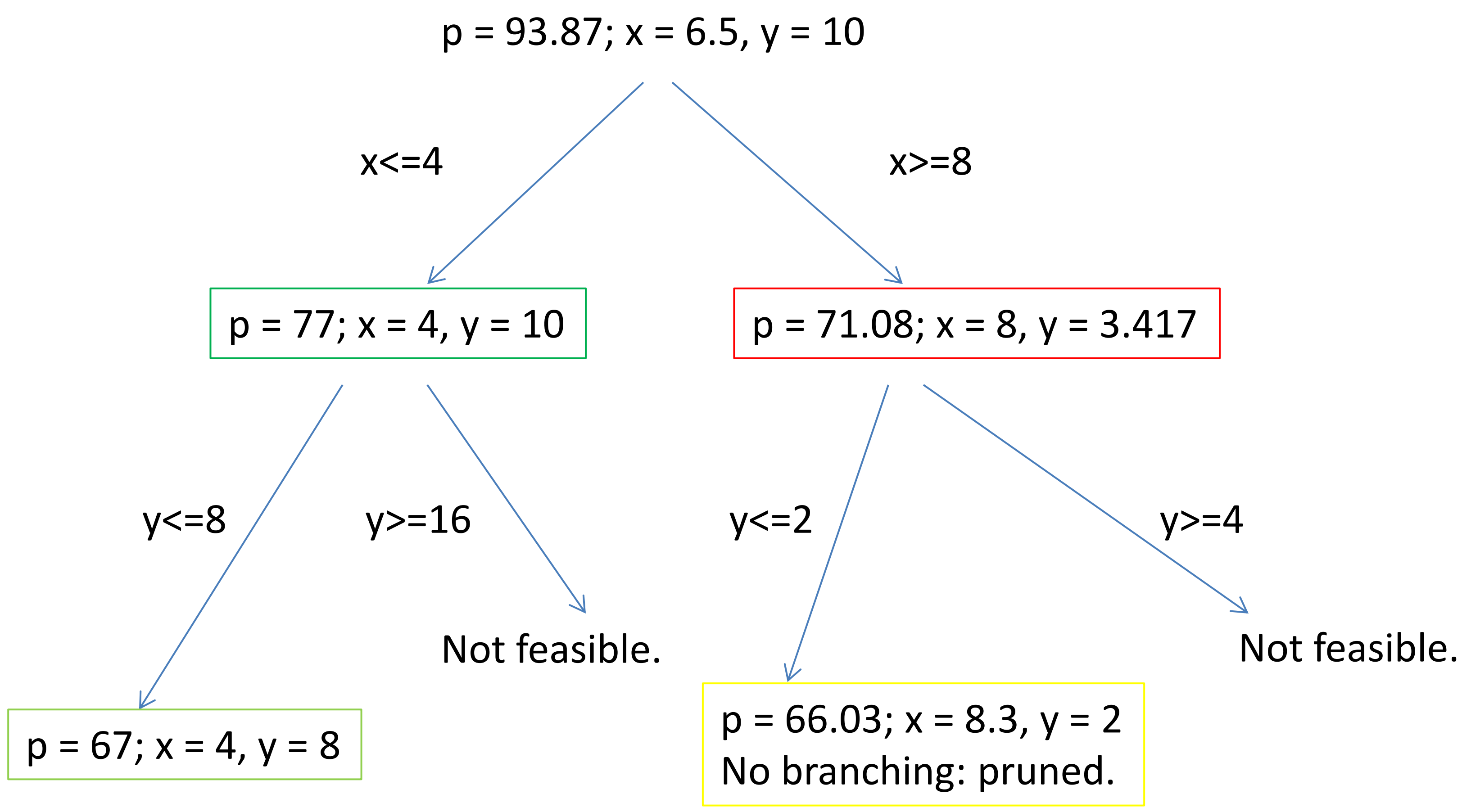
Maximize p = 6.75x + 5y subject to
-x <= 0
-y <= 0
y <= 10
x <= 8.3
79x + 18y <= 693.5
x >= 8
y <= 2

2C: Optimal Solution:
p = 66.03; x = 8.3, y = 2

66.03 < 67 (best complete solution) so:
No further branching.

2D: Not feasible (x >= 8 and y >= 4).





So final answer is $p=67$ at $x=4$, $y=8$.