Gravitational Birefringence

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Renewed interest in massive gravity, originally expressed in terms of the graviton field:

$$\mathcal{L}_{FP} = -\frac{m_G^2 m_{\mathrm{Pl}}^2}{8}(h_{\mu\nu}h^{\mu\nu} - (h^\mu_\mu)^2)$$

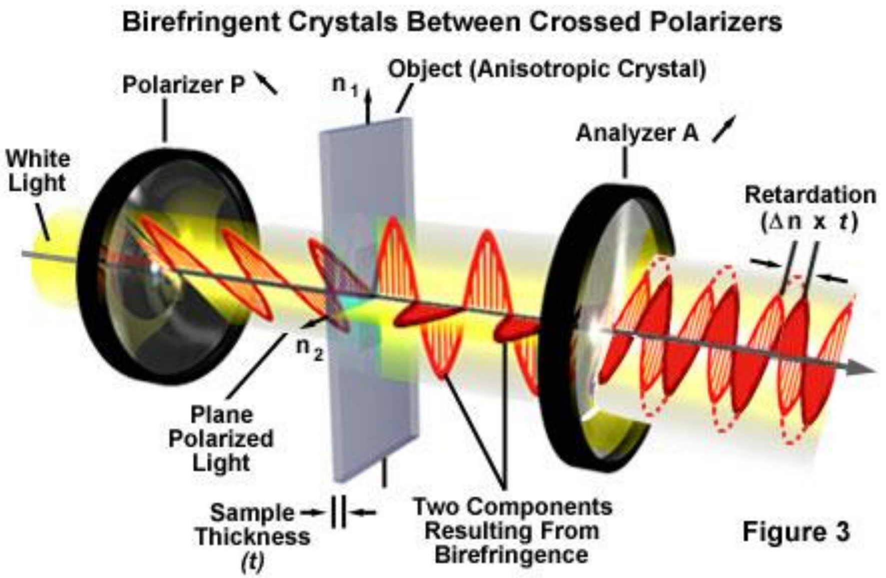
Fierz-Pauli (1939)

Unfortunately suffers from vDVZ discontinuity and has a ghost in curved space

$$h_{\mu\nu} = \frac{16\pi G_N}{p^2 + m_G^2}(T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu}) \longrightarrow h_{\mu\nu} = \frac{16\pi G_N}{p^2}(T_{\mu\nu} - \frac{\eta_{\mu\nu}}{2}T)$$

Recent direct observation of GW could become a good testing ground for models beyond GR. Should be sensitive to the existence of multi-gravitons.

If multigravity, could observe birefringence: oscillations between the gravitons (not between the polarisations of a single graviton).



Bimetric Gravity

One way to cure these problems is to consider a non-linear version of massive gravity with two dynamical metrics:

$$S = \int d^4x (e_1 \frac{R_1}{16\pi G_N} + e_2 \frac{R_2}{16\pi G_N} + \Lambda^4 \sum_{ijkl} m_{ijkl} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^{ai}_\mu e^{bj}_\nu e^{ck}_\rho e^{dl}_\sigma)$$

where the graviton mass is of order:

$$m_g^2 \sim \frac{\Lambda^4}{m_{\mathrm{Pl}}^2} \sim H_0^2$$

Because of the presence of two dynamical metrics, there are two types of gravitational perturbations of space-time. In particular, in a Minkowski background, there is one massive and one massless graviton. In a cosmological FRW background, the two waves are coupled and lead to gravitational birefringence.

$$ds_1^2 = a_1^2(-d\eta^2 + d\vec{x}^2) \qquad ds_2^2 = a_2^2(-b^2 d\eta^2 + d\vec{x}^2)$$

In this background, the two gravitational perturbations are:

$$\delta e^i_{\alpha j} = a_\alpha h^i_{\alpha j}, \ \alpha = 1, 2$$

and the mass matrix for the two gravitons reads:

$$M^2_{\alpha\beta} = -24 m^2 (b_\alpha b_\beta)^{1/2} \sum_{\delta\gamma} m_{\alpha\beta\gamma\delta} \tilde{a}_\gamma a_\delta \qquad b_1 = 1, \ b_2 = b, \tilde{a}_\alpha = b_\alpha a_\alpha$$

which depends on the coefficients of the potential in the Lagrangian.

In vacuum the waves obey two coupled equations:

$$\frac{d^2\bar{h}_1}{d\eta^2} - \Delta\bar{h}_1 + (M_{11}^2 - \frac{1}{a_1}\frac{d^2 a_1}{d\eta^2})\bar{h}_1 + M_{12}^2\bar{h}_2 = 0$$
$$\frac{d^2\bar{h}_2}{d\eta^2} - b^2\Delta\bar{h}_2 + (M_{22}^2 - \frac{b^{1/2}}{a_2}\frac{d^2(a_2 b^{-1/2})}{d\eta^2})\bar{h}_2 + M_{21}^2\bar{h}_1 = 0$$

Notice that one of the waves has a speed which is not unity when b is not equal to one. The normalised gravitons are

$$\bar{h}_{ij}^1 = m_{\rm Pl} a_1 h_{ij}^i, \quad \bar{h}_{ij}^2 = m_{\rm Pl}\frac{a_2}{b^{1/2}}h_{ij}^2$$

Over time-scales much shorter than the age of the Universe and for wave-numbers larger than the masses, there are two eigen-frequencies:

$$\omega_+^2 = \vec{k}^2 + \tilde{M}_{11}^2 + \frac{\tilde{M}_{12}^4}{(1-b^2)\vec{k}^2 + \tilde{M}_{11}^2 - \tilde{M}_{22}^2}$$
$$\omega_-^2 = b^2\vec{k}^2 + \tilde{M}_{22}^2 - \frac{\tilde{M}_{12}^4}{(1-b^2)\vec{k}^2 + \tilde{M}_{11}^2 - \tilde{M}_{22}^2}$$

The propagation of these waves depends on the background cosmology which depends intrinsically on the coupling to matter of the two metrics. The coupling to matter is determined by the combined vielbein, the most general and "healthy" combination at low energy:

$$e^a_\mu = \beta_1 e^a_{1\mu} + \beta_2 e^b_{2\mu}$$

From which one constructs the Jordan metric:

$$g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\mu$$

The theory is a metric theory, not of the unconstrained vielbeins, provided the symmetric condition is satisfied:

$$\eta_{ab} e^a_{1\mu} e^b_{2\nu} = \eta_{ab} e^a_{2\mu} e^b_{1\nu}$$

This guarantees that the two Einstein equations are well defined and when specialised to the background evolution allows one to study the cosmology of doubly coupled bigravity.

There are two Friedman equations and two Raychaudhuri equations:

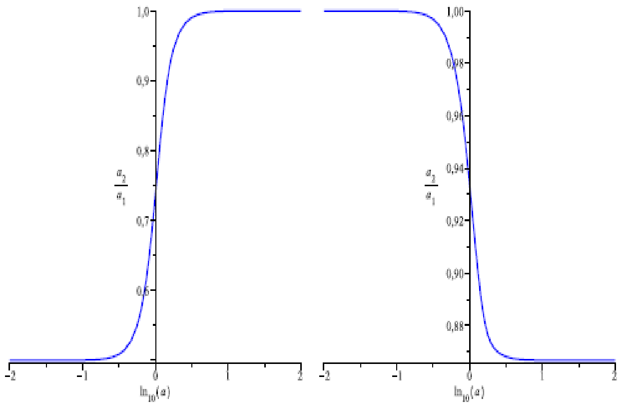
$$3H_1^2 m_{\mathrm{Pl}}^2 = \beta_1 \frac{a_J^3}{a_1^3}\rho + 24\Lambda^4 m^{1jkl}\frac{a_j a_k a_l}{a_1^3}$$
$$\frac{3H_2^2 m_{\mathrm{Pl}}^2}{b^2} = \beta_2 \frac{a_J^3}{a_2^3}\rho + 24\Lambda^4 m^{2jkl}\frac{a_j a_k a_l}{a_2^3}$$

The Raychaudhuri equations imply that there are two branches of solutions, one such that:

$$b = \frac{a_2 H_2}{a_1 H_1}$$

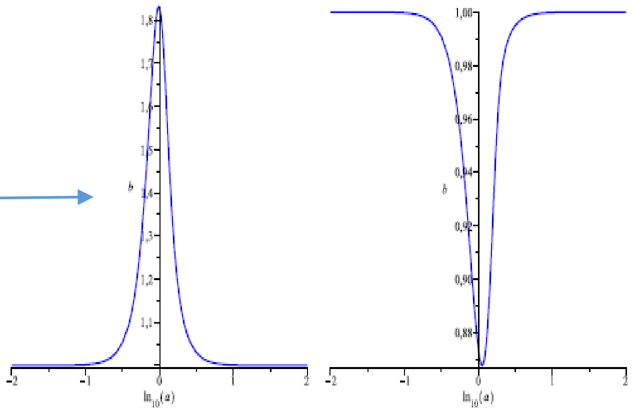
On this branch, the ratio between the two scale factors goes to a constant in the matter, radiation and dark energy eras, with b=1 in each case.

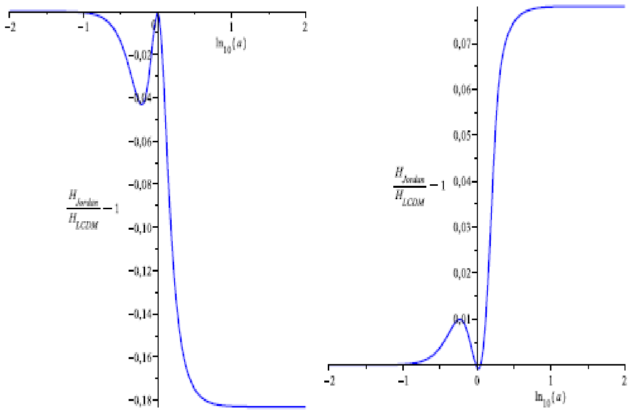
Variation of a2/a1 as a function of the Jordan frame redshift



Variation of b as a function of the Jordan frame redshift

Notice that b only differs from one in the recent Universe





Deviation of the Jordan frame Hubble rate from concordance

Models with eithers all m’s=1 and different couplings or same coupling and one differing m.

The two gravitons couple to matter and their wave equations are:

$$\frac{d^2\bar{h}_1}{d\eta^2} - \Delta\bar{h}_1 + (M_{11}^2 - \frac{1}{a_1}\frac{d^2 a_1}{d\eta^2})\bar{h}_1 + M_{12}^2\bar{h}_2 = \beta_1 \frac{\tilde{a}_J}{m_{\rm Pl}}\bar{T}_{ij}$$

$$\frac{d^2\bar{h}_2}{d\eta^2} - b^2\Delta\bar{h}_2 + (M_{22}^2 - \frac{b^{1/2}}{a_2}\frac{d^2(a_2 b^{-1/2})}{d\eta^2})\bar{h}_2 + M_{21}^2\bar{h}_2 = \beta_2 \frac{b^{1/2}\tilde{a}_J}{m_{\rm Pl}}\bar{T}_{ij}$$

where the source is the traceless energy momentum in the Jordan frame. On time scales much shorter than the age of the Universe, one can neglect the role of the mass terms in the emission process from a source:

$$h_{ij}^J = G_{\rm loc}\frac{a_J^3}{\tilde{a}_J^3}\frac{\beta_1^2 + b\beta_2^2}{\beta_1^2+\beta_2^2}\frac{1}{|x|}\Lambda_{ijkl}\frac{d^2\bar{I}^{kl}}{d\eta^2}$$

$$ds^2 = -\tilde{a}_J^2 d\eta^2 + a_J^2 d\vec{x}^2$$

$$a_J = \beta_1 a_1 + \beta_2 a_2$$

$$\tilde{a}_J = \beta_1 a_1 + \beta_2 b a_2$$

$$a_J h^J = \beta_1 a_1 h^1 + \beta_2 a_2 h^2$$

In bigravity Newton's constant is rescaled as:

$$G_{\rm loc} = (\beta_1^2 + \beta_2^2) G_N$$

This is the GR result up to a prefactor which differs from unity is b is not equal to unity, i.e. the emission is anomalous in the present Universe as we are not yet in the dark energy dominated era and at present b is not one.

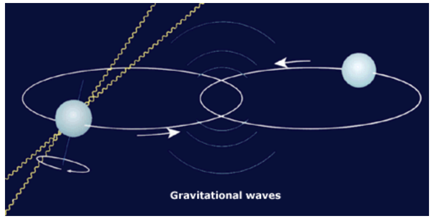
The emitted rate from the system is also proportional to the GR result, more precisely

$$\frac{dE}{dt_J} = \left(\frac{\beta_1^2 + b\beta_2^2}{\beta_1^2 + \beta_2^2}\right)^2 \frac{a_J^4}{\tilde{a}_J^4} \frac{dE}{dt_{\rm GR}}$$

There is a tight bound from binary pulsars:

Beltran-Jimenez et al . (2015)

$$0.995 < \left(\frac{\beta_1^2 + b\beta_2^2}{\beta_1^2 + \beta_2^2}\right)^2 \frac{a_J^4}{\tilde{a}_J^4} < 1$$

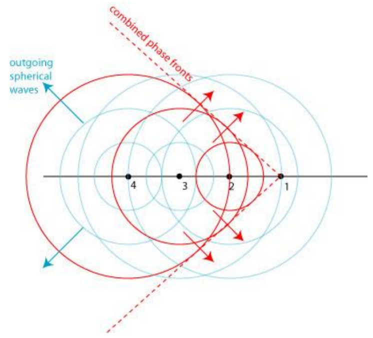


which implies that b is constrained at the per mil level now, i.e. the speed of gravitational waves cannot differ by more than 1/1000 from the speed of light.

When the speed of gravitational waves is so close to one, the two waves mix and effectively behave like one wave with a modified dispersion relation and a phase shift. This is the gravitational analogue of birefringence well-known in optics and expected for axions.

$$A_J = \Re(h_J) = A\cos(\omega t - \vec{k}.\vec{x} - \delta)$$

The new dispersion relation becomes:



$$w = c_T k + \frac{\tilde{M}_{11}^2}{4k} + \frac{\tilde{M}_{22}^2}{4bk}$$

where as expected the effective speed depends on b:

$$c_T = \frac{1+b}{2}$$

When this speed is less than one, high energy cosmic rays would emit gravitational Cerenkov radiation and would be absent on earth unless:

$$1 - b < 10^{-17}$$

So essentially, speeds smaller than the speed of light are excluded.

Another interesting entity is the transfer function:

$$|h_J|^2 = (\frac{a_J^4}{\tilde{a}_J a_{\rm GR}^3})^2 (\frac{\beta_1^2 + b\beta_2^2}{\beta_1^2 + \beta_2^2})^2 \left(1 - 4\frac{(\beta_1 + \beta_2 b^{1/2} C)^2 (\beta_2 b^{1/2} - \beta_1 C)^2}{(\beta_1^2 + b\beta_2^2)^2} \sin^2(\frac{(\omega_+ - \omega_-)}{2} t)\right) |h_{\rm GR}|^2$$

Essentially, at a given distance for a source t=d, the GR signal is modulated and the variation of the amplitude should not be too rapid compared to the one of the GR signal, if not then the sine term gets averaged out to a half; and the modulation becomes simply a change of amplitude. On the other hand if

$$|b - 1| \leq \frac{1}{k_{\rm exp} d}$$

The modulation could be significant. For nano-Herz sensitivities and 100 Mpc distances, this is one part of a millionth… four order of magnitude lower than the pulsar bound. Hence not all parameter space covered.

Are these results valid? There are several provisos:

1) The theory has a strong coupling scale of order:

$$\Lambda_3 = (m^2 m_{\rm Pl})^{1/3}$$

which implies that for redshifts before BBN and for scales less than 1000 km's, the theory may require a UV completion which could modify the results. Hence no possibility of studying the merging of astrophysical objects.

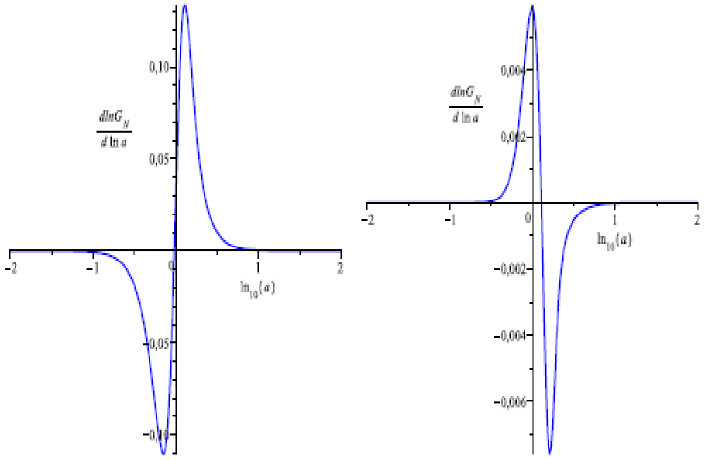
2) The bound on the speed of gravitational waves from cosmic rays may not apply as the energies are way higher than the cut-off.

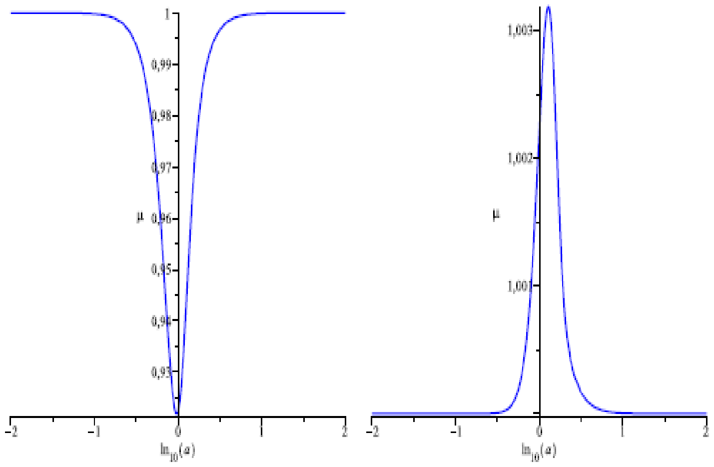
3) The theory needs a UV completion above the strong coupling scale. Such a UV completion might have new degrees of freedom in particular generate a scalar charge for compact bodies. This could generate dipolar radiation on top of the quadrupolar one.

The only safe regime where the theory is trustable/testable is at late time and low energy where the effects of a UV completion should be negligible. On top of gravitational waves, these models have an effect on the growth of structure. For instance the Poisson equation in the Jordan frame becomes:

$$\Delta\Phi_N = 4\pi G_{\mathrm{local}}\mu\delta\rho$$

in local coordinates. The μ parameter depends on time and would induce a local variation of Newton's constant. This is tightly constrained, although with the bounds on b at the per mil level this should be easily satisfied.





The redshift variation of μ would lead to more or less growth, although with small values of b, this may be hardly detectable.

CONCLUSIONS

The gravitational sector of bigravity is characterised by unusual birefringence features from the two propagating gravitational waves. This happens in the reliable sector of the theory at low energy in the late time Universe.

There is a non-trivial correlation between the gravitational wave signals and growth of structure.

Could be interesting to look for multi-graviton effects in future data.