List of Publications

Stefan Forcey

Ph.D. Dissertation

- *Loop Spaces and Higher-Dimensional Iterated Enrichment*

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Abstract

There is an ongoing massive effort to link category theory and geometry, especially homotopy coherence and categorical coherence. This constitutes just a part of the broad undertaking known as categorification as described by Baez and Dolan. This effort has as a partial goal that of understanding the categories and functors that correspond to loop spaces and their associated topological functors. Progress towards this goal has been advanced greatly by the recent work of Balteanu, Fiedorowicz, Schwänzl, and Vogt who show a direct correspondence between k–fold monoidal categories and k–fold loop spaces through the categorical nerve.

This thesis pursues the hints of a categorical delooping that are suggested when enrichment is iterated. At each stage of successive enrichments, the number of monoidal products seems to decrease and the categorical dimension to increase, both by one. This is mirrored by topology. When we consider the loop space of a topological space, we see that paths (or 1–cells) in the original are now points (or objects) in the derived space. There is also automatically a product structure on the points in the derived space, where multiplication is given by concatenation of loops. Delooping is the inverse functor here, and thus involves shifting objects to the status of 1–cells and decreasing the number of ways to multiply.

Enriching over the category of categories enriched over a monoidal category is defined, for the case of symmetric categories, in the paper on A_∞–categories by Lyubashenko. It seems that it is a good idea to generalize his definition first to the case of an iterated monoidal base category and then to define $\mathcal{V}$–$(n+1)$–categories as categories enriched over $\mathcal{V}$–n–Cat, the $(k-n)$–fold monoidal strict $(n+1)$–category of $\mathcal{V}$–n–categories where $k < n \in \mathbf{N}$.

Section 1 reviews enrichment and Section 2 investigates just what obstacles arise when defining a product based on a braiding and attempting to define further a braiding of that derived product. Section 3 goes over the recursive definition of k–fold monoidal categories, altered here to include a coherent associator. The immediate question is whether the delooping phenomenon happens in general for these k–fold monoidal categories. The answer is yes, once enriching over a k–fold monoidal category is carefully defined in Section 4. The concept of higher dimensional enrichment is important in its relationship to double, triple and further iterations of delooping. All the information included in the axioms for the k–fold monoidal category is exhausted in the process of defining this notion, as described in Section 5.

Publications

- *Enrichment as Categorical Delooping I. Enrichment over Iterated Monoidal Categories*

submitted to Algebraic and Geometric Topology, preprint available math.CT/0304026

Abstract

Joyal and Street note in their paper on braided monoidal categories that the 2–category $\mathcal{V}$–Cat of categories enriched over a braided monoidal category $\mathcal{V}$ is not itself braided in any way that is based upon the braiding of $\mathcal{V}$. What is meant by "based upon" here will be made more clear in the present paper. The exception that they mention is the case in which $\mathcal{V}$ is symmetric, which leads to $\mathcal{V}$–Cat being symmetric as well. The symmetry in $\mathcal{V}$–Cat is based upon the symmetry of $\mathcal{V}$. The motivation behind this paper is in part to describe how these facts relating $\mathcal{V}$ and $\mathcal{V}$–Cat are in turn related to a categorical analogue of topological delooping first mentioned by Baez and Dolan. To do so I need to pass to a more general setting than braided and symmetric categories – in fact the k–fold monoidal categories of Balteanu, Fiedorowicz, Schwänzl and Vogt. It seems that the analogy of loop spaces is a good guide for how to define the concept of enrichment over various types of monoidal objects, including k–fold monoidal categories and their higher dimensional counterparts. The main result is that for $\mathcal{V}$ a k–fold monoidal category, $\mathcal{V}$–Cat becomes a $(k-1)$–fold monoidal 2–category in a canonical way. I indicate how this process may be iterated by enriching over $\mathcal{V}$–Cat, along the way defining the 3–category of categories enriched over $\mathcal{V}$–Cat. In the next paper I hope to make precise the n–dimensional case and to show how the group completion of the nerve of $\mathcal{V}$ is related to the loop space of the group completion of the nerve of $\mathcal{V}$–Cat.

- *Higher–Dimensional Enrichment*

submitted to Theory and Applications of Categories, preprint available math.CT/0306086

Abstract

Lyubashenko has described enriched 2–categories as categories enriched over $\mathcal{V}$–Cat, the 2–category of categories enriched over a symmetric monoidal $\mathcal{V}$. Here I generalize this to a k–fold monoidal $\mathcal{V}$. The latter is defined as by Balteanu, Fiedorowicz, Schwänzl and Vogt but with the addition of making visible the coherent associators α^i. The symmetric case can easily be recovered. The introduction of this paper proposes a recursive definition of $\mathcal{V}$–n–categories and their morphisms. Then I consider the special case of $\mathcal{V}$–2–categories and give the details of the proof that with their morphisms these form the structure of a 3–category.

- *Axiomatic Topological Quantum Field Theory* (with F. Quinn, J. Siehler)

to appear in Encyclopedia of Mathematical Physics, Elsevier Ltd.

Abstract

(Draft) Recently topological quantum field theory (TQFT) has been considered seriously as the mathematical foundation for certain versions of quantum gravity. The origins of TQFT also lie in physics – the intuitive definition of the concept is a quantum field theory that depends only on the topology of the space-time as opposed to reflecting the metric as well. The path integral examples from physics, however, lack mathematical rigor. Thus there is reason to think that a correctly envisioned axiomatic mathematical definition of a TQFT could shed light on both existing physics and the possibilities of a future unified theory. Extreme caution is needed to make sure that this process of defining axioms is not unnaturally swayed by early guesses at their desired consequences. The goal of this article is to discuss and compare the existing axiomatic systems. We will also describe the importance of using categorical limits, and what is still lacking in various approaches to defining a multi-dimensional extended TQFT. Finally we will mention some possible ways that TQFT's might be used to attack open problems, both physical and topological. Suggested further reading follows.

- *Weak Higher–Dimensional Enrichment*

(in preparation)

Abstract

(Draft) There are currently more than a dozen (more or less rigorous) competing general definitions of n–category. The basic idea is to define higher–dimensional analogues of a bicategory, where composition of morphisms is only associative up to higher morphisms, and so on, and where the top level morphisms obey a commuting diagram. There is an accepted recursive definition of a strict n–category, given in terms of enrichment. This paper outlines a new approach to weak n–categories in terms of a weakened version of higher–dimensional enrichment. The latter is based upon labeling the vertices of Stasheff's associahedra with products of hom–objects. Then we fill the associahedron in with enriched morphisms that fit a subdivision generated by composition morphisms with domains at the vertices and a common range at the center. In fact, there is given a more general definition of weak $\mathcal{V}$–n–Cat, for k–fold monoidal $\mathcal{V}$, which has the structure of a weak $(n+1)$–category. The new notion is shown to give the standard definitions when $\mathcal{V} =$ **Set** and $n = 0, 1, 2, 3$. Prospects of comparison and contrast with existing definitions of n–category are discussed.