Optical Lab, Astronomy 120
Lab 4, Doppler Measurement of Solar Rotation

Sameen Yunus
sfsyunus@berkeley.edu
Anthony Khodanian & Pavan Muddukrishna

12/05/2017

Abstract

The aim of this lab was to use solar observing and spectral analysis to compute the Astronomical Unit: the distance between the earth the sun. We took spectral data of the sun as it moved across the optical fiber using the solar spectrometer. We use a neon lamp and a 653nm LED to calibrate the spectrometer. We perform cross correlation of solar spectra in order to find the Doppler shift due to solar rotation. From the Doppler shift we can find the rotational velocity of the sun and knowing the period of the sun, finding $R_\odot$ and AU is fairly straightforward through simple geometric relations. We find $\mathbf{R_\odot}$ = **685569 km** with 1.45% error and **AU** = **145382455 km** with 2.82% error.

Introduction

The Astronomical Unit is an important constant; it forms an astronomical length scale for measuring distances between solar system objects and allows us to determine larger distances in the universe by the method of parallax cause by the earth's rotation around the sun. Therefore calculating the earth-sun distance, or the astronomical unit, is a crucial step in determining the cosmological distance ladder.
In order to determine the AU we first calibrated the spectrometer by taking spectra of a neon lamp 2a and a red LED. We find centroids in specific Echelle orders and compare them to known wavelength peaks of neon to perform a least squares fit for the pixel to wavelength mapping. We also took spectra of the sun as it moves across an optical fiber and get multiple frames for the entire transit. We cross-correlated each frame from the solar transit with respect to the center of the transit which allowed us to determine the Doppler shift of the Sun's spectrum. Using the Doppler shift, we were able to find the rotational velocity of the sun along its equator, and with the known rotational period of the sun; we determined the radius of the sun. Finally, with the radius and angular diameter of the sun we were able to find the astronomical unit.

Data and Apparatus

We acquired solar data on November 28th and used test data from November 20th for our wavelength calibration. Neon, LED and halogen spectra were measured using lamps in the the dark room. Solar spectra were measured by a telescope set up outside 541 NCH with the telescope pointed at the sun and a fiber optic cable that ran from the telescope to the spectrometer. The earth's rotation causes the sun to move across the telescope aperture and we measure the flux from a small region along the diameter of the sun before, during and after the transit. We used an integration time of 1 second so as to get high enough counts for analysis but not saturate the CCD. The average flux from frames before and after the transit were used to measure the scattered light and create a subtraction image for the solar transit images. The transit lasted about 137 seconds; we determine this is the analysis section using the Eddington Approximation.

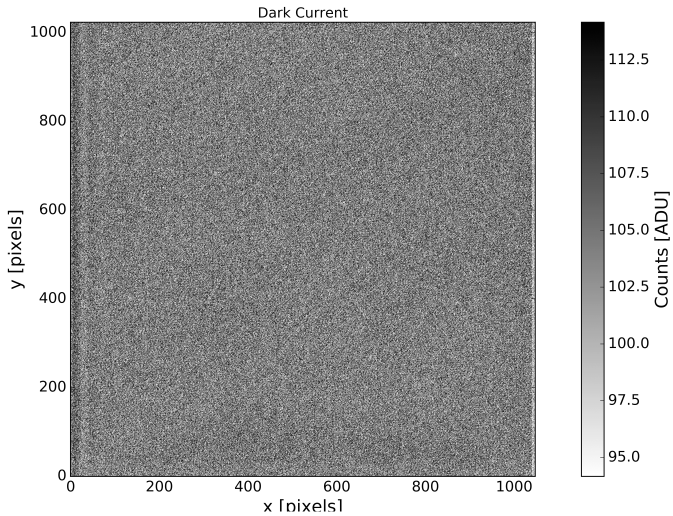


Figure 1: *Averaged dark current for the solar spectrometer. This image is the average of 30 files of 1 second exposures with the shutter closed so that no light falls on the CCD. Dark pixels show a lot of dark current is present while bright pixels show low dark current in the CCD.*

The dark current was measured for all integration times for which we recorded data. Figure 1 shows the dark current for an integration time of 1 second which we used in the subtraction for neon calibration. The average dark current was 104.58 ADU however since it's a non-uniform effect; it wasn't the same for the calibration images and the solar images. One of the image processing steps we did for solar spectra was to subtract the average flux of each pixel from frames before and after the transit. This accounts for both scattered sunlight and the dark current.

The instrument setup involved light falling on a collimating lens from the fiber optic cable; this focused the beam onto a cross-disperser that separates orders and an Echelle grating that provides high spectral resolving power. This results in a two dimensional spectral image that falls onto a 135mm camera and is recorded by a 1048×1024 pixel APOGEE CCD. This results in a CCD image with the spectrum spread over a large number of pixels resulting in a resolution high enough to resolve small Doppler shifts.

The spectral resolving power of the spectrometer, R, describes the ability of the equipment to differentiate between two points of light. This depends on the echelle order and the rest wavelength. Using equations from the handout, we can calculate R for a wavelength of interest. Table 1 summarizes all the variables and their nominal values.

$$R = \frac{\lambda}{\delta\lambda} = \frac{m\lambda}{\sigma\cos(\alpha)\delta\alpha} \qquad m = \frac{2\sigma}{\lambda}\sin\delta\cos\vartheta \qquad R = \frac{2\sin\delta\cos\vartheta}{\cos(\alpha)\delta\alpha} \tag{01}$$

Variable	Description	Value
λ	rest wavelength	$653nm^*$
$\delta\lambda$	Doppler shift	$0.0232nm^*$
$\delta\alpha$	angular diameter of source	0.0005
m	Echelle order	34^*
σ^{-1}	groove density	$8\times10^{-5}nm^{-1}$
α	angle of incidence	75.4°
δ	blaze angle	64.4°
ϑ	half the angle between incident and diffracted beams	11°

Table 1: *Nominal values of the spectrometer and * denotes quantities we calculate based on known values.*

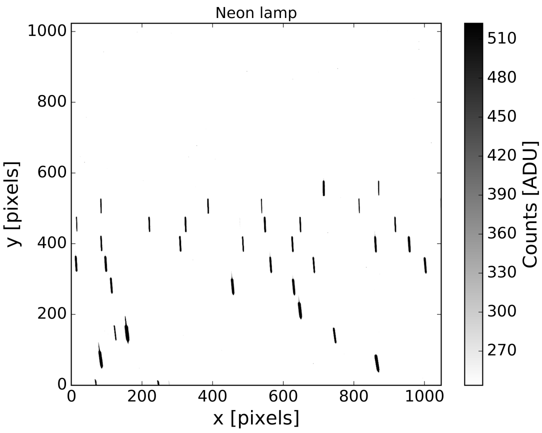
We choose $\lambda = 653$nm as this line in the CCD image of the LED will correspond to the neon peak at the same wavelength, giving us a reference peak for which we know both the pixel and wavelength value. We find that the Echelle order this corresponds to is m = 34 and **R = 14050**. This gives us a velocity resolution $\delta v = c/R$ of about $21 kms^{-1}$ which is too low to resolve the tangential velocity of the sun. However, a high signal to noise ratio and multiple line measurements [1] from the solar spectra return a velocity resolution up to 1000 orders of magnitude lower.
We can also find a nominal dispersion relation; this is the number of wavelengths measured per pixel by assuming $\delta\lambda = \lambda/R$ is the number of wavelengths collected over two pixels and by the Nyquist sampling theorem; we expect the dispersion: $\frac{\delta\lambda}{\delta p} = 0.0232$nm/pixel. Our least squares fit in the neon calibration returns a measured dispersion relation that agrees with this within 8.2 %.

Data Analysis Methods

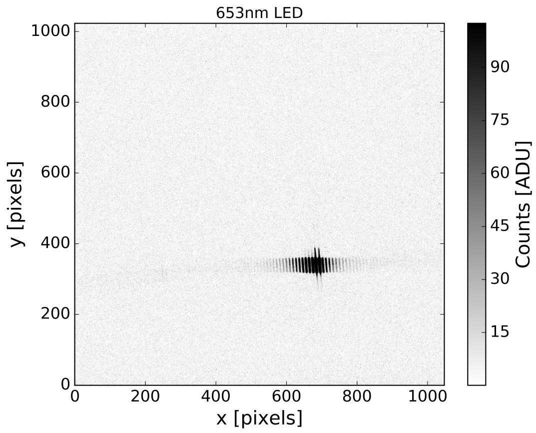
Neon Calibration

The pixel to wavelength calibration we performed in this lab was very similar to that done in lab 2 however, we first have to extract 1D spectra from the 2D CCD images. Figures 2a and 2b demonstrate dark subtracted neon and LED images; we can see the same lines around x = 680 pixels and y = 350 pixels. Both these lines correspond to 653nm which lies in echelle order 34. I used the nominal dispersion value: $\frac{\delta\lambda}{\delta p} = 0.0232$nm/pixel to find what known wavelengths lie in each order however, the analysis focuses on orders 34, 35 and 36 only since they have the most lines in neon and the most useful lines in the solar spectra. In order to extract 1D spectra from the dark subtracted CCD images, we find the Echelle bands in the y pixels by eye. The orders are not aligned perfectly, however we average over about 50-60 pixels for each band so the extracted flux values have low error. Once we have the average flux across each Echelle band, we can then plot the flux versus x pixel values which returns a typical 1D spectrum. We can find the centroids of prominent peaks in each value and compare them to known wavelengths to perform a linear least squares fit.

[1] $\delta v = \frac{c}{SNR\sqrt{M}\,R}$ from page 7; Lab IV: DOPPLER MEASUREMENT of SOLAR ROTATION, James R. Graham November 7, 2017



(a) *Dark subtracted, averaged neon CCD image.*



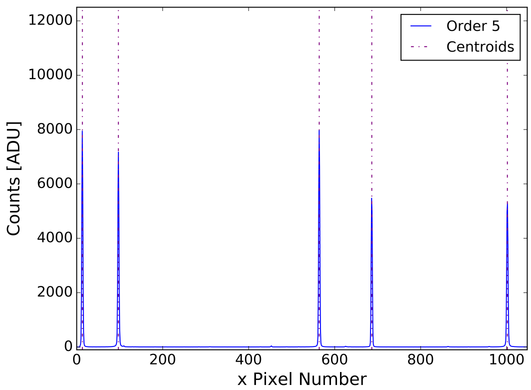
(b) *Dark subtracted, averaged LED CCD image.*

Figure 2: *Neon and LED images, note the 653nm line occurring in both images. Wavelengths increase from left to right and top to bottom.*

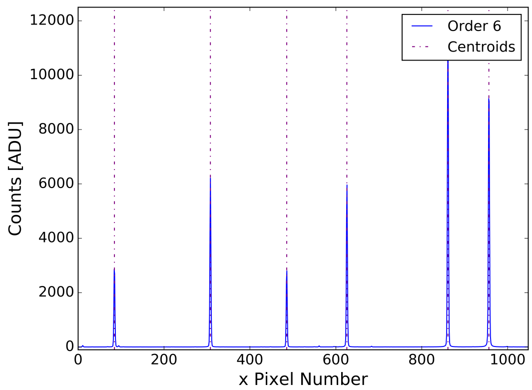
By the method described above; we extract 1D spectra for various orders and calculate the centroids and centroid errors in a 20 pixel threshold around each peak. Here $\langle x \rangle$ is the position of the centroid, x_i is the pixel location of each point used to calculate the centroid, I_i is the corresponding pixel value and $\sigma^2_{\langle x \rangle}$ is the error in each centroid position. We calculated these centroids and errors for each centroid position for the neon spectra and then found the mean of these errors: $\mathbf{\bar{s}^2_{\langle x \rangle} = 0.00245\ pixels^2}$.

$$\langle x \rangle = \sum_i x_i I_i \Big/ \sum_i I_i \qquad \sigma^2_{\langle x \rangle} = \left(\langle x^2 \rangle - \langle x \rangle^2\right) \Big/ \sum_i I_i \tag{02}$$

The 1D spectra extracted from Figure 2a and the calculated centroids are then plotted for orders 34 and 35 in Figures 3a and 3b. We then perform a linear least squares fit of the centroid values and known neon wavelengths to find the pixel to wavelength calibration.



(a) *Echelle order 34 peaks and centroids. The peak around 680 pixels corresponds to the 653nm peak.*



(b) *Echelle order 34 peaks and centroids. This echelle order is used for the solar spectrum analysis.*

Figure 3: *34th and 35th Echelle Order 1D spectra and centroids.*

We perform both a linear and quadratic best fit for the calculated centroids and known wavelengths for m = 34, 35 and 36 however we find that a linear fit is a very good approximation and use that for the pixel to wavelength calibration. The linear fit line and residuals for all three orders are shown in Figure 4. Table 2 summarizes the fit parameters and the residual errors.

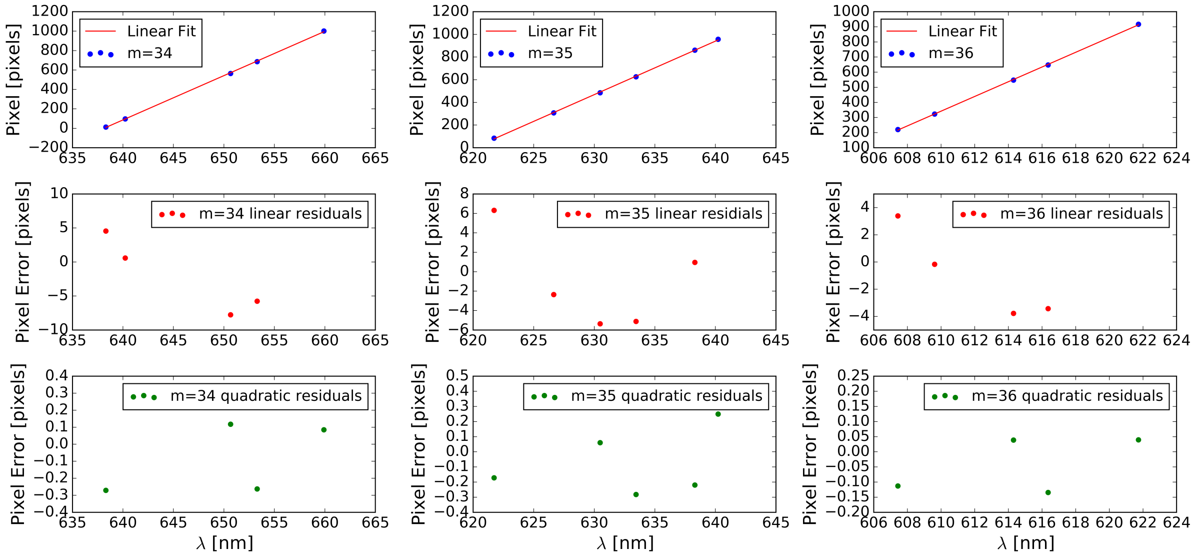


Figure 4: *Linear best fit and fit residuals from Neon I calibration for m = 34, 35, 36*

The independent parameter is the known Neon wavelengths in nm and the dependent parameter are the measured centroids we calculated previously. The inverse slope of the linear fit returns the dispersion. We use the 35th order for the solar spectra analysis and calculation of $R_\odot$ and AU; therefore our measured dispersion is: $\frac{\delta\lambda}{\delta p} = 0.02118$nm/pixel.

Order	m [pix/nm]	c [pix]	a2 [pix/nm^2]	a1 [pix/nm]	a0 [pix]
34	45.59 ± 0.055	-29097 ± 1653	0.129 ± 0.0024	-121.28 ± 3.21	25005 ± 953
35	47.18 ± 0.041	-29258 ± 1589	0.133 ± 0.0034	-120.72 ± 4.36	23728 ± 1056
36	48.68 ± 0.042	-29354 ± 1577	0.148 ± 0.0025	-132.78 ± 3.10	26392 ± 1376

Table 2: *Linear and quadratic best fit parameters using neon calibration peaks.*

Solar Spectra

Once we've performed our pixel to wavelength calibration; we can begin analyzing our solar data. Figure 5 shows a raw CCD frame from the center of the transit when the flux from the sun is at its brightest. We can observe some common spectral features: the strong H−α line in the order 5th from the bottom and the sodium doublet in the 8th order from the bottom. We can also see the sodium doublet repeated in the next order; this is a good example of the overlapping orders at bluer wavelengths. There are also strong peaks from atmospheric H_20 and O_2 in the 2nd and 3rd orders. These lines show no Doppler shift since they're within the earth's atmosphere. The H−α order, m = 34, is not ideal either because the relative flux of the absorption line is much deeper than the other features. This returns an inaccurate cross-correlation result. We choose Echelle order 35 where there are plenty of solar absorption features with close to the same relative flux and also a lot of neon lines for a good calibration.

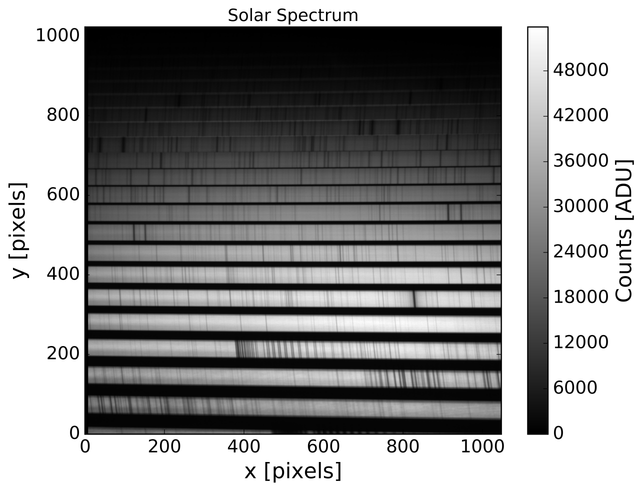
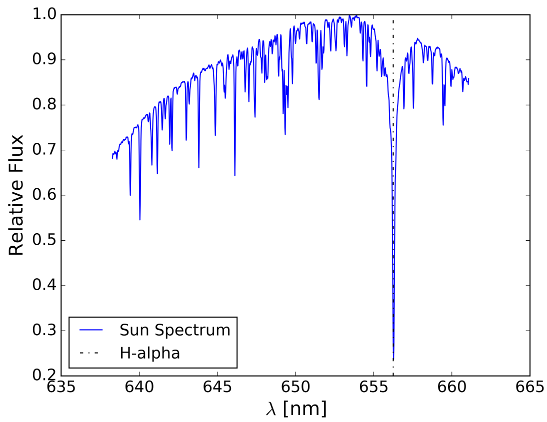


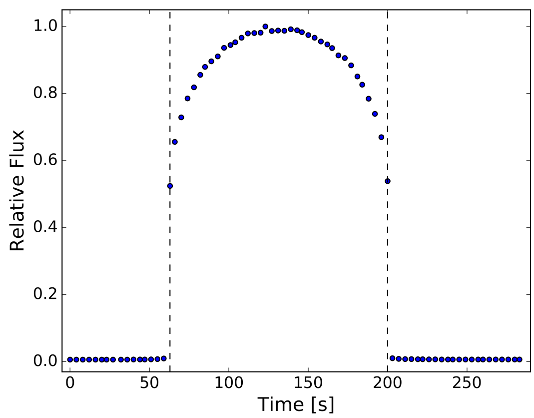
Figure 5: *Solar spectrum CCD Image, note the strong H-alpha line and sodium doublet lines. Dark lines show high intensities and bright areas are low intensities.*

In order to remove unnecessary noise from the solar images; we take the average of all frames before and after the transit which include dark current and scattered light falling onto the optical fiber; and we subtract this average frame from all solar frames before extracting spectra. In addition to subtracting this average frame; we also divide each spectrum by it's maximum therefore we get all of our spectra plotted with relative flux instead of absolute flux. This makes the cross-correlation simpler and makes the figures look better.

In order to reassure ourselves that the pixel to wavelength is accurate; we plot Figure 6a which shows the 34th solar Echelle order containing the $H-\alpha$ peak. The vertical line is plotted at 656.28 nm which lines up with the peak. This spectrum has been noise subtracted and normalized however, we perform some additional image processing before performing the cross-correlation.



(a) *m = 34 order, we can see the strong* $H-\alpha$ *line and reassure ourselves that the calibration is accurate*



(b) *Solar transit time series image, we can observe the limb darkening effect and use this plot to find the transit time.* $\Delta t = 137s$

Figure 6: *Example 1D solar spectra and time series of transit.*

In Figure 6b we plot the averaged flux from each frame versus the time in seconds from the beginning of the transit (from the fits header). This plot results from the Eddington Approximation where I is the measured incident intensity, I_0 is the maximum solar intensity and ϑ is the angle between the line of sight and the surface of the sun:

$$I = \frac{I_0}{5}(2 + 3\cos\vartheta) \tag{03}$$

We can see when $\vartheta = 0°$ the incident intensity is the observed maximum. Additionally from this figure we can obtain the duration of the transit where is begins when the curve starts to rise and ends when the curve falls. We find $\Delta t = 137 \pm 0.2$seconds, the error arises from the fact that there is an average time of 0.2 seconds between each frame.

Cross-correlation

Before performing the cross-correlation; we must perform some additional processing on the images. We subtract the mean of each spectra from the spectra because we expect them to line up around the same intensities. We also fit a second order polynomial to the spectra and subtract the fit from the spectra so that the data are centered at intensity 0. We also apply a Hanning window function, which applies a cosine window to the spectrum because we don't want the edge effects of the spectrometer to add error to our cross correlation. We can see the processed spectra in Figure 7 where we plot a spectrum in the 35th Echelle order from the left limb of the sun and the center of the sun before and after processing. The sub-pixel shift is difficult to observe with the naked eye however when I zoomed in to the image on several peaks; I could see the offsets caused by the rotation of the sun.

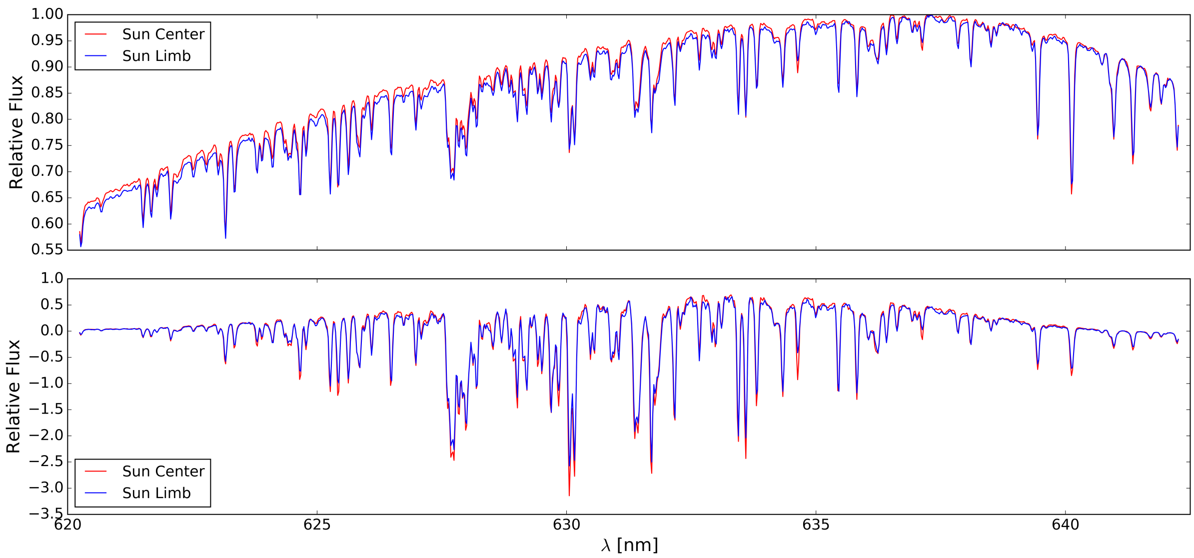
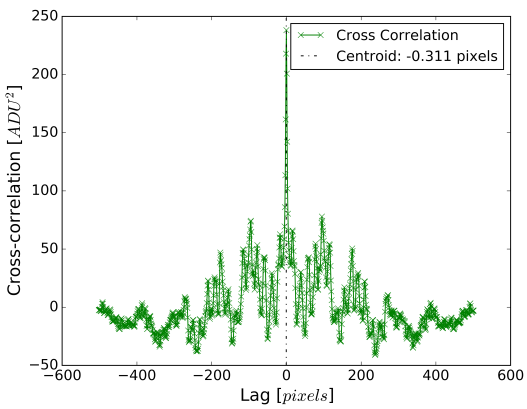


Figure 7: *Spectra from solar limb and center before and after image processing. We can see both spectra are centered at zero and weighted heavily in the middle.*

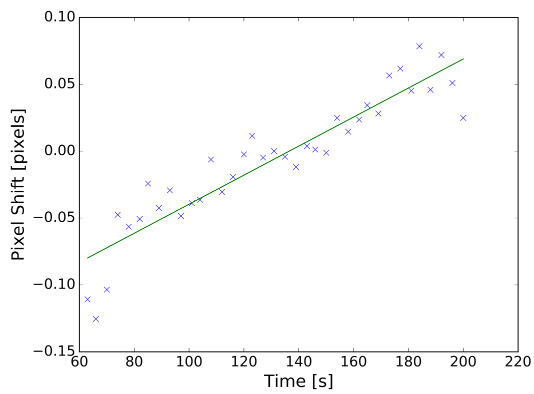
With our spectra ready for cross-correlation; we can use the *Python Numpy* Library to find the cross-correlation peaks. The algorithm adopts the following equation:

$$s_j = \frac{1}{N-1}\sum_i \left(x_i\, y_{i+j}\right) - \frac{N}{N-1}\left(\bar{x}\bar{y}\right) \tag{04}$$

Here, x is the reference data set, y is the data set to be correlated, N is the number of points and $\bar{x}$, $\bar{y}$ and the respective means of the data. We perform the cross-correlation around several peaks, but we focus on the peak at 548 pixels or 631 nm close to the center of the spectrum. We plot the cross correlation peak in Figure 8a and as expected, observe a sharp peak around 0. However this peak is shifted slightly from 0 which results in the pixel offset. In order to find the Doppler shift; we perform the cross correlation for all frames in the solar transit with relation to the central frame. And we then find the centroids and errors of each of the cross-correlation peaks. We can extract the time of the observations from the fits file headers and we plot the pixel shifts relative to the center of the sun versus time to get the Doppler shift in Figure 8b. The figure looks exactly as expected, we see negative offsets at the beginning of the transit as the spectrum is blueshifted since the left solar limb is moving towards us. The data crosses through zero and then shows some positive pixel offsets as the spectra are redshifted towards the right solar limb. We measure the slope of the pixel shift plot which returns $\frac{\delta p}{\delta t} = 0.00109$ however this is in units of pixels per second and we need the Doppler shift in units of nm - we find this in the next section.



(a) *Cross correlation peak for Echelle order 35, centered at 631nm*



(b) *Pixel offsets are used to calculate the Doppler shift.*

Figure 8: *Cross correlation results and pixel offsets*

Measuring $R_\odot$ and the AU

The radial Doppler shift is given by the following equation:

$$\frac{v_r}{c} = \frac{\Delta\lambda}{\lambda_0} \qquad (05)$$

Where v_r is the radial velocity, c is the speed of light, $\Delta\lambda$ is the Doppler shift and λ_0 is the rest wavelength. However we must correct for the fact that our Doppler shift is measured in pixels per second and that the sun's axis is tilted relative to the earth. Accounting for these corrections we find;

$$v_{rot} = \frac{\frac{\delta p}{\delta t}\frac{\delta\lambda}{\delta p}\Delta t}{\lambda_0 \cos\xi \cos\eta} c \qquad (06)$$

Where $\frac{\delta p}{\delta t}$ is the slope from the pixel shift plot in Figure 8b, $\frac{\delta\lambda}{\delta p}$ is the dispersion found from the slope of the least squares fit of the Neon wavelengths, Δt is the transit time, ξ is the tilt of the solar spin axis from the ecliptic, and η accounts for the tilt of the solar spin axis from the N/S axis. We use ephemeris values from JPL Horizons for November 28th 2017 and calculated values summarized in Table 3. We get a value for $\mathbf{v_{rot}}$ = **1.8937 $\mathbf{kms^{-1}}$** with a 5.17% percent error from the true value of 1.997 kms^{-1}.

Variable	Description	Value
$\frac{\delta p}{\delta t}$	pixel offset	0.001086 pix/sec
$\frac{\delta\lambda}{\delta p}$	dispersion	0.02118 nm/pixel
Δt	transit time	137 sec
λ_0	rest wavelength	631.685nm
ξ	tilt from ecliptic axis	1.26°
η	tilt from N/S axis	73.94°
ϑ_D	angular diameter of sun	1945 arcsec
c	speed of light	299792 kms^{-1}

Table 3: *Values needed to calculate* v_{rot} *for the sun found from our analysis or from JPL Horizons Ephemeris generator.*

Finding the radius of the sun and the astronomical unit after we have the rotational velocity is quite simple. We simply apply the relation:

$$R_\odot = \frac{1}{2\pi} v_{rot} T_\odot \tag{07}$$

Where v_{rot} is the value we calculated, and $T_\odot$ is the period of the sun. We do have to account for the earth's relative rotation; therefore instead of using the nominal value of 25 days for the sun, we use the synodic period of 26.24 days which considers the observer's rotational velocity. Finding the AU from the radius just requires some simple geometry where $R_\odot$ was calculated and ϑ_D is the angular diameter of the sun given by JPL Horizons in radians (see Table 3);

$$D = \frac{2R_\odot}{\sin\vartheta_D} \tag{08}$$

Our computed values and errors are summarized in Table 4:

	Calculated Value	Known Value	Percent Error
v_{rot}	1.8937 kms^{-1}	1.9970 kms^{-1}	5.17 %
$R_\odot$	685569 km	695700 km	1.45 %
AU	145382455 km	14959787 km	2.82 %

Table 4: *Calculated values of the solar constants with relative percentage errors.*

Sources of Error

There are various sources of error in this lab, starting with the data acquisition and quality. We did not perform a flat field correction for the non-uniform pixel response across each order. In addition; each Echelle order was tilted by some small angle and we did not correct for that which likely resulted additional error in the centroid positions. we performed three least squares fits; first for the neon calibration; then for the polynomial subtraction before we performed the cross correlation, then to fit the peak of the cross-correlation. Finally, we assumed that wavelength shifts were constant, however any non-linearity in the CCD would result in non-linear wavelength shifts which would create an error in our Doppler shift measurement.

Conclusion

We applied our knowledge of least squares fitting and CCD arrays to perform a pixel to wavelength calibration;and found that the dispersion: wavelengths per pixel for the Echelle order 35 was 0.02188 nm/pix, with an error of 8.6 %. We then studied several properties of solar spectra during a transit to understand how we can arrive at values for solar constants; R_{odot} and the astronomical unit. There were several sources of errors introduced along the lab, several of which I did not account for in my analysis however despite this; we find $R_\odot = 6.86 \times 10^5$ km and the AU = 1.458×10^8 km with reasonable errors.