

Calculus II. Review for Test 2.

1. Show the correct form for a partial fraction decomposition of these functions. Don't actually solve for the variables.

a)  $\frac{x^2 + 1}{x^2(x + 2)}$

b)  $\frac{x + 3}{x^2 - 4}$

c)  $\frac{5x + 1}{x^3 - 3x - 2}$  Note that  $x=2$  makes the denominator = 0.

2. Decompose the function into its partial fractions. (Actually solve for the variables.)

a)  $\frac{7x}{(x - 1)(x^2 + 3)}$

b)  $\frac{x + 3}{(x - 2)(x + 3)^2} = \frac{A}{x - 2} + \frac{B}{(x + 3)^2} + \frac{C}{x + 3}$

3. Find the indefinite integrals:

a)  $\int \frac{x^2 + 2x + 3}{x(x + 1)} dx$

b)  $\int \frac{5x + 1}{x^3 - 3x - 2} dx$

4. Find these definite integrals and classify as "divergent" or "convergent":

a)  $\int_3^\infty x e^{(-x^2)} dx$

b)  $\int_{-1}^0 \frac{3}{x^5} dx$

c)  $\int_{-2}^{14} \frac{1}{\sqrt[4]{x + 2}} dx$

5. For each of these sequences, find the limits, if they exist, and decide "diverges" or "converges."

a)  $\lim_{n \rightarrow \infty} \frac{2^n + n}{3^n + 1}$

b)  $\lim_{n \rightarrow \infty} \frac{n + 4n^3}{2n^4 + 1}$

c)  $\lim_{n \rightarrow \infty} \frac{n^3 + n^2}{7n^3 + 1}$

d)  $\lim_{n \rightarrow \infty} \frac{7}{\cos(n\pi)}$

e)  $\lim_{n \rightarrow \infty} \frac{\tan^{-1}(n)}{3}$

f)  $\lim_{n \rightarrow \infty} \frac{(-1)^n}{3^n}$

g)  $\lim_{n \rightarrow \infty} \frac{3^n(-1)^n}{2^n}$

6. For each series, what does the limit test for divergence tell us? [converge, diverge, or inconclusive] Show your work by performing the test.

$$a) \sum_{n=1}^{\infty} \frac{e^{2n} + 3n}{5e^{2n} - 6}$$

$$b) \sum_{n=1}^{\infty} \frac{2^n}{3^n}$$

$$c) \sum_{n=1}^{\infty} \frac{3}{e^{2n}}$$

7. For each series, what does the geometric series test tell us? [not applicable, converge, diverge, or inconclusive] Show your work, and find the value if it converges.

$$a) \sum_{n=1}^{\infty} \frac{3^n}{\pi^n}$$

$$b) \sum_{n=1}^{\infty} \frac{5^n}{(\sqrt{3})^n}$$

$$c) \sum_{n=1}^{\infty} \frac{3}{(0.5)^n}$$

$$d) \sum_{n=1}^{\infty} \frac{1}{(-2)^n}$$

$$e) \sum_{n=1}^{\infty} \frac{3}{e^{2n}}$$

$$f) \sum_{n=1}^{\infty} \frac{n}{e^{2n}}$$

8. For each series, what does the  $p$ -series test tell us? [not applicable, converge, diverge, or inconclusive] Show your work.

$$a) \sum_{n=1}^{\infty} \frac{5^n}{(\sqrt{3})^n}$$

$$b) \sum_{n=1}^{\infty} \frac{3}{n^{(0.5)}}$$

$$c) \sum_{n=1}^{\infty} \frac{2}{n}$$

$$d) \sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^3$$

9. For each series, what does the integral test tell us? [not applicable, converge, diverge, or inconclusive] Show your work.

$$a) \sum_{n=1}^{\infty} \frac{\sqrt{n} + 4}{n^2}$$

$$b) \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$$

$$c) \sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^3$$

$$d) \sum_{n=1}^{\infty} \frac{1}{(-2)^n}$$

$$e) \sum_{n=1}^{\infty} \frac{1}{\cos^2(n)}$$

10. For each series, what does the comparison test tell us? [not applicable, converge, or diverge] Show your work.

$$a) \sum_{n=1}^{\infty} \frac{1}{2n^3 + 1}$$

$$b) \sum_{n=1}^{\infty} \frac{9^n}{3 + 10^n}$$

$$c) \sum_{n=1}^{\infty} \frac{6^n}{-4 + 5^n}$$

$$d) \sum_{n=1}^{\infty} \frac{(-1)^n}{-4 + 5^n}$$

11. For each series, what does the limit comparison test tell us? [not applicable, converge, or diverge] Show your work.

$$(a) \sum_{n=1}^{\infty} \frac{1}{2n + 1}$$

$$b) \sum_{n=1}^{\infty} \frac{n + 2}{(n + 1)^3}$$

$$c) \sum_{n=1}^{\infty} \frac{2^n}{5^n - n}$$

12. For each series, use the alternating series test or the limit test for divergence to decide: [converge, or diverge]. Show your work.

$$a) \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$$

$$b) \sum_{n=1}^{\infty} \frac{(-1)^n}{4n+1}$$

$$c) \sum_{n=1}^{\infty} \frac{(-1)^n}{e^{-n}}$$

13. Decide if the sums converge or diverge, explain why. If there is a formula for the sum, find the value.

$$a) \sum_{n=1}^{\infty} n^2 e^{-n^3}$$

$$b) \sum_{n=1}^{\infty} e^{2n}$$

$$c) \sum_{n=1}^{\infty} \frac{2^n}{e^{3n}}$$

14. For each series, what does the ratio test tell us? [converge absolutely, diverge, or inconclusive]. Show your work by finding the limit.

$$(a) \sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$$

$$(b) \sum_{n=1}^{\infty} \frac{2^{(n^2)}}{(2n)!}$$

15. For each series, what does the root test tell us? [converge absolutely, diverge, or inconclusive]. Show your work by finding the limit.

$$(a) \sum_{n=1}^{\infty} \left( \frac{-2n}{3n+1} \right)^n$$

$$(b) \sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n}} \right)^n$$

16. Also study the quizzes, and the homework questions. These are good test questions too!