Instrumentation Lab, Physics 111A

Lab 7, Op Amps II

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Signature Card

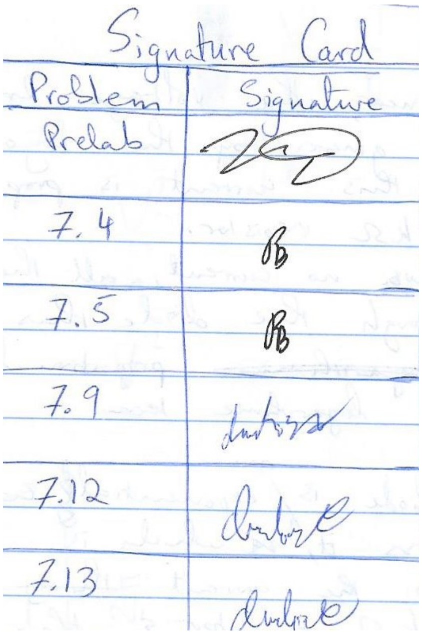


Figure 1: Pre-lab and Problems 7.4, 7.5, 7.9, 7.12

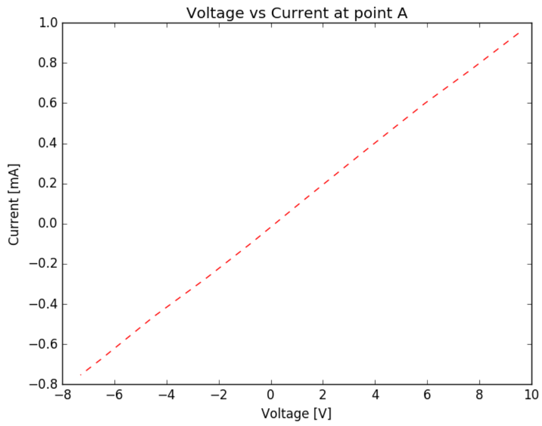
Problems

Problem 7.1 - Negative Impedance Converter (NIC)

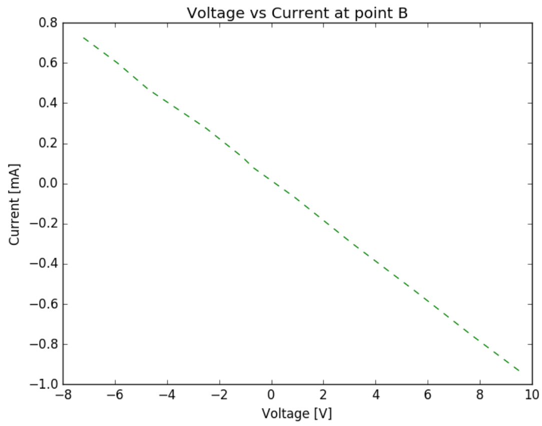
We checked that the resistors were matched to within 1% and we found both the 10 kΩ resistors to be rated at 10.02 kΩ and the 2.21 kΩ resistors to be rated at 2.202 kΩ.
We saw that the circuit behaved as expected when connected to point A and point B.

Point A			Point B		
V/ V	I/ mA	Z kΩ	V/ V	I/ mA	Z kΩ
9.5	0.948	10.02	9.5	-0.932	-10.19
7.7	0.769	10.01	7.7	-0.757	-10.17
5.9	0.598	9.87	5.5	-0.534	-10.29
3.5	0.351	9.97	3.1	-0.298	-10.40
1.3	0.121	10.74	0.9	-0.069	-13.04
0.9	0.0785	11.46	-0.7	0.0796	-8.79
-0.7	-0.0917	7.63	-1.1	0.131	-8.39
-2.5	-0.274	9.12	-2.5	0.275	-9.09
-4.5	-0.464	9.69	-4.7	0.467	-10.064
-6.5	-0.674	9.64	-5.9	0.600	-9.83
-7.3	-0.754	9.68	-7.3	0.735	-9.93

Table 1: NIC impedance measurements



(a) ≈10 kΩ Z at point A



(b) ≈-10 kΩ Z at point A

Problem 7.2 - Gyrator

We found the resonant frequency of the circuit to be at $\mathbf{f_0=4.838}$ kHz. We found the calculated resonance using the following;

$$f = \frac{1}{2\pi\sqrt{LC}}$$
$$L = CR^2$$
$$f = \frac{1}{2\pi\sqrt{C^2R^2}}$$
$$f = \frac{1}{330.2\ \Omega * 0.103\ \mu\text{F}}$$
$$\boxed{f = 4.822\ \text{kHz}}$$

Our measured resonance was quite close to the calculated resonance.
In order to maximize Q, we used the ring-down method to find the actual measured Q which is outlined later in this question and we also found the calculated values of Q using the following method where R_{pot} is the potentiometer resistance, R=330.2 Ω and C=0.103 µF;

$$Q = \frac{\omega_0}{\Delta\omega}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$
$$\omega_0 = \frac{1}{R^2C^2}$$
$$\omega_0 = 2.94\ \text{k}\Omega$$

$$\Delta\omega = \frac{1}{2R_{pot}C}$$
$$from\ lab\ 2\ for\ large\ Q$$

$R_{pot}/\ \Omega$	ω_0/s^{-1}	$\Delta\omega/s^{-1}$	Q_{calc}	N_{rings}	Q measured
26680	29400	187.4	156.8	9	153.7
18900	29400	264.6	111.1	7	119.6
17420	29400	287.0	102.4	6	102.5
11420	29400	437.8	67.15	4	68.32
7300	29400	684.9	42.93	2.5	42.69
4100	29400	1219.5	24.11	1.5	25.62

Table 2: Finding maximum Q for the gyrator circuit

We can see that maximum Q is found for the maximum R_{pot} at 26.68 kΩ.
To use the ring-down method to find Q, we set the signal generator to output a high amplitude, low frequency square wave - some signal to make the circuit "ring". The image below shows the output on channel two. We count the number of rings it takes before the energy stored drops by $\frac{1}{e}$ and then use the formula below to find the Q.

$$Q = \frac{2\pi E_{stored}}{E_{lost\ per\ cycle}}$$
$$Q = \frac{2\pi E_{stored}}{\frac{E_{stored}}{e*N_{ring}}}$$
$$Q = 2\pi e N_{ring}$$
$$\boxed{Q_{max} = 153.7}$$

We counted 9 rings and the calculated **Q=156.8**. Our measured Q was quite close to this.

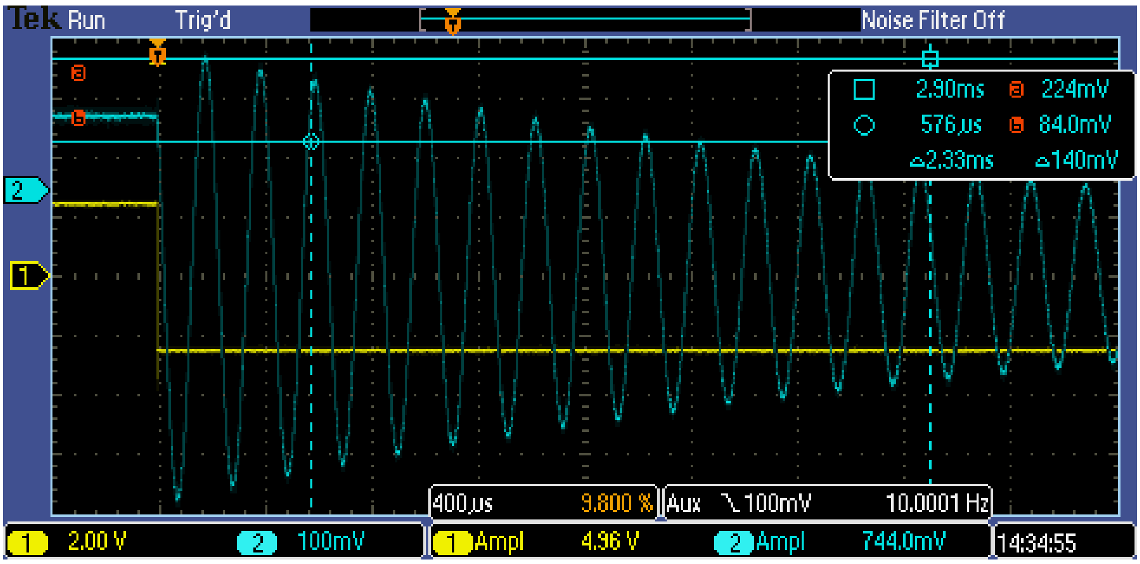


Figure 3: Q max for ring-down method

We then switched C_1 for a 1 µF capacitor. To find the new resonance we found ω_0 again;

$$\begin{aligned}
\omega_0 &= \frac{1}{\sqrt{LC_2}} \\
\omega_0 &= \frac{1}{\sqrt{R^2 C_1 C_2}} \\
\omega_0 &= \frac{1}{\sqrt{(330\ \text{k}\Omega)^2 (1\ \mu\text{F})(0.1\ \mu\text{F})}} \\
\omega_0 &= 9583 s^{-1} \\
f_0 &= \frac{\omega_0}{2\pi} \\
&\boxed{f_0 = 1.525\ \text{kHz}}
\end{aligned}$$

And we scanned to find the actual resonance of our circuit to be at $\mathbf{f_0}$**=1.527** kHz. These agree very well so we know our circuit behaves as expected.

Problem 7.3 - Relaxation Oscillator

We got the following scope trace with the op-amp output on channel 1 and the capacitor voltage on channel 2.

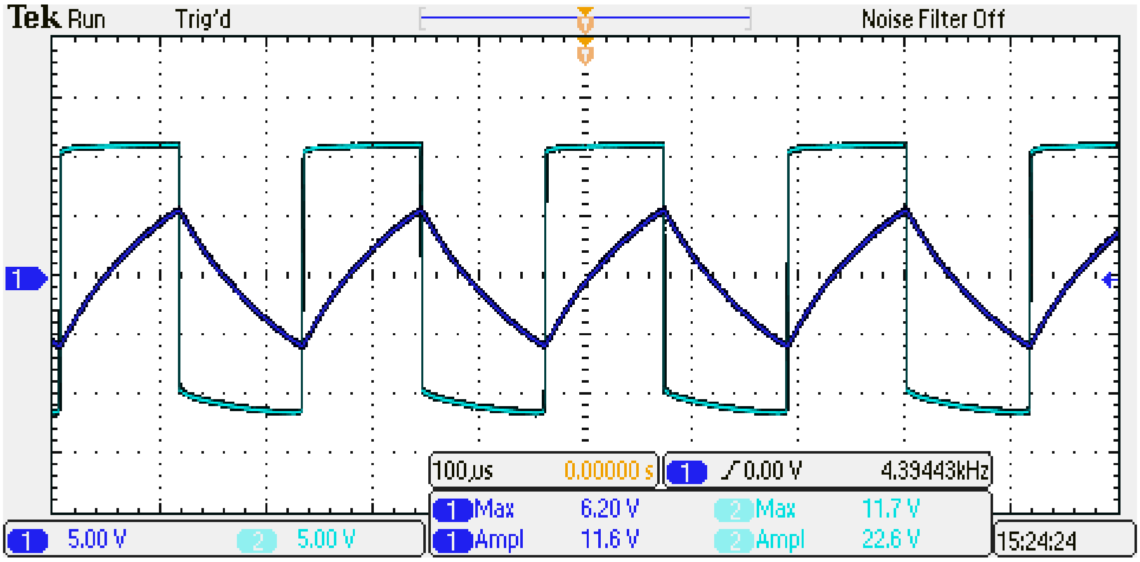


Figure 4: Relaxation Oscillator scope traces

The oscillator frequency is about 4.39 kHz as seen from the image. This is very close to the frequency we calculated in the pre-lab of 4.55 kHz.
The positive feedback loop creates hysteresis and the addition of the RC filter in the negative feedback loop causes the circuit to oscillate.

When the output is at some non-zero voltage above zero, the circuit reaches saturation at +12 V because of the positive feedback loop. The RC circuit connected to the inverting input causes the input to approach the output voltage with the period depending on RC. When the inverting input is larger than the non-inverting input the output decreases. As the difference between the inputs gets negative the positive feedback loop causes the output to reach saturation again and this is how the circuit oscillates.

Problem 7.4 - Filters

Refer to signature card, figure 4.

Problem 7.5 - Absolute Value

Refer to signature card, figure 4.

Problem 7.6 - Logarithm

The circuit expects negative input so we take the absolute value of the input voltages and plot them to get a good fit to a natural log. I did this in Microsoft Excel and it gave me an equation which gives the scaling factor and offset. The following image is the output we saw for a ramp input;

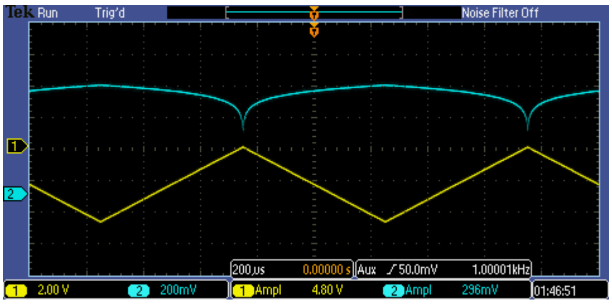


Figure 5: Logarithmic Converter V_{out}

V_{in}/mV	$\lvert V_{in}/\text{mV}\rvert$	V_{out}/mV	V_{in}/mV	$\lvert V_{in}/\text{mV}\rvert$	V_{out}/mV
-48	48	490	-960	960	622
-96	96	514	-1900	1900	656
-144	144	534	-2860	2860	678
-192	192	546	-3800	3800	692
-288	288	570	-4760	4760	704
-384	384	582	-5700	5700	710
-480	480	592	-6640	6640	718
-572	572	598	-7600	7600	722
-668	668	606	-8560	8560	728
-760	760	614	-9520	9520	736
-864	864	618			

Table 3: Measurements for logarithmic converter

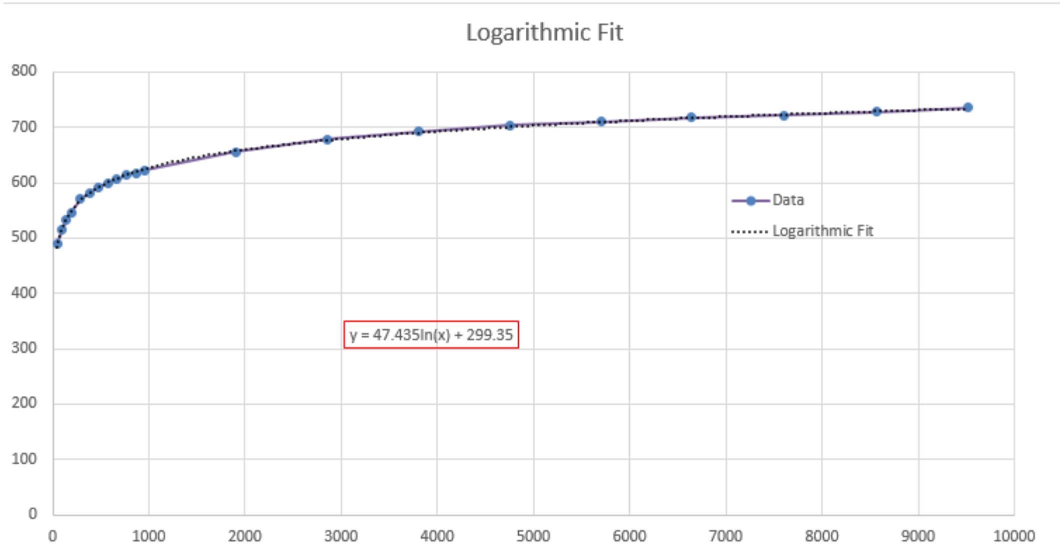


Figure 6: ln fit using $|V_{in}/\mathrm{mV}|$ V_{out}/mV

The figure above shows our data using the absolute value of V_{in} and the best logarithmic fit. The equation for the fit is given by;

$$V_{out} = 47.435\ ln\ |V_{in}| + 299.35$$

The following is the plot we see from our data plotted in python;

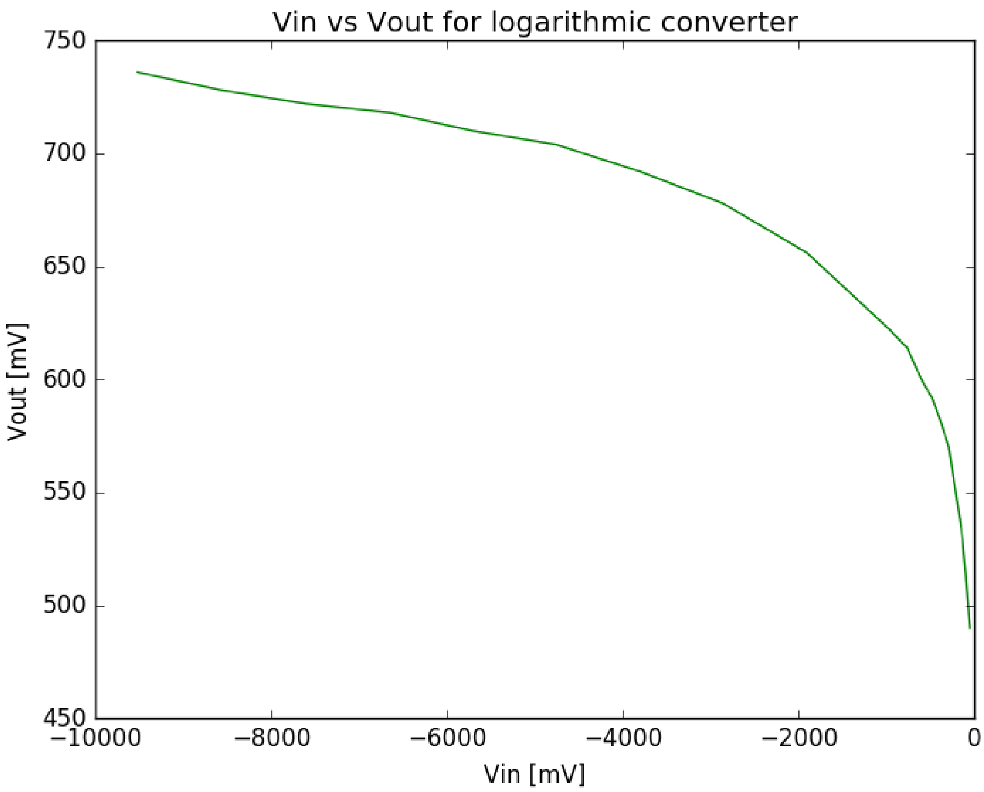


Figure 7: V_{in} against V_{out}

Problem 7.7 - Multiplier and Shifter

For our multiplier and shifter the output from the first op-amp was **-0.996** V and the output of the circuit was **+0.620** V. We got the following output for the ramp input;

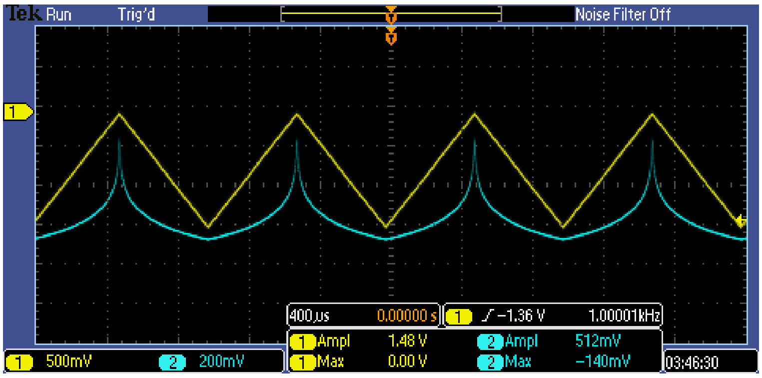


Figure 8: Multiplier and Shifter output

Problem 7.8 - Exponentiator

The output voltage we got from a ramp function is shown in the figure below;

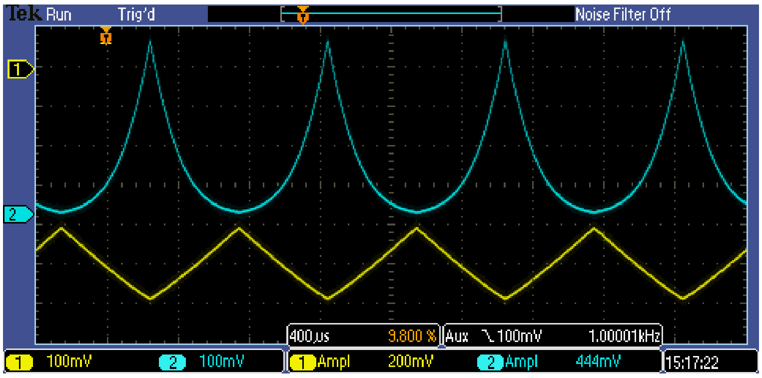


Figure 9: Exponentiator V_{out}

V_{in}/mV	$\lvert V_{in}/\text{mV}\rvert$	V_{out}/mV	V_{in}/mV	$\lvert V_{in}/\text{mV}\rvert$	V_{out}/mV
-42	42	12	-564	564	464
-94	94	12	-580	580	648
-146	146	14	-594	594	860
-182	182	17	-608	608	1100
-282	282	19	-616	616	1380
-384	384	27	-652	652	2720
-499	499	81	-676	676	4240
-524	524	220	-692	692	5880
-544	544	328			

Table 4: Measurements for exponentiator

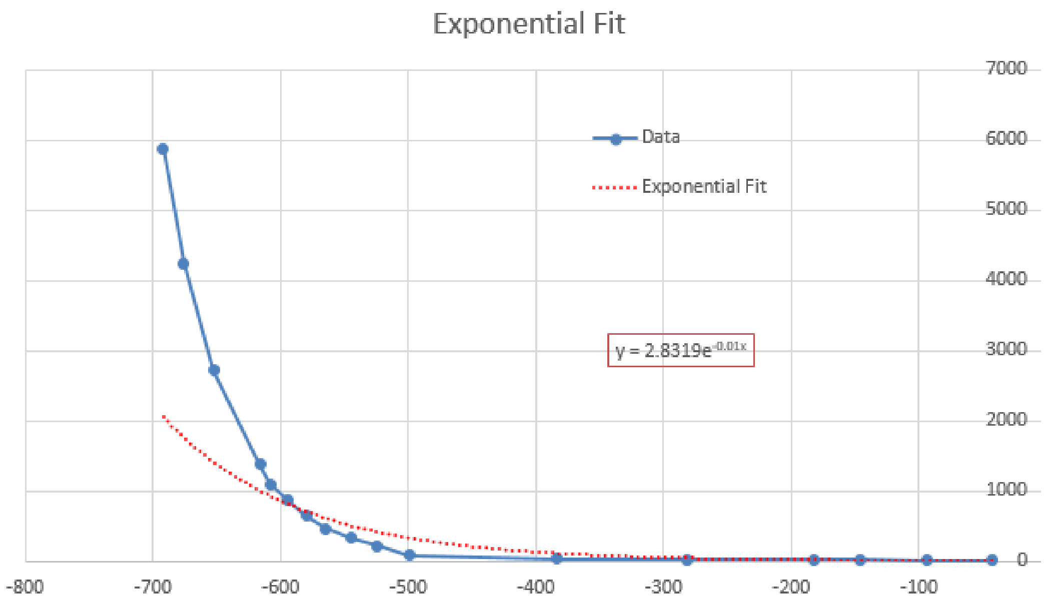


Figure 10: Exponential fit using our data for exponentiator

The figure above shows our exponential fit using the data for V_{in} and V_{out}. The equation for the fit is given by;

$$V_{out} = 2.8319\ e^{-0.01V_{in}}$$

The circuit stops behaving like an exponential around an input of 700 mV because of the limitations of the op amp. It starts to reach saturation and the expected output is higher than the power supply.

The following is the plot we see from our data plotted in python;

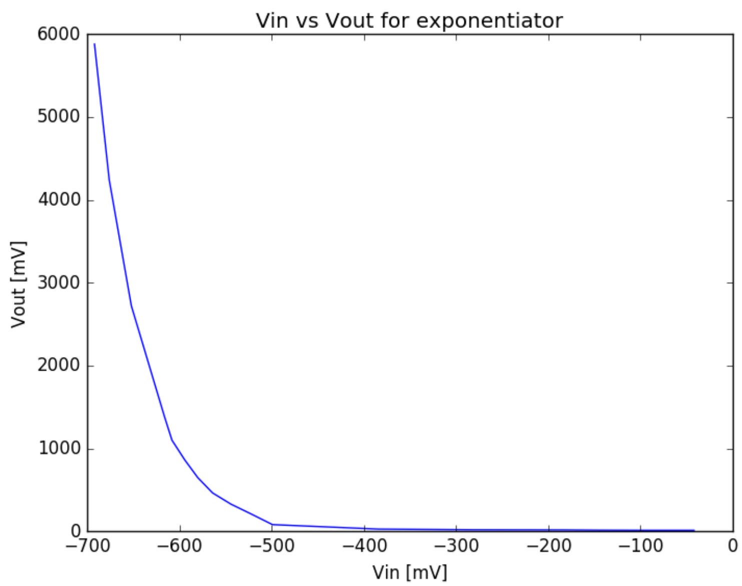


Figure 11: V_{in} against V_{out}

Problem 7.9 - Squarer

Refer to signature card, figure 4.

Problem 7.10 - Time Averager

We saw that the circuit had an upper frequency limit set by the scope however the output only got better with higher frequencies. There was however a lower limit that showed distortion at lower frequencies around 500 Hz.
For a sine wave, we saw that the output was very small and resembled a cosine wave as expected from the equation given. The integral of a sine wave is a cosine wave.

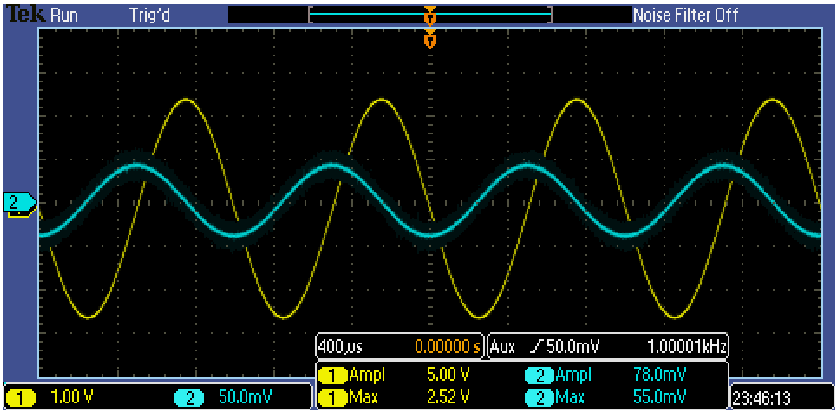


Figure 12: Time averager sine wave

For the square wave input we saw a very small amplitude ramp function output. This is as expected because the square wave equation is represented by some constant oscillating values of V_{max} and V_{min} then the integral of that would be a straight line segment.

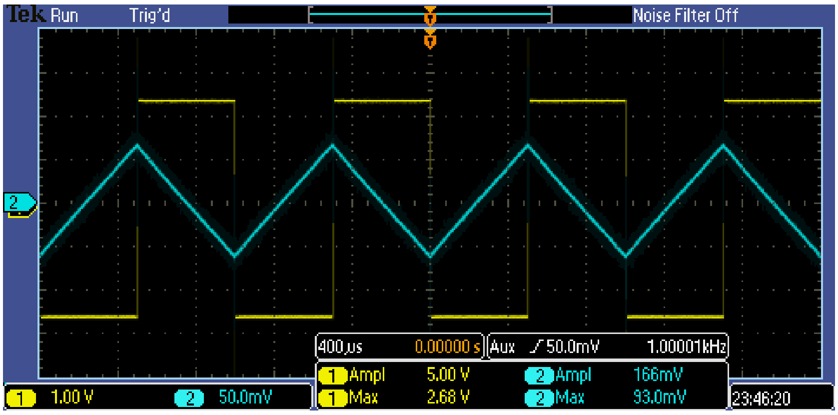


Figure 13: Time averager square wave

For a ramp function input, we saw an output that looked like a sine wave but it's actually represented by an oscillating second order polynomial which is the integral of a straight line segment (ramp function).

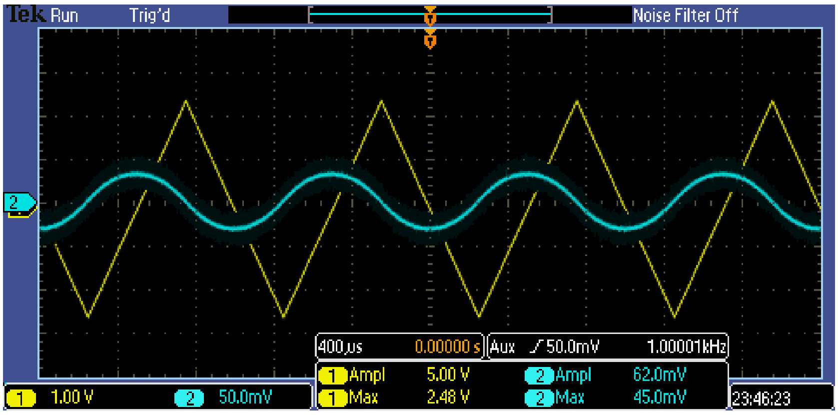
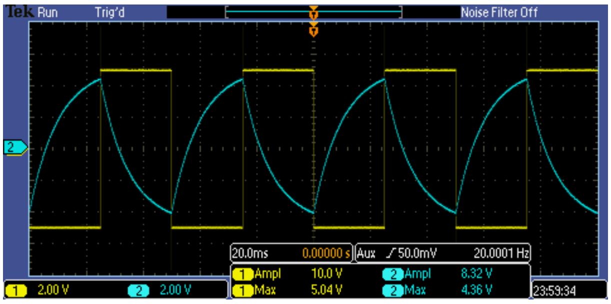
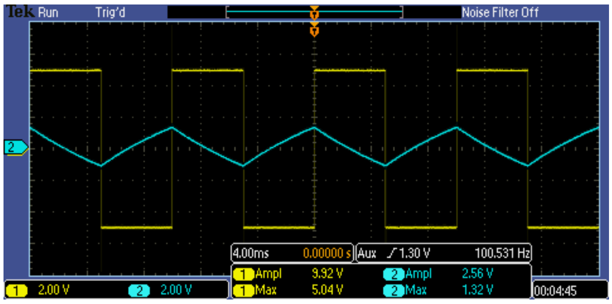


Figure 14: Time averager ramp function

At low frequencies, we saw that the amplitude of the time averager output was much higher and the output signal was distorted, it did not agree with the integral equation.

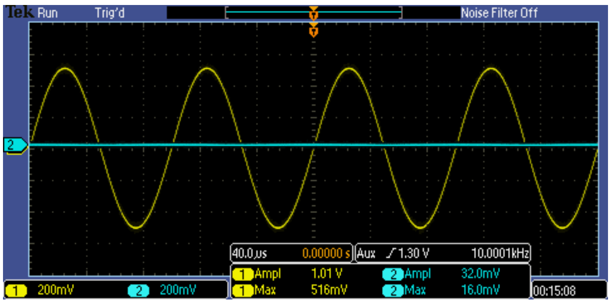


(a) Distortion of output at 20 Hz

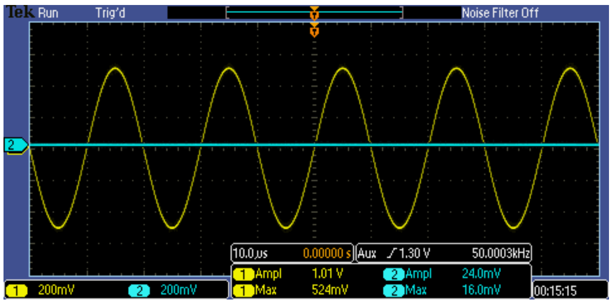


(b) Distortion of output at 100 Hz

At high frequencies, the amplitude remained very small and the signal kept being attenuated.



(a) 10 kHz



(b) 100 kHz

Problem 7.11 - Square Rooter

The output voltage we got from a ramp function is shown in the figure below;

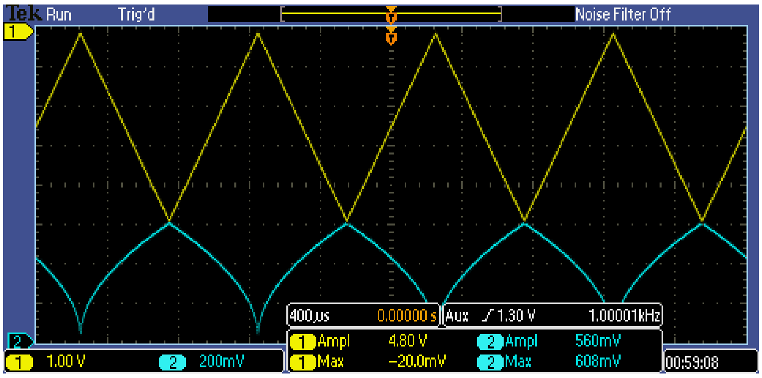


Figure 17: Square rooter V_{out}

V_{in}/mV	$\|V_{in}$/mV$\|$	V_{out}/mV	V_{in}/mV	$\|V_{in}$/mV$\|$	V_{out}/mV
-46	46	78	-2860	2860	470
-94	94	104	-3320	3320	514
-142	142	121	-3800	3800	550
-189	189	137	-4280	4280	578
-286	286	165	-4760	4760	606
-378	378	188	-5600	5600	658
-476	476	204	-6160	6160	690
-572	572	226	-6640	6640	714
-666	666	238	-7120	7120	742
-760	760	255	-7600	7600	766
-856	856	272	-8080	8080	790
-952	952	282	-8560	8560	812
-1430	1430	337	-9040	9040	830
-1900	1900	384	-9520	9520	862
-2380	2380	430			

Table 5: Measurements for square rooter

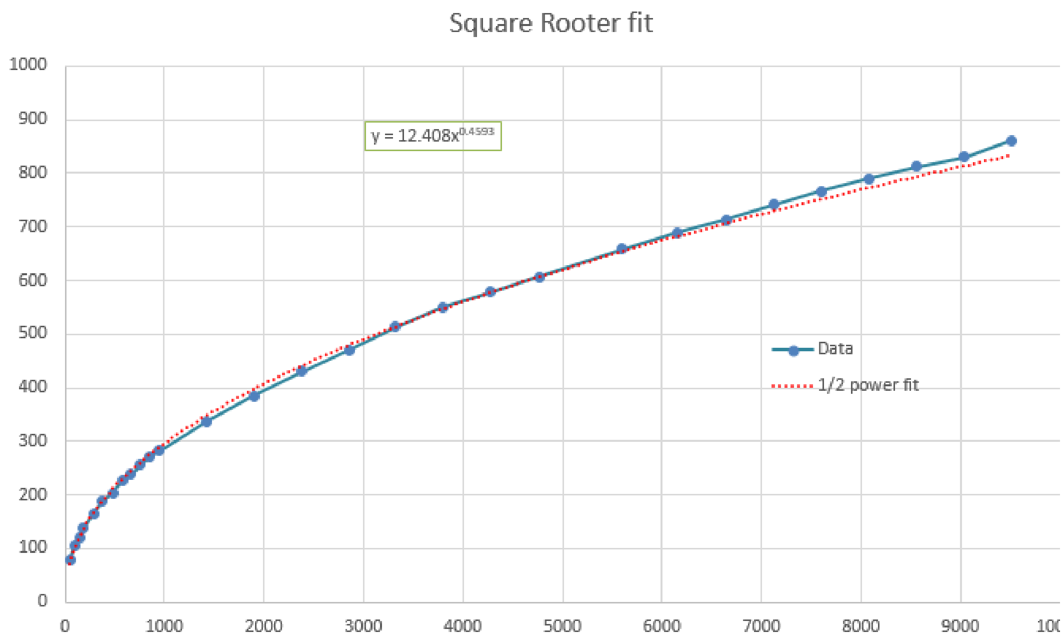


Figure 18: Power of $\frac{1}{2}$ fit using our data for square rooter

The figure above shows our exponential fit using the data for V_{in} and V_{out}. The equation for the fit is given by;

$$V_{out} = 12.408V_{in}^{0.4593}$$

The following is the plot we see from our data plotted in python;

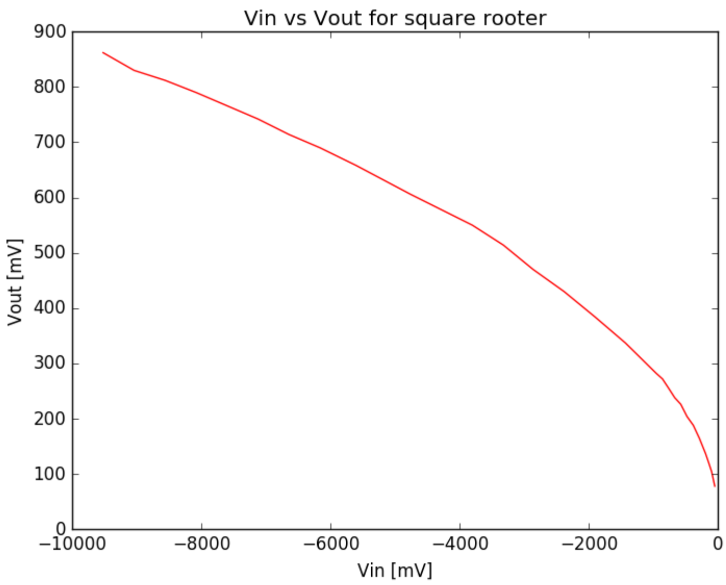


Figure 19: V_{in} against V_{out} for square rooter

Problem 7.12 - RMS Converter: No Averaging

Refer to signature card, figure 4.

Problem 7.13 - RMS Converter

Refer to signature card, figure 4.

Problem 7.14 - Gyrator Analysis

We know that the NIC has impedance equivalent to the negative resistance of the components directly below it. So on the right hand side of the circuit we have Z_{right}=-R. Then the lower half of the circuit has equivalent impedance Z_{eq} and we can then combine this with the top resistor to get Z_{in};

$$Z_{eq} = \frac{(-R)(Z+R)}{(-R)+(R+Z)} \qquad Z_{in} = -(R+Z_{eq})$$

$$Z_{eq} = \frac{-RZ-R^2}{Z} \qquad Z_{in} = -R+R+\frac{R^2}{Z}$$

$$Z_{eq} = -R-\frac{R^2}{Z} \qquad \boxed{Z_{in} = \frac{R^2}{Z}}$$

Problem 7.15 - Absolute Value Analysis

For a negative input signal, since we know that the virtual ground nodes created by the grounded non-inverting op-amp inputs (V_+) are held at 0, there has to be an equivalent voltage drop from V_{in} V to 0 V across both the 1 kΩ resistors connected to V_{in}. Thus there is no voltage drop across the 500 Ω resistor and the current passing through it has to be 0.
Since the second op-amp's negative input is also at a virtual ground, it is held at 0 V but in order to do this it must drive a +I current to the left and there is -I flowing to the right from the negative V_{in}. I here is given by;

$$I = \left|\frac{V_{in}}{1\ \mathrm{k}\Omega}\right|$$

In order for this current to be carried away and the negative input of the second op-amp to remain at 0 V the output voltage has to be V_{out}=-V_{in}.
For a positive input signal, we should have V=-V_{in} in order to keep the second op-amp's negative input at virtual ground. This leads to +I going through both the first 1 kΩ resistors and then -I going through the top 1 kΩ resistor in left loop. By Kirchoff's law we should get -2I flowing through the 500 Ω resistor. But we still have +I coming in from the bottom 1 kΩ resistor and these combine to give -I at the first node held at ground.
Then to keep the negative input of the second op-amp at 0 V, there has to be +I flowing right through the 1 kΩ resistor with the second op-amp. This combined with the -I from the left side of the circuit to give 0 V at the virtual ground. This also leads to V_{out}=V_{in}.

Problem 7.16 - Squarer Analysis

We can find the output voltage by breaking down the circuit into its modules.
First we have the log amplifier;

$$V_{out-log} = -V_T \; ln\left(\frac{V_{in}}{I_{sat}R}\right)$$

From the offset adder we have the output from the first op-amp equal -1 V and the output through the log amplifier is given by the above equation using V_{in}=-1 V.

$$V_{offset} = -V_T \; ln\left(\frac{-1\text{ V}}{I_{sat}R}\right) \qquad\qquad V_{offset} = V_T \; ln\left(I_{sat}R\right)$$

Then we know from Problem 7.7 that the total output of the Multiplier and Shifter is given by the following equation, we also plug in V_{offset} and V_{in-MS}=$Vout-log$;

$$V_{out-MS} = -2V_{in-MS} + V_{offset}$$
$$V_{out-MS} = -2\left(-V_T \; ln\left(\frac{V_{in}}{I_{sat}R}\right)\right) + V_T \; ln\left(I_{sat}R\right)$$
$$V_{out-MS} = 2V_T \; ln\left(\frac{V_{in}}{I_{sat}R}\right) + V_T \; ln\left(I_{sat}R\right)$$

Then finally the output from the exponentiator at the right is the output of our circuit and it is given by;

$$V_{out-exp} = RI_{sat}exp\left(\frac{V_{out-MS}}{V_T}\right)$$
$$V_{out-exp} = RI_{sat}exp\left(\frac{1}{V_T}\left(2V_T \; ln\left(\frac{V_{in}}{I_{sat}R}\right) + V_T \; ln\left(I_{sat}R\right)\right)\right)$$
$$V_{out-exp} = RI_{sat}exp\left(ln\left(\frac{V_{in}}{I_{sat}R}\right)^2 + ln\left(I_{sat}R\right)\right)$$
$$V_{out-exp} = RI_{sat}\left(\frac{V_{in}}{I_{sat}R}\right)^2 \left(I_{sat}R\right)$$
$$\boxed{V_{out-exp} = V_{out} = V_{in}^2}$$

Problem 7.17 - Averager Analysis

We know since the negative input draws no current and is held at a virtual ground by the golden rules; the current going through the first resistor has to be equal to the sum of the currents through the second resistor and the capacitor.

$$I_1 = I_2 + I_C \tag{0.1}$$
$$\frac{V_{in}}{R_1} = \frac{V_{out}}{R_2} + C\frac{dV_{out}}{dt} \tag{0.2}$$
$$\frac{V_{in}}{RC} = \frac{V_{out}}{RC} + \frac{dV_{out}}{dt} \tag{0.3}$$
$$\tag{0.4}$$

And then we can solve this differential equation using the method of an integrating factor;

$$Q = \frac{dy}{dx} + Py \qquad \frac{V_{in}}{RC} = \frac{V_{out}}{RC} + \frac{dV_{out}}{dt}$$
$$I = e^{\int P dx} \qquad I = e^{\int \frac{1}{RC} dt}$$
$$IQ = I\frac{dy}{dx} + IPy \qquad I = e^{\frac{t}{RC}}$$
$$Iy = \int IQdx \qquad e^{\frac{t}{RC}} V_{out} = \int_{-\infty}^{t} e^{\frac{t'}{RC}} \frac{V_{in}}{RC} dt'$$

$$\boxed{V_{out} = \frac{1}{RC}\int_{-\infty}^{t} V_{in}(t')dt' e^{\frac{t'-t}{RC}}}$$

We can compare this to equation (1) in Problem 7.10;

$$\langle H \rangle (t) = \frac{1}{\tau}\int_{-\infty}^{t} H(t')e^{\frac{t'-t}{\tau}} t'$$

Then comparing the two;

$$\boxed{V_{in}(t') = H(t')}$$
$$\boxed{\tau = RC}$$