

# Liquidity, Default Risk, and the Information Sensitivity of Sovereign Debt

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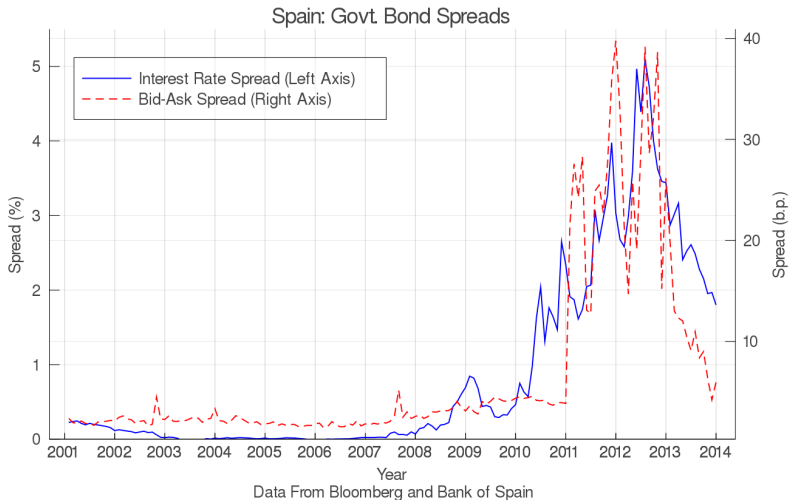
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# This Paper

- Document empirical relationships between interest rate spreads, liquidity, and default risk in Spain.
  - ▶ Bid-Ask Spread =  $Y\bar{T}M_{Bid} - Y\bar{T}M_{ASK}$
- Explain variation in liquidity measures as the equilibrium result of some traders having private information.
- Match business cycle patterns of debt accumulation in a developed country using more flexible preferences.

# Bid-Ask Spreads and Interest Rates: Spain



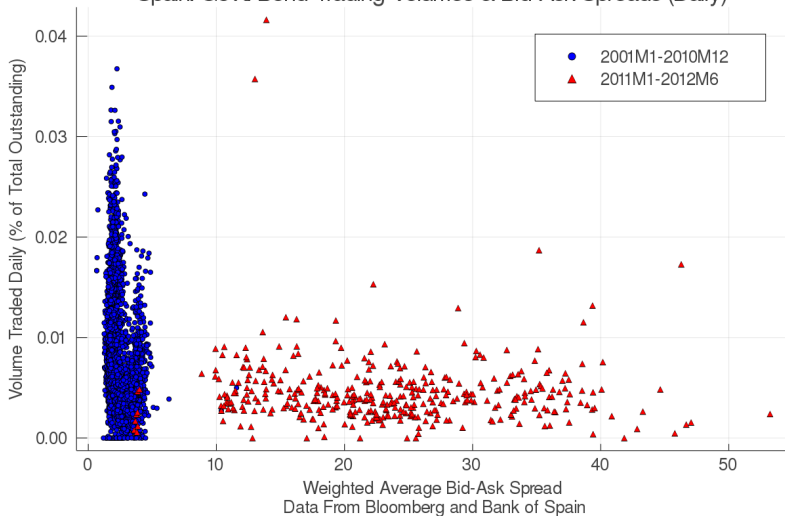
► vs. CDS S

► Bid-Ask Spread Time Series

► Interest Rate Spread Time Series

# Liquidity and Bid-Ask Spreads: Spain

Spain: Govt. Bond Trading Volumes & Bid Ask Spreads (Daily)



# Literature Review

- Passadore and Xu (2018) and Chaumont (2018):
  - ▶ This paper has no search frictions in secondary markets.
  - ▶ Differences in valuations not driven by permanent changes in investor preferences (good investor vs. bad investor).
- Gorton and Ordonez (2014 and 2019) and Dang, Gorton, and Holmstrom (2015):
  - ▶ This paper implements a version of their “information sensitivity” concept.

# Key Ingredients

- Model of external sovereign debt a la Eaton Gersovitz (1981).
- Add model of secondary market interactions with:
  - ① Ability of some agents to acquire private, payoff-relevant information
  - ② Anonymous trading
  - ③ Random differences in fundamental valuations of bonds between buyers and sellers

# Environment

- Small open economy.
- Output is a Markov Process  $y(s)$ .
- Benevolent government and representative consumer. Recursive preferences.
- Single long term bond: maturity rate  $\lambda$  & coupon rate  $\kappa$
- While in default, output is reduced.
- Continuum  $[0, \bar{B}]$  of risk neutral, competitive international investors, each of whom can hold a unit of debt.
- Current investors may spend  $f(\pi)$  to access information about  $y(s')$  one period ahead of time with probability  $\pi$ .

# Timing

- ① Income and reentry realized.
- ② Default decisions.
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Current investors may attempt to access it at cost.
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- ⑤ Current investors' random taste shocks are realized.
- ⑥ Secondary market opens:
  - ▶ Random matching.
  - ▶ Bid and ask prices submitted simultaneously.
  - ▶ If  $p_{bid} \geq p_{ask}$ , the transaction clears at  $p_{bid}$ .
  - ▶ New purchasers replace exiting sellers.

## Government Problem

$$W(s, b) = \max_{d \in \{0,1\}} (1 - d)W^R(s, b) + dW^D(s) \quad (1)$$

Conditional on repayment:

$$W^R(s, b) = \max_{c, b'} U(c, \bar{W}(s, b')) \quad (2)$$

such that

$$c + (\lambda + (1 - \lambda)\kappa)b = y(s) + q(s, b')(b' - (1 - \lambda)b) \quad (3)$$

Conditional on default:

$$W^D(s) = U(y(s) - \phi(s), \bar{W}^D(s)) \quad (4)$$

where  $\mu(\cdot)$  is a certainty equivalent operator and:

$$\bar{W}(s, b') = \mu(W(s', b')|s) \quad \bar{W}^D(s) = \mu(W(s', 0), W^D(s')|s) \quad (5)$$

## Secondary Markets

- Risk neutrality and competitiveness of lenders:

$$q(s, b') = \max_{\pi} (1 - \pi)q_U(s, b') + \pi q_I(s, b') - f(\pi) \quad (6)$$

- $\pi_S(s, b')$  = equilibrium proportion of current investors who obtain access to  $\hat{y}'$ .
- $q_U(\cdot), q_I(\cdot)$  = value of being uninformed or informed, respectively.
- $\pi_S(s, b') \in (0, 1)$  implies:

$$q_I(s, b') - f'(\pi) = q_U(s, b') \quad (7)$$

## Secondary Markets - Notation

- $v$  denotes the undiscounted unit value of the asset to an uninformed agent.

$$v(s, b') = E[(1 - d(s', b'))(\lambda + (1 - \lambda)(\kappa + q(s', b''(s', b')))) | s] \quad (8)$$

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- $\hat{v} \sim G(.)$  denotes the random variable which is the undiscounted unit value of the asset to an informed agent (and of course  $E[\hat{v}] = v$ ).

$$\hat{v}(s, \hat{y}', b') = E[(1 - d(s', b'))(\lambda + (1 - \lambda)(\kappa + q(s', b''(s', b')))) | s, \hat{y}'] \quad (9)$$

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- $\hat{\delta} \sim F(\cdot)$  denotes the random taste shock of current investors.
- $\delta$  denotes the constant, known taste shock of new investors.

## Secondary Markets - Sellers

Given any bid strategy of buyers and their own  $\hat{\delta}$ , sellers solve:

$$q_U(v|\hat{\delta}) = \max_{p_{S,U}} \mathbf{1}\{p_{S,U} > p_B\} \hat{\delta} v + \mathbf{1}\{p_{S,U} \leq p_B\} p_B \quad (10)$$

or:

$$q_I(\hat{v}|\hat{\delta}) = \max_{p_{S,I}} \mathbf{1}\{p_{S,I} > p_B\} \hat{\delta} \hat{v} + \mathbf{1}\{p_{S,I} \leq p_B\} p_B \quad (11)$$



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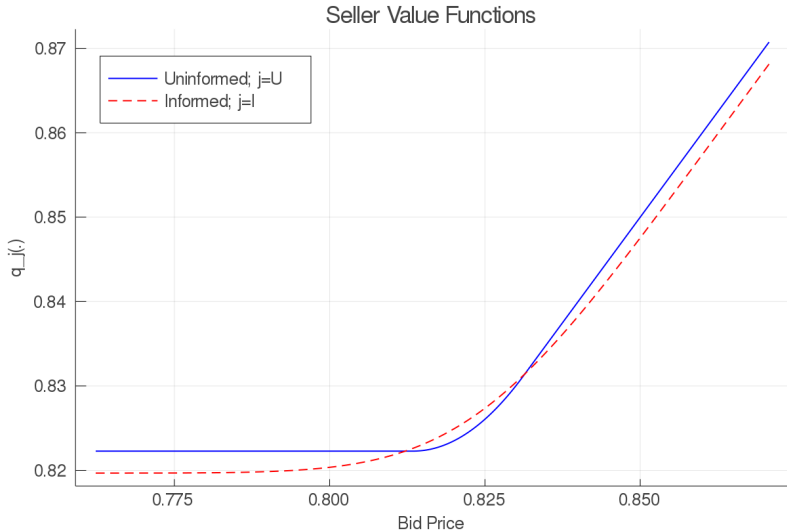
Since transactions clear at the bid price:

$$p_{S,U}^*(\hat{\delta}, v) = \hat{\delta} v \qquad p_{S,I}^*(\hat{\delta}, \hat{v}) = \hat{\delta} \hat{v} \quad (12)$$

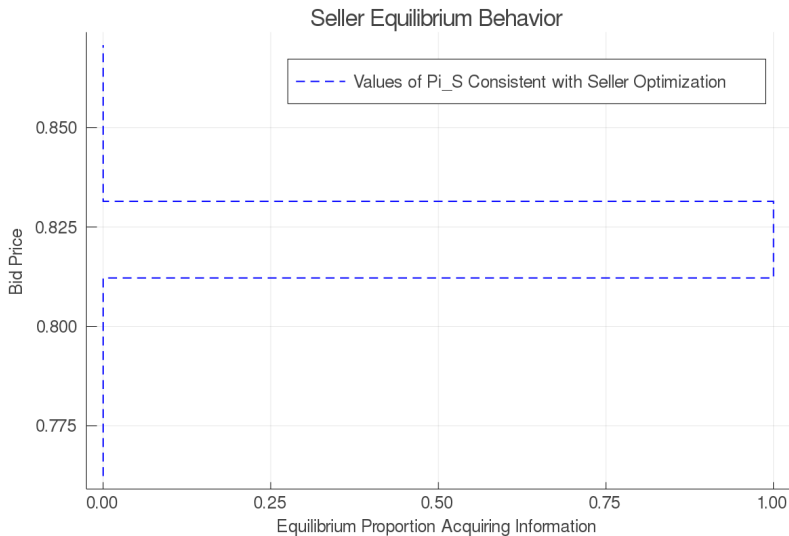
Probabilities of trading at a given bid price  $p_B$ :

$$Pr(\text{Trade}|U, v)(p_B) = F\left(\frac{p_B}{v}\right) \quad Pr(\text{Trade}|I, \hat{v})(p_B) = F\left(\frac{p_B}{\hat{v}}\right) \quad (13)$$

# Secondary Markets - Seller Values



# Secondary Markets - Seller Equilibrium Behavior



## Secondary Markets - Buyers

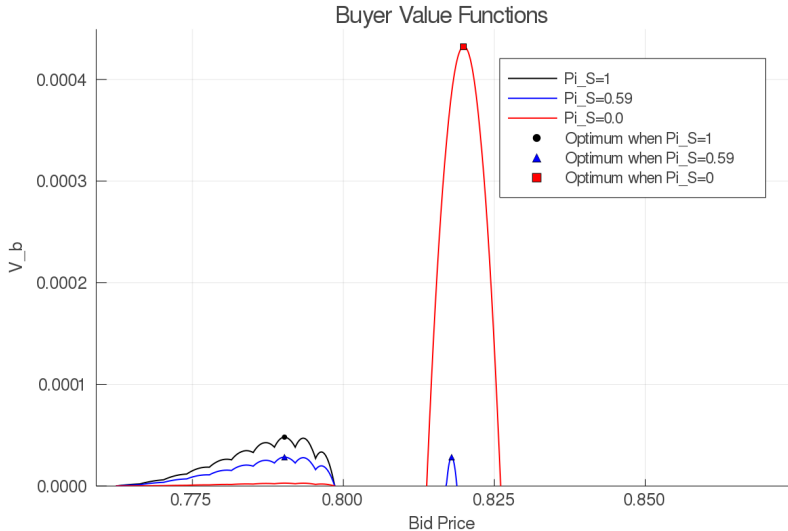
- Buyers then solve:

$$\begin{aligned} & \max_{p_B} (1 - \pi_S)(\delta v - p_B)F\left(\frac{p_B}{v}\right) \\ & + \pi_S \left( -Pr(\hat{v} = 0)p_B + \int_V (\delta \hat{v} - p_B)F\left(\frac{p_B}{\hat{v}}\right)dG(\hat{v}) \right) \end{aligned} \quad (14)$$

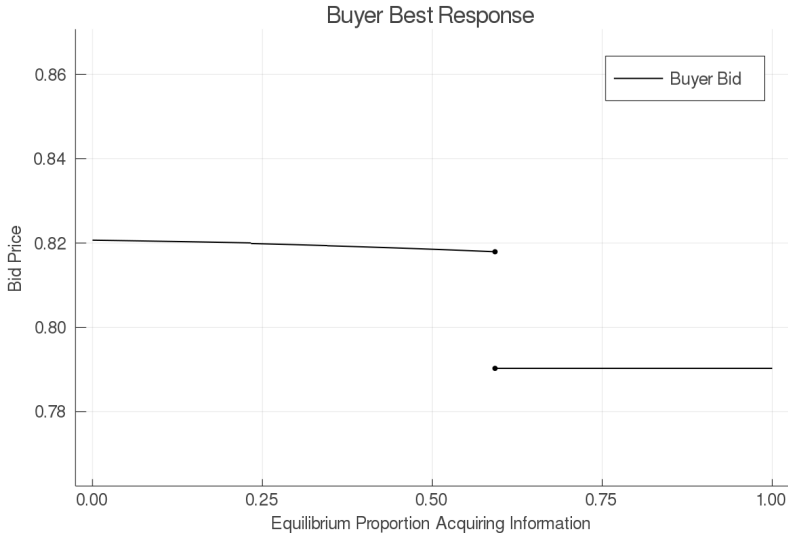
- Mechanism driving bid ask spreads:

$(\delta \hat{v} - p_B)$  negatively correlated with  $F\left(\frac{p_B}{\hat{v}}\right)$

# Secondary Markets - Buyer Values

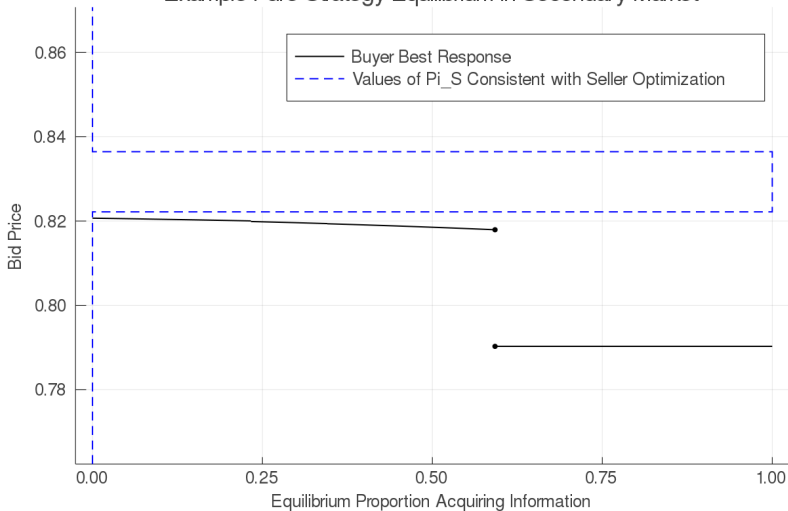


# Secondary Markets - Buyer Best Response



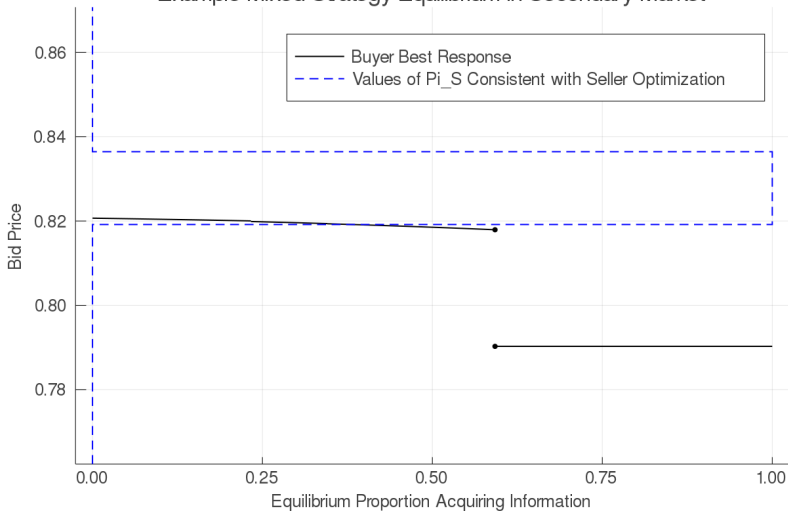
# Secondary Markets - Equilibrium

Example Pure Strategy Equilibrium in Secondary Market



# Secondary Markets - Equilibrium

Example Mixed Strategy Equilibrium in Secondary Market





# Functional Forms

- Epstein-Zin Preferences:

$$U(c, \bar{W}'(s)) = ((1 - \beta)c^{1-\psi} + \beta \bar{W}'(s)^{1-\psi})^{\frac{1}{1-\psi}} \quad (15)$$

$$\bar{W}'(s) = E[W(s')^{1-\gamma} | s]^{\frac{1}{1-\gamma}} \quad (16)$$

- $y(s) = \tilde{y} + m$

$$\tilde{y}' = \rho \tilde{y} + \eta \quad \eta \sim^{iid} N(0, \sigma_\eta^2) \quad m \sim^{iid} TN(0, \sigma_m^2, -\bar{m}, \bar{m}) \quad (17)$$

- $\hat{\delta} \sim U(\underline{\delta}, \bar{\delta})$

- $\hat{y}'$  parametrized as the true  $\tilde{y}'$  plus a noise term:

$$\hat{y}' = \tilde{y}' + \epsilon \quad \epsilon \sim^{iid} N(0, \sigma_\epsilon^2) \quad (18)$$

# Calibration

All parameter values are monthly, where applicable.

Table 1: Fixed Parameters

Parameter	Value	Notes
$\rho$	0.9918	SE: 0.007
$\sigma_{\eta}$	0.0049	SE: 0.0005
$\sigma_m$	0.0015	SE: 0.0004
$\bar{m}$	0.0031	
$\theta$	0.0130	CE 2012
$\underline{\delta}$	0.990	Fix implied $r_f = 0.33\%$ when $\pi_S = 0$
$\bar{\delta}$	0.999	Fix B-A Spread = 2.5 b.p. when $\pi_S = 0$
$\bar{\delta}$	1.001	Fix volumes=37% when $\pi_S = 0$
$\lambda$	0.0122	Weighted Average Maturity of Debt
$\kappa$	0.0041	Average Coupon of Debt

# Calibration

This leaves the parameters below.

Table 2: Calibrated Parameters

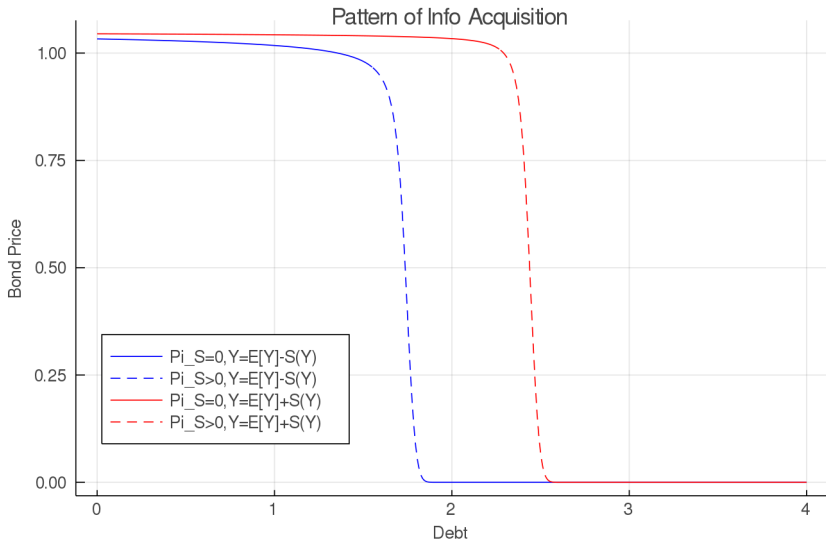
Parameter	Value	Notes
$\psi$	11.73	Govt Inverse IES
$\gamma$	4.83	Govt Risk Aversion
$\beta$	0.992	Govt Discount Factor
$d_0$	-0.110	Linear Default Cost
$d_1$	0.142	Quadratic Default Cost
$f$	0.000125	Cost of Information (Linear)
$\sigma_\epsilon$	0.037	SD of Noise in $\hat{y}$

# Results

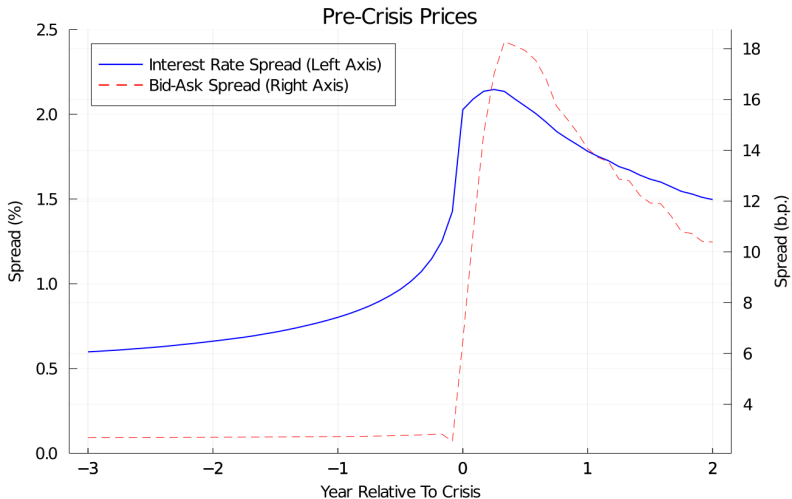
Table 3: Targeted Moments (Annualized Values)

Moment	Period	Data	Model
$E[B'/Y]$	Jan 1 2001 - June 30 2012	11.9%	13.5%
$\rho(B'/Y, \ln(Y))$	Jan 1 2001 - June 30 2012	-0.76	-0.49
$\rho(NX/Y, \ln(Y))$	Jan 1 2001 - June 30 2012	-0.78	-0.10
$E[r - r^f]$	Jan 1 2001 - June 30 2012	0.72%	0.83%
$\sigma(r - r^f)$	Jan 1 2001 - June 30 2012	1.13%	1.05%
$E[BA]$	Jan 1 2001 - June 30 2012	5.5 b.p.	5.4 b.p.
$\rho(BA, r - r^f)$	Jan 1 2001 - June 30 2012	0.84	0.80

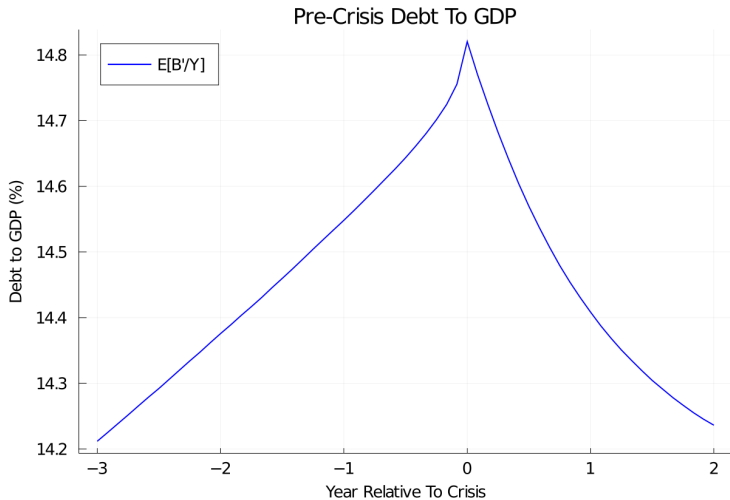
# Results - Mechanism



# Results - Crises



# Results - Crises

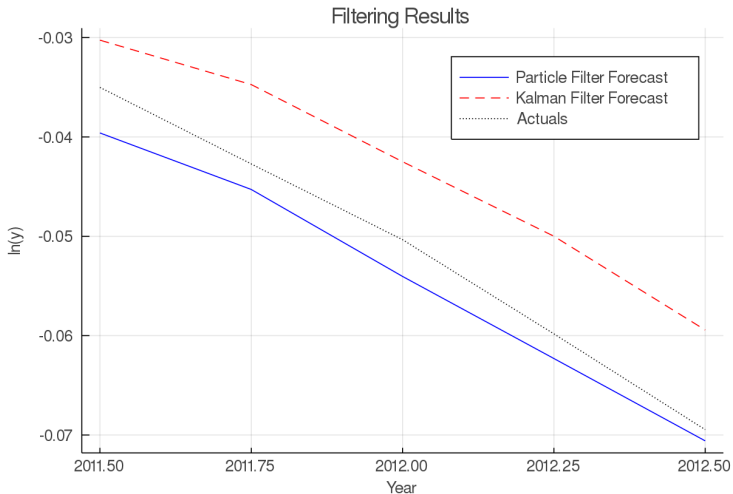


## Results - Validation

- In the model, realized bid-ask spreads depend on the distribution of forecasts obtained by investors.
- Those forecasts in turn depend on the true value of future output.
- Therefore, bid-ask spreads should provide information on future output.
- Does including this information improve forecasts of Spanish output during the crisis relative to the one-step ahead prediction of the Kalman Filter?



# Results - Validation

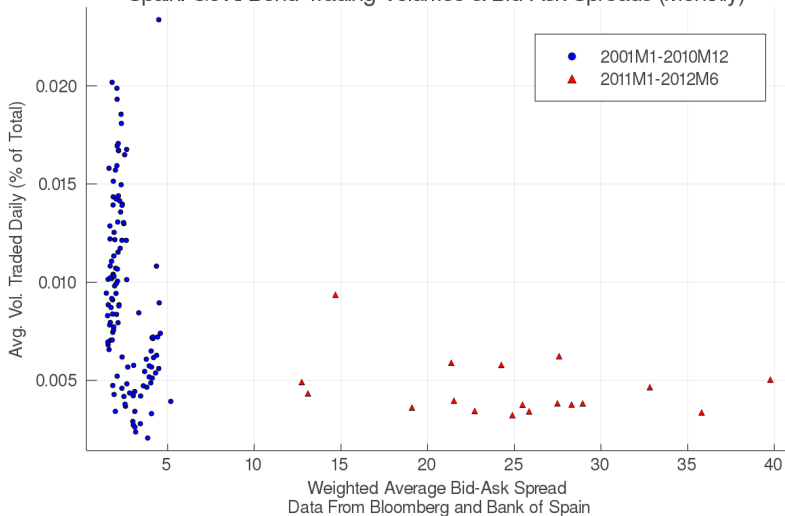


# Conclusions

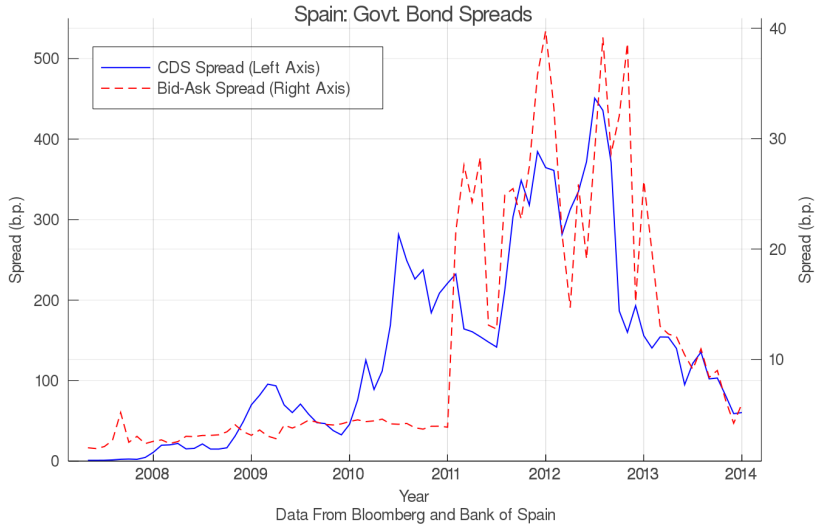
- A model of costly acquisition of private information by traders can rationalize the type of relationship between bid-ask spreads and interest rate spreads/default risk observed in the data.
- Predictions the model makes about the relationship between bid-ask spreads and future realizations of output are borne out in the data.

# Liquidity and Bid-Ask Spreads: Spain

Spain: Govt. Bond Trading Volumes & Bid-Ask Spreads (Monthly)

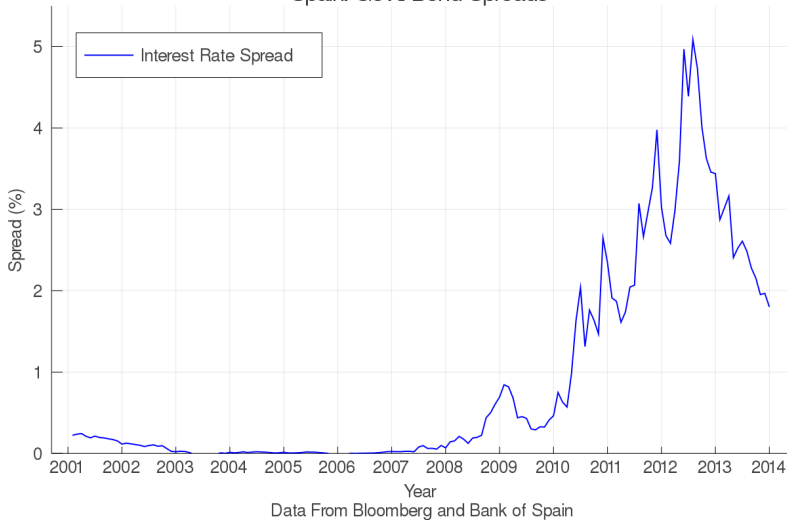


# Bid-Ask Spreads and CDS Spreads: Spain



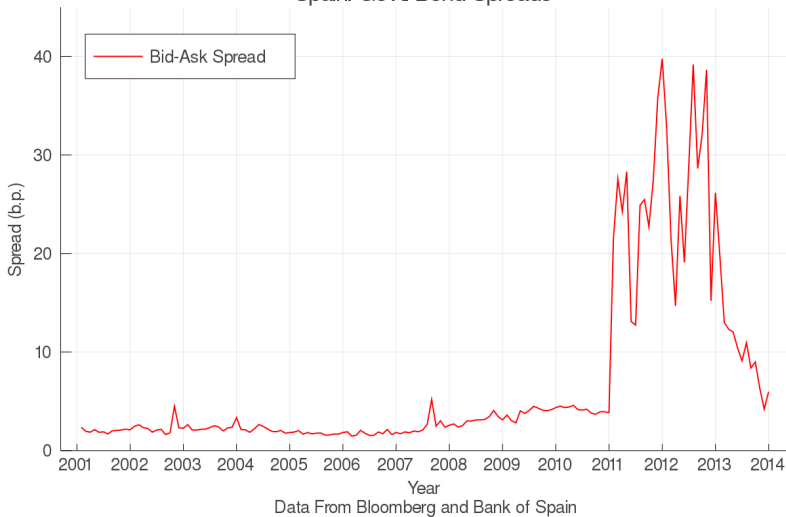
# Interest Rates: Spain

Spain: Govt. Bond Spreads



# Bid-Ask Spreads: Spain

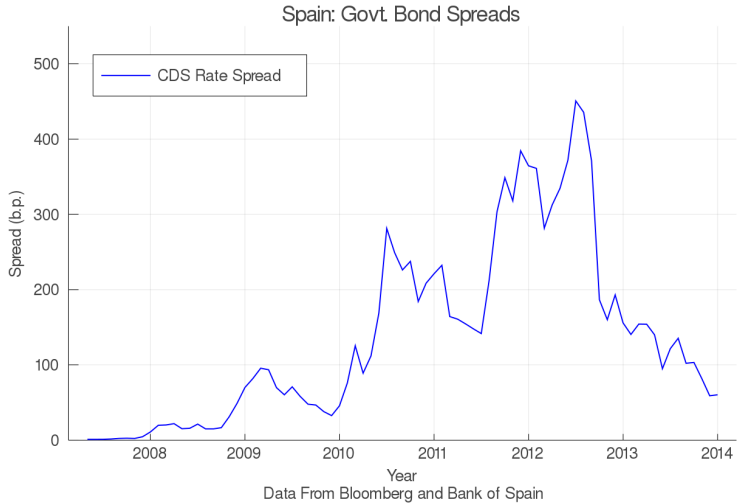
Spain: Govt. Bond Spreads



[◀ Back to Int/BA](#)

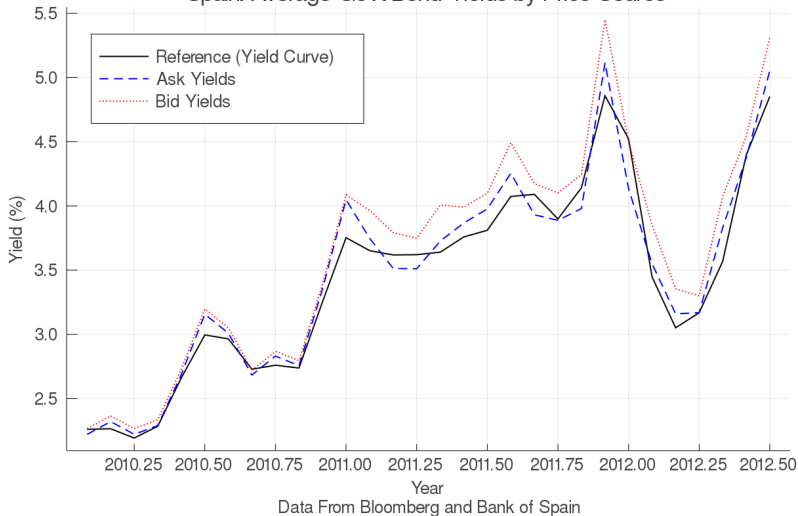
[◀ Back to CDS/BA](#)

# CDS Spreads: Spain



# Bid-Ask Spreads: Spain

Spain: Average Govt Bond Yields by Price Source





# Secondary Markets - Equilibrium

Example Mixed Strategy Equilibrium in Secondary Market

