

→ Geometric meaning and 3 types of solution.

For any number of linear equations,
with any number of variables,
there can only be one of 3 possibilities

→ Zero solutions

→ One solution

→ ∞ solutions

Why?

1) dimension of a space

is a counting number $0, 1, 2, 3, 4, 5, 6, \dots$
which describes a collection of
points. It tells:

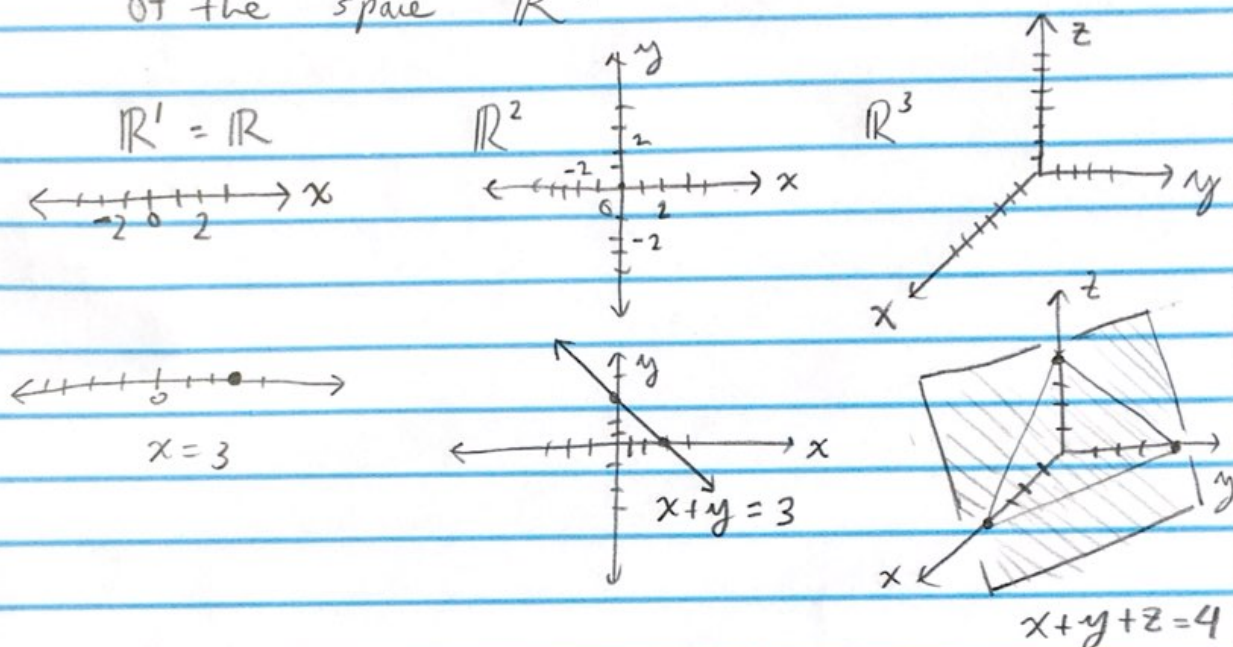
→ the number of independent, free
decisions for perpendicular motions

→ the number of real numbers needed
to describe a single point location
in that space

degrees
of freedom

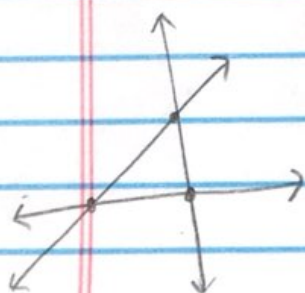
coordinates,
axes \mathbb{R}

2) Each single (affine) linear equation describes the points in a hyperplane of the space \mathbb{R}^d

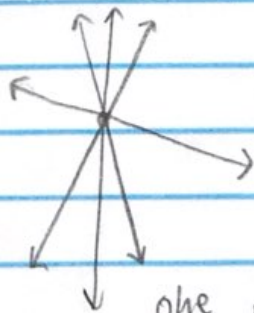


3) Several hyperplanes in \mathbb{R}^d

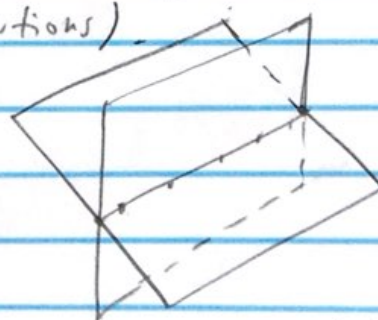
- can either:
- not share a common point of intersection (0 solutions)
 - share exactly one common point (needs at least d hyperplanes but no guarantee)
 - or, all intersect in a lower-dimensional plane, line, or hyperplane (∞ solutions)



no point in common - no solution



one point solution

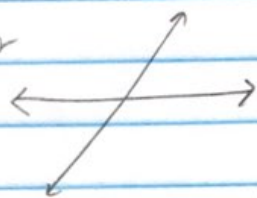


line of solution points → ∞ solutions

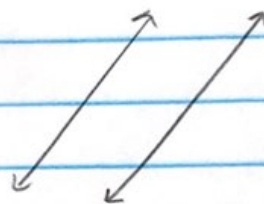
Goal # 1: be able to look at equations & know what the picture is, and vice versa: look at the picture and know things about the equations.

Two lines in \mathbb{R}^2 :

either



or



crossing = different slopes

parallel = same slope

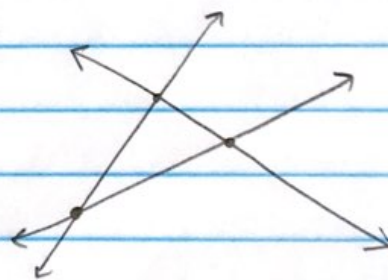
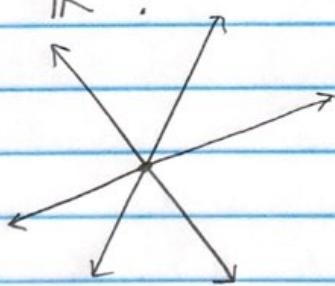
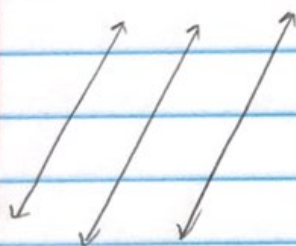
ex:

$$y = \frac{2}{3}x + 1, \quad y = \frac{2}{3}x - 5$$

$$3y - 2x = 3, \quad 3y - 2x = -15$$

Slope = coefficients

Three lines in \mathbb{R}^2 :



Four lines: 9 different pictures

Five lines: 47

Six lines: 791

Seven lines: 37,830

Eight lines: 4134,940

Nine lines: Unknown

In general n lines?

→ open research question.