Sovereign Default and Government Reputation

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Abstract

In this paper, I build a flexible theoretical model of sovereign borrowing, default, and renegotiation with borrower reputation. There is asymmetric information about the government's "type", and reputation is the market belief that it is "responsible" and therefore less likely to default. Every government decision informs market beliefs about this "type". I calibrate the model using data on how countries' credit histories affect the prices they face. Using the model, I show that countries that have recently defaulted have poor reputations because they rapidly run up their debts prior to default, not because the default decision itself is revealing. I also show that, for countries facing non-trivial levels of default risk, the reputational costs of default are less than 0.2 basis points of consumption. I then validate the model by showing that its predictions about the effects of borrowing behavior on interest rate spreads through the reputation channel are borne out in the data. Finally, I show that transparency initiatives and audit programs have significant, negative implications for welfare, because they weaken the signaling mechanisms that prevent, to some extent, overborrowing by the government.

JEL Codes: F34, F41, H63

1 Introduction

Governments value easy access to cheap credit. Being able to borrow at low interest rates allows them to smooth consumption and make optimal investments when opportunities present themselves. One key determinant of access to cheap credit is whether a country has a good reputation for paying its debts back. There are countries like Germany, which have built strong reputations for being frugal and consistently paying back their debts, and there are countries like Argentina, which have become notorious for how frequently they default. Reinhart et al. (2003) show that such patterns are systematic. Having been in default more frequently and more recently is correlated with markets' perceived likelihood a country will default again. However, if countries avoid defaulting for long enough, they appear to "graduate" and shed their old reputation (Qian et al., 2015).

There is evidence in statements going back hundreds of years that policy makers are interested in building and maintaining a good reputation for their countries in financial markets. For example, right after the federal government of the United States was established, Alexander Hamilton pushed for it to assume the debts of the states that had been accrued in waging the American Revolution. His reasoning was that, by shouldering a large debt burden and steadily, consistently paying it back, the country could establish a good reputation in international markets, ensuring that it could borrow at cheaper rates should it need to, in the future (Hall et al., 2021).

The purpose of this paper is to build a theory of how governments build and maintain a good reputation, and how concerns about this reputation influence their borrowing, default, and renegotiation decisions. In doing so, I fill a gap in the literature on sovereign default, the role of reputation. Quantitative research on this issue has been rather sparse, but that does not reflect doubts that reputation is important. Rather, it follows from a limitation of existing models to capture reputational effects in a tractable way and discipline them using data.

In this paper, I build a state of the art model of sovereign borrowing, default, and renegotiation which incorporates a notion of reputation. The model contains a number of features that the literature has shown to be important for matching the data on sovereign borrowing, including long term debt and a flexible renegotiation bargaining protocol.

In my model, reputation is the market belief about whether a government is believed to be "responsible" or 'irresponsible". The true "type" of the government is known only to the government itself. In this model, the government makes a wide variety of decisions, including how much to borrow, whether to default, and what type of renegotiation offers to make to lenders (or accept from them). All of those decisions can affect its reputation. This flexibility about which decisions affect reputation is a key theoretical improvement over the existing literature (which has focused almost exclusively on the default vs repayment margin).

In order to discipline my model, I augment a relatively standard set of targeted moments with a set of regression coefficients estimated by Cruces and Trebesch (2013) that describe how historical restructurings affect current spreads up to seven years after the fact. I show how the parameters controlling the stochastic government type process and the flow of information in the model are informed by the patterns in the data. Once I have estimated the model, I describe its key features and the effects of the information asymmetry on government behavior. One key result that emerges is that the default decision in and of itself is not very informative. It is the case, in the model, that countries which have recently defaulted have very poor reputations, but this is because they had run their reputation into the ground by borrowing very large amounts prior to defaulting. This rapid borrowing both destroys the country's reputation and raises its probability of default, so the vast majority of countries which do default already have poor reputations.

In a related result, I quantify the potential reputational costs of default for countries facing non-trivial levels of short term default risk. I find that countries exposed to even $1-2\%$ three month default risk have very poor reputations, and the average value of their remaining reputation is less than 0.002% (or 0.2 basis points) of consumption. For countries facing higher levels of default risk, these potential reputational costs are even smaller. Furthermore, this is not because the model implies having a good reputation is never valuable. Indeed, it is possible for the value of a good reputation to be worth almost 2% of consumption. However, countries whose reputations are worth quite a lot are almost invariably very far from default. By the time the possibility of default is truly on the horizon, countries have already revealed themselves as likely to be irresponsible, and the value of any remaining ambiguity is negligible. This suggests that, in policy debates about whether to default, concerns about reputational costs should not play the outsize role they often have.

Having used information on how default/restructuring histories affect spreads to calibrate the model, I then validate the model by verifying that its predictions about how incremental borrowing decisions affect spreads are borne out by the data. Specifically, I use my model to estimate how issuance choices affect reputation. Then I use data on debt issuances for a large set of countries to construct a measure of those countries' reputations. This model-filtered reputation provides significant additional explanatory power in reduced form estimates of the determinants of interest rate spreads and near term default probabilities. I consider this evidence that the out of sample predictions made by my model are quantitatively sound.

I then use my model to evaluate the effects of transparency initiatives, audit programs, and accountability offices on both government payoffs and consumer welfare. In this section, I find that transparency (interpreted as an exogenous flow of information about the government's type) has significant, negative consequences on both government payoffs and consumer welfare. This result follows from overborrowing in the setting with transparency and the strength of signalling incentives in the setting without transparency. Long term debt introduces dilution motives that make the allocation without asymmetric information inefficient (see Aguiar and Amador (2019) for a detailed discussion of this phenomenon). This inefficiency manifests as overborrowing by the government. The signalling motives under asymmetric information induce the government to borrow significantly less, partially correcting the underlying inefficiency. As a result, default, which is quite costly for both government payoffs and consumer welfare, occurs significantly less frequently. While the government suffers some from the inability to run up its debt quite as fast (relative to what it could do under increased transparency), it issues debt at prices which are uniformly higher than they would be in the case with transparency. This helps ameliorate the losses it faces from lower overall borrowing levels.

2 Literature Review

This paper builds on three related strands of literature. The first is the overall quantitative sovereign default literature. The second is a broad set of papers focusing on models of borrowing and default with private information. The third is a large set of empirical papers examining the correlation between credit history and sovereign bond spreads.

2.1 Sovereign Default

The seminal paper, Eaton and Gersovitz (1981), marks the beginning of the modern literature on sovereign default. The authors construct a model in which a risk averse government facing income risk can borrow using non-state contingent contracts from a pool of lenders, but cannot commit to repay them. Since then, many pieces have been added to this model in order to study various features of government borrowing, but the core of the environment almost always remains the same. Early notable theoretical contributions include Bulow et al. (1989), who study what types of punishments for default can support positive levels of debt in equilibrium and Cole and Kehoe (2000), who provide a theoretical framework for studying self-fulfilling panics in government debt markets.

The modern quantitative sovereign default literature traces its roots to Aguiar and Gopinath (2006) and Arellano (2008). Both study how income fluctuations affect interest rates and borrowing patterns in emerging economies using models with stochastic output, risk neutral lenders, one period bonds, 100% haircuts, and a physical output loss during default. While these are important contributions, especially in terms of their computational implementations, both fail to rationalize observed levels of debt and patterns of interest rates. Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) show that modifying the baseline model to include long term debt allows the model to achieve debt levels consistent with the data and interest rate spreads which are both as large and as volatile as those observed in emerging economies. These models with long term debt provide a workhorse model that many strands of the literature build on.[1] I incorporate their environment as a

[1] Among these are papers focused on the government's choice of maturity for its debt (Arellano and Ramanarayanan (2012), Sánchez et al. (2018), Bocola and Dovis (2019), and Dvorkin et al. (2021)), papers focused on the interaction between government borrowing and default decisions and a domestic production

baseline and then build on it by introducing asymmetric information and explicitly modelling the renegotiation process. For a detailed survey of the overall sovereign default literature, I refer the reader to Aguiar and Amador (2014) and Aguiar and Amador (2021).

One strand of the literature explicitly models the renegotiation process. One early contribution here is Yue (2010), which employs a Nash Bargaining framework in order to study how the renegotiation process and the determination of recovery rates ex post affect the government's borrowing and default behavior ex ante. My paper builds on two newer, similar papers in this literature on sovereign default with endogenous renegotiation. The first is Benjamin and Wright (2013), who study why there exist delays, sometimes taking years, between default and the completion of the renegotiation process. In doing so, they develop a flexible quantitative model of the renegotiation process as a game with an alternating offers structure, building on the general bargaining environment described by Merlo and Wilson (1995). The second is Dvorkin et al. (2021) who update the model of Benjamin and Wright (2013) to encompass sunspot equilibria (a la Cole and Kehoe (2000)) as well as more modern features of quantitative models of sovereign default (long term debt with variable maturity structures, for example). Their focus is on explaining the existence of delays as well as the role of maturity extensions in enabling more efficient settlements. This paper will largely abstract from the reasons underlying delays and focus on the patterns observed after the renegotiation process is completed. That said, my model incorporates this bargaining protocol.

2.2 Default with Private Information

Cole et al. (1995) is a paper motivated by the observation that most defaulting sovereigns in the 1800's repaid their old debts before issuing any new debts, even if the defaulted debts were decades old. This is especially puzzling because, at the time, sovereign immunity was interpreted quite broadly in the jurisdiction where most bonds were issued, so investors had essentially zero recourse if the government simply decided not to pay. The authors propose

economy (Mendoza and Yue (2012), Bocola (2016), and Gordon and Guerron-Quintana (2018)), and papers focused on the role of rollover risk and self-fulfilling crises (Conesa and Kehoe (2017) and Bocola and Dovis (2019)).

that settling old debts may be a signaling mechanism for the government. By settling old debts, the government may be able to prove to investors that it is less likely to default in the future. This reputational gain is valuable to the government because it affects the prices of any new debt issuances. The authors build a simple model of default in which there are two types of government, one more patient and one myopic. Using an example set of parameters, they show that the patient one does in fact repay defaulted debts in order to signal to lenders that it is the patient type and take advantage of the better prices offered to it afterwards.

English (1996) is closely related to Cole et al. (1995) in its focus. It documents the borrowing and defaults of various US states in the mid 1800's. Similarly to the cases of sovereigns documented in the 1995 paper, states which defaulted settled their debts before issuing new debts (in one case, this settlement occurred over 50 years after the original default). Furthermore, there was not a particularly strong correlation between the debt levels of the states which borrowed and defaulted and those that borrowed and repaid, suggesting that factors beyond easily observable fundamentals may have been involved. The paper does not build a model to justify the behavior documented but suggests that the one constructed by Cole et al. (1995) or another, similar model of reputational gains through settling old debts may be a fruitful line for research. He points out that, in the context of such models, reputation is more than just a coordinating device for lenders, but actually contains information relevant for their payoffs.

These two papers focus primarily on the role of settling the debt as the primary signal by which lenders learn about the government's type, and ignore the potential role of borrowing and/or default. Furthermore, in their quantitative sections, they focus on matching stylistic facts (i.e. governments settle debt before borrowing again), rather than explaining the more specific patterns observed in the data (how long settlement takes, what the average settlement looks like, the dynamics after settlement, etc.). In this paper, I do not specify ex ante which of the governments actions (i.e. borrowing, default, renegotiation, and restructuring) are informative to lenders, nor what the relative informational content must be. By calibrating the model to match the patterns of debt accumulation, default, restructuring, and interest rate spreads, I discipline the parameters governing how informative the various actions of the government are to lenders.

In a classic paper, Bulow et al. (1989) show that reputation, construed as simply the entire history of government actions, is not sufficient to sustain positive levels of debt in equilibrium under fairly general assumptions. Since then, there have been a number of attempts to resurrect the role of reputation in such models. One story, put forward by Cole and Kehoe (1998), is that there may be spillovers to the government's reputation in different relationships when it defaults on foreign creditors. Costs incurred by damage to those other relationships may then be able to sustain positive levels of debt. Another set of stories involves asymmetric information and the role of government actions in communicating that information to lenders in a credible way. Cole et al. (1995) is actually a member of this group. Others include Sandleris (2008), Phan (2016), Phan (2017a), and Phan (2017b). All of them rely on the default vs. repay margin being a way for the government to signal to foreign lenders good news or bad news. While the model I build in this paper shares their focus on the informational content of government decisions, the information is not about economic fundamentals, but rather about the preferences of the decision maker.

Amador and Phelan (2021b) and Amador and Phelan (2021a) are a pair of recent papers which develop stylized models of how governments build and maintain a reputation for repaying their debts. Their setting includes a sovereign borrower who can be either a commitment type or an optimizing type. The type changes over time and is known only to the government itself. In both papers, the only difference between the two types is their default behavior. Conditional on repayment, the two types behave exactly the same way. In Amador and Phelan (2021b), the commitment type never defaults, while the optimizing type may default whenever it wishes. In Amador and Phelan (2021a), there is an exogenously specified – and not necessarily optimal – default policy function (and haircut policy function, conditional on defaulting) for the commitment type, and the optimizing type may default and choose haircuts however it likes. Relative to these papers, my model is more flexible about which decisions affect reputation. And I show that flexibility matters. Once the model is disciplined with the data, the default decision itself is not very informative, and instead the borrowing decisions prior to the default convey the vast majority of the information.

Morelli and Moretti (2021) is a quantitative paper that is more flexible than most of the existing literature in terms of which choices can affect a country's reputation. They study how the Argentinian Government's reports (and misreports) of the inflation rate affect the interest rates it pays on dollar-denominated debt. In their empirical work, they find that misreports are in fact positively correlated with those interest rates. Since the reported inflation rate does not directly affect the value of payments on such debts, they interpret this relationship as evidence of reputational effects. Specifically, they hypothesize that misreports of inflation are informative about whether the government is a type that is likely to default or not. To test this theory, they build a sovereign default model with asymmetric information about whether the government is a commitment type or a strategic type. The key differences between the two types are that the strategic type finds default less painful than the commitment type does, and that the strategic type may freely choose its report of inflation (which affects the cost of servicing inflation indexed debts) while the commitment type must tell the truth. Lenders do not directly observe this report, but see a noisy signal about it. In their setting, only the default decision and the inflation report can affect a country's reputation. They impose that the strategic type, conditional on not defaulting, behave exactly like the commitment type, so borrowing choices provide no information to lenders. Unlike their paper, I do not consider nominal debts and the role of inflation. Furthermore, government actions in my model are perfectly observed by lenders, rather than communicated via a noisy signal. In addition, as mentioned above, I will allow all of the government's actions (including its borrowing decisions) to affect its reputation.

In terms of its setting and flexibility about which choices affect a country's reputation, D'Erasmo (2011) is the closest paper to mine in the sovereign default literature. In particular, it allows all government decisions (default, borrowing, and whether to initiate the restructuring process) to affect the government's reputation. That paper, however, had very different goals (1. allow high levels of debt in equilibrium and 2. explain sovereign credit ratings). Furthermore, it does not include long term debt, which the quantitative literature since 2011 has shown to be a crucial tool for matching the behavior of debt levels and interest rates (see, for example Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012)). It also uses Nash Bargaining to determine the outcome of the restructuring process, and implements it in a way that forces the outcome to be independent of the actual type of the government. I will instead use the alternating offers protocol developed by Benjamin and Wright (2013) and Dvorkin et al. (2021) to model the renegotiation process. This protocol is much better suited to a setting with asymmetric information than the Nash Bargaining protocol implemented in D'Erasmo (2011). Finally, the model of D'Erasmo (2011) is solved computationally in pure strategies. Since pure strategy equilibria of this model always feature either perfect pooling or perfect separation, belief updates have a bang-bang characterization. The government's actions are either perfectly informative or perfectly uninformative. This makes the model hard to quantify and take to the data. To remedy this, I borrow methods from the consumer credit literature, namely Chatterjee et al. (2020), to which I now turn.

Chatterjee et al. (2020) is the closest paper, methodologically, to mine. They have a similar focus on reputation about the preferences of the borrower. That paper is focused on consumer debt and the role of credit scores. It uses one period debt and has zero recovery to lenders in the event of a default. In their model, borrowers can be patient or impatient. This type is persistent but not permanent. Based on their actions (borrowing and default), lenders form beliefs about what the borrower's underlying type is. Historically, models like this had always needed to arbitrarily specify how beliefs evolved off of the equilibrium path (see Egorov and Fabinger (2016) or D'Erasmo (2011) for example). Chatterjee et al. (2020) avoid this issue by borrowing a technique from the Industrial Organization literature. They introduce preference shocks that ensure that every feasible action is played with nonzero probability in equilibrium. Therefore, there is no such thing as feasible actions off the equilibrium path, and no need to specify how beliefs evolve there. In this paper, I rely on the same technique. In addition, I add long term debt and endogenous renegotiation, as well as enriching the type space to allow for the cost of default as well as the patience rate to vary by type. I show that these additions are fundamental for the model's performance in the sovereign debt setting.

2.3 Empirical Papers

There are several empirical papers that study the effects of past defaults on interest rate spreads. Dell'Ariccia et al. (2006) are primarily focused on the effects of news about official lending on emerging market government bond spreads. Among their control variables, however, is a dummy indicating whether the country has been in default at any point within the past three years. Whenever included, its coefficient is consistently positive and statistically significant. Borensztein and Panizza (2009) are interested in a wide range of consequences of default, including changes in interest rates paid on government debt. They find that there are significant, positive effects on interest rate spreads over the first 2 years after a default.

Cruces and Trebesch (2013) is the primary source of the empirical targets for this paper. The authors are interested in the relationship between default, as well as haircut magnitude conditional on default, and post-restructuring spreads, as well as the duration of countries exclusion from international financial markets. Whereas most previous papers found relatively short lived ($1-3$ years) effects, the authors find that there are significant effects of restructuring outcomes up to 7 years after the event occurs. In particular, they find that controlling for both the extensive margin of whether a restructuring occurred as well as the intensive margin of how severe the haircut imposed on investors was yields a series of effects which is generally decreasing in the time since the restructuring, given a haircut, but increasing in the haircut, at any given time since the restructuring. While the effects of restructurings with relatively small or even average haircuts may attentuate to zero within the first couple years, the effects of restructurings with higher haircuts may still be economically significant after seven years. On the duration side of the results, they find that higher haircuts are also associated with longer periods of exclusion from international financial markets.

These papers focus primarily on describing the patterns in interest rate spreads after restructurings observed in the data and are agnostic as to the reason why those correlations exist. I take the existence of these patterns as given, and build a structural model that can be disciplined by setting its parameters in order to reproduce these patterns.

3 Model

Time is discrete and infinite. There is a small open economy populated by a representative consumer and a government. Both have expected utility preferences and period flow utility given by $u(c)$ with u a nice function. The public, exogenous state of the world is s, which is a Markov Process (and governs the country's endowment $y(s)$). The private, exogenous state of the world includes T, the government's type. This type is independent of s and also a Markov Process. T is persistent and known only to the government, but its transition rule is public knowledge.

The government's type T determines the its current discount factor β_T and the cost of default in the current period $\phi_T(s, d_t)$. The government understands that its type may change over time, and incorporates that possibility into its optimization process. Note that there are other ways of implementing this idea of government types. One alternative version would be that a government does not care at all about what happens after a transition. Another would be that each type always gets utility from the country's consumption, but the type making the decisions changes over time. Both of these interpretations are more consistent with transitions being observable events (i.e. one political party loses an election and cedes power to their opponents). In this paper, I construe "type" to mean something deeper about the way decisions are made – including dominant schools of thought among policy makers – the organization of domestic interest groups and their influence in the policy-making process, as well as the preferences of the populace overall.

The private, exogenous state of world also includes preference shocks ϵ, η, or ν for the government that are independent of the its type and i.i.d. over time. As we will see in the full model's exposition, the preference shocks enter additively in the government's decision problems. They will be unbounded and therefore will serve to ensure that every feasible action is played with nonzero probability in equilibrium. Since the equilibrium concept will be Bayesian Perfection, this is valuable because it ensures that I never need to arbitrarily specify how beliefs evolve off the equilibrium path. Lender beliefs about T are denoted by π and are updated using Bayes Law and the transition rule for T. Specifically, whenever an action a is observed, there is a belief update Γ^A associated with that action:

$$\Gamma^A(a, \pi|s, b)(T_0) = \frac{\pi(T_0)Pr(a^\star(s, T_0, \epsilon, \pi, b) = a)}{\int_{\mathscr{T}} \pi(x)Pr(a^\star(s, x, \epsilon, \pi, b) = a)dx}$$

At the end of a period, beliefs are updated in order to account for possible changes in the government's type between periods. This second kind of update will be denoted as $\Gamma(\pi)$.

The government may borrow from a continuum of international lenders using a defaultable long term bond. There is a finite set $\mathscr{B}$ of values that the government's debt level can take. Following Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009), I model this debt as a contract promising a stream of exponentially declining coupon payments. Specifically, at time t, a unit of the bond promises to pay $(1-\lambda)^{t+l-1}(\lambda+\kappa)$ of the consumption good in period $t + l$. If the government chooses to default, the country enters financial autarky and begins suffering a flow utility penalty of $\phi_T(s, d_t)$. This penalty depends on the exogenous, public state of the world s, the government's type T, and whether the country entered default in the current period d_t (as opposed to having defaulted in some previous period but not yet completed the renegotiation process). In order to resolve a default, the government must negotiate an agreement with bondholders. I describe this process in detail below, starting in the "Renegotiation" section of the model exposition.

There is an issuance cost $i(s, b, \pi', b')$ (possibly 0) incurred when the government adjusts its debt level. This is standard in models with long term debt and positive recovery rates (see Dvorkin et al. (2021) or Chatterjee and Eyigungor (2015) for example). If it is not included, then whenever there is a positive recovery value for its debt, the government has an incentive to issue enormous amounts of debt the period before a default in order to complete extract the value of legacy bondholders' asset holdings. Since this type of "maximum" dilution behavior is counterfactual, issuance costs are included in the model to prevent it from occurring in equilibrium. Quantitatively, the amount of resources spent financing the issuance costs is quite small (see the calibration section for an extended discussion of the functional form and its effects).

3.1 Repayment

If the country enters the period in good standing, the timing of events is as follows:

1. The states s, T, and $\epsilon = \{\epsilon^D, \{\epsilon^R(b')\}_{b' \in \mathscr{B}}\}$ are realized.

2. The government chooses whether to default (d). A belief update based on that decision $\Gamma^D(d, \pi | s, b)$ occurs.

   - If the government has chosen not to default, it chooses the new level of debt b'. A belief update based on that decision $\Gamma^R(b', \pi | s, b)$ occurs.

   - If the government has chosen to default, it enters bad standing and possibly begins negotiations with lenders. The exact sequence of events and actions which occur in this case will be specified later, in the sections on renegotiation and restructuring.

3. A belief update based on the transition rule for T, $\Gamma(\pi)$, occurs (updating π to be the prior belief at the beginning of the next period).

At the beginning of the period, if the country is not in default, its problem is:

$$V(s, T, \epsilon, \pi, b) = \max_{d \in \{0,1\}} (1 - d) V^R(s, T, \epsilon, \hat{\pi}, b) + d(V_0^D(s, T, \hat{\pi}, b) + \epsilon^D)$$

$$\text{where:}$$

$$\hat{\pi} = \Gamma^D(d, \pi | s, b)$$

where V^R is the repayment value function, Γ^D is the law for updating lender beliefs based on the default decision, and V_0^D is the value of entering default. In this problem, the government decides whether or not to default, taking into account the reputational consequences of that choice. Conditional on choosing to repay its debt, the government's problem is:

$$V^R(s,T,\epsilon,\pi,b) = \max_{c,b' \in \mathscr{B}} (1-\beta_T)u(c) + \beta_T \mathbb{E}[V(s',T',\epsilon',\pi',b')|s,T] + \epsilon^R(b')$$

$$\text{such that}$$

$$c + (\lambda+\kappa)b = y(s) + q(s,\pi',b')(b' - (1-\lambda)b) - i(s,b,\pi',b')$$

$$\text{where:}$$

$$\pi' = \Gamma\Big(\Gamma^R(b',\pi|s,b)\Big)$$

where $\Gamma^R(.)$ represents the law for updating lender beliefs conditional on seeing the government make specific borrowing decisions. In this problem, the government chooses its optimal borrowing level b' taking into account how that choice affects both the revenue raised in the auction today $q(.)(b' - (1-\lambda)b) - i(.)$ as well as the continuation value that it will receive starting in the following period $\mathbb{E}[V(.)|s,T]$. In a model without reputation, there would be limited channels for each of these. The choice of borrowing would affect the price of the debt by changing next period's policies, and it would affect the continuation values through future policies, debt service, and auction revenue. In my model, the choice of borrowing affects both the price and the continuation value through an additional channel, reputation. Different choices of debt induce different belief updates by lenders. Independent of the actual level of debt the country exits the period with, these have an effect on current prices and future beliefs. Through their effects on future beliefs, they also affect future values.

Before I move to describing in detail the government's default values and the the renegotiation process, I will provide some specific definitions for the belief update process and prices while in good credit standing. Let Γ denote the mapping from beliefs at the end of one period to a prior at the beginning of the next implied by the probability law of θ. Set Γ^D and Γ^R by:

$$\Gamma^D(d,\pi|s,b)(T_0) = \frac{\pi(T_0)Pr(d^\star(s,T_0,\epsilon,\pi,b)=d)}{\int_{\mathscr{T}} \pi(x)Pr(d^\star(s,x,\epsilon,\pi,b)=d)dx}$$

And set $\Gamma^R(b', \pi|s, b)(\theta)$ by:

$$\Gamma^R(b', \pi|s, b)(T_0) = \frac{\pi(T_0)Pr(b'^{\star}(s, T_0, \epsilon, \pi, b) = b')}{\int_{\mathscr{T}} \pi(x)Pr(b'^{\star}(s, x, \epsilon, \pi, b) = b')dx}$$

Since lenders are competitive and risk neutral, the price of a bond will be exactly equal to the expected present value of the sequence of payments holding the bond entitles them to. The value of this sequence of payments depends on the exogenous state of the world s, lenders' prior belief about the government's type at the beginning of the next period π', and the country's debt level b'. We can define repayment prices $q(s, \pi', b')$ recursively by:

$$\begin{aligned} q(s, \pi', b') = \frac{1}{R}\mathbb{E}\Bigg[\int_{\mathscr{T}} \Big(& d'q^D(s', \pi''_D, b') \\ &+ (1 - d'))(\lambda + \kappa + (1 - \lambda)q(s', \pi''_R, b''))\Big)\pi'(T')dT'|s\Bigg] \\ &\text{where} \\ &d' = d^{\star}(s', T', \epsilon', \pi', b') \\ &\pi''_D = \Gamma^D(1, \pi'|s', b') \\ &b'' = b'^{\star}(s', T', \epsilon', \pi', b') \\ &\pi''_R = \Gamma\Big(\Gamma^R\big(b'', \Gamma^D(0, \pi'|s', b')|s', b'\big)\Big) \end{aligned}$$

Here we see how reputation is reflected in prices. First of all, it changes lenders' perceptions of which type the government will be tomorrow. This is reflected in the integration with respect to $\pi(T')dT'$. If a different revision to beliefs occurs today, then this term will take different values. There is also a second channel in this functional equation by which reputation is reflected in prices. In particular, the government's optimal policies in the following period d' and b'' depend on the reputation it has at the beginning of the period, so the terms being integrated are also directly affected by reputation.

3.2 Renegotiation

I follow Dvorkin et al. (2021) in using an alternating offers structure of renegotiation. If the government enters the period in bad standing, then the first events of the period are the realization of s and T. After that, (or if the government had entered the period in good standing but then defaulted), the following occurs.

1. With constant probability ψ, an opportunity for renegotiation arises this period.

2. If an opportunity for renegotiation arises, then the identity of the party proposing the deal $P \in \{G, L\}$ is drawn with μ_G the constant probability that the proposer is the government.

If no opportunity for renegotiation arises, then the period ends and lenders update their beliefs to account for possible changes in the government's type between periods (using $\Gamma(\pi)$).

If an opportunity does arise, the government and lenders begin the renegotiation process. The proposer makes a take it or leave it offer to the other party. Offers consist of an ex post unit value to lenders Q, so a lender holding a unit of the bond will receive Q of the consumption good if the deal is agreed. There is a finite set $\mathscr{Q}$ of values that this offer value can take. If the other party accepts the deal Q, the country enters the restructuring process having committed to deliver a total payment of Qb of the consumption good (where b was the level of defaulted debt) to lenders this period. Otherwise, the country remains in default, the period ends, and lenders update their beliefs to account for changes in the government's type between periods (using $\Gamma(\pi)$).

There are preference shocks η_D^P associated with actions of the proposer and preference shocks η_D^R associated with the actions of the party receiving the offer. The government's preference shocks here are private and serve the same purpose that they did in the repayment problem (ensuring that all feasible actions are played with nonzero probability). The lender's preference shocks are public and are included to ensure computational tractability. The specific timing of these events is:

1. The proposer's preference shocks $\eta_D^P = \{\eta^O(Q)\}_{Q\in\mathscr{Q}}$ are realized.

2. The proposer chooses which offer Q to make. If the government is proposing the deal, a belief update $\Gamma_G^Q(Q,\pi|s,b)$, based on this decision, occurs.

3. The receiver's preference shocks $\eta_D^R = \{\eta^Y, \eta^N\}$ are realized.

4. The receiving party chooses A, whether to accept (Y) or reject (N) the offer. If the lender is proposing the deal, a belief update $\Gamma_L^Q(A,\pi|s,b,Q)$, based on this decision, occurs.

5. If offer was accepted, the restructuring process begins. If the offer was rejected, a belief update $\Gamma(\pi)$ based on transition rule for T occurs.

Before describing the details of exactly how the debt is restructured once a deal has been reached, I will describe the renegotiation process itself more fully. The value of a government in default is given by:

$$V^D(s,T,\pi,b) = \psi(\mu_G \mathbb{E}[V_G^D(s,T,\eta_D^P,\pi,b)] + (1-\mu_G)V_L^D(s,T,\pi,b)) + (1-\psi)V_N^D(s,T,\pi,b)$$

where $V_P^D(.)$ indicates the value to the government in default when an opportunity to renegotiate arises and party P can propose an offer, and $V_N^D(.)$ indicates the value to the government in default no opportunity to renegotiate arises. The last is simply:

$$V_N^D(s,T,\pi,b) = (1-\beta_T)(u(y(s)) - \phi_T(s,0)) + \beta_T \mathbb{E}[V^D(s',T',\pi',b)|s,T]$$

$$\text{where:}$$

$$\pi' = \Gamma(\pi)$$

Similarly, the value to the lender of holding a unit of the bond at the beginning of a period is:

$$q^D(s,\pi,b) = \psi(\mu_G \bar{q}_G^D(s,\pi,b) + (1-\mu_G)\mathbb{E}[\hat{q}_L^D(s,\eta_D^P,\pi,b)]) + (1-\psi)q_N^D(s,\pi,b)$$

where $\bar{q}_G^D(.)$ indicates the value to lenders if an opportunity to renegotiate arises and the government is chosen to make an offer, $\hat{q}_L^D(.)$ indicates the value to lenders if an opportunity arises and they are chosen to make an offer, and $q_N^D(s, \pi, b)$ indicates the value to lenders if no opportunity for renegotiation arises this period. The last is simply:

$$q_N^D(s, \pi, b) = \frac{1}{R}\mathbb{E}[q^D(s', \pi', b)|s]$$

$$\text{where:}$$

$$\pi' = \Gamma(\pi)$$

When an opportunity for renegotiation does in fact arise and the government is the proposer, the government solves:

$$\begin{aligned} V_G^D(s, T, \eta_D^P, \pi, b) = &\max_{Q \in \mathcal{Q}} Pr(A_L = 1|s, \hat{\pi}, b, Q)\mathbb{E}[V^{RS}(s, T, \nu^{RS}, \hat{\pi}, Qb)] \\ &+ Pr(A_L = 0|s, \hat{\pi}, b, Q)V_N^D(s, T, \hat{\pi}, b) \\ &+ \eta^O(Q) \\ \hat{\pi} = &\Gamma_G^Q(Q, \pi|s, b) \end{aligned}$$

When the government makes an offer, lenders beliefs are updated from π to $\hat{\pi}$ using the belief update function $\Gamma_G^Q(.)$. The first term in the maximization, $V^{RS}(.)$, represents the value to the government of entering the restructuring process with this updated value of reputation having agreed to a deal entailing a total payment of Qb to lenders, weighted by the probability that lenders accept the offer of Q, $Pr(A_L = 1|.)$. The second term in the maximization, $V_N^D(.)$, represents the value to the government of remaining in default with that same updated value of reputation weighted by the probability that lenders reject the deal Q, $Pr(A_L = 0|.)$. The final term is simply the preference shock associated with choosing to propose the offer Q.

After the government makes an offer and lenders update their beliefs based on that decision, the receiver's vector of preference shocks $\eta_D^R = \{\eta^Y, \eta^N\}$ is drawn and lenders solve:

$$\hat{q}_G^D(s, \eta_D^R, \pi, b, Q) = \max_{A_L \in \{0,1\}} A_L \Big[Q + \eta^Y\Big]$$
$$+ (1 - A_L)\Big[q_N^D(s, \pi, b) + \eta^N\Big]$$

If the deal is agreed, the value to the lender will simply be $Q + \eta^Y$. If it is not agreed, then the lender retains their claim and the value associated with it, $q_N^D(s, \pi, b)$, and receives the preference shock associated with rejecting the deal η^N. The ex ante value to lenders when the government is chosen to propose a deal is then:

$$\bar{q}_G^D(s, \pi, b) = \mathbb{E}[\hat{q}_G^D(s, \eta_D^R, \hat{\pi}, b, Q_G^\star(s, T, \eta_D^P, \pi, b))]$$

where:

$$\hat{\pi} = \Gamma_G^Q(Q_G^\star(s, T, \eta_D^P, \pi, b), \pi | s, b)$$

If lenders, on the other hand, are the party chosen to propose a deal, then they solve:

$$\hat{q}_L^D(s, \eta_D^P, \pi, b) = \max_{Q \in \mathscr{Q}} Pr(A_G = 1 | s, \pi, b, Q)Q$$
$$+ (1 - Pr(A_G = 1 | s, \pi, b, Q))q_N^D(s, \hat{\pi}, b) + \eta^O(Q)$$

where:

$$\hat{\pi} = \Gamma_L^A(A_G, \pi | s, b, Q)$$

Once lenders have made an offer, the government solves the following problem in order to decide whether or not to accept the offer:

$$\hat{V}_L^D(s, T, \eta_D^R, \pi, b, Q) = \max_{A_G \in \{0,1\}} A_G(\mathbb{E}[V^{RS}(s, T, \nu^{RS}, \hat{\pi}, Qb)] + \eta^Y)$$
$$+ (1 - A_G)(V_N^D(s, T, \hat{\pi}, b) + \eta^N)$$

where:

$$\hat{\pi} = \Gamma_L^A(A_G, \pi | s, b, Q)$$

Should the government accept the offer, i.e. $A_G = 1$, it gets the expected value $\mathbb{E}[V^{RS}(.)]$ associated with restructuring its debt when promising to pay lenders Qb and the taste shock associated with accepting the offer. Lender beliefs would also update to reflect the observation that the government accepted the deal. If the government rejects the offer $A_G = 0$, it gets the value associated with remaining in default, $V_N^D(.)$, as well as the taste shock associated with rejecting the offer. Again, lender beliefs also update to reflect the government's rejection of the deal. The ex ante value to the government when lenders are chosen to propose a deal is then:

$$V_L^D(s, T, \pi, b) = \mathbb{E}[\hat{V}_L^D(s, T, \eta_D^R, \pi, b, Q_L^\star(s, \eta_D^P, \pi, b))]$$

The initial value of default considered by the government in the repayment problem is:

$$V_0^D(s, T, \pi, b) = V^D(s, T, \pi, b) + \phi_T(s, 0) - \phi_T(s, 1)$$

The difference in penalties appears in this definition because the various definitions of other default value functions assume that the country defaulted in a prior period.

3.3 Restructuring

Once a deal Q has been agreed, the government has committed to paying lenders $W = Qb$ and moves to the restructuring process (i.e. deciding exactly how to deliver that value). At this point, the government immediately regains access to international markets and can use a new auction of debt to fund the payment W. Also, another vector of preference shocks $\nu = \{\nu^{RS}(b')\}_{b' \in \mathscr{B}}$ is realized and the government solves:

$$V^{RS}(s, T, \nu^{RS}, \pi, W) = \max_{c, b' \in \mathscr{B}} (1 - \beta_T)(u(c) - \phi_T(s, 0)) + \beta_T \mathbb{E}[V(s', T', \epsilon', \pi', b') | s, T] + \nu^{RS}(b')$$

$$\text{such that}$$

$$c + W = y(s) + q(s, \pi', b')b'$$

$$\text{where:}$$

$$\pi' = \Gamma\Big(\Gamma^{RS}(b', \pi | s, b)\Big)$$

This renegotiation/restructuring protocol is significantly more flexible than the more common version involving face value haircuts (i.e. there is a reduction of the debt level from b_{old} to b_{new}, and no other transfer of value from one party to the other; see D'Erasmo (2011) or Sunder-Plassmann (2018) for examples of this). In particular, it allows for both an exchange of old bonds for new bonds as well as a cash transfer at the time of the exchange. Since such transfers are very common elements in real world restructuring deals,[2] it is important to allow for them. Furthermore, it allows measured haircuts in the model to be different from the face value reductions in the debt.

3.4 Equilibrium

An stationary recursive competitive equilibrium for this environment consists of:

1. Value functions $V, V^R, V_0^D, V^D, V_G^D, V_L^D, \hat{V}_L^D, V_N^D, V^{RS}$;
2. Price functions $q, q^D, q_N^D, \bar{q}_G^D, \hat{q}_G^D, \hat{q}_L^D$;
3. Policy functions $d^\star, b'^\star, Q_G^\star, Q_L^\star, A_G^\star, A_L^\star, b'^\star_{RS}$;
4. Belief update functions $\Gamma^D, \Gamma^R, \Gamma_G^Q, \Gamma_L^A, \Gamma^{RS}$.

which satisfy the following conditions:

1. Default decision optimality: given Γ_D, V_0^D, and V^R, $d^\star$ solves the government's default or repay decision problem and V is the resulting value function.
2. Borrowing decision optimality: given V, Γ^R, and q, $b'^\star$ solves the government's repayment problem and V^R is the resulting value function.
3. Zero profits: given q^D, Γ_D, Γ^R, $d^\star$, and $b'^\star$, q satisfies the functional equation defining prices while in good standing.
4. Offset of initial value of default: given V^D, V_0^D satisfies the equation defining the value of default in the period of default.

[2] For example, in the Greek Government Debt Restructuring of 2012, short term notes guaranteed by the EFSF made up a significant fraction of the portfolio lenders received in exchange for their existing bonds (Zettelmeyer et al., 2013). Since these notes were extremely safe and quite liquid, they can reasonably be construed as a cash transfer.

5. Government default value if no deal agreed: given V^D, V_N^D is the value function for the government when no deal with lenders is agreed.

6. Lender default value if no deal agreed: given q^D, q_N^D is the defaulted bond price function when no deal is agreed.

7. Ex ante government default value: given V_N^D, V_G^D, and V_L^D, V^D is the value of being in default before resolution of the uncertainty about whether an opportunity to renegotiate arises.

8. Ex ante lender default value: given q_N^D, $\bar{q}_G^D$, and $\hat{q}_G^D$, q^D is the defaulted bond price before resolution of the uncertainty about whether an opportunity to renegotiate arises.

9. Government deal proposal optimality: given $A_L^\star$, V^{RS}, V_N^D, and Γ_G^Q, $Q_G^\star$ solves the problem of the government when deciding what offer to propose to lenders and V_G^D is the resulting value function.

10. Lender deal acceptance optimality: given q_N^D, $A_L^\star$ solves the lender's problem when deciding whether to accept a deal proposed by the government and $\hat{q}_G^D$ is the resulting price function.

11. Ex ante lender value if receiving proposal: given $\hat{q}_G^D$, Γ_G^Q, and $Q_G^\star$, $\bar{q}_G^D$ is the ex ante price of the bond when an opportunity to renegotiate arises and the government is chosen to be the proposer.

12. Lender deal proposal optimality: $A_G^\star$, Γ_L^A, and q_N^D, $Q_L^\star$ solves the problem of lenders when deciding what offer to propose to the government and $\hat{q}_L^D$ is the resulting price function.

13. Government deal acceptance optimality: V^{RS}, V_N^D, and Γ_L^A, $A_G^\star$ solves the government's problem when deciding whether to accept a deal proposed by lenders and $\hat{V}_L^D$ is the resulting price function.

14. Given $\hat{V}_L^D$ and $Q_L^\star$, V_L^D is the ex ante value to the government when an opportunity to renegotiate arises and lenders are chosen to be the proposer.

15. Government restructuring choice optimality: q, V, and Γ^{RS}, $b'^{\star}_{RS}$ solves the government's problem of restructuring its debt and V^{RS} is the resulting value function.

16. Bayesian updating: belief updates $\Gamma^D, \Gamma^R, \Gamma^Q_G, \Gamma^A_L, \Gamma^{RS}$ are consistent, respectively, with the policy functions $d^{\star}, b'^{\star}, Q^{\star}_G, A^{\star}_G, b'^{\star}_{RS}$ and Bayes' Law.

3.5 Existence

In this section, I prove that, under certain assumptions, an equilibrium, as defined above, must exist. Some of these assumptions are simply to ease the notational burden of the proof and have no impact on its generality. Others have a material impact on its generality. Whenever an assumption falls into this second category, I note it explicitly. Furthermore, in order to make the exposition of this proof easier to follow, I adjust some of the notation I have been using. None of these notational adjustments have a material impact on the proof. The most significant of these adjustments is in dealing with belief updates which occur within a period. Instead of using the within period posterior belief as a state variable, I use the beginning of period prior belief as well as the history of actions observed during the current period. Since beliefs are update using Bayes's Law, these two formulations are exactly equivalent.

This section proceeds as follows. First, I define the assumptions I need to make about functional forms, distributions, and state spaces. Then I state the main theorem on existence, and sketch its proof (the full details are in the appendix). The first assumption I must make involves the cardinality of the choice sets for b' and Q:

Assumption 1. ***Finiteness of choice sets:*** *$\mathscr{B} \subset \mathbb{R}$ and $\mathscr{Q} \subset R$ are both nonempty and satisfy $|\mathscr{B}| = N_b < \infty$ and $|\mathscr{Q}| = N_Q < \infty$. $0 \in \mathscr{Q}$ and $\min\{\mathscr{B}\} = 0$.*

The first part of this assumption is just restating that the choice sets facing all parties are always finite. The second set of conditions is here merely to ease the exposition. $0 \in \mathscr{Q}$ guarantees that there is always a feasible choice for the government during the renegotiation process[3], and the assumption that 0 is the lowest debt value eases the notation involved in

[3]Instead of this assumption, I could assume that the government can decline to make an offer, but that would complicate the exposition.

defining revenue from auctions.

Assumption 2. ***Preference shock distributions:***

1. *The preference shocks under repayment are distributed Generalized Type One Extreme Value with scale parameter σ_ϵ and correlation parameter ρ_ϵ.*

2. *The preference shocks while in default are distributed Type One Extreme Value with scale parameters $\sigma^Y_{\eta,X}$ where $X \in \{G, L\}$ and $Y \in \{P, R\}$.*

3. *The preference shocks while completing the restructuring process are distributed Type One Extreme Value with scale parameter σ^{RS}_ν.*

4. *The location parameters of every distribution are set such that the mean value of each individual shock is zero (i.e. $\mu = -\gamma\sigma$ where γ is the Euler-Mascheroni constant).*

This assumption reduces the generality of the proof somewhat. It is required to ensure that ex ante values and choice probabilities are continuous functions of the values associated with each feasible choice. This in turn guarantees that posterior beliefs at each feasible choice are continuous functions of those values. In addition, this assumption allows me to show that there is an assignment rule for beliefs at infeasible choices that preserves the continuity of the update operator defined later.

Assumption 3. ***Finiteness of state space:***

1. *$s \in \mathscr{S}$ with $\mathscr{S}$ nonempty and finite.*

2. *$T \in \{H, L\}$ and the transition matrix for T has entries $p_{HH} \in (0, 1)$ and $p_{LL} \in (0, 1)$ on its main diagonal. $\beta_H > \beta_L$.*

3. *$\pi \in \Pi$ with $\Pi \subset [0, 1]$, $|\Pi| < +\infty$, and Π satisfying $\{0, 1\} \subset \Pi$.*

Part 2 of this assumption is here just to ease the notational burden in the proof. Parts 1 and 3 of this assumption also reduces the generality of the following existence proof. It is required to ensure that the set of equilibrium objects can be construed as a single vector in a high but finite dimensional Euclidean space, so that certain results about continuous mappings between such spaces can be applied. Working in a similar environment, Chatterjee et al. (2020) make the same assumptions on the cardinality of $\mathscr{S}$ and Π. In order to deal with how beliefs evolve from one period to the next when the posterior belief at the end of a period $\hat{\pi}$ would evolve to a value $\hat{\pi}' \notin \Pi$, I follow Chatterjee et al. (2020) in defining the randomization rule $g(\pi', |\hat{\pi}')$ and modified expectation operator $\hat{\mathbb{E}}[.|\hat{\pi}']$ as follows:

Definition 1. ***Randomization rule for belief evolution:*** *For any function $f(\pi')$, set $\hat{\mathbb{E}}[.|\hat{\pi}']$ by:*

$$\hat{\mathbb{E}}[f(\pi')|\hat{\pi}'] = \sum_{\pi' \in \Pi} g(\pi'|\hat{\pi}')f(\pi')$$

So g assigns weights to on grid points in Π for any point $\pi \in [0, 1]$. I make the following assumptions about g:

Assumption 4. ***Randomization function properties:***

1. *For every $\hat{\pi}'$, $\hat{\mathbb{E}}[\pi'|\hat{\pi}'] = \hat{\pi}'$.*
2. *For every $\pi' \in \Pi$ $g(\pi'|\hat{\pi}')$ is continuous in $\hat{\pi}'$.*

The first part of this is simply the requirement that g be not disturb the consistency of Bayesian updating, in expectation. The second part will be important in guaranteeing that certain key expected value terms be continuous in posterior beliefs. Finally, I make some assumptions about the issuance cost function.

Assumption 5. ***Functional form of issuance cost:*** *$i(s, \pi, b, h, b')$ can be expressed as:*

$$i(s, \pi, b, h, b') = \begin{cases} 0 & b' < (1 - \lambda)b \\ \hat{i}(\delta(s, \hat{\pi}', b'))q(s, \hat{\pi}', b')(b' - (1 - \lambda)b) & b' \geq (1 - \lambda)b \end{cases}$$

where $\hat{\pi}'$ is the next period prior after beliefs are updated from π, taking into account other previously observed actions this period h as well as the debt choice b', $\delta(s, \hat{\pi}', b')$ is the probability of a default occurring in the next period, and $\hat{i} : [0, 1] \to [0, 1]$ is a continuous function.

In other words, the cost of issuance debt is a fraction of the revenue from auctioning that debt which depends only on the next period default probability. This fraction is required to be a continuous function of the next period default probability.

I make the following assumptions about the utility of consumption and the penalties for defaulting:

Assumption 6. ***Utility function:*** *$u : \mathbb{R}_{++} \to \mathbb{R}$ is continuous, increasing, and has $\lim_{x \downarrow 0} u(x) = -\infty$.*

These are standard restrictions on utility functions. We can now state the main theorem of this section

Theorem 1. ***Equilibrium Existence:*** *Suppose that assumptions 1, 2, 3, 4, 5, and 6. Then an equilibrium exists.*

Proof: see appendix.

While I relegate the full details of the proof to the appendix, I will sketch its main components here. The strategy is partially based on the existence proof in Chatterjee et al. (2020). Finiteness of the (s, π, b) states combined with finiteness of the choice sets guarantees that there are only finitely many within-period sequences of actions, and all values, prices, and belief updates can be construed simply as vectors in some high (but finite) dimensional Euclidean Space. This gives me access to certain theoretical results about continuous mappings between compact, convex sets of such spaces (in particular, Brouwer's Fixed Point Theorem). The assumptions on the issuance cost function and the randomization rule guarantee that consumption values are continuous in prices and posterior beliefs, and the assumptions on the parametrizations of the preference shocks ensure that both choice probabilities and ex ante values can be written as continuous functions of the values associated with the individual choices. This ensures continuity of the mapping which updates the values of all equilibrium objects at any feasible choice sequence (feasibility here refers to whether the consumption value associated with a choice sequence is strictly positive).

A particular complication that arises in the proof is choosing how beliefs evolve after infeasible choices to ensure that, as a choice becomes just infeasible (i.e. $c \downarrow 0$), the limit of the sequence of posterior beliefs matches the value assigned when that choice is infeasible. While it is easy to show that values and choice probabilities remain continuous in this situation, the definition for the posterior belief reads $\frac{0}{0}$ in the limit. The correct assignment rule for posterior beliefs after an infeasible choice sequence occurs will turn out to be putting full weight on type with the highest β. While all types play an action that is just barely feasible only very rarely, the type with the highest beta puts the smallest weight on the extremely large negative number associated with utility from consumption in the current period, since values are written given by:

$$(1-\beta_T)u(c) + \beta_T EV(T)$$

Placing less weight on $u(c)$ makes the just feasible option not quite as bad for the type with the highest β as it is for the other types, and that difference grows as c falls to 0. Therefore, as $u(c)$ goes to $-\infty$, the type with the highest β chooses the choice in question infinitely more frequently than the other types, ensuring that the limit of the posterior beliefs puts full weight on the type with the highest β, just as the assignment rule at infeasible choices does.

4 Calibration

In this section, I describe the patterns in the data that I use to identify the reputation-related components of the model. Then I describe the functional forms I specify in the quantitative implementation of the model and detail the calibrated parameter values. After that, I show how well the models fits the data.

4.1 Identification

After completing negotiations with creditors and reaccessing international debt markets, countries which have defaulted pay higher interest rates than would appear to be justified by their levels of debt and the state of their economies. There is a long list of empirical papers that verify that a regression of interest rate spreads on economic, political, and other relevant observables, as well as credit history variables, will yield a set of jointly significant effects for the credit history variables. Specifically, the coefficients α_τ on dummy variables $d_{it,\tau}$ indicating that at time t, country i defaulted (or restructured) τ years ago in the specification

$$spread_{it} = X_{it}\beta + \sum_{\tau \in \mathscr{T}} \alpha_\tau d_{it,\tau} + \epsilon_{it}$$

will be significant. In general, they have positive signs and are declining in magnitude as τ rises (i.e. the effect of a default on spreads fades over time). Furthermore, the data show that it is not just the extensive margin of default vs. repayment which matters for this effect. Rather, the intensive margin of how severely lenders suffered in the restructuring will affect its scale . Cruces and Trebesch (2013) show that under a wide variety of ways of measuring investor losses, often called "haircuts," h_{it}, the slope coefficients γ_τ in the augmented specification

$$spread_{it} = X_{it}\beta + \sum_{\tau \in \mathscr{T}} d_{it,\tau}(\alpha_\tau + \gamma_\tau h_{it}) + \epsilon_{it}$$

will also be significant. In their work, the extensive margin coefficients associated with the effect at a τ lag fall over time, while the intensive margin ones generally rise. Whenever they are individually significant, the slopes are always positive. Using their point estimates for the various parameters, we can construct the following picture:

Figure 1: Post-Restructuring Spread Premia

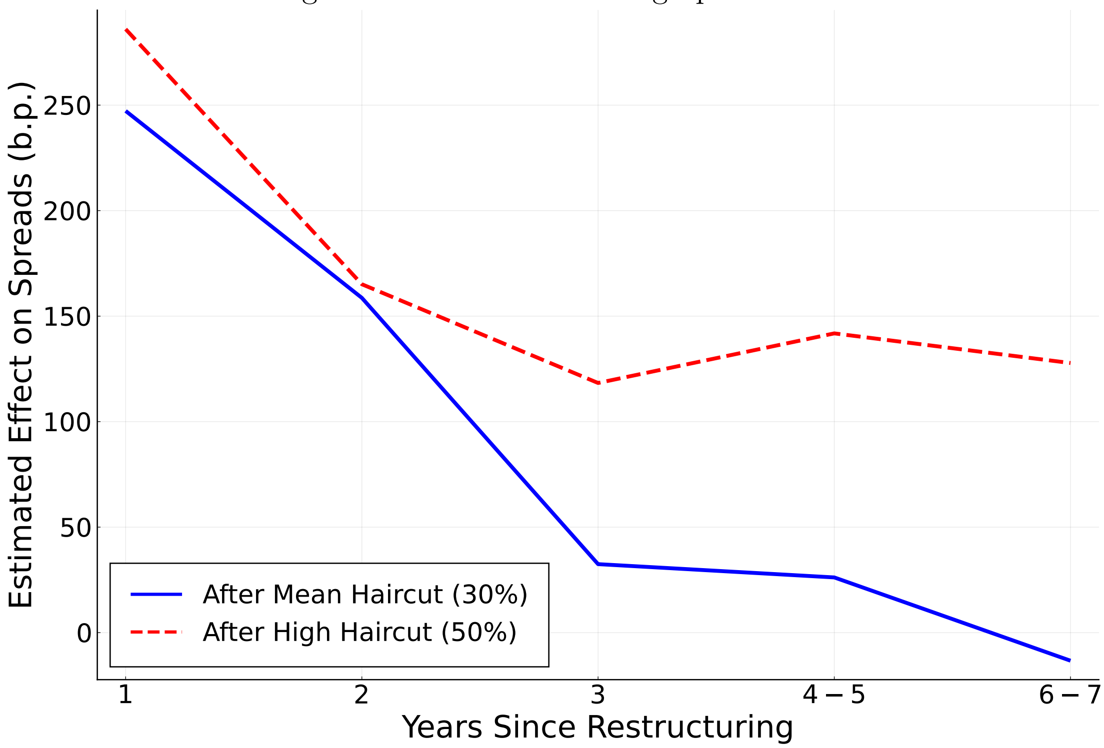


Figure 1 shows that there are economically significant effects of default across most levels of haircuts observed in the data over the first two years following a restructuring. Furthermore, restructurings featuring larger haircuts are associated with economically significant effects up to seven years after the restructuring takes place. That is, countries that treat foreign creditors badly face longer lasting increases in interest rate spreads.

There are two features in figure 1, encoded in the estimates of the extensive margin effects α_τ and the intensive margin effects γ_τ, that will identify the reputation-related parameters of my model. The first key feature is the immediate increase in spreads following a restructuring. The second key feature of these data is that restructurings featuring relatively higher haircuts are followed by relatively higher spreads, especially at longer time horizons.

There are two dimensions along which the types in my model differ, the impatience β_T and the default cost $\phi_T(s, d_t)$. Each of those differences is the primary driver (but not the only cause) of a specific difference in behavior which allows the model to match these key features of the data. I will now explain these.

The difference in impatience leads to the more impatient type borrowing more quickly. Since it borrows more quickly, it reaches higher levels of debt more frequently, and therefore ends up defaulting more often. Since the set of countries which end up restructuring their debt is necessarily similar (in type) to the set of countries which default, countries that have recently restructured their debts are more likely to be the more impatient type. Therefore, countries which have recently restructured their debts are more likely to be exactly the type of country which will accumulate debt quickly again and default again. This results in spreads being significantly elevated in the immediate aftermath of a restructuring, the first key feature of these data.

The difference in default costs leads to the type with the lower default cost tolerating default better. Since it does not mind staying in default as much as the high cost type does, it holds out for better deals from lenders. It uniformly makes lower offers to lenders, and lenders make lower offers to it when lenders believe they are more likely facing the type with a low default cost. This results in a correlation between type and observed haircuts. The fact that higher haircuts are followed by even higher spreads in the data also informs how these differences in preferences should be paired. In particular, in the calibration, the irresponsible type will be more impatient and have a lower default cost, while the responsible type will be less impatient and have a higher default cost. I emphasize that this pairing is an outcome of the calibration and not imposed ex ante.

In addition to these specific differences in preferences, there are several other pieces of the model that these patterns help identify. The persistence of the types is closely associated with how long lasting the effects are. The preference shock parameters associated with the renegotiation and restructuring process help govern the flow of information when the government is in default. Conditional on specific differences in preferences, these therefore govern how precise (or imprecise) beliefs are when a restructuring is completed, which moderates how much the differences in preferences are translated into the reduced form effects on prices we see in the data. Finally, the distribution of the preference shocks during repayment help control the flow of information when the country is in good standing, which affects how much more (or less) accurate beliefs become in the years following a restructuring, therefore affecting the trends of the effects.

4.2 Functional Forms and Parameters

The model is calibrated to match the experience of Argentina since 1993. The quarterly real interest rate, r is set to 0.01, a standard value. The parameters governing the debt payment structure, the maturity rate λ and coupon value κ, are set to the values used by Chatterjee and Eyigungor (2012) (who also study Argentina during mostly the same period). The income process is assumed to be an $AR(1)$ with autocorrelation coefficient $\rho_y = 0.95$ and innovation standard deviation $\sigma_y = 0.03$, the same estimates used by Chatterjee and Eyigungor (2012). The functional form of utility was assumed to be constant relative risk aversion:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

The relative risk aversion coefficient γ is set to 2, a standard value in macroeconomics. Table 1 summarizes the parameters set outside the model.

Table 1: Parameters Set Independently

Parameter	**Value**	**Source**
γ	2.00	Standard
r	0.01	
λ	0.05	Chatterjee & Eyigungor (2012)
κ	0.03	
ρ_y	0.95	
σ_y	0.03	

Other functional forms that I must specify are the flow utility cost of default and the issuance cost function. I use a flow utility cost of default, rather than an output cost of default (which is used by Arellano (2008) and Chatterjee and Eyigungor (2012), among many others), in order to avoid the realization of output while in default perfectly communicating the government's type to lenders. Note that this assumption does not imply that there are no real output costs of default, just that those costs are felt differently by the two government types. In order to make clear the relationship between the default costs in my model and those employed in the literature, I define the utility cost of default implicitly by:

$$u(y(s)) - \phi_T(s, d_t) := u\Big(y(s) - \max\{(h_0 + \tilde{\phi}_{d_t} + \hat{\phi}_T)y(s) + h_1 y(s)^2, 0\}\Big)$$

One of the most common parametrizations of the cost of default in the sovereign default literature is a linear-quadratic cost in output (see Chatterjee and Eyigungor (2012), for example). This functional form replicates that. Furthermore, it allows there to exist:

1. a constant percent difference in the as-if output costs experienced by the two types (specified by $\hat{\phi}_T$);
2. a constant percent difference in the as-if output costs between the period the country enters default and subsequent periods (specified by $\tilde{\phi}_{d_t}$).

The issuance cost function is assumed to have the following form:

$$i(s, b, \pi', b') = \begin{cases} 0 & b' \leq \hat{b} \text{ or } Pr(d'^{\star} = 1) \leq p_d \\ q(s, \pi', b')(b' - \hat{b})\hat{i}(s, \pi', b') & b' > \hat{b} \text{ and } Pr(d'^{\star} = 1) > p_d \end{cases}$$

where $\hat{b} = \max\{(1 - \lambda)b, 0\}$.[4] The purpose of issuance cost functions in this type of model are to prevent a behavior Chatterjee and Eyigungor (2015) termed "maximum dilution." Essentially, in the period prior to default, the maturity structure of the model's debt gives the government an incentive to issue an enormous amount of debt, completely extracting the value of existing bondholders' securities. Issuance cost functions counteract these incentives. The above function combines elements of two main types of issuance cost functions used in the literature. The first, used by Chatterjee and Eyigungor (2015), as well as others, is a strict limit on the one period ahead default probability (or spread) (i.e. cost is 0 up until some value and then infinite thereafter). The second, used by Dvorkin et al. (2021), as well as others, is a continuous, convex cost in the scale of the issuance. $\hat{i}(s, \pi', b')$ combines the

[4] The specific functional form of $\hat{i}(s, \pi', b')$ is given by:

$$\hat{i}(s, \pi', b') = \frac{1}{2}\left(1 + \sin\left(\pi\Big(\frac{Pr(d'^{\star} = 1) - p_d}{1 - p_d} - \frac{1}{2}\Big)\right)\right)$$

i.e. a sine wave shifted and scaled to rise from 0 to 1 as it travels from p_d to 1.

0-up-to-a-threshold property of the the first class of functions with the continuity (and some of the convexity, at least for lower values of $Pr(d'^{\star} = 1)$) of the second.

The distribution of the preference shocks during repayment is assumed to be Generalized Type 1 Extreme Value. The distributions of all the other preference shocks in the model are assumed to be Type 1 Extreme Value. These distributions are chosen because of their computational tractability. In particular, both choice probabilities and ex ante expected values can be written analytically in terms of the values associated with the choices (McFadden, 1978).

Apart from the parameters specified in Table 1, all parameters are calibrated by simulated method of moments. The targeted moments are the mean and volatility of the external debt to GDP ratio while not in default, the mean and volatility of spreads while not in default, the default rate, the average haircut, the average delay between a default and a restructuring, and the average rise in rise in the debt to GDP ratio in the one year preceding a default, as well as the five extensive margin effects α_τ and five intensive margin effects γ_τ from the regression of Cruces and Trebesch (2013):

$$spread_{it} = X_{it}\beta + \sum_{\tau \in \mathscr{T}} d_{it,\tau}(\alpha_\tau + \gamma_\tau h_{it}) + \epsilon_{it}$$

The target values of the mean and volatility of external debt to GDP were calculated using annual Argentinian data from 1993 to 2019 from the World Bank's International Debt Statistics (IDS) (formerly called Global Development Finance data), excluding years in default. The target values of mean and volatility of spreads were calculated based on monthly Argentinian EMBIG spreads from 1997 to 2019, excluding months in default, from the World Bank's Global Economic Monitor (GEM). Argentina's default rate was taken from Chatterjee and Eyigungor (2012).

Due to the relative rarity of default events, the remaining targets were calculated using cross-country evidence. The target average haircut was calculated using the dataset provided in Cruces and Trebesch (2013), excluding donor-funded restructurings. The target average delay was calculated using the dataset provided in Asonuma and Trebesch (2016), again excluding donor-funded restructurings. The target average rise in debt to GDP over the year preceding a default was calculated based on data presented in Benjamin and Wright (2013). The target values for the regression coefficients were taken directly from the results of Cruces and Trebesch (2013).

While all of the parameters affect all moments and are jointly calibrated, I will provide some insight, when I can, into which parameters are identified by which moments informed by a sensitivity analysis. The first five parameters are:

$$(\mathbb{E}[\beta_T], \mathbb{E}[h_0 + \hat{\phi}_T], h_1, \sigma_\epsilon, \rho_\epsilon)$$

These govern the average impatience of the government, the average penalty for defaulting, and the distribution of the preference shocks under repayment (σ_ϵ is the scale parameter for Generalized Type 1 Extreme Value distribution of preference shocks under repayment, and ρ_ϵ is its correlation parameter (for the repayment nest)). These five parameters are closely tied to the mean and volatility of the debt to GDP ratio B'/Y, the mean and volatility of interest rate spreads $r - r^f$, and the default rate. As mentioned earlier, σ_ϵ and ρ_ϵ also effect the patterns in the α_τ and γ_τ as τ changes.

The next three parameters are:

$$(p_d, \psi, \mu_G)$$

These govern the shape of the issuance cost function, the frequency at which renegotiation opportunities arise, and the bargaining power of the government during renegotiation. p_d, the value of the expected probability of default in the next period at which the issuance cost begins rising away from 0, is very tightly tied to the rise in debt over the one year preceding a default. The other two parameters in this group, ψ and μ_G, control two key characteristics of the renegotiation process. In particular, ψ is the rate at which opportunities to make an offer arise, and μ_G is the probability that the government will be the proposer of the deal, should such an opportunity arise. The value of ψ is closely tied to the delay between default and the completion of the restructuring process. The parameter μ_G governs the bargaining power of the government during the renegotiation process and is therefore pretty closely linked to the average haircut imposed on lenders.

The last ten parameters are:

$$(p_{HH}, p_{LL}, \beta_H - \beta_L, \hat{\phi}_H - \hat{\phi}_L, \tilde{\phi}_1, \sigma_\eta^{P,G}, \sigma_\eta^{R,G}, \sigma_\eta^{P,L}, \sigma_\eta^{R,L}, \sigma_\nu^{RS})$$

The first two, p_{HH} and p_{LL}, are the probability of remaining the high type and the probability of remaining the low type. The next two, $\beta_H - \beta_L$ and $\hat{\phi}_H - \hat{\phi}_L$ are the difference between the impatience rates of the two types and the difference between their as-if output costs of default. $\tilde{\phi}_1$ controls the extra penalty associated with the initial period of default, and is closely associated with the average haircut. $\tilde{\phi}_0$ is normalized to 0. The final five are preference shock parameters associated with the renegotiation process. $\sigma_\eta^{X,Y}$ is the scale parameter for the Type 1 Extreme Value preference shocks for party X when that party has role Y in the renegotiation process (so, for example, $\sigma_\eta^{P,G}$ is the scale parameter for the preference shocks of the government when the government proposes a deal). The final parameter in this block, σ_ν^{RS}, is the scale parameter for the Type 1 Extreme Value preference shocks associated with the restructuring choice of the government. As discussed in the previous section, these parameters are identified by regression coefficients α_τ and γ_τ.

The full set of parameters calibrated jointly is detailed in table 2 below:

Table 2: Calibrated Parameters

Parameter	**Value**	**Parameter**	**Value**
$\mathbb{E}[\beta_T]$	0.947	σ_ϵ	$1.7e-4$
$\mathbb{E}[h_0 + \hat{\phi}_T]$	-0.159	ρ_ϵ	0.378
h_1	0.219	$\sigma_{\eta,G}^P$	$6.7e-4$
p_{HH}	0.986	$\sigma_{\eta,L}^P$	$3.2e-3$
p_{LL}	0.984	$\sigma_{\eta,G}^R$	$1.8e-4$
$\beta_H - \beta_L$	0.043	$\sigma_{\eta,L}^R$	$1.4e-2$
$\hat{\phi}_H - \hat{\phi}_L$	0.022	σ_ν	$2.2e-3$
p_d	0.322	ψ	0.080
μ_G	0.906	$\tilde{\phi}_1$	0.136

The calibrated average impatience is very close to what is the calibrated impatience used in other work on sovereign default in emerging market economies (for example, Chatterjee &

Eyigungor (2012) estimate $\beta = 0.954$). The average penalty scale is also pretty similar to their calibration ($h_0 = -0.19, h_1 = 0.25$). Since both models are calibrated to the Argentinian economy, this should not come as a surprise.

The most interesting features of the calibration are the parameters describing the two types (highlighted in red in Table 2). Both the high and low type are quite persistent, with the high type lasting 17.5 years on average and the low type lasting 15.5 years on average. Since the high type is slightly more persistent, time in power is split 53% to 47% in its favor. There is also a relatively large difference in how impatient the two types are, with the difference in discount factors over 4%. In fact, the discount factor of the high type $\beta_H = 0.967$ is actually closer to the lender discount factor of $\frac{1}{R} = 0.990$ than it is to the discount factor of the low type $\beta_L = 0.924$. There is a somewhat smaller difference in how painful the two types find default. In particular, the high type finds default (in terms of the as-if output cost) just 2.2 percentage points more painful. For reference, the cost for the low type at the mean level of output is 6.0% (after the initial period of default).

4.3 Targeted Moments

Let me know discuss how the model fits the data. Data and model values for the non-regression coefficient targeted moments are detailed in Table 3 below:

Table 3: Targeted Moments (Annualized Values)

Group	Moment	Data	Model
1	$\mathbb{E}[B'/Y]$	21.89%	19.44%
	$\sigma(B'/Y)$	6.19%	5.44%
	$\mathbb{E}[r - r^f]$	7.57%	5.80%
	$\sigma(r - r^f)$	4.71%	6.55%
	$\mathbb{E}[d]$	12.50%	12.55%
2	$\Delta_1(B'/Y \mid d = 1)$	6.0 p.p.	5.0 p.p.
	$\mathbb{E}[\text{delay}]$	3.23	3.30
	$\mathbb{E}[h]$	29.73%	31.87%

Group 1 contains moments calculated using only data from Argentina. Group 2 contains moments calculated based on data from a large sample of countries. Figures 2 and 3 below show the model fit of the regression coefficients visually:

Figure 2: Model Fit: Intercepts

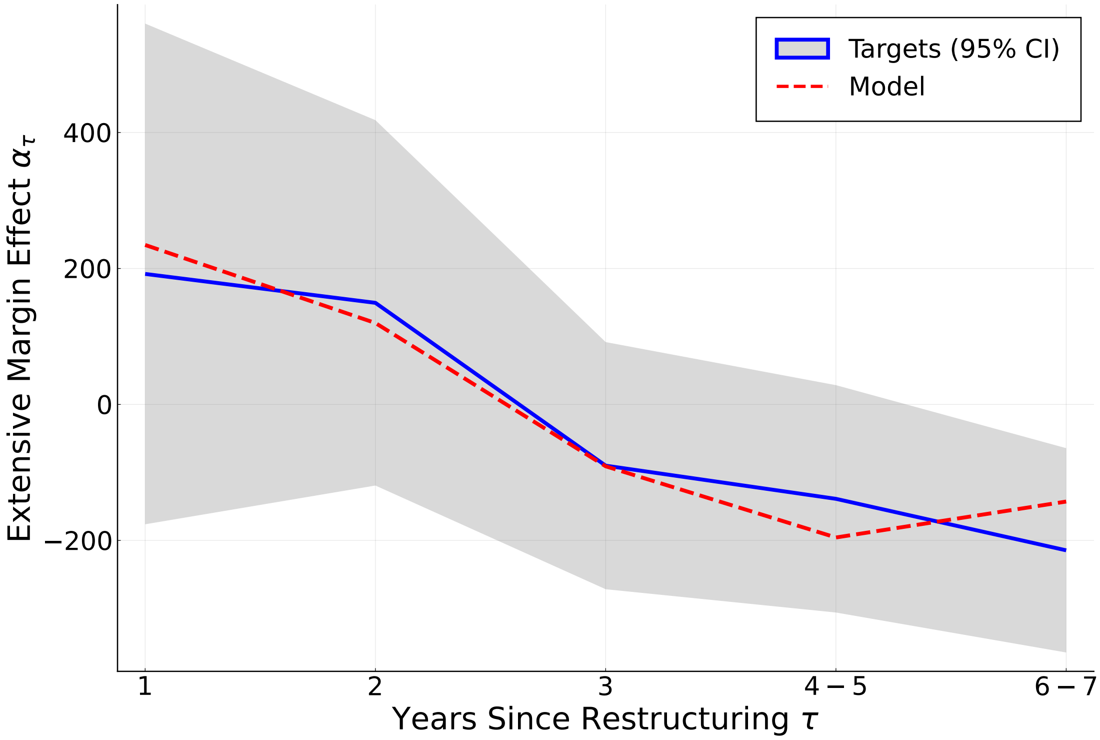
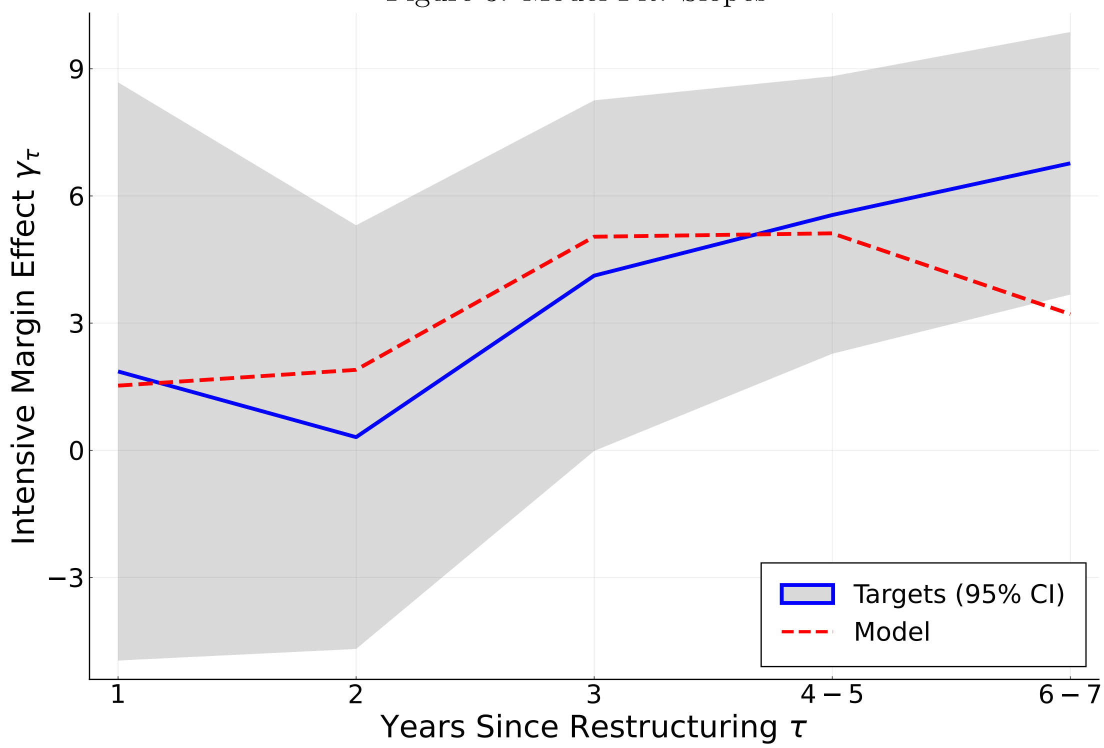


Figure 3: Model Fit: Slopes



In the first two blocks, there are only two medium size misses, on the mean and volatility of spreads. Since the model does not feature any risk premia, it is not surprising that it struggles to match the full average level of spreads in the data. Among the targeted regression coefficients, there is only one miss which is statistically significant according to the standard errors reported by Cruces and Trebesch (2013). In particular, it is the slope coefficient on the haircut effect after $6-7$ years. This is due to the model targeting a significantly higher default rate than the average in the sample for those author's regression, resulting in significantly quicker attrition of the the riskier members of the sample.

5 Results

Table 4 contains a selection of untargeted moments. In particular, it contains data and model values for the first and second moments of debt to GDP and interest rate spreads when the sample is not restricted to periods out of default. The model slightly underestimates long run levels of debt and significantly underestimates the long run volatility of debt to GDP. On the other hand, the model's estimates of the mean long run spread and the volatility of long run spreads are very close to their data counterparts.

Table 4: Other Moments

Moment	**Data**	**Model**
$\mathbb{E}[B'/Y]$ (including while in def.)	28.95%	23.6%
$\sigma(B'/Y)$ (including while in def.)	19.76%	7.1%
$\mathbb{E}[r - r^f]$ (including while in def.)	15.14%	13.1%
$\sigma(r - r^f)$ (including while in def.)	17.42%	12.9%
$\rho(r_t - r^f, r_{t-1} - r^f)$ (including while in def.)	0.92	0.95
% of defaults with $Y < E[Y]$	61%	61%
Effect of q_0^D on haircut	-0.60	-1.35

The final two entries in this table are the autocorrelation of interest rate spreads, the percent of defaults which occur when output is below trend, and the estimated effect on future haircuts of the price of a bond that has just been defaulted on, in a linear regression. The model just slightly overestimates the autocorrelation of spreads. Furthermore, it manages to replicate the data fact that only a small majority of defaults occur when default is below trend. In fact, almost 40% occur when output is above its long run mean. The specific definition of the final moment is δ_1 in the regression:

$$h_j = \delta_0 + \delta_1 q_{0,j}^D + e_j$$

where h_j is the haircut observed in restructuring j and $q_{0,j}^D$ is the average price of the bonds restructured, measured one month after the default occurred.[5] Meyer et al. (2021) report a highly significant value of -0.60 for the effect δ_1 and use this result to argue that bond prices directly after default forecast haircuts in the eventual restructurings (possibly years away) rather well. This relationship is nontrivial because haircuts measure the ex post difference between the value of old debt and the value of new debt (a relative quantity), while the bond

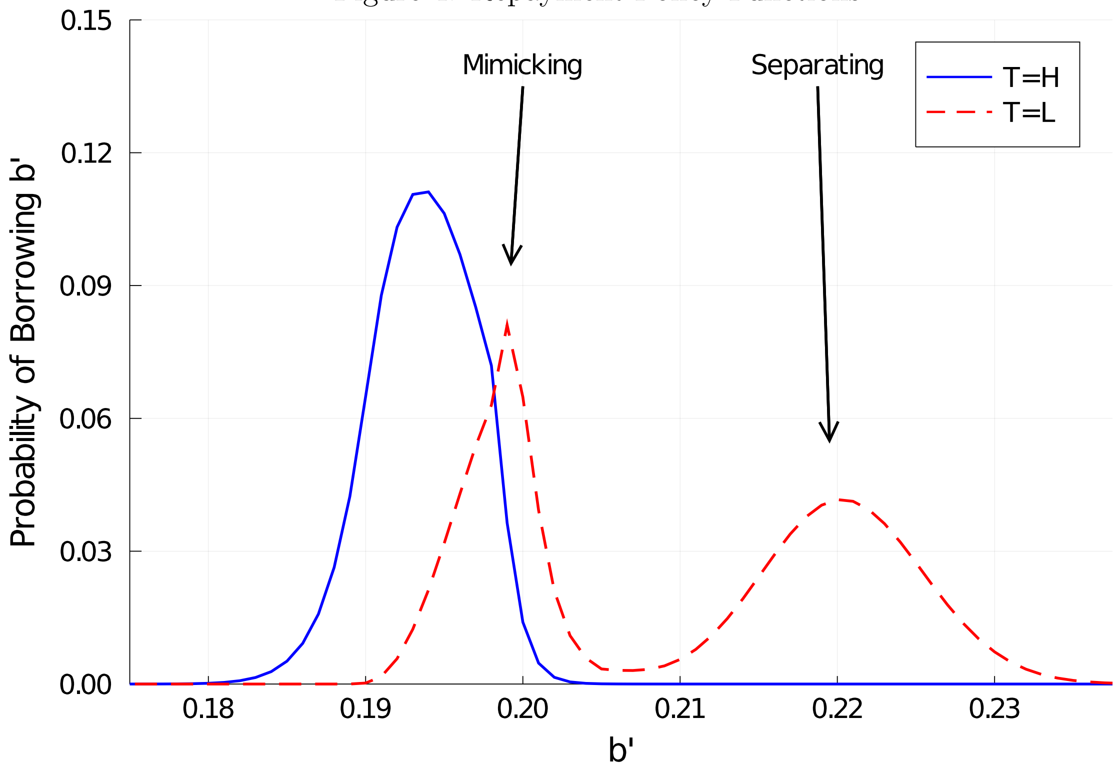
[5] I map $q_{0,j}^D$ to the price of the bond immediately after the government makes its default decision (and before the resolution of any uncertainty about whether there will be an opportunity to renegotiate in the current period). Since the quarterly probability of such an opportunity arising is relatively small, this result is robust to other assumptions about the timing of this measurement.

price right after default measures the actual ex ante value of the debt (an absolute quantity). My model replicates the sign of this relationship, but the association is somewhat sharper. To my knowledge, my paper is the first to replicate this pattern. While the first five data values in this table were calculated using only data from Argentina, the data values for the sixth and seventh untargeted moments reported in this table were calculated by Benjamin and Wright (2013) and Meyer et al. (2021), respectively, using cross country samples.

5.1 Properties of the Model

I now describe some features of behavior in the calibrated model, the forces driving them, and how these features lead to the post-restructuring patterns in spreads used to fit the model. An example set of borrowing policy functions for the government is plotted below in figure 4:

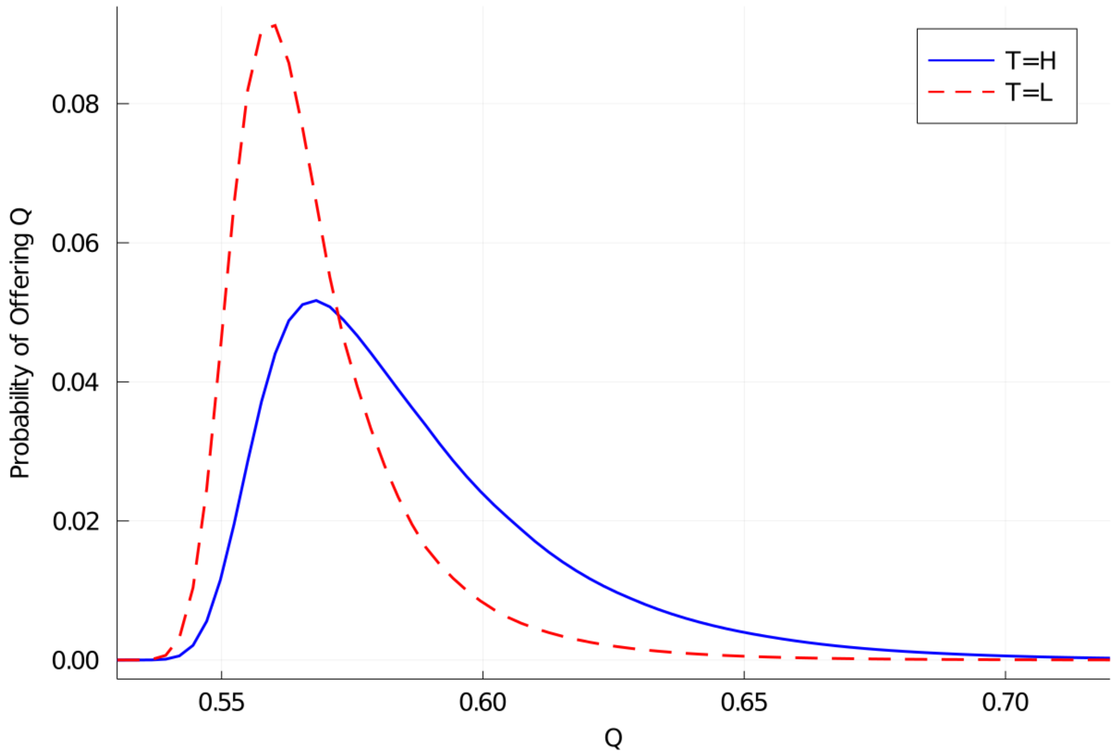
Figure 4: Repayment Policy Functions



The b' axis here is b' divided by the average annual value of GDP, 4, to make its values comparable to the moments reported above. These policies correspond to a level of debt close to the mean, the mean value of income, and a high value of beginning-of-period reputation. Here, we can see that the low type is torn between revealing itself entirely and borrowing relatively more, or preserving some of its reputation by choosing relatively lower borrowing levels that the high type also chooses frequently. The high type in turn knows that the low type chooses those borrowing levels sometimes and in turn tilts its choices even lower, often signalling to lenders that it is without a doubt the responsible type. This signalling mechanism is the key to one of major roles of asymmetric information in this model. The higher patience level of the high type provides an initial, basic incentive not to borrow as much as the low type. The signalling mechanism reinforces this by providing extra curvature to the price function faced by the government as it adjusts the level of borrowing. In the long run, this signalling mechanism will guarantee that the high type occupies a relatively low debt region of the state space, and therefore rarely defaults. The low type, on the other hand, occupies a much higher debt region of the state space and therefore ends up defaulting much more often.

An example set of renegotiation policy functions for the government is plotted below in figure 5:

Figure 5: Renegotiation Policy Functions



In this case, the policy functions of both types are unimodal. However, they display similar patterns to those associated with repayment. In particular, the low type makes relatively low offers to lenders most of the time, since it is less worried than the high type about lenders rejecting its offer. That said, it does sometimes offer values that the high type plays with nontrivial probability. It does this in order to ensure that lenders accept the offer and to be able to take advantage of the relatively higher prices offered to it in the restructuring phase if it enters with relatively higher reputation. The high type finds default quite painful and evaluates the balance of 1) getting a good deal from lenders, and 2) the probability of exiting default, differently from the low type. In particular, the high type tilts its offer towards higher values in order to ensure that lenders do not reject the deal. In simulations, the high type makes offers which are accepted 99.7% of the time while the low type makes offers which are accepted 96.7% of the time. When the lender is proposing the deal, the difference is significantly wider. The high type accepts 98.8% of the offers it receives while the low type accepts just 86.9% of the offers it receives.

Both of these features play into the effect of reputation on prices. First of all, the low type is relatively more likely to default in the near future. Since default entails an immediate suspension of cash flows to bondholders, lower reputation depresses the price of the bond. Note that, whenever default might occur in the next period, this statement would be true of short term (one period) debt as well. However, if default would never occur in the next period, regardless of the government's type and reputation, the price of short term debt is invariant to the government's reputation. With long term debt, on the other hand, the entire infinite sequence of future government actions is relevant to lenders valuing the bond today, so reputation can affect prices even when default is many years away. Furthermore, once the government defaults, the payout is relatively lower if the government is the low type, and settlement takes longer if the government is the low type. Therefore, low reputation also depresses the value of the bond in default. Together, these mean that lower reputation corresponds with a higher probability of transitioning to a lower value state.

In order to get from this observation to the effects observed in the regression, we need only observe that the majority of defaults (almost 98%) are the doing of the low type. Since type is quite persistent (lasting $15 - 17$ years) relative to the average delay before reentry (just over 3 years), the distribution of types which have recently restructured their debts is quite skewed towards the low type (relative to the long run split of 53% high/47% low). Furthermore, after controlling for the level of debt chosen during the restructuring, haircuts associated with deals reached by the low type are significantly higher than haircuts associated with deals reached by the high type.

5.2 Role of Asymmetric Information

I now want to take a moment to highlight the role of asymmetric information in the model. Specifically, I want to illustrate the dramatic effect on the behavior of the high type that is induced by the signalling motives in the baseline model. To that end, I re-solve a model with the exact same set of parameters under the assumption that the government's type is public information. I then calculate a few moments of interest, conditional on type. These are listed below in Table 5:

Table 5: Comparison: Differences Across Types

Moment	Baseline Model		Full Info Model	
	T=H	T=L	T=H	T=L
$\mathbb{E}[B'/Y\|T]$	16.67%	25.85%	21.49%	26.99%
$Pr(T\|d_t = 1)$	2.28%	97.72%	14.51%	85.49%
$\mathbb{E}[\pi\|T]$	0.890	0.147	1	0

The first line of this table shows that, when we move from the baseline to a full information setting and therefore remove signalling motives, both types borrow more in the long run. This is expected, since both types are impatient relative to lenders. However, borrowing by the high type rises almost 5%, while borrowing by the low type rises by only about 1%. In short, the high type was disciplined much more by the signalling incentives in the asymmetric information setting. When those incentives are removed, it borrows more and begins to be responsible for a larger share of the country's defaults. In particular, as we see in the second row, the share of defaults performed by the responsible type rises from about 2% to almost 15%, because it now more frequently reaches levels of debt at which default occurs with nontrivial probability. Finally, all of this occurs despite the fact that beliefs are quite accurate in the long run in the asymmetric information setting, as we can see in the table's third row. But even when entering the period with very high reputation, the signalling motives are still strong enough that the responsible type still wants to re-prove to lenders that it is indeed this type.

Before we continue on to the validation section, I want to take a moment to note another implication of these results. The stark separation of the long run distributions of debt across type means that, in the asymmetric information version, the responsible type is only responsible for just over 2% of defaults. Furthermore, when the responsible type defaults, it is almost always the case that both types would have defaulted. In fact, the average beginning of period reputation value, conditional on default occurring in the current period is 2.28%. The posterior after that default is taken into consideration by lenders is 2.31%, quantitatively indistinguishable from the prior. Thus the model can deliver the pattern that countries which have recently defaulted have relatively poor reputations. However, this is not because the default decision itself revealed that they were the irresponsible type. Rather, all of the borrowing decisions required to run up the debt to the point where default was a nontrivial possibility were what destroyed the country's reputation in international markets. Figure 6, below, illustrates this pattern.

Figure 6: Pre-Default Paths

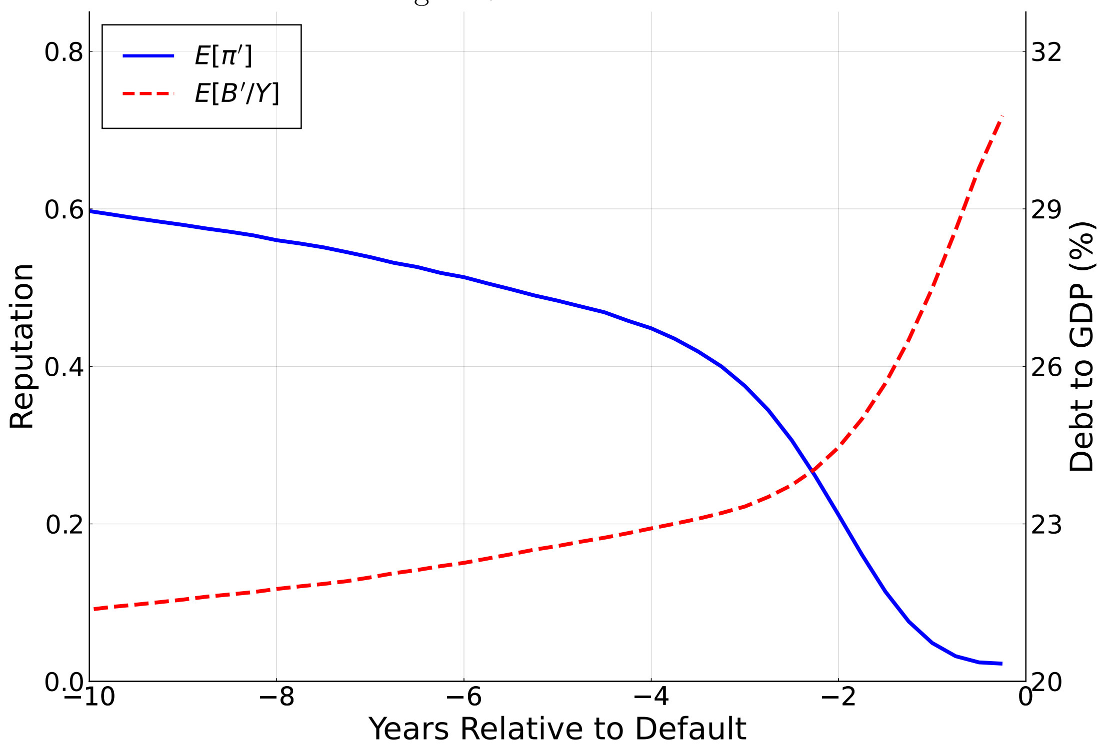


Figure 6 plots the the average end of period reputation π' and end of period debt to GDP ratio B'/Y during the 10 years prior to a default. Over these years, countries monotonically increase their debt levels[6] at the expense of their reputation. Both sides of this process accelerate markedly in the last 3 years prior to a default. One force driving this acceleration is that, as the country's reputation falls, it becomes more and more painful for the irresponsible type to convincingly imitate the responsible type. Eventually, such mimicry becomes costly enough that the irresponsible type simply gives up entirely and just borrows more. The result, that reputations are lost already by the time of default, is one of the core contributions

[6]The pattern in Figure 6 is driven primarily by increases in the debt stock B' rather than decreases in output Y. If the picture is reproduced using $\mathbb{E}[B']$ instead of $\mathbb{E}[B'/Y]$, it looks extremely similar.

of my paper.

This in turn leads to one key implication of my paper for policy. In debates about whether to default, concerns about the reputational costs of defaulting have often played an outsize role. In order to provide guidance on how relevant such concerns are, I examine the relationship between reputation and one period ahead default risk. In the model, there is a strong negative association between these quantities, and the vast majority of countries facing any nontrivial risk of default within 3 months already have very poor reputations, which sharply limits the reputational costs of defaulting (there is barely any reputation left to lose). Specifically, the average reputation of a country with just a $1-2\%$ risk of defaulting within three months is just under 10%, and the average value of the reputation they still have is only about 2 tenths of a basis point (in consumption equivalent terms) of the government's payoff. The average reputation of countries at more than 20% risk of default within 3 months is below 3%, and the average value of their remaining reputation is about 5 hundredths of a basis point. These results indicate that, in the vast majority of cases, reputation is not a very important consideration for troubled countries. By the time they find themselves exposed to any nontrivial level of default risk, they have almost no reputation left to lose, and whatever does remain is not worth very much. Furthermore, note that by conditioning on default risk, rather than default, in this analysis, I control for the fact that countries which face large reputational costs of default may not end up defaulting for exactly that reason, and therefore would not be included in my original analysis.

5.3 Measuring Reputation

Now that I have described some of the key internal mechanisms of the calibrated model, I move on to a section validating my approach. Specifically, I will use the model develop a method to use real world data on debt issuance to measure reputation. I then show that this model-implied measure has real quantitative bite.

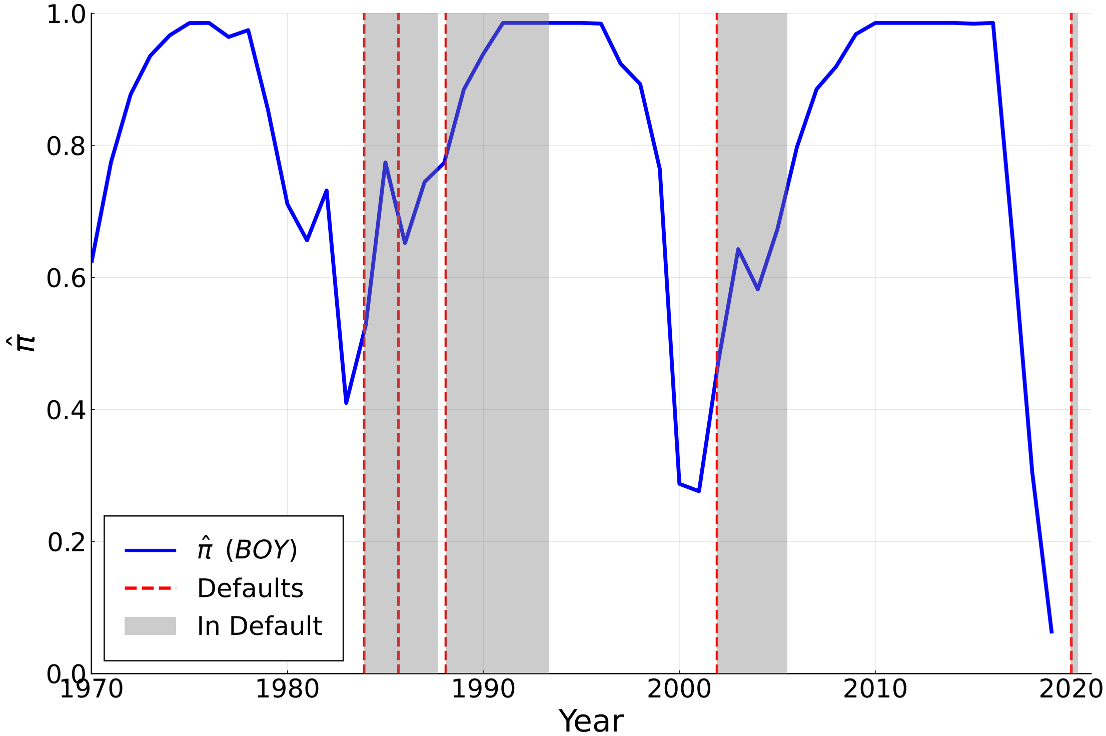
This model was fit based by matching patterns in the correlations between historical default/restructuring choices and current interest rate spreads. This data is all based, naturally, on post default facts. However, the model also makes a rich set of predictions about how debt issuance behavior, conditional on not defaulting, should affect reputation and therefore affect spreads. In this section, I check whether these predictions are borne out in the data.

In order to do this, I first compress all the model's belief update functions into a parsimonious functional form which only depends on current period reputation and current period gross issuances of debt divided by GDP GI. This focus on debt issuance is motivated by the fact, evidenced in the previous section of this paper, that borrowing decisions are one of the most important channels by which information is conveyed to lenders, and, on average, far more important than default itself. Furthermore, since no relationship between debt issuance histories and current spreads was targeted when fitting the model, so the relationship between debt issuances and reputation constitutes a non-targeted moment. To derive the abovementioned parsimonious approximation of the belief update function, I estimate the following regression equation using simulated data:

$$ln\left(\frac{\pi_t}{1-\pi_t}\right) = \beta_0 + \beta_1 ln\left(\frac{\pi_{t-1}}{1-\pi_{t-1}}\right) + \beta_2 GI_t + \beta_3 ln\left(\frac{\pi_{t-1}}{1-\pi_{t-1}}\right)\beta_3 GI_t + \epsilon_t$$

This resulting approximate belief update equation is very easy to take to the data and is actually pretty accurate within the model (the R^2 is about 82%). With the estimated coefficients $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\beta}_4)$ in hand, I then proceed to do the a few exercises. First, using data on Argentina debt issuance and output (from the ISD dataset used in the calibration section), I generate the sequence of reputation values for Argentina from 1970 to 2019 (the initial $\frac{\pi_{t-1}}{1-\pi_{t-1}}$ is set to the long run mean of the term in the model). This produces the following picture:

Figure 7: Reputation of Argentina (1970-2019)



The three troughs in reputation coincide with the onset of all three of Argentina's default episodes over the past 50 years. Note that the model was calibrated primarily to match the patterns of behavior following a restructuring, not before a default. The only pre-default behavior that was targeted was the rise in debt to output over the one year preceding a default. That said, the falls into each of the three notable troughs begins significantly more than one year before them, implying that this calibration target is not the only reason the model is detecting these deteriorations in Argentina's credit standing.

In addition to this more stylized test, I also do more systematic ones. First, I used the same ISD data on debt issuance and GDP (augmented with OECD data for a few countries) to produce full sequences of model-filtered reputation for most of the countries in the Cruces and Trebesch (2013) sample. I then estimate this augmented version of their specification:

$$spread_{it} = X_{it}\beta + \sum_{\tau \in \mathscr{T}} d_{it,\tau}(\alpha_\tau + \gamma_\tau h_{it,\tau}) + \beta_\pi \hat{\pi}_{it} + \epsilon_{it}$$

The results of this estimation (and an alternative version where gross issuances of debt divided by GDP is added to the original specification instead of filtered reputation) are detailed below in Table 6:

Table 6: Regression Results: Interest Rate Spreads

	EMBIG Spread		
$\hat{\pi}$	–	-225^{**} (98)	–
GI	–	–	1081 (797)
Other Controls	Yes	Yes	Yes
Country & Year Fixed Effects	Yes	Yes	Yes
R^2	0.3516	0.3578	0.3532
Observations	2964	2964	2964

Here, we see that the filtered measure of reputation informed by the model provides significant additional explanatory power when compared to the reference specification. Furthermore, the fact that current gross issuances alone do not pass the same test show that this is not just due to the fact that reputation incorporates them. The way the model aggregates the history of gross issuances into a single term is providing significant additional information. Furthermore, the effect of reputation has the predicted sign and is economically significant in magnitude. The difference in interest rate spreads between two otherwise identical countries, one with the worst possible reputation and one with the best possible reputation, is about 2 percentage points.

Finally, I evaluate how well this filtered measure of reputation predicts future defaults. Specifically, I estimate the following logit model with fixed effects:

$$\log\left(\frac{p_{it}}{1-p_{it}}\right) = X_{it}\beta + \sum_{\tau\in\mathscr{T}} d_{it,\tau}\alpha_\tau + \beta_\pi\hat{\pi}_{it}$$

where p_{it} is the probability of defaulting within the next year.[7] The results of three regressions specifications are detailed below in Table 7:

[7] I have omitted the intensive margin effects of haircuts from this specification because, within the reduced sample that the fixed effects logit model requires, they are extremely collinear with the extensive margin dummies and, under some specifications, the estimation fails to converge when they are included. In the cases where it does converge, the increase in the log likelihood relative to the case without them is extremely small (so their exclusion should not be very worrisome).

Table 7: Regression Results: Default Probability

	Default within the Next Year		
$\hat{\pi}$	–	-42.21^{***} (15.31)	–
Debt to GDP	1.14^{***} (0.19)	0.005 (0.543)	1.25^{***} (0.22)
GI	–	–	-60.71^{*} (32.28)
Other Controls	Yes	Yes	Yes
Country Fixed Effects	Yes	Yes	Yes
LL	-48.03	-13.34	-46.30
Observations	990	990	990

I have included the estimated coefficient on debt to GDP in this table in order to highlight how it changes when different sets of covariates are used. In particular, the effect of debt to GDP essentially vanishes when filtered reputation is included in the specification. Furthermore, when included, reputation is highly significant and dramatically increases how well the model can fit the data. While current period gross issuances alone is marginally significant and slightly improves the fit of the model, it does not add nearly as much information as the full history of debt issuances, aggregated through the lens of my model.

5.4 Welfare Consequences of Transparency

Having established that my approach to modelling sovereign borrowing with reputational concerns is validated by the data, I now move on to the main policy result of this paper. In this section, I evaluate the welfare effects of policies such as transparency initiatives, audit programs, and accountability offices. In the context of my model, one of the results of such policies is a periodic public signal about the type of the government. In general, this type of program will add an exogenous flow of information to the model, weakening the effects of any signalling motives (or mimicking motives). In this draft, I consider only signals which are perfectly informative. Therefore, such a policy change takes us from the asymmetric information benchmark all the way to the analogous full information model, completely eliminating signalling motives. First, I evaluate the change in payoffs to the government associated with such a change. Then I evaluate the changes in the welfare of a representative consumer with preferences that potentially differ from the preferences of the government.

In order to provide an exact decomposition of the sources of these variations following Aguiar et al. (2020), I first adjust baseline consumption streams $\hat{c}(.)$ to account for default costs and the period by period effects of the preference shocks on values, if necessary. For example, consider the contribution to the government's value from events occurring in the current period when it enters the period in good standing and decides not to default:

$$U^R(s, T, \epsilon, \pi, b) = (1 - \beta_T)u(\hat{c}(s, T, \epsilon, \pi, b)) + \epsilon^R(b'^{\star}(s, T, \epsilon, \pi, b))$$

In this case I set $c(.)$ such that:

$$(1 - \beta_T)u(c(s, T, \epsilon, \pi, b)) = U^R(s, T, \epsilon, \pi, b)$$

so the value of consumption is adjusted to account for the effect of the preference shock on current period utility.[8] The effects of this adjustment are negligible. Throughout this section, $c(.)$ will refer to this adjusted consumption value.

Given an initial type $T_0 \in \{H, L, N\}$ (where N indicates the the initial type is randomly drawn from the long run distribution of type), I then define the government payoff or consumer welfare in this case as the expected value over GDP states and preference shocks to an agent when the government starts with zero debt and reputation at the long run probability the government is the high type. For example, the value to the low type under asymmetric information would be:

$$\mathbb{E}_{s_0,\epsilon}[V_{AI}(s_0, L, \epsilon, \bar{\pi}, 0)] = \mathbb{E}_0\left[\sum_{t=0}^{+\infty}\left(\Pi_{l=0}^{t-1}\beta_{T(l)}\right)(1 - \beta_{T(t)})(u(c_t^{AI}) - \phi_{T(t)}(s_t, d_t))|T(0) = L\right]$$

Since utility is CRRA with relative risk aversion coefficient γ, I can define an overall con-

[8]Since the preference shocks are unbounded, this mapping is not always well defined. However, such states, i.e. those in which no value of consumption c can satisfy this equation, are vanishingly rare in practice.

sumption equivalent change in welfare as ζ in the equation:

$$(1+\zeta)^{1-\gamma}\mathbb{E}_{s_0,\epsilon}\left[V^{AI}(s_0,T_0,\epsilon,\bar{\pi},0)\right]=\mathbb{E}_{s_0,\epsilon}\left[V^{FI}(s_0,T_0,\epsilon,0)\right]$$

Some rearrangement yields:

$$1+\zeta=\left(\frac{\bar{V}^{FI}_{T_0}}{\bar{V}^{AI}_{T_0}}\right)^{\frac{1}{1-\gamma}}$$

where $\bar{V}^{AI}_{T_0}$ is the time 0 value under asymmetric information if the initial type is T_0 and $\bar{V}^{FI}_{T_0}$ is its full information counterpart. I also follow Aguiar et al. (2020) by defining a breakdown of this variation into:

1. changes due to different incidence of default costs;
2. changes due to different variability of consumption streams;
3. changes due to different trends in the time path of average. consumption.

To do this, note that we can rewrite $(1+\zeta)$ as:

$$\left(\frac{\bar{V}^{FI}_{T_0}}{\bar{V}^{FI,ND}_{T_0}}\frac{\bar{V}^{AI,ND}_{T_0}}{\bar{V}^{AI}_{T_0}}\right)^{\frac{1}{1-\gamma}}*\left(\frac{\bar{V}^{FI,ND}_{T_0}}{\bar{V}^{FI,NDV}_{T_0}}\frac{\bar{V}^{AI,NDV}_{T_0}}{\bar{V}^{AI,ND}_{T_0}}\right)^{\frac{1}{1-\gamma}}*\left(\frac{\bar{V}^{FI,NDV}_{T_0}}{\bar{V}^{AI,NDV}_{T_0}}\right)^{\frac{1}{1-\gamma}}$$

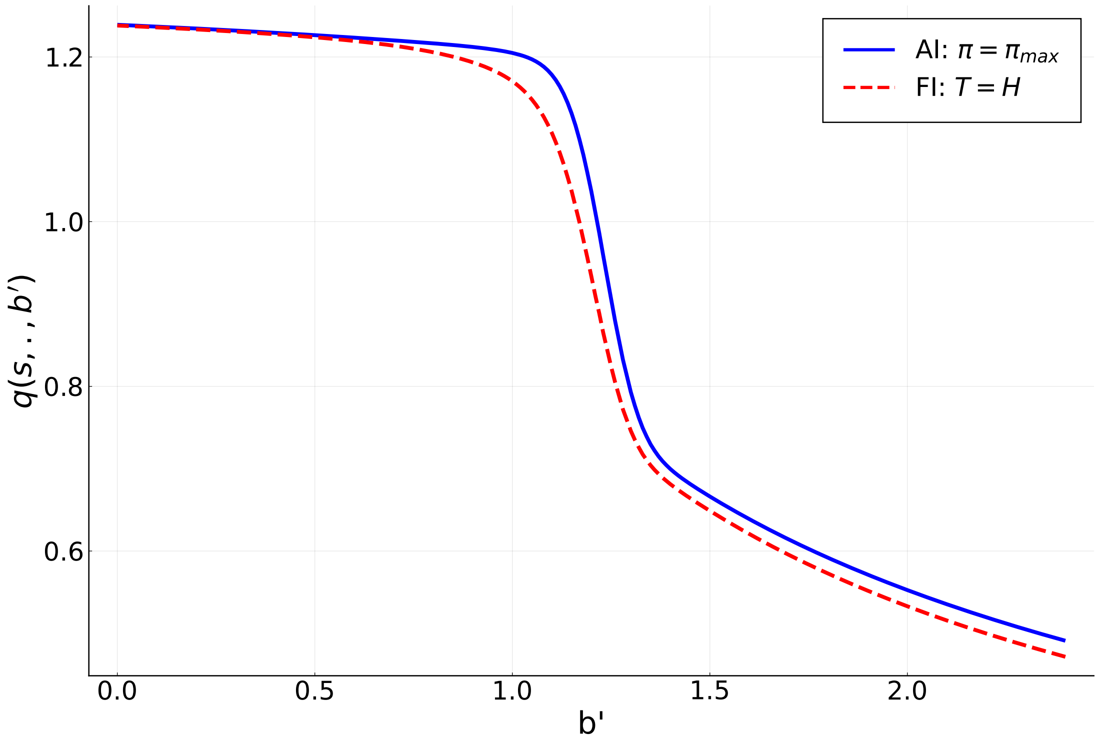
where the value functions with the additional ND superscript are the value functions when default costs are removed and those with the additional NDV superscript are the value functions when default costs are removed and consumption at each time t is set to its mean value across possible paths. The first term is $1+\zeta_D$, the welfare effects of changes in default costs, the second term is $1+\zeta_V$, the welfare effects of changes in the variability of consumption, and the third term is $1+\zeta_T$, the welfare effects of changes in the trend of average consumption over time. Table 5.4 details the effects on government payoffs of moving from asymmetric information to full information:

Welfare Change	$T_0=L$	$T_0=H$	$T_0=N$
ζ^G_D	−0.13%	−0.73%	−0.45%
ζ^G_V	−0.05%	−0.10%	−0.08%
ζ^G_T	−0.06%	+0.34%	+0.15%
ζ^G	−0.23%	−0.50%	−0.38%

Here we see that increased transparency has negative effects on the average payoffs for both the responsible and the irresponsible type. The losses for the responsible type are roughly twice as large as the losses for the irresponsible type. All three channels produce negative effects for the irresponsible type. For the responsible type, however, we see that it does see a gain from moving to full information associated with the average time trend of consumption. Since it no longer needs to signal to lenders that it is the responsible type, it can accumulate debt faster and consume more right away. Since it is more impatient than international investors (although not as impatient as the irresponsible type), this is a valuable feature for it.

However, it will end up defaulting sooner and more frequently in the full information setting, and those costs end up outweighing the benefits of being able to borrow more in the near future. Furthermore, the future effects of those signalling motives (or their absence) is constantly reflected in prices. Below, in Figure 8, I plot the price function at the mean level of output both in the case with full information when the government is the high type as well as the case with asymmetric information when the government is known with certainty to be the high type today:

Figure 8: Price Functions



The difference between the two price functions is the cumulative impact of signalling motives throughout the future. We can see that they result in the asymmetric information schedule of prices being uniformly higher than full information schedule. Thus, while the government ends up borrowing less under asymmetric information due to the signalling motives, the price at which it borrows is consistently higher, which lessens the impact of the lower borrowing level on auction revenue and therefore consumption.

An alternative way to think about this result is to remember that this long bond model, even without types and without endogenous renegotiation, features inefficiently high levels of borrowing (Aguiar and Amador, 2019). There are feasible allocations which are Pareto improving relative to the competitive equilibrium allocation. Since this is a model with incomplete markets, feasibility here is a stricter condition than usual. Specifically, a feasible allocation is one attainable by committing to an infinite sequence of debt issuances while still being subject to the lack of commitment when it comes to default. The signalling motives in the baseline model help push equilibrium borrowing levels down in the long run, moving the competitive equilibrium allocation closer to the allocation a planner would choose when that limited commitment (with respect to the default decision) constraint is imposed. When phrased this way, it becomes clear that the signalling motives induced my information asymmetry are acting as a substitute for various macroprudential policies intended to limit borrowing, such as fiscal rules.

The equivalent set of values for a representative consumer who is just as patient as the international investors (i.e. has $\beta = \frac{1}{R}$) are:

Welfare Change	$T_0 = L$	$T_0 = H$	$T_0 = N$
ζ_D^C	-0.78%	-1.06%	-0.93%
ζ_V^C	$+0.03\%$	$+0.01\%$	$+0.02\%$
ζ_T^C	$+0.06\%$	$+0.07\%$	$+0.06\%$
ζ^C	-0.69%	-0.98%	-0.84%

Overall, the welfare changes experienced by this representative consumer are dominated by the default cost channel. Since this consumer is significantly more patient than either government and perhaps the most important differences between the two settings is that default happens sooner and more frequently under full information, this should not come as a surprise. This decomposition does illustrate, however, how asymmetric information can shield consumers from the political economy frictions that make their government more impatient than them. The signalling motives induce both government types to act as if they were slightly more patient and had preferences more similar to those of their citizens.

6 Conclusion

In this paper, I built a flexible model of sovereign borrowing, default, and renegotiation with asymmetric information. I then estimated this model, disciplining the reputation-related parameters using post-restructuring patterns of interest rate spreads observed in the data. Having estimated the model, I examine which decisions are considered to be important for conveying information according to the data. Here, I find that the most important set of decisions in determining a country's reputation are its borrowing decisions. This suggests that the literature's focus on the default decision as the key part of behavior which distinguishes responsible types from irresponsible types may be incomplete, and that more work should be devoted to how differences in borrowing patterns, conditional on repayment, feed into differences in reputation.

After estimating the model and analyzing some notable patterns of behavior present in the calibration, I move on to validating the model by showing that the predictions it makes about how debt issuance decisions feed into a country's reputation have real quantitative bite. I show that I can use the model to develop a way to measure countries' reputations using real world data on debt issuance. I then do a few tests of this measure, some stylized and some systematic. In particular, I show that this model-filtered measure of reputation provides significant additional information in explaining interest rate spreads and default probabilities. This shows that the model is picking up on important patterns in the determination of reputation in the real world.

Finally, I use the model to examine the implications of programs which increase transparency in policy making. Here, I show that by weakening or even removing the signalling motives in place in the benchmark asymmetric information case, such policies can have significant negative effects on both government payoffs and consumer welfare. These effects arise because the presence of signalling motives gives the government a type of pseudo-commitment, a way to discipline its future selves against borrowing too much. Of course, there may exist other channels by which such policies could have positive effects, but it is important to account for this channel in considering whether to implement audit programs when extending debt relief to troubled countries.

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7 Appendix

7.1 Proof of Theorem 1

In this section, I prove that under the assumptions made in the text, an equilibrium must exist. In order to do that, I must first define (and redefine) some quantities and functions. This section will proceed as follows. First, I define some bounds for value functions and price functions. Second, I define a reduced set of equilibrium objects from which the full set defined above can be recovered. Third, I define an operator which updates that set of equilibrium objects. Fourth, I show that this operator is a continuous mapping of a compact, convex set into itself, and therefore, according to Brouwer's Fixed Point Theorem, must have a fixed point.

Definition 2. *1. Set q^{max} by:*

$$q^{max} = \max\{q^{RF}, \max\{\mathscr{Q}\}\} + \frac{1}{1-\frac{1}{R}}\left(\sigma^{R}_{\eta,G} log(2) + \sigma^{P}_{\eta,L} log(N_Q)\right)$$

where q^{RF} is the unique solution to:

$$q^{RF} = \frac{1}{R}(\lambda + \kappa + (1-\lambda)q^{RF})$$

i.e. the risk free price of the bond.

2. Set V^{max} by:

$$u(\max_{s\in\mathscr{S}} y(s) + q_{max} \max_{b'\in\mathscr{B}} b') + \frac{1}{1-\beta_H}\left(\sigma_{\epsilon} log(N_b+1) + \sigma^{P}_{\eta,G} log(N_Q) + \sigma^{R}_{\eta,L} log(2) + \sigma^{RS}_{\nu} log(N_b)\right)$$

3. Set V^{min} by:

$$\min_{s\in\mathscr{S}}\{u(y(s))\} - \max_{(s,T,d)\in\mathscr{S}\times\{H,L\}\times\{0,1\}}\{\phi_T(s,d)\}$$

The upper bound for q is simply the largest sequence of raw payments lenders can ever receive for it plus the maximum possible contribution of preference shocks to their values. The upper bound for V is simply the fundamental value (i.e. without preference shocks) of consuming the highest value of consumption possible plus the maximum contribution of preference shocks to the government's value. The lower bound on V is simply the value of being in default in the worst output state with the largest penalty forever. Since always defaulting, always proposing $Q = 0$ and then choosing $b' = 0$, and always declining deals from lenders is a feasible sequence of actions, this provides a lower bound on values for the government.

With these assumptions and definitions in hand, I now define the reduced set of objects which I will use to prove that an equilibrium must exist. They are:

Definition 3. *Let X denote a generic member of the set $\mathscr{X}$, to be defined below:*

$$X = \Big(\bar{V}(s,T,\pi,b), \bar{V}^D(s,T,\pi,b), \bar{q}(s,\pi',b'), \bar{q}^D(s,\pi,b), \bar{\delta}(s,\pi',b'), \\ \Gamma^R(b',\pi|s,b), \Gamma^D(\pi|s,b), \Gamma^D_G(d,Q,\pi|s,b), \Gamma^D_L(d,Q,\pi|s,b), \\ \Gamma^{RS}_G(d,Q,b',\pi|s,b), \Gamma^{RS}_L(d,Q,b',\pi|s,b) \Big)$$

where:

1. $\bar{V} : \mathscr{S} \times \{H,L\} \times \Pi \times \mathscr{B} \to [V^{min}, V^{max}]$
2. $\bar{V}^D : \mathscr{S} \times \{H,L\} \times \Pi \times \mathscr{B} \times \to [V^{min}, V^{max}]$
3. $\bar{q} : \mathscr{S} \times \Pi \times \mathscr{B} \to [0, q^{max}]$
4. $\bar{q}^D : \mathscr{S} \times \Pi \times \mathscr{B} \to [0, q^{max}]$
5. $\bar{\delta} : \mathscr{S} \times \Pi \times \mathscr{B} \to [0,1]$
6. $\Gamma^R : \mathscr{B} \times \Pi \times \mathscr{S} \times \mathscr{B} \to [0,1]$
7. $\Gamma^D : \times \Pi \times \mathscr{S} \times \mathscr{B} \to [0,1]$
8. $\Gamma^D_G : \{0,1\} \times \mathscr{Q} \times \Pi \times \mathscr{S} \times \mathscr{B} \to [0,1]$
9. $\Gamma^D_L : \{0,1\} \times \mathscr{Q} \times \Pi \times \mathscr{S} \times \mathscr{B} \to [0,1]$
10. $\Gamma^{RS}_G : \{0,1\} \times \mathscr{Q} \times \mathscr{B} \times \Pi \times \mathscr{S} \times \mathscr{B} \to [0,1]$

11. $\Gamma_L^{RS} : \{0,1\} \times \mathscr{Q} \times \mathscr{B} \times \Pi \times \mathscr{S} \times \mathscr{B} \to [0,1]$

Set $\mathscr{X}$ to be the set of mappings from the product of these 11 domains to the the product of these 11 codomains.

Now, define the feasible sets of choices for the government as:

Definition 4. ***Feasible sets***

1. $\mathscr{F}^R(s,\pi,b) = \{b' \in \mathscr{B} | c(s,\pi,b,b') > 0\}$
2. $\mathscr{F}(s,\pi,b) = \begin{cases} \{0,1\} & \mathscr{F}^R(s,\pi,b) \neq \emptyset \\ \{1\} & \mathscr{F}^R(s,\pi,b) = \emptyset \end{cases}$
3. $\mathscr{F}_G^{RS}(s,\pi,b,d,Q) = \{b' \in \mathscr{B} | c_G^{RS}(s,\pi,b,d,Q,b') > 0\}$
4. $\mathscr{F}_L^{RS}(s,\pi,b,d,Q) = \{b' \in \mathscr{B} | c_L^{RS}(s,\pi,b,d,Q,b') > 0\}$
5. $\mathscr{F}_G^{D}(s,\pi,b,d) = \{Q \in \mathscr{Q} | \mathscr{F}_G^{RS}(s,\pi,b,d,Q) \neq \emptyset\}$
6. $\mathscr{F}_L^{D}(s,\pi,b,d,Q) = \begin{cases} \{0,1\} & \mathscr{F}_L^{RS}(s,\pi,b,d,Q) \neq \emptyset \\ \{0\} & \mathscr{F}_L^{RS}(s,\pi,b,d,Q) = \emptyset \end{cases}$

where the repayment consumption function and auction prices are given by:

$$\begin{aligned} c(s,\pi,b,b') &= y(s) - (\lambda+\kappa)b + q(s,\hat{\pi}',b')(b' - (1-\lambda)b) - i(s,\pi,b,b') \\ q(s,\hat{\pi}',b') &= \hat{\mathbb{E}}[\bar{q}(s,\pi',b')|\hat{\pi}'] \\ \hat{\pi'} &= \Gamma(\Gamma^R(b',\pi|s,b)) \end{aligned}$$

and the restructuring consumption functions and auction prices, for $X \in \{G,L\}$, are given by:

$$\begin{aligned} c_X^{RS}(s,\pi,b,d,Q,b') &= y(s) - Qb + q(s,\hat{\pi}',b')b' - i(s,\pi,b,d,Q,b') \\ q(s,\hat{\pi}',b') &= \hat{\mathbb{E}}[\bar{q}(s,\pi',b')|\hat{\pi}'] \\ \hat{\pi'} &= \Gamma(\Gamma_X^{RS}(d,Q,b',\pi|s,b)) \end{aligned}$$

I can now define the operator T which updates X.

Definition 5. ***The update operator:*** *Set* $T : \mathscr{X} \to \mathscr{X}$ *by:*

$$T(X) = (\bar{V}_{new}, \bar{V}^{D}_{new}, \bar{q}_{new}, \bar{q}^{D}_{new}, \bar{\delta}_{new}, \Gamma^{R}_{new}, \Gamma^{D}_{new}, \Gamma^{D}_{G,new}, \Gamma^{D}_{L,new}, \Gamma^{RS}_{G,new}, \Gamma^{RS}_{L,new})$$

Where the objects in this list are obtained as follows:

First, I obtain the ex ante values of restructuring, choice probabilities associated with restructuring choices, and new restructuring belief update functions. To do this, for each $Q \in \mathscr{F}^{D}_{G}(s, \pi, b, d)$, set $V^{RS}_{G}(s, \pi, b, d, Q)$ by:

$$\begin{aligned} V^{RS}_{G}(s, T, \nu^{RS}, \pi, b, d, Q) = &\max_{b' \in \mathscr{F}^{RS}_{G}(s,\pi,b,d,Q)} (1 - \beta_T)(u(c^{RS}_{G}(s, \pi, b, d, Q, b')) - \phi_T(s, d)) \\ &+ \beta_T \hat{\mathbb{E}}[\bar{V}(s', T', \pi', b')|s, T, \hat{\pi}'] + \nu_{RS}(b') \\ \hat{\pi}' = &\,\Gamma(\Gamma^{RS}_{G}(d, Q, b', \pi|s, b)) \end{aligned}$$

Then define the government's ex ante values $\bar{V}^{RS}_{G}(s, T, \pi, b, d, Q)$ and its choice probabilities $p^{RS}_{G}(s, T, \pi, b, d, Q, b')$ by:

$$\begin{aligned} \bar{V}^{RS}_{G}(s, T, \pi, b, d, Q) &= \mathbb{E}[V^{RS}_{G}(s, T, \nu^{RS}, \pi, b, d, Q)] \\ p^{RS}_{G}(s, T, \pi, b, d, Q, b') &= Pr(b'^{RS,\star}_{G}(s, T, \nu^{RS}, \pi, b, d, Q) = b') \end{aligned}$$

Similarly, for each $Q \in \mathscr{Q}$ such that $\mathscr{F}^{D}_{L}(s, \pi, b, d, Q) = \{0, 1\}$ (i.e. accepting the offer Q when that offer is made by lenders results in there being a feasible way to deliver Qb of the consumption good to them), set $V^{RS}_{L}(s, \pi, b, d, Q)$ by:

$$\begin{aligned} V^{RS}_{L}(s, T, \nu^{RS}, \pi, b, d, Q) = &\max_{b' \in \mathscr{F}^{RS}_{L}(s,\pi,b,d,Q)} (1 - \beta_T)u(c^{RS}_{L}(s, \pi, b, d, Q, b')) \\ &+ \beta_T \hat{\mathbb{E}}[\bar{V}(s', T', \pi', b')|s, T, \hat{\pi}'] + \nu_{RS}(b') \\ \hat{\pi}' = &\,\Gamma(\Gamma^{RS}_{L}(d, Q, b', \pi|s, b)) \end{aligned}$$

Then define the government's ex ante value $\bar{V}^{RS}_{L}(s, T, \pi, b, d, Q)$ and its choice probabilities

$p_L^{RS}(s, T, \pi, b, d, Q, b')$ by:

$$\bar{V}_L^{RS}(s, T, \pi, b, d, Q) = \mathbb{E}[V_L^{RS}(s, T, \nu^{RS}, \pi, b, d, Q)]$$
$$p_L^{RS}(s, T, \pi, b, d, Q, b') = Pr(b_L'^{RS,\star}(s, T, \nu^{RS}, \pi, b, d, Q) = b')$$

This completes the construction of the set of new objects associated with the restructuring process and that I need in order to update the objects in $T(X)$ associated with the renegotiation and restructuring process, as well as the repayment problem.

I now move on to the updates associated with the renegotiation. I first consider the case where the government has been chosen to propose a deal. First, for each $Q \in \mathscr{F}_G^D(s, \pi, b, d)$, define the value to the government of exiting the period without a deal as:

$$\begin{aligned} V_{G,N}^D(s, T, \pi, b, d, Q) &= (1 - \beta_T)(u(y(s)) - \phi_T(s, d)) \\ &\quad + \beta_T \hat{\mathbb{E}}[\bar{V}^D(s', T', \pi', b)|s, T, \hat{\pi}'] \\ \hat{\pi}' &= \Gamma(\Gamma_G^D(d, Q, \pi|s, b)) \end{aligned}$$

Similarly, define on the same set the value to lenders of exiting a period without a deal as:

$$\begin{aligned} q_{G,N}^D(s, \pi, b, d, Q) &= \frac{1}{R}\hat{\mathbb{E}}[\bar{q}^D(s', \pi', b)|s, \hat{\pi}'] \\ \hat{\pi}' &= \Gamma(\Gamma_G^D(d, Q, \pi|s, b)) \end{aligned}$$

Now define lender lender values and policies after the government has made the feasible offer Q as:

$$\begin{aligned} \hat{q}_G^D(s, \eta_D^R, \pi, b, d, Q) = \max_{A_L \in \{0,1\}} & A_L \Big[Q + \eta^Y\Big] \\ & + (1 - A_L)\Big[q_{G,N}^D(s, \pi, b, d, Q) + \eta^N\Big] \end{aligned}$$

Then define the ex ante lender value and average policies in this case as:

$$\begin{aligned} q_G^D(s,\pi,b,d,Q) &= \mathbb{E}[q_G^D(s,\eta_D^R,\pi,b,d,Q)] \\ \bar{A}_L(s,\pi,b,d,Q) &= Pr(A_L^\star(s,\eta_D^R,\pi,b,d,Q)=1) \end{aligned}$$

Then I can define the government's problem as:

$$\begin{aligned} V_G^D(s,T,\eta_D^P,\pi,b,d) = \max_{Q\in\mathscr{F}_G^D(s,\pi,b,d)} & \bar{A}_L(s,\pi,b,d,Q)\bar{V}_G^{RS}(s,T,\pi,b,d,Q) \\ & + (1-\bar{A}_L(s,\pi,b,d,Q))V_{G,N}^D(s,T,\pi,b,d,Q) + \eta^O(Q) \end{aligned}$$

Using this, I can then define, for the case when the government is chosen to be the proposer, the government's ex ante value and choice probabilities:

$$\begin{aligned} \bar{V}_G^D(s,T,\pi,b,d) &= \mathbb{E}[V_G^D(s,T,\eta_D^P,\pi,b,d)] \\ p_G^D(s,T,\pi,b,d,Q) &= Pr(Q_G^\star(s,T,\eta_D^P,\pi,b,d)=Q) \end{aligned}$$

From the lenders' perspective, this implies choice probabilities and ex ante values are:

$$\begin{aligned} \bar{p}_G^D(s,\pi,b,d,Q) &= \Gamma_G^D(d,Q,\pi|s,b)p_G^D(s,H,\pi,b,d,Q) + (1-\Gamma_G^D(d,Q,\pi|s,b))p_G^D(s,L,\pi,b,d,Q) \\ \bar{q}_G^D(s,\pi,b,d) &= \sum_{Q\in\mathscr{F}_G^D(s,\pi,b,d)} \bar{p}_G^D(s,\pi,b,d,Q)q_G^D(s,\pi,b,d,Q) \end{aligned}$$

That completes the construction of required new values and beliefs when the government is chosen to be the proposer. I now move to the case where the lender is chosen to be the proposer. Again, I begin with definitions of values for both parties if they exit the period without a deal. The value to the government in this case is:

$$\begin{aligned} V_{L,N}^D(s,T,\pi,b,d,Q) &= (1-\beta_T)(u(y(s))-\phi_T(s,d)) \\ &\quad + \beta_T\hat{\mathbb{E}}[\bar{V}^D(s',T',\pi',b)|s,T,\hat{\pi}'] \\ \hat{\pi}' &= \Gamma(\Gamma_L^D(d,Q,\pi|s,b)) \end{aligned}$$

Similarly, the value to lenders of exiting a period without a deal is:

$$\begin{aligned} q^D_{L,N}(s,\pi,b,d,Q) &= \frac{1}{R}\hat{\mathbb{E}}[\bar{q}^D(s',\pi',b)|s,\hat{\pi}'] \\ \hat{\pi}' &= \Gamma(\Gamma^D_L(d,Q,\pi|s,b)) \end{aligned}$$

I now define the government's problem once lenders have made an offer:

$$\begin{aligned} \hat{V}^D_L(s,T,\eta^R_D,\pi,b,d) = \max_{A_G\in\mathscr{F}^D_L(s,\pi,b,d,Q)} & A(\bar{V}^{RS}_L(s,T,\pi,b,d,Q)+\eta^Y) \\ & + (1-A_G)(V^D_{L,N}(s,T,\pi,b,d,Q)+\eta^N) \end{aligned}$$

These values and the associated policy functions let me define the government's ex ante value when lenders make a given offer and its choice probabilities by:

$$\begin{aligned} V^D_L(s,T,\pi,b,d,Q) &= \mathbb{E}[\hat{V}^D_L(s,T,\eta^R_D,\pi,b,d)] \\ \bar{A}_G(s,T,\pi,b,d,Q) &= Pr(A^\star_G(s,T,\eta^R_D,\pi,b,d)=1) \end{aligned}$$

Using these choice probabilities, I can then write, from the lender's perspective, the probability that an offer will be accepted as:

$$\tilde{A}_G(s,\pi,b,d,Q) = \begin{cases} \pi\bar{A}_G(s,H,\pi,b,d,Q)+(1-\pi)\bar{A}_G(s,L,\pi,b,d,Q) & d=0 \\ \Gamma^D(\pi|s,b)\bar{A}_G(s,H,\pi,b,d,Q)+(1-\Gamma^D(\pi|s,b))\bar{A}_G(s,L,\pi,b,d,Q) & d=1 \end{cases}$$

which in turn lets me define the problem of the lender when proposing an offer as:

$$\begin{aligned} q^D_L(s,\eta^P_D,\pi,b,d) = \max_{Q\in\mathscr{Q}} & \tilde{A}_G(s,\pi,b,d,Q)Q \\ & + (1-\tilde{A}_G(s,\pi,b,d,Q))q^D_{L,N}(s,\pi,b,d,Q)+\eta^O(Q) \end{aligned}$$

Finally, I can define lenders' ex ante values when chosen to be the proposer, their choice probabilities, and the government's ex ante values when lenders are chosen to be the proposer:

$$\begin{aligned}
\bar{q}_L^D(s,\pi,b,d) &= \mathbb{E}[q_L^D(s,\eta_D^P,\pi,b,d)] \\
\bar{p}_L^D(s,\pi,b,d,Q) &= Pr(Q_L^\star(s,\eta_D^P,\pi,b,d)=Q) \\
\bar{V}_L^D(s,T,\pi,b,d) &= \sum_{Q\in\mathscr{Q}} \bar{p}_L^D(s,\pi,b,d,Q)V_L^D(s,T,\pi,b,d,Q)
\end{aligned}$$

Finally, I can then define values to both sides before it is determined whether a renegotiation will arise in the current period. These are:

$$\begin{aligned}
V^D(s,T,\pi,b,d) &= \psi(\mu_G\bar{V}_G^D(s,T,\pi,b,d)+(1-\mu_G)V_L^D(s,T,\pi,b,d))+(1-\psi)V_N^D(s,T,\pi,b,d) \\
q^D(s,\pi,b,d) &= \psi(\mu_G\bar{q}_G^D(s,\pi,b,d)+(1-\mu_G)\bar{q}_L^D(s,\pi,b,d))+(1-\psi)q_N^D(s,\pi,b,d)
\end{aligned}$$

where the values if no opportunity arises in this period V_N^D and q_N^D are given by:

$$\begin{aligned}
V_N^D(s,T,\pi,b,d) &= (1-\beta_T)(u(y(s))-\phi_T(s,d)) \\
&\quad + \beta_T\hat{\mathbb{E}}[V^D(s',T',\pi',b,0)|s,T,\hat{\pi}'] \\
q_N^D(s,\pi,b,d) &= \frac{1}{R}\hat{\mathbb{E}}[q^D(s',\pi',b)|s,\hat{\pi}'] \\
\hat{\pi}' &= \begin{cases} \Gamma(\Gamma^D(\pi|s,b)) & d=1 \\ \Gamma(\pi) & d=0 \end{cases}
\end{aligned}$$

Immediately, this allows me to define updated versions of $\bar{V}^D$ and $\bar{q}^D$ as:

$$\begin{aligned}
\bar{V}_{new}^D(s,T,\pi,b) &= V^D(s,T,\pi,b,0) \\
\bar{q}_{new}^D(s,\pi,b) &= q^D(s,\pi,b,0)
\end{aligned}$$

I now move to characterize the set of updates associated with values and policies when the government enters the period in good standing. After that I will define the full set of new belief update functions. Given $\bar{V}$, Γ^R, and q, I can define the government's problem if it chooses to repay its debt as:

$$\begin{aligned} V^R(s,T,\epsilon,\pi,b) = \max_{b'\in\mathscr{F}^R(s,\pi,b)} &(1-\beta_T)u(c(s,\pi,b,b')) \\ &+ \beta_T\hat{\mathbb{E}}[V(s',T',\pi',b')|s,T,\hat{\pi}'] + \epsilon^R(b') \\ \hat{\pi}' &= \Gamma(\Gamma^R(b',\pi|s,b)) \end{aligned}$$

and, given this and V^D, the government's problem of deciding whether to default is:

$$V(s,T,\epsilon,\pi,b) = \max_{d\in\mathscr{F}(s,\pi,b)} d(V^D(s,T,\pi,b,d) + \epsilon^D) + (1-d)V^R(s,T,\epsilon,\pi,b)$$

These let us define update ex ante government values when entering the period in good standing, and the government's choice probabilities:

$$\begin{aligned} \bar{V}_{new}(s,T,\pi,b) &= \mathbb{E}[V(s,T,\epsilon,\pi,b)] \\ p_d(s,T,\pi,b) &= Pr(d^\star(s,T,\epsilon,\pi,b) = 1) \\ p_{b'}(s,T,\pi,b) &= Pr(b'^\star(s,T,\epsilon,\pi,b) = b') \end{aligned}$$

These type-specific choice probabilities then allow us to define the choice probabilities from the point of view of the lenders, an updated one period ahead default probability, and an updated price function $\bar{q}$:

$$\begin{aligned} \bar{p}_d(s,\pi,b) &= \pi p_d(s,H,\pi,b) + (1-\pi)p_d(s,L,\pi,b) \\ \bar{\delta}_{new}(s,\pi',b') &= \mathbb{E}[\bar{p}_d(s',\pi',b')|s] \\ \bar{p}_{b'}(s,\pi,b) &= \pi(1-p_d(s,H,\pi,b))p_{b'}(s,H,\pi,b) + (1-\pi)(1-p_d(s,L,\pi,b))p_{b'}(s,L,\pi,b) \\ \bar{q}_{new}(s,\pi',b') &= \frac{1}{R}\mathbb{E}[\bar{p}_d(s',\pi',b')q^D(s',\pi',b',1) + (1-\bar{p}_d(s',\pi',b'))(\lambda+\kappa) \\ &\quad + \sum_{b''\in\mathscr{F}^R(s',\pi',b')} \bar{p}_{b''}(s',\pi',b')q(s',\hat{\pi}'',b'')|s] \\ \hat{\pi}'' &= \Gamma(\Gamma^R(b'',\pi'|s',b')) \end{aligned}$$

This completes the definitions of new versions of the non-belief update components of X, i.e. $\bar{V}, \bar{V}^D, \bar{q}, \bar{q}^D, \bar{\delta}$. Now, I use the choice probabilities to derive new versions of the belief update functions.

Whenever a choice is not feasible, set the value of the new belief update function to 1 (i.e. certainty that the government is the high type). This assignment rule will be crucial when establishing that the operator T is continuous at points in the space $\mathscr{X}$ where the sets of feasible actions change (i.e. as points where there is at least one consumption value which is exactly 0, so infinitesimal adjustments to the various objects which define consumption may result in that consumption value becoming feasible). The intuition for why this will work proceeds as follows. The high type is the one which places relatively less weight on utility from consumption in the current period. As a choice becomes close to being infeasible, the consumption associated with it gets very close to zero, and the value associated with the choice becomes arbitrarily negative. However, since the high type places less weight on that large negative utility from consumption in the current period, the gap in value for the high type between its optimal choice and the barely feasible grows more slowly than the same difference does for the low type. Therefore, although both choose this barely feasible choice very rarely, the high type chooses it infinitely more often than the low type as the choice becomes infeasible. This intuition will be borne out more rigorously in the main proof of this section. For choices which are feasible, set Γ^R_{new} and Γ^D by:

$$\Gamma^R_{new}(b', \pi | s, b) = \frac{\pi(1 - p_d(s, H, \pi, b))p_{b'}(s, H, \pi, b)}{\bar{p}_{b'}(s, \pi, b)}$$
$$\Gamma^D_{new}(\pi | s, b) = \frac{\pi p_d(s, H, \pi, b)}{\bar{p}_d(s, \pi, b)}$$

Now, set the type specific probabilities of following certain choice paths when in/entering default by:

$$P_G^D(s,T,\pi,b,d,Q) = \begin{cases} p_G^D(s,T,\pi,b,d,Q) & d=0 \\ p_d(s,T,\pi,b)p_G^D(s,T,\pi,b,d,Q) & d=1 \end{cases}$$

$$P_L^D(s,T,\pi,b,d,Q) = \begin{cases} p_L^D(s,T,\pi,b,d,Q) & d=0 \\ p_d(s,T,\pi,b)(1-\bar{A}_G(s,T,\pi,b,d,Q)) & d=1 \end{cases}$$

$$P_G^{RS}(s,T,\pi,b,d,Q) = \begin{cases} p_G^D(s,T,\pi,b,d,Q)p_G^{RS}(s,T,\pi,b,d,Q,b') & d=0 \\ p_d(s,T,\pi,b)p_G^D(s,T,\pi,b,d,Q)p_G^{RS}(s,T,\pi,b,d,Q,b') & d=1 \end{cases}$$

$$P_L^{RS}(s,T,\pi,b,d,Q) = \begin{cases} p_L^D(s,T,\pi,b,d,Q)p_L^{RS}(s,T,\pi,b,d,Q,b') & d=0 \\ p_d(s,T,\pi,b)\bar{A}_G(s,T,\pi,b,d,Q)p_L^{RS}(s,T,\pi,b,d,Q,b') & d=1 \end{cases}$$

Then, at every feasible choice path, I set $\Gamma^D_{G,new}, \Gamma^D_{L,new}, \Gamma^{RS}_{G,new}, \Gamma^{RS}_{L,new}$ by:

$$\Gamma^D_{G,New}(d,Q,\pi|s,b) = \frac{\pi P_G^D(s,H,\pi,b,d,Q)}{\pi P_G^D(s,H,\pi,b,d,Q)+(1-\pi)P_G^D(s,L,\pi,b,d,Q)}$$

$$\Gamma^D_{L,New}(d,Q,\pi|s,b) = \frac{\pi P_L^D(s,H,\pi,b,d,Q)}{\pi P_L^D(s,H,\pi,b,d,Q)+(1-\pi)P_L^D(s,L,\pi,b,d,Q)}$$

$$\Gamma^{RS}_{G,New}(d,Q,b',\pi|s,b) = \frac{\pi P_G^{RS}(s,H,\pi,b,d,Q,b')}{\pi P_G^{RS}(s,H,\pi,b,d,Q,b')+(1-\pi)P_G^{RS}(s,L,\pi,b,d,Q,b')}$$

$$\Gamma^{RS}_{L,New}(d,Q,b',\pi|s,b) = \frac{\pi P_L^{RS}(s,H,\pi,b,d,Q,b')}{\pi P_L^{RS}(s,H,\pi,b,d,Q,b')+(1-\pi)P_L^{RS}(s,L,\pi,b,d,Q,b')}$$

This completes the definition of the operator T. Next, I establish a key property of the consumption functions in this environment, before moving on to the main proof of this section.

Lemma 1. ***Continuity of consumption functions:*** *Let assumptions 4 and 5 hold. Then the values of consumption are continuous in X.*

Proof: Since products of continuous functions and compositions of continuous functions are also continuous, this follows immediately from the assumption that $g(\pi'|\hat{\pi}')$ is continuous in $\hat{\pi}'$ and $\hat{i}(\bar{\delta})$ is continuous.

Before I begin the main proof of this section, I state some definitions for choice probabilities and ex ante values when preference shocks are distributed Generalized Type One Extreme Value or Type One Extreme Value. For Type One Extreme Value shocks, when there is a set of choices $i \in \{1, ..., N\}$ with associated choice values $V_i + e_i$, Dvorkin et al. (2021) show that ex ante values are given by:

$$\bar{V} = E[V_i + e_i] = V_{max} + \sigma log\left(\sum_{j=1}^{N} exp\left(\frac{V_j - V_{max}}{\sigma}\right)\right)$$

and choice probabilities are given by:

$$p_i = \frac{exp\left(\frac{V_i - V_{max}}{\sigma}\right)}{\sum_{j=1}^{N} exp\left(\frac{V_j - V_{max}}{\sigma}\right)}$$

where $V_{max} = \max_{iin\{1,...,N\}} V_i\}$. Note that I have defined these choice probabilities in order to require that denominator be at least 1 (it must be that $V_j - V_{max} = 0$ for at least one element j).

For Generalized Type One Extreme Value shocks, when there is a binary choice $d \in \{0, 1\}$ and a set of choices $i \in \{1, ..., N\}$ which are made when $d = 0$, with associated choice values $V^D + e^D$ and $V_i^R + e_i^R$, respectively, Dvorkin et al. (2021) show that ex ante values are given by:

$$\begin{aligned}\bar{V} = E[V_i + e_i] = V_{max} + \sigma log\Bigg(& exp\left(\frac{V^D - V_{max}}{\sigma}\right) \\ & + exp\left(\frac{V_{max}^R - V_{max}}{\sigma}\right)\left(\sum_{j=1}^{N} exp\left(\frac{V_j^R - V_{max}^R}{\sigma\rho}\right)\right)^{\rho}\Bigg)\end{aligned}$$

and choice probabilities are given by:

$$p^D = \frac{exp\left(\frac{V^D - V_{max}}{\sigma}\right)}{exp\left(\frac{V^D - V_{max}}{\sigma}\right) + exp\left(\frac{V^R_{max} - V_{max}}{\sigma}\right)\left(\sum_{j=1}^N exp\left(\frac{V^R_j - V^R_{max}}{\sigma\rho}\right)\right)^\rho}$$

$$p^R_i = (1 - p^D)\frac{exp\left(\frac{V^R_i - V^R_{max}}{\sigma\rho}\right)}{\sum_{j=1}^N exp\left(\frac{V^R_j - V^R_{max}}{\sigma\rho}\right)}$$

where $V^R_{max} = \max_{i\in\{1,\dots,N\}} V^R_i$ and $V_{max} = \max\{V^D, V^R_{max}\}$. The fact that Type One Extreme Value shocks (and generalized ones) lead to these types of expressions for ex ante values and choice probabilities will be extremely useful in what follows. Note here that in both cases, ex ante values and choice probabilities are continuous in the choice values V_i (or $V^D, \{V^R_i\}_{i\in\{1,\dots,N\}}$). Furthermore, note that when I take the limit as one of those values V_k goes to negative infinity, the ex ante values and choice probabilities converge to exactly their values when choice k is removed from the choice set. I now proceed to the proof of the main theorem of this section.

Theorem 2. ***Equilibrium Existence:*** *Suppose that assumptions 1, 2, 3, 4, 5, and 6. Then there exists $X \in \mathscr{X}$ such that X=T(X).*

Proof: Since T maps a compact, convex set into itself, it will be sufficient to establish that T is continuous. First, I show that this is the case for the pieces of $T(X)$ which are not belief updates, i.e. only $\bar{V}_{new}, \bar{V}^D_{new}, \bar{q}_{new}, \bar{q}^D_{new}, \bar{\delta}_{new}$.

As noted above, whenever choice sequences become infeasible, the probability that they are chosen converges to 0 and their influence on the ex ante value of the agent making the choice also converges to 0. Since the consumption functions are continuous in X and the various expected values taken using $\hat{\mathbb{E}}$ are continuous, all the various individual choice values are continuous whenever they are feasible. Furthermore, since the ex ante values and choice probabilities are continuous at points where a choice sequence becomes infeasible, they are then continuous everywhere. Therefore $\bar{V}_{new}, \bar{V}^D_{new}, \bar{q}_{new}, \bar{q}^D_{new}, \bar{\delta}_{new}$ are continuous in X.

Establishing that the belief update functions are also continuous everywhere is a little more difficult. At points in $\mathscr{X}$ where the feasible sets of choices are invariant to small perturbations, the exact same logic invoked above holds. In particular, if there is no choice sequence which has associated consumption function value equal to exactly 0, then I can require that X' be close enough to X that every feasible set always remain the same. This is possible because the consumption values can be uniformly bounded away from zero (either above or below) due to their being only finitely many of them. Given that the feasible set is fixed, there are no choice sequences which, for X, are assigned using Bayes Law but for X' are infeasible and therefore have the posterior belief associated with them update to 1 (the reverse case is also impossible). Since both of these update rules are continuous functions of the consumption whenever consumption is bounded away from 0, T must be continuous at such points in $\mathscr{X}$

The only real difficulty is posed by points in $\mathscr{X}$ where the one of the feasible sets of choices changes. It is not obvious that, at such points, the mapping of old belief update to new belief update will be continuous. These points in $\mathscr{X}$ are those where at least one of the values taken by the consumption functions c, C_G^{RS}, c_L^{RS} is exactly 0. Moving towards such a point along a path with that consumption value strictly greater than 0, the belief update is defined as $\frac{\pi p_{i,H}}{\pi p_{i,H}+(1-\pi)p_{i,L}}$ and both the numerator and the denominator converge to 0. In order to show that the new belief update functions defined by T are continuous in X, I must show that in every case where this can happen, the mapping to the new belief updates is continuous.

Here, there are four possible cases:

1. There is one choice b' which has $c(s,\pi,b,b'|X) = 0$ and $\mathscr{F}^R(s,\pi,b|X) \neq \emptyset$.

2. There is at least one choice b' which has $c(s,\pi,b,b'|X) = 0$ and $\mathscr{F}^R(s,\pi,b|X) = \emptyset$.

3. There is one choice b' which has $c_X^{RS}(s,\pi,b,d,Q,b'|X) = 0$ and $\mathscr{F}_X^{RS}(s,\pi,b,d,Q,b'|X) \neq \emptyset$ for $X \in \{G, L\}$.

4. There is at least one choice b' which has $c_X^{RS}(s,\pi,b,d,Q,b'|X) = 0$ and $\mathscr{F}_X^{RS}(s,\pi,b,d,Q,b'|X) = \emptyset$ for $X \in \{G, L\}$.

The first and third cases are very similar, and the second and fourth cases are very similar. For this reason, I first consider cases 1 and 3 before moving on to cases 2 and 4.

I begin with case 1. Fix (s, π, b) and the name of the choice i which is just infeasible (had $c = 0$) at X_0. And let X' be a point in $\mathscr{X}$ at which choice i is feasible, and suppose that such points X' occur arbitrarily close X_0 (otherwise, continuity holds since the feasible set remains constant in some small enough neighborhood of X_0). In case 1, the choice set for repayment is nonempty at X_0. Therefore, I can uniformly bound away from $-\infty$ the values associated with choices that are feasible at X_0. Let $W > -\infty$ be such a bound. The value of default is bounded by the V^{min} and V^{max}. For any X', let $V_i^R(T)$ be the value to type T when it chooses choice i. The posterior likelihood ratio when i is chosen is given by:

$$\frac{\pi^{post}}{1 - \pi^{post}} = \frac{\pi}{1 - \pi} \frac{p_i^R(H)}{p_i^R(L)}$$

i.e. the posterior likelihood ratio is equal to the prior likelihood ratio multiplied by the action likelihood ratio. In order to prove that the assignment rule for infeasible choice sequences that sets the posterior to 1 yields a continuous mapping from X to the new belief update function, I must show that for X' close enough to X_0, the action likelihood ratio can be made arbitrarily large. The action likelihood ratio can be written as:

$$\frac{p_i^R(H)}{p_i^R(L)} = \frac{1 - p^D(H)}{1 - p^D(L)} * \frac{\sum_{j=1}^N exp\left(\frac{V_j^R(L) - V_{max}^R(L)}{\sigma\rho}\right)}{\sum_{j=1}^N exp\left(\frac{V_j^R(H) - V_{max}^R(H)}{\sigma\rho}\right)} * \frac{exp\left(\frac{V_i^R(H) - V_{max}^R(H)}{\sigma\rho}\right)}{exp\left(\frac{V_i^R(L) - V_{max}^R(L)}{\sigma\rho}\right)}$$

Since V^D and the values of other feasible choices are continuous in X, the ratio of repayment probabilities can be uniformly bounded. Both the numerator and the denominator of the second piece (the summations), are bounded below by 1 and above by the number of possibly feasible choices N_B, so their ratio cannot lie outside $[\frac{1}{N_B}, N_B]$. Therefore, it is sufficient to show that the final piece becomes unbounded as X' gets close to X. Since the exponential is a strictly increasing function, I apply this principle to its argument:

$$\frac{V_i^R(H) - V_i^R(L) - (V_{max}^R(H) - V_{max}^R(L))}{\sigma\rho}$$

Again, since the values of all choices besides i can be uniformly bounded below by W and above by V^{max}, the only term of interest is the difference in values across types at choice i

$(V_i^R(H) - V_i^R(L))$. This can be written as:

$$\frac{V_i^R(H) - V_i^R(L)}{\sigma\rho} = \frac{((1-\beta_H)u(c_i) + \beta_H EV_i(H)) - ((1-\beta_L)u(c_i) + \beta_L EV_i(L))}{\sigma\rho}$$
$$= \frac{(\beta_L - \beta_H)u(c_i) - (\beta_L EV_i(L) - \beta_H EV_i(T))}{\sigma\rho}$$

Since $\bar{V}$ is uniformly bounded, the continuation value terms are uniformly bounded. Since consumption is continuous in X and utility is continuous in consumption and has $\lim_{c\downarrow 0} u(c) = -\infty$, the $u(c_i)$ term can be made to be a negative number of arbitrary magnitude. Since $\beta_L - \beta_H < 0$, this means that their product can be made arbitrarily large. Therefore, the distance between X' and X_0 can be chosen such that the posterior belief at choice i is arbitrarily close to 1. So in case 1, T is continuous. Note that specifying that only a single choice had been just infeasible was not in fact a restriction. The only points at which such a value would have entered this proof are in the default likelihood ratio, and the ratio of the denominators of the repayment choice probabilities, both of which can be uniformly bounded to begin with.

The proof for case 3 (the analogue when a single restructuring choice is just infeasible and the feasible set at X_0 was nonempty) is almost identical (the bounded terms of the action likelihood ratio are different). In that case, when $d = 1$, the action likelihood ratio contains the ratio of default probabilities, the ratio of probabilities that a given Q is offered or accepted, and the ratio of probabilities that b' is chosen. When $d = 0$, it just contains the last two of these ratios. Near X_0, the ratio of default probabilities can be bounded because the default probabilities themselves are both continuous in X. Furthermore, since the ex ante values of restructuring at the Q in question are continuous in X, the ratio of probabilities that this Q is chosen are continuous in X. Therefore that piece can be bounded. This leaves us with just the ratio of probabilities that a given b' value is chosen during the restructuring process:

$$\frac{exp\left(\frac{V_i^{RS}(H) - V_{max}^{RS}(H)}{\sigma^{RS}}\right)}{exp\left(\frac{V_i^{RS}(L) - V_{max}^{RS}(L)}{\sigma^{RS}}\right)} \frac{\sum_{j=1}^N exp\left(\frac{V_j^{RS}(L) - V_{max}^{RS}(L)}{\sigma^{RS}}\right)}{\sum_{j=1}^N exp\left(\frac{V_j^{RS}(H) - V_{max}^{RS}(H)}{\sigma^{RS}}\right)}$$

As before, the second piece of this ratio (the one involving summations) can be uniformly bounded. Combining the remaining two and then removing the exponential leads to:

$$\frac{V_i^{RS}(H) - V_i^{RS}(L) - (V_{max}^{RS}(H) - V_{max}^{RS}(L))}{\sigma^{RS}}$$

As before, the $V_{max}^{RS}(T)$ terms can be bounded since there are feasible choices at X_0. Substituting for the remaining terms then leads us to a familiar difference:

$$\frac{V_i^{RS}(H) - V_i^{RS}(L)}{\sigma^{RS}} = \frac{(\beta_L - \beta_H)u(c_i) - (\beta_L EV_i(L) - \beta_H EV_i(T))}{\sigma^{RS}}$$

Again, $u(c_i)$ is the only term which is unbounded as X' gets close to X_0 and c_i becomes arbitrarily close to 0. Since it is multiplied by a negative number, this term therefore becomes arbitrarily large as as X' gets close to X_0, meaning that the posterior belief becomes arbitrarily close to 1. Therefore, in case 3, T is continuous.

Now I move to case 2. Fix (s, π, b) and the name of the choice i which is just infeasible (had $c = 0$) at X_0. In this case, at X_0, there were no feasible choices in this state. Set $\hat{V}(T)$ by:

$$\hat{V}^R(T) = \sum_{j=1}^{N} exp\left(\frac{V_j^R(T) - V_{max}^R(T)}{\sigma\rho}\right)$$

When I write the action likelihood ratio now, I need to be more careful with the ratio of repayment probabilities, since at X_0, default occurs with certainty (so the ratio becomes $\frac{0}{0}$). This ratio is:

$$\frac{1 - p^D(H)}{1 - p^D(L)} = \frac{exp\left(\frac{V_{max}^R(H) - V_{max}(H)}{\sigma}\right)\hat{V}^R(H)^\rho}{exp\left(\frac{V_{max}^R(L) - V_{max}(L)}{\sigma}\right)\hat{V}^R(L)^\rho} * \frac{exp\left(\frac{V^D(L) - V_{max}(L)}{\sigma}\right) + exp\left(\frac{V_{max}^R(L) - V_{max}(L)}{\sigma}\right)\hat{V}^R(L)^\rho}{exp\left(\frac{V^D(H) - V_{max}(H)}{\sigma}\right) + exp\left(\frac{V_{max}^R(H) - V_{max}(H)}{\sigma}\right)\hat{V}^R(H)^\rho}$$

Since, in this case, I know that for X' close enough to X_0, the value of default will be the largest of the fundamental choice values for both types, this can be simplified to:

$$\frac{1 - p^D(H)}{1 - p^D(L)} = \frac{exp\left(\frac{V_{max}^R(H) - V^D(H)}{\sigma}\right)}{exp\left(\frac{V_{max}^R(L) - V^D(L)}{\sigma}\right)} * \frac{\hat{V}^R(L)^{-\rho} + exp\left(\frac{V_{max}^R(L) - V_{max}(L)}{\sigma}\right)}{\hat{V}^R(H)^{-\rho} + exp\left(\frac{V_{max}^R(H) - V_{max}(H)}{\sigma}\right)}$$

The second part of this can be uniformly bounded. The first part, however, cannot necessarily be uniformly bounded, because the $V^R_{max}(T)$ will by assumption become arbitrarily large negative numbers as X' gets close to X_0. Combine this with the problematic piece of the action likelihood ratio considered in the previous case to obtain:

$$\frac{exp\left(\frac{V^R_{max}(H)-V^D(H)}{\sigma}\right)}{exp\left(\frac{V^R_{max}(L)-V^D(L)}{\sigma}\right)} * \frac{exp\left(\frac{V^R_i(H)-V^R_{max}(H)}{\sigma\rho}\right)}{exp\left(\frac{V^R_i(L)-V^R_{max}(L)}{\sigma\rho}\right)}$$

or, more compactly:

$$exp\left(\frac{V^D(L)-V^D(H)}{\sigma} + \left(1-\frac{1}{\rho}\right)\frac{V^R_{max}(H)-V^R_{max}(L)}{\sigma} + \frac{V^R_i(H)-V^R_i(L)}{\sigma\rho}\right)$$

The V^D terms, as well as the continuation value terms inside the V^R's are uniformly bounded and therefore will not play any further role. Removing the exponential and multiplying through by $\sigma * \rho$, the piece of interest is then:

$$\begin{aligned}
&(\rho-1)\Bigg((1-\beta_H)u(c^\star(H)) - (1-\beta_L)u(c^\star(L))\Bigg) + (\beta_L-\beta_H)u(c_i) \\
&= (\rho-1)\Bigg((1-\beta_H)\Big(u(c^\star(H)) - u(c^\star(L))\Big) - (\beta_H-\beta_L)u(c^\star(L))\Bigg) + (\beta_L-\beta_H)u(c_i) \\
&= (\rho-1)(1-\beta_H)\Big(u(c^\star(H)) - u(c^\star(L))\Big) + (\beta_L-\beta_H)\Big(u(c_i) - u(c^\star(L))\Big) + \rho(\beta_L-\beta_H)u(c^\star(L))
\end{aligned}$$

To complete the proof, I now need merely show that the difference in utility terms are bounded. Although it is not necessary, I will provide uniform bounds for both. Suppose that $c^\star(T)$ is the highest value choice for type T. Then for any alternate choice yielding consumption c^{alt} and continuation value $EV^{alt}(T)$:

$$\begin{aligned}
(1-\beta_T)u(c^\star(T)) + \beta_T EV^\star(T) &\geq (1-\beta_T)u(c^{alt}(T)) + \beta_T EV^{alt}(T) \\
\frac{\beta_T}{1-\beta_T}(EV^\star(T) - EV^{alt}(T)) &\geq u(c^{alt}(T)) - u(c^\star(T))
\end{aligned}$$

Since the expected value terms are uniformly bounded, this provides an explicit upper bound on difference between flow utilities from two choices when one of those two choices is the highest value choice for at least one of the types. Thus the term:

$$(\rho-1)(1-\beta_H)\Big(u(c^\star(H))-u(c^\star(L))\Big)$$

can be uniformly bounded both above and below. Furthermore, since $u(c_i)-u(c^\star(L))$ can be bounded above and $(\beta_L-\beta_H)<0$, the term:

$$(\beta_L-\beta_H)\Big(u(c_i)-u(c^\star(L))\Big)$$

can be uniformly bounded below. The only remaining term is then:

$$\rho(\beta_L-\beta_H)u(c^\star(L))$$

As was the case before, this term becomes unboundedly large as X' gets close to X_0 and the largest possible consumption value converges to 0. Therefore, the distance between X' and X_0 can be chosen such that the posterior belief at choice i is arbitrarily close to 1. So in case 3, T is continuous. The proof for case 4 is almost identical.

In case 4, the action likelihood ratio contains some different pieces. If $d=1$, it contains the ratio of default probabilities. Since the default probabilities are continuous, this term can be bounded. Regardless of the value of d, the action likelihood ratio contains the probability of the government offering (or accepting) choice Q, and the probability that it then chooses b' during the restructuring process. These are the probabilities of interest. Labeling the Q choice which is just infeasible at X_0 i and one of the just infeasible b' choices k, the likelihood ratio for choosing Q_i is:

$$\frac{exp\left(\frac{\bar{V}_i^{RN}(H)-\bar{V}_{max}^{RN}(H)}{\sigma^{RN}}\right)}{exp\left(\frac{\bar{V}_i^{RN}(L)-\bar{V}_{max}^{RN}(L)}{\sigma^{RN}}\right)}\frac{\sum_{j=1}^{N}exp\left(\frac{\bar{V}_j^{RN}(H)-\bar{V}_{max}^{RN}(L)}{\sigma^{RN}}\right)}{\sum_{j=1}^{N}exp\left(\frac{\bar{V}_j^{RN}(H)-\bar{V}_{max}^{RN}(H)}{\sigma^{RN}}\right)}$$

where $\bar{V}_i^{RN}(T)$ is the ex ante value of proposing or accepting Q_i, and the ex ante value of the highest deal is $\bar{V}_{max}^{RN}(T)$. The second ratio (the summations) can be uniformly bounded, as usual. Since proposing $Q=0$ and rejecting an offer is always a feasible choice, the

$\bar{V}_{max}^{RN}(T)$ terms are uniformly bounded. So the only term of concern here are the two $\bar{V}_i^{RN}(T)$ values.

If the government is accepting the deal, these are simply this ex ante values of proceeding to the restructuring phase having agreed to deliver Q to every bondholder. If the government is proposing the deal, then these are the ex ante restructuring values multiplied by the probability lenders accept the deal plus the values of remaining in default multiplied by the probability that lenders reject the deal, so in both cases, we can write them as:

$$\bar{V}_i^{RN}(T) = \alpha \bar{V}_i^{RS}(T) + (1-\alpha) V_i^D(T)$$

with $\alpha \in [0,1]$ and $V_i^D(T)$ denoting the value of remaining in default after the government makes its choice and the ex ante value of entering the restructuring process is:

$$\bar{V}_i^{RS}(T) = V_{i,max}^{RS}(T) + \sigma^{RS} log \left(\sum_{l=1}^{M} exp \left(\frac{V_{i,l}^{RS} - V_{i,max}^{RS}}{\sigma^{RS}} \right) \right)$$

As before, the second term can be uniformly bounded. Then the only piece of the renegotiation phase action likelihood ratio which can potentially be unbounded can be written as:

$$exp \left(\alpha \frac{V_{i,max}^{RS}(H) - V_{i,max}^{RS}(L)}{\sigma^{RN}} \right)$$

The action likelihood ratio associated with the restructuring process itself is:

$$\frac{exp\left(\frac{V_{i,k}^{RS}(H) - V_{i,max}^{RS}(H)}{\sigma^{RS}}\right)}{exp\left(\frac{V_{i,k}^{RS}(L) - V_{i,max}^{RS}(L)}{\sigma^{RS}}\right)} \frac{\sum_{l=1}^{M} exp\left(\frac{V_{i,l}^{RS}(L) - V_{i,max}^{RS}(L)}{\sigma^{RS}}\right)}{\sum_{l=1}^{M} exp\left(\frac{V_{i,l}^{RS}(H) - V_{i,max}^{RS}(H)}{\sigma^{RS}}\right)}$$

As has been true throughout this proof, the piece involving summations can be uniformly bounded. This leaves us with:

$$exp \left(\frac{(V_{i,k}^{RS}(H) - V_{i,k}^{RS}(L)) - (V_{i,max}^{RS}(H) - V_{i,max}^{RS}(L))}{\sigma^{RS}} \right)$$

Combining this with the piece from the renegotiation process, removing the exponential, and multiplying through by σ^{RS} then yields:

$$\left(V_{i,k}^{RS}(H) - V_{i,k}^{RS}(L)\right) - \left(1 - \frac{\alpha\sigma^{RS}}{\sigma^{RN}}\right)\left(V_{i,max}^{RS}(H) - V_{i,max}^{RS}(L)\right)$$

I then substitute using the definitions of each of these values (and drop the continuation value terms, since they, again, can be uniformly bounded) to obtain:

$$\Big((1-\beta_H)u(c_{i,k}) - (1-\beta_L)u(c_{i,k})\Big) - \left(1 - \frac{\alpha\sigma^{RS}}{\sigma^{RN}}\right)\Big((1-\beta_H)u(c_i^\star(H)) - (1-\beta_L)u(c_i^\star(L))\Big)$$

Similarly to how things worked in case 2, rearranging terms yields:

$$\begin{aligned}&\Big(\beta_L - \beta_H\Big)\Big(u(c_{i,k}) - u(c_i^\star(L)\Big) - \left(1 - \frac{\alpha\sigma^{RS}}{\sigma^{RN}}\right)(1-\beta_H)\Big(u(c_i^\star(H)) - u(c_i^\star(L))\Big)\\ &+ \frac{\alpha\sigma^{RS}}{\sigma^{RN}}\Big(\beta_L - \beta_H\Big)u(c_i^\star(L))\end{aligned}$$

Since differences of the form $u(c_i^{alt}(T)) - u(c_i^\star(T))$ can be uniformly bounded above in the exact same way they were in case 2, the only term here which can potentially be unbounded is:

$$\frac{\alpha\sigma^{RS}}{\sigma^{RN}}\Big(\beta_L - \beta_H\Big)u(c_i^\star(L))$$

As X' gets arbitrarily close to X_0, all feasible consumption values become arbitrarily close to 0, so the utility from consumption becomes an arbitrarily large negative number. Since it is multiplied by another negative number $\beta_L - \beta_H < 0$, this term becomes arbitrarily large. Therefore, as X' becomes arbitrarily close to X_0, the new posterior belief becomes arbitrarily close to 1, which is exactly its value at X_0. Therefore, in case 4, T is continuous. Note that by proving case 4 and showing that the mapping to $\Gamma_{X,new}^{RS}$ is always continuous, I also show that the mapping to $\Gamma_{G,new}^{RN}$ (used when the government proposes Q but lenders decline the offer) is continuous when some Q is just infeasible at X_0. This follows because the individual action likelihood ratios associated with every choice the government could make after proposing Q all become infinitely large as X' gets close to X_0. Therefore the ratio of the sums of those likelihoods must also become infinitely large, so the posterior belief which occurs just upon seeing Q also converges to 1 as X' becomes arbitarily close to X_0.

Since I have now shown that in all four potentially problematic cases, T is continuous, I have now established that T is a continuous operator mapping a compact, convex subset of a finite dimensional Euclidean Space into itself. Therefore, by Brouwer's Fixed Point Theorem, the operator T has a fixed point, so an equilibrium must exist. This completes the proof.

The definition of T shows how to recover the full set of equilibrium objects described in the main text from a given X. Therefore, the existence of a fixed point of T is equivalent to the existence of an equilibrium, proving Theorem 1.