Experimentation Lab, Physics 111B
Hall Effect in a Plasma, HAL*

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Abstract

In this lab we explore the Hall effect in a low-density, weakly-ionized plasma discharge tube. Plasmas are unstable, neutral gases that consist of free electrons and positive ions at low, pressures or very high temperatures. The low number density of charge carriers (compared to metals) in the discharge tube produces a large and easily observable Hall effect. We first determine the response of the magnet field to applied magnet current and discover a weak hysteresis effect. Then, by controlling various external parameters such as pressure, discharge current, and magnetic field, we measure the Hall Voltage and analyze the characteristics of the electron gas. Electron drift velocity, density, and collision frequency are inferred from the Hall field. Given the collision cross section for scattering of electrons by helium atoms, we then find the electron temperature and average energy. To first order we find $\Delta\vec{u} \approx 10^4\ ms^{-1}$, $n_e \approx 10^{16}\ m^{-3}$, $\nu = 10^9\ s^{-1}$, $T = 10^3\ K$

Introduction

The Hall effect arises from a voltage difference produced across a conducting medium when a magnetic field is applied perpendicular to the motion of charge carriers. The Lorentz force: $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ drives positive and negative charges to opposite boundaries which establishes a measurable potential difference known as the *Hall voltage*. The Hall effect has many applications from magnetometers, to quantum computers, and semiconductor physics. In addition, it allows us to measure several properties of conducting media such as drift velocities, collision frequencies and temperatures of charge carriers.
The Hall effect in metals is quite small, as the amplitude of the Hall voltage is inversely proportional to the number density of charge carriers. In metals this is of order $10^{29} m^{-3}$ [1] while we find later, that in plasmas and semiconductors n_e is several orders of magnitude smaller thus the observable V_H is much larger. Using this fact, we measure the Hall voltage in a gaseous discharge tube as a function of gas pressure, discharge current, and

*see Appendix A for signature sheet

magnetic field strength.

In order to do so, we first explore the voltage range over which the discharge tube plasma is stable for given pressures. We then map the magnetic field as a function of the magnet current because we cannot run our high voltage power supply and a Digital Multimeter at the same time due to voltage limitations. And finally, we make measurements of the Hall voltage with the external magnetic field applied. Using all of the above measurements, we then explore the relevant plasma properties in the analysis section.

Theory

Hall Effect

A single type of charge carrier (an electron) in a macroscopic conductor can be described by its charge q, number density, n_e, and drift velocity $\Delta\vec{u}$. The current density of the charge carrier is given by

$$\mathbf{j} = q n_e \Delta\vec{\mathbf{u}} \tag{01}$$

We are interested in the measurable electric current: $\mathbf{I}$ which is simply related to the cross sectional area, A of the conductor $\mathbf{I} = A\mathbf{j}$. In order to find the Hall field, we can consider the resistivity of the conductor as described by the friction experienced by charge carriers in their motion through the material. This is conveniently expressed by the frictional force on the charge q of mass m_e by a momentum loss due to collisions with frequency ν. The frictional force is then given by,

$$\mathbf{F_f} = -m_e \nu \Delta\vec{\mathbf{u}} \tag{02}$$

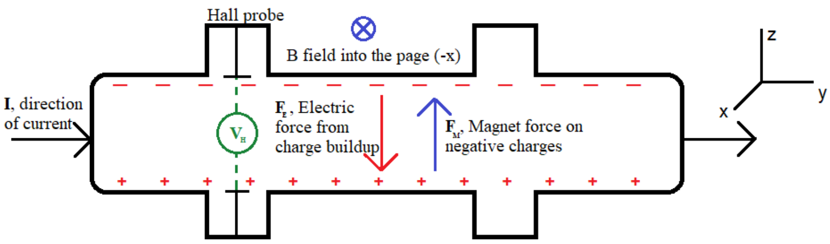


Figure 1: *Motion of charge carriers due to Hall effect*

To maintain a steady current, the frictional force is balanced by an electric force, and an additional magnetic force when an external field is applied 1. In addition, the presence of the magnetic field induces a potential difference across the conductor which produces the Hall force. The force balance at steady state can be described,

$$q\mathbf{E_o} + q\mathbf{E_H} + q\Delta\vec{\mathbf{u}} \times \mathbf{B} - m_e \nu \Delta\vec{\mathbf{u}} = 0. \tag{03}$$

Here $\mathbf{E_o}$ is the Ohmic field described by the resistivity ρ of the material and $\mathbf{E_H}$ is the Hall field.

$$\mathbf{E_o} = \rho\mathbf{j} \qquad \rho = \frac{m_e \nu}{n_e q^2} \qquad \mathbf{E_H} \cong -\Delta\vec{\mathbf{u}} \times \mathbf{B} = \mathbf{B} \times \mathbf{j}/q n_e \tag{04}$$

This describes Ohm's law for a total electric field $\mathbf{E} = \mathbf{E_o} + \mathbf{E_H}$ incorporating the Hall effect.

Hall Effect in Plasma Tube

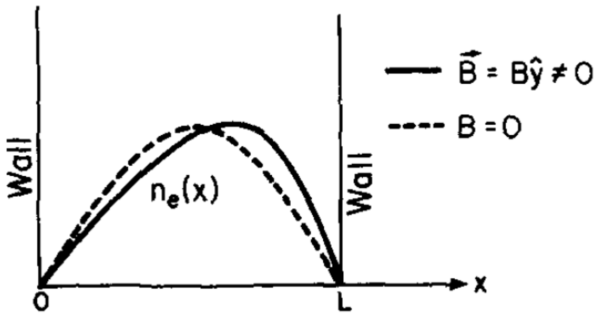


Figure 2: *Density distribution of charge carriers in plasma tube. The dashed line is without any external magnetic field applied and the solid line is with a field perpendicular to the current direction. Both show a distribution with a central maximum that falls off towards the walls of the tube. [1]*

We are interested in the Hall field across a low-density discharge tube in which the primary current carriers are free electrons. The electrons are produced in collisions with the gas molecules throughout the tube, however the charged particles recombine only at the tube walls resulting in a non-uniform density distribution across the tube cross section. The electron and ion densities are highest at the center and fall off to zero near the walls as shown in Figure 2. This distortion of the density distribution brings about a decrease in the effective Hall voltage by a factor of exactly 1/2 for a system with a short mean free path and slab geometry. This non-uniformity results in an additional pressure gradient which must be accounted for in the force balance. This comes about from the high temperature of the electrons.

$$qn_e\mathbf{E_o} + qn_e\mathbf{E_H} + qn_e\Delta\vec{\mathbf{u}} \times \mathbf{B} - n_e m_e v \Delta\vec{\mathbf{u}} - \nabla n_e k_b T_e = 0. \tag{05}$$

Where T_e is the temperature of the electrons. Now we can rewrite the total field as $\mathbf{E} = \mathbf{E_o} + \mathbf{E_H} + \mathbf{E_p}$ where $\mathbf{E_p} = -(\nabla n_e k_b T_e)/e$. Using one dimensional slab geometry, Kunkel [1] finds:

$$\Delta V_H = \int_{x_1}^{x_2} E_x dx \;=\; \frac{(x_2 - x_1)}{2} E_H \tag{06}$$

Kunkel finds that, if x_1 and x_1 are equidistant from the center of the tube, as is true of the geometry of our setup, the ambipolar field due to the pressure gradient, E_p cancels out exactly half of the Hall field as we stated previously.

Plasma Instabilities

Plasmas are able to support many types of waves, some of which grow spontaneously into *instabilities* from thermal fluctuations induced by combinations of discharge and gas parameters. These instabilities resemble whistle tones excited in air and they can introduce large errors so it is important to make measurements at steady state. We limit the range of our parameters: gas flow, pressure, voltage, current[3], to oscillation free operation. Despite this, stationary structures known as *striations* can appear especially at low pressures. These appear as alternating bright and dim bands arising from large amplitude ionization waves. However, these stationary striations come primarily from variations in the tail of the electron energy distribution so we can ignore them for our analysis of the Hall field. In addition, the distance between our probes is large enough that we can average over several wavelengths of striations.

Moving striations on the other hand can travel across the discharge tube at speeds of

$10^3 - 10^5$ cm/s [2]. This causes the electric potential to increase nonlinearly throughout the tube. Potential jumps increase the electron temperature and the degree of ionization in front of the striation while the tail end has insufficient electron energy to be ionized and does not glow. Plasma instabilities vary across values of I_d for each pressure and we can find the experimental upper and lower bounds for the existence of striations. We study the ranges of discharge current over which we observe quiescent operation, these are explored further in the methods and analysis sections.

Measurable quantities

Starting with Equation 01, we can find the drift velocity and number density of the electrons,

$$\Delta u = \frac{E_H}{B} \qquad n_e = \frac{I_d B}{qAE_H} = \frac{I_d}{qA\Delta u} \tag{07}$$

The collision frequency of electrons can be found from the two equations relating the resistivity of the plasma to the Ohmic field and the collisions of the electrons. Note $\Omega_s \approx 2\times 10^{11} B$ $[T]$ is the cyclotron frequency of electrons. Our analysis is valid when the cyclotron frequency is smaller than the collision frequency. In this regime, the electron's motion is interrupted by a collision before it can complete a cyclotron orbit.

$$\rho = \frac{E_o}{j_d} = \frac{m_e \nu}{n_e q^2} \qquad \Rightarrow \qquad \nu = \frac{E_o}{E_H}\Omega_s = \frac{qE_o}{\Delta u m_e} \tag{08}$$

Electron Temperature

We can constrain the free electrons to obey a Maxwell-Boltzmann velocity distribution since the plasma tube operates at high electron temperatures. Then, we can find average electron temperatures from the average velocities of electrons in a MB distribution. The collision frequency for a weakly ionized gas is given by the following [3]:

$$\nu \cong N_g \langle \sigma u \rangle_e \tag{09}$$

$$N_g = \frac{P}{k_b T} \tag{010}$$

Here N_g is the Helium gas density that we find from the ideal gas law assuming the plasma tube operates at roughly room temperature. where $\langle \sigma u \rangle_e$ is an average over the distribution of thermal speeds of the free electrons. Since Helium gas is used in this experiment, the cross section $\sigma$is fairly constant[1] at $\sigma \approx 3.8\times 10^{-20} m^2$.

$$\langle \sigma u \rangle_e \approx \langle \sigma \rangle \langle |u| \rangle_e \tag{011}$$

We must note that for the Maxwell-Boltzmann distribution, the square of the average speed is not the same as the RMS speed of particles. We can find this relation from the thermally averaged velocities and the RMS velocity [4];

$$\sqrt{\langle u^2 \rangle} = \sqrt{\frac{3k_b T_e}{m_e}} \qquad \sqrt{\langle u \rangle^2} = \sqrt{\frac{8k_b T_e}{\pi m_e}} \tag{012}$$

$$\sqrt{\langle u^2 \rangle} \approx 1.085 \langle u \rangle \tag{013}$$

For a particle has three translational degrees of freedom we can relate the kinetic energy to thermal energy and find the electron temperature, T_e,

$$\epsilon = \frac{1}{2}m_e\langle u^2\rangle = \frac{3}{2}k_bT_e \qquad \Rightarrow \qquad T_e = \frac{m_e\langle u^2\rangle}{3k_b} \tag{014}$$

It can also be shown that the energy the free electrons has an upper bound given by

$$E = \frac{1}{2}m_g\Delta u \tag{015}$$

Where Δu is the drift velocity of particles and m_g is the mass of the gas atom or molecule. The upper limit is reached when electron energy loss due to inelastic collisions (excitation and ionization) is negligible compared to that caused by elastic collisions [3].

Experimental Setup

The block diagram for the experiment is shown in Figure 3, we can split the experimental set up into two parts: the gas flow and vacuum system, and the electrical circuit that supplies the voltage through the discharge tube and measures the Hall effect. The pumping system allows a continuous stream of gas to flow through the discharge tube to create a stable plasma. We have a gas cylinder with 98.9% Helium, 1% Argon, and 0.1% Nitrogen, with the majority of free electrons coming from ionized Argon. The diagram shows a series of valves and pressure gauges that allows us to regulate the gas flow. For our technique, the most useful valves were v2 and v4 which allowed us to finely adjust the gas inflow and outflow respectively. This regulates the discharge pressure and the presence of instabilities and striations.

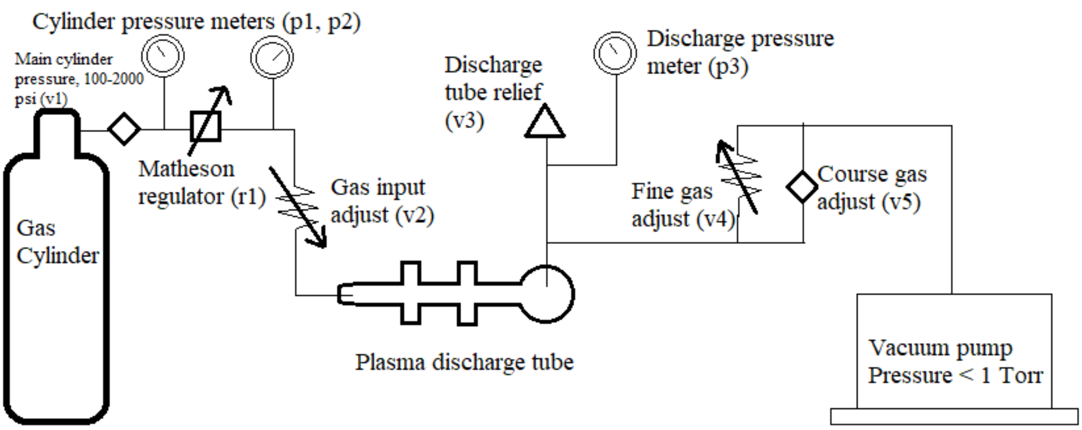
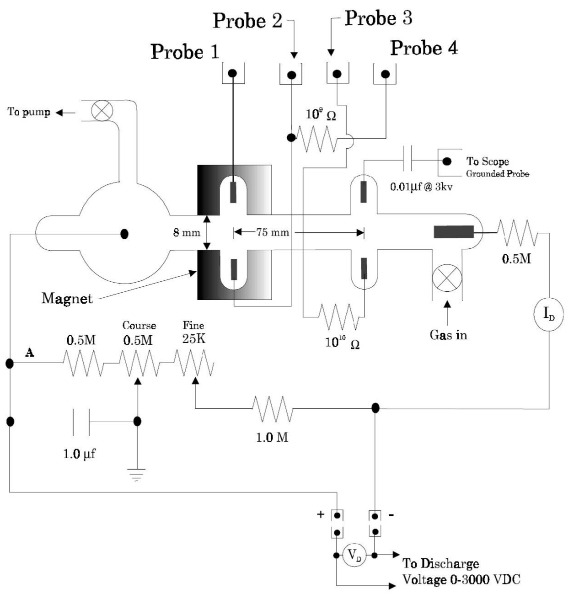
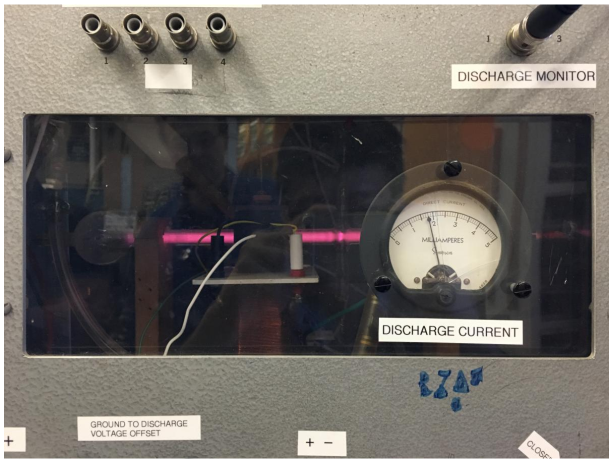


Figure 3: *Block diagram of experimental setup showing gas cylinder, gas flow valves, discharge tube and vacuum pump.*

Once we have a continuous gas glow in the discharge tube, we can create the plasma. To do so, we apply a strong discharge voltage across the tube, measured between probes 2 and 3, 75mm apart Figure 4a. This is the discharge voltage and it is supplied by the high voltage power supply at the base of the electronics rack. There is a potentiometer circuit in place to measure the discharge voltage and Hall voltage (between probes 1 and 2, 8mm apart); and there is a current meter shown in Figure 4b to measure the discharge current which should be regulated between $0.5mA$ to $2mA$. In addition, we can apply an external magnetic field that allows us to measure the Hall effect. The field is created by sending a current through a Helmholtz coil around region lateral to the Hall probes. Varying the magnet current varies the field through the coils thus the Hall voltage. Note, it is important to make sure the oscilloscope doesn't read any variations in signal larger than $50mV$ peak to peak, if it does, we have significant instabilities in our plasma and thus, large error in our measurements.



(a) *Electrical system showing discharge tube, associated potential differences.*



(b) *Image showing the discharge tube with stable plasma.*

Figure 4: *Electrical experimental setup and gaseous discharge tube.*

Data and Procedure

In this section we explore the various data that we measured and their associated errors. We will look at the non-Ohmic relation of the discharge current as a function of discharge voltage. From this data we will acquire the range of high voltage values over which the plasma is stable. Following this, we look at the weak, but evident hysteresis effect in the ferromagnet used to generate the external magnetic field. We can use this result to map our magnet currents to magnetic field strengths which we use for the next set of measurements. Lastly, we measure the Hall voltage as a function of the magnet current and use our theoretical understanding of Hall effect in discharge tubes to get the Hall field,

$$E_H = \frac{2V_H}{d} \; where \; d = 8mm \tag{016}$$

Discharge Voltage vs Discharge Current for Various Pressures

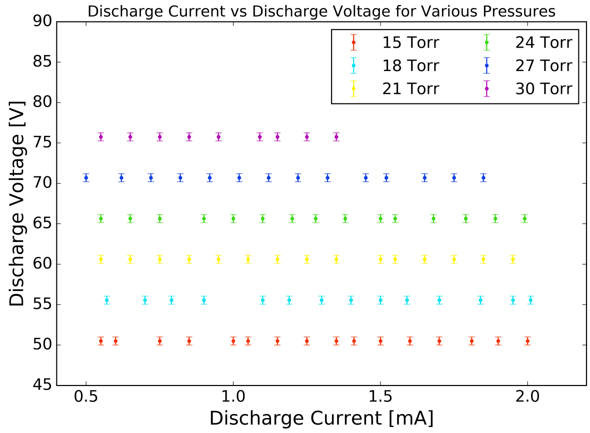
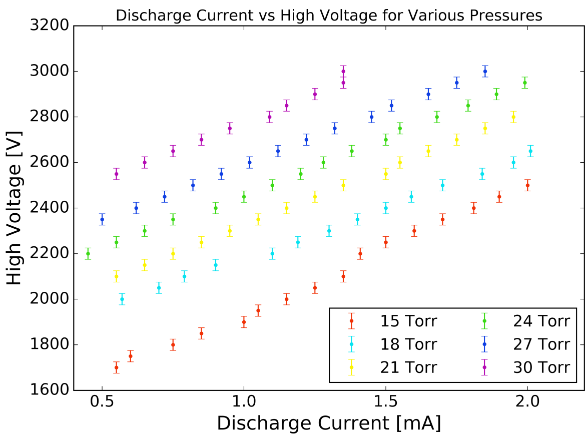
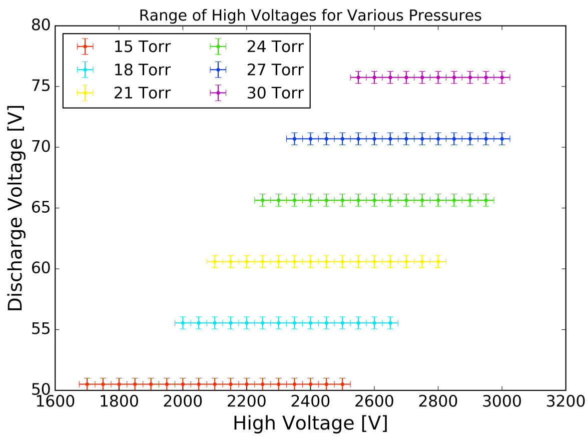


Figure 5: *Non Ohmic relation for V_o and I_d. Systematic error in V_o of $\pm 0.5V$*

First, to illustrate the nonlinear resistivity of the plasma discharge we measure the discharge voltage V_o as a function of discharge current I_d. The discharge voltage is measured between probes 2 and 3 for pressures ranging from 15 $Torr$ to 30 $Torr$. We know normal conductors obey Ohm's Law with a linear relationship between voltage and current. This is decidedly not the case for the discharge tube as we can see in Figure 5. This highlights the atypical relationship between V_o and I_d in a plasma discharge. We find the voltage is independent, within an error, of the discharge current. The overall voltage level increases linearly with pressure Figure 6a. Since increased pressure means a higher number density of charge carriers, the plasma requires a higher voltage to discharge. We make note of the range of high voltages over which the plasma is stable Figure 6b. We make sure to keep the voltage centered in this ideal glow range for each pressure when performing our measurements. Note, the discharge current is limited by a $0.5M\Omega$ impedance to keep the discharge current below $3mA$.



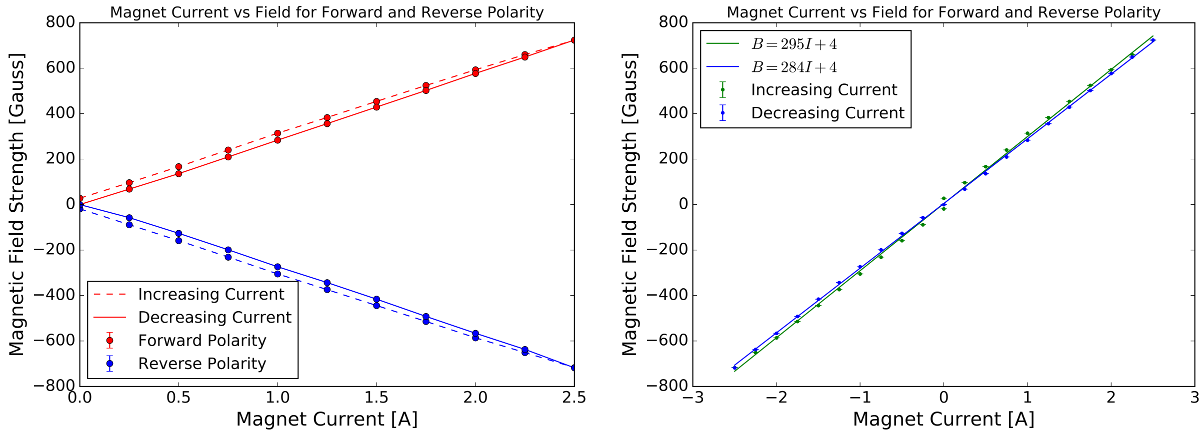
(a) *Discharge current versus high voltage. We note that it takes a higher voltage to have stable plasma with increasing gas pressure.*



(b) *High voltage versus discharge voltage to see the range of voltages over which the plasma is stable. It clearly increases within our boundaries.*

Figure 6: *Exploring the range of valid high voltage values with the pressure dependence. Systematic error in V_o of $\pm 0.5V$, and in high voltage of $\pm 25V$*

Hysteresis: Magnet Current vs Magnetic Field Strength

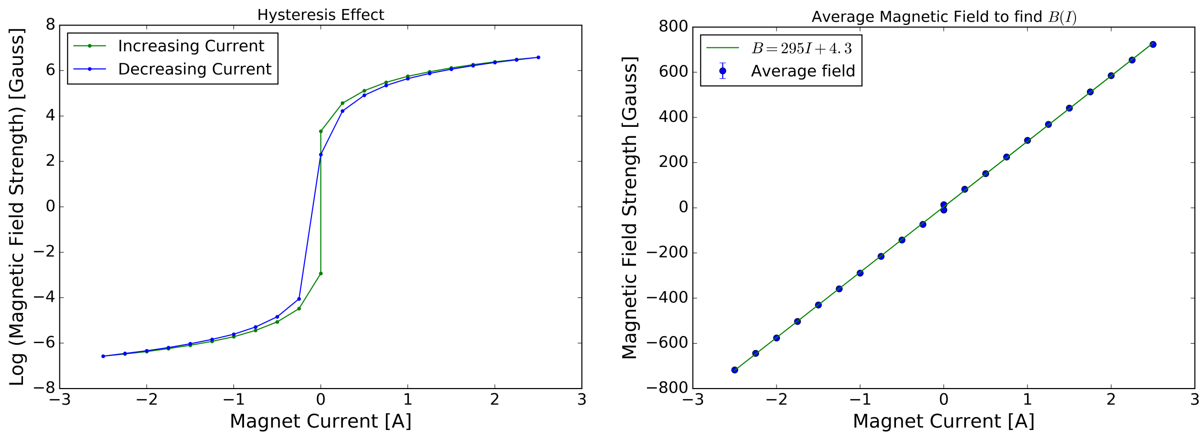


(a) *Magnetic field strength B_M vs magnetic current I_M, for both normal and reverse polarity, and for increasing and decreasing current.*

(b) *Magnetic field strength B_M as a function of magnetic current I_M, with currents increasing and decreasing to get a better fit line through 0.*

Figure 7: *Magnetic field versus magnetic current. We plot linear fits to get B(I).*

We explore the magnetic field produced as a function of the magnetic current. We measured the field response from -2.5 to $+2.5A$ in $0.25A$ increments. The maximum magnetic field strength recorded was 723 $Gauss$, the field response appears to be linear as seen in Figure 7b. The difference of y-intercept for the normal and reverse polarity fits is 20 $Gauss$ for decreasing current, and 49 $Gauss$ for increasing current. This is indicative of a small hysteresis effect in the magnet. To better quantify this effect and possible errors incurred as a result, we traced the magnetic field strength from zero current, up to the maximum, and back to zero; then we flipped the magnet polarity and traced the current from zero to the minimum and back to zero. This continuous trace of current versus magnetic field produces the typical Hysteresis effect plot in Figure 8a.



(a) *The hysteresis curve observed for continuous variation of magnet current.*

(b) *Average line of I_M vs B_M to create a line of best fit.*

Figure 8: *Hysteresis effect and line of best fit.*

The Hysteresis in this case results from the alignment of ferromagnetic domains in the iron dielectric used to increase the strength of the magnet. This is not a memoryless process and the domains “remember” the history of applied magnetic fields. Reducing the current to zero does not bring the magnetic field to zero. To zero the field, an AC current should be applied and slowly reduced to zero. However, for the purposes of this experiment, the hysteric effect is minimal and we disregard it for further analysis.
We also use this data to map the magnetic field strength as a function of the magnet current by performing a conventional least squares fit. To do so, we use the magnetic field averaged over the forward, reverse polarities, and over the increasing, decreasing currents as seen in Figure 8b. This function is necessary since we cannot run the gauss-meter and directly measure the magnetic field while running the high voltage power supply.

Linear Least Squares Fit

Here we make a quick statistical detour to explore the method of getting a linear fit and its associated errors, along with a goodness fit. To do so we can use the following equations 017 to find m and c using a conventional least squares method [6].

$$m = \left(\overline{I_M B_M} - \overline{I_M}\ \overline{B_M}\right) / \left(\overline{I_M^2} - \overline{I_M}^2\right) \qquad c = \overline{B_M} - m\overline{I_M} \tag{017}$$

Where the notation $\overline{I_M}$ and $\overline{B_M}$ represents the statistical mean of our data. Furthermore, in order to find the errors in our fit we can use Equation 6.23 from Bevington [5]. We will later scale these errors through error propagation to find parameters associated with the electron gas.

$$\sigma_m^2 = \frac{\sigma^2}{N\sum I_M^2 - \sum I_M{}^2} \qquad \sigma_c^2 = \frac{\sigma^2 \sum I_M^2}{N\sum I_M^2 - \sum I_M{}^2} \qquad \sigma^2 = \frac{1}{N}\sum (fit - data)^2 \tag{018}$$

Here σ^2 is the variance calculated from the residual errors and the errors in m and c follow from error propagation. Lastly we want to compute a goodness of fit statistic to ensure that our data is reliable, for linear regression we can determine the R^2 statistic, or the Coefficient of Determination which is the *"proportion of the variance in the dependent variable that is predictable from the independent variables"*[7] given by the following equation where the closer R^2 is to 1, the better your fit:

$$R^2 = 1 - \frac{\sum (fit - B)^2}{\sum (fit - \overline{B})^2} \tag{019}$$

To be completely thorough, we also look at the reduced chi-squared statistic which can perform a goodness of fit test for various fitting functions, unlike the Coefficient of Determination which is effective only for linear regression. v here represents the number of effective degrees of freedom which is $(N - fitting\ paramters)$ where N is the number of data points. The reduced chi-square is interpreted as showing a good fit when the value is close to 1 [5].

$$\chi_v^2 = \frac{1}{v}\sum_i^N \frac{(fit - B)^2}{fit} \tag{020}$$

Table 1 summarizes the best fit lines, errors, and goodness of fits for the magnetic field as a function of magnet current.

m [$Gauss/A$]	c [$Gauss$]	σ_m [$Gauss/A$]	σ_c [$Gauss$]	R^2	χ^2_ν
289.767	4.55681	1.02845	0.695364	0.999884	0.961568

Table 1: *Summary of slopes, intercepts, errors and goodness of fit for the magnet current to magnetic field strength mapping.*

For the following sections we use;

$$\mathbf{B(I) = 289I + 4.56} \tag{021}$$

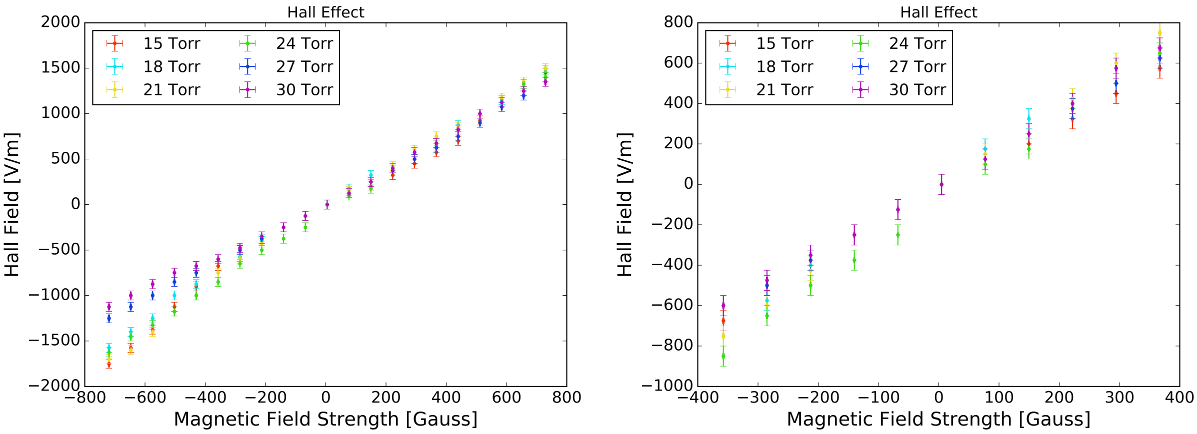
To get the total error on magnetic field, we combine the statistical error with the systematic error which we found to be $\pm 0.3\ Gauss$:

$$\sigma_B = \sqrt{\sigma_{sys}^2 + \sigma_{stat}^2} \tag{022}$$

$$\sigma_B = \sqrt{(I\sigma_m)^2 + \sigma_c^2 + \sigma_{sys}^2} \tag{023}$$

Hall field versus Magnetic field

Finally, we go on to measure the Hall field E_H as a function of magnetic field strength over a range of pressures to explore functional dependence. The Hall voltage V_H is measured directly between probes 1 and 2 4a. The voltage is converted to the Hall field in units of V/m by using Equation 016. We take measurements for the full range of magnet current between $-2.5A$ to $2.5A$, as well as both positive and negative polarities of the magnet coils. It is important to adjust the potentiometer bridge after each change of parameters to keep the probes floating near ground potential. We convert the current value to magnetic field using our relation from Equation 021. The results are plotted in Figure 9a. We expect to see a linear relation between the Hall field and the external magnetic field in the low field limit shown in Figure 9b ($B_M \leq 300\ Gauss$). The relation is roughly linear over the entire range of magnetic fields however, we constrain the rest of our analysis to the linear regime. The deviation from linearity at high magnetic fields is likely due to induced distortions of the charge distribution which occurs when a high enough magnetic field nullifies the assumptions we made about the axial geometry of the setup. We plot the best fit lines for each pressure in the linear regime in Figure 10. The error in Hall field is introduced by systematic uncertainties in the Hall voltage $\sigma_{V_H} \approx 0.2V$ which propagates to the Hall field as $\sigma_{E_H} = 2\sigma_{V_H}/8mm$; while the error in magnetic field is found as described in the previous section. The residuals read up to 150 $Gauss$ for some of the pressures. This is a high percentage error and it might be a source of error in our analysis later. With the relations explored in this section, we can now calculate several properties of the plasma and compare with theoretical predictions.



(a) *B_H vs E_H over the full range of magnet currents.*

(b) *B_H vs E_H over the linear regime: ± 366 Gauss.*

Figure 9: *Linear relationship between magnetic field and Hall field.*

P *Torr*	m $[V/m \cdot G]$	c $[V/m]$	σ_m $[V/m \cdot G]$	σ_c $[V/m]$	R^2
15	1.67532	-24.3008	0.0470248	1.031611	0.993736
18	2.02356	5.36229	0.0304948	0.668982	0.998186
21	2.02356	-7.13770	0.0346498	0.760132	0.997659
24	2.02984	-77.9996	0.0601279	1.319060	0.993029
27	1.70983	-5.70808	0.0096592	0.211901	0.999744
30	1.76630	10.7012	0.0343407	0.753351	0.996985

Table 2: *Summary of slopes, intercepts, errors and goodness of fit for the Hall field as a function of magnetic field strength*

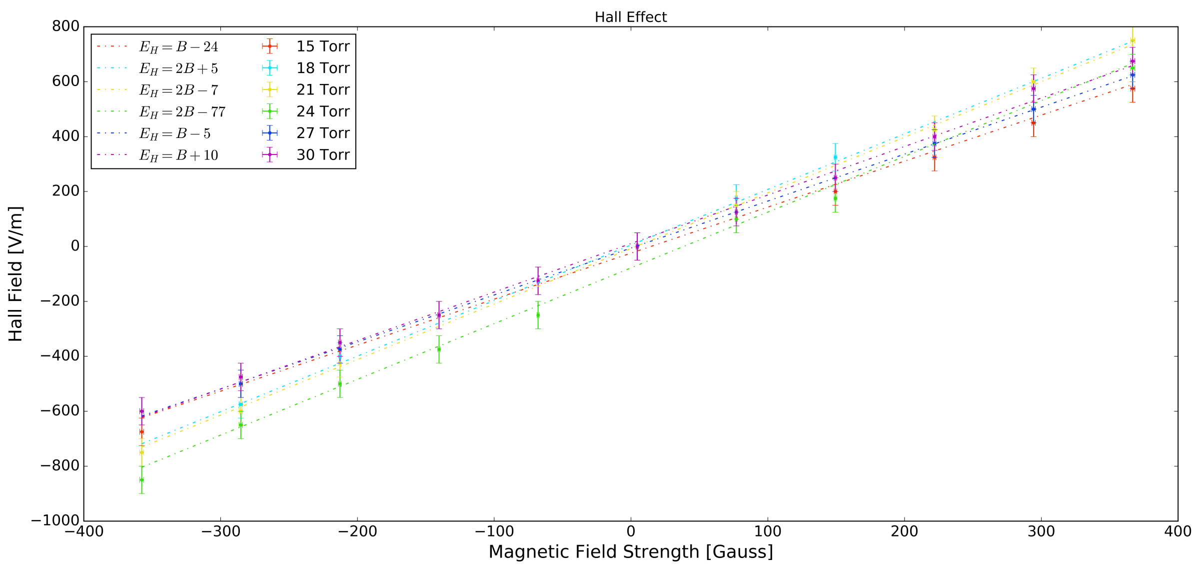


Figure 10: *Linear regime in low field limit with lines of best fit for each pressure.*

Analysis and Results

In this section we analyze various plasma parameters as functions of pressure and try to understand the associated error. The Figure below summarizes all the quantities we will explore.

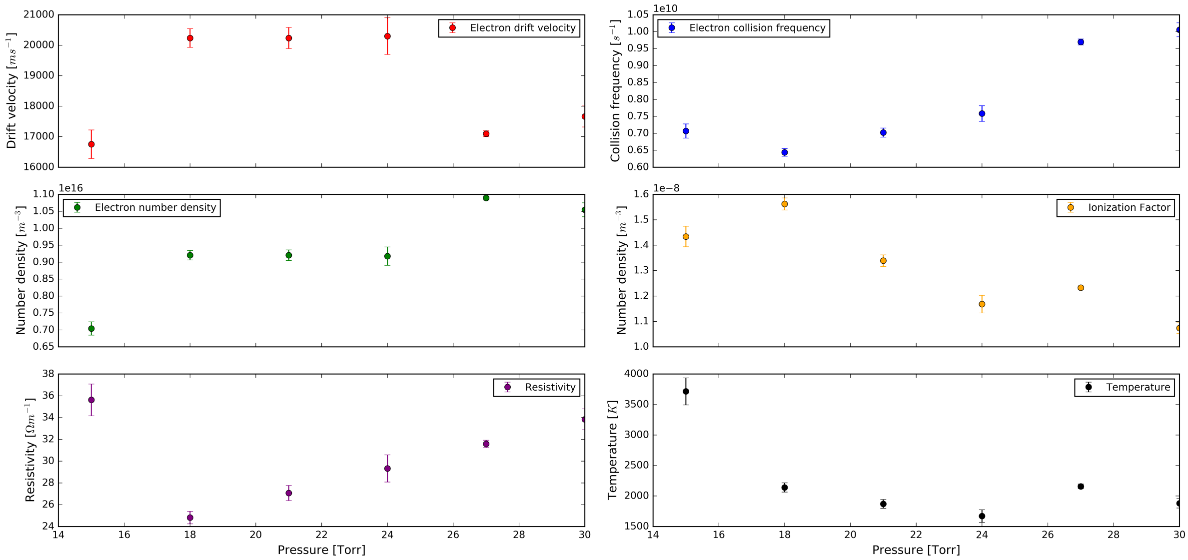


Figure 11: *Pressure dependence of plasma parameters*

Electron Drift Velocity

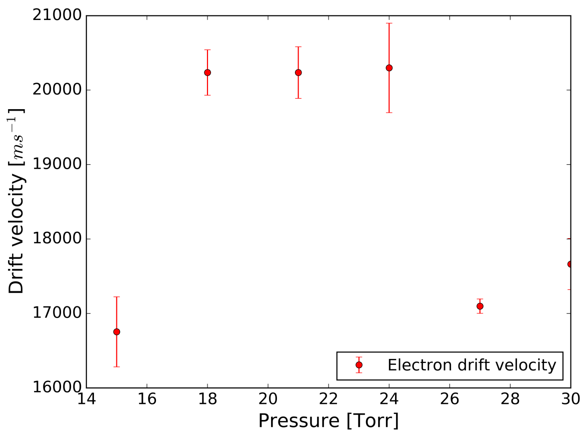


Figure 12: Calculated drift velocities.

P $[Torr]$	Δu_e $[m/s]$	$\sigma_{\Delta u_e}$ $[m/s]$
15	16753.2	470.248
18	20235.6	304.948
21	20235.6	346.498
24	20298.4	601.279
27	17098.3	96.5925
30	17663.0	343.407

Table 3: Electron drift velocities and errors for a range of pressures.

Referring back to Equation 07 we can see that the drift velocity of electrons in the plasma is related to the Hall field and the external magnetic field, by E_H/B. We need to scale our results in Table 2 by a factor of 10^{-4} to convert into MKS units of velocity. But we can easily find the drift velocities for each pressure value from the slopes of the lines in Figure 10. The error in our drift velocity comes from errors introduced by the Hall field and external magnetic field as outlined previously.
We can compare our calculated velocities to the theoretical value for the drift velocity,

$\Delta u_e = 30000 m/s$. Our values are on the same order of magnitude however, there doesn't appear to be a trend with pressure. We expect the drift velocity to decrease with pressure since higher pressures have correspondingly higher charge carrier densities, and so more frequent collisions and lower momenta. In order to better see the trends, we could repeat the experiment several times to reduce errors and consider other systematic errors which were not accounted for.

Electron Collision Frequency

From Equation 08 we can find a relation for the mean electron collision frequency ν in terms of the drift velocity, Δu_e and the Ohmic field E_o which we determined previously is maintained at a constant value for each pressure. Theoretically, we expect the collision frequencies to increase with pressure as the number of particles in the plasma increases. This trend can be observed in Figure 13, except for the point at 15 $Torr$ which proves to be anomalous throughout our analysis likely due to a measurement error or plasma instability when measuring the Hall voltage.
From the sample values in the lab writeup [3] we can find a theoretical value of $\nu \approx 2.9 \times 10^{10}\ s^{-1}$. On average, our collision frequencies were on order of $10^9\ s^{-1}$ which is off by one order of magnitude. We can attribute this error to an inexplicable problem we came across: the writeup quotes an average Ohmic field of 5000 V/m while on average, we found that the Ohmic fields we were operating with were 800 V/m, or roughly an order of magnitude lower.

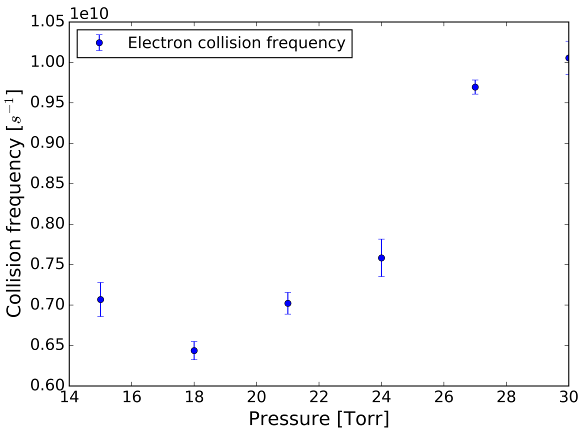


Figure 13: Electron collision frequencies.

P [$Torr$]	ν [$1/s$]	σ_ν [$1/s$]
15	7.07E+09	2.10E+08
18	6.44E+09	1.13E+08
21	7.02E+09	1.33E+08
24	7.58E+09	2.32E+08
27	9.70E+09	8.78E+07
30	1.01E+10	2.06E+08

Table 4: Electron collision frequencies and errors for a range of pressures.

The errors in collision frequencies are found by propagating the error in Ohmic field measurements and electron drift velocities.

$$\nu = \frac{qE_o}{m_e \Delta u_e} \qquad \Rightarrow \qquad \sigma_\nu = \sqrt{\left(\frac{\partial \nu}{\partial E_o}\right)^2 \sigma_{E_o}^2 + \left(\frac{\partial \nu}{\partial \Delta u_e}\right)^2 \sigma_{\Delta u_e}^2} = \sqrt{\left(\frac{q\sigma_{E_o}}{m_e \Delta u_e}\right)^2 + \left(\frac{qE_o \sigma_{\Delta u_e}}{m_e \Delta u_e^2}\right)^2} \tag{024}$$

Electron Number Density

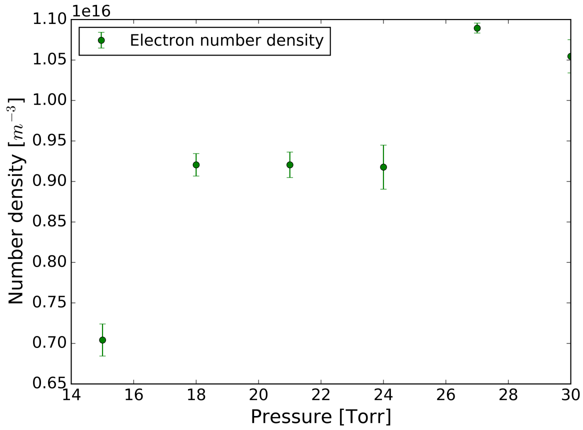


Figure 14: Electron number densities.

P [$Torr$]	n_e [$1/m^3$]	σ_{n_e} [$1/m^3$]
15	7.04E+15	1.98E+14
18	9.21E+15	1.39E+14
21	9.21E+15	1.58E+14
24	9.18E+15	2.72E+14
27	1.09E+16	6.15E+13
30	1.05E+16	2.05E+14

Table 5: Electron number densities and errors for a range of pressures.

The electron number density is evaluated from the current density across the tube cross section of $A \approx 0.5\ cm^{-2}$,

$$\vec{j} = qn_e\Delta\vec{u}_e = \vec{I}A \qquad \Rightarrow \qquad n_e = \frac{I_d B}{qAE_H} = \frac{I_d}{qA\Delta u_e} \tag{025}$$

We find that the number density of electrons is inversely proportional to the electron drift velocity which makes sense theoretically. Thus we expect to see the number density of electrons increase with pressure, once again, we don't quite see this trend in our data without repeated measurements for better accuracy. The theoretical value from the lab writeup [3] was given to be $n_e \approx 2 \times 10^{15}\ m^{-3}$. Our values ranged from $10^{15} - 10^{16}\ m^{-3}$ producing that pesky order of magnitude offset once again. We computed the error by propagating the uncertainty in drift velocity;

$$n_e = \frac{I_d}{qA\Delta u_e} \qquad \Rightarrow \qquad \sigma_{n_e} = \frac{I_d \sigma_{\Delta u_e}}{qA\Delta u_e^2} \tag{026}$$

Ionization Factor

The ionization factor, or the degree of ionization tells us what fraction of the gaseous particles have been ionized to charged particles, which in this case is given by n_e/N_g where $N_g = P/k_b T$ from the ideal gas law and T is the temperature of the plasma gas which we approximate as room temperature, 295 K. The theoretical ionization factor for this setup is about 2×10^{-9}, and our measurement on average is about 10^{-8}. The error in ionization factor is simply σ_{n_e}/N_g.

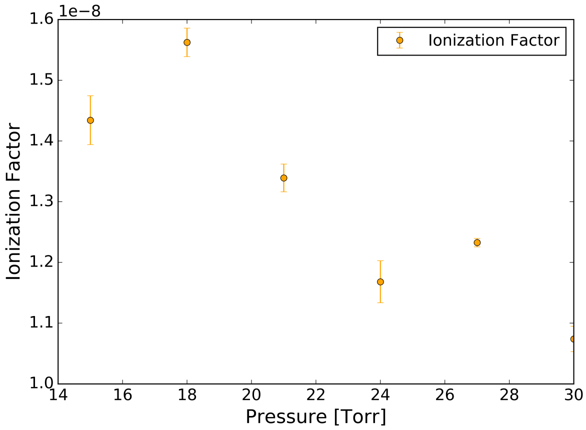


Figure 15: Ionization factors.

P $[Torr]$	n_e/N_g	σ_{n_e/N_g}
15	1.43E-08	4.03E-10
18	1.56E-08	2.35E-10
21	1.34E-08	2.29E-10
24	1.17E-08	3.46E-10
27	1.23E-08	6.96E-11
30	1.07E-08	2.09E-10

Table 6: Ionization factors and errors for a range of pressures.

While it is difficult to observe an obvious decreasing trend in the experimental ionization factors, we can consider the theoretical distribution with pressures over a range of allowed discharge currents. We can see that, as the discharge current increases, the number density also increases, thus ionization current scales linearly with I_d. More interestingly, the ionization factor decreases with pressure since at higher pressures, N_g is greater however, n_e is roughly the same. At low pressures, the ionization factor is higher and the plasma is unstable thus it is more susceptible to instabilities and striations.

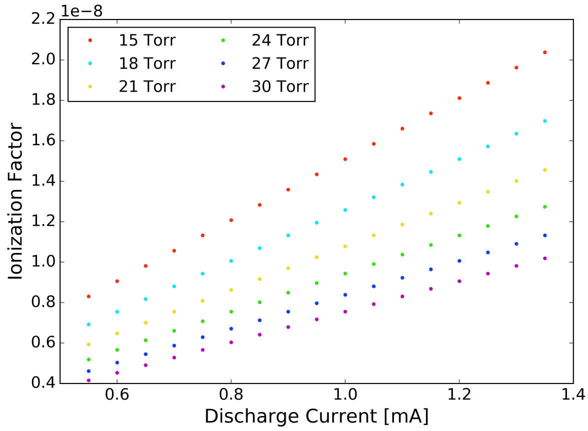


Figure 16: *Ionization factor versus discharge current.*

Resistivity

We recall the resistance to electrical charges in the plasma arises from the frictional force due to collisions of particles, then from Equation 04 we get the following relations for the resistivity and the associated error,

$$\rho = \frac{m_e \nu}{n_e q^2} \quad \Rightarrow \quad \sigma_\rho = \sqrt{\left(\frac{m_e \sigma \nu}{q^2 n_e}\right)^2 + \left(\frac{m_e \nu \sigma_{n_e}}{q^2 n_e^2}\right)^2} \tag{027}$$

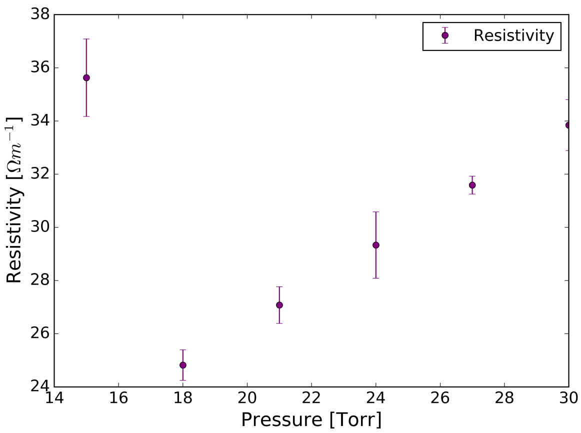


Figure 17: Plasma resistivity.

P $[Torr]$	ρ $[\Omega/m]$	σ_ρ $[\Omega/m]$
15	35.6267	1.45755
18	24.8199	0.57420
21	27.0763	0.69268
24	29.3327	1.24894
27	31.5890	0.33704
30	33.8454	0.95702

Table 7: Plasma resistivity and errors for a range of pressures.

In general, we expect the resistivity to increase with pressure. The resistance to the flow of current will increase as the pressure increases because the number of gas particles will be greater, thus the overall drift velocity of the electrons will be slower and the resistivity will increase. The theoretical value of resistivity is calculated to be $\rho \approx 533\ \Omega/m$, whereas our average resistivity is about 30 Ω/m. Once again, we come across the order of magnitude offset that is possible introduced by a discrepancy in the Ohmic field.

Electron Temperature and Energy

One of the most interesting phenomenon of the plasma discharge tube is in the temperature of electrons. We can expect T_e to go up to $10,000\ K$ yet the tube is barely warm to the touch. This is because the number density of electrons is very small compared to the number density of gas particles (small ionization factor), thus there are few collisions to allow for momentum and energy transfer from electrons to gas molecules. In addition, the timescale of interactions is so small that very little energy is transferred to the gas and to the tube walls.

We can calculate the electron temperature by assuming a Maxwell-Boltzmann distribution of electron velocities. We use the relation: $\nu \cong N_g \langle \sigma u \rangle_e$ to find the mean electron velocities from our knowledge of the collision frequency of electrons. We must note that the average squared electron velocity: $\langle u \rangle^2$ is not the same as the RMS velocity of the MB Distribution, we explored this relation in the Theory section, so now we can find the electron temperatures from Equation 014. We find that our temperatures are about an order of magnitude lower than the theoretical prediction, however, we would expect the temperatures to generally increase with pressure assuming the ideal gas law but there are some uncertainties because of the motion under electric fields in the plasma.

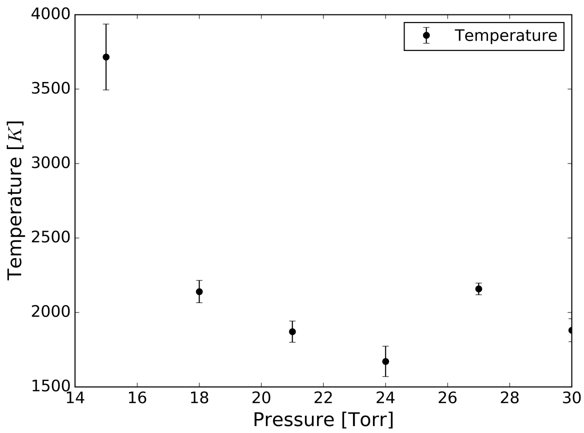


Figure 18: Electron temperatures.

P $[Torr]$	T_e $[K]$	σ_{T_e} $[K]$
15	3715.59	221.182
18	2140.00	75.1279
21	1871.10	71.1293
24	1670.89	102.209
27	2157.88	39.0640
30	1880.25	77.2108

Table 8: Electron temperatures and errors for a range of pressures.

The average kinetic energy of electrons is given by Equation 028 and we can find an upper bound on these energies in Equation 029 where m_g is the mass of Helium molecules.

$$\epsilon = \frac{1}{2} m_e \langle u^2 \rangle \tag{028}$$

$$E = \frac{1}{2} m_g \Delta u \tag{029}$$

The upper limit is reached when electron energy loss due to inelastic collisions (excitation and ionization) is negligible compared to that caused by elastic collisions [3] which is the regime in which our analysis is valid, therefore it is reassuring to know that the average electron energies are all lower than the upper limits for each pressure shown in Table 9.

P $Torr$	ϵ_{avg} $[eV]$	ϵ_{up} $[eV]$
15	0.480329	5.86082
18	0.276646	8.55058
21	0.241884	8.55058
24	0.216003	8.60369
27	0.278958	6.10477
30	0.243067	6.51468

Table 9: *Average energies are on the order of* 10^{-1} eV *and upper bounds are about* 10 eV*, we are well within the limits for our analysis to be valid.*

Additional sources of error

"*The stability of a plasma is dependent on almost anything imaginable, not only the obvious: current, voltage, pressure, magnetic field and mass flow rate; but potentially also the non-obvious: contaminant gases, ambient temperature (untested), ambient light (not observed), and time (very important).*" [3] There are multiple sources of error in an experiment involving plasmas, in our case, particularly the reliability of our Hall voltage and Ohmic field measurements are in question given that all of our analysis values seemed to be off by an order of magnitude, and that the plasma parameters did not follow the trends with pressure as we might expect them to.

Conclusion

The goal of this lab was to examine the physics of the Hall effect in order to determine the relations between variables to understand characteristics of plasma discharge. We explored the response of the plasma to discharge current and found the discharge voltage V_o to be independent of I_d exhibiting a non-Ohmic response in plasmas. We measured the magnetic field strength of the coils as a function of the magnet current to explore the hysteresis effect in our ferromagnetic material. We find a small but noticeable hysteric effect, moreover, we find the magnetic field as a function of the current. Finally, we measured the Hall voltage as a function of magnet current and used our current to magnetic field mapping, and our knowledge of the specific geometry of our setup to find the Hall field as a function of the magnetic field. We found the response fairly linear, and from this we were able to find the electron drift velocity, collision frequency, number density, ionization factor, resistivity, and electron temperature in addition to a few other results. However, due to the unstable nature of plasmas, there are large errors in our measurements and our plasma parameters don't scale quite as we expect them to. In order to accurately determine a trend of the characteristics with the pressure, we must allow repeated experiments.

References

[1] W. B. Kunkel, *Hall Effect In A Plasma*, American Journal of Physics 49, 733 (1981).

[2] A.V. Nedospasov, *Striations*, Soviet Physics Uspekhi 11, 174-187 (1968)

[3] D. Orlando. Hall Effect in Plasma, 2018. `http://experimentationlab.berkeley.edu/sites/default/files/writeups/HAL.pdf`

[4] Kittel, C., Kroemer, H., *Thermal and Statistical Physics*. Wiley, 2nd ed., 1980.

[5] Bevington, P., *Data Reduction and Error Analysis for the Physical Sciences.* McGraw-Hill

[6] Graham, J., R., *Conventional Linear Least-Squares Fitting* `https://drive.google.com/file/d/0B40Ynk22SiBpSU1DN2dPN3pzNXc/view`

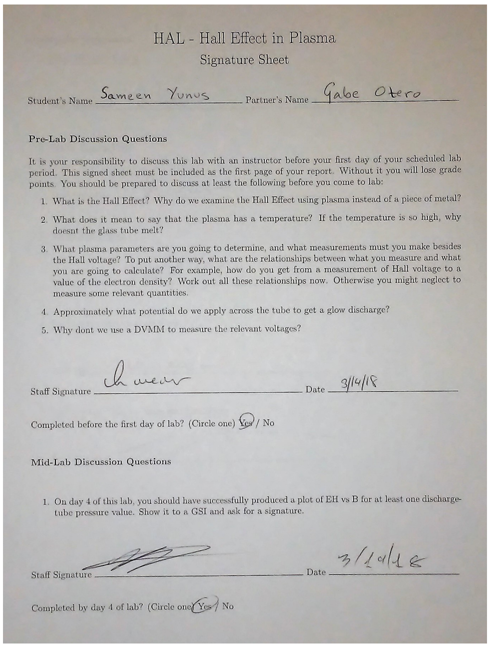
[7] Coefficient of determination, `https://en.wikipedia.org/wiki/Coefficient_of_determination`

[8] Hughes, I. G. and Hase, T. P. A., *Measurements and their Uncertainties, A Practical Guide to Modern Error Analysis.* Oxford University Press, 2010

[9] National Centers for Environmental Information, National Oceanic and Atmospheric Administration, `https://www.ngdc.noaa.gov/geomag-web/#igrfwmm`

Appendix A

Signature Sheet



HAL - Hall Effect in Plasma
Signature Sheet

Student's Name Sameen Yunus Partner's Name Gabe Otero

Pre-Lab Discussion Questions

It is your responsibility to discuss this lab with an instructor before your first day of your scheduled lab period. This signed sheet must be included as the first page of your report. Without it you will lose grade points. You should be prepared to discuss at least the following before you come to lab:

1. What is the Hall Effect? Why do we examine the Hall Effect using plasma instead of a piece of metal?
2. What does it mean to say that the plasma has a temperature? If the temperature is so high, why doesnt the glass tube melt?
3. What plasma parameters are you going to determine, and what measurements must you make besides the Hall voltage? To put another way, what are the relationships between what you measure and what you are going to calculate? For example, how do you get from a measurement of Hall voltage to a value of the electron density? Work out all these relationships now. Otherwise you might neglect to measure some relevant quantities.
4. Approximately what potential do we apply across the tube to get a glow discharge?
5. Why dont we use a DVMM to measure the relevant voltages?

Staff Signature ______ Date 3/14/18

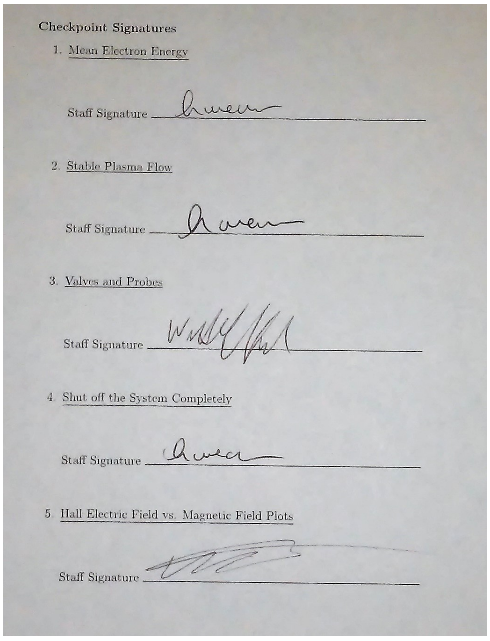
Completed before the first day of lab? (Circle one) Yes / No

Mid-Lab Discussion Questions

1. On day 4 of this lab, you should have successfully produced a plot of EH vs B for at least one discharge-tube pressure value. Show it to a GSI and ask for a signature.

Staff Signature ______ Date 3/19/18

Completed by day 4 of lab? (Circle one) Yes / No



Checkpoint Signatures

1. Mean Electron Energy

   Staff Signature ______

2. Stable Plasma Flow

   Staff Signature ______

3. Valves and Probes

   Staff Signature ______

4. Shut off the System Completely

   Staff Signature ______

5. Hall Electric Field vs. Magnetic Field Plots

   Staff Signature ______

Appendix B

Pre-lab

Problem 1

If we apply a transverse electric field to a conductor, and a magnetic field perpendicular to the transverse electric field, the charge carriers in the conducting material experience a force driving them to the boundaries of the medium. This creates a potential difference between the walls of the conductor which is known as the Hall voltage.

Problem 2

The electrons in the plasma have temperatures of up to $10^4\ K$, but the tube is not hot to the touch. This is because the number density of electrons is very small compared to the number density of gas particles (small ionization factor), thus there are few collisions to allow for momentum and energy transfer from electrons to gas molecules. In addition, the timescale of interactions is so small that very little energy is transferred to the gas and to the tube walls.

Problem 3

We measure:

- discharge current I_d versus discharge voltage V_o
- magnet current I_M versus magnetic field strength B_M
- magnet current I_M versus Hall voltage V_H

We determine the following plasma parameters:

- electron drift velocity
- electron collision frequency
- electron number density
- gas number density
- ionization factor
- resistivity
- electron temperature
- average energies

Problem 4

The plasma tube needs a minimum of 1.5 kV to discharge, and we can go up to 3 kV.

Problem 5

The digital magnetometer cannot be operated in the setup while the high voltage power supply is running because the potential difference is too great.

Appendix C

Raw Data

High Voltage /V	V_o /V	I_d /mA
1700	50.5	0.55
1750	50.5	0.6
1800	50.5	0.75
1850	50.5	0.85
1900	50.5	1
1950	50.5	1.05
2000	50.5	1.15
2050	50.5	1.25
2100	50.5	1.35
2150	50.5	1.32
2200	50.5	1.41
2250	50.5	1.5
2300	50.5	1.6
2350	50.5	1.7
2400	50.5	1.81
2450	50.5	1.9
2500	50.5	2

Table 10: I_d vs V_o, 15 $Torr$

High Voltage /V	V_o /V	I_d /mA
1850	55.55	0.22
1900	55.55	0.33
1950	55.55	0.41
2000	55.55	0.57
2050	55.55	0.7
2100	55.55	0.79
2150	55.55	0.9
2200	55.55	1.1
2250	55.55	1.19
2300	55.55	1.3
2350	55.55	1.4
2400	55.55	1.5
2450	55.55	1.59
2500	55.55	1.7
2550	55.55	1.84
2600	55.55	1.95
2650	55.55	2.01

Table 11: I_d vs V_o, 18 $Torr$

High Voltage /V	V_o /V	I_d /mA
2100	60.6	0.55
2150	60.6	0.65
2200	60.6	0.75
2250	60.6	0.85
2300	60.6	0.95
2350	60.6	1.05
2400	60.6	1.15
2450	60.6	1.25
2500	60.6	1.35
2550	60.6	1.5
2600	60.6	1.55
2650	60.6	1.65
2700	60.6	1.75
2750	60.6	1.85
2800	60.6	1.95

Table 12: I_d vs V_o, 21 $Torr$

High Voltage /V	V_o /V	I_d /mA
2200	65.65	0.45
2250	65.65	0.55
2300	65.65	0.65
2350	65.65	0.75
2400	65.65	0.9
2450	65.65	1
2500	65.65	1.1
2550	65.65	1.2
2600	65.65	1.28
2650	65.65	1.38
2700	65.65	1.5
2750	65.65	1.55
2800	65.65	1.68
2850	65.65	1.79
2900	65.65	1.89
2950	65.65	1.99

Table 13: I_d vs V_o, 24 $Torr$

High Voltage /V	V_o /V	I_d /mA
2350	70.7	0.5
2400	70.7	0.62
2450	70.7	0.72
2500	70.7	0.82
2550	70.7	0.92
2600	70.7	1.02
2650	70.7	1.12
2700	70.7	1.22
2750	70.7	1.32
2800	70.7	1.45
2850	70.7	1.52
2900	70.7	1.65
2950	70.7	1.75
3000	70.7	1.85

Table 14: I_d *vs* V_o, 27 $Torr$

High Voltage /V	V_o /V	I_d /mA
2550	75.75	0.55
2600	75.75	0.65
2650	75.75	0.75
2700	75.75	0.85
2750	75.75	0.95
2800	75.75	1.09
2850	75.75	1.15
2900	75.75	1.25
2950	75.75	1.35
3000	75.75	1.35

Table 15: I_d *vs* V_o, 30 $Torr$

Forward polarity				**Reverse polarity**			
I_{inc}		I_{dec}		I_{inc}		I_{dec}	
I_M /A	B_M /G	I_M /A	B_M /G	I_M /A	B_M /G	I_M /A	B_M /G
0	0	0	27.9	0	-0.1	0	-18.8
0.25	68.1	0.25	96.4	0.25	-57.5	0.25	-88.5
0.5	136.1	0.5	166.8	0.5	-126.4	0.5	-158.6
0.75	209.6	0.75	240.4	0.75	-198.8	0.75	-231
1	283.4	1	314	1	-273.5	1	-305
1.25	356	1.25	383	1.25	-343	1.25	-374
1.5	429	1.5	454	1.5	-416	1.5	-444
1.75	502	1.75	524	1.75	-492	1.75	-514
2	577	2	593	2	-566	2	-586
2.25	649	2.25	660	2.25	-637	2.25	-651
2.5	724	2.5	723	2.5	-718	2.5	-717

Table 16: *Magnet current versus Magnet field strength*

Forward Polarity		**Reverse Polarity**	
I_B /A	V_H /V	I_B /A	V_H /V
0	0	0	0
0.25	0.7	0.25	-0.5
0.5	0.8	0.5	-1
0.75	1.3	0.75	-1.5
1	1.8	1	-2
1.25	2.3	1.25	-2.7
1.5	2.8	1.5	-3.6
1.75	3.7	1.75	-4.5
2	4.3	2	-5.5
2.25	5	2.25	-6.3
2.5	5.8	2.5	-7

Table 17: *Hall voltage vs Magnet Current,* 15 $Torr$

Forward Polarity		**Reverse Polarity**	
I_B /A	V_H /V	I_B /A	V_H /V
0	0	0	0
0.25	0.7	0.25	-0.5
0.5	1.3	0.5	-1
0.75	1.7	0.75	-1.6
1	2.4	1	-2.3
1.25	3	1.25	-3
1.5	3.5	1.5	-3.5
1.75	4	1.75	-4
2	4.6	2	-5
2.25	5.4	2.25	-5.6
2.5	5.9	2.5	-6.3

Table 18: *Hall voltage vs Magnet Current,* 18 $Torr$

Forward Polarity		**Reverse Polarity**	
I_B /A	V_H /V	I_B /A	V_H /V
0	0	0	0
0.25	0.6	0.25	-0.5
0.5	1	0.5	-1
0.75	1.7	0.75	-1.7
1	2.4	1	-2.4
1.25	3	1.25	-3
1.5	3.4	1.5	-4
1.75	4	1.75	-4.7
2	4.7	2	-5.6
2.25	5.4	2.25	-6.4
2.5	6	2.5	-6.8

Table 19: *Hall voltage vs Magnet Current*, 21 $Torr$

Forward Polarity		**Reverse Polarity**	
I_B /A	V_H /V	I_B /A	V_H /V
0	0	0	0
0.25	0.4	0.25	-1
0.5	0.7	0.5	-1.5
0.75	1.5	0.75	-2
1	2	1	-2.6
1.25	2.6	1.25	-3.4
1.5	3.3	1.5	-4
1.75	3.6	1.75	-4.7
2	4.5	2	-5.3
2.25	5.3	2.25	-5.8
2.5	5.6	2.5	-6.5

Table 20: *Hall voltage vs Magnet Current*, 24 $Torr$

Forward Polarity		**Reverse Polarity**	
I_B /A	V_H /V	I_B /A	V_H /V
0	0	0	0
0.25	0.5	0.25	-0.5
0.5	1	0.5	-1
0.75	1.5	0.75	-1.5
1	2	1	-2
1.25	2.5	1.25	-2.4
1.5	3	1.5	-3
1.75	3.6	1.75	-3.4
2	4.3	2	-4
2.25	4.8	2.25	-4.5
2.5	5.4	2.5	-5

Table 21: *Hall voltage vs Magnet Current*, 27 $Torr$

Forward Polarity		**Reverse Polarity**	
I_B /A	V_H /V	I_B /A	V_H /V
0	0	0	0
0.25	0.5	0.25	-0.5
0.5	1	0.5	-1
0.75	1.6	0.75	-1.4
1	2.3	1	-1.9
1.25	2.7	1.25	-2.4
1.5	3.3	1.5	-2.7
1.75	4	1.75	-3
2	4.5	2	-3.5
2.25	5	2.25	-4
2.5	5.4	2.5	-4.5

Table 22: *Hall voltage vs Magnet Current*, 30 $Torr$