A step in the direction of solving both!

(We think this is true, but we need to test (and prove) it!)

Assume the wires in the box are not planar, that is, some of them cross each other (and you cannot make an electrical circuit yielding the same pairwise resistances that is planar).

Then there must be some set of 4 terminals (wire-ends) which are not planar: that is, even after pushing all the terminals except those 4 back inside the box, the network stays tangled. There is still no way to get those 6 pairwise resistances without using a non-planar network.

Experiment: Find the troublesome 4.
Pattern: how can we spot those 4?
Proof: show that those 4 always exist.

1) show that the pattern is correct; it spots trouble accurately
2) show that the pattern is always there