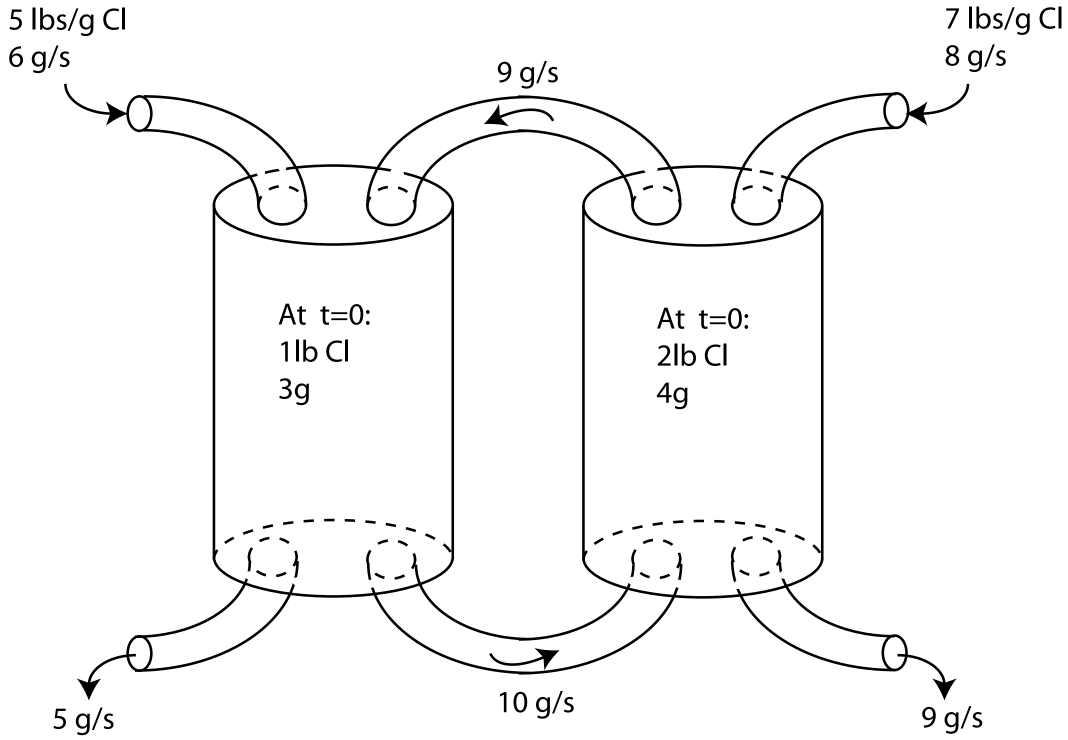
Differential Equations. Review for Test 2.

Also study all the homework and quizzes, as well as examples in class notes.

Note: Some questions on the actual test may state "Set up the differential equation only."
Note: Don't forget that the answer will have an unknown constant or constants, unless it is an IVP.

1. Set up (but don't solve) the vector diff. eq. for $\vec{x}(t)$ which give the lbs of Cl at time t in the two tanks shown below. Your answer should be written $\vec{x}' = A\vec{x} + \vec{b}$ where you find A and $\vec{b}$.



$$\vec{x}' = \begin{pmatrix} -5 & \frac{9}{4} \\ \frac{10}{3} & \frac{-9}{2} \end{pmatrix} \vec{x} + \begin{pmatrix} 30 \\ 56 \end{pmatrix}; \quad \vec{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

2. Solve the differential equation generally: $4x^2y'' + y = 0$. You are given that one solution is $y_1 = \sqrt{x}\ln x$.

$$y = c_1\sqrt{x}\ln x + c_2\sqrt{x}.$$

3. Solve the diff. eq.: $y'' + 8y' + 16y = 0$.

$$y = c_1 e^{-4x} + c_2 x e^{-4x}.$$

4. Solve the diff. eq.: $y'' + 9y = 0$.

$$y = c_1 \cos 3x + c_2 \sin 3x.$$

Solve the diff. eq.: $y'' - 4y' + 5y = 0$.

$$y = e^{2x}(c_1 \cos x + c_2 \sin x).$$

5. • Given a diff. eq. $a_2 y'' + a_1 y' + a_0 y = 7 \sin x + 5xe^{2x}$ which has the complimentary solution $y_c = c_1 e^{2x} + c_2 x e^{2x}$. Find the form of the particular solution y_p using only the variables A, B, C, D.

$$y_p = A \sin x + B \cos x + Cx^3 e^{2x} + Dx^2 e^{2x}.$$

• Given the diff. eq. $y'' + 4y' - 2y = 2x^2 - 3x + 6$ and the form of the particular solution $y_p = Ax^2 + Bx + C$. Find the particular solution.

$$y_p = -x^2 - \frac{5}{2}x - 9.$$

6. • Solve the diff. eq. $y'' + y = \sec x$ by variation of parameters.

$$y = c_1 \cos x + c_2 \sin x + (\ln |\cos x|) \cos x + x \sin x.$$

7. • Given a diff. eq. $y'' + P(x)y' + Q(x)y = e^{-x}\sqrt{x}$ which has the complimentary solution $y_c = c_1 e^{-x} + c_2 x e^{-x}$.

• Check that the Wronskian of y_1 and y_2 is $W = e^{-2x}$.

$$\begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & e^{-x} - xe^{-x} \end{vmatrix} = e^{-2x}.$$

- Find the particular solution y_p.

$$y_p = \frac{4}{15}e^{-x}x^{\frac{5}{2}}.$$

8. • Solve the differential equation (generally): $x^2y'' + 3xy' = 0$.

$$aux.: \quad m(m-1) + 3m = 0; \quad y = c_1 + c_2x^{-2}.$$

• Given that the solution to the above diff. eq. is $y = c_1 + c_2x^{-2}$, solve the initial value problem where $y(1) = 0$; $y'(1) = 4$.

$$y = 2 - 2x^{-2}.$$

9. Solve the diff. eq. $25x^2y'' + 25xy' + 4y = 0$.

$$y = c_1 \cos(\frac{2}{5}\ln x) + c_2 \sin(\frac{2}{5}\ln x).$$

10. Set up (but don't solve) the differential equation for the motion $x(t)$ of a 4lb weight attached to a spring, which is stretched 15 inches by the weight. It is damped by friction with strength equalling $\frac{2}{5}$ of the vertical velocity at any time, and driven by a force $f(t) = \sin 7t$.

$$\frac{1}{8}x'' + \frac{2}{5}x' + \frac{48}{15}x = \sin 7t.$$

11. The solution to a spring-mass system is $x(t) = -2\cos(5t) - \sqrt{3}\sin(5t)$. Write the solution in the alternate form $x(t) = A\sin(\omega t + \phi)$. What are the amplitude and the period?

$$x(t) = \sqrt{7}\sin(5t - 2.285); \quad A = \sqrt{7}; \quad P = \frac{2\pi}{5}.$$

12. Solve (find $\vec{x}(t)$) for the vector diff. eq. $\vec{x}' = \begin{pmatrix} -6 & 2 \\ -3 & 1 \end{pmatrix} \vec{x}$ using eigenvalues and eigenvectors. Your answer, written as a linear combination of vectors, will have two unknown constants!

$$\vec{x}(t) = c_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-5t}$$