An Introduction to Gravitational Waves

Testing Gravity 2017, Vancouver BC

Dimitrios Psaltis
U. Arizona/Harvard U.

Nice pedagogical introduction at

Flanagan & Hughes,

New Journal of Physics **7** (2005) 204

The basic picture (in words):

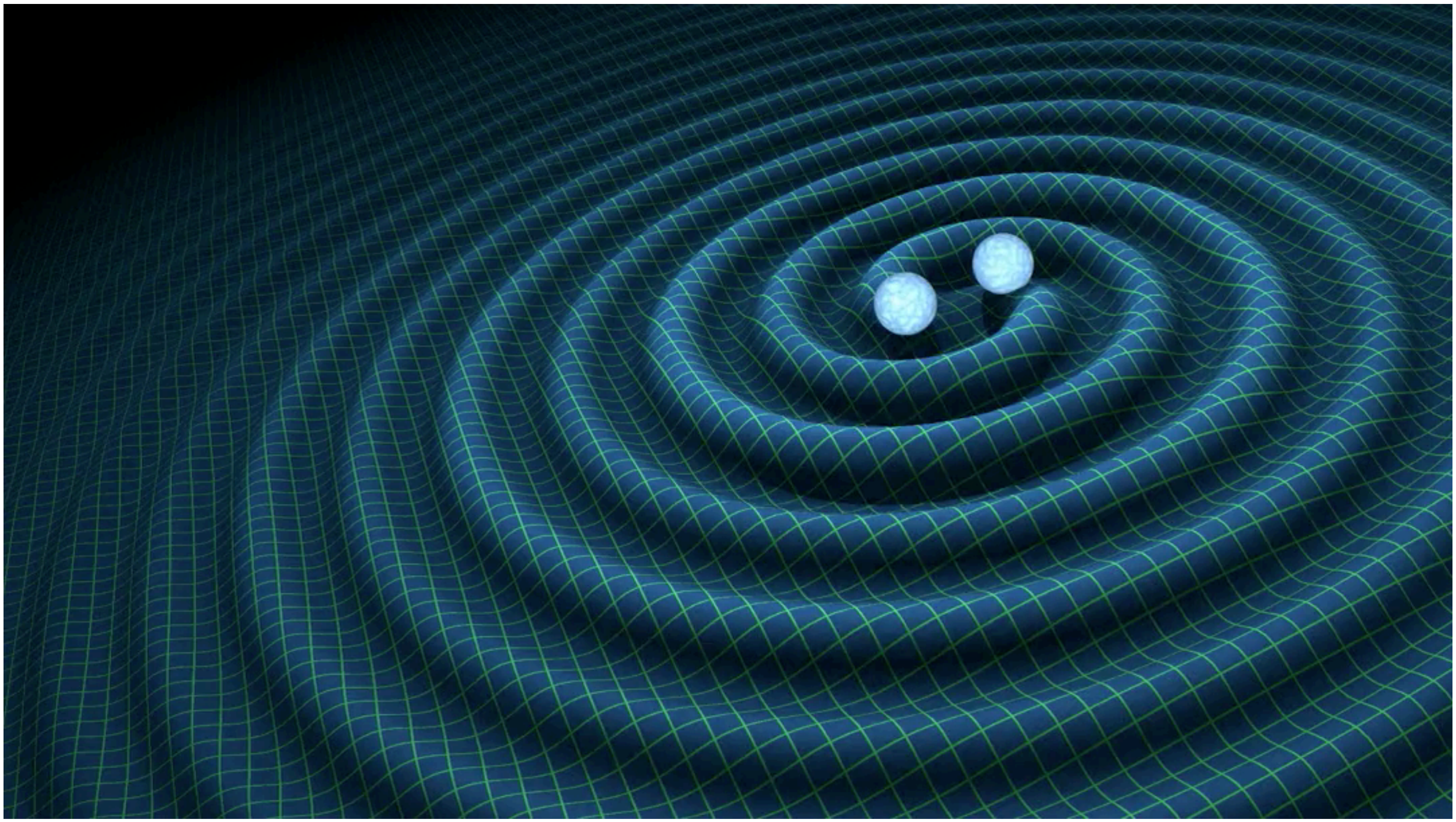
The Newtonian field equation (the Poisson equation) has no explicit time dependence.

$$\nabla^2 \phi = 4\pi G \rho.$$

In Newtonian gravity, the gravitational potential reacts instantaneously to any rearrangement of the masses that generate the field.

This is not true in General Relativity (GR), which has a finite speed of propagation of changes in the gravitational field.

Just as in E&M the retardation of the field leads to E&M waves, the same mechanism leads to gravitational waves.



credit: LIGO

The Einstein Field Equation in Vacuum

Einstein tensor

Ricci scalar

$$G_{ab} = R_{ab} - \tfrac{1}{2} g_{ab} R = 0$$

Ricci tensor

metric tensor

The Einstein Field Equation in Vacuum

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R = 0$$

Let's linearize the metric around the Minkowski space,

$$g_{ab} = \eta_{ab} + h_{ab}, \qquad \|h_{ab}\| \ll 1.$$

make a change of variables to a slightly different tensor

$$\bar{h}_{ab} = h_{ab} - \frac{1}{2}\eta_{ab} h$$

evaluate the Christoffel symbols, the Ricci metric, and plug everything into the vacuum field equation. The result is…

$$G_{ab} = \frac{1}{2}(\partial_c \partial_b \bar{h}^c{}_a + \partial^c \partial_a \bar{h}_{bc} - \Box \bar{h}_{ab} - \eta_{ab} \partial_c \partial^d \bar{h}^c{}_d) = 0$$

In General Relativity, spacetime coordinates are arbitrary.

To make progress (and make calculations easier), we need to make a choice of a gauge.

It is customary to use the Lorentz gauge

$$\partial^a \bar{h}_{ab} = 0$$

which allows us to write the Einstein field equation in vacuum as

$$\Box \bar{h}_{ab} = 0.$$

The linearized Einstein field equation in vacuum in the Lorentz gauge

$$\Box \bar{h}_{ab} = 0.$$

This is a special case of the Klein-Gordon equation

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi - \nabla^2\psi + \frac{m^2c^2}{\hbar^2}\psi = 0.$$

where

$$\Box = -\eta^{\mu\nu}\partial_\mu\partial_\nu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$$

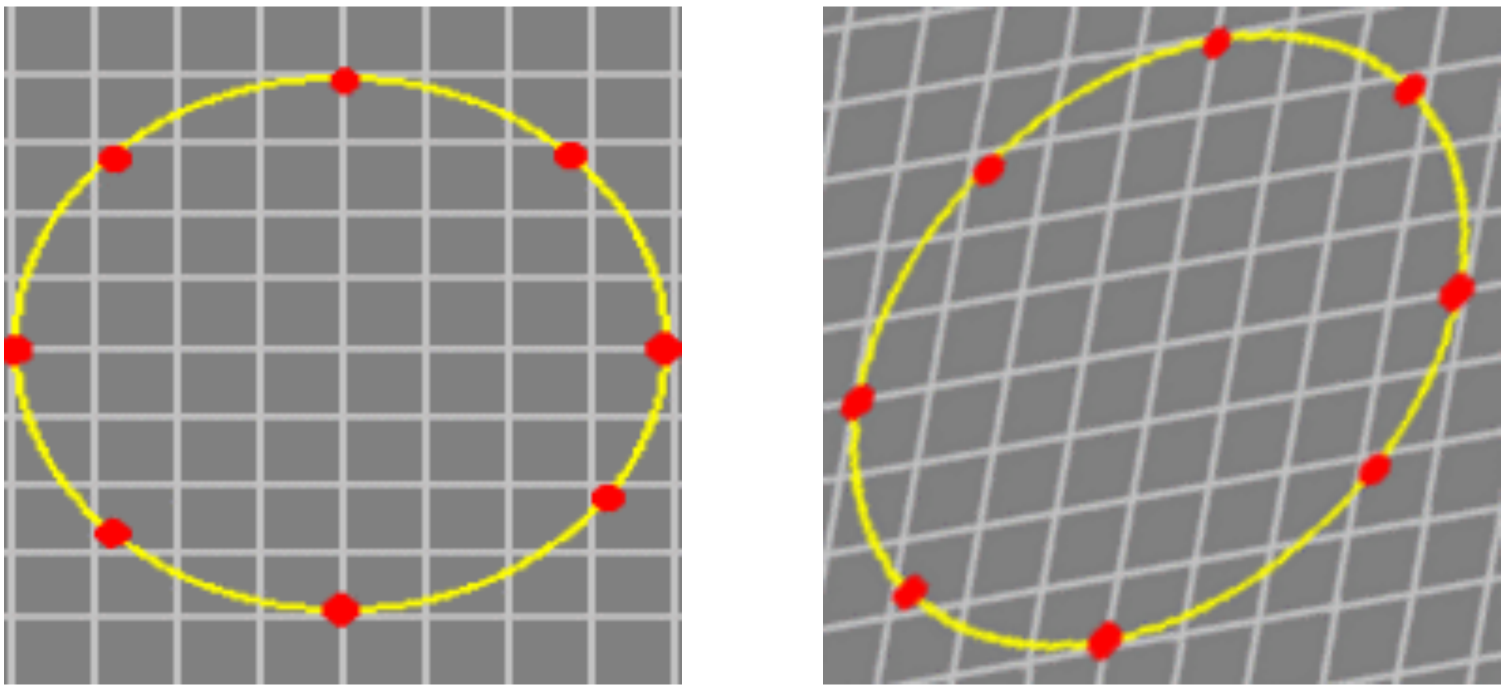
d'Alembert operator

mass of graviton

It shows that (in principle), gravitational waves

- propagate at the speed of light
- show no dispersion (massless gravitons)

GR gravitational waves can be decomposed into two polarization modes



Because of gauge freedom, we need to be very careful whether particular “radiative modes” are physical.

Physical radiative modes will have observable consequences for real systems.

For example,

(i) They will affect the orbits of binary stars

(ii) They will couple to (i.e., be detected by) gravitational wave detectors.

The linearized Einstein field equation with matter

$$\Box \bar{h}_{ab} = -16\pi T_{ab}$$

Skipping a lot of algebra steps, here is the solution to this equation:

We first define the second moment of the mass distribution:

$$I_{ij}(t) = \int \mathrm{d}^3x' \, \rho(t, \mathbf{x}')x'^i x'^j,$$

and from that, its quadrupole:

$$\mathcal{I}_{ij} = I_{ij} - \tfrac{1}{3}\delta_{ij} I, \qquad I = I_{ii}.$$

Then use Green's function to find the solution as

$$\bar{h}_{ij}(t, \mathbf{x}) = \frac{2}{r} \frac{\mathrm{d}^2 I_{ij}(t-r)}{\mathrm{d}t^2}.$$

$$\bar{h}_{ij}(t, \mathbf{x}) = \frac{2}{r}\frac{\mathrm{d}^2 I_{ij}(t-r)}{\mathrm{d}t^2}.$$

Gravitational waves are sourced (in the weak field) when the quadrupole moment of the mass distribution is time dependent.

No gravitational waves from:

- spherical collapse of a star (Birkhoff's theorem)
- rotation of an axisymmetric mass distribution

$$\bar{h}_{ij}(t, \mathbf{x}) = \frac{2}{r}\frac{\mathrm{d}^2 I_{ij}(t-r)}{\mathrm{d}t^2}.$$

For a binary stellar system of total mass M, separation a, and period P=2π/Ω,

$$h \sim \frac{2}{r}\frac{Ma^2}{P^2} \qquad a \sim M^{1/3}P^{2/3}$$

so the order-of-magnitude result (at a distance r) is

$$h = \frac{M^{5/3}\Omega^{2/3}}{r}.$$

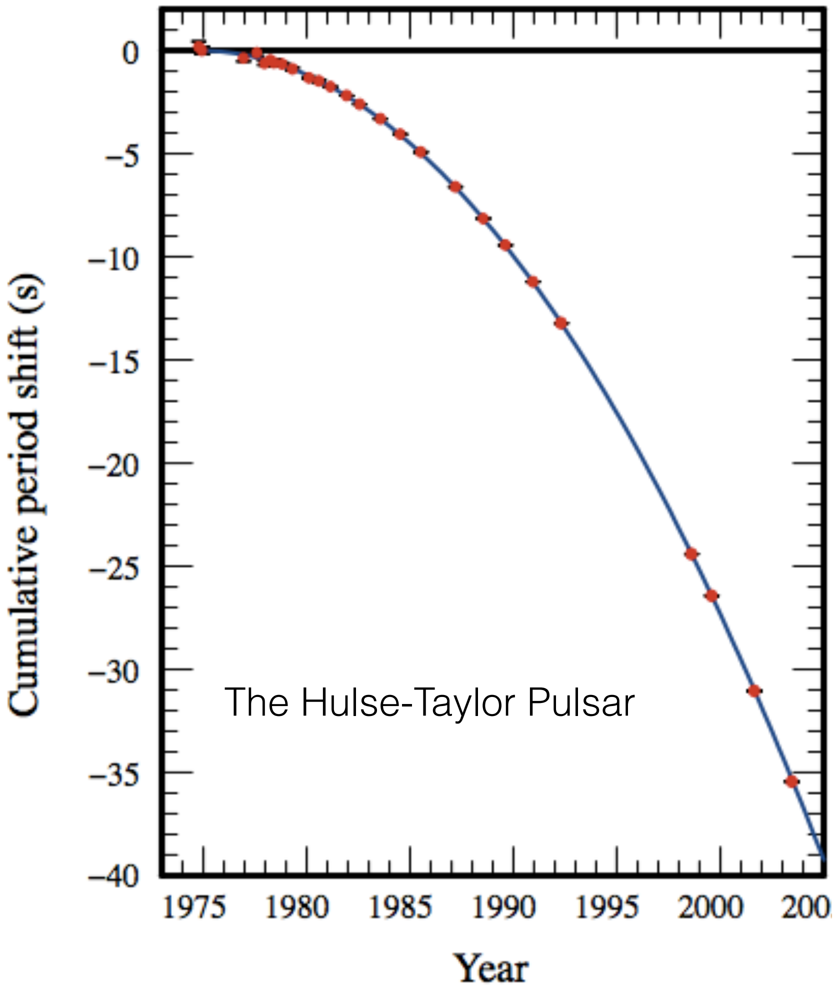
Putting the units in the previous relation, we get

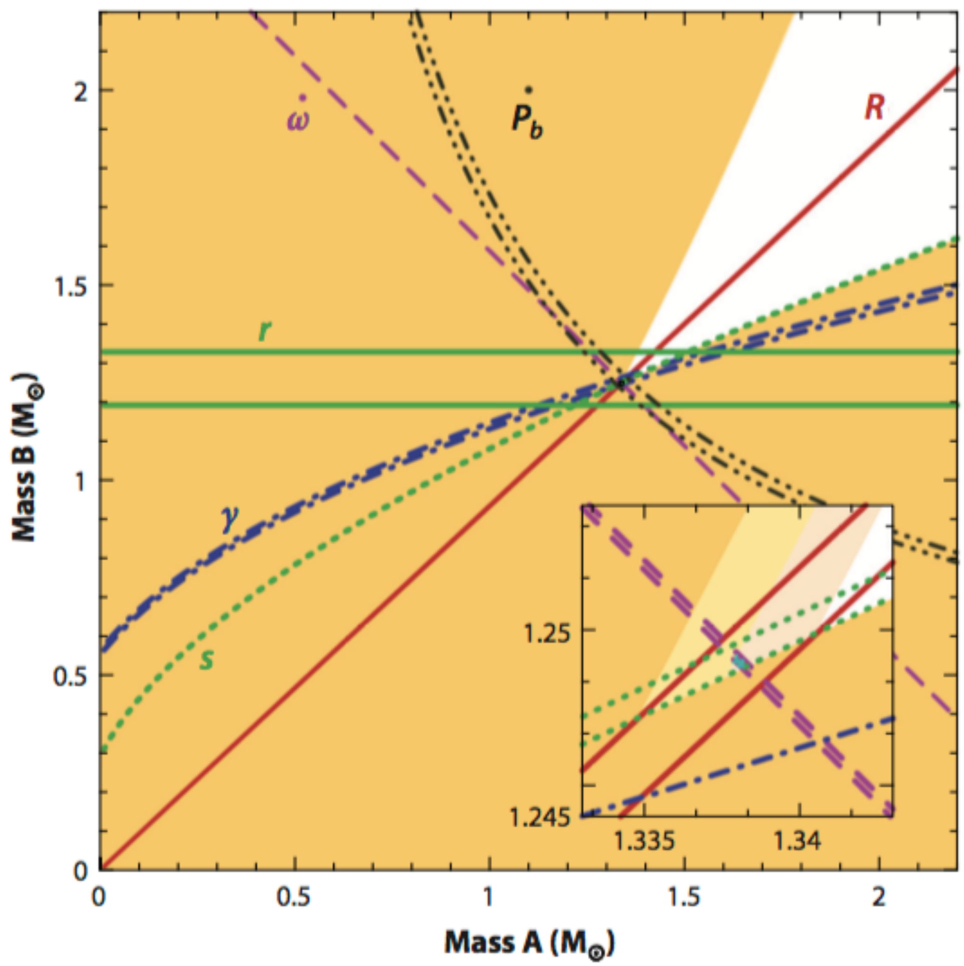
$$h \simeq 10^{-21} \left(\frac{M}{2M_\odot}\right)^{5/3} \left(\frac{1\,\mathrm{h}}{P}\right)^{2/3} \left(\frac{1\,\mathrm{kiloparsec}}{r}\right)$$
$$\simeq 10^{-22} \left(\frac{M}{2.8M_\odot}\right)^{5/3} \left(\frac{0.01\,\mathrm{second}}{P}\right)^{2/3} \left(\frac{100\mathrm{megaparsecs}}{r}\right)$$

This is an extraordinarily small number.

However, it can be shown that the stress-energy tensor of the gravitational waves is non-zero and that they carry away from the binary energy and angular momentum causing the orbit to evolve.

Detection of orbital decay of binaries due to emission of gravitational waves





The Double Pulsar

Kramer et al. 2016

What if GR is modified?

Many (most?) modifications to GR can be written in an equivalent way as Scalar-Tensor theories.

The presence of the scalar field leads to dipole gravitational radiation and scalar waves.

OBSERVABLE EFFECTS OF A SCALAR GRAVITATIONAL FIELD IN A BINARY PULSAR*

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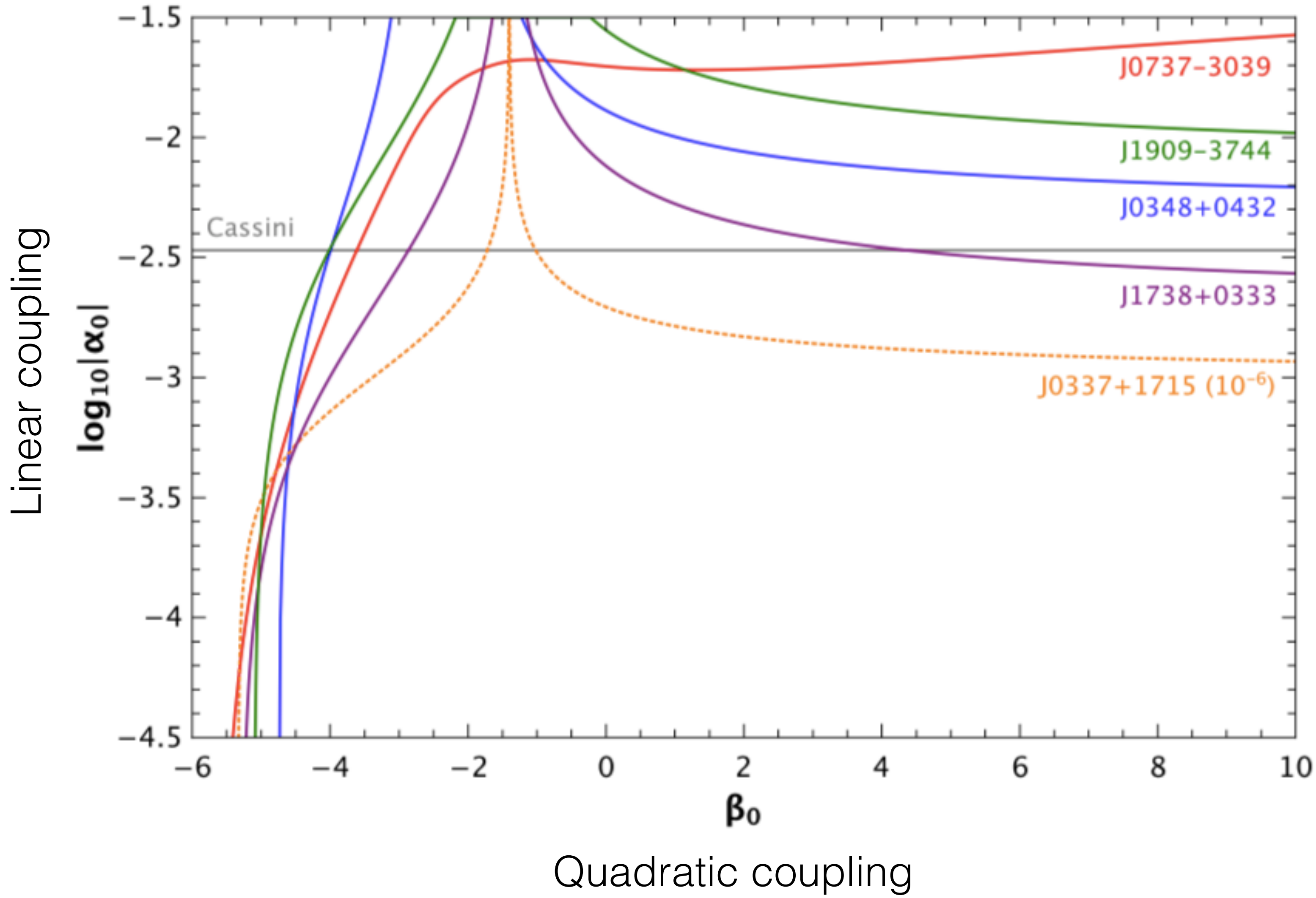
Received 1974 November 7

ABSTRACT

It is remarked that the Dicke-Brans-Jordan theory of gravitation predicts *dipole* gravitational radiation from certain binary systems that contain a neutron star or black hole, causing decay of the orbit. Further, it predicts that the true rate of a pulsar clock should vary with distance from a binary companion (other than a black hole). At least one of these effects would be observable in PSR 1913+16.

The dipolar energy loss depends on the coupling of the scalar field to the metric but also on the mass asymmetry (dipole) in the binary.

Current constraints on Scalar-Tensor Gravity from pulsars in binaries



Kramer 2016
based on work by Damour & Esposito-Farese

How do gravitational waves couple to a detector?

Let's consider the motion of a test particle (e.g., the mirror of a detector), when a gravitational wave passes by.

$$\frac{d^2 x^a}{d\tau^2} + \Gamma^a{}_{bc} \frac{dx^b}{d\tau} \frac{dx^c}{d\tau} = 0,$$

Plugging in for the metric the wave solution of the linearized Einstein field equations, one can show that

$$\frac{d^2 x^a}{d\tau^2} = 0,$$

This means that the *coordinate* location of a test particle does not change!

How do gravitational waves couple to a detector?

But how about the *proper* distance between two test particles?

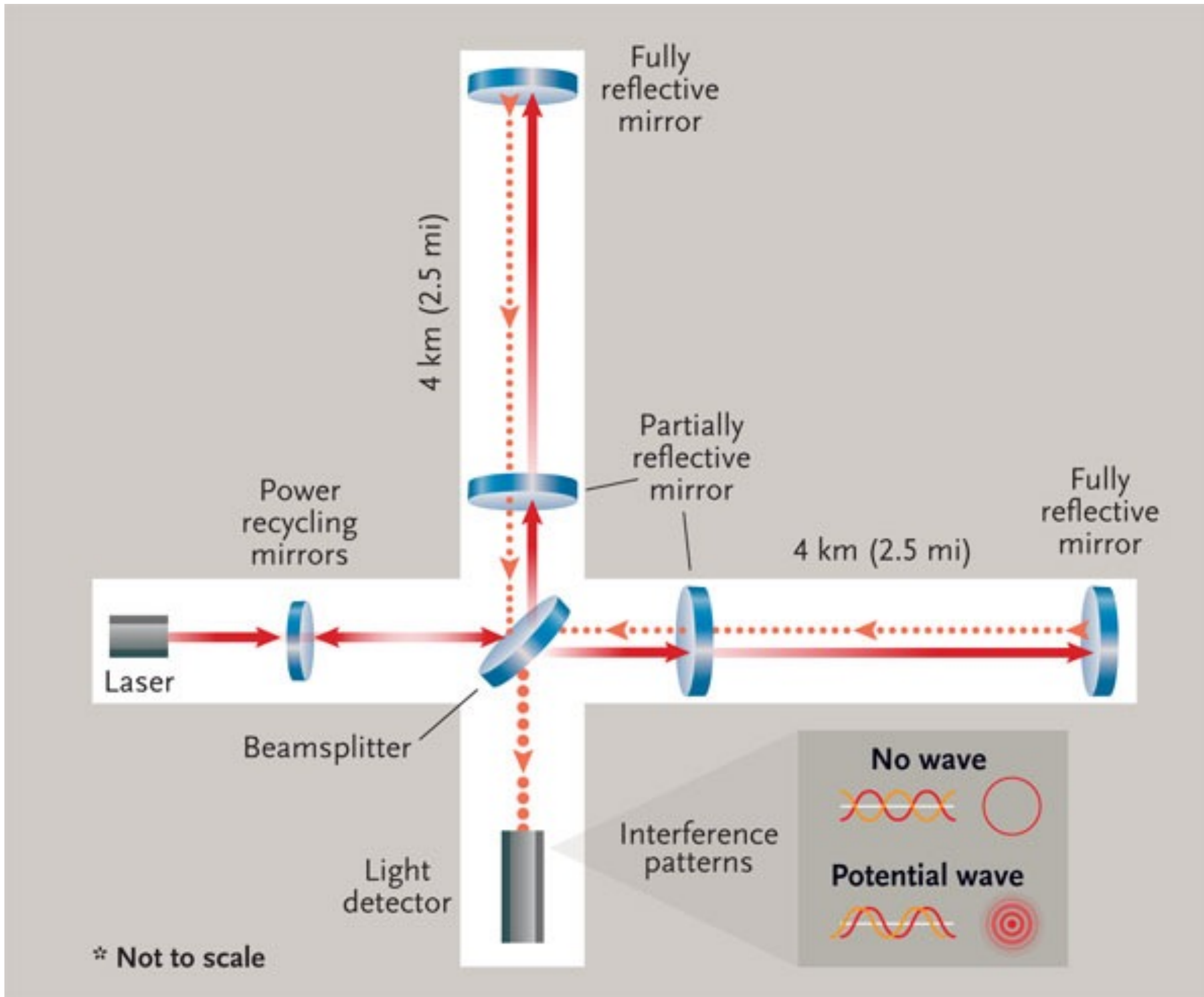
Using a particular coordinate system that obeys the Lorentz gauge, one can show that

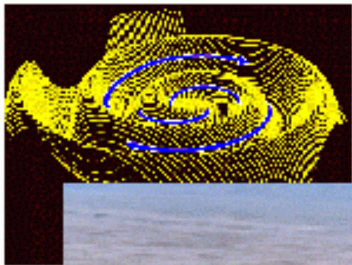
$$L = \int_0^{L_c} \mathrm{d}x \sqrt{g_{xx}} = \int_0^{L_c} \mathrm{d}x \sqrt{1 + h_{xx}^{\mathrm{TT}}(t, z = 0)}$$

$$\simeq \int_0^{L_c} \mathrm{d}x [1 + \tfrac{1}{2} h_{xx}^{\mathrm{TT}}(t, z = 0)] = L_c [1 + \tfrac{1}{2} h_{xx}^{\mathrm{TT}}(t, z = 0)].$$

If you measure distances and time intervals between the two test particles using light, you should see an observable effect due to the gravitational waves.

The concept of interferometric gravitational wave detection





Hanford, WA

Laser Interferometer Gravitational-wave Observatory (LIGO)

Livingston, LA

-two instalations

-two 4 km arms

-laser interferometers

What limits the sensitivity of a GW interferometer?

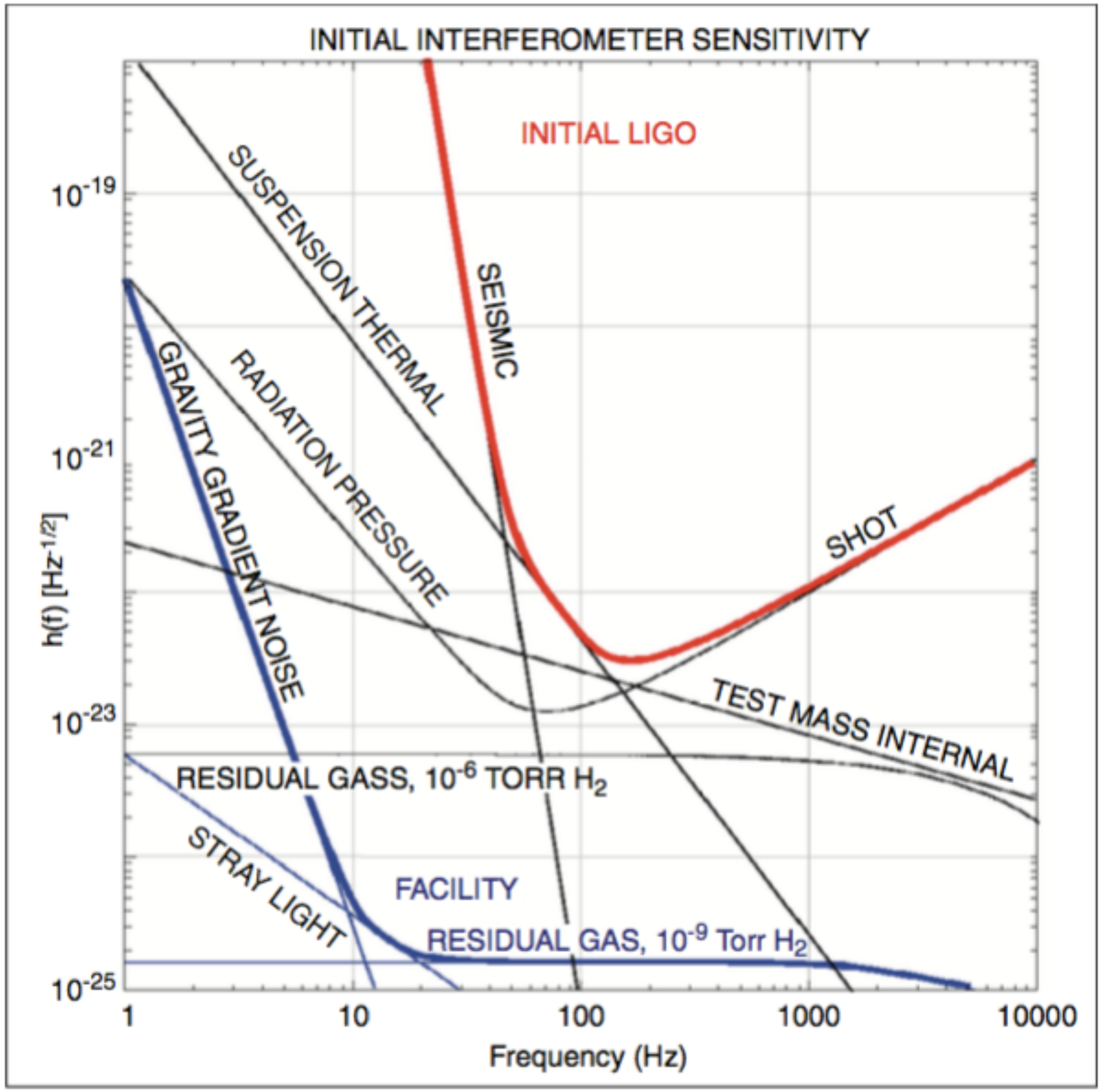
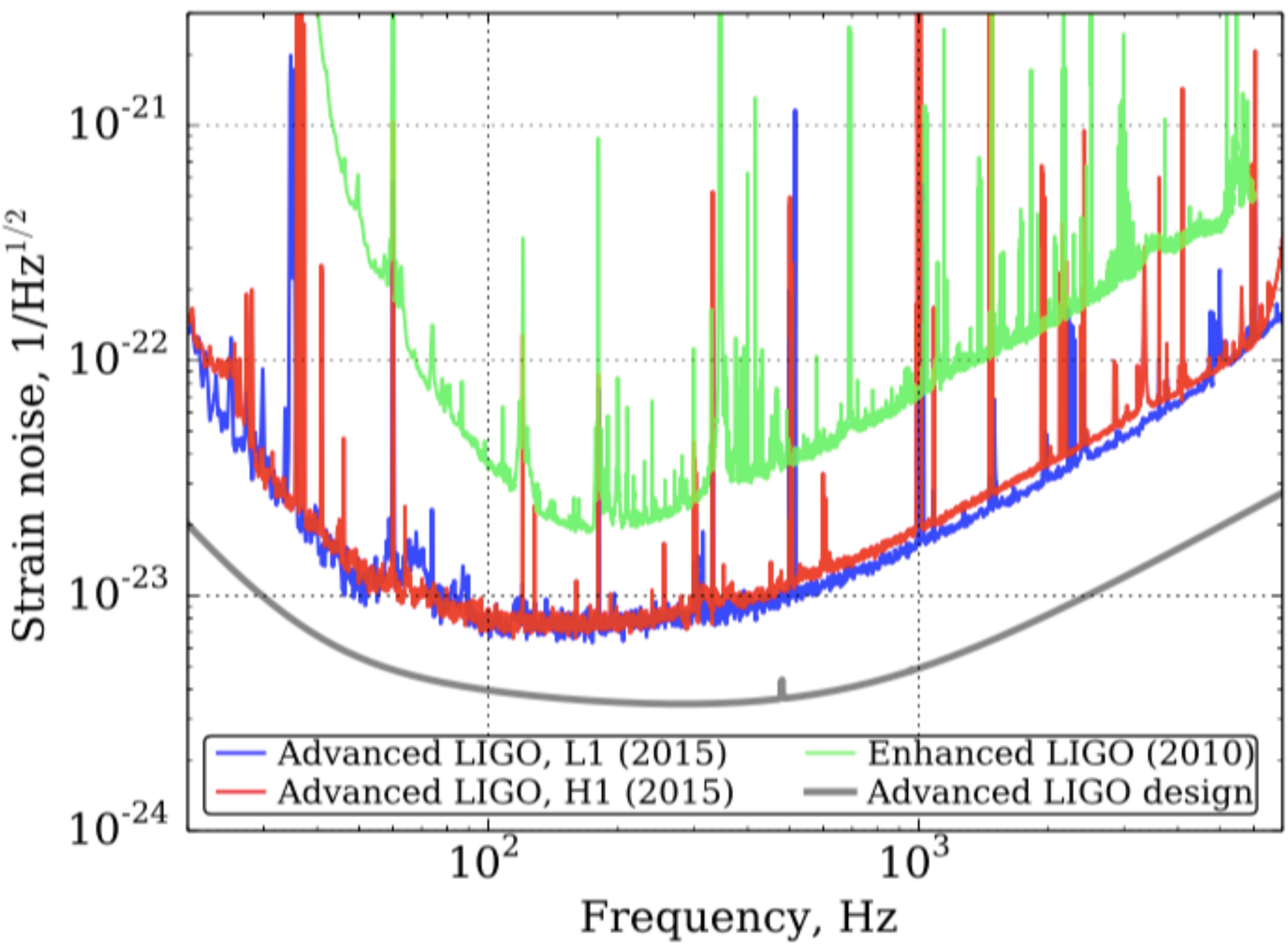
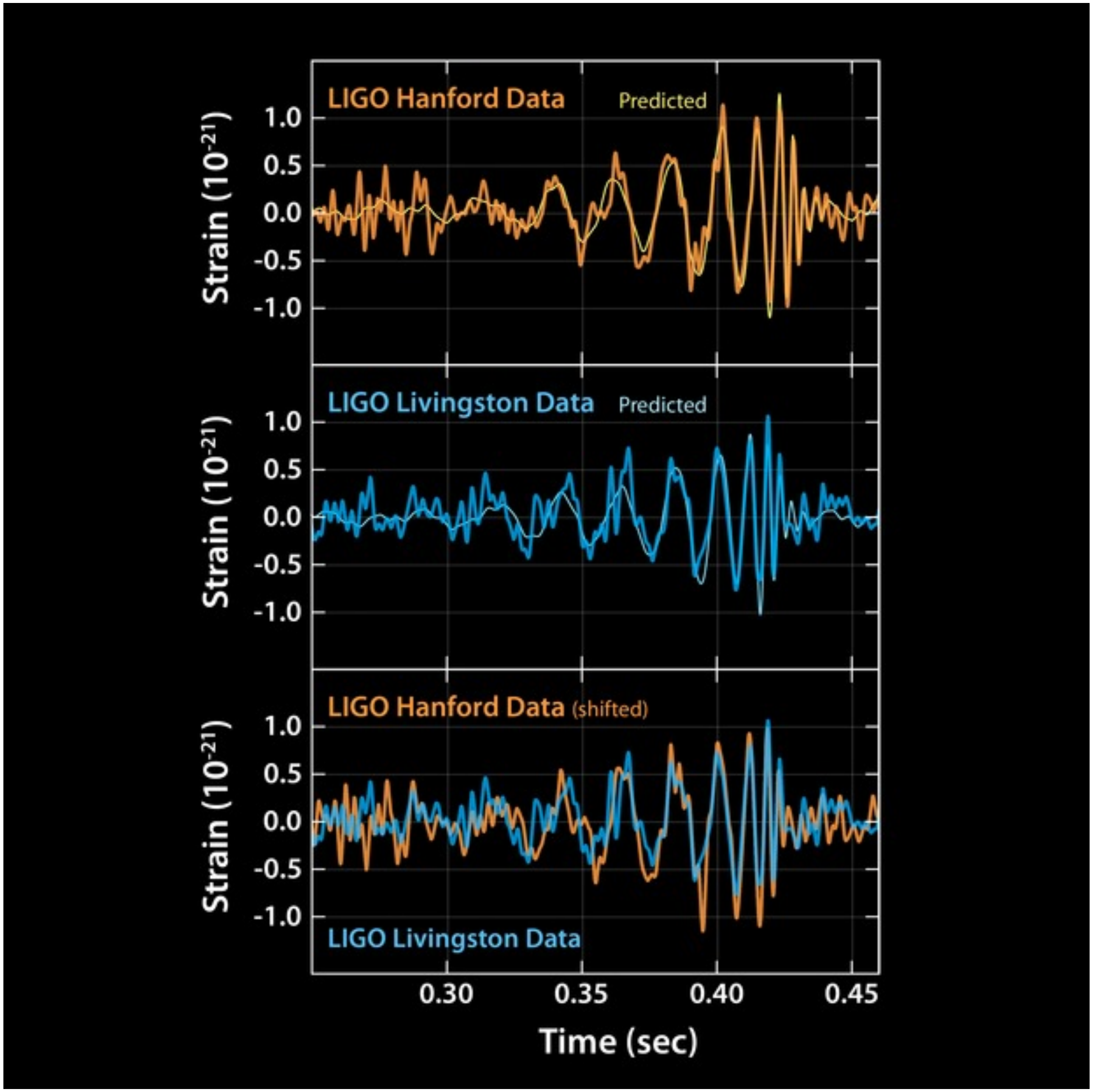


Figure 2. Sensitivity goals of the initial LIGO interferometers, and facility limits on the LIGO sensitivity (taken from [25]).

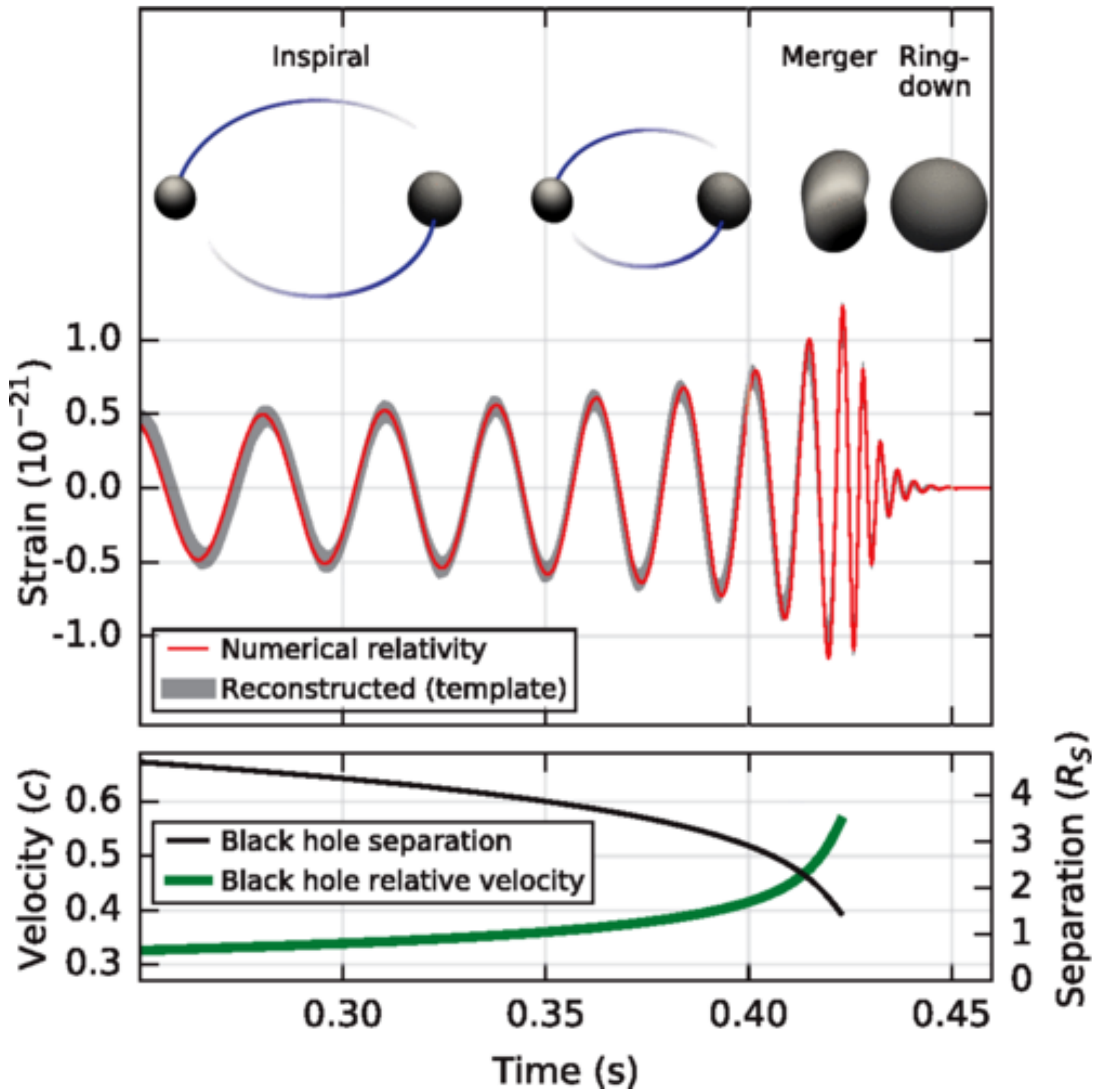
What limits the sensitivity of a GW interferometer?



Martynov et al. 2016



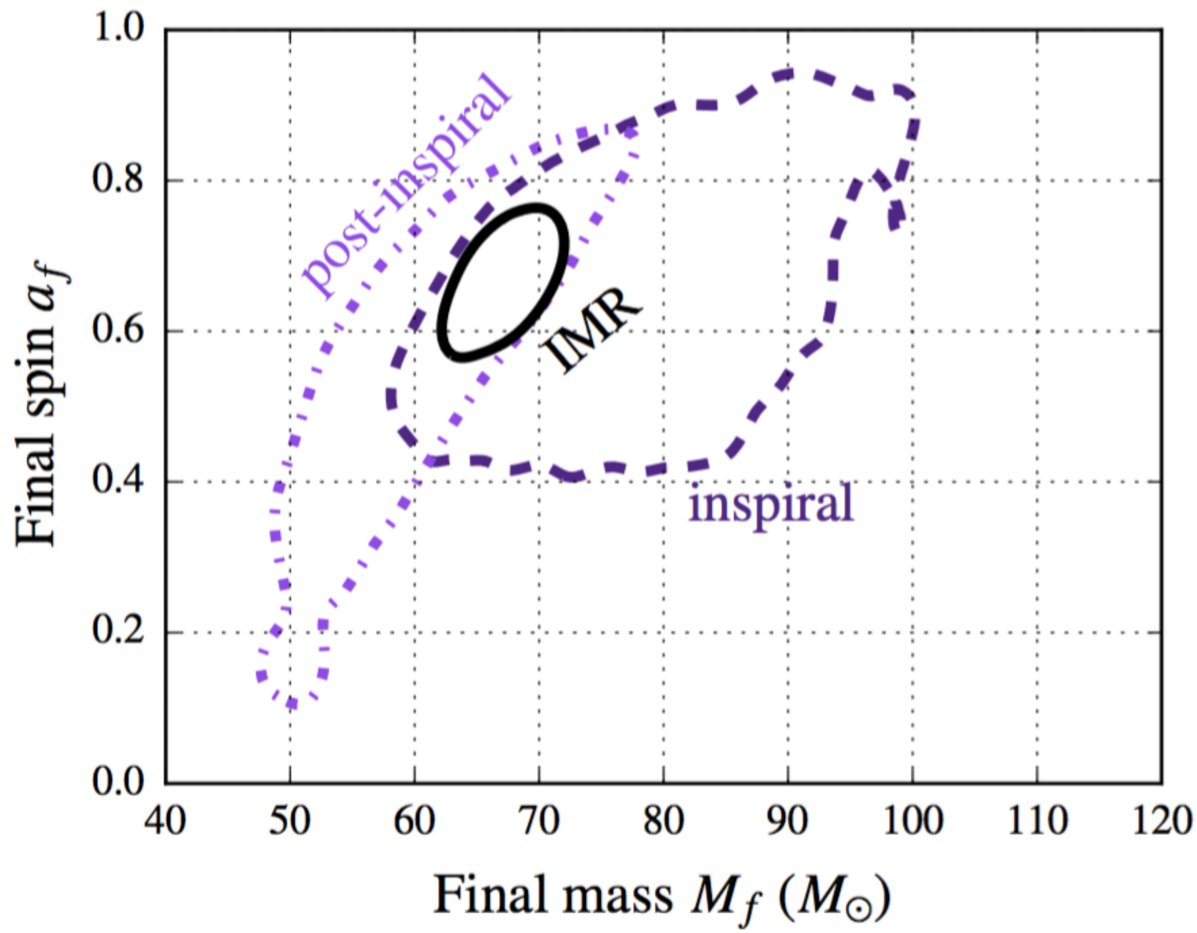
First direct detection with LIGO



The three phases of the GW event

GR Tests with LIGO

1. Are the parameters inferred from the inspiral and the ringdown consistent with each other?



GR Tests with LIGO

2. Are the PPN parameters consistent with GR?

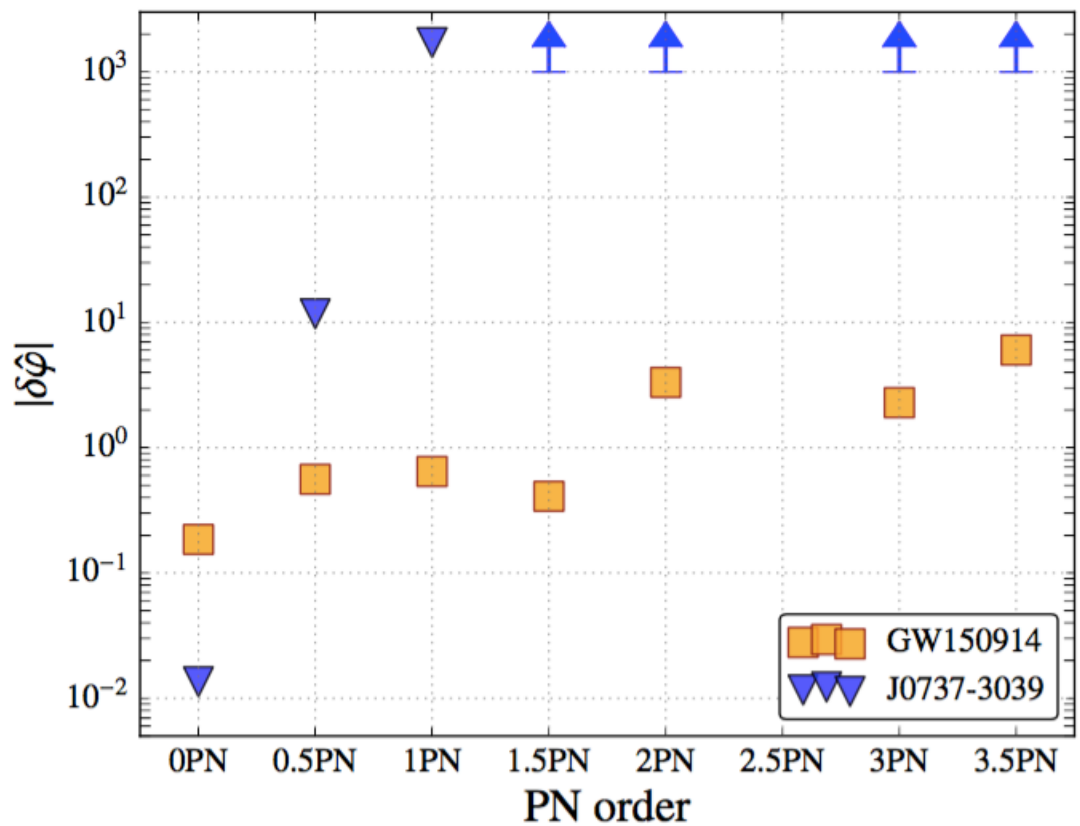
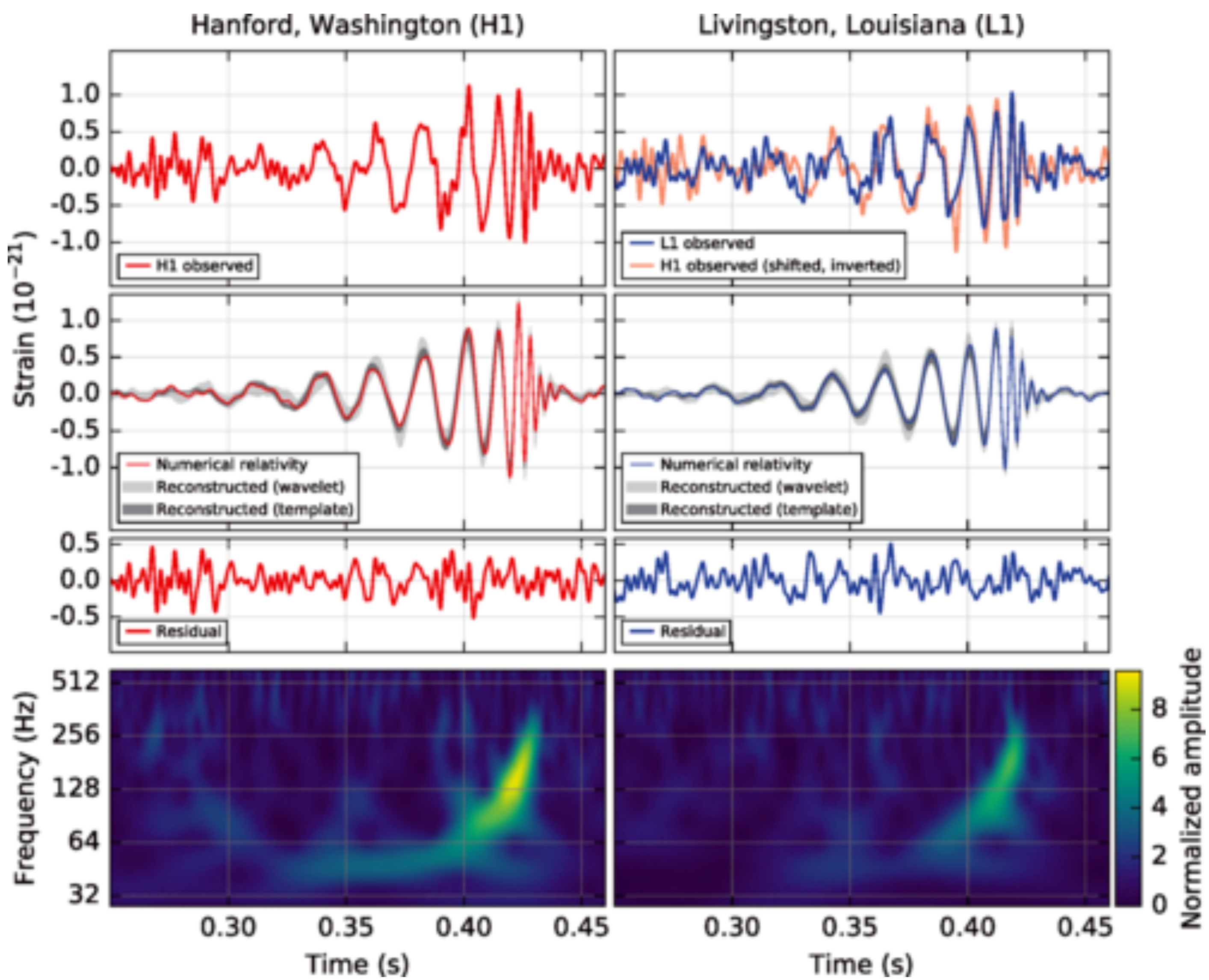


FIG. 6. 90% upper bounds on the fractional variations of the known PN coefficients with respect to their GR values.



The chirp of the GW frequency

GR Tests with LIGO

3. Do gravitational waves propagate at the speed of light?

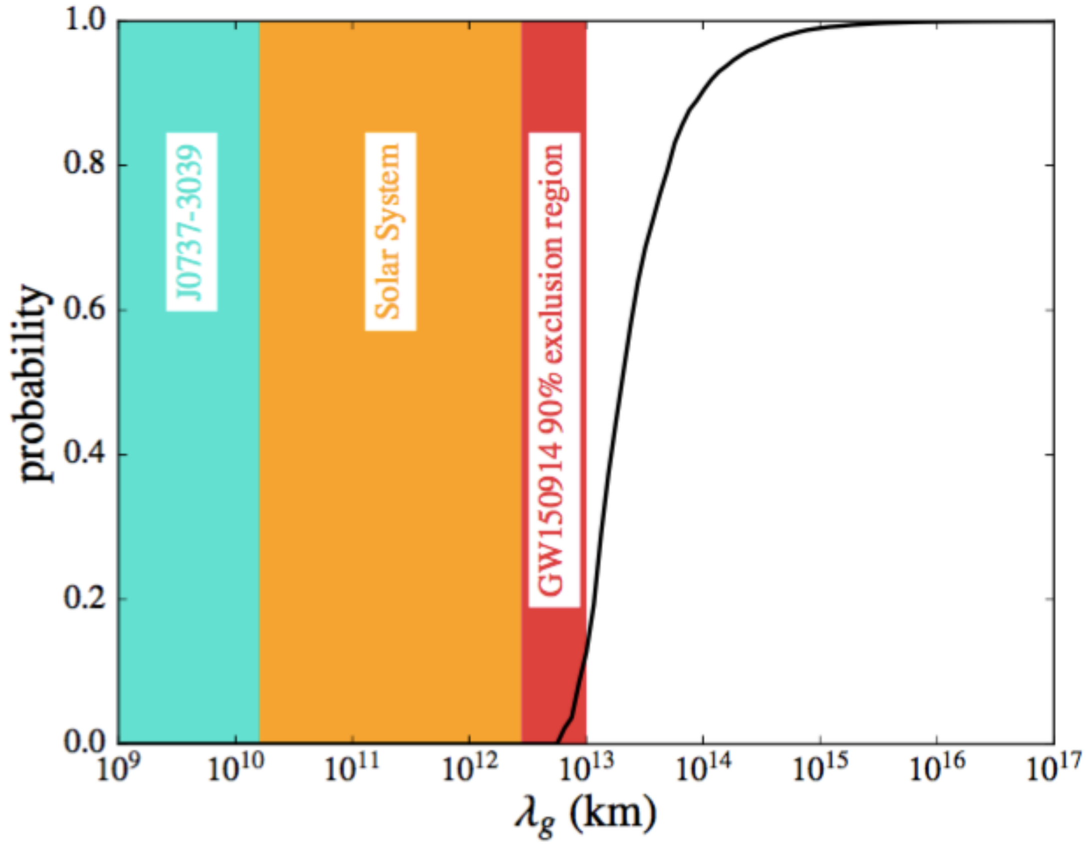
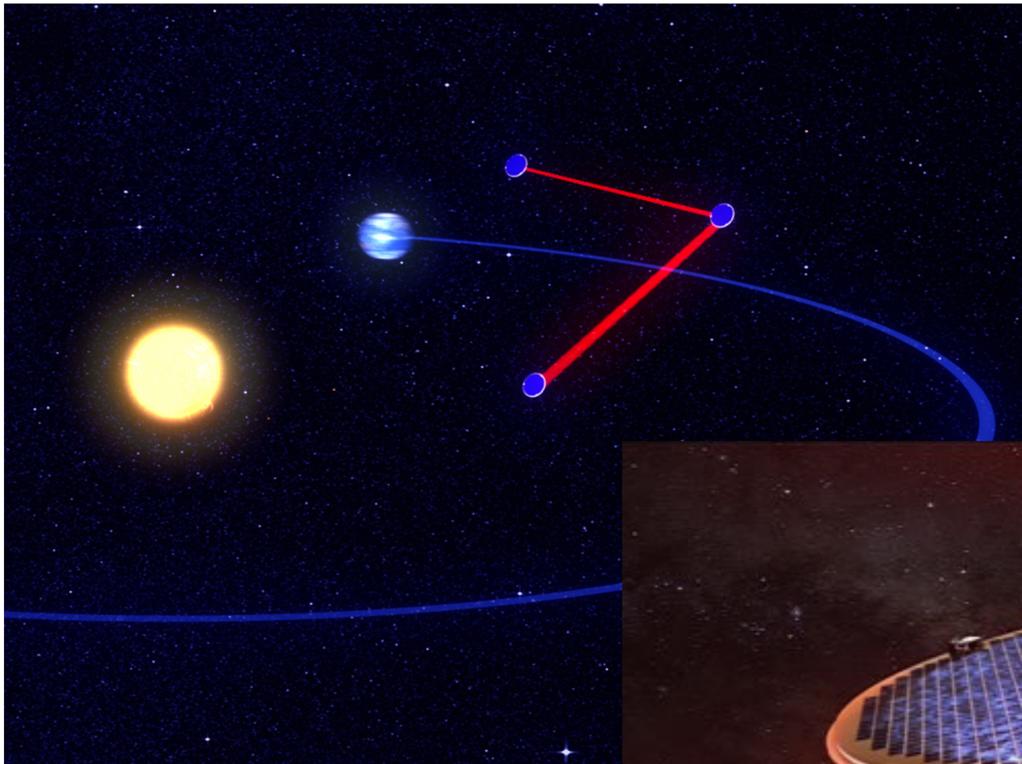
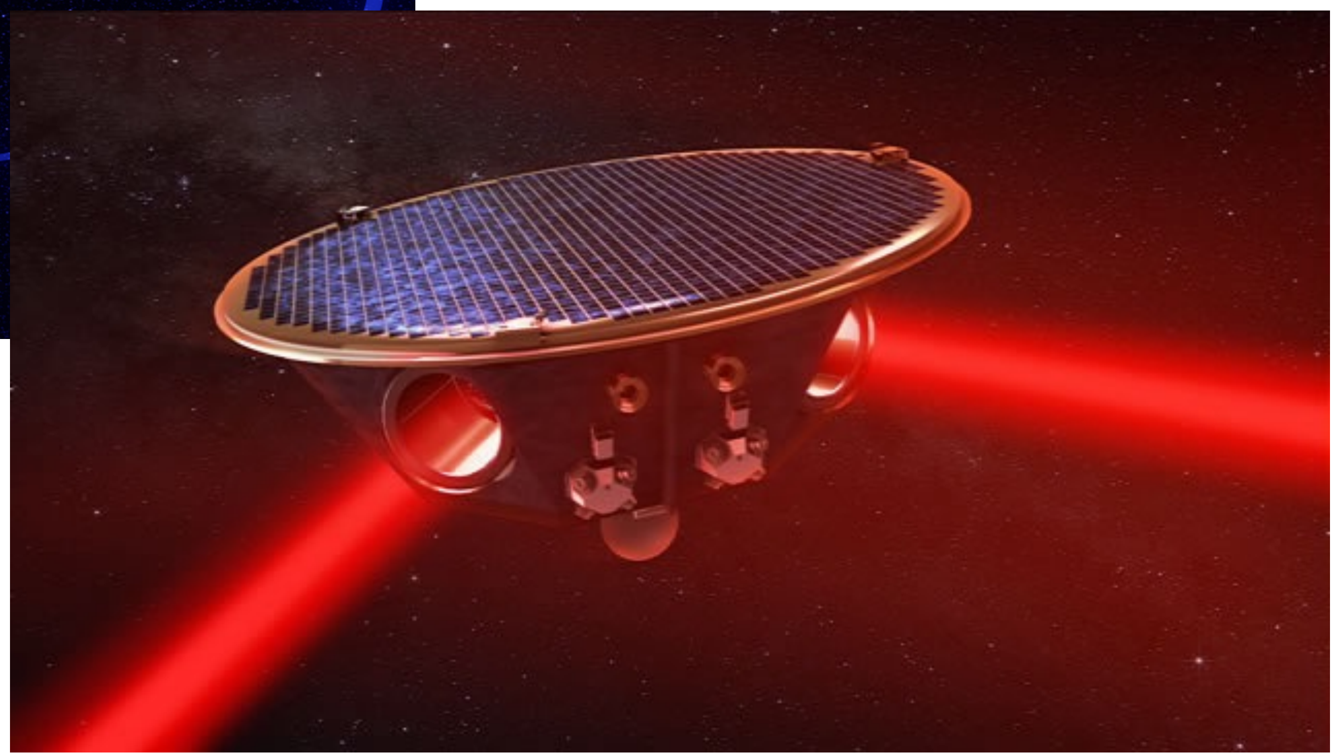


FIG. 8. Cumulative posterior probability distribution for λ_g (black curve) and exclusion regions for the graviton Compton wavelength λ_g from GW150914. The shaded areas show exclusion regions from the double pulsar observations (turquoise), the static Solar System bound (orange) and the 90% (crimson) region from GW150914.

Gravitational Wave Detection from Space



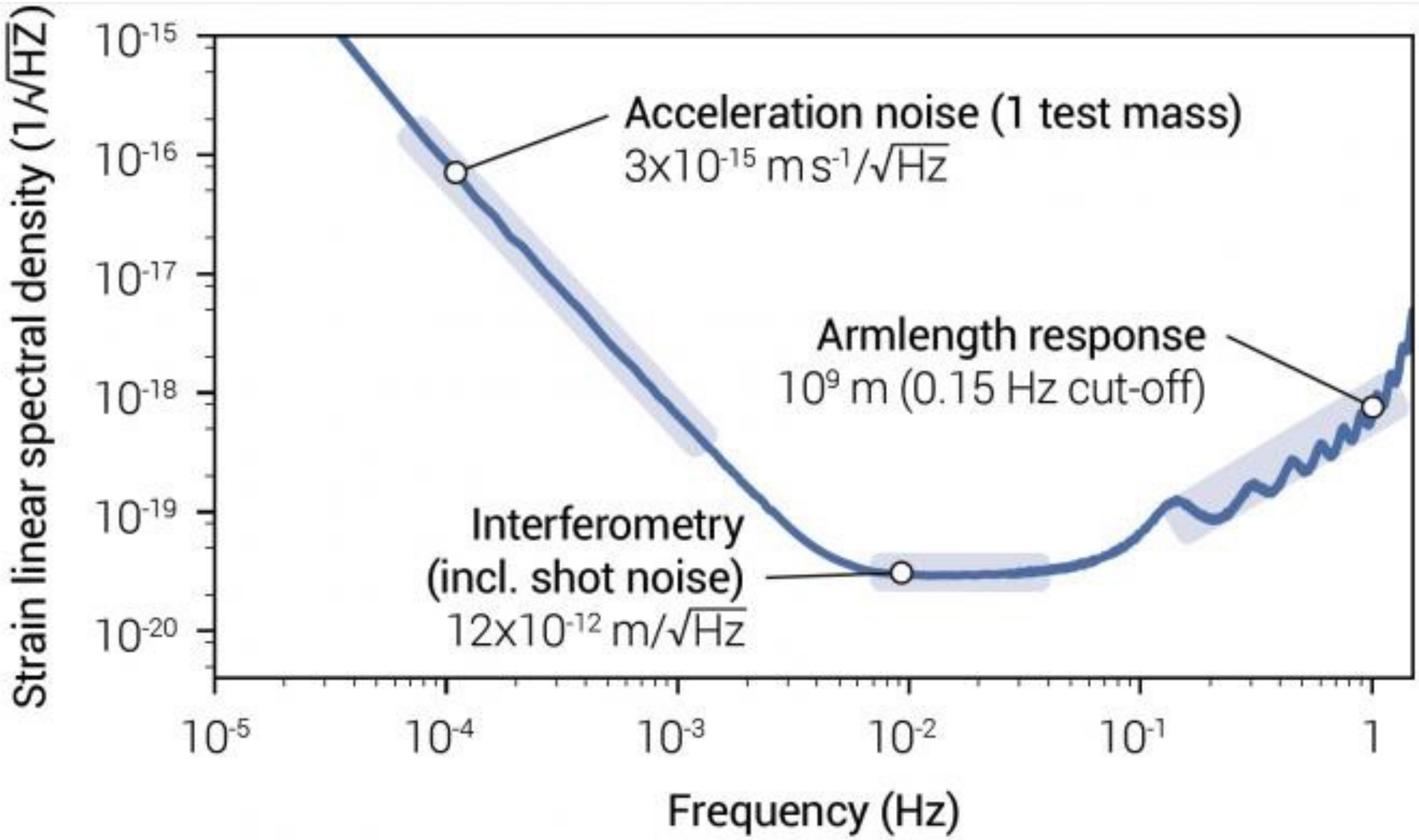
Laser
Interferometer
Space
Antenna



www.elisascience.org

Gravitational Wave Detection from Space

Laser Interferometer Space Antenna



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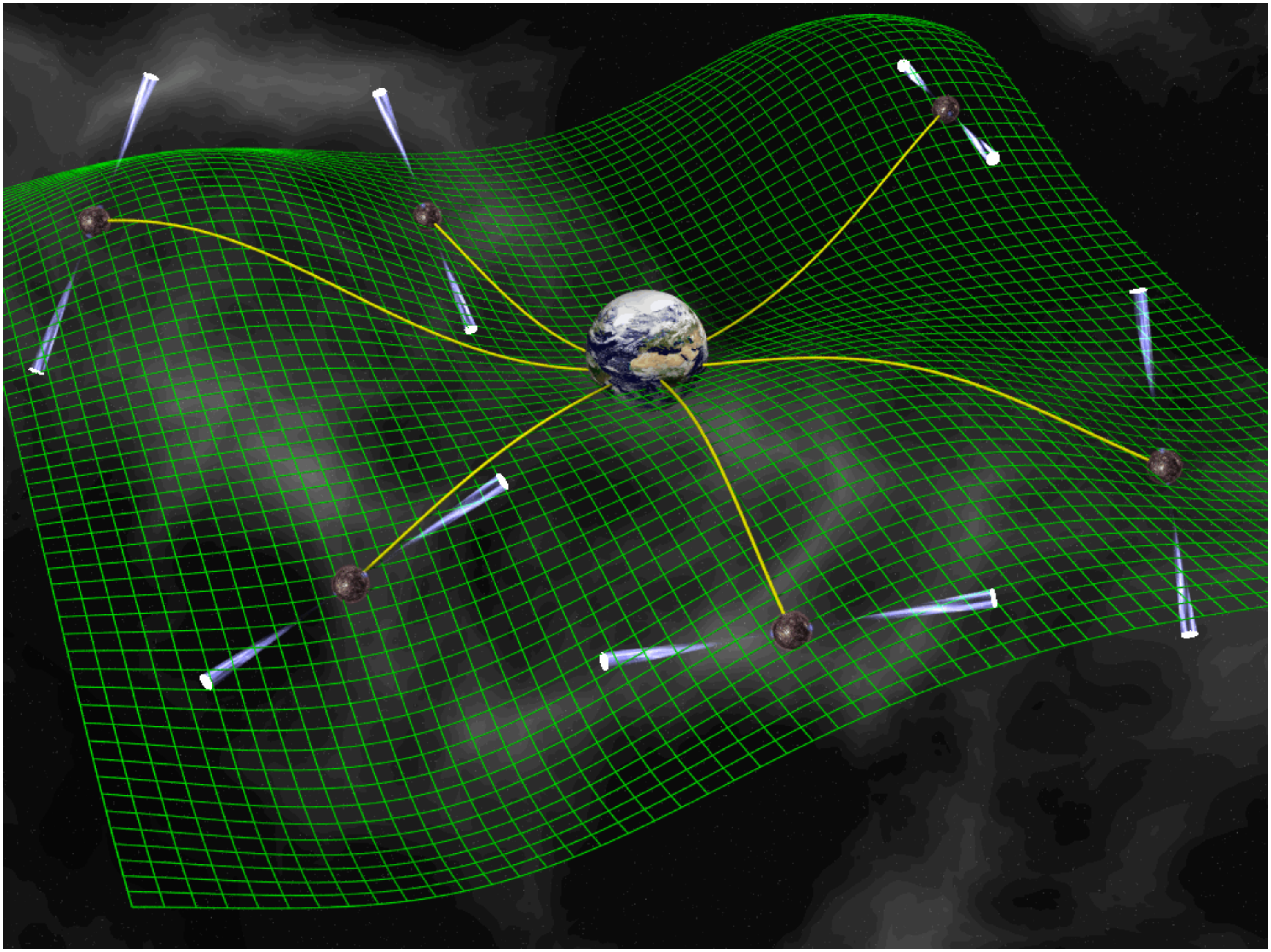
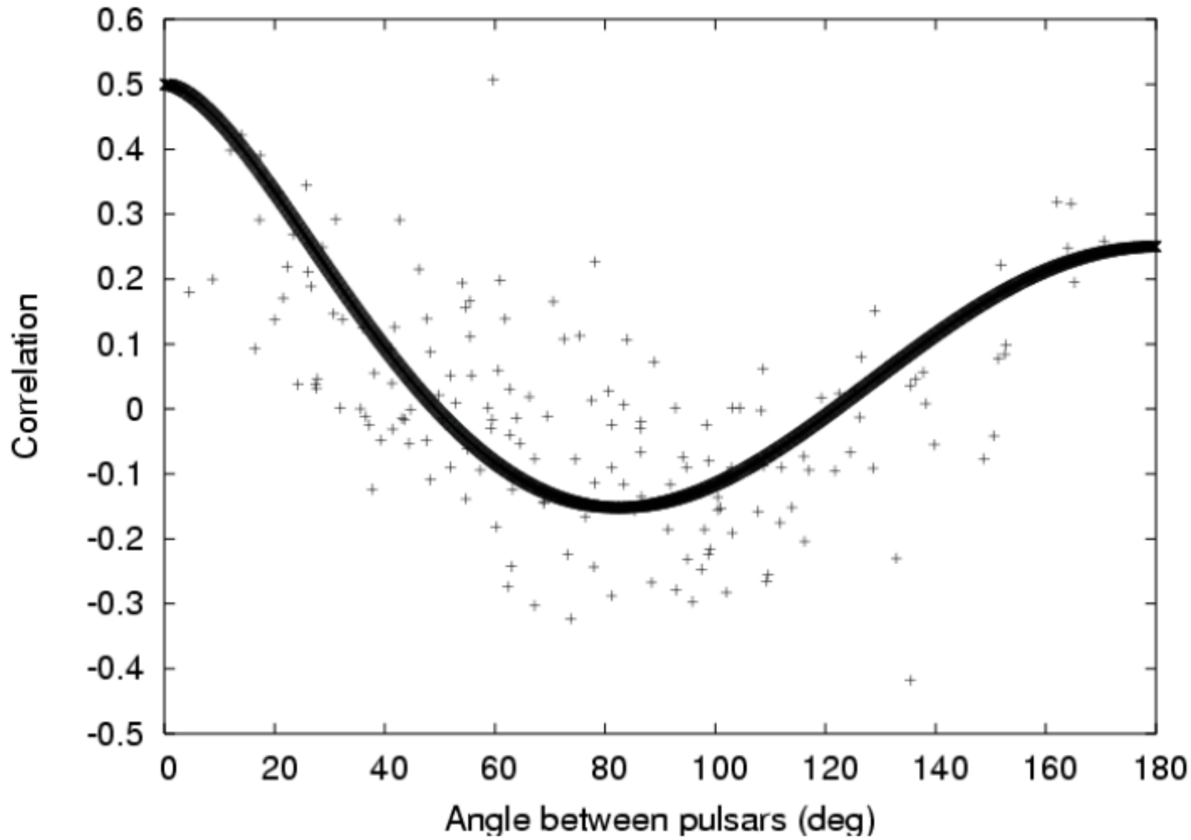
Observing Gravitational Waves with Pulsar Timing Arrays



Observing Gravitational Waves with Pulsar Timing Arrays

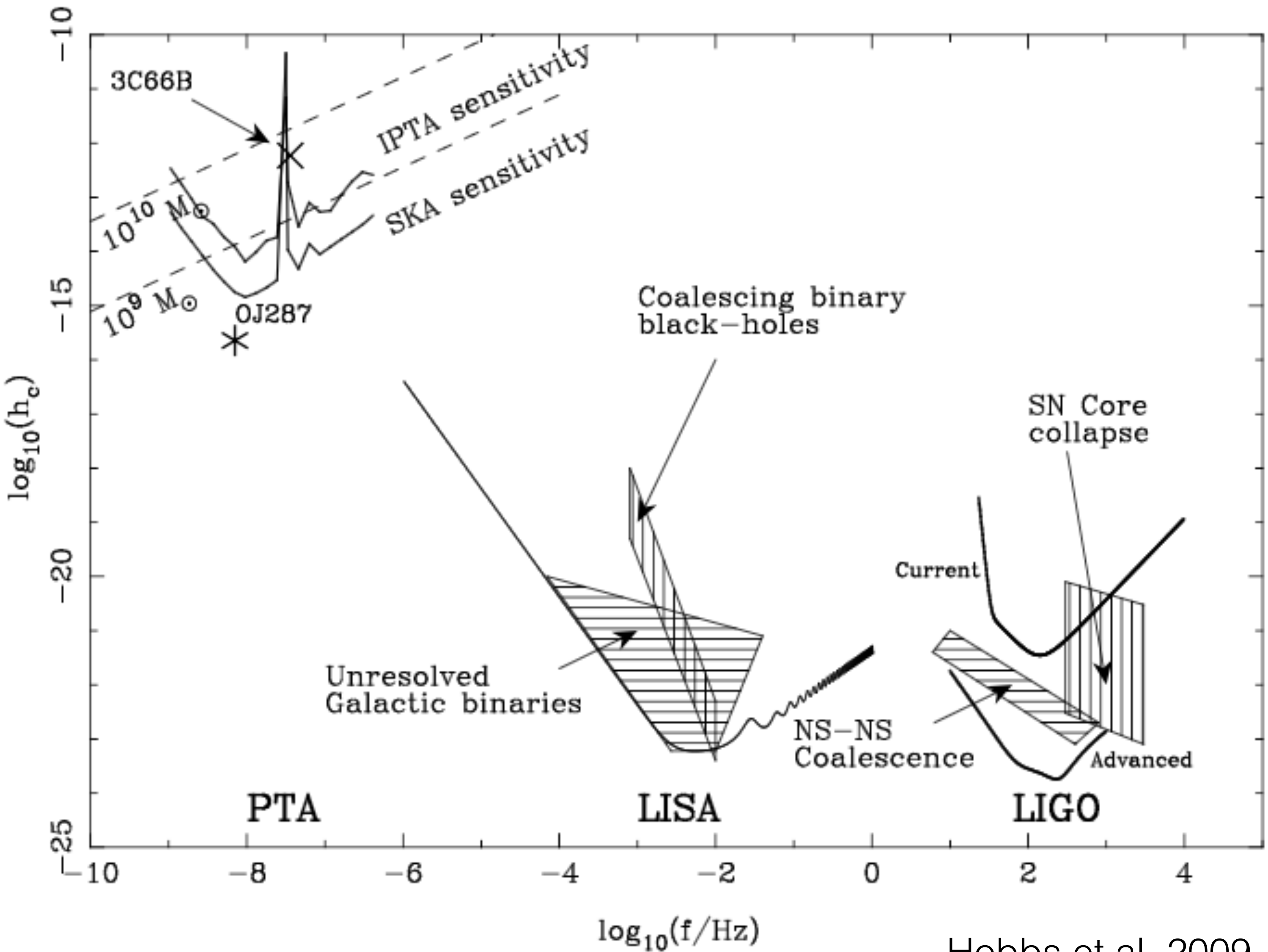


Hellings & Downs (1983)
curve

Figure 1. The expected correlation in the timing residuals of pairs of pulsars as a function of angular separation for an isotropic GW background.

nice pedagogical introduction at arXiv:1412.1142v2

Pulsar Timing Arrays



Hobbs et al. 2009

The International Pulsar Timing Array

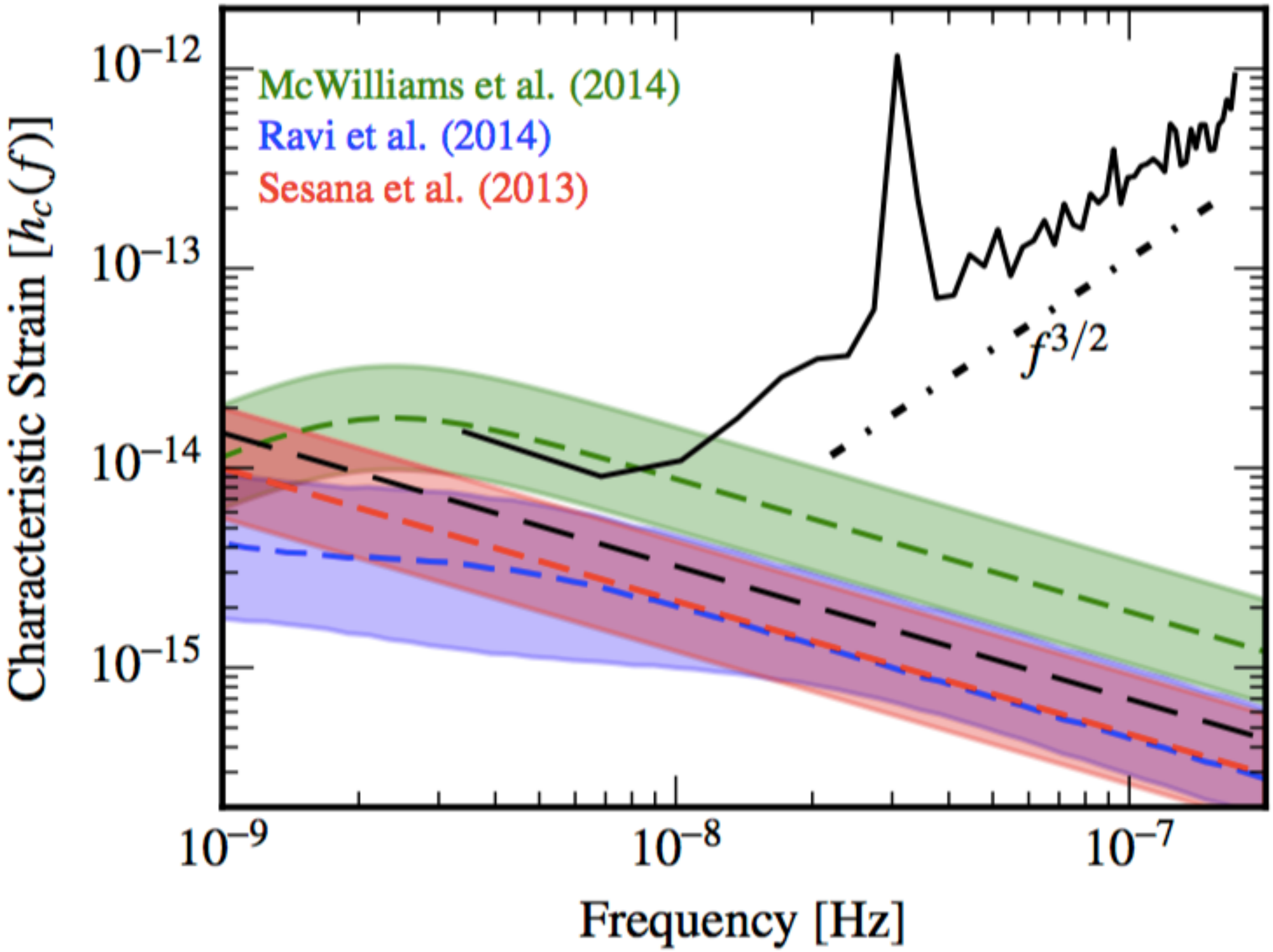
EPTA

NANOGrav

PPTA

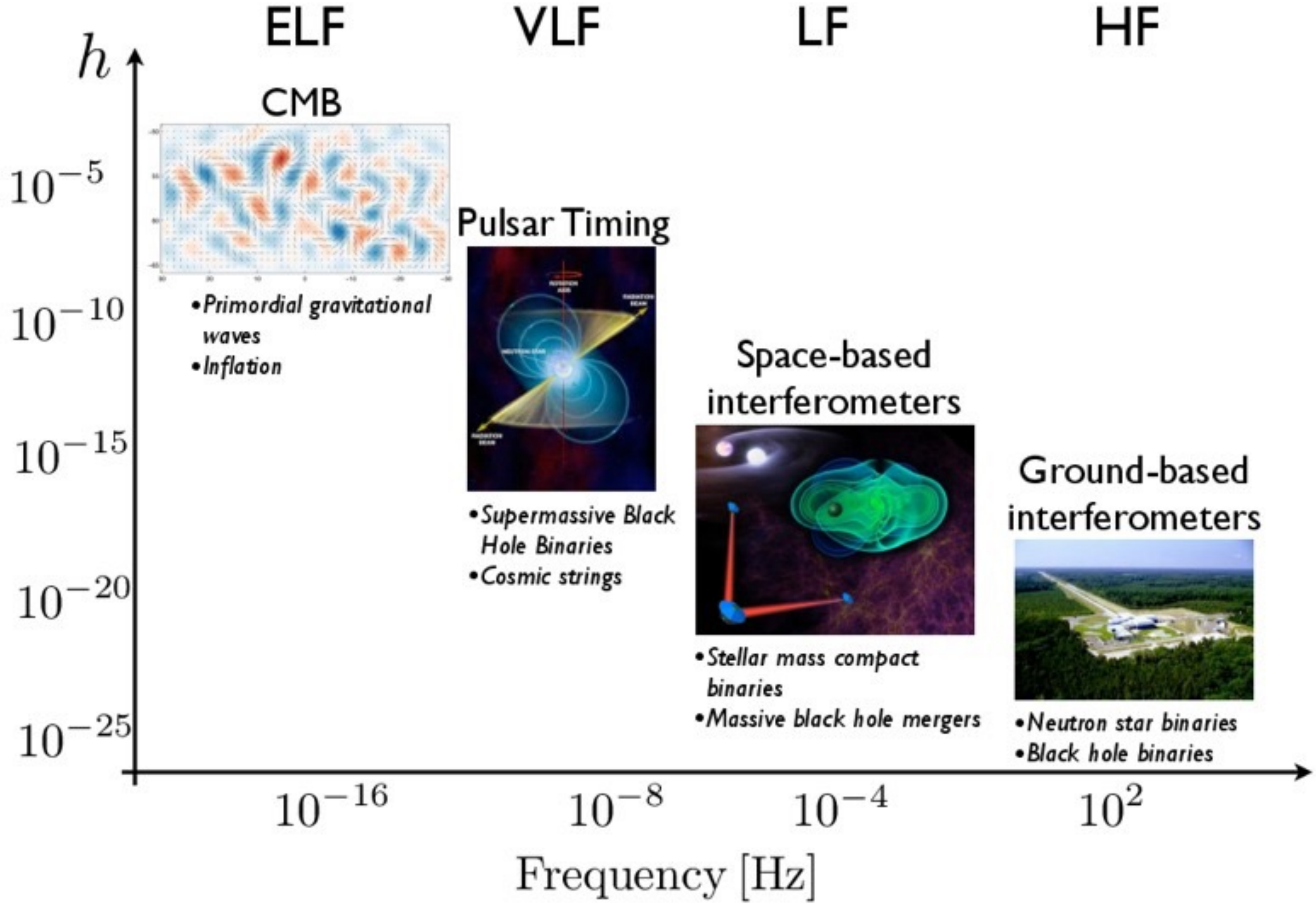
ipta4gw.org

Pulsar Timing Arrays Results



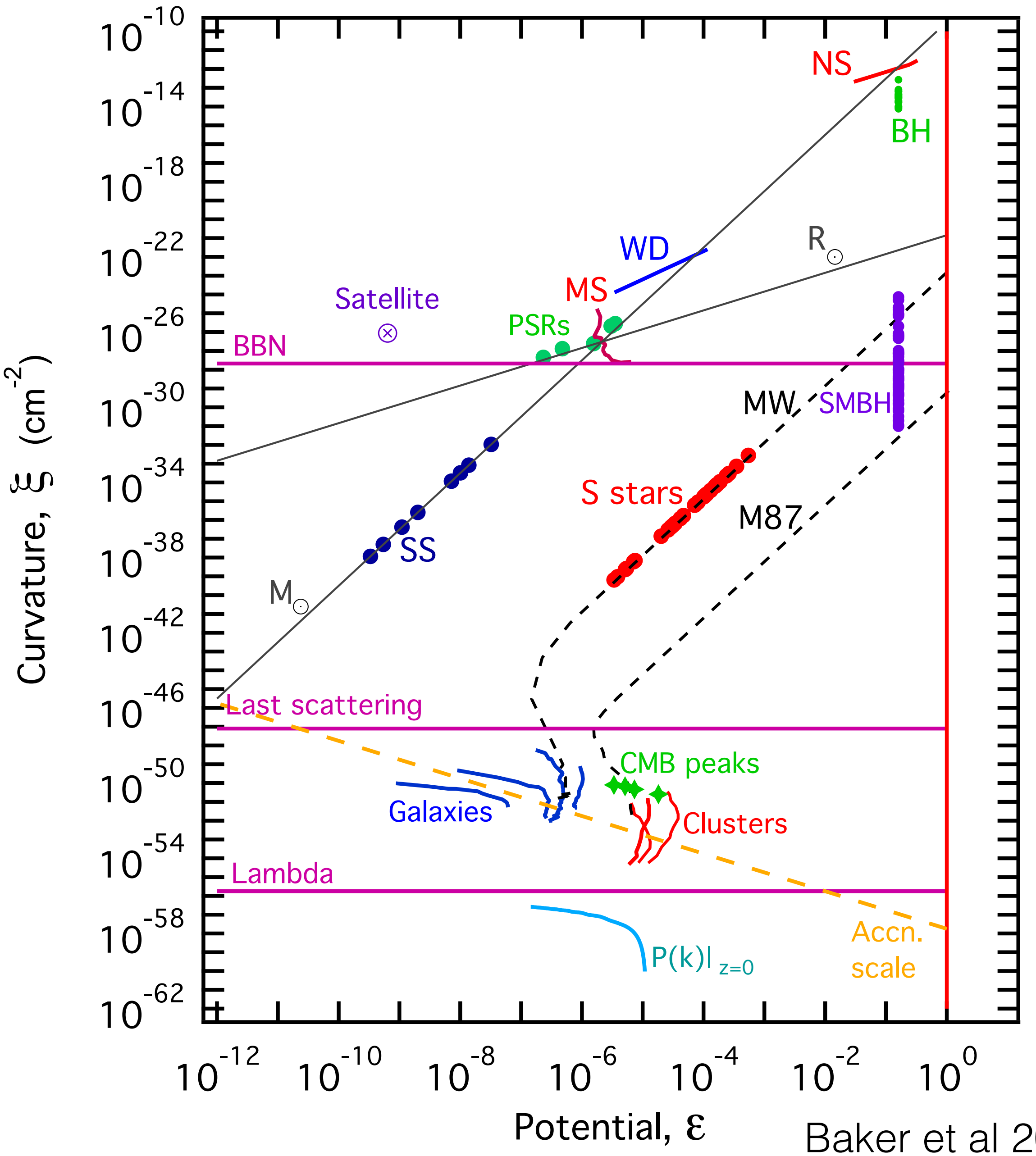
Arzoumanian et al. 2016, NanoGRAV

The big picture of gravitational wave astronomy



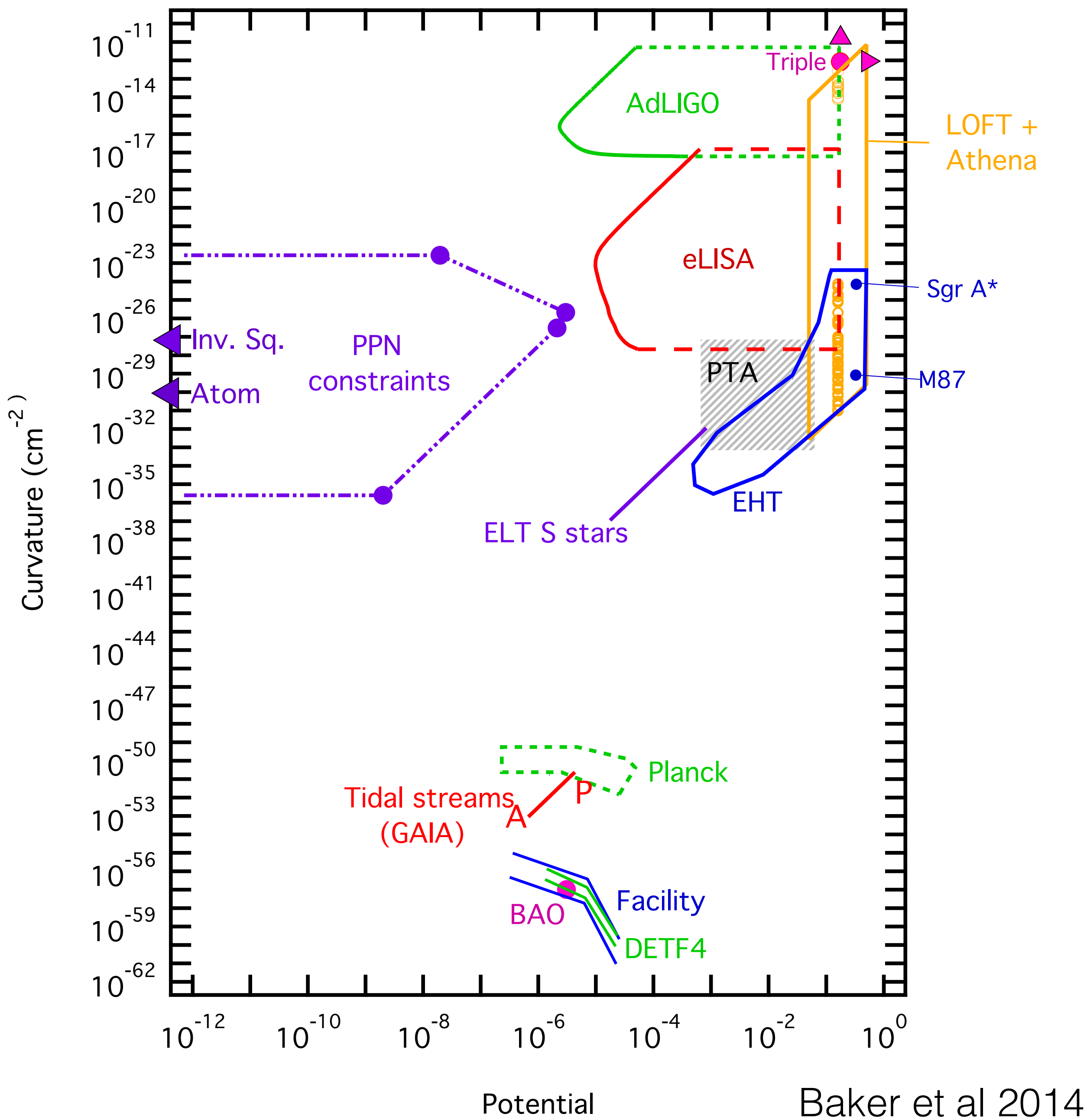
credit: NanoGRAV

Gravitational fields probed by different experiments



Baker et al 2014

Gravitational fields probed by different experiments



Baker et al 2014