ECON G6905

Topics in Trade

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Columbia University
in the City of New York

This week: The canonical urban model

- Monocentric city model
- Contrast with quantitative spatial models
- Embedded in a system of cities

Overview of monocentric city models

The monocentric city model is spatial equilibrium in the simplest commuting geography: a single, exogenous central workplace

- Basic 1: linear geography with continuum of locations and unit housing demand
- Basic 2: linear geography with continuum of locations and continuum of identical individuals
- Multiple commuting modes/technologies (LeRoy and Sonstelie 1983)
- Endogenize firm location (Fujita and Ogawa 1982)

The basic monocentric city model

All employment is at the city center, $\tau = 0$. Commuting costs rise with distance to the center, τ. What is the equilibrium rent at τ?

Better presentations of this material:

- Video: Kevin Murphy. "Location Choice: An Introduction to Compensating Differences"
- Handbook chapter: Gilles Duranton and Diego Puga. Section 8.2 of "Urban Land Use"

Kevin uses unit housing demand, opportunity cost of time, and heterogeneous workers. Gilles and Diego use general preferences, commuting costs in units of the numeraire good, and homogenous workers. The beauty of the economic logic comes through in both presentations, but these details matter for empirical investigations.

Basic monocentric city model: Setup

Like Murphy, our formulation in Davis and Dingel (2020) uses unit housing demand, opportunity cost of time, and heterogeneous workers.

- Locations within cities vary in their desirability τ; schedule is $S(\tau)$
- Relative to DD (2020), assume only one city and ignore sectoral choice σ
- Utility is consumption of the numeraire final good, which is income minus locational cost:

$$U(c, \tau; \omega) = T(\tau)G(\omega) - r(c, \tau)$$

- In DD (2020), $T'(\tau) < 0$ may be interpreted as commuting to CBD, proximity to productive opportunities, or consumption value
- Commuting time $T(\tau) = 24 - L - \tau$ and $G(\omega)$ is wage of skill ω
- Linear city $S(\tau) = \tau$; disc city $S(\tau) = \pi\tau^2$
- More skilled are more willing to pay for more attractive locations (Net income is supermodular in $G(\omega)$ and $T(\tau)$)

Basic monocentric city model: Equilibrium conditions

Individuals maximize their utility by their choices of city, location, and sector such that

$$f(c, \omega, \tau) > 0 \iff \{c, \tau\} \in \arg\max U(c, \tau; \omega)$$

Profit maximization by absentee landlords engaged in Bertrand competition causes unoccupied locations to have rental prices of zero,

$$r(c, \tau) \times \left(S'(\tau) - L \int_{\omega \in \Omega} f(\omega, c, \tau) \mathrm{d}\omega \right) = 0 \ \forall c \ \forall \tau$$

Single (closed) city has an exogenous population $L(c)$ and skill distribution $F(\omega)$.

Prior to using a differential equation, sketch the outcome for homogenous workers using the dual approach $\bar{U} = U(\tau; \omega) = T(\tau)G(\omega) - r(\tau)$. Then consider discrete types of workers.

(The dual approach is powerful here, as it is in neoclassical trade theory)

Basic monocentric city model: Equilibrium solution

Lemma 6. In autarkic equilibrium, there exists a continuous and strictly decreasing locational assignment function $N : \bar{\mathcal{T}}(c) \to \Omega$ such that $f(\omega, c, \tau) > 0 \iff N(\tau) = \omega$, $N(0) = \bar{\omega}$ and $N(\bar{\tau}(c)) = \underline{\omega}$.

This assignment function is obtained by equating supply and demand of locations:

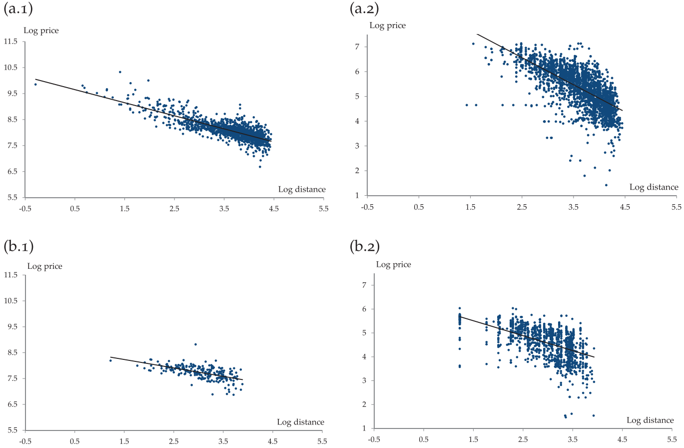
$$S(\tau) = L \int_0^\tau \int_{\omega \in \Omega} f(\omega, c, x, \sigma) \mathrm{d}\omega \mathrm{d}x \implies N(\tau) = F^{-1}\left(\frac{L(c) - S(\tau)}{L(c)}\right)$$

Lemma 7. In autarkic equilibrium, $r(c, \tau)$ is continuously differentiable on $\tau \geq 0$ and given by $r(c, \tau) = -A(c) \int_\tau^{\bar{\tau}(c)} T'(t) G(N(t)) \mathrm{d}t$ for $\tau \leq \bar{\tau}(c)$.

The properties of interest in a competitive equilibrium are characterized by the assignment function $N : \bar{\mathcal{T}}(c) \to \Omega$.

Rent and density gradients

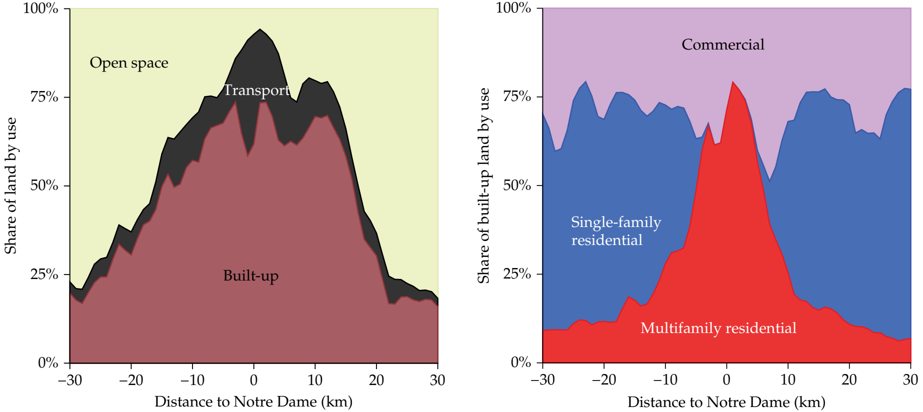
The monocentric geography produces a negative, convex rent gradient (works qualitatively, what about quantitatively?)



Combes, Duranton, Gobillon - The Costs of Agglomeration: House and Land Prices in French Cities

Rent and density gradients

The monocentric geography produces a negative, convex rent gradient and not yet an adequate account of the density gradient



Duranton and Puga (2015): "Since monocentricity can always be rejected, the more interesting question is: How monocentric are cities?"

Introducing housing quantities

Homogeneous workers, commuting costs in terms of goods, and general preferences:

$$\max U(q,z) \text{ s.t. } r(\tau)q + z + \tau \leq w$$

The indirect utility function is $V(r(\tau), w-\tau)$. Spatial equilibrium means

$$V(r(\tau), w-\tau) = \bar{V} \implies \frac{\partial V}{\partial r}\frac{\mathrm{d}r}{\mathrm{d}\tau} + \frac{\partial V}{\partial y}\frac{\partial y}{\partial \tau} = 0 \implies \frac{\mathrm{d}r}{\mathrm{d}\tau} = \frac{\partial V/\partial y}{\partial V/\partial r} = \frac{-1}{q(\tau)} < 0$$

by Roy's identity. The Alonso-Becker insight: transform consumption problem into production problem by putting commuting costs in the budget constraint.

(Note τ is commuting cost, so $\frac{\partial y}{\partial \tau} = -1$. If using distances, $\frac{\partial(w-\tau)}{\partial \text{distance}}$ is in numerator.)

Substitution effect from Hicksian demand h for housing:

$$\frac{\partial q(r(\tau), \bar{V})}{\partial \tau} = \frac{\partial q(r(\tau), \bar{V})}{\partial r(\tau)}\frac{\mathrm{d}r}{\mathrm{d}\tau} \geq 0$$

See Brueckner (1987) and Duranton and Puga (2015) chapters for details, housing supply, and 3 more gradients: land price, housing capital intensity, population density.

Density gradients have flattened over time

$$\ln D(\tau) = \ln D(0) - \gamma\tau$$

TABLE 3

Averages of Gradients by Sector and Year[a]

Sector	1948	1954	1958	1963	1970/1972[b]	1977/1980[b]
Population	0.58	0.47	0.42	0.38	0.29	0.24
Manufacturing	0.68	0.55	0.48	0.42	0.34	0.32
Retailing	0.88	0.75	0.59	0.44	0.35	0.30
Services	0.97	0.81	0.66	0.53	0.41	0.38
Wholesaling	1.00	0.86	0.70	0.56	0.43	0.37

[a]1948–1963 data: [7, Table 12, p. 42].
[b]Noncommensurate years (see text).

Macauley (1985)

Population density gradient is flatter than employment density gradient

See Ken Jackson's *Crabgrass Frontier: The Suburbanization of the United States*

Multiple transportation technologies

- Consider a set of transportation technologies with a trade-off between fixed and variable costs (as a function of distance).
- For example, walking has the highest variable cost and no fixed cost, buses have a lower variable cost and a higher fixed cost, and cars have the lowest variable cost and the highest fixed cost.
- This makes τ as a function of distance the upper envelope of these technologies' costs as a function of distance.

Multiple transport modes and multiple income levels

LeRoy and Sonstelie (1983): Use the monocentric city model to think about changing income gradients (poor centers and rich suburbs vs urban revivals of 1970s and 2000s)

- Contrast cars and buses in terms of fixed monetary cost f^i, variable monetary cost v^i, and variable time cost t^i. Total cost is $f^i + (wt^i + c^i)d$ where d is distance.
- Assume $f^{\text{car}} > f^{\text{bus}}$, $v^{\text{car}} > v^{\text{bus}}$, $t^{\text{car}} < t^{\text{bus}}$.
- If someone uses cars, it will be the high-wage workers (higher value of time).

Bid-rent function is maximum price one would pay for housing at distance d while achieving utility $\bar{u}$:

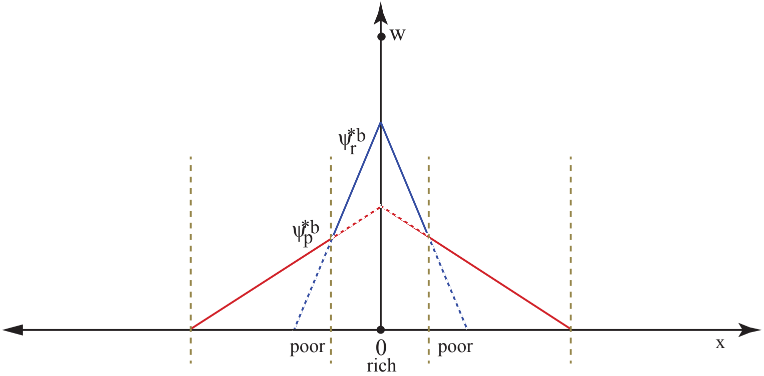
$$\Psi(w, t^i, d, \bar{u}) = \frac{1}{\bar{h}} \left(w - wt^i d - f^i - u^{-1}(\bar{u}) \right)$$

where $\bar{h}$ is fixed housing consumption [see Matt Turner's notes]

LeRoy and Sonstelie (1983): Possible equilibria

A world without cars ("paradise")

Figure 1: Equilibrium with rich and poor households and only bus travel

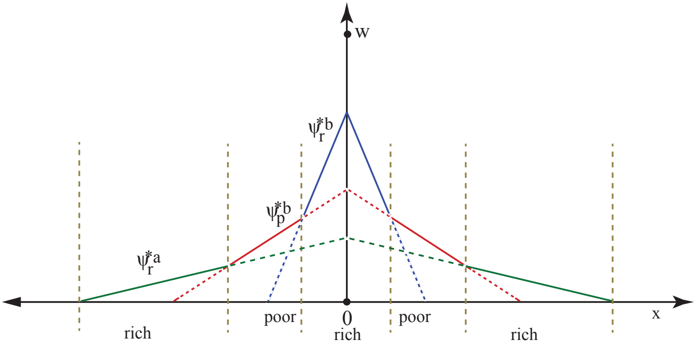


$w_r > w_p$ means rich bus riders outbid poor bus riders for central locations

LeRoy and Sonstelie (1983): Possible equilibria

A world with cars has rich at center and in suburbs

Figure 2: Equilibrium with rich and poor households and cars for the rich



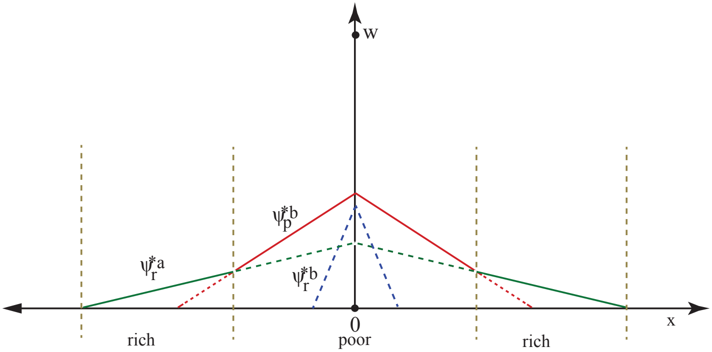
Rich bus riders at center (small d):

$$\frac{1}{\bar{\bar{h}}}\left(w_r - w_r t^{\text{bus}} d - f^{\text{bus}} - u^{-1}(\bar{u})\right) > \frac{1}{\bar{\bar{h}}}\left(w_r - w_r t^{\text{car}} d - f^{\text{car}} - u^{-1}(\bar{u})\right) \iff -w_r t^{\text{bus}} d - f^{\text{bus}} > -w_r t^{\text{car}} d - f^{\text{car}}$$

LeRoy and Sonstelie (1983): Possible equilibria

A center without rich if they demand more housing ($\bar{h}_r > \bar{h}_p$)

Figure 3: Equilibrium with rich and poor households and cars and more money for the rich



The evolution of income gradients and natural amenities as anchors

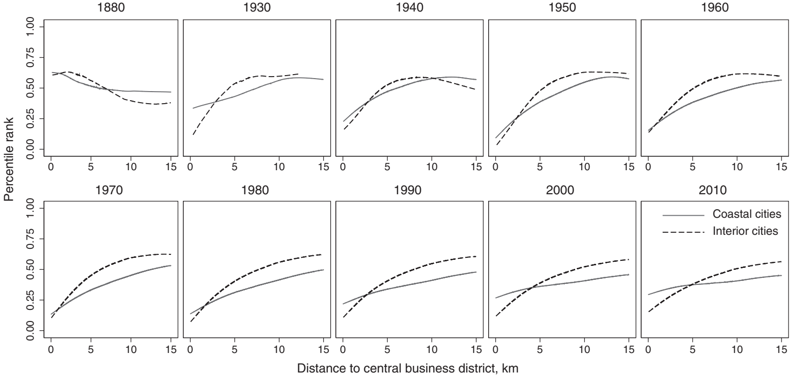


FIGURE 7

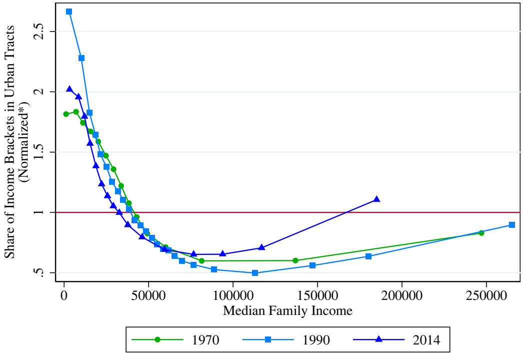
Income and residential location for coastal and interior cities, 1880–2010

Notes: Each plot shows, for a different census year, the pattern of neighbourhood average household income on the vertical axis versus neighbourhood distance to the city centre (up to 15 km) on the horizontal axis. The smoothed lines are from lowess regressions. Two groups of cities are shown in each panel: coastal cities are represented by a solid line and interior cities by a dashed line. The sample is a fixed group of twenty-nine metropolitan areas with some missing city observations from 1930 to 1950. Coastal and interior cities are classified in Online Appendix Table B2.

Lee and Lin (2018)

Urban revival and income sorting

COUTURE *ET AL.* DISTRIBUTIONAL EFFECTS OF URBAN SPATIAL SORTING 3



Normalized by aggregate urban share: 0.174 in 1970, 0.103 in 1990, and 0.077 in 2014

FIGURE 1

Downtown residential income propensity by income

Note: This figure shows sorting by income in 100 large CBSAs in 1970, 1990, and 2014. The 100 CBSAs are those with the largest population in 1990. Each point plots the share of families in a given Census income bracket who reside downtown in a given year—normalised by the share of all families who reside downtown that year—against the median family income (in 1999 dollars) of that Census income bracket in that year. The downtown area of each CBSA is defined as the set of tracts closest to the centre of each CBSA that account for 10% of that CBSA's population in 2000. The number of points on the graph is limited by the number of income brackets reported by the Census for tract-level data. We compute the median income in each bracket using IPUMS microdata (Ruggles *et al.* 2018) for the corresponding year in the 100 largest CBSAs. The IPUMS microdata are adjusted for top-coding using the generalised Pareto method, as described in online Appendix C.

Model with 2 neighborhoods, 2 income levels, and endogenous amenities

Agglomeration economies and firm locations

- We might want w at the CBD to depend on the number of workers (agglomeration economies at a single point)
- Straightforward to have firms use land at center (disc and donut of land use)
- More complex is producing commercial, residential, and mixed-use locations endogenously as a function of local productivity spillovers and commuting costs (Ogawa and Fujita 1980; Fujita and Ogawa 1982; Lucas and Rossi-Hansberg 2002)
- For Ogawa and Fujita (1980) and Fujita and Ogawa (1982), see Fujita and Thisse (2003) monograph and Duranton and Puga (2016) chapter, respectively.
- Leverage the bid-rent logic of land allocation across multiple types when firms are one of the bidders
- Monocentric city is outcome when productivity spillovers are large relative to commuting costs

Contrast with quantitative spatial models

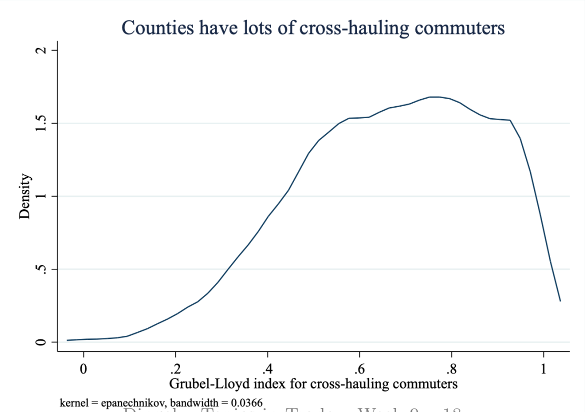
Thisse, Turner, Ushchev (2021) contrast quantitative spatial models and canonical urban model

- QSM features cross-hauling of homogeneous labor, canonical model does not
- Canonical model: Commuting costs are a centralizing force when employment is concentrated (e.g., the monocentric case)
- Quantitative spatial models: Commuting costs are a centralizing force even when net labor flows are zero

Cross hauling is prevalent in commuting matrices

Grubel-Lloyd (1971) index for intraindustry trade with flows X_{ij}:

$$\text{GL}_i \equiv 1 - \frac{|\sum_j X_{ij} - \sum_j X_{ji}|}{\sum_j X_{ij} + \sum_j X_{ji}}$$



Cross-hauling example: Alameda County CA

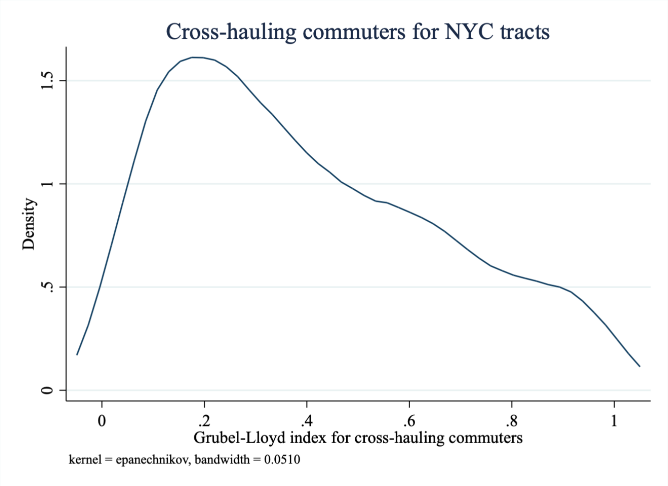
Alameda (home to Oakland) has a Grubel-Lloyd index of 0.995.

Residence	Workplace	Commuters	MOE	Distance (km)
06001	06001	468,181	4,506	0
06013	06001	92,797	2,340	30.36452
06001	06075	71,861	2,017	98.8513
06001	06085	64,696	1,865	51.42117
06001	06013	39,883	1,528	30.36452
06085	06001	38,339	1,513	51.42117
06001	06081	34,369	1,272	48.02481
06077	06001	26,121	1,276	64.74002
06075	06001	22,009	1,012	98.8513
06081	06001	13,417	739	48.02481

American Community Survey 2006-2010 county-to-county commuting flows

Cross hauling in NYC tract-to-tract matrix

Looking at tracts, employment is much more concentrated.



Locational assignments with multiple cities

Davis and Dingel (2020) solve an assignment model in which locational choice is between and within cities. The key is to define assignments in terms of a single index and then unveil quantities

Let's start by the reviewing differential rents model. (see Sattinger *JEL* 1993) In the spirit of Ricardo's analysis of rent, start with land and labor:

- A plot of land has fertility $\gamma \in \mathbb{R}$
- A farmer has skill $\omega \in \mathbb{R}$
- Profits are $\pi(\gamma, \omega) = p \cdot y(\gamma, \omega) - r(\gamma)$

Which farmer will use which plot of land?

- Farmers optimize: $\gamma^*(\omega) \equiv \arg\max_\gamma \pi(\gamma, \omega)$
- Equilibrium prices $r(\gamma)$ must support the equilibrium assignment of farmers to plots

Supermodularity

Definition (Supermodularity)

A function $g : \mathbb{R}^n \to \mathbb{R}$ is *supermodular* if $\forall x, x' \in \mathbb{R}^n$

$$g\left(\max\left(x, x'\right)\right) + g\left(\min\left(x, x'\right)\right) \geq g(x) + g(x')$$

where max and min are component-wise operators.

- Supermodularity means the arguments of $g(\cdot)$ are complements
- $g(x)$ is SM in (x_i, x_j) if $g(x_i, x_j; x_{-i,-j})$ is SM
- $g(x)$ is SM $\iff$ $g(x)$ is SM in (x_i, x_j) $\forall i, j$
- If g is C^2, $\frac{\partial^2 g}{\partial x_i \partial x_j} \geq 0 \iff g(x)$ is SM in (x_i, x_j)

Supermodularity implies PAM

Positive assortative matching:

- If $g(x,t)$ is supermodular in (x,t), then $x^*(t) \equiv \arg\max_{x \in X} g(x,t)$ is increasing in t
- If $y(\gamma,\omega)$ is strictly supermodular (fertility and skill are complements), then $\gamma^*(\omega)$ is increasing
- More skilled farmers are assigned to more fertile land

Why? Suppose not:

- Suppose $\exists \omega > \omega', \gamma > \gamma'$ where $\gamma' \in \gamma^*(\omega), \gamma \in \gamma^*(\omega')$
- $\gamma' \in \gamma^*(\omega) \Rightarrow p \cdot y(\gamma',\omega) - r(\gamma') \geq p \cdot y(\gamma,\omega) - r(\gamma) \; \forall \gamma$
- $\gamma \in \gamma^*(\omega') \Rightarrow p \cdot y(\gamma,\omega') - r(\gamma) \geq p \cdot y(\gamma',\omega') - r(\gamma') \; \forall \gamma'$
- Summing: $p \cdot (y(\gamma',\omega) + y(\gamma,\omega')) \geq p \cdot (y(\gamma,\omega) + y(\gamma',\omega'))$
- Would contradict strict supermodularity of $y(\cdot)$

Log-supermodularity (1/2)

Definition (Log-supermodularity)

A function $g : \mathbb{R}^n \rightarrow \mathbb{R}^+$ is *log-supermodular* if $\forall x, x' \in \mathbb{R}^n$

$$g\left(\max\left(x, x'\right)\right) \cdot g\left(\min\left(x, x'\right)\right) \geq g(x) \cdot g(x')$$

where max and min are component-wise operators.

- Example: $A : \Sigma \times \mathbb{C} \rightarrow \mathbb{R}^+$, where $\Sigma \subseteq \mathbb{R}$ and $\mathbb{C} \subseteq \mathbb{R}$, with $\sigma > \sigma'$ and $c > c'$

$$A(\sigma, c)A(\sigma', c') \geq A(\sigma', c)A(\sigma, c')$$

- $g(x)$ is LSM in (x_i, x_j) if $g(x_i, x_j; x_{-i,-j})$ is LSM
- $g(x)$ is LSM $\iff$ $g(x)$ is LSM in (x_i, x_j) $\forall i, j$
- $g > 0$ and g is $C^2 \Rightarrow \frac{\partial^2 \ln g}{\partial x_i \partial x_j} \geq 0 \iff g(x)$ is LSM in (x_i, x_j)

Log-supermodularity (2/2)

Three handy properties:

1. If $g, h : \mathbb{R}^n \to \mathbb{R}^+$ are log-supermodular, then gh is log-supermodular.
2. If $g : \mathbb{R}^n \to \mathbb{R}^+$ is log-supermodular, then $G(x_{-i}) \equiv \int g(x_i, x_{-i})dx_i$ is log-supermodular.
3. If $g : \mathbb{R}^n \to \mathbb{R}^+$ is log-supermodular, then $x_i^*(x_{-i}) \equiv \arg\max_{x_i \in \mathbb{R}} g(x_i, x_{-i})$ is increasing in x_{-i}.

Individual optimization

Perfectly mobile individuals simultaneously choose

- A sector σ of employment
- A city with total factor productivity $A(c)$
- A location τ (distance from ideal) within city c

The productivity of an individual of skill ω is

$$q(c, \tau, \sigma; \omega) = A(c)T(\tau)H(\omega, \sigma)$$

Utility is consumption of the numeraire final good, which is income minus locational cost:

$$\begin{aligned} U(c, \tau, \sigma; \omega) &= q(c, \tau, \sigma; \omega)p(\sigma) - r(c, \tau) \\ &= A(c)T(\tau)H(\omega, \sigma)p(\sigma) - r(c, \tau) \end{aligned}$$

Sectoral choice

- Individuals' choices of locations and sectors are separable:

$$\arg\max_{\sigma} \underbrace{A(c)T(\tau)}_{\text{locational}} \underbrace{H(\omega,\sigma)p(\sigma)}_{\text{sectoral}} - r(c,\tau) = \arg\max_{\sigma} H(\omega,\sigma)p(\sigma)$$

- $H(\omega,\sigma)$ is log-supermodular in ω,σ and strictly increasing in ω
- Comparative advantage assigns high-ω individuals to high-σ sectors
- Absolute advantage makes more skilled have higher incomes ($G(\omega) = \max_{\sigma} H(\omega,\sigma)p(\sigma)$ is increasing)

Locational choice

- A location's attractiveness $\gamma = A(c)T(\tau)$ depends on c and τ
- $T'(\tau) < 0$ may be interpreted as commuting to CBD, proximity to productive opportunities, or consumption value
- More skilled are more willing to pay for more attractive locations
- Equally attractive locations have same rental price and skill type
- Location in higher-TFP city is farther from ideal desirability

$$\gamma = A(c)T(\tau) = A(c')T(\tau')$$
$$A(c) > A(c') \Rightarrow \tau > \tau'$$

- Locational hierarchy: A smaller city's locations are a subset of larger city's in terms of attractiveness: $A(c)T(0) > A(c')T(0)$

Single-index assignment function

- Label cities from 1 to C so $A(C) \geq A(C-1) \geq \cdots \geq A(2) \geq A(1)$.
- Denote attractiveness levels occupied in equilibrium by $\Gamma \equiv [\underline{\gamma}, \bar{\gamma}]$, where $\underline{\gamma} \equiv A(C)T(\bar{\tau}(C))$ and $\bar{\gamma} \equiv A(C)T(0)$.
- In equilibrium, there exists a continuous and strictly increasing locational assignment function $K : \Gamma \to \Omega$ such that (i) $f(\omega, c, \tau) > 0 \iff A(c)T(\tau) = \gamma$ and $K(\gamma) = \omega$, and (ii) $K(\underline{\gamma}) = \underline{\omega}$ and $K(\bar{\gamma}) = \bar{\omega}$.
- Denote the supply of locations across all cities combined with attractiveness γ or greater by

$$S_\Gamma(\gamma) = \sum_{c:A(c)T(0)\geq\gamma} S\left(T^{-1}\left(\frac{\gamma}{A(c)}\right)\right).$$

$S_\Gamma(\bar{\gamma}) = 0$ and $S_\Gamma(\underline{\gamma}) = L$. $S_\Gamma(\gamma) = L\int_\gamma^{\bar{\gamma}} f(K(x))K'(x)\mathrm{d}x$, so $K(\gamma) = F^{-1}\left(\frac{L-S_\Gamma(\gamma)}{L}\right)$.

Equilibrium distributions

- Skill and sectoral distributions reflect distribution of locational attractiveness: Higher-γ locations occupied by higher-ω individuals who work in higher-σ sectors
- Locational hierarchy $\Rightarrow$ hierarchy of skills and sectors
- The distributions $f(\omega, c)$ and $f(\sigma, c)$ are log-supermodular if and only if the supply of locations with attractiveness γ in city c, $s(\gamma, c)$, is log-supermodular

$$s(\gamma, c) = \begin{cases} \frac{1}{A(c)} V\left(\frac{\gamma}{A(c)}\right) & \text{if } \gamma \leq A(c)T(0) \\ 0 & \text{otherwise} \end{cases}$$

where $V(z) \equiv -\frac{\partial}{\partial z} S\left(T^{-1}(z)\right)$ is the supply of locations with innate desirability τ such that $T(\tau) = z$

When is $s(\gamma, c)$ log-supermodular?

Proposition (Locational attractiveness distribution)

The supply of locations of attractiveness γ in city c, $s(\gamma, c)$, is log-supermodular if and only if the supply of locations with innate desirability $T^{-1}(z)$ within each city, $V(z)$, has a decreasing elasticity.

- Links each city's exogeneous distribution of locations, $V(z)$, to endogenous equilibrium locational supplies $s(\gamma, c)$
- Informally, ranking relative supplies is ranking elasticities of $V(z)$

$$s(\gamma, c) \propto V\left(\frac{\gamma}{A(c)}\right) \Rightarrow \frac{\partial \ln s(\gamma, c)}{\partial \ln \gamma} = \frac{\partial \ln V\left(\frac{\gamma}{A(c)}\right)}{\partial \ln z}$$

- Satisfied by the canonical von Thünen/monocentric geography

Next week and the week after

Next week: No class

November 16: Spatial sorting of skills and sectors