Teaching Statement

Stefan Forcey

1. Introduction

For inspiration when teaching, I draw on my positive past experiences both as a teacher and as a student. I recall how good teachers have motivated me and I remember the successes I have enjoyed while in front of the classroom myself. I have had complete responsibility for a total of 46 sections of courses, 38 at Tennessee State University and 8 at Virginia Tech. These include several flavors of calculus and vector geometry, both point-set and algebraic topology, linear algebra at three different levels, and a course I designed about structured category theory. My teaching–related experience includes tutoring various levels of mathematics from precalculus to abstract algebra. I also have experience assisting students involved in computer–based courses in calculus and linear algebra. All this has served to educate me in the art of teaching. Some lessons I carry foremost in my mind are as follows:

(1) Set concrete goals
This includes goals set by the department and by myself, for the students corporately and individually. In standard courses I use as primary guides the department documented course goals and suggested homework sets. As I organize lectures I make the concepts included in these resources central. In the past when the course I taught included a standardized exam, I verbally set the goal for my students to master the material well enough for our section to pass with flying colors. I then helped them towards that goal by assigning sample test problems throughout the semester in order to keep the exam constantly in mind and review material as we proceeded. On an individual level I try to challenge students to stretch themselves.

(2) Challenge each student
When giving attention to individuals, I gauge their learning styles and potential and then try to convince them to outdo themselves. Sometimes this involves helping them to understand their own responsibility for their mastery of the subject and for their grade. At other times that is not an issue. Then I am free to challenge them further – to excel as opposed to simply pass. I have fond memories of teachers who were not content to let me simply learn the material but dared me to expand my horizons by researching ideas that the class did not cover.

(3) Instill confidence
Of anything that can be directly said to a class, the most effective speech I have given is the one that lets the students know how much self confidence their teacher has. A marvelous way to instill confidence in those under my tutelage is to convince them that *I* believe in *my* abilities: first, that I can solve without trouble every problem in their text, and second, that if they will do what I ask, I can bring them to that point as well. Of course, to follow through on my half of this deal requires effort and careful preparation.

(4) Prepare for everything
For many years it has been impressed upon me that the value of a given lecture is directly proportional to the amount of time and effort spent in preparing it. I owe it to my audience to spend adequate time to ensure that the notes are well-organized, clearly presented, and given with vivid examples. I have had success with varying my presentation, including visual aids such as upturning my bicycle on the lectern to demonstrate the chain rule. I also believe that planning for interactive learning including group work and presentations by individual students is of value. I have been privileged to work with a broad spectrum of students, traditional and nontraditional, mathematically inclined science majors and those who would rather not be required to use numbers, students looking for fun and those looking for a challenge and driven by curiosity. Each student has potential, and it is important to first meet them at their level and then begin to spur them on to a greater understanding. Part of preparation is to use diagnostic tools to find out what that starting point is.

(5) Communicate enthusiasm
I welcome any chance to impart my love of mathematics and physics. There is opportunity in the classroom to harness my sharing impulse to motivate the students. I believe that good grades and prerequisite knowledge for future experiences are important for students to achieve. It is also important for them to have a grander motivation than mere necessity. Indeed, it is often a means to the more pedantic end that the students feel excited about what they are studying. Thus I try to communicate the importance of mathematics and physics as realms of beauty and discovery. I try to introduce the bigger picture by hints of what lies behind the concepts that are covered on the test, as well as by optional assignments encouraging exploration.

These are the ideas that have been impressed upon my mind by experiences in classrooms so far. I am very interested in learning more and plan to continue to enrich my knowledge of what it means to teach well through literature, interaction with other educators, and experimentation of my own based on what I read and hear. I am always interested in finding new methods and tools to help me encourage students to be creative in their approach to problems while insisting on correctness in their solutions.

It is always important in this process to get feedback such as student evaluations and comments, scores from standardized testing, and peer reviews. My student evaluations have been very encouraging.

Student comments include:

"[Mr. Forcey] is very thorough. He goes over many examples and happily answers any questions we have,"

"Mr. Forcey is an excellent teacher and makes an above average effort to help you succeed; he is also very fair and makes the material as stimulating as possible."

" [Dr. Forcey is a] talented mathematician that can actually teach! 'Here's what the theorem says... Now here's what it means...' "

" He is an exceptional teacher who gives students the benefit of the doubt. If you try, he will bend over backwards to help you!"

2. Integrating research and teaching

I have had the privilege of directing four masters theses and three senior theses, the enjoyable responsibility of organizing undergraduate research experience courses, and the honor of chairing the research seminar committee. The roles of researcher and instructor in mathematics are often seen to be at odds, one suffering from emphasis on the other. This is an unfortunate perception, since the quality and the motivational power of teaching at the university level is directly proportional to the instructor's involvement in leading-edge research. The teacher/researcher is the link for the student between an esoteric world of developing science and the more familiar sphere of the classroom. Not only does research activity keep teaching relevant by forcing the professor to stay abreast of recent developments, but glimpses of new results and unanswered questions energize students with a larger view of their studies than afforded by the more mundane problems in their homework.

In teaching and advising at the undergraduate and graduate levels, I have found the principle of research integration to be of universal importance. There should be no surprise here since the goals of inspiring the students' curiosity and motivating their study are inseparable. Of course, our desire to enrich our courses with research should not eclipse the need for solid instruction in the fundamentals of concept, calculation and proof. Mathematical experiments are both easier and more difficult than those in chemistry or biology. The equipment is simpler, since a mathematical experiment is just a calculation for which the outcome is not yet known. The difficulty lies in that the hypotheses and observations are harder to clearly describe. Great amounts of preparation should lie behind each lecture in order to optimize communication of ideas, instill confidence in the instructor and remove as many obstacles from the path to understanding as possible.

The educational value of experimentation is evident at each stage of a research project. First there is the design of the experiment, in which the mathematician must communicate clearly the definitions of the objects and the processes to be performed upon them. This teaches the importance of understanding definitions, which will be reinforced when it comes time to prove theorems. Second, there is the performance of the experiment which teaches the importance of careful calculation possibly including programming and visual representation. Third, there is the repetition with variation of the experiment, which allows the experimenter to hone problem-solving skills in the process of trying to decide what variations might demonstrate a cause and effect relation. Fourth, there is the actual pattern-finding stage, including making predictions and performing further experiments to test them. Finally, there is the stage unique to mathematics, where the researcher must either logically prove the observed pattern of cause and effect or else disprove it by a counterexample. This last stage is one in which the student finds the opportunity to practice reasoning skills.

In the remainder of this statement I will describe some of the courses that I have had the privilege of teaching as well as some that I have designed or modified myself. Due to the requirement that a specified body of knowledge be learned in certain standard classes, not all of these have room for a fully realized research component. However, there is always room to insert individual components of research, such as designing or performing an experiment, or more often suggesting experimentation to be performed on the student's own time. This can be motivated by examples of the sort that are sadly missing in most traditional courses –

examples of open questions. It is unfortunate that we have so little time to teach, since that often means we neglect the larger portion of the subject – the part that marks the boundary of our present knowledge.

3. General education courses.

I have occasionally taught courses designed for non-science majors, such as Contemporary Mathematics. This course has two goals: it is intended in part to engender appreciation of mathematical methods, and in part to introduce the practical mathematics of probability and statistics. A nice segue between the two goals is accomplished by an introduction to combinatorics. Experimentation plays a major role throughout. At first the experiments are prearranged for illustration. For example, the students are asked to plot the path of billiard balls on rectangular tables. The end result of many experiments is their discovery of how the (45 degree) path of the ball depends only on the ratio of the table dimensions, written as a fraction in lowest terms. I then encourage further experimentation on other sorts of tables, even going so far as to discuss the open questions regarding which 2-dimensional figures lead to chaotic situations. Later in the course the experiments are more freely designed by the students, as in the problems of tiling a checkerboard with combinations of various shapes. In this section the goal is to not only see patterns but then to prove them. There are also experiments that lead to false patterns, such as counting the regions of a disk cut by n chords. This allows introduction of counterexamples.

4. Service courses.

In the calculus sequence and other standard classes taken by engineering and science majors, there needs to be emphasis on technical proficiency and exposure to applications as well as mathematical research. The argument that links these together is well known. Concrete examples are often found in the physical world of real objects and real rules (physical, chemical, biological, economical) that correspond nearly perfectly to that abstract theory. Then the patterns proven by mathematicians can be used (together with observed physical laws) to help predict what the outcome of a physical experiment will be. This mathematical modeling demonstrates that mathematical beauty and utility often coincide. A theory that is applicable often turns out to be elegant, and vice versa.

I certainly take the time to stress applications. This is most evident in the discussion of optimization of design in Calculus I, where I have assigned a design project. The students were allowed, after developing their own mousetrap, to consult with me on how calculus could be used to optimize it – then they had to actually perform the mathematics.

There is also value in letting the students get glimpses of the big picture, including open questions related to the topic at hand. In Calculus III I mention many of the applications of convergent series, including finding approximate solutions to unsolved differential equations. This affords an opportunity to mention the Navier-Stokes equations. When we cover the p-series test for convergence I lead a discussion of the Riemann Zeta function and the Riemann hypothesis.

5. Courses for math majors.

An exciting opportunity that I have recently enjoyed is to co-develop with my colleague M. Reed a two-semester sequence entitled Mathematics Research Experience. Some of the content is relatively pedestrian: we practice finding, reading and understanding mathematical literature, typesetting in LaTeX, and giving presentations. Even in this segment the students are exposed to exotic mathematics; they choose an unfamiliar structure about which to write a short exposition.

The central goal however is to direct the students in the discovery and creation of new mathematics. This includes experimentation, the formulation of conjectures, the finding of counterexamples and the writing of original proofs. Some of the work is done corporately. For instance we recently experimented in combinatorial geometry by constructing square figures from a pair of shapes – a right isosceles triangle and a rhombus as pictured in Figure 1. The simpler problem is to develop a recipe for how many of each shape is needed to construct a given size square, and to prove this recipe well defined. Much harder is the stomachion-type problem of enumerating the dissimilar arrangements of shapes which yield that square. In the past I have also assigned experiments based on operads in the natural numbers. An operad in the natural numbers is just a sequence that grows in a particular recursive way. The goal is to find closed-form formulas for the n^{th} term, which is still an open question.

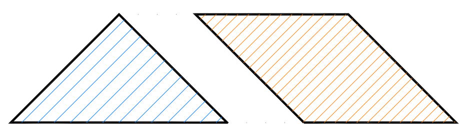

Figure 1. The pieces of the parquetry problem.

As individuals, the students are also asked to design their own experiments and make their own conjectures about some aspect of the mathematical object they wrote about earlier in the course. This exercise can often be a huge help in starting their eventual senior projects. We finish the first semester with a section about the creation of new theory – new axioms, new structures, new properties – and the pitfalls of doing this without proper motivation.

6. Graduate courses

In point-set topology I focus on the importance of understanding the axioms of each definition. Examples are crucial, and one class which allows unambiguous visual representation is the class of the finite spaces. Thus each concept, from quotient spaces to the compact-open topology, is demonstrated first on finite spaces. One problem I pose students early on is to investigate the geometric combinatorics of finite spaces of n points. These spaces form a poset with the covering relation described by containment of topologies. The simpler question is whether their Hasse diagram can be the 1-skeleton of a polytope. The harder question is the enumeration of finite topologies of a given size. Of course many examples and counterexamples in topology require infinite or even uncountable spaces. An open question I never miss the opportunity to discuss is the question of path connectedness of the Mandelbrot set.

In Algebraic topology the exercises are usually a little less experimental, designed rather to demonstrate the usefulness of homotopy and homology groups. I find it helpful to include examples that are homotopic but not homeomorphic, as well as examples that are homeomorphic but hard to visualize. Classification projects often work well as group efforts. Another more experimental group project I assign is to find the knot group of a braid closure, in terms of a choice of generators of the braid group. Markov moves should turn out to correspond to group isomorphisms.

Other interesting injections of research topics into my graduate courses include a discussion of determinants as used in the Alexander knot polynomial (in Linear Algebra) and a discussion of Nash equilibria as predicted by the Brouwer fixed point theorem (in Algebraic topology). As a final example, in my course on structured categories, we cover the Mac Lane coherence theorem. Before going on to braided and iterated monoidal categories, the students are assigned the task of constructing the 1-skeleton of the three-dimensional associahedron, and building the polytope either as a 3d model or a Schlegel diagram.