

2. Find a power series which converges to the following functions, in the form  $\sum_{n=k}^{\infty} a_n x^n$  where  $k \geq 0$ .

(a)  $f(x) = \frac{x^3}{(1-x)^2}$

$$\begin{aligned} \sum_{n=0}^{\infty} x^n &= \frac{1}{1-x} \\ \frac{d}{dx} \left( \frac{1}{1-x} \right) &= \sum_{n=0}^{\infty} n x^{n-1} \\ \cdot x^3 \Rightarrow \frac{x^3}{(1-x)^2} &= \sum_{n=0}^{\infty} n x^3 x^{n-1} \\ &= \sum_{n=0}^{\infty} n x^{n+2} = \sum_{n=0}^{\infty} (n+1) x^{n+3} \\ &= \sum_{n=2}^{\infty} (n-2) x^n \\ \text{OR } \sum_{n=3}^{\infty} (n-2) x^n \end{aligned}$$

(b)  $f(x) = e^{2x}$

$$\begin{aligned} e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \\ \Rightarrow e^{2x} &= \sum_{n=0}^{\infty} \frac{(2x)^n}{n!} \\ &= \sum_{n=0}^{\infty} \frac{2^n x^n}{n!} \end{aligned}$$