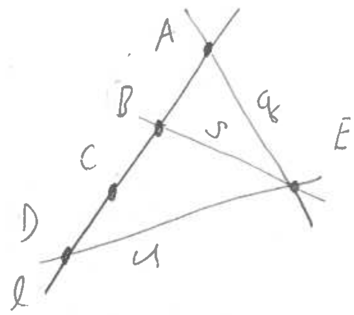
Geometry Test 2 Review: first study quizzes!

Formulas $d_S(A,B) = R\cos^{-1}\left(\frac{A\cdot B}{R^2}\right)$. $d_H(A,B) = \ln\left(\frac{1-A\cdot B+d_E(A,B)}{1-A\cdot B-d_E(A,B)}\right)$

Given point set $\mathcal{P} = \{A,B,C,D,E\}$;

(1) For $\mathcal{P}$ with lines $\mathcal{L} = \{l,q,s,u\}$,
let $\mathcal{I} = \{(A,l),(B,l),(C,l),(D,l),(A,q),(B,s),(D,u),(E,q),(E,s),(E,u)\}$.

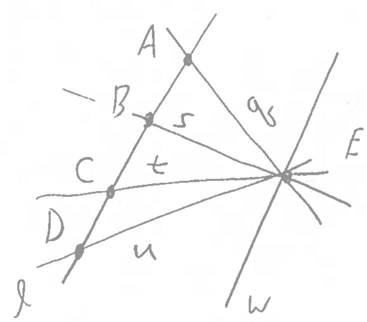
Is this an abstract incidence geometry or not? Draw a diagram and explain.



No, missing line through C, E.

(2) For $\mathcal{P}$ with lines $\mathcal{L} = \{l,q,s,u,t,w\}$,
let $\mathcal{I} = \{(A,l),(B,l),(C,l),(D,l),(A,q),(B,s),(C,t),(D,u),(E,q),(E,s),(E,t),(E,u),(E,w)\}$.

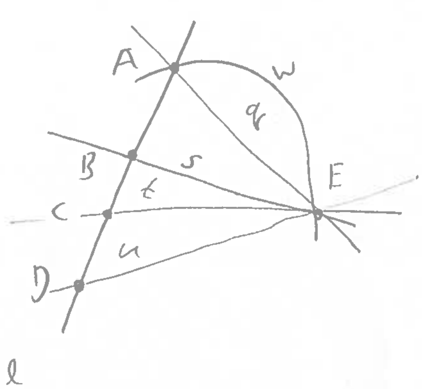
Is this an abstract incidence geometry or not? Draw a diagram and explain.



No, line w has only one point.

(3) For $\mathcal{P}$ with lines $\mathcal{L} = \{l,q,s,u,t,w\}$,
let $\mathcal{I} = \{(A,l),(B,l),(C,l),(D,l),(A,q),(A,w),(B,s),(C,t),(D,u),(E,q),(E,s),(E,t),(E,u),(E,w)\}$.

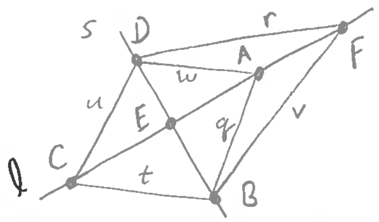
Is this an abstract incidence geometry or not? Draw a diagram and explain.



No, 2 lines w, q through A, E.

(4) For $\mathcal{P} = \{A, B, C, D, E, F\}$ with lines $\mathcal{L} = \{l, q, s, u, t, w, r, v\}$,
let $\mathcal{I} = \{(A,l), (A,q), (A,w), (B,q), (B,s), (B,t), (C,t), (C,l), (C,u), (D,u), (D,s), (D,w), (E,l), (E,s), (F,l), (F,r), (F,v), (D,r), (B,v)\}$.

Is this an abstract incidence geometry or not? Draw a diagram and explain.



Yes, I1, I2, I3 all obeyed.

(5) For number (4) above, find the line cardinality vector *LCV*. If another incidence geometry has a different LCV, can you find an isomorphism between them? No

$\langle 4, 3, 2, 2, 2, 2, 2, 2 \rangle = \langle 4, 3, 6 \cdot 2 \rangle$

(6) For number (4) above, find the automorphism f such that $f(A) = C$, $f(B) = B$, and $f(C) = A$.

x	A	B	C	D	E	F
$f(x)$	C	B	A	D	E	F

(7) For number (4) above, find the automorphism f such that $f(A) = A$, $f(B) = D$, and $f(C) = F$.

x	A	B	C	D	E	F
$f(x)$	A	D	F	B	E	C

(8) Consider the three points given: $A = (1/2, 0)$, $B = (1/4, 1/4)$, and $C = (1/2, 1/2)$,
Find the 12 distances: Euclidean, Taxicab, Max, Bus, Post-Office, and Hyperbolic between the two points.

$d_E(A,B) = \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2} = \sqrt{\frac{1}{8}}$ $\quad d_E(B,C) = \sqrt{\frac{1}{8}}$

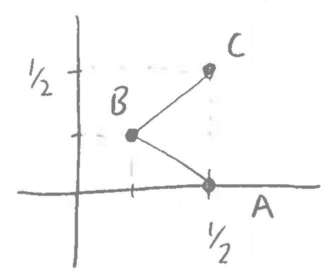
$d_T(A,B) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$, $\quad d_T(B,C) = \frac{1}{2}$

$d_M(A,B) = \frac{1}{4}$, $\quad d_M(B,C) = \frac{1}{4}$

$d_B(A,B) = \frac{1}{2} + \sqrt{\frac{1}{8}}$, $\quad d_B(B,C) = \sqrt{\frac{1}{8}}$

$d_P(A,B) = \frac{1}{2} + \sqrt{\frac{1}{8}}$, $\quad d_P(B,C) = \sqrt{\frac{1}{8}} + \sqrt{\frac{1}{8}} + \sqrt{\frac{1}{8}} = 3\sqrt{\frac{1}{8}}$

$d_H(A,B) = \ln\left(\frac{7/8 + \sqrt{1/8}}{7/8 - \sqrt{1/8}}\right) = 0.85$, $\quad d_H(B,C) = \ln\left(\frac{6/8 + \sqrt{1/8}}{6/8 - \sqrt{1/8}}\right) = 1.02$



For Euclidean, Taxicab, Max, and Hyperbolic, what are the equivalence classes of the two segments $\overline{AB}$ and $\overline{BC}$?

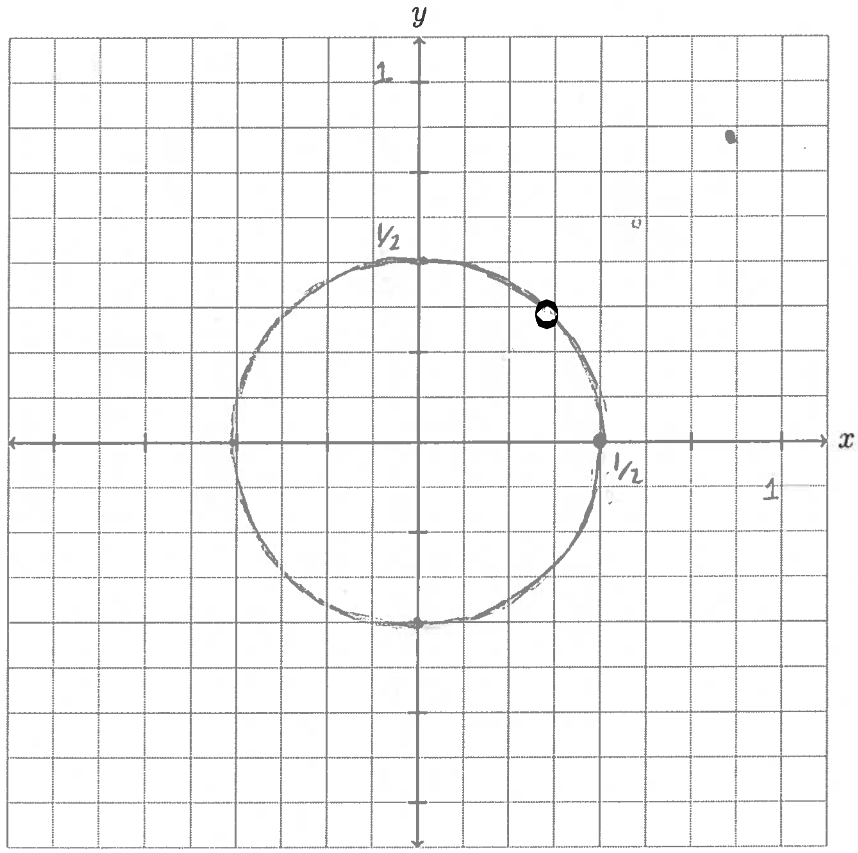
E $\{ \{ \overline{AB}, \overline{BC} \} \}$

T $\{ \{ \overline{AB}, \overline{BC} \} \}$

M $\{ \{ \overline{AB}, \overline{BC} \} \}$

H $\{ \{ \overline{AB} \}, \{ \overline{BC} \} \}$

(9) Draw the circle for each metric centered at B through the point A. Use compass and straightedge.



Bus Metric

d_B

(10) Find the three distances between points $A = (2, 10, 25)$, $B = (2, 14, 23)$, and $C = (7, 14, 22)$ on the sphere with radius $= 27$.

$d_S(A, B) =$ 4.477, $d_S(B, C) =$ 5.10, $d_S(A, C) =$ 7.09.

$$27 \cos^{-1}\left(\frac{2 \cdot 2 + 10 \cdot 14 + 25 \cdot 23}{27^2}\right)$$