Testing gravity workshop, Vancouver, Jan, 2017

Cosmology in generalized Proca theories

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Collaboration with
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arXiv:1602.00371, arXiv:1603.05806, arXiv:1605.05066, arXiv:1605.05565

Massive vector theories (Proca theories)

(i) Maxwell field (massless)

Lagrangian: $\mathcal{L}_F = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

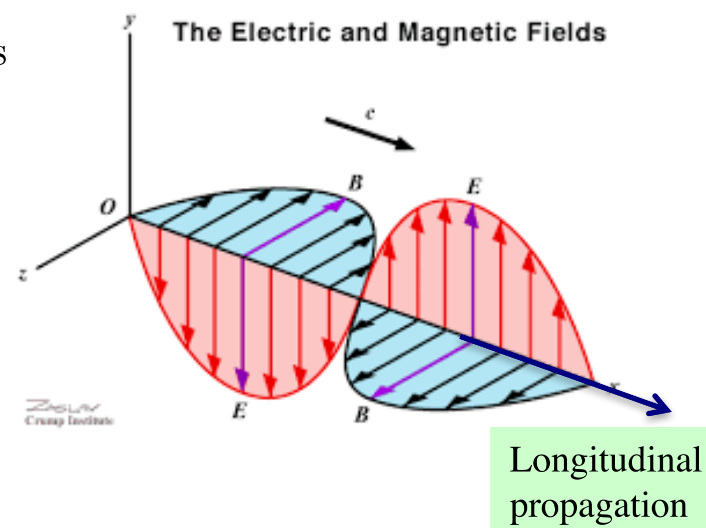
There are two transverse polarizations (electric and magnetic fields).

(ii) Proca field (massive)

Lagrangian: $\boxed{\mathcal{L}_F = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 A_\mu A^\mu}$

Introduction of the mass m of the vector field A_μ allows the propagation in the longitudinal direction due to the breaking of $U(1)$ gauge invariance.

2 transverse and 1 longitudinal = 3 DOFs



Generalized Proca theories

On general curved backgrounds, it is possible to extend the massive Proca theories to those containing three DOFs (besides two tensor polarizations).

Heisenberg Lagrangian (2014)

See also Tasinato (2014)

$$
\begin{aligned}
\mathcal{L}_2 &= G_2(X,F,Y)\,,\\
\mathcal{L}_3 &= G_3(X)\nabla_\mu A^\mu\,,\\
\mathcal{L}_4 &= G_4(X)R + G_{4,X}(X)\left[(\nabla_\mu A^\mu)^2 - \nabla_\rho A_\sigma \nabla^\sigma A^\rho\right]\,,\\
\mathcal{L}_5 &= G_5(X)G_{\mu\nu}\nabla^\mu A^\nu - \frac{1}{6}G_{5,X}(X)[(\nabla_\mu A^\mu)^3 - 3\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma]\\
&\quad - g_5(X)\tilde{F}^{\alpha\mu}\tilde{F}^\beta{}_\mu \nabla_\alpha A_\beta\,,\\
\mathcal{L}_6 &= G_6(X)L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2}G_{6,X}(X)\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}\nabla_\alpha A_\mu \nabla_\beta A_\nu\,,
\end{aligned}
$$

Intrinsic vector mode

where $X = -\frac{1}{2}A_\mu A^\mu\,,\quad F = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,,\quad Y = A^\mu A^\nu F_\mu{}^\alpha F_{\nu\alpha}$

$$L^{\mu\nu\alpha\beta} = \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}R_{\rho\sigma\gamma\delta}\,,\qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

The non-minimal derivatives couplings like $G_4(X)R$ are required to keep the equations of motion up to second order.

Taking the scalar limit $A^\mu \to \nabla^\mu \pi$, the above Lagrangian recovers a sub-class of Horndeski theories (with $\mathcal{L}_6$ vanishing).

Cosmology in generalized Proca theories

De Felice et al, 1605.05066

Can we realize a viable cosmology with the late-time acceleration?

Vector field: $A^\mu = (\phi(t), 0, 0, 0)$ (which does not break spatial isotropy)

Variation of the Heisenberg action with respect to $g_{\mu\nu}$ on the flat FLRW background leads to

$$G_2 - G_{2,X}\phi^2 - 3G_{3,X}H\phi^3 + 6G_4H^2 - 6(2G_{4,X} + G_{4,XX}\phi^2)H^2\phi^2 + G_{5,XX}H^3\phi^5 + 5G_{5,X}H^3\phi^3 = \rho_M\,,$$

$$G_2 - \dot{\phi}\phi^2 G_{3,X} + 2G_4\left(3H^2 + 2\dot{H}\right) - 2G_{4,X}\phi\left(3H^2\phi + 2H\dot{\phi} + 2\dot{H}\phi\right) - 4G_{4,XX}H\dot{\phi}\phi^3$$
$$+ G_{5,XX}H^2\dot{\phi}\phi^4 + G_{5,X}H\phi^2(2\dot{H}\phi + 2H^2\phi + 3H\dot{\phi}) = -P_M\,.$$

The matter density ρ_M and the pressure P_M obey the continuity equation

$$\dot{\rho}_M + 3H(\rho_M + P_M) = 0$$

Variation of the action with respect to A^μ leads to

$$\phi\left(G_{2,X} + 3G_{3,X}H\phi + 6G_{4,X}H^2 + 6G_{4,XX}H^2\phi^2 - 3G_{5,X}H^3\phi - G_{5,XX}H^3\phi^3\right) = 0\,.$$

The branch $\phi \neq 0$ gives the solution where ϕ depends on H alone, which allows the existence of de Sitter solutions with constant ϕ and H.

Vector Galileons

The Lagrangian of vector Galileons which recover the Galilean symmetry in the scalar limit $(A_\mu \to \partial_\mu \pi)$ on the flat space-time is given by

$$G_2(X) = b_2 X\,, \qquad G_3(X) = b_3 X\,, \qquad G_4(X) = \frac{M_{\rm pl}^2}{2} + b_4 X^2\,, \qquad G_5(X) = b_5 X^2.$$

We substitute these functions into the vector-field equation:

$$\boxed{G_{2,X} + 3G_{3,X} H\phi + 6G_{4,X} H^2 + 6G_{4,XX} H^2\phi^2 - 3G_{5,X} H^3\phi - G_{5,XX} H^3\phi^3 = 0}$$

Taking note that $X = \phi^2/2$, the background EOM admits the solution

$$\phi H = \text{constant}.$$

$$\Downarrow$$

The temporal component ϕ is small in the early cosmological epoch, but it grows with the decrease of H.

The solution finally approaches the de Sitter attractor characterized by

$$\phi = \text{constant}, \qquad H = \text{constant}.$$

Generalizations of vector Galileons

Let us consider the case in which ϕ is related with H according to

$$\phi^p \propto H^{-1} \qquad (p > 0)$$

This solution can be realized for

$$G_2(X) = b_2 X^{p_2}\,, \qquad G_3(X) = b_3 X^{p_3}\,, \qquad G_4(X) = \frac{M_{\rm pl}^2}{2} + b_4 X^{p_4}\,, \qquad G_5(X) = b_5 X^{p_5}\,,$$

where

$$p_3 = \frac{1}{2}(p + 2p_2 - 1)\,, \qquad p_4 = p + p_2\,, \qquad p_5 = \frac{1}{2}(3p + 2p_2 - 1)\,.$$

The vector Galileon corresponds to $p_2 = p = 1$.

The dark energy and radiation density parameters obey

$$\Omega_{\rm DE}' = \frac{(1+s)\Omega_{\rm DE}(3+\Omega_r - 3\Omega_{\rm DE})}{1 + s\,\Omega_{\rm DE}}\,,$$
$$\Omega_r' = -\frac{\Omega_r[1 - \Omega_r + (3+4s)\Omega_{\rm DE}]}{1 + s\,\Omega_{\rm DE}}\,,$$

where $s \equiv \dfrac{p_2}{p}\,.$

There are 3 fixed points:

(a) $(\Omega_{\rm DE}, \Omega_r) = (0, 1)$
(b) $(\Omega_{\rm DE}, \Omega_r) = (0, 0)$
(c) $(\Omega_{\rm DE}, \Omega_r) = (1, 0)$

Dark energy equation of state

$$w_{\rm DE} = -\frac{3(1+s) + s\,\Omega_r}{3(1+s\,\Omega_{\rm DE})}\,.$$

(a) $w_{\rm DE} = -1 - 4s/3$ in the radiation era,
(b) $w_{\rm DE} = -1 - s$ in the matter era,
(c) $w_{\rm DE} = -1$ in the de Sitter era

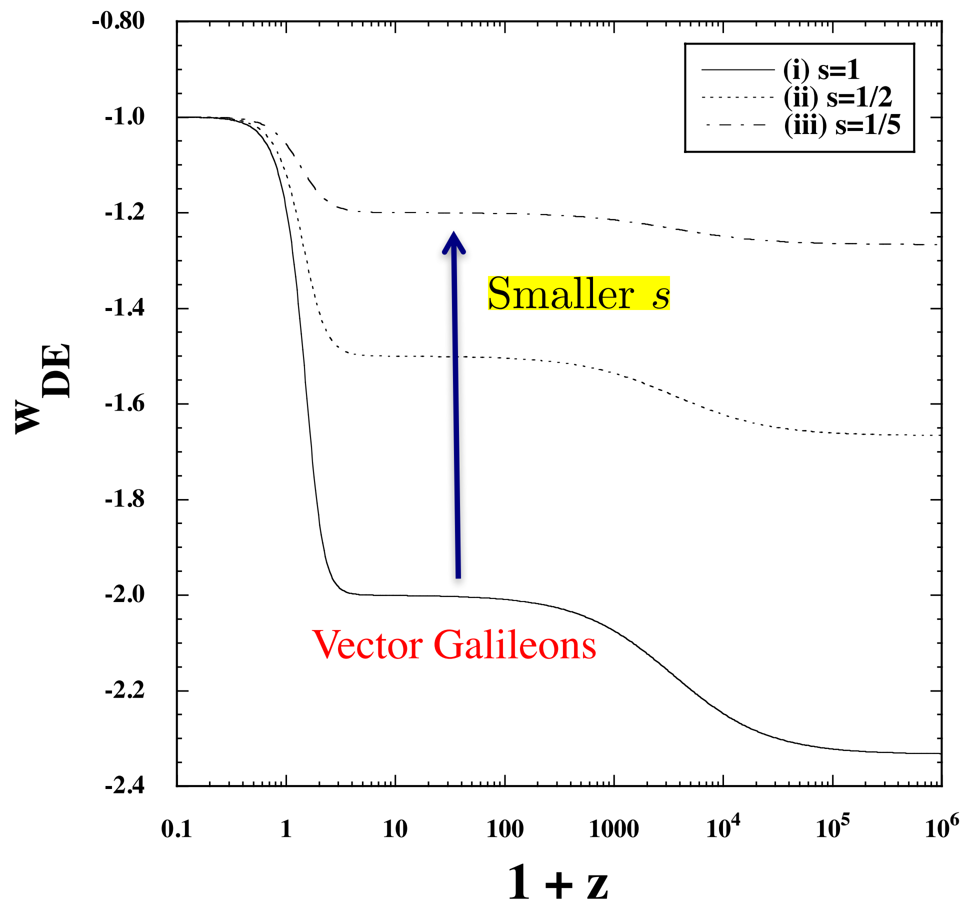
For smaller $s = p_2/p$ close to 0, $w_{\rm DE} = -1 - s$ approaches -1.

The joint data analysis of SNIa, CMB, and BAO give the bound

$$0 \le s \le 0.36 \quad (95\,\%\mathrm{CL})$$

(De Felice and ST, 2012)

For larger p the field ϕ evolves more slowly as $\phi \propto H^{-1/p}$, so $w_{\rm DE}$ approaches -1.



Theoretical consistency and observational signatures

- There are 6 theoretically consistent conditions associated with tensor, vector, and scalar perturbations:

No ghosts: $q_t > 0, \ q_v > 0, \ q_s > 0$

No instabilities: $c_t^2 > 0, \ c_v^2 > 0, \ c_s^2 > 0$

See arXiv:1603.05806 for details.

There exists a wide range of parameter space consistent with these conditions.

- The effective gravitational coupling associated with the growth of large-scale structures can be smaller than the Newton constant.

The existence of the intrinsic vector mode can lead to

$$G_{\rm eff} < G$$

See arXiv:1605.05066 for details.

Weak gravity in generalized Proca theories

De Felice et al, 1605.05066 (2016)

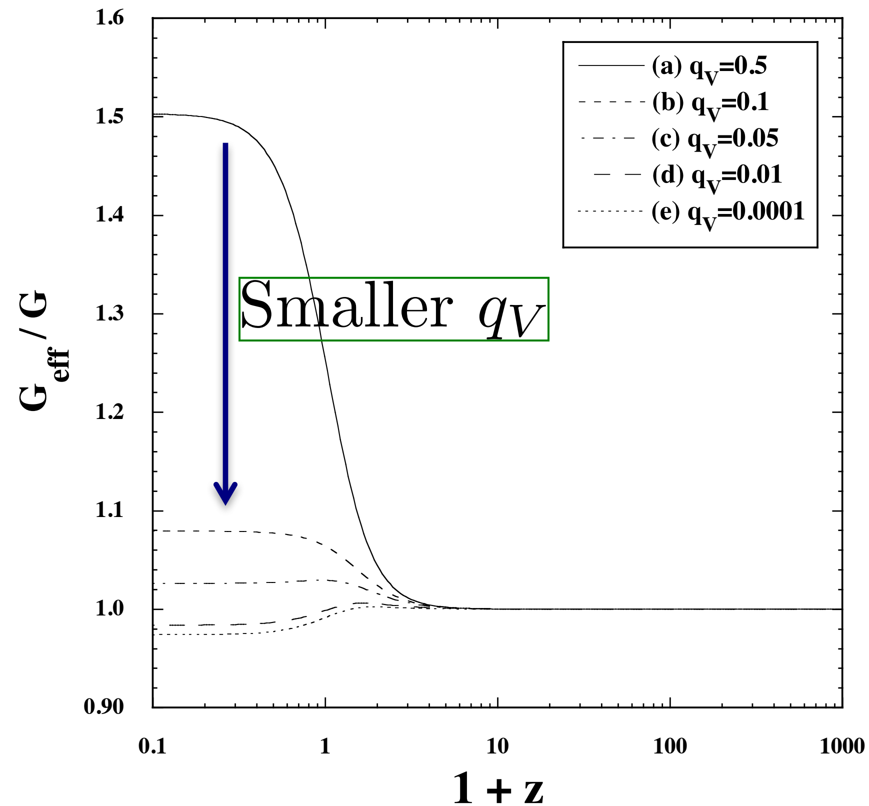
$G_{\rm eff}$ is modified through the intrinsic vector mode through the quantity q_V.

For a massive vector field with $G_2 = F + m^2 X$ we have

$$q_V = 1 \underbrace{- 4g_5 H\phi + 2G_6 H^2 + 2G_{6,X} H^2 \phi^2}_{\text{Effect of the intrinsic vector mode}}$$

For smaller q_V approaching 0, the effect of the vector field tends to reduce the gravitational attraction.

It is possible to see signatures of the intrinsic vector mode in redshift-space distortion measurements.



Vainshtein screening

De Felice et al.
arXiv:1602.00371

The screening of fifth forces works well in generalized Proca theories.

Consider the model

$$G_4(X) = \frac{M_{\rm pl}^2}{2} + \beta_4 X^2$$

On the spherically symmetric background

$$ds^2 = -e^{2\Psi(r)}dt^2 + e^{2\Phi(r)}dr^2 + r^2 d\Omega^2$$

Vector field: $A^\mu = (\phi(r), \chi'(r), 0, 0)$

There is a screened solution where the longitudinal mode vanishes:

$$\boxed{\chi'(r) = 0}$$

The post-Newtonian parameter reads

$$\boxed{\gamma = -\frac{\Phi}{\Psi} \simeq 1 - \frac{2\beta_4\phi^4}{M_{\rm pl}^2}}$$

The solar-system constraint $|\gamma - 1| < 2.3 \times 10^{-5}$ is satisfied under the mild bound $|\beta_4|\phi^4 < 10^{-5} M_{\rm pl}^2$.

Conclusions and outlook

1. Generalized Proca theories give rise to interesting cosmological solutions with a late-time de Sitter attractor.

2. Existence of the intrinsic vector mode allows an interesting signature like the realization of $G_{\rm eff}$ smaller than G.

3. The Vainshtein screening also allows the consistency with local gravity experiments.

It will be of interest to put observational and experimental constraints on our models.