Optical Lab, Astronomy 120
Lab 3, Astrometry from CCD Images

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Abstract

In this lab we used raw CCD images from the 30-inch Leuschner Observatory telescope to measure the distance to a minor planet using the method of parallax. We tracked the asteroid, 25 Phocaea over a period of three weeks and using our measured data and comparison to standard star catalogs we were able to calibrate our CCD by performing a linear least squares fit to find the plate constants which form the conversion matrix between pixels and standard celestial coordinates. This allows us to map pixels in our two dimensional image to points on the celestial sphere. Using our calibration and comparison to sky charts, we are able to find the asteroid in our CCD image and calculate the parallax from second and third epoch measurements of the asteroid to find the distance between earth and 25 Phocaea, and the sun and 25 Phocaea.

Introduction

This lab constituted two components, first we found the CCD calibration between pixels and celestial coordinates, and then using this calibration we calculated the distance to the bright asteroid 25 Phocaea. We calculated centroids to find the brightest objects in our raw CCD data and compared these to Aladin skymaps to find our approximate center of field in units of Right Ascension (RA) and Declination (Dec). We then used the coordinates of the field center to query the United States Naval Observatory catalog to find the brightest stars in that field. Using the USNO stars as our independent variable and our CCD centroids as our dependent variable, we then performed a linear least squares fit to find the mapping between pixels and celestial coordinates. The method of parallax is used to determine astronomical distances by comparing relative motions of bodies. The asteroid measured during the first epoch has a different background of distant stars as opposed to the measurement in the second and third epoch. With the known celestial coordinates of the background stars and our CCD calibration, we can measure the position of the asteroid as it moves across the sky. The distance to the asteroid is then found by using the equations of motion of the asteroid with respect to the sun and the earth, and by iteratively solving for r and ρ which are the distances to the sun and the earth respectively.

Data and Apparatus

All data used in this lab was taken with the Leuschner Observatory telescope which uses a 30-inch diameter mirror which focuses the light onto a two dimensional CCD array of 4008 pixels by 2672 pixels. However, the data recorded is in a 2004 by 1336 array because it is binned to 2 pixels. Binning combines charges from adjacent pixels for a faster readout speed and lower noise. The charges measured on the pixels are recorded in Analog to Digital Units (ADU) with a gain factor and some readout noise. Each image used[1] in analysis was an 8 second exposure in the R-band taken over the course of October 10, 2017 to October 26, 2017. Data was recorded as Flexible Image Transport System (fits) files which save header data and pixel data. The pixel data is the 2004 by 1336 array of ADU counts that we are interested in however the header data contains some useful astronomical data such as the CCD temperature, RA, Dec, date of observation, coordinated universal time and local sidereal time. Each element of the header can be accessed by index using the *astropy.io.fits* Python package.

Data Analysis Methods

Dark current

Dark current is the non-uniform accumulation of charges across the pixels due to the residual electric current flowing in the CCD photodiode and thermal excitations of electrons. We can see in Figure 1 that there are a few pixels that generate more dark current than the others; therefore we average over each pixel in 22 files for data taken on October 10th, 2017. We measure dark current for six dates between 10/10 and 10/26 and then calculate the variance of the mean dark current: 28.306 ADU^2. Since the dark current varies per night, it is important for our analysis that we have a number of good 8-second exposure darks per night in order to average it out. Measuring dark current also includes the bias offset; this is a minimum positive voltage at each pixel introduced by the device in order to prevent negative voltage that can't be measured by the ADC which is limited by a 12-bit number. We can then simply subtract out the averaged darks from each subsequent image used in the analysis to get counts that represent only the light falling on the pixels.

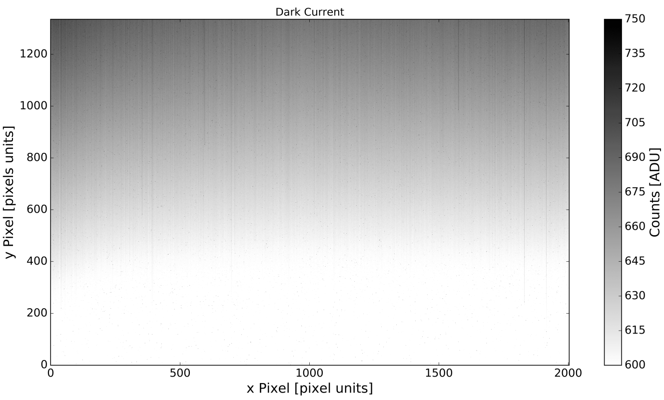
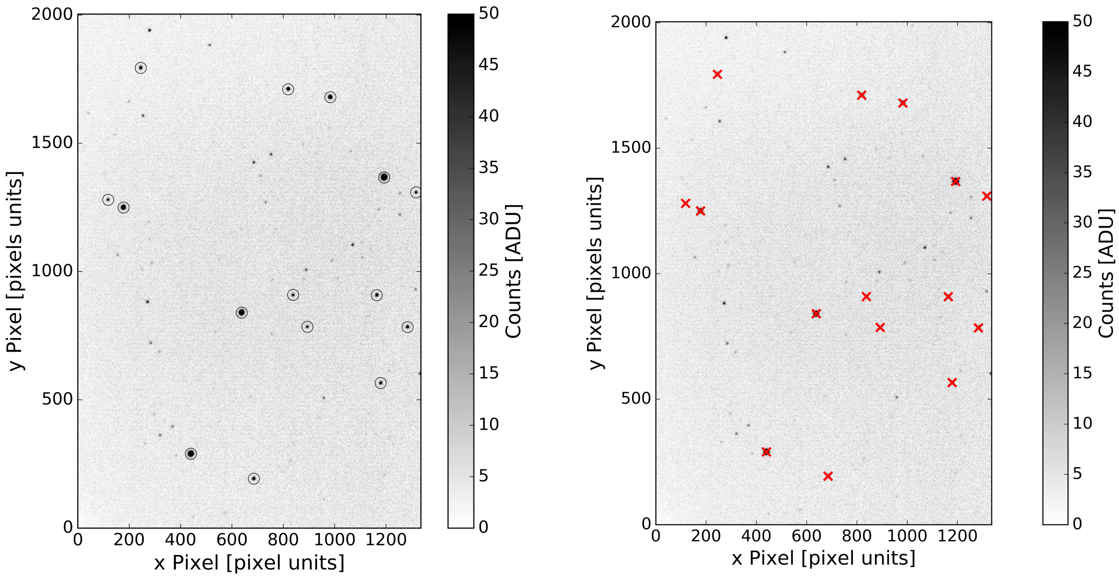


Figure 1: *Averaged dark current measured for the Leuschner Observatory Telescope on 10-10-2017. This image is the average of 22 files of 8 second exposures with the shutter closed so that no light falls on the CCD. Dark pixels show a lot of dark current is present while bright pixels show low dark current in the CCD.*

[1] Data used was from 10-10-2017, 10-18-2017 and 10-26-2017 taken by the groups: Supernova 2.0 Remastered, Let's Get Sirius and Supernovae, and James Graham, Robert Citron and Jordan Sligh.

Centroids and centroid error

Before calibrating our CCD, we need to find the positions of bright objects in our dark subtracted CCD images. The best method to do this is by locating the centroids[2] (weighted mean) of the peak pixel locations. We start by calculating the maximum pixel value; this is found by looping through the CCD x and y pixel data and finding the local maxima for the first 15 bright stars. This is shown in Figure 2a. The orientation of the image is not the same as Figure 1, this is because the dark current was measured for the raw CCD data however in order to understand our images and make sure we are viewing the correct field, we have to rotate and flip the arrays to get them in the standard astronomical orientation with north pointing up and east pointing left.



(a) *Star positions of the brightest objects in the image from 10-10 marked by circles. These points represent the brightest pixels in the image*

(b) *Centroid positions using maximum pixel values and finding the weighted mean over some radius surrounding each bright object.*

Figure 2: *Dark subtracted field, 0032.fts from 10-10-2017. We find the approximate star positions and then the centroid locations.*

The local maxima of the 15 brightest objects can then be used to to find the centroids. We do so by considering a radius of 20 pixels around each maxima and applying Equation 01 where x_i and y_i are the x and y values running over the radius in which we calculate the centroid, and I_i is the sum of the flux in the vicinity measured in ADU.

$$\langle x \rangle = \sum_i x_i I_i \Big/ \sum_i I_i \qquad \langle y \rangle = \sum_i y_i I_i \Big/ \sum_i I_i \tag{01}$$

In addition to the weighted mean in the radius around the star, we also correct for the sky background. To do so we create a mask around the initial radius and generate an annulus of width 10. We then calculate the mean value of the ADU counts within this annulus and subtract it from our raw ADU counts before calculating the centroids. This makes sure that the star (and asteroid) flux is calculated only for the body of interest and we aren't

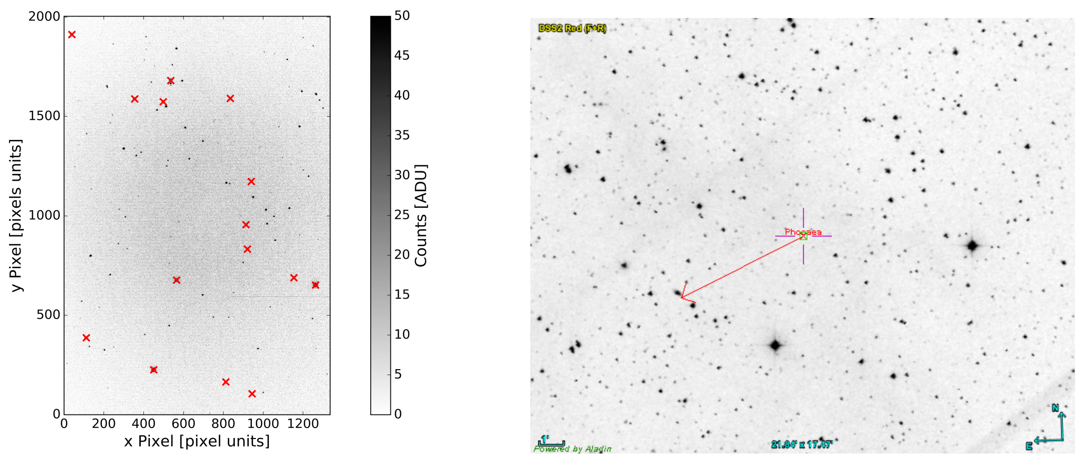
[2]Python function *star_centroids()* by James R. Graham 11/7/2011 University of Toronto

getting any additional counts from the brightness in the background sky.
We calculate the RMS errors of the centroids by using Equation 02. Here $\sigma^2_{\langle x\rangle}$ and $\sigma^2_{\langle y\rangle}$ are the RMS errors in x and y. $\langle x\rangle$ and $\langle y\rangle$ are the centroids as defined in Equation 01. The centroid values and their errors for 10-10-2017, 0032.fts are summarized in Table 2 in the section: CCD calibration. However the mean and standard deviation of x and y RMS errors are: $\mu_{xRMS} = 0.347$ pixels, $\mu_{yRMS} = 0.369$ pixels, $\sigma_{xRMS} = 4.775$ pixels and $\sigma_{yRMS} = 5.145$ pixels.

$$\sigma^2_{\langle x\rangle} = \left(\langle x^2\rangle - \langle x\rangle^2\right) / \sum_i I_i \qquad \sigma^2_{\langle y\rangle} = \left(\langle y^2\rangle - \langle y\rangle^2\right) / \sum_i I_i \tag{02}$$

CCD calibration

Once we have our centroids, we use standard star catalogs to find the calibration between pixels and standard celestial coordinates. We start by locating the field center of our image and compare it to a standard field and confirm that we see the same star patterns. I used the desktop version of the Aladin Sky Atlas and queried it using the SkyBot catalog which takes in an approximate field center and an epoch which is the date and UTC time from the header of the fits file. We did this process for all three epochs, 10-10, 10-18 and 10-25. Figure 3 shows the centroid positions and approximate field center from Aladin Sky Atlas for 10-25-2017. We can trace the same patterns of stars in both figures, the cluster of stars in the middle of the field, offset to the right is easiest to compare. We can also guess (and tell from SkyBot) that the bright object around (600, 700) pixels is the asteroid.



(a) *Centroid positions of field from showing brightest stars*

(b) *Aladin Sky Atlas using DSS2 red plates and SkyBot catalog*

Figure 3: *Matching field centers using SkyBot for 10-25. We did this process for all three epochs.*

10-10-2017	10-18-2017	10-25-2017
RA = 20H:49M:06.93S	RA = 21H:00M:21.40S	RA = 21H:10M:44.45S
Dec = +09°:22M:39.8S	Dec = +07°:18M:19.9S	Dec = +05°:50M:29.5S

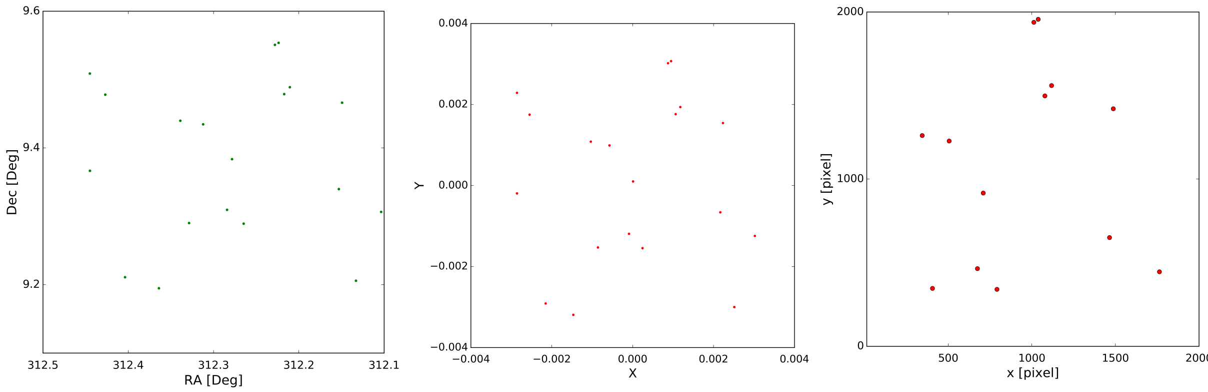
Table 1: *Approximate field centers from Aladin Sky Atlas for all three epochs.*

Using our RA, Dec we then query[3] the United States Naval Observatory using our approximate field centers. We use our field centers as the Equinox J2000 coordinates, first converting them to degrees and USNO returns bright stars, less than 12 Mag, in a 22 arcminute window in degree units of RA and Dec shown in Figure 4a for 10-10-2017; note that RA is conventionally plotted so that it is decreasing towards the right. We then convert the RA and Dec in degrees to standard celestial coordinates using the following transformation where α and δ are the RA and dec of the star, and α_0 and δ_0 are the RA and Dec of the field center, this is shown in Figure 4b.

$$X = -\frac{\cos\delta\sin(\alpha-\alpha_0)}{\cos\delta_0\cos\delta\cos(\alpha-\alpha_0)+\sin\delta\sin\delta_0} \qquad Y = -\frac{\sin\delta_0\cos\delta\cos(\alpha-\alpha_0)-\cos\delta_0\sin\delta}{\cos\delta_0\cos\delta\cos(\alpha-\alpha_0)+\sin\delta\sin\delta_0} \tag{03}$$

Once we have the stars in standard coordinates we can use nominal plate constants and offset by eye to get the field in pixels and compare it to our centroids. The equations to convert from standard coordinates are given by Equation 04 where f is the nominal focal length of the camera, we use $f = 6300$ mm and p is the size of the pixel, we use $p = 0.018$ mm because each pixel is 9μm but binning is two x_0 and y_0 are the x and y offsets, we initially offset to the field center. This transformation of coordinates then returns Figure 4c.

$$x = f(X/p) + x_0 \qquad y = f(Y/p) + y_0 \tag{04}$$



(a) *USNO bright stars RA and Dec in units of degrees* (b) *USNO bright stars in standard coordinates* (c) *USNO bright stars in units of pixels*

Figure 4: *USNO field for 10-10-2017 using field center from Table 1. The plots show transformations between different coordinate systems, we are interested in comparing (c) to the centroids we found.*

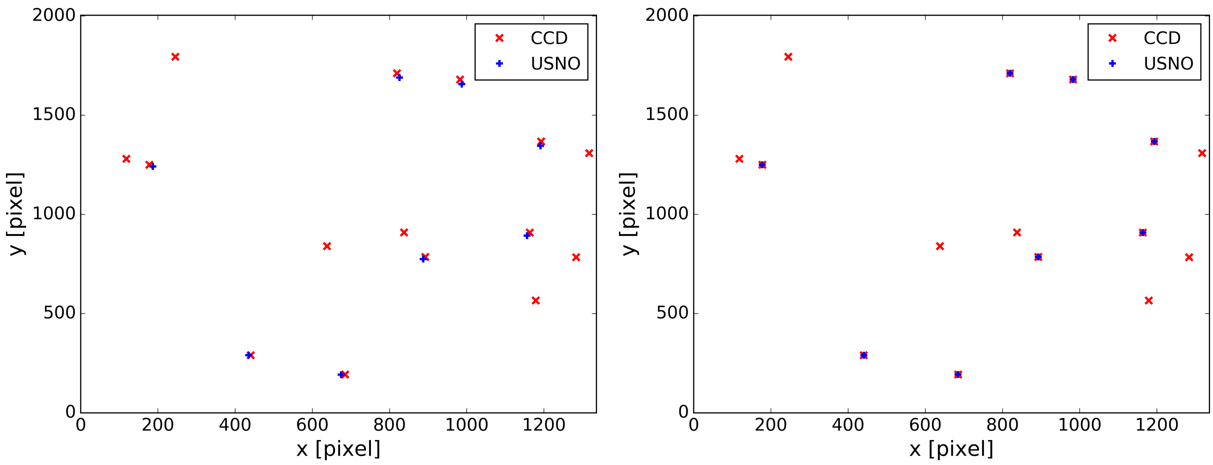
Table 2 summarizes this data for 10-10-2017. It gives us the x and y centroids, the RMS error in these centroids in addition to all the USNO stars in standard coordinates and in pixel units. For succinctness, we don't list the coordinates in degrees and we only list here the coordinates we use for the least squares fit later and the asteroid.

[3]Using Python function *usno()* by James R. Graham 2013/10/13 UC Berkeley

x_{cent}	y_{cent}	x RMS	y RMS	x USNO	y USNO	X USNO	Y USNO
1192.8	1367.6	4.638	4.903	1190.8	1345.8	8.28×10^{-06}	1.02×10^{-4}
440.15	290.26	5.022	5.096	434.85	291.05	-2.15×10^{-03}	-2.91×10^{-3}
177.63	1250.1	4.772	4.966	186.73	1242.3	-2.86×10^{-03}	-1.93×10^{-4}
982.54	1680.2	4.701	4.844	986.94	1657.1	-5.74×10^{-04}	9.91×10^{-4}
819.15	1711.6	4.816	5.066	825.83	1689.7	-1.03×10^{-03}	1.02×10^{-3}
1163.4	908.58	5.265	4.825	1155.9	893.48	-9.14×10^{-05}	-1.19×10^{-3}
685.09	193.45	4.328	4.648	674.29	192.66	-1.46×10^{-03}	-3.18×10^{-3}
892.63	785.66	4.694	5.221	887.13	775.22	-8.59×10^{-04}	-1.52×10^{-3}
637.78	*840.53*	*4.786*	*4.989*				

Table 2: *Summary of centroid positions and centroid RMS errors for 10-10-2017. We also list the star locations returned by a USNO query in pixels (x, y) and standard coordinates (X, Y). These values are from fitting by eye before the least squares calculation. The values in italics are the asteroid centroids and errors.*

Table 2 summarizes only the values used for the least squares fit. In Figure 5a we plot all available measured centroids and USNO stars in pixel units to compare their locations. There is clearly a magnification and rotation problem, therefore we perform a least squares fit that should correct this error.



(a) *Centroids and USNO data where offset is roughly* $x_0 = -40$ pixels, $y_0 = 1004$ pixels

(b) *Centroids and USNO data, using least squares fit to get plate constants.*

Figure 5: *Comparison of USNO data and measured centroids for 10-10-2017 before and after least squares fit. The fit returns plate constants that define a transformation matrix between pixels and standard coordinates.*

Least Squares Fit - We present the results of the least squares transformation in Figure 5b and outline here how this was performed. The transformation $\mathbf{x} = \mathbf{TX}$ is given by Equation 05 where f is the nominal focal length: 6300 mm, p is the pixel size: 0.018 mm, x_0 and y_0 are the offsets, and a_{ij} are the constants that refine the shear, scale and orientation.

$$\mathbf{T} = \begin{bmatrix} (f/p)a_{11} & (f/p)a_{12} & x_0 \\ (f/p)a_{21} & (f/p)a_{22} & y_0 \\ 0 & 0 & 1 \end{bmatrix} \tag{05}$$

x are the pixel coordinates and **X** are the standard celestial coordinates. We can then use $\mathbf{a} = \mathbf{Bc}$ to find the plate constants. **a** is the x pixel coordinates, **c** is our plate constants for x pixels and **B** is as defined below. We can compute the plate constants for y with a similar equation.

$$\mathbf{a} = \begin{bmatrix} x_1 \\ x_2 \\ x_N \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} (f/p)X_1 & (f/p)Y_1 & 1 \\ (f/p)X_2 & (f/p)Y_2 & 1 \\ (f/p)X_N & (f/p)Y_N & 1 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} a_{11} & a_{12} & x_0 \end{bmatrix} \tag{06}$$

Some simple linear algebra as outlined in the lab handout Equations 8, 9, 10 and 11 demonstrate how to perform the χ^2 fit with the pixels as our dependent variable and the USNO coordinates as our independent variable to get the plate constants: $\mathbf{c} = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{a}$. Once we have the plate constants for both x and y we can use the inverse transformation, $\mathbf{X} = \mathbf{T}^{-1}\mathbf{x}$ to get the pixels of our centroids in units of standard coordinates or vice versa. We demonstrated this in Figure 5b and the constants for the least squares fit for 10-10-2017 are summarized in Table 3. We perform the process of centroiding, querying USNO and performing the least squares fit three times, for each epoch and then plot the residuals of all three fits in Figure 6. We can see that the residuals vary from day to day implying that the plate constants also vary. Therefore it is necessary to perform three fits.

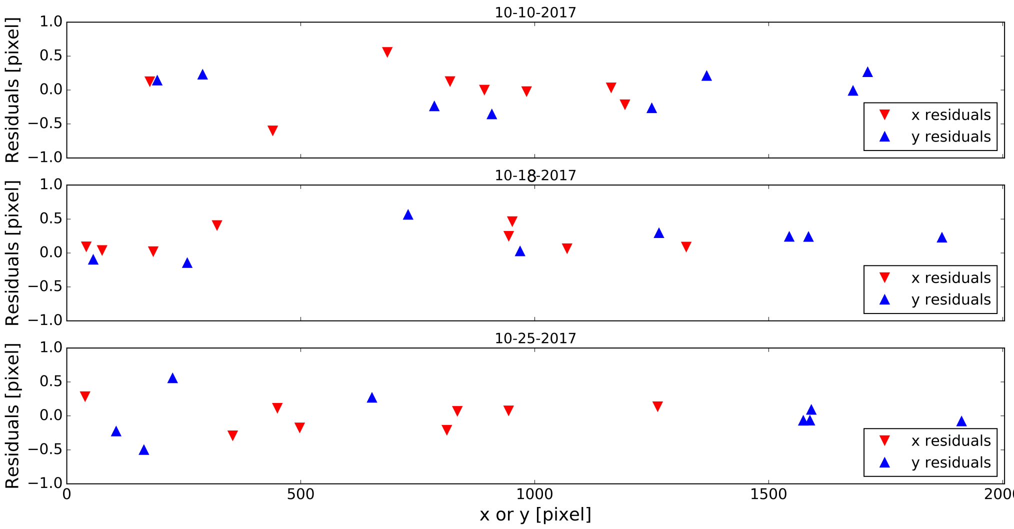


Figure 6: *Residuals of least squares fit from all three epochs used in our analysis. We remove some points to get a better fit and reduce the RMS error.*

The RMS error is calculated by taking the root-mean-square of the sum of residuals for each epoch. The true value for f/p is given by $\sqrt{\det(\mathbf{T})}$, we use this later to calculate the error in our asteroid position.

10-10-2017	10-18-2017	10-25-2017
a_{11}=1.0126	a_{11}=1.0129	a_{11} =1.0131
a_{12}=0.2837	a_{12}=-0.01296	a_{12}=-0.0124
a_{21}=-0.2812	a_{21}=0.01279	a_{21}=0.01324
a_{22}=-0.1898	a_{22}=1.0116	a_{22}=1.0130
x_0=616.097 pix	x_0=609.148 pix	x_0=712.391 pix
y_0=1094.13 pix	y_0=1019.017 pix	y_0=886.261 pix
x_{RMS}=0.3061 pix	x_{RMS}=0.5232 pix	x_{RMS}=0.1884 pix
y_{RMS}=0.2347 pix	y_{RMS}=0.5263 pix	y_{RMS}=0.2971 pix
f/p =354464	f/p =354325	f/p =354620

Table 3: *Plate constants computed using least squares fit for all three epochs. Nominal value for f/p was 350000*

Using our plate constants table, we can find the position of the asteroid in each epoch. We know the position of each asteroid from our centroids and comparing the fields. We then convert the position in pixels to standard coordinates using our least squares fit plate constants for the transformation: $\mathbf{X} = \mathbf{T}^{-1}\mathbf{x}$. We can find the errors in asteroid positions by calculating the number of arc second per pixel: $206265/\sqrt{\det(\mathbf{T})}$ where 206265 is the conversion from radians to arc seconds. We know our our RMS errors for RA and Dec in pixels from the fit residuals (from Table 3). Then assuming that the errors propagate linearly in our transformation, we can get the error in the RA and Dec in arc seconds and seconds.

10-10-2017	10-18-2017	10-25-2017
RA = 20H:49M:28.89S	RA = 21H:00M:21.21S	RA = 21H:10M:50.28S
Dec = +09°:17M:28.2S	Dec = +07°:18M:17.2S	Dec = +05°:48M:28.9S
err_{RA} = 0.018 S	err_{RA} = 0.020 S	$err_{R}A$ = 0.0073 S
err_{Dec} = 0.12"	err_{Dec} = 0.31"	err_{Dec} = 0.17"

Table 4: *Asteroid positions in RA and Dec with associated errors. Compare to Table 1 for field centers.*

Parallax

The last part of the lab involved using linear algebra and the method of parallax to find the orbit of the asteroid. However I ran out of time and since the full orbital determination was an optional component, I was only able to find the distance between the asteroid and the sun, and the distance between the asteroid and the earth.

The method for determining parallax varies based on what coordinate system we use. The simplest coordinate system is the geocentric ecliptic reference frame which is defined by the orbit of the earth. Figure 7 shows the relative position of the sun, earth and asteroid in the heliocentric frame, for our coordinate system, the vector **R** goes in the opposite direction. From our data, we can find **s** since that is simply the unit vector in the direction from the earth to the asteroid, we can find **R** from the Horizones Ephemeris calculator for all of our epochs. The quantities of interest are **r** and ρ which represent the distance to 25 Phocaea from the sun and the earth respectively..

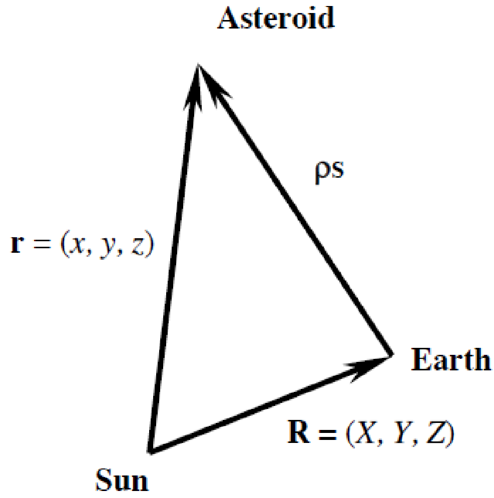


Figure 7: *Relative position of the bodies in our parallax problem from the lab handout. The* **R** *vector should be reversed for the ecliptic coordinate system.*

The unit vector **s** is given by Equation 7 where α and δ are the RA and Declination of our asteroid in standard celestial coordinates We use $\dot{\mathbf{s}}$ and $\ddot{\mathbf{s}}$ from Equation 8 to find ρ and the scalar r. The velocity calculation requires two epochs and the acceleration calculation requires three. $\tau_1 = t_2 - t_1$ and $\tau_3 = t_3 - t_1$ where t_i are the times measured in days from UTC = 00:00:00 on 10-10-2017. We do our calculation in days and AU for simplicity.

$$\mathbf{s} = \begin{bmatrix} \cos\alpha\cos\delta \\ \sin\alpha\cos\delta \\ \sin\delta \end{bmatrix} \tag{07}$$

$$\dot{\mathbf{s}}_2 = \frac{\tau_3(s_2 - s_1)}{\tau_1(\tau_1+\tau_3)} + \frac{\tau_1(s_3 - s_2)}{\tau_3(\tau_1+\tau_3)} \qquad \ddot{\mathbf{s}}_2 = \frac{2(s_3 - s_2)}{\tau_3(\tau_1+\tau_3)} + \frac{2(s_2 - s_1)}{\tau_1(\tau_1+\tau_3)} \tag{08}$$

The only **R** of interest to us is for the second epoch because our calculation is based in that frame. The relevant vectors for the parallax calculation are summed up below. Units are $[\mathbf{s}] = [\mathbf{R}] = \text{AU}$, $[\dot{\mathbf{s}}_2] = \text{AU/day}$, $[\ddot{\mathbf{s}}_2] = \text{AU/day}^2$. And finally using these quantities we can find r and ρ from Equations 10.

$$\mathbf{s_1} = \begin{bmatrix} 0.66 \\ -0.72 \\ 0.16 \end{bmatrix} \quad \mathbf{s_2} = \begin{bmatrix} 0.70 \\ -0.70 \\ 0.12 \end{bmatrix} \quad \mathbf{s_3} = \begin{bmatrix} 0.73 \\ -0.67 \\ 0.10 \end{bmatrix} \quad \dot{\mathbf{s}}_2 = \begin{bmatrix} 0.0049 \\ 0.0041 \\ -0.0042 \end{bmatrix} \quad \ddot{\mathbf{s}}_2 = \begin{bmatrix} -1.76\times10^{-4} \\ -4.98\times10^{-5} \\ 2.47\times10^{-4} \end{bmatrix} \quad \mathbf{R_2} = \begin{bmatrix} 0.89 \\ 0.40 \\ 0.17 \end{bmatrix} \tag{09}$$

$$\rho = k^2\left(\frac{1}{R^3} - \frac{1}{r^3}\right)\frac{\dot{\mathbf{s}}\cdot(\mathbf{R}\times\mathbf{s})}{\dot{\mathbf{s}}_2\cdot(\ddot{\mathbf{s}}\times\mathbf{s})} \qquad r^2 = \rho^2 + R^2 + 2\rho\mathbf{R}\cdot\mathbf{s} \tag{010}$$

Then using our vectors and all the equations summarized in this section, we can iteratively calculate r and ρ using a simple *Python for loop*. We assume an initial guess of r=5 AU

and plug that into the ρ equation. We then use that value of ρ to find r and repeat this process until both r and ρ converge.
Assuming our error in the least squares fit propagates linearly to the error in RA and declination, we can calculate the error[4] in the distance calculation to the asteroid by assuming RA and Dec are normally distributed with the standard deviation from found from error propagation of the linear fits error. This would be an interesting statistical analysis however I ran out of time and could not study this relationship between RA and Dec therefore the errors I calculated are simply found from the Horizons Ephemeris calculation of 25 Phocaea for epoch 2. The iterations of r and ρ, and the errors are summarized in Table 5.

	r / AU	ρ / AU
iter 1	5	2.076456708
iter 2	2.076456708	1.484668207
iter 3	1.929317127	1.31856274
iter 4	1.893502084	1.277586513
iter 5	1.883075726	1.26561218
iter 6	1.879893556	1.261953337
actual	1.904	1.329
errors	0.0241	0.067

Table 5: *Iterations over r and ρ using Equation 10 and the errors in the final values using Ephemeris values.*

Conclusion

We conducted a very interesting and essential astronomical calculation in this lab starting with only raw CCD data. We were able to correct our raw data for noise sources by removing the averaged dark current. Calculating our centroids for three different dates, we compared these to standard sky catalogs to perform a linear least squares fit that translates between pixels and standard celestial coordinates. Our calibration was accurate to under 0.5 pixels in both x and y for all three epochs. Using our least squares fit we used a transformation matrix to find the asteroid location in RA and Dec over the three epochs and used this in our parallax calculation. Using linear algebra, our measured positions of the asteroid and ephemeris calculations from NASA's Horizons, we were able to find the distance from earth to the asteroid on 10-18-2017 to within 0.07 AU and from the sun to the asteroid to within 0.02 AU.

[4]explained to me by Samantha Wu