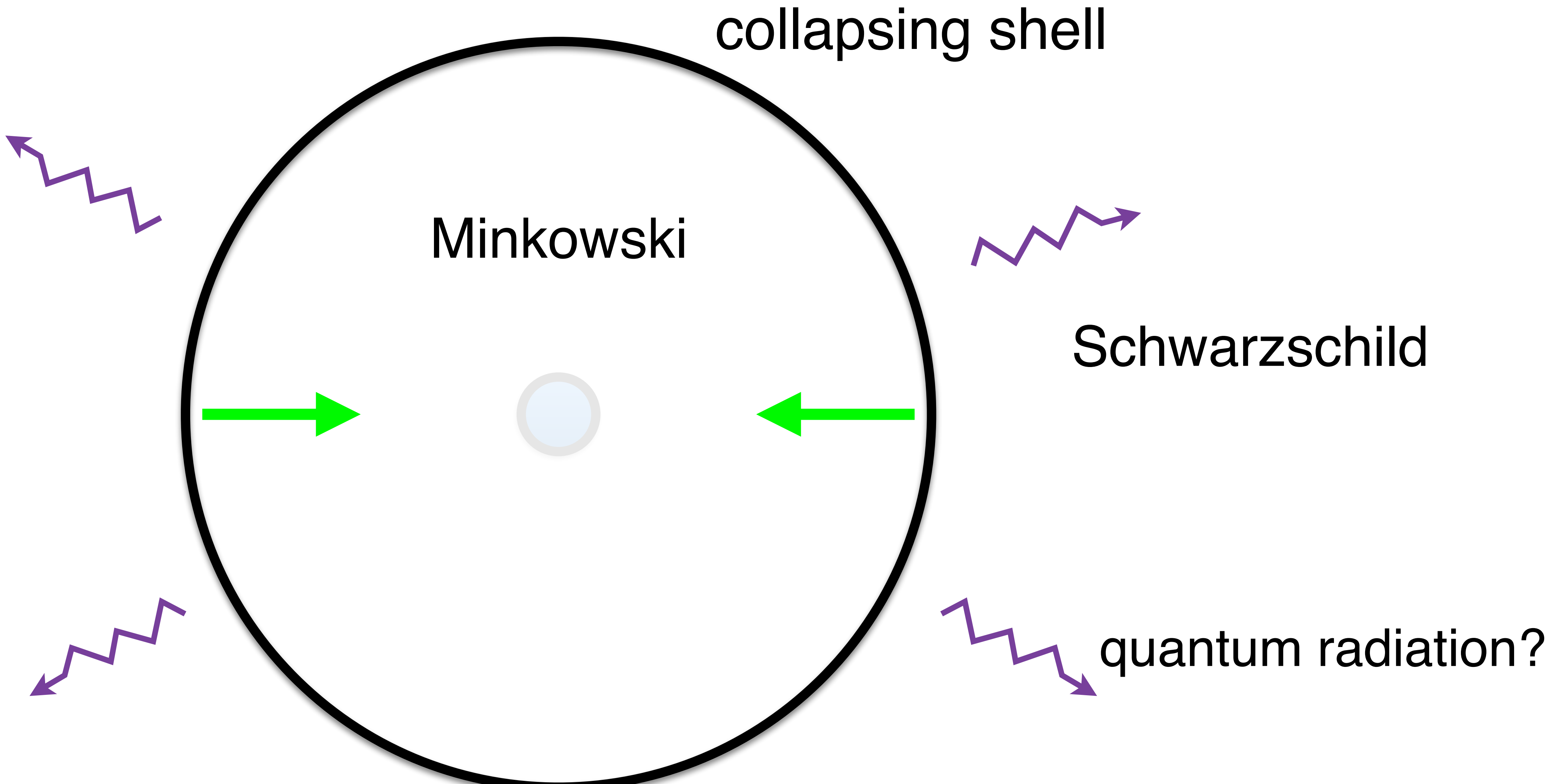
Quantum Radiation During Gravitational Collapse

Tanmay Vachaspati

Arizona State University
University of Maryland (through August 2017)

Setup



Kolopanis & TV, 2013
TV, Stojkovic & Krauss, 2007

Technique

$$\phi(t,\mathbf{x}) = \sum_{\mathbf{k}} b_{\mathbf{k}}(t)\phi_{\mathbf{k}}(\mathbf{x})$$

expansion in modes

$$S = \sum_k \int d\eta \left[\frac{1}{2}{b'_k}^2 - \frac{v_k(\eta)}{2B(\eta)} b_k^2\right]$$

action for quantum modes

$$\left[-\frac{1}{2}\frac{\partial^2}{\partial b^2} + \frac{1}{2}\omega^2(\eta) b^2\right]\psi(b,\eta) = i\frac{\partial}{\partial\eta}\psi(b,\eta)$$

Schrodinger equation for modes

$$\psi(b,\eta) = e^{i\alpha(\eta)}\left(\frac{1}{\pi\rho^2}\right)^{1/4} \exp\left[\frac{i}{2}\left(\frac{\rho_\eta}{\rho} + \frac{i}{\rho^2}\right) b^2\right]$$

wave function for modes

$$\boxed{\rho_{\eta\eta} + \omega^2(\eta)\rho = \frac{1}{\rho^3}}$$

$$\rho(0) = \frac{1}{\sqrt{\omega_0}}\ , \quad \rho_\eta(0) = 0$$

auxiliary function

$$\alpha(\eta) = -\frac{1}{2}\int_0^\eta \frac{d\eta'}{\rho^2(\eta')}$$

phase of wavefunction

Technique continued

$$f_{\eta\eta} + \omega^2(\eta) f = 0 \longrightarrow (\xi, \chi)$$

classical solution of two-dimensional SHO
with suitable initial conditions

$$\rho_{\eta\eta} + \omega^2(\eta)\rho = \frac{1}{\rho^3} \qquad \longrightarrow \qquad \rho = \frac{1}{\sqrt{\omega_0}}\sqrt{\xi^2 + \chi^2}$$

quantum solution

For gravitational collapse problem:

$$\omega(\eta) = \frac{\omega_0}{\sqrt{B(\eta)}} = \frac{\omega_0}{\sqrt{1-\eta}}$$

and solutions are known in terms of Bessel functions.

Results

$$\xi = \frac{\pi u}{2}\left[Y_0(2\omega_0)J_1(u) - J_0(2\omega_0)Y_1(u)\right]$$
$$\chi = \frac{\pi u}{2}\left[Y_1(2\omega_0)J_1(u) - J_1(2\omega_0)Y_1(u)\right]$$
$$\xi_\eta = -\pi\omega_0^2\left[Y_0(2\omega_0)J_0(u) - J_0(2\omega_0)Y_0(u)\right]$$
$$\chi_\eta = -\pi\omega_0^2\left[Y_1(2\omega_0)J_0(u) - J_1(2\omega_0)Y_0(u)\right]$$

$$u \equiv 2\omega_0\sqrt{1-\eta}$$

$$N(t,\omega_0) = \frac{\sqrt{1-\eta}}{4(\xi^2+\chi^2)}\left[\left(\frac{\xi^2+\chi^2}{\sqrt{1-\eta}} - 1\right)^2 + \frac{1}{\omega_0^2}(\xi\xi_\eta + \chi\chi_\eta)^2\right]$$

occupation numbers for quantum modes

Results continued

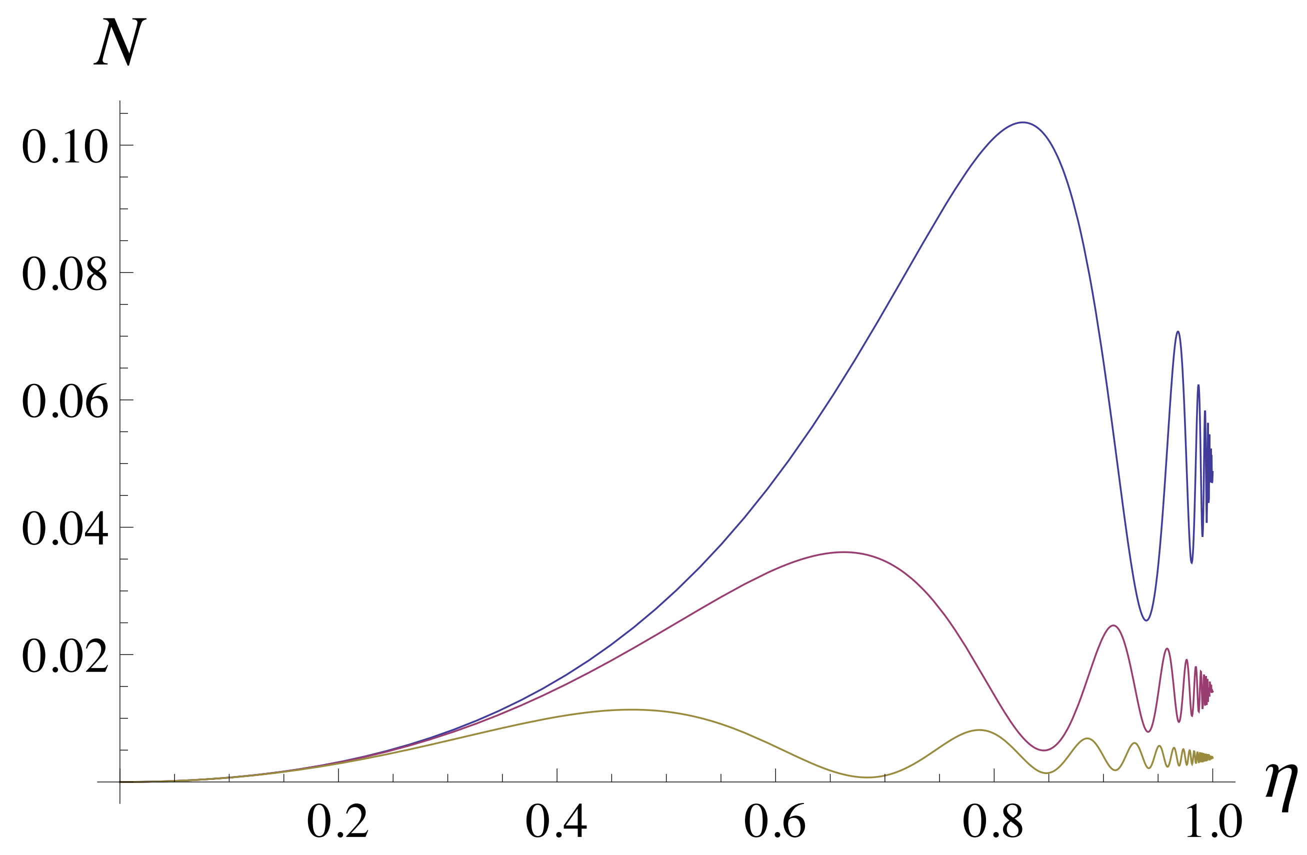


FIG. 3: Occupation number versus η for $\Omega = 0.5$, 1, and 2 (highest to lowest curve).

Results continued

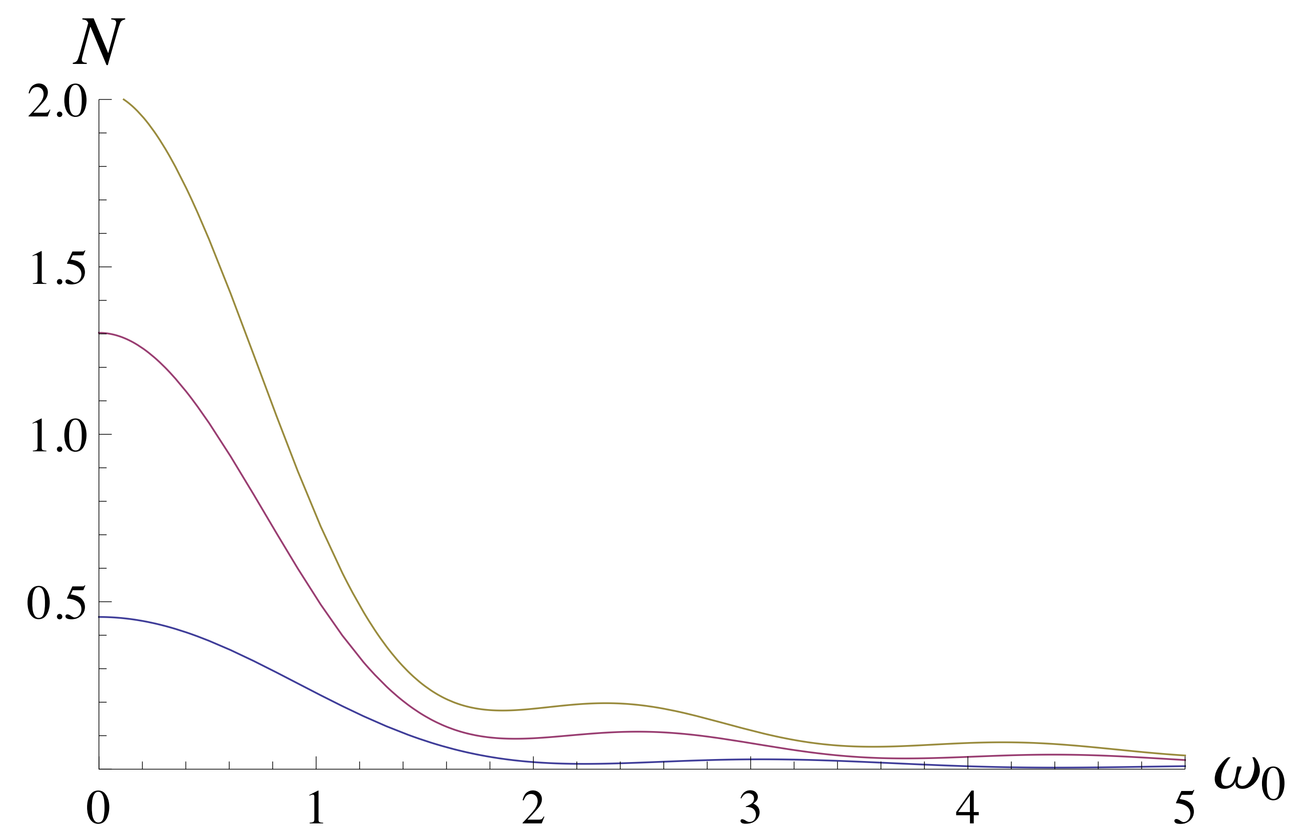


FIG. 1: Occupation number, N, as a function of ω_0 for $\eta = 0.92$, 0.98, and 0.99 (lowest to highest curve).

As $t \to \infty$, we recover thermal spectrum of excitations with $T = 1/(4\pi)$ (Hawking temperature) at long wavelength.

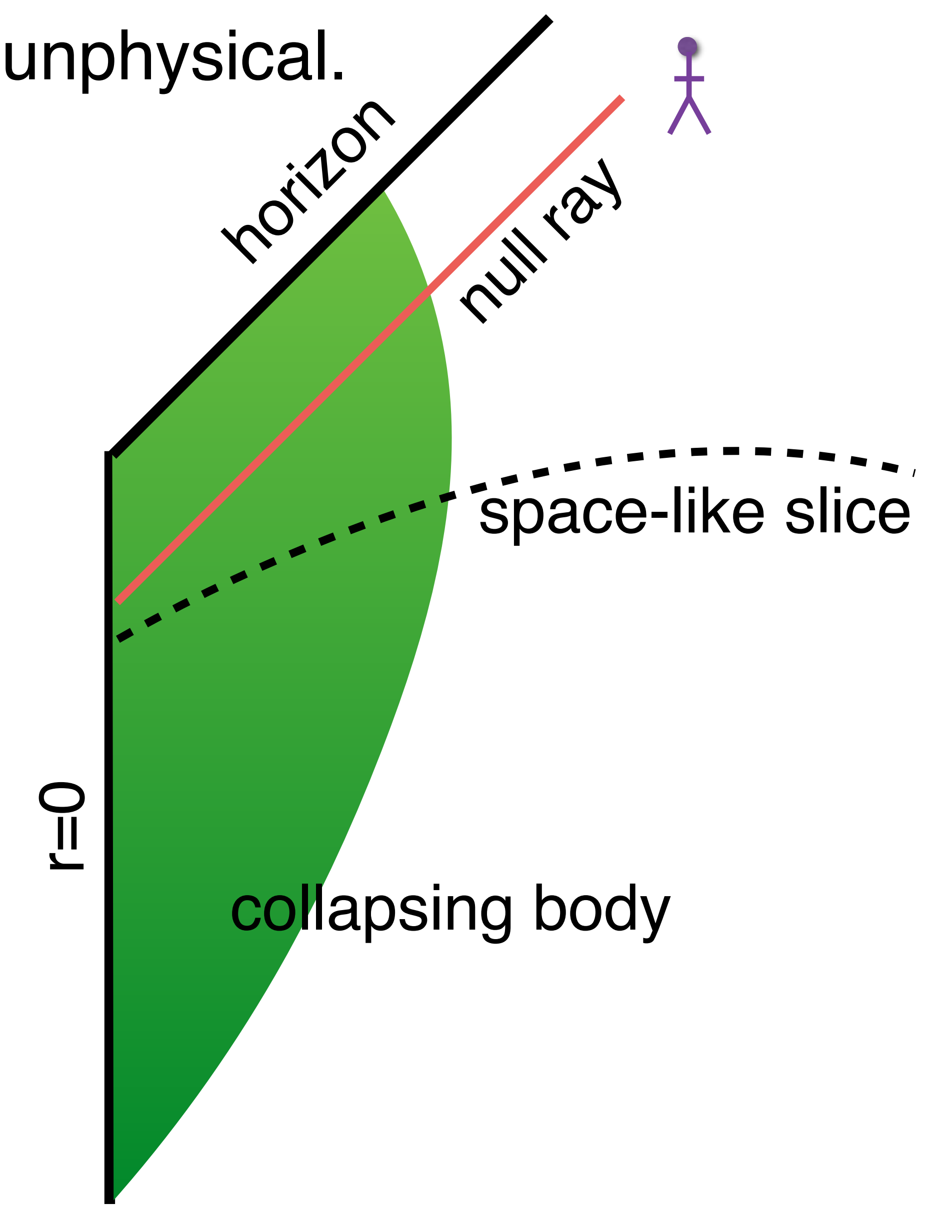
Consistency & Differences

Consistent with Hawking's calculation of Hawking radiation.

Three differences:

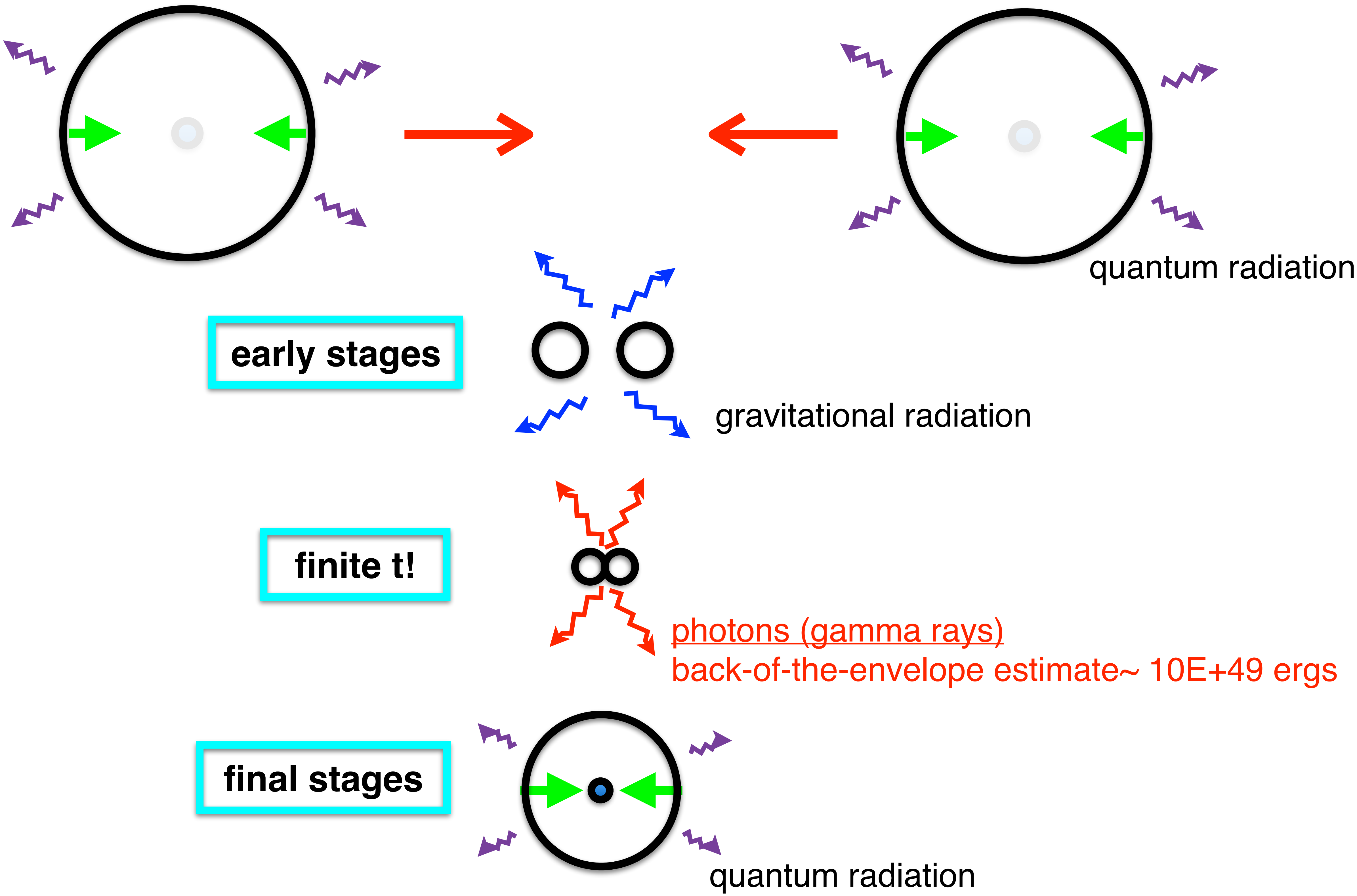
1. Excitations include all modes, not just radiative modes.
2. Full time-dependent solution, not just at t=infinity.
3. Evaporation during collapse means t=infinity limit is unphysical.

Observer "sees" an evaporating "**black star**"
(a.k.a. frozen star) at any finite time.



Collisions at *finite* t.

TV, 2007; 2016



Conclusions

- Time-dependent quantum excitations are produced during quantum collapse and build-up towards Hawking radiation.

- Thermal Hawking radiation is emitted but only in the unphysical t=infinity limit.

- Collapsing, colliding objects emit gravitational waves *<u>followed</u>* by E&M counterparts. (Ref: LIGO 1602.03837 & Fermi 1602.03920.)

- Absence of delayed E&M counterparts would indicate *primordial* black holes.