Name ______________________________

14. Use any method to find the first 4 nonzero terms of the Maclaurin series for $f(x) = \frac{1}{(1-2x)^2}$.

14: 10 pts

Method 1

Use $\frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} n x^{n-1}$

$\Rightarrow \frac{1}{(1-2x)^2} = \underbrace{\sum_{n=0}^{\infty} n(2x)^{n-1}}$

Then

$n=0 \longrightarrow 0(2x)^{-1} = 0$

$n=1 \longrightarrow 1(2x)^0 = 1$

$n=2 \longrightarrow 2(2x)^1 = 4x$

$n=3 \longrightarrow 3(2x)^2 = 12x^2$

$n=4 \longrightarrow 4(2x)^3 = 32x^3$

Method 2.

Use $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)\,x^n}{n!} = f(x)$

$f^{(0)}(x) = (1-2x)^{-2}$

$f^{(1)}(x) = -2(1-2x)^{-3}(-2)$

$f^{(2)}(x) = 6(1-2x)^{-4}(-2)(-2)$

$f^{(3)}(x) = -24(1-2x)^{-5}(-2)(-2)(-2)$

$f^{(0)}(0) = 1$ | $1x^0/0! = 1$

$f^{(1)}(0) = 4$ | $4x^1/1! = 4x$

$f^{(2)}(0) = 24$ | $24x^2/2! = 12x^2$

$f^{(3)}(0) = 24(8)$ | $24(8)x^3/3! = 32x^3$

15. Find the first 4 nonzero terms of the Maclaurin series for $f(x) = \frac{e^{x^2} - (1+x^2)}{x}$.

15: 10 pts

Method 1

Use $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\Rightarrow e^{x^2} = \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} = 1 + x^2 + \sum_{n=2}^{\infty} \frac{(x^2)^n}{n!}$

$\Rightarrow e^{x^2} - (1+x^2) = \sum_{n=2}^{\infty} \frac{(x^2)^n}{n!}$

$\Rightarrow \frac{e^{x^2} - (1+x^2)}{x} = \sum_{n=2}^{\infty} \frac{x^{2n}}{x\,n!} = \sum_{n=2}^{\infty} \frac{x^{2n-1}}{n!}$

$n=2 \longrightarrow x^{4-1}/2! = \frac{x^3}{2}$

$n=3 \longrightarrow x^{6-1}/3! = \frac{x^5}{6}$

$n=4 \longrightarrow x^{8-1}/4! = \frac{x^7}{24}$

$n=5 \longrightarrow x^{10-1}/5! = \frac{x^9}{120}$

Method 2

Use $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)\,x^n}{n!} = f(x)$

(first few terms are 0, start at $n=2$)

Pg 8: 20 pts