Differential Equations. Review chapters 6-7, Spring '13.

•The Final Exam will cover material from test 1, test 2, review 1, review 2, this review, and quizzes. Questions on the exam may use different functions and constants than in the reviews and previous tests. For extra examples also study similar problems from the homework and quizzes, as well as examples from class notes.

•Note: Some questions on the actual test may state "Set up the differential equation only."

•Note: Don't forget that the answer will have an unknown constant or constants, unless it is an IVP.

1. (Approximately) solve the following diff. eq. (IVP) using power series, and finding c_0 through c_6.

$$y'' = \frac{1}{1-x}; \quad y(0) = 4; \quad y'(0) = 7.$$

$$y = 4 + 7x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{20}x^5 + \frac{1}{30}x^6 + \ldots$$

2. (Approximately) solve the following diff. eq. (IVP) using power series, and finding c_0 through c_6.

$$(1-x)y' + xy = 2; \quad y(0) = 5.$$

$$y = 5 + 2x + \frac{-3}{2}x^2 + \frac{-5}{3}x^3 + \frac{-7}{8}x^4 + \frac{-11}{30}x^5 + \frac{-23}{144}x^6 + \ldots$$

3. Find $\mathscr{L}^{-1}\left\{\frac{5}{2s+8}\right\}$.

$$f(t) = \frac{5}{2}e^{-4t}.$$

4. Find $\mathscr{L}\left\{(t^2+1)^2 e^{-5t}\right\}$.

$$F(s) = \frac{24}{(s+5)^5} + \frac{4}{(s+5)^3} + \frac{1}{s+5}.$$

5. Find $\mathscr{L}\left\{7 - \mathscr{U}(t-4) + (t-1)\mathscr{U}(t-1)\right\}$.

$$F(s) = \frac{7}{s} + \frac{-e^{-4s}}{s} + \frac{e^{-s}}{s^2}.$$

6. Find

$$\mathscr{L}^{-1}\left\{\frac{3}{s^2+1} + \frac{s}{4s^2+9}\right\}.$$

$$f(t) = 3\sin t + \frac{1}{4}\cos\left(\frac{3}{2}t\right).$$

7. Solve the diff. eq. using Laplace transforms: $y' + y = \mathscr{U}(t-5)$; $\quad y(0) = 0$.

$$y(t) = \mathscr{U}(t-5) - e^{-(t-5)}\mathscr{U}(t-5).$$

8. Solve the diff. eq. using Laplace transforms: $y' + y = 1 - 2\mathscr{U}(t-1)$; $\quad y(0) = 0$.

$$y(t) = 1 - e^{-t} - 2(1 - e^{-(t-1)})\mathscr{U}(t-1)$$

9. Solve the diff. eq. using Laplace transforms: $y'' = 1 - \mathscr{U}(t-3)$; $\quad y(0) = 0$; $\quad y'(0) = 0$.

$$y(t) = \frac{1}{2}t^2 - \frac{1}{2}(t-3)^2\mathscr{U}(t-3)$$