Cosmological (in)consistency tests of gravity theory and cosmic acceleration

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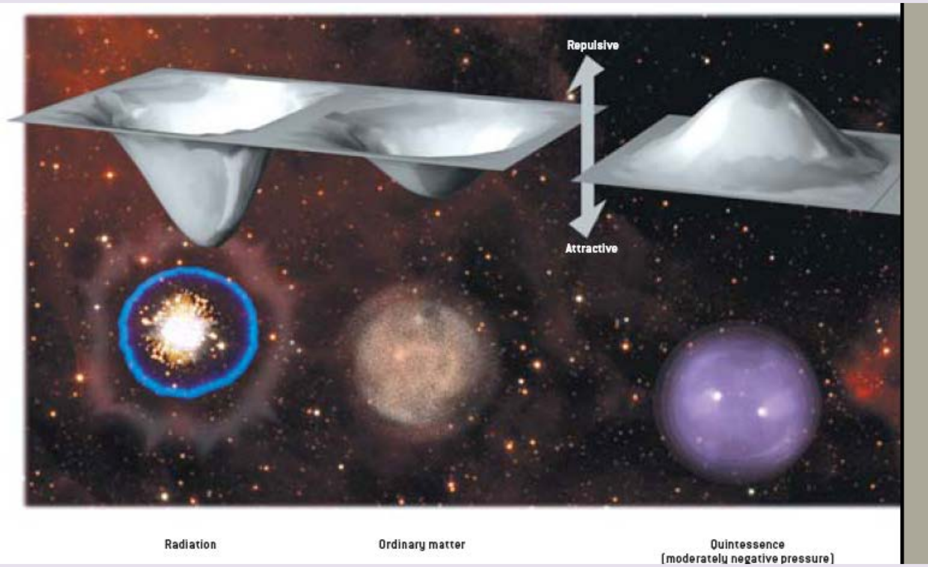
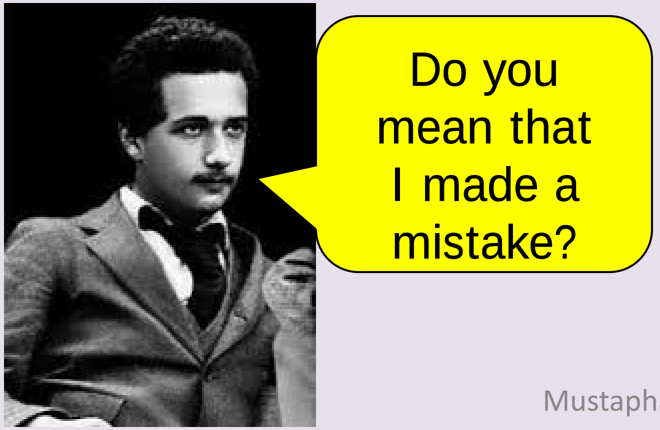
Testing gravity theory at cosmological scales for (at least) two reasons:



(1) Is cosmic acceleration due to Dark Energy or Modified Gravity?

(2) Is General Relativity modified or extended at cosmological scales?



Einstein's Equations:

$$G^a_b + \Lambda \delta^a_b = 8\pi G T^a_b$$

Using the growth rate of large scale structure

At least two methods to test gravity using cosmology have been used:

1) Looking for inconsistencies in the cosmological parameter spaces as determined by the growth data versus the geometry/expansion data (e.g. MI, Upadhye, and Spergel, PRD 2006; Wang *et al.*, 2007; Ruiz & Huterer, 2015; Bernal, Verdi, Cuesta, 2016, …)

2) Defining parameters for the growth rate and constraining them using data sets (e.g. Linder, 2005; Koyama, 2006; Bertschinger and Zukin, 2008; and many others in this meeting…)

We use method (1) here

Example from sometime ago: Consistency between the growth rate and the expansion history as a test of cosmic acceleration

(MI, Upadhye, and Spergel, PRD 2006, astro-ph 2005)

- For a dark energy *wCDM* model, the expansion history is given by:

$$H(z) = Ho\sqrt{(1-\Omega_{de})(1+z)^3 + \Omega_{de}\varepsilon(z)} \qquad (1)$$

- and the Growth rate *G(a=1/(1+z))* is given by integrating:

$$G'' + \left[\frac{7}{2} - \frac{3}{2}\frac{w(a)}{1+X(a)}\right]\frac{G'}{a} + \frac{3}{2}\frac{1-w(a)}{1+X(a)}\frac{G}{a^2} = 0; \qquad G(a) = \frac{D(a)}{a}; \quad D(a) = \frac{\delta(a)}{\delta(1)} \qquad (2)$$

- For Modified Gravity DGP models and k=0, the expansion history is given by

$$H(z) = Ho\left[\frac{1}{2}(1-\Omega_m) + \sqrt{\frac{1}{4}(1-\Omega_m)^2 + \Omega_m(1+z)^3}\right] \qquad (3)$$

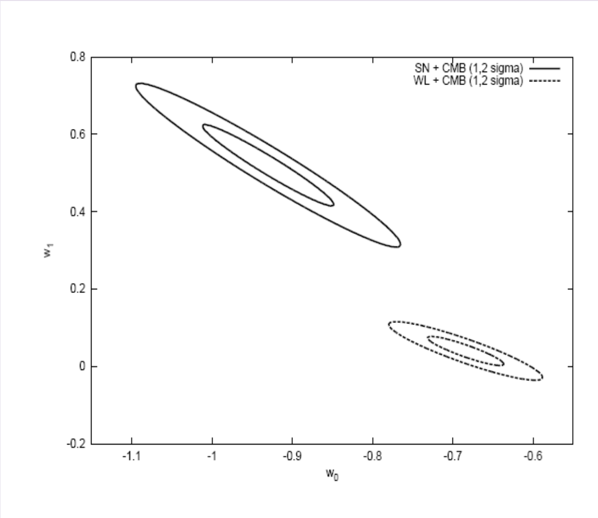
- and the growth rate of function is given by

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho\left(1 + \frac{1}{3\beta}\right)\delta = 0 \qquad \beta = 1 - 2r_c H\left(1 + \frac{\dot{H}}{3H^2}\right) \qquad (4)$$

- Equation (1) and (2) must be mathematically consistent one with another via General Relativity.
- Equation (3) and (4) must be consistent one with another via DGP theory

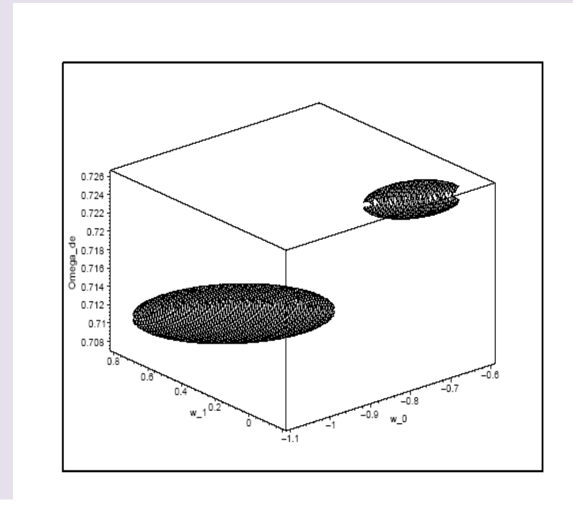
Examples: MI, Upadhye, and Spergel, Phys.Rev. D74 (2006) 043513).

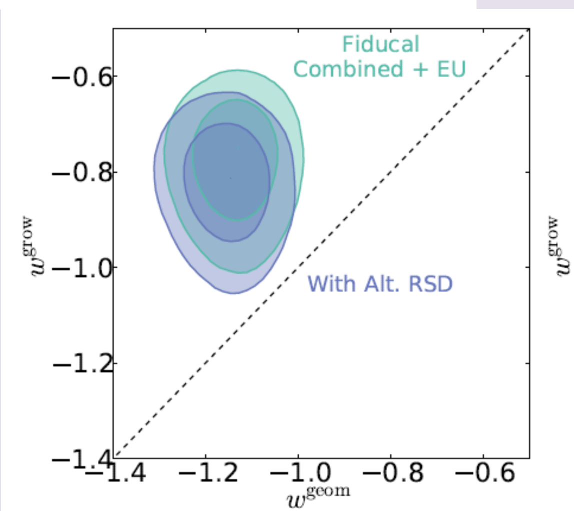
"Is Cosmic Acceleration a Symptom of the Breakdown of General Relativity?"

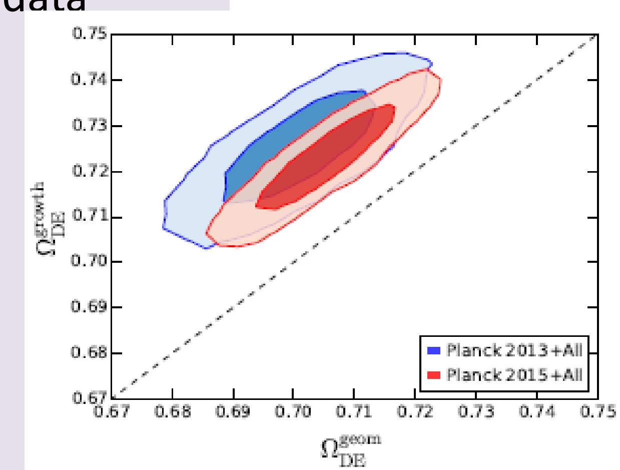


We simulated the data using modified gravity (DGP) but then we fit the data to dark energy models

=> a detectable inconsistency in the simulated data







Ruiz, Huterer, Phys. Rev. D 91, 063009 (2015)
Banana Split: Testing the Dark Energy Consistency with Geometry and Growth

Bernal, Verde, Cuesta , JCAP02(2016)059, "Parameter splitting in dark energy: is dark energy the same in the background and in the cosmic structures?"

New work: How to quantify the degree of inconsistency? (W. Lin and MI, in prep. 2017)

Need to define a mathematical measure that takes into account 3 aspects of inconsistencies:

a) deviation between likelihood maxima

b) volume of covariance matrices (ellipsoid sizes)

c) degeneracy directions (ellipsoid orientations)

d) Other practical properties (e.g. invariance)

Index of Inconsistency (IOI)

(W. Lin and MI, in prep. 2017)

- We consider two experiments and define

$$\frac{1}{2}\Delta\chi^2(\boldsymbol{\mu}) \equiv \frac{1}{2}\Delta\chi^2_{(1)}(\boldsymbol{\mu}) + \frac{1}{2}\Delta\chi^2_{(2)}(\boldsymbol{\mu}) \ ,$$

where

$$\Delta\chi^2_{(i)}(\boldsymbol{\mu}) = \chi^2_{(i)}(\boldsymbol{\mu}) - \chi^2_{(i)}(\boldsymbol{\mu}^{(i)}).$$

$\frac{1}{2}\Delta\chi^2_{(1)}(\boldsymbol{\mu})$: The 'difficulty' for the 1st experiment to support the mean of joint analysis.

$\frac{1}{2}\Delta\chi^2_{(2)}(\boldsymbol{\mu})$: The 'difficulty' for the 2nd experiment to support the mean of joint analysis.

Index of Inconsistency (*IOI*)

- In the Gaussian limit $\Delta\chi^2_{(i)} = (\boldsymbol{\lambda} - \boldsymbol{\mu}^{(i)})\boldsymbol{L}^{(i)}(\boldsymbol{\lambda} - \boldsymbol{\mu}^{(i)})$
- We define the IOI as: $\frac{1}{2}\Delta\chi^2 \xrightarrow{Gaussian} \frac{1}{2}\boldsymbol{\Delta G \Delta} \equiv \mathrm{I\,OI}$

where $\boldsymbol{\Delta} = \boldsymbol{\mu}^{(2)} - \boldsymbol{\mu}^{(1)}$, and $\boldsymbol{G} = \left((\boldsymbol{L}^{(1)})^{-1} + (\boldsymbol{L}^{(2)})^{-1}\right)^{-1} = (\boldsymbol{C}^{(1)} + \boldsymbol{C}^{(2)})^{-1}$.

- And for multiple experiments:

$$\frac{1}{2}\sum_i \Delta\chi^2_{(i)}(\boldsymbol{\mu}) \xrightarrow{Gaussian} \frac{1}{2}\left(\sum_i \boldsymbol{\mu}^{(i)}\boldsymbol{L}^{(i)}\boldsymbol{\mu}^{(i)} - \boldsymbol{\mu L \mu}\right) \equiv \mathrm{IOI}$$

- Where $\boldsymbol{\mu} = \boldsymbol{L}^{-1}\left(\sum_i \boldsymbol{L}^{(i)}\boldsymbol{\mu}^{(i)}\right)$ and $\boldsymbol{L} = \sum \boldsymbol{L}^{(i)}$.
- Other works defined other quantities (e.g. Marshall, Rajguru, Slosar, 2006; March, Trotta, Amendola, Huterer, 2011; Verdi, Protopapas, Jimenez, 2013; Seehars, Grandis, Amara, Refregier, 2016; Grandis, Rapetti, Saro, Mohr, 2016)

Comparison to other measures of consistency/inconsistency in the literature

- Comparison between IOI and these quantities provided in the paper

Quantities	Symbols	Relevant concept	Gaussian and weak prior limit
Index of Inconsistency	IOI	$\Delta\chi^2_{(1)}(\mu) + \Delta\chi^2_2(\mu)$	$\frac{1}{2}\Delta G \Delta$ (definition)
Robustness [34, 35]	$-\ln R$	Bayes. evid. ratio	$\mathrm{IOI} - \frac{1}{2}\ln\left(\frac{\lvert L^{(1)}L^{(2)}\rvert}{\lvert LP\rvert}\right)$
Normalized Robustness [35]	$-\ln R_N$	Bayes. evid. ratio	$\mathrm{IOI} - \frac{1}{2}\ln\left(\frac{2\lvert L^{(2)}L^{(1)}\rvert}{\lvert L^{(1)}(L^{(1)}+L^{(2)})\rvert}\right)$
Tension [33]	$\ln \mathcal{T}$	Bayes. evid. ratio	IOI
Surprise[a] [36, 38, 39]	$S(\mathscr{P}^{(2)} \vert\vert \mathscr{P}^{(1)})$	Relative entropy	$\frac{1}{2}\Delta L^{(1)}\Delta - \frac{1}{2}\mathrm{tr}(I_N + (L^{(2)})^{-1}L^{(1)})$
and its deviation	$\sigma^2(D)$		$\frac{1}{2}\mathrm{tr}\left(\left(I_N + (L^{(2)})^{-1}L^{(1)}\right)^2\right)$
Calibrated Evid. Ratio [40]	$-\mathrm{CER}$	Bayes. evid. ratio	IOI $-$ N/2
and its deviation	$\sigma^2(R)$		$N/2$

[33] L. Verde, P. Protopapas, and R. Jimenez, Physics of the Dark Universe **2**, 166 (2013), arXiv:1306.6766 [astro-ph.CO].

[34] P. Marshall, N. Rajguru, and A. c. v. Slosar, Phys. Rev. D **73**, 067302 (2006).

[35] M. C. March, R. Trotta, L. Amendola, and D. Huterer, Mon. Not. R. Astron. Soc. **415**, 143 (2011), arXiv:1101.1521.

[36] S. Seehars, S. Grandis, A. Amara, and A. Refregier, Phys. Rev. D **93**, 103507 (2016), arXiv:1510.08483.

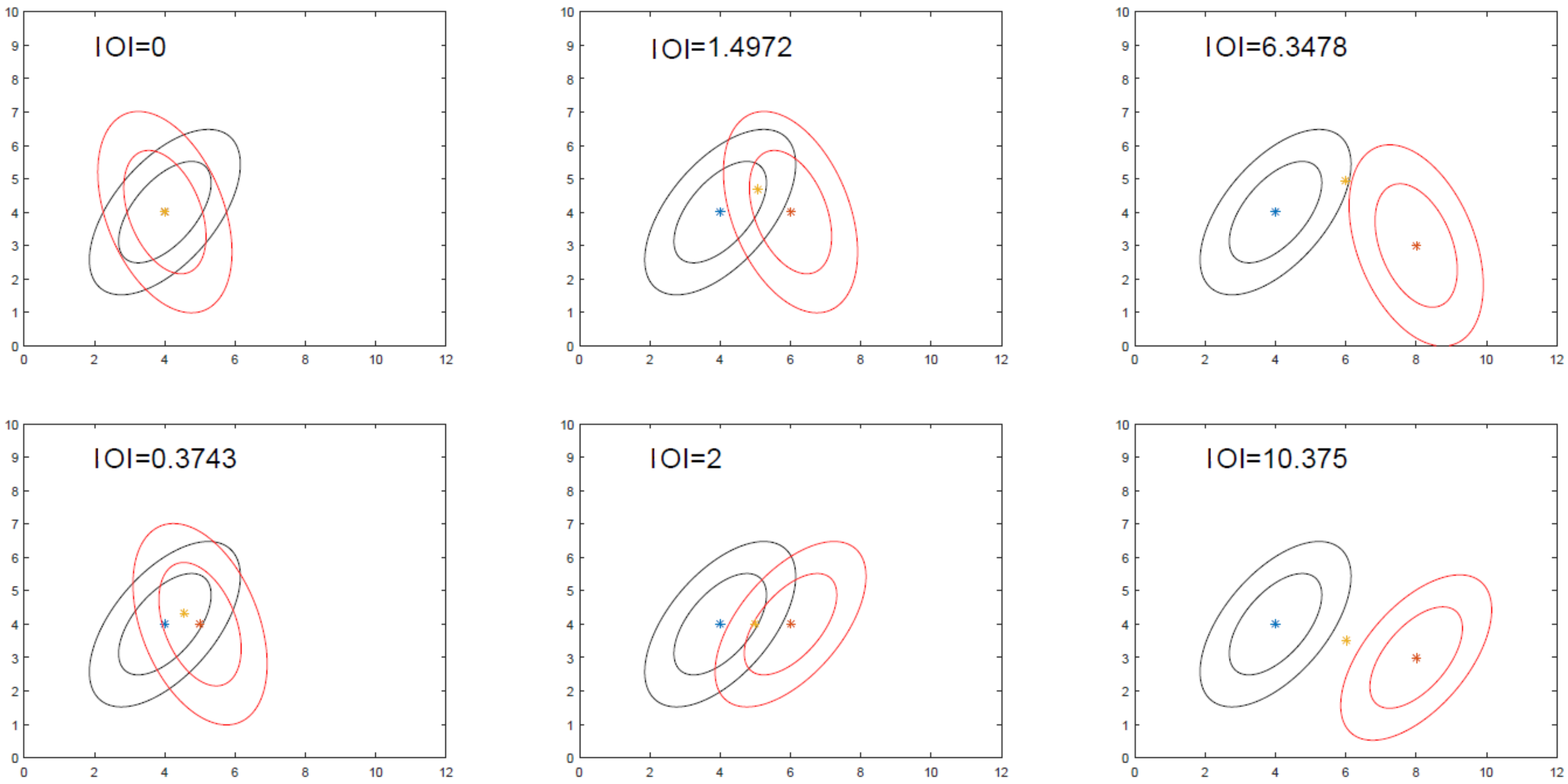
[38] S. Seehars, A. Amara, A. Refregier, A. Paranjape, and J. Akeret, Phys. Rev. D **90**, 023533 (2014), arXiv:1402.3593.

[39] S. Grandis, S. Seehars, A. Refregier, A. Amara, and A. Nicola, J. Cosmol. Astropart. Phys. **5**, 034 (2016), arXiv:1510.06422.

[40] S. Grandis, D. Rapetti, A. Saro, J. J. Mohr, and J. P. Dietrich, ArXiv e-prints (2016), arXiv:1604.06463.

IOI Captures various cases of inconsistencies.

Here are two toy experiments for each type of inconsistency.



Zooming in an individual cosmological parameter

- ΔIOI_i : Relative drop in the index of inconsistency after marginalization over a given parameter p_i

$$\Delta IOI_i = \frac{IOI - IOI_{marg.\,over\,p\_i}}{IOI}$$

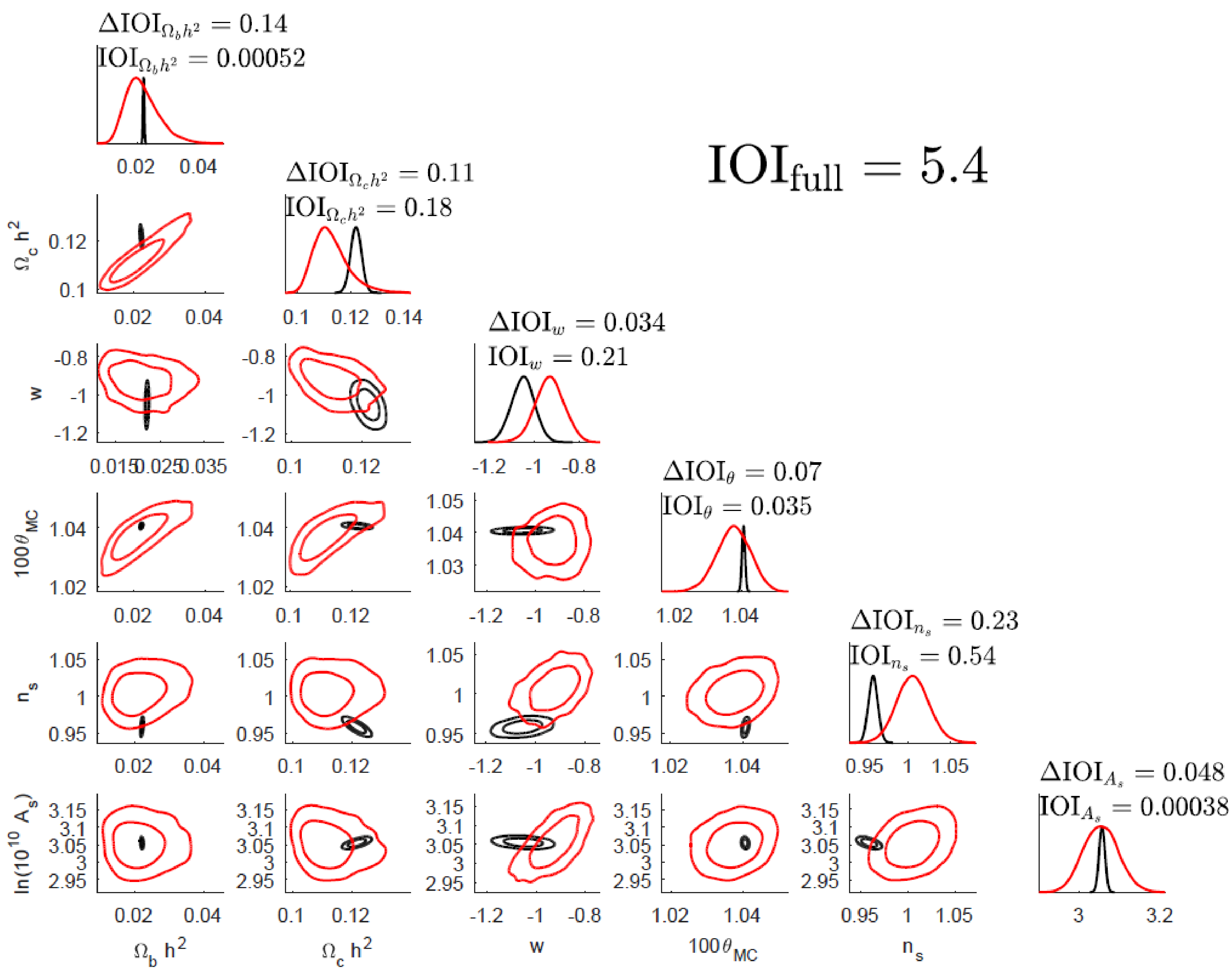
- IOI_i : Relative residual index of inconsistency for a given parameter p_i after marginalization over all the other parameters

- For consistency between two or more experiments for a given parameter, both of the corresponding relative drop and relative residual IOIs must be small.

Application to current data sets: Geometry versus Growth

Geometry		Growth	
Supernovae Type Ia [42]		Low ℓ CMB temperature and polarization [5]	
BAO	6dF ($z_{eff} = 0.106$) [43]	CMB lensing [44]	
	MGS ($z_{eff} = 0.15$) [47]	Sunyaev–Zel'dovich effect [28]	
	Lyman-α ($z_{eff} = 2.34$) [20]	galaxy weak lensing [12]	
High ℓ CMB temperature [5]		RSD	WiggleZ_MPK [11, 45]
			SDSS DR12 CMASS and LOWZ catalogs [46]

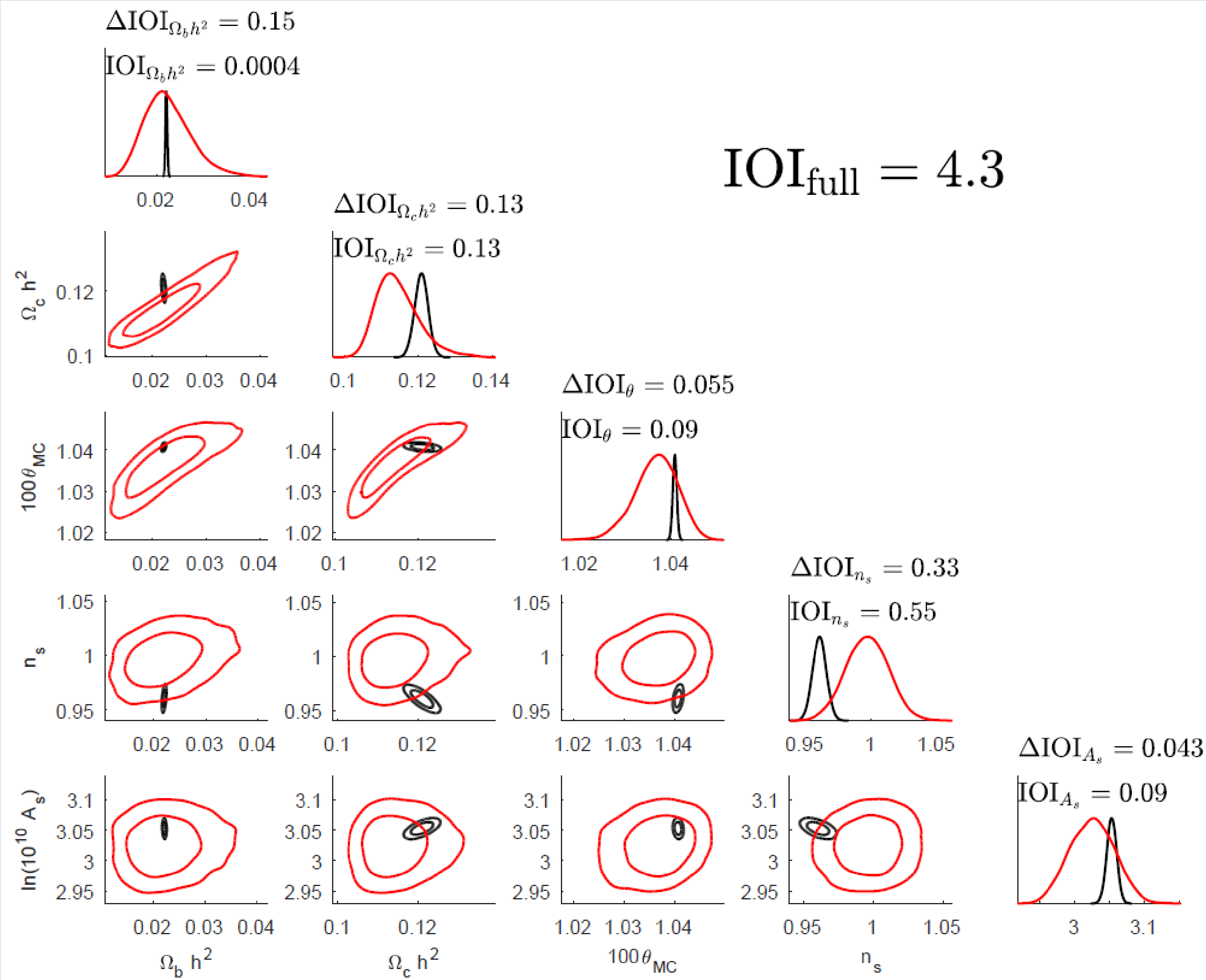
IOIs for the wCDM model: geometry vs growth



Concluding remarks

- Incoming and future large data sets will allow one to perform (in-)consistency tests.
- Inconsistencies can be of physical source or due to systematics
- *IOI*, relative drop, and relative residual *IOIs* have useful features to look for inconsistencies among cosmological parameters if present.
- Applying *IOI* measures to current data indicate a moderate-to-strong inconsistency on the Jeffrey scale between growth and geometry data (consistent with some other works).
- More details to come very soon in the paper (W. Lin and MI, in prep. 2017)

IOI for growth versus geometry for the ΛCDM model



Relative drop in IOI and relative residual in IOI for individual parameters in the wCDM model

Parameters	$\Omega_b h^2$	$\Omega_c h^2$	θ	w	A_s	n_s
IOI_i	0.0005	0.18	0.21	0.035	0.54	0.0004
$\Delta\mathrm{IOI}_i$	0.14	0.11	0.034	0.07	0.23	0.048

Definitions and quantities used in the work

Distributions	Notations	Fisher mat.	Elem. of Fisher mat.	Means	Elem. of means
ith Likelihood	$\mathcal{L}^{(i)}$	$\boldsymbol{L}^{(i)}$	$\ell^{(i)}_{jk}$	$\boldsymbol{\mu}^{(i)}$	$\mu^{(i)}_{j}$
Prior	$\mathcal{P}$	$\boldsymbol{P}$	p_{jk}	$\boldsymbol{\mu}^{(p)}$	$\mu^{(p)}_{j}$
ith Posterior	$\mathscr{P}^{(i)}$	$\boldsymbol{F}^{(i)}$	$f^{(i)}_{jk}$	$\bar{\boldsymbol{\mu}}^{(i)}$	$\bar{\mu}^{(i)}_{j}$

TABLE I. Table of notations: Probability distributions, their means and elements of means, Fisher matrices and elements of the Fisher matrices for the likelihood of the ith experiment, prior, and the posterior of the ith experiment. Likelihoods are *not* normalized in the parameter space, while the Prior and Posteriors are.

Parameter vector	Observable vector	Mean-difference	Covariance matrix
$\boldsymbol{\lambda}$	$\boldsymbol{Q}$	$\boldsymbol{\Delta}$	$\boldsymbol{C}$

TABLE II. Other frequently used notations in this work.

Ranges	FOI< 1	1 <FOI< 2.5	2.5 <FOI< 5	FOI> 5
Interpretation	no significant inconsistency	weak inconsistency	moderate inconsistency	strong inconsistency