Gravity from Cosmology

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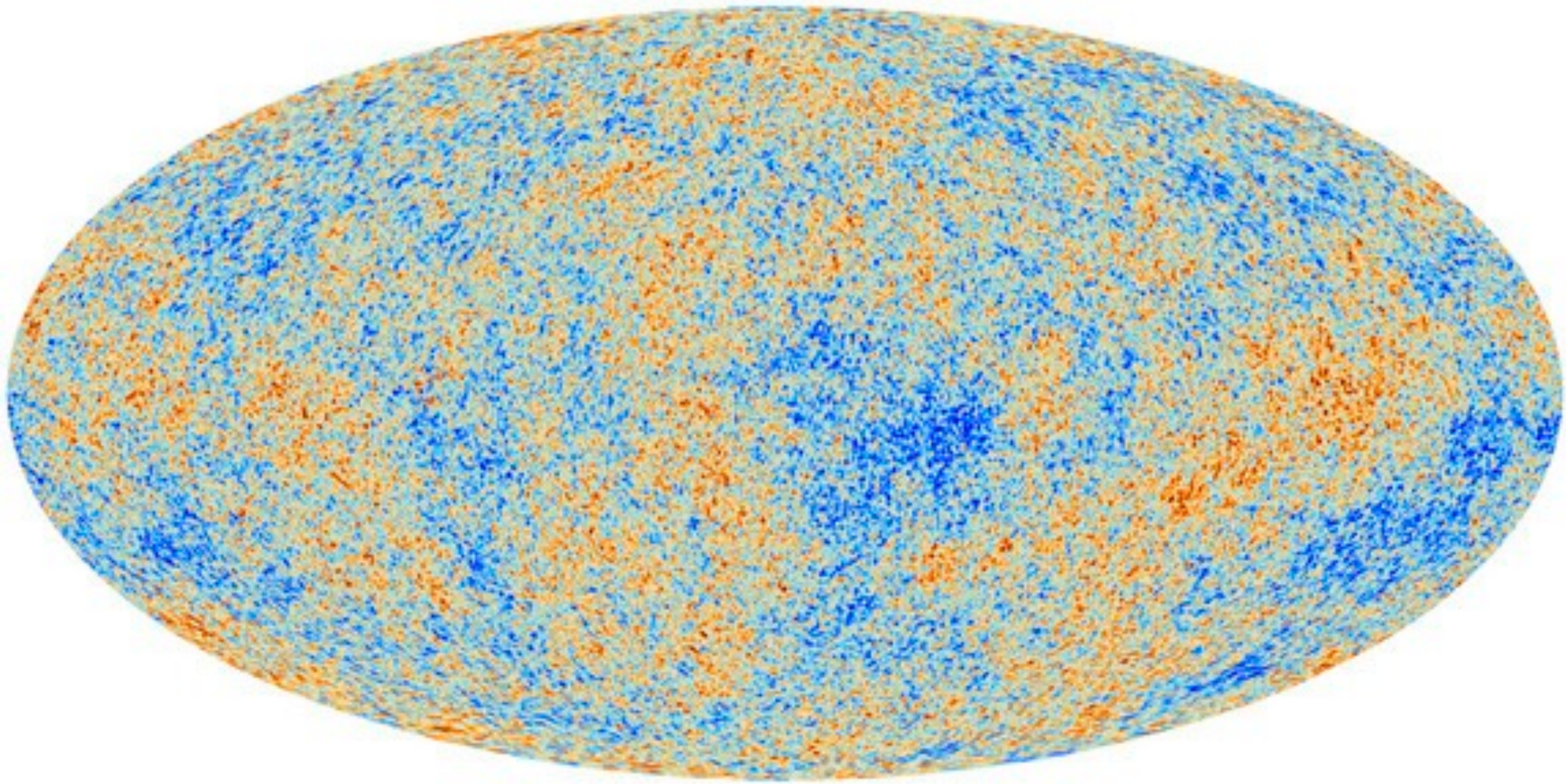
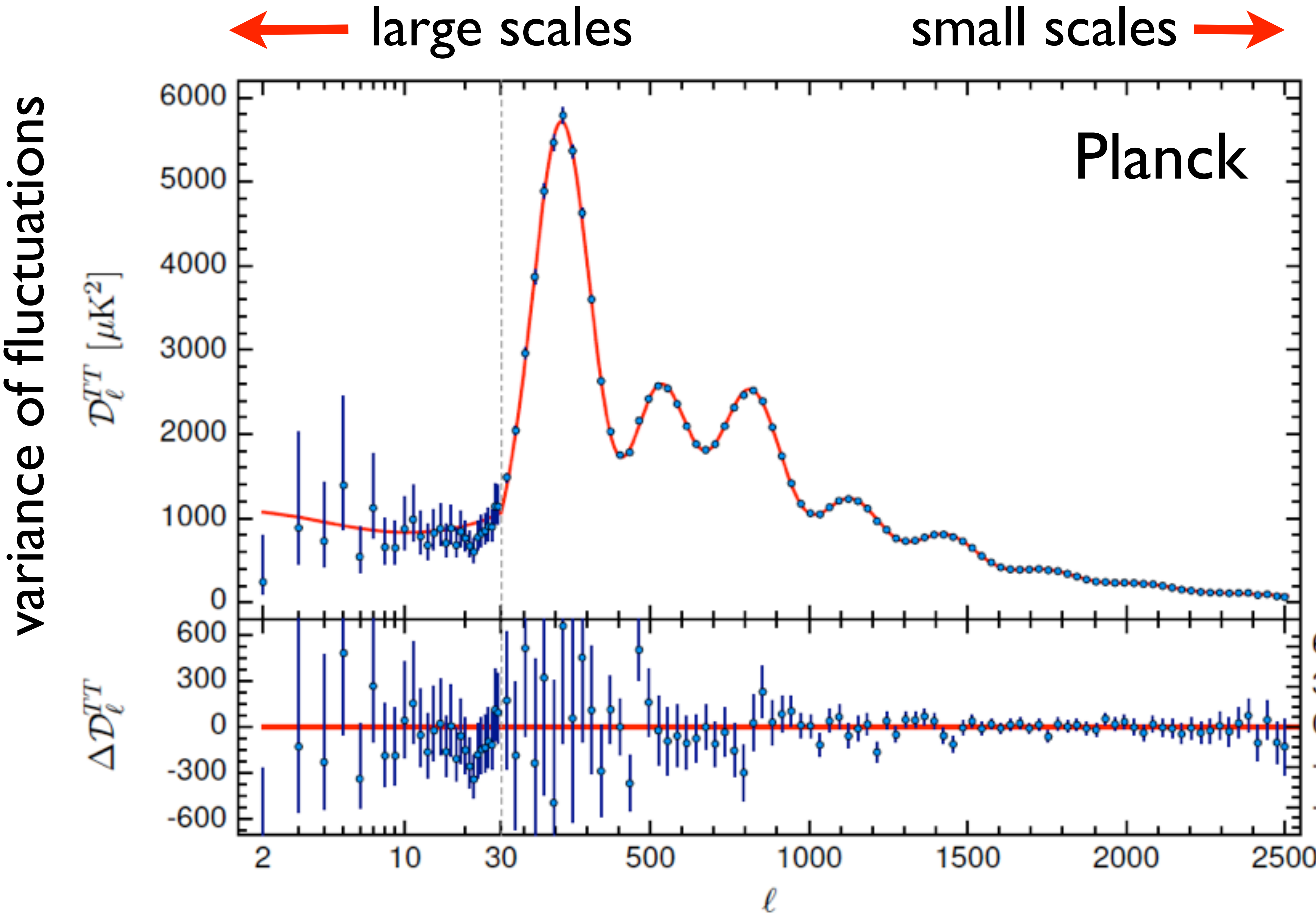
The state of General Relativity in 1957

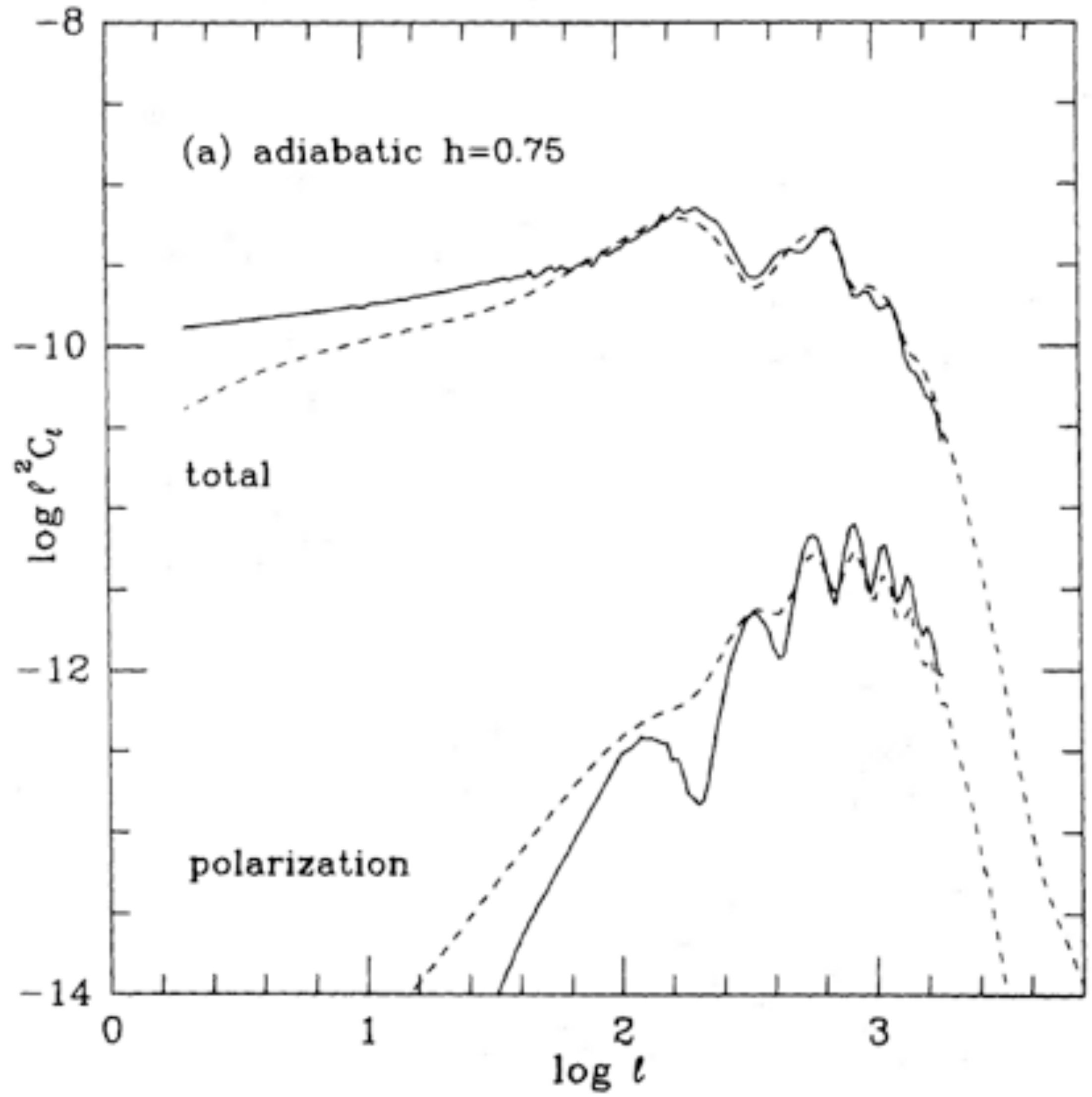
"There exists. . . one serious difficulty, and that is the lack of experiments. Furthermore, we are not going to get any experiments, so we have to take the viewpoint of how to deal with the problems where no experiments are available. ... the best viewpoint is to pretend that there are experiments and calculate. In this field we are not pushed by experiments but pulled by imagination."

R. Feynman, Chapel Hill workshop on gravitation (1957)

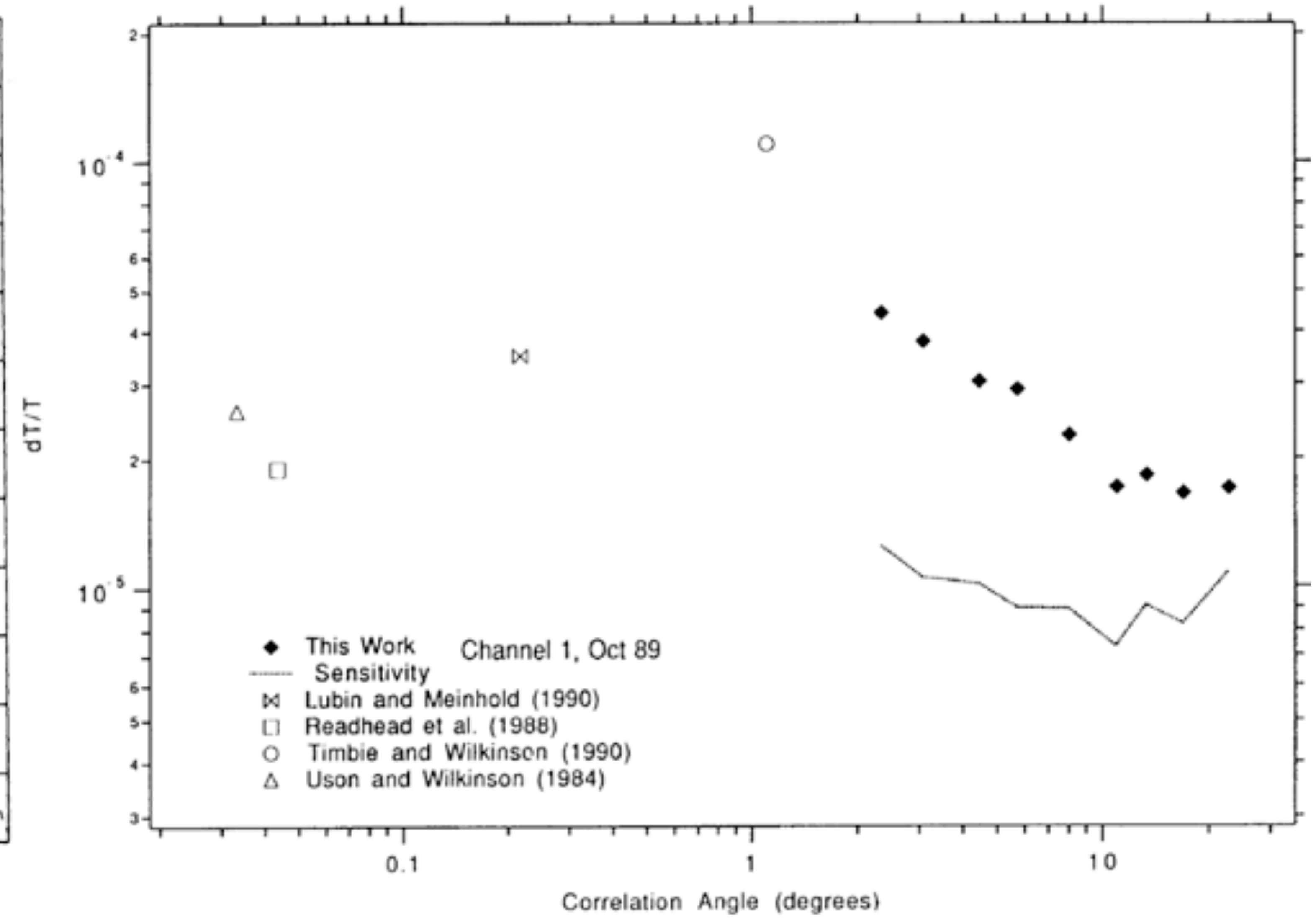


Planck 2015

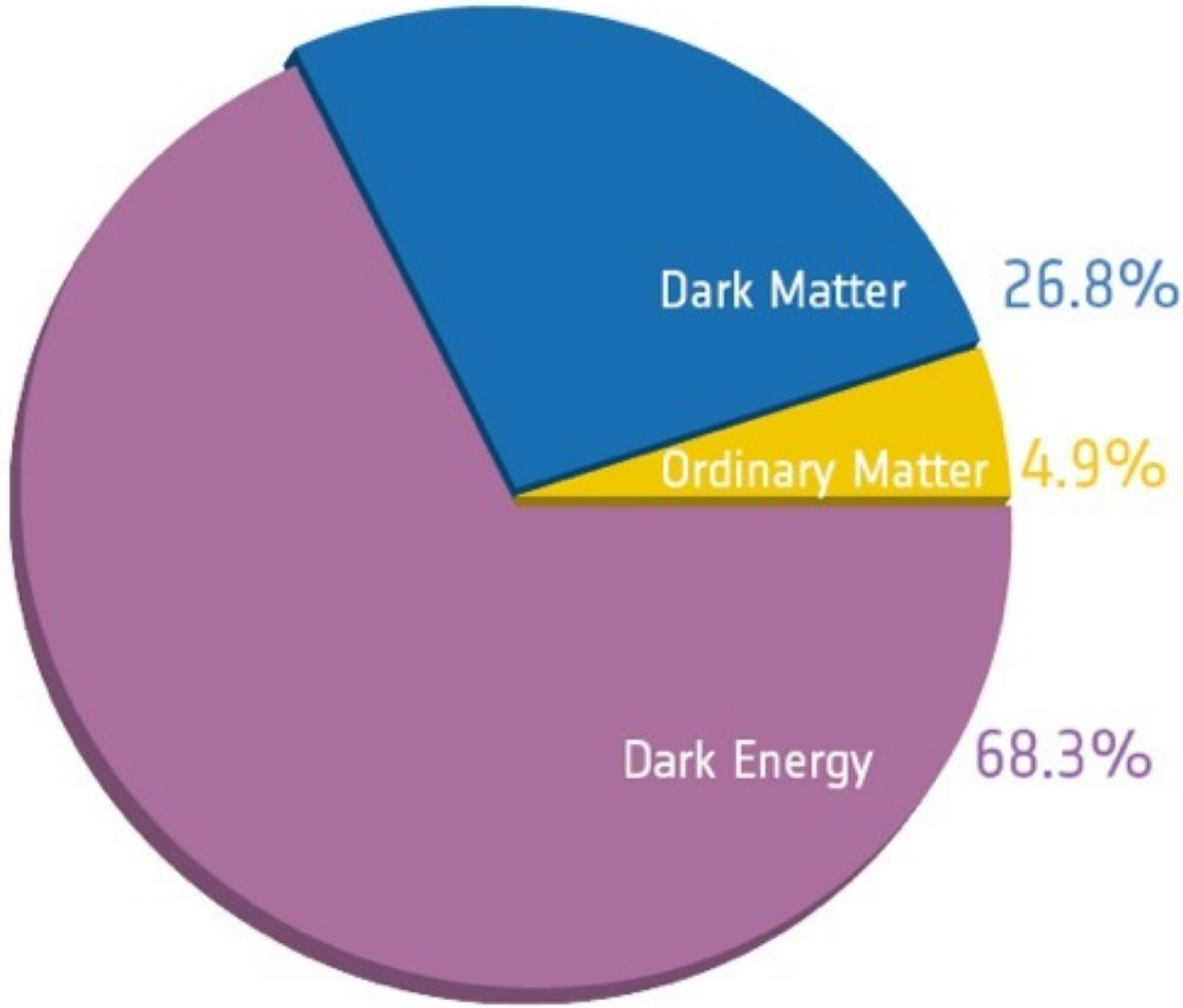

large scales
small scales
variance of fluctuations
Planck
$\mathcal{D}_\ell^{TT}$ $[\mu K^2]$
$\Delta\mathcal{D}_\ell^{TT}$
6000
5000
4000
3000
2000
1000
0
600
300
0
-300
-600
60
30
0
-30
-60
2
10
30
500
1000
1500
2000
2500
ℓ



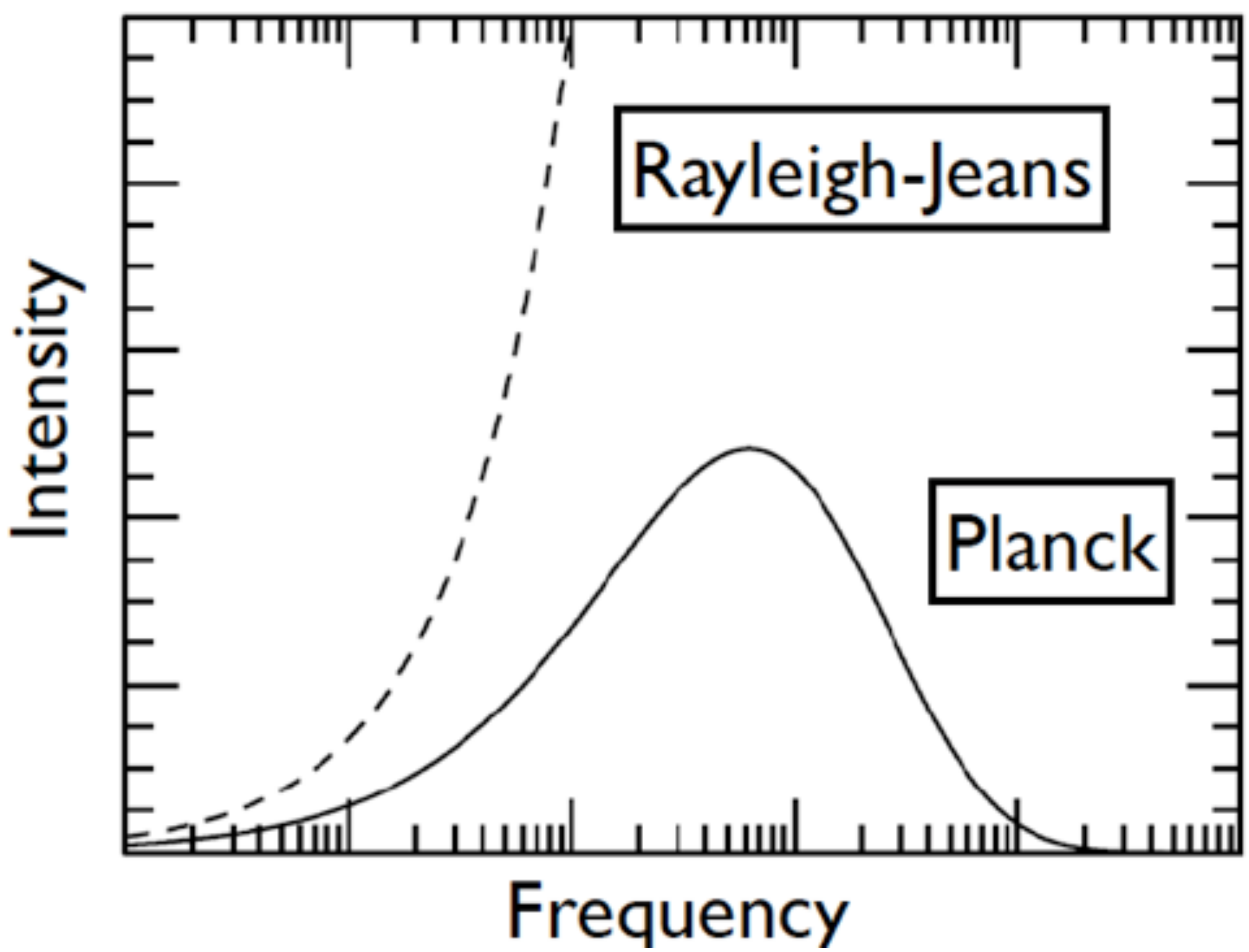
Bond & Efstathiou 1987



Meyer, Cheng and Page 1991


Dark Matter
26.8%
Ordinary Matter
4.9%
Dark Energy
68.3%

Big puzzles



Small inconsistencies

- Lamb shift
- Wu parity violation experiment
- Fitch-Cronin CP violation
- Precession of perihelion of Mercury

The Theory

Einstein Gravity: a field theory view

Feynman/Weinberg Theorem

Spin-2 field

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

$$\Box h_{\mu\nu} = 16\pi G(T^M_{\mu\nu} + T^G_{\mu\nu})$$

Feynman (1963)
Weinberg (1965)
Deser (1970)

Unique non-linear completion is GR...

Einstein Gravity: a relativists view

Curvature

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}[R(g) - 2\Lambda] + \int d^4x\sqrt{-g}\mathcal{L}(g, \mathrm{matter})$$

Metric of space-time

Lovelock's theorem (1971) :*"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."*

See also Hojman, Kuchar & Teitelboim (1976)

Jordan-Brans-Dicke Theory

One free parameter

$$S = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega_{\rm BD}}{\phi}\nabla_\mu\phi\nabla^\mu\phi + V + \mathscr{L}_M[g_{\mu\nu},\varphi]\right\}$$

Cassini (Bertotti et al 2003) $\omega_{\rm BD} \quad > \quad 40,000$

Extra degrees of freedom

metric → add ϕ , A^{μ} , $f_{\alpha\beta}$ etc.

4D → e.g. in 5 dimensions:

$$g_{AB} = \begin{pmatrix} g_{\alpha\beta} & A_{\alpha} \\ A_{\beta} & \phi \end{pmatrix}$$

2nd order → e.g. if $\int d^4x\sqrt{-g}f(R)$ define $\phi = \frac{df}{dR}$.

Local → e.g. $\phi = \frac{R}{\Box}$.

All transform differently under diffeomorphisms

Background $ds^2 = g^{(0)}_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t)(d\vec{x})^2$

$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta} + \underline{U_{\alpha\beta}}$$

where $U_{\alpha\beta}(a, \dot{a}, \rho_M, P_M, \phi, \cdots)$ $\qquad P_X = \underline{w}\rho_X$

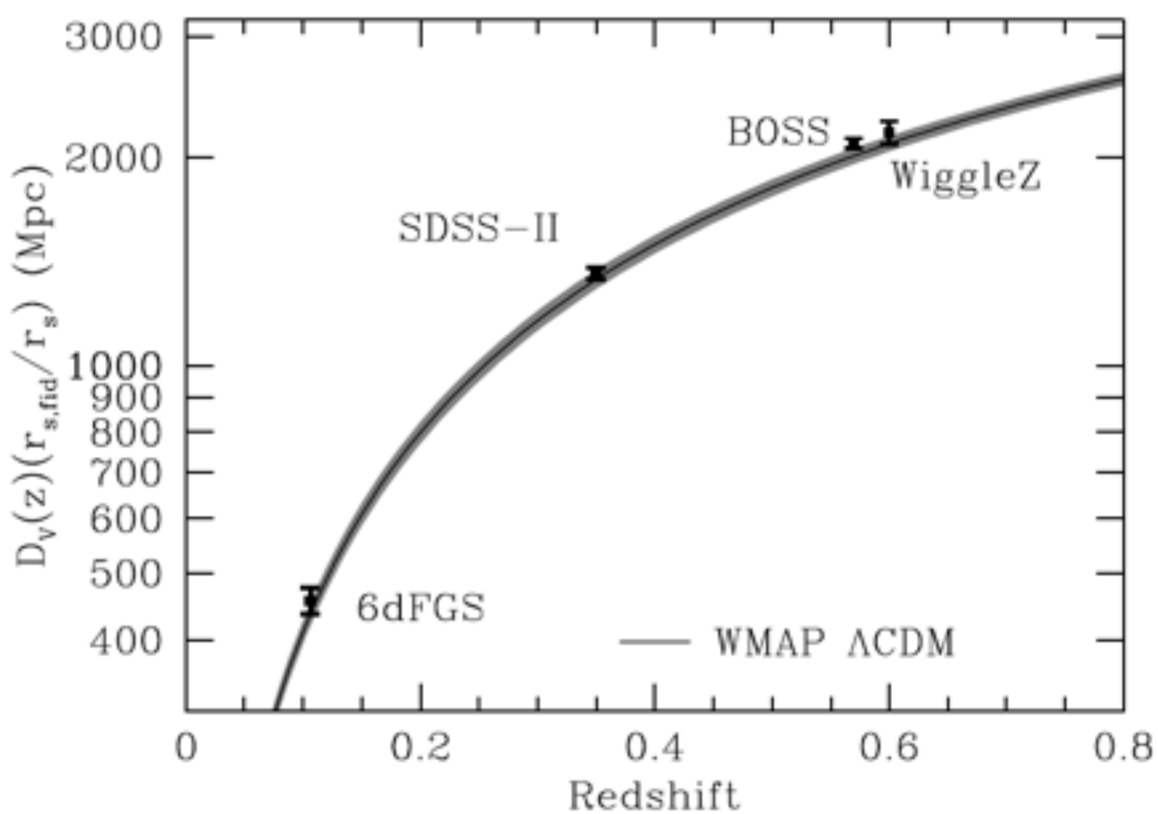
Homogeneity and isotropy

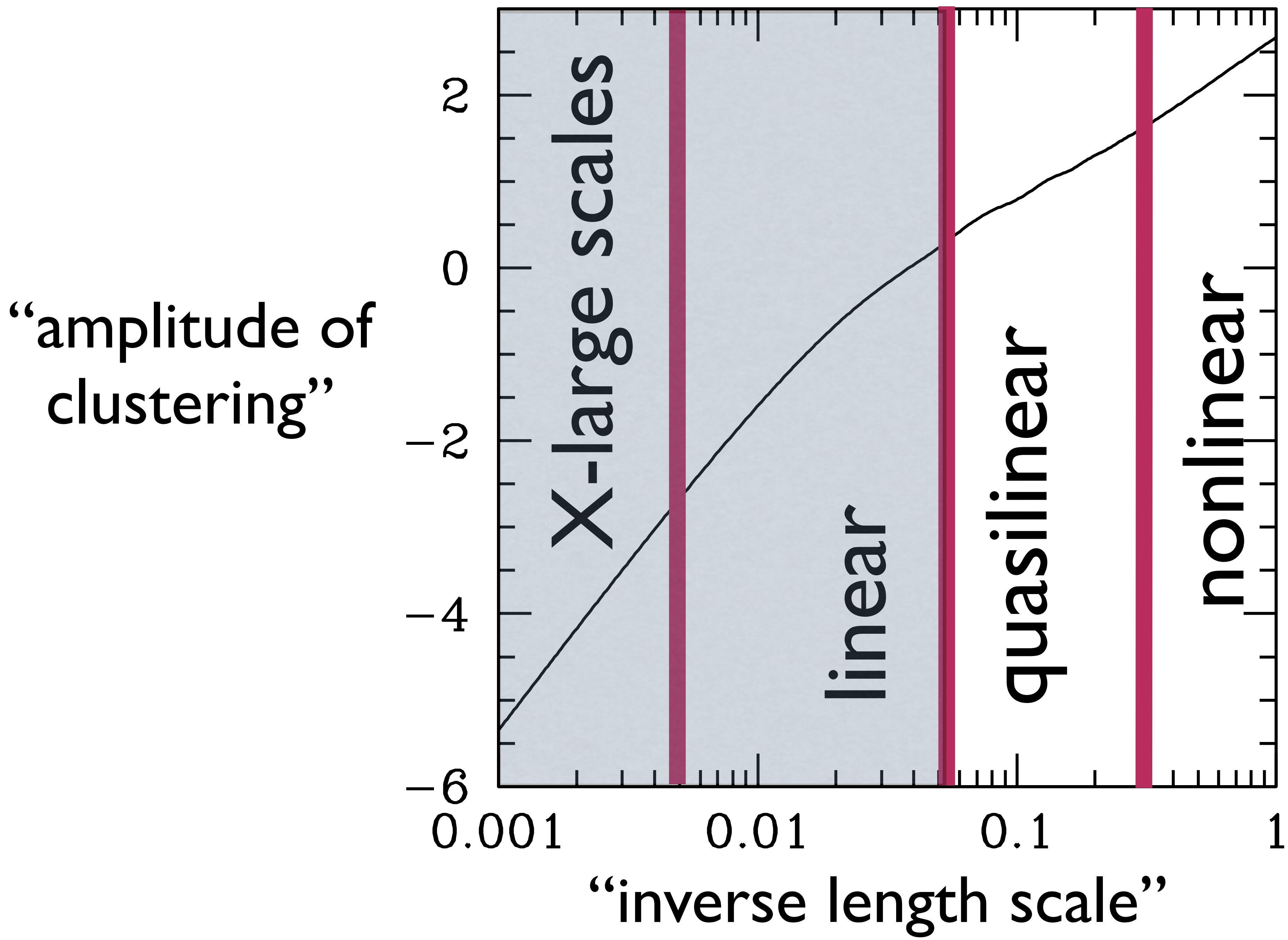
$$U_{\alpha\beta} = 8\pi G \begin{pmatrix} \rho_X & 0 \\ 0 & a^2 P_X \delta_{ij} \end{pmatrix}$$

Bianchi identities

$$\nabla^\alpha [8\pi G T_{\alpha\beta} + U_{\alpha\beta}] = 0$$

BOSS, Anderson et al 2013.





$N(k) \propto k^3$

More statistical power

Linear Perturbations

$$
\begin{aligned}
g_{\alpha\beta} &= g^{(0)}_{\alpha\beta} + h_{\alpha\beta} \\
\rho_M &= \bar{\rho}_M(1+\delta_M) \\
\phi &= \phi_0 + \delta\phi
\end{aligned}
$$

Construct most general quadratic action which has:

- upto 2nd order in time derivatives
- $h_{\alpha\beta} \to h_{\alpha\beta} + \nabla_\alpha \xi_\beta + \nabla_\beta \xi_\alpha$ where $x^\alpha \to x^\alpha + \xi^\alpha$
- inherits symmetries of the background

Lagos et al 2016

Linear Perturbations

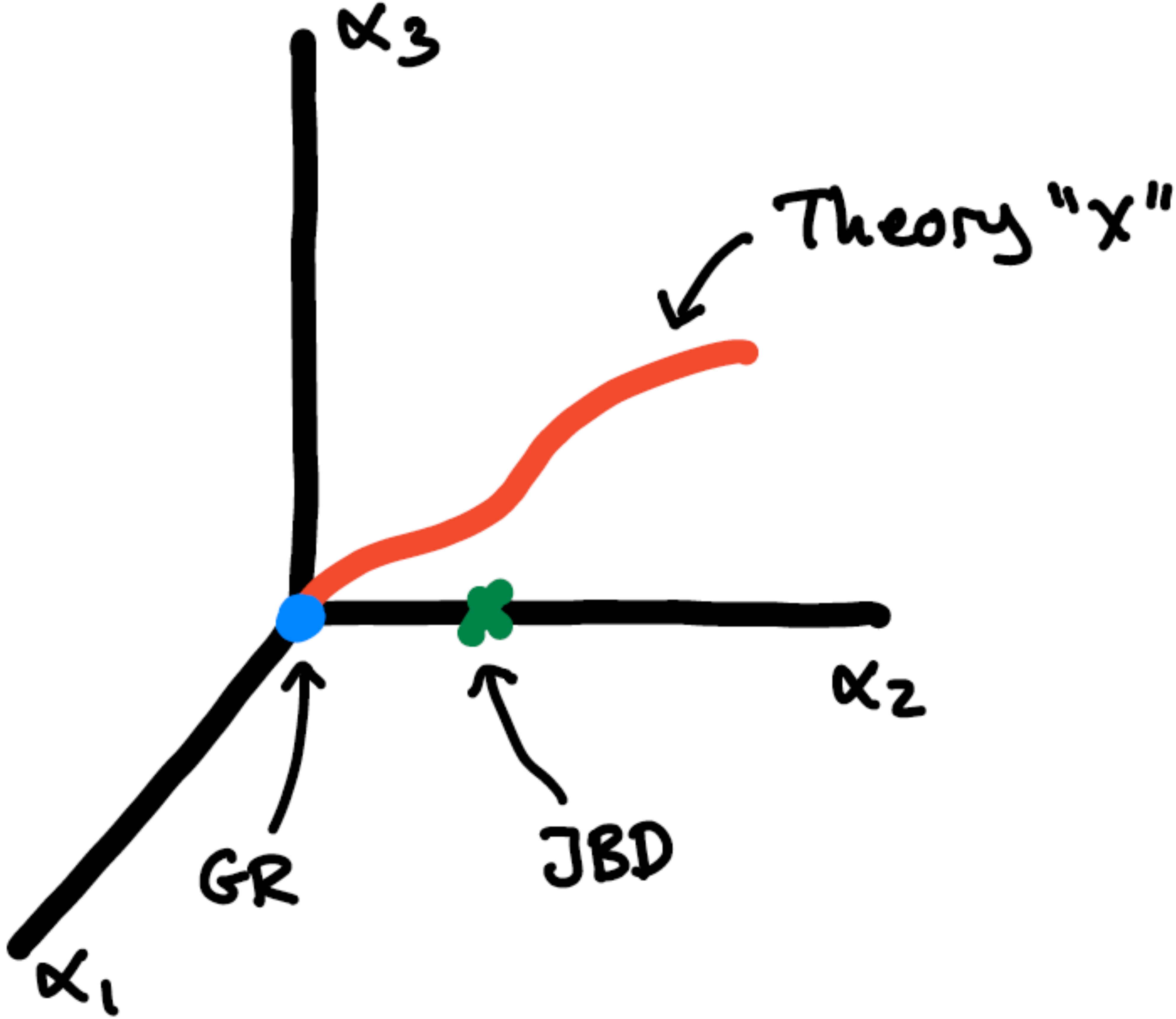
$$S=\int d^4x\sqrt{-g^{(0)}}\,[\alpha_1\nabla^\mu h^{\alpha\beta}\nabla_\mu h_{\alpha\beta}+\alpha_2\nabla^\mu h^{\alpha\beta}\nabla_\alpha h_{\mu\beta}$$
$$+\alpha_3\nabla^\mu h\nabla^\alpha h_{\mu\alpha}+\alpha_4\nabla^\mu h\nabla_\mu h+\alpha_5\nabla^\mu h_{\mu\alpha}\nabla^\alpha\delta\phi+\cdots]$$

Properties:

- $(\alpha_1,\alpha_2,\cdots)$ are functions of t
- $\alpha_X(t)$ depend on transf. props of extra fields
- clear mapping theory $\longleftrightarrow$ $\alpha_X(t)$
- clear physical interpretation of each $\alpha_X(t)$

Examples:

- Scalar-tensor (Horndeski): five $\alpha_X(t)$
- Vector-tensor (Einstein-Aether, Proca): nine $\alpha_X(t)$
- Tensor-tensor (Bigravity, massive gravity): three $\alpha_X(t)$


α_3
Theory "x"
α_2
GR
JBD
α_1

The Data

A preferred length scale- the horizon

$$\mathcal{H}^{-1} \equiv \left(\frac{\dot{a}}{a}\right)^{-1} \propto \tau \simeq 3000 h^{-1}\mathrm{Mpc}$$

Most surveys $\ll \tau$ so that $k\tau \gg 1$

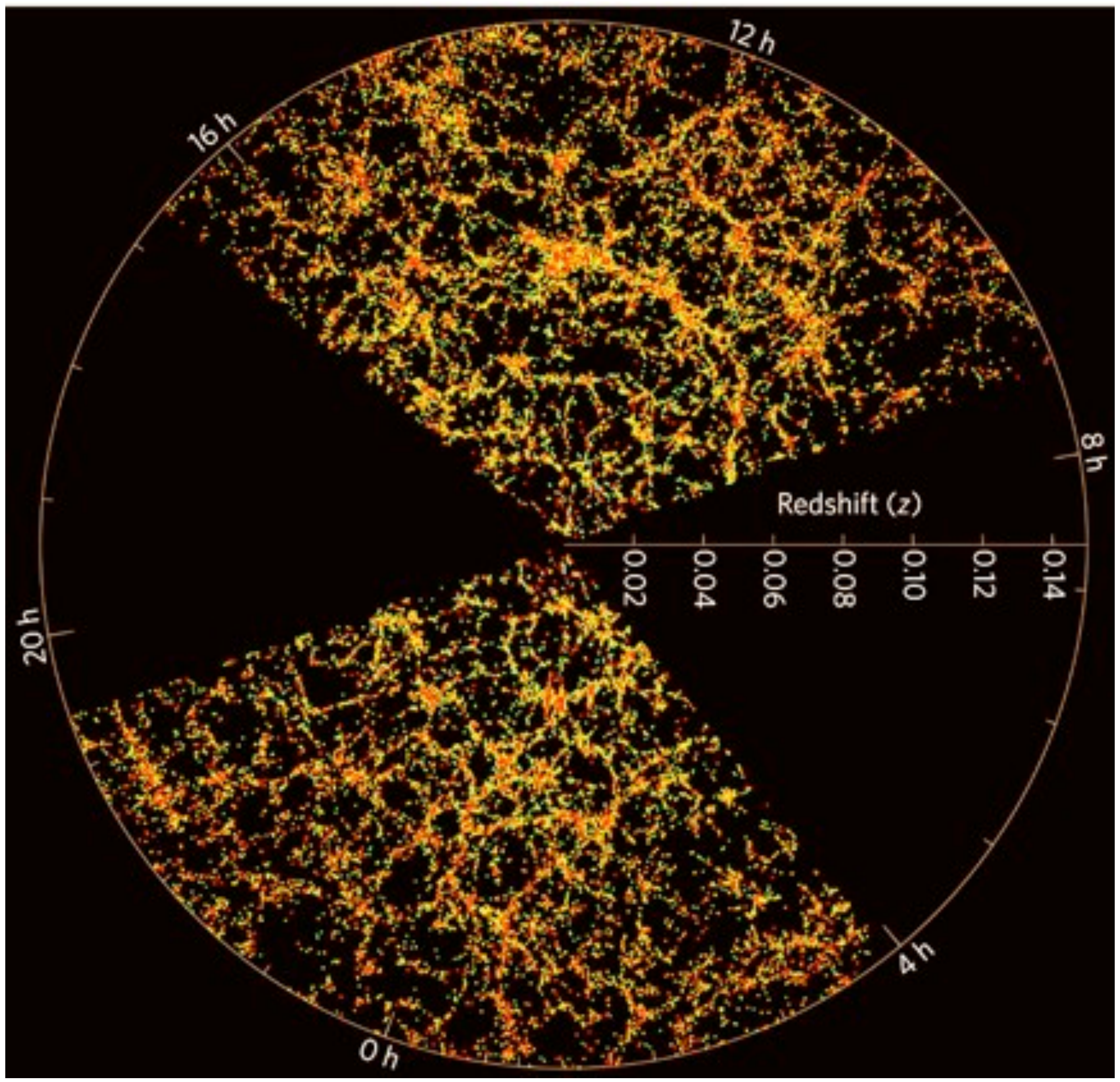
Newtonian potentials: $h_{\alpha\beta} = 2\begin{pmatrix} \Phi & 0 \\ 0 & a^2\Psi\delta_{ij} \end{pmatrix}$

Einstein equations: $-k^2\Phi = 4\pi G\underline{\mu} a^2\rho\Delta$

$$\underline{\gamma}\Psi = \Phi$$

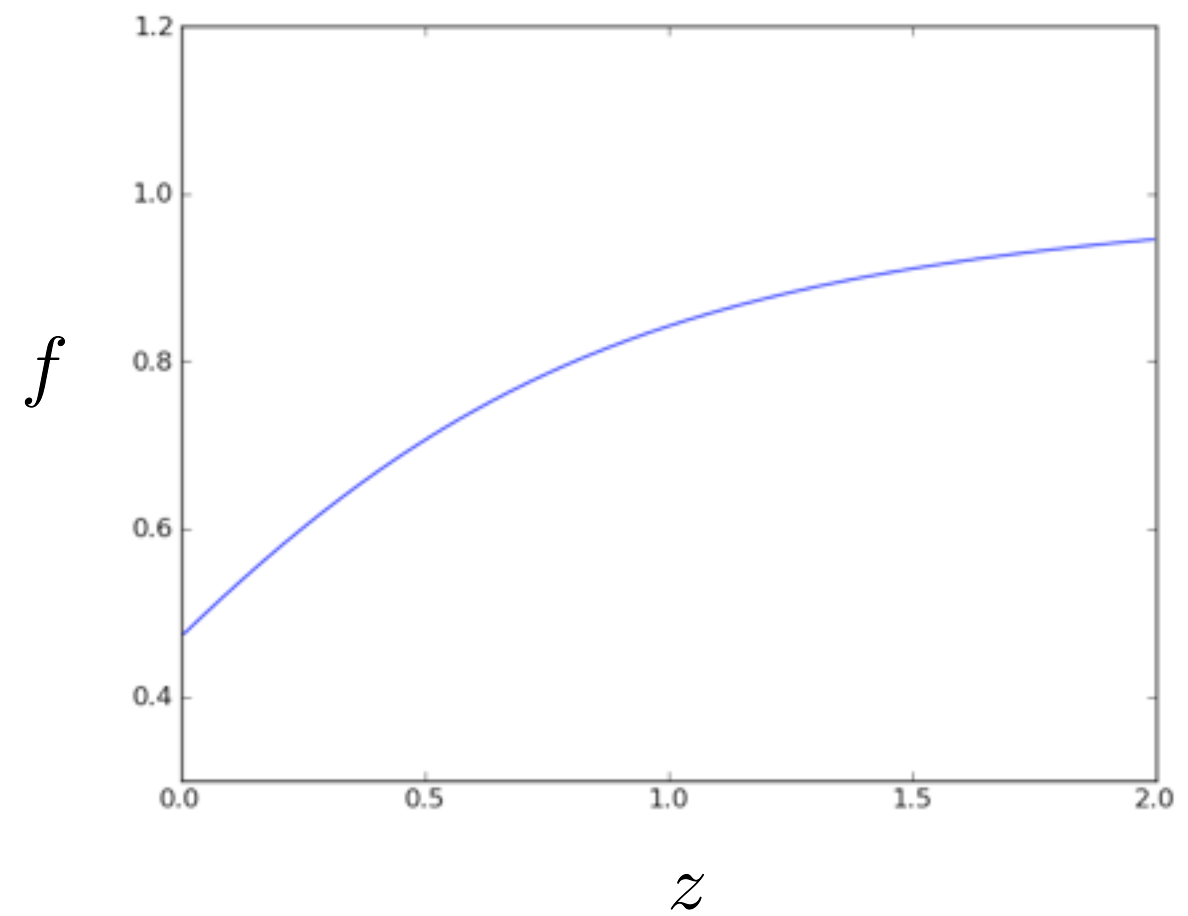
(μ, γ) are rational functions of $\alpha_X(t)$ and k^2

We measure matter and light.



Growth rate

$$f(k,a) = \frac{d\ln\delta_M(k,a)}{d\ln a}$$

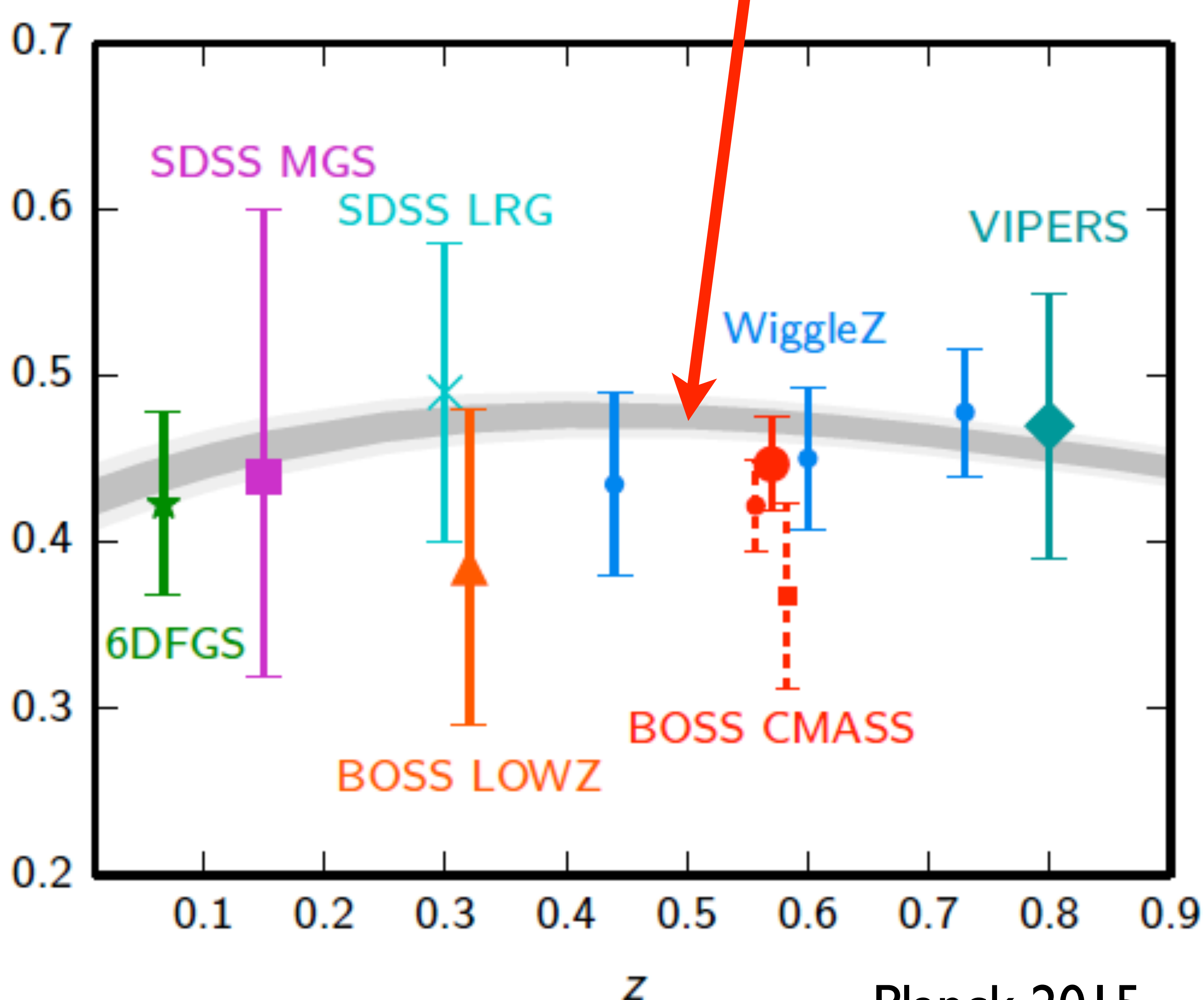


f satisfies a simple ODE

$$\frac{df}{d\ln a} + qf + f^2 = \frac{3}{2}\Omega_M\xi$$

with $q = \frac{1}{2}[1 - 3w(1-\Omega_M)]$ and $\xi = \frac{\mu}{\gamma}$

Planck “prediction”

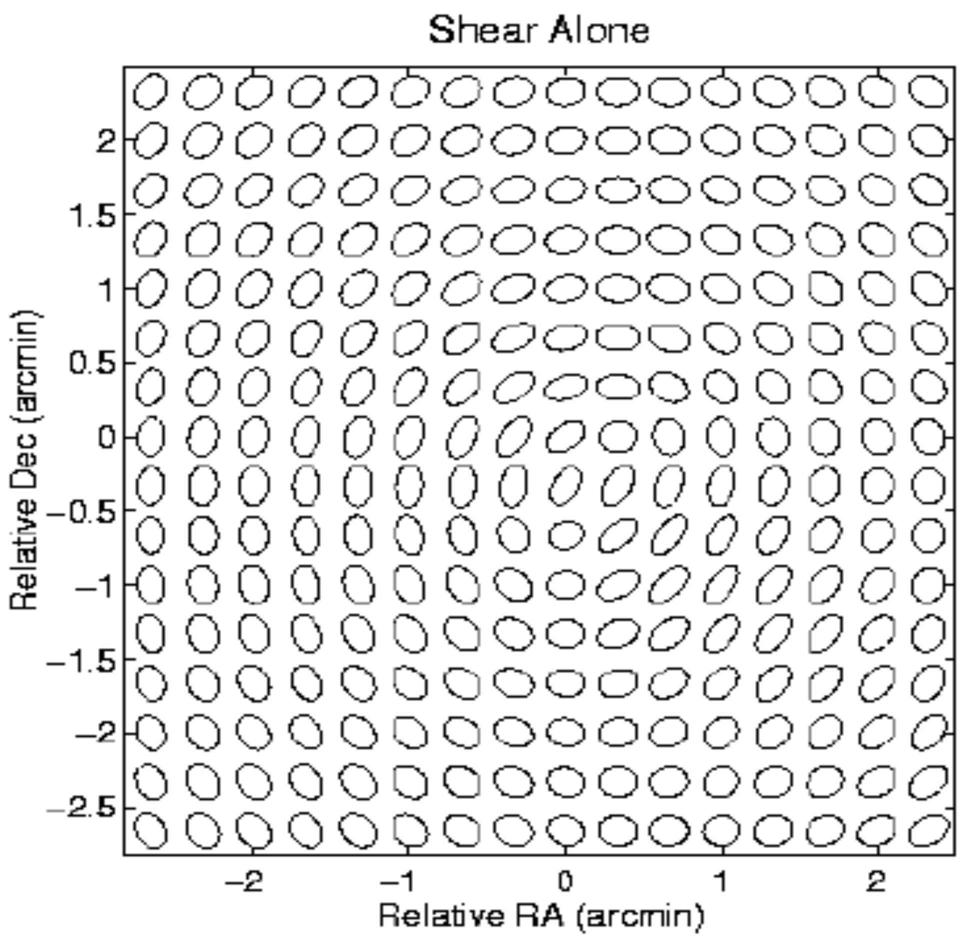


“growth rate of structure”

$$f\sigma_8 \propto \frac{d\delta}{d\ln a}$$

Planck 2015

Weak Lensing



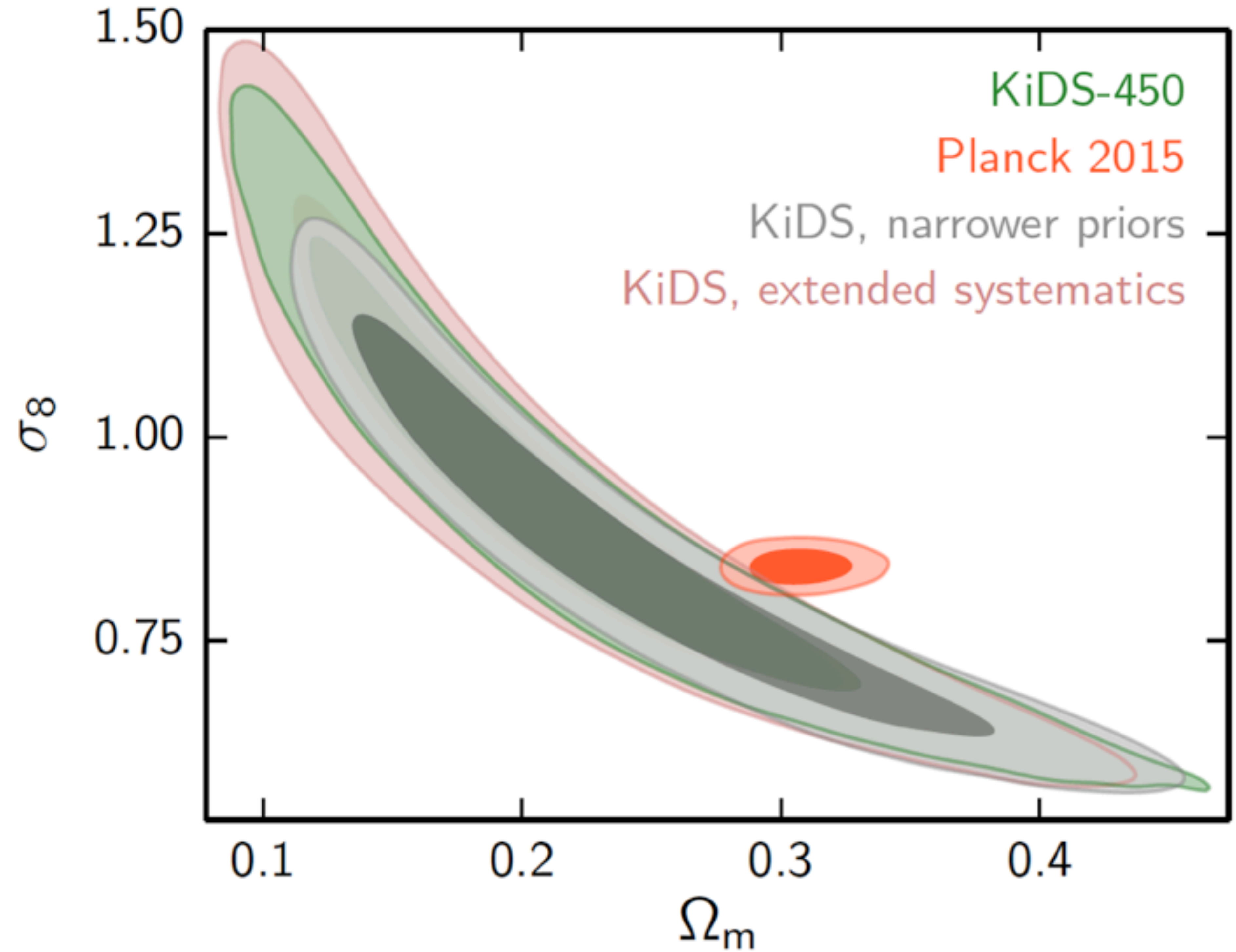
shear $\simeq \int_0^{\chi} \nabla_{\perp}^2[\Phi + \Psi](\chi') \left[\chi' \left(1 - \frac{\chi'}{\chi}\right)\right] d\chi'$

shear $\sim \Sigma \equiv \mu \left(1 + \frac{1}{\gamma}\right)$

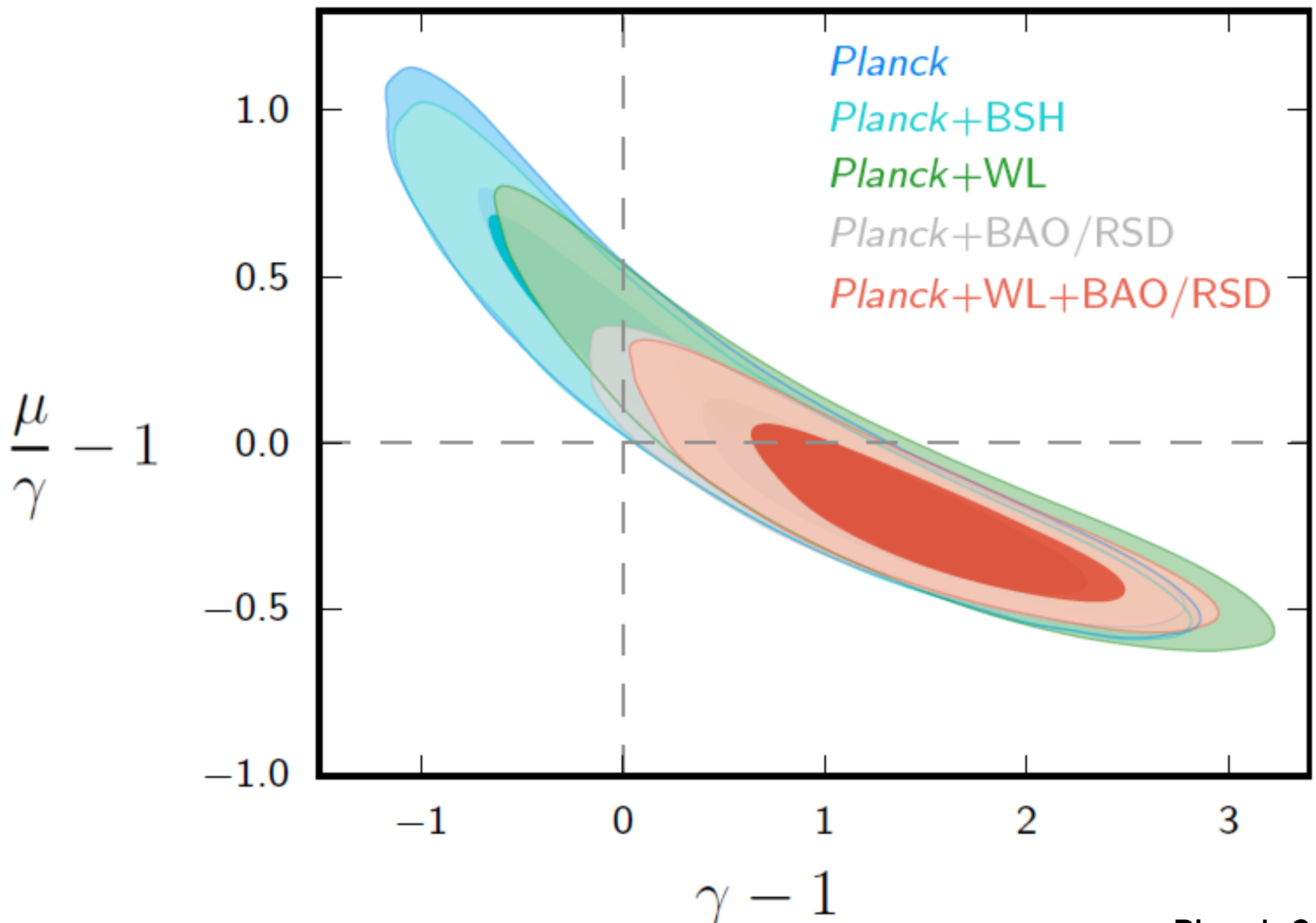
Sarah Bridle lectures (2003)

Joudaki et al 2016

“amplitude of clustering at 8 h^{-1} Mpc”



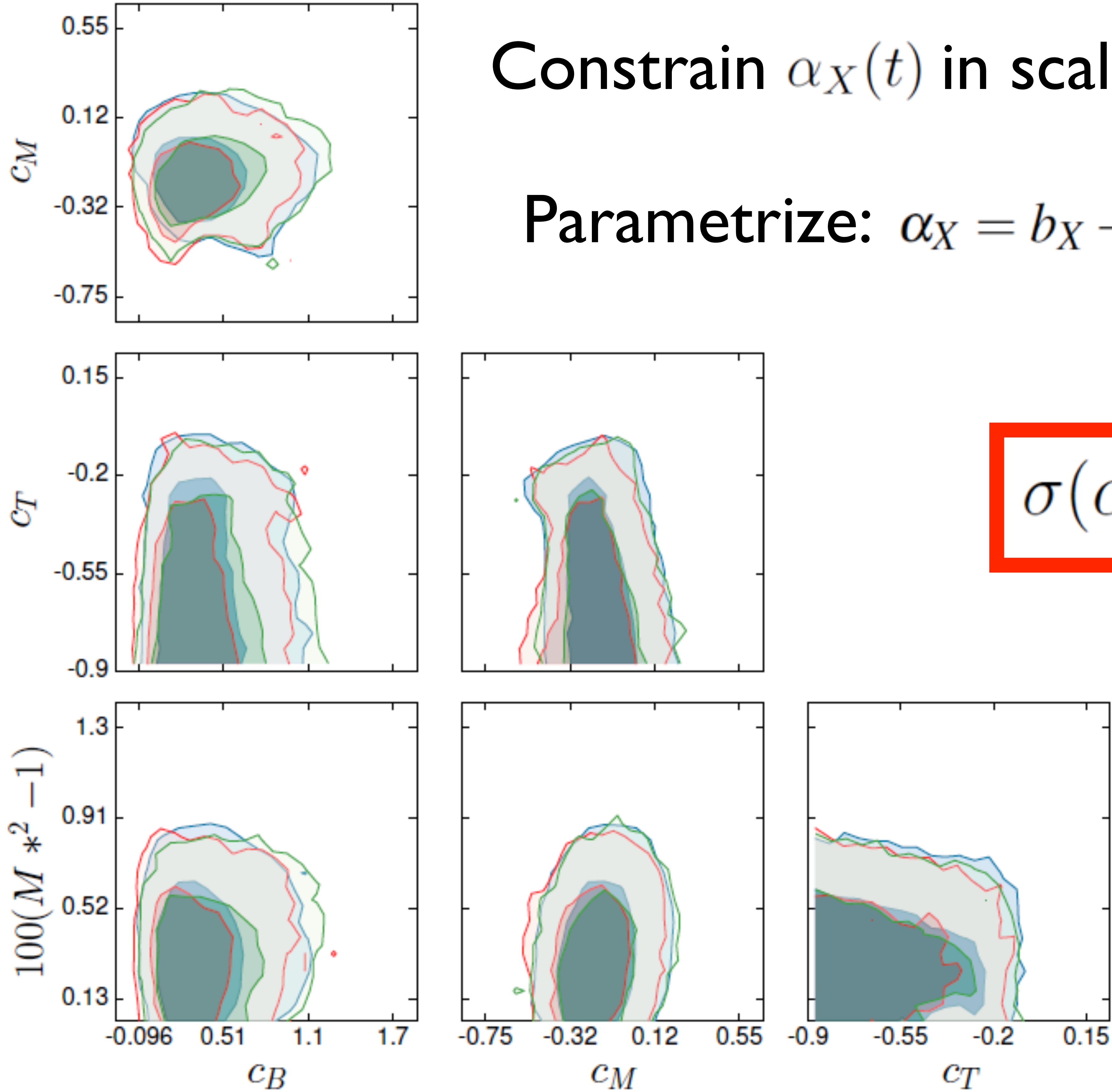
“matter density”


Planck
Planck+BSH
Planck+WL
Planck+BAO/RSD
Planck+WL+BAO/RSD
$\frac{\mu}{\gamma} - 1$
$\gamma - 1$
1.0
0.5
0.0
−0.5
−1.0
−1
0
1
2
3

Planck 2015

Constrain $\alpha_X(t)$ in scalar-tensor theories

Parametrize: $\alpha_X = b_X + c_X \frac{\Omega_{\rm DE}(z)}{\Omega_{\rm DE}(z=0)}$



$$\boxed{\sigma(\alpha_X) \sim 0.5}$$

Bellini et al 2016

Jordan-Brans-Dicke Theory

One free parameter

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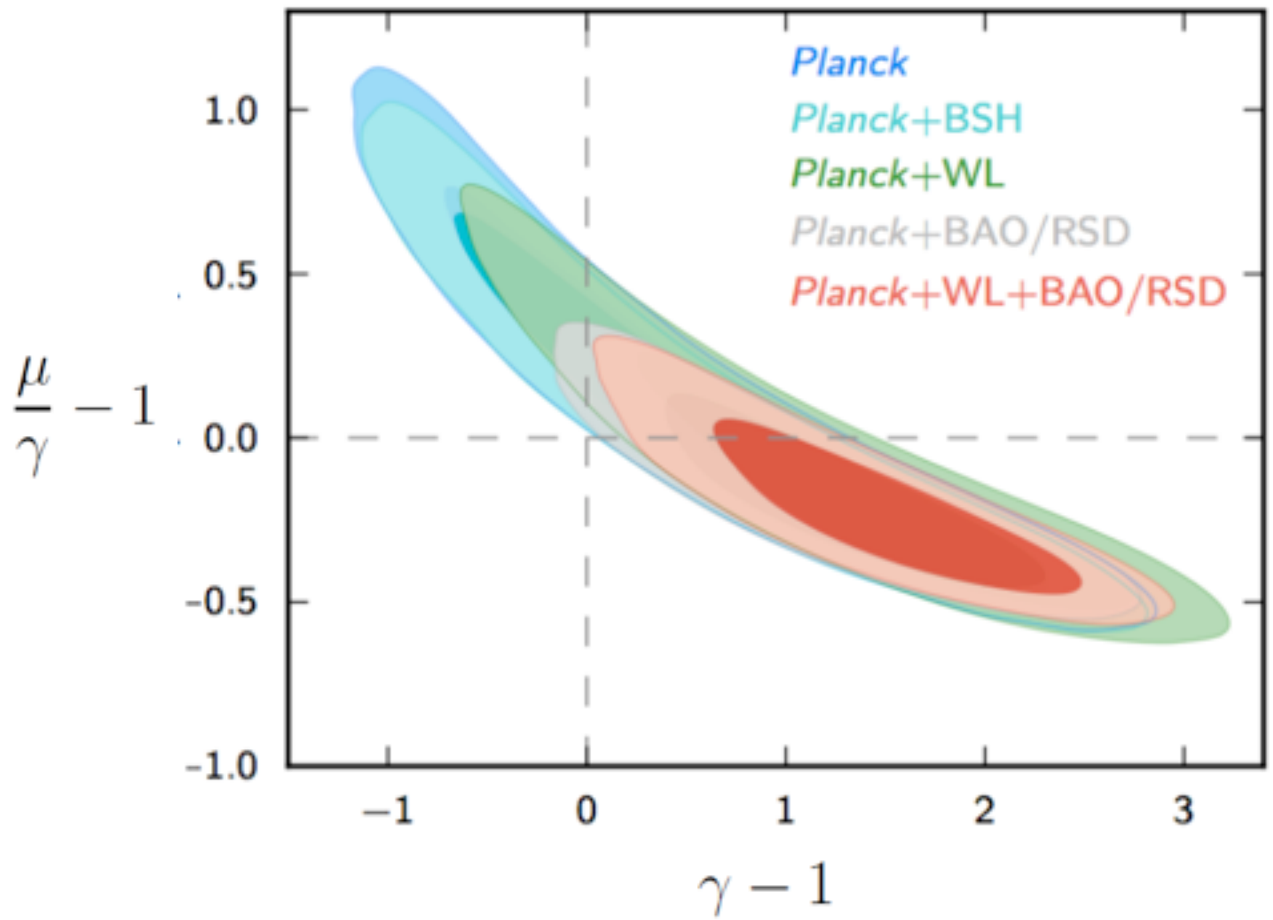
Cassini (Bertotti et al 2003) $\omega_{\rm BD} > 40,000$

Planck (Avilez & Skordis 2015) $\omega_{\rm BD} > 1,000$

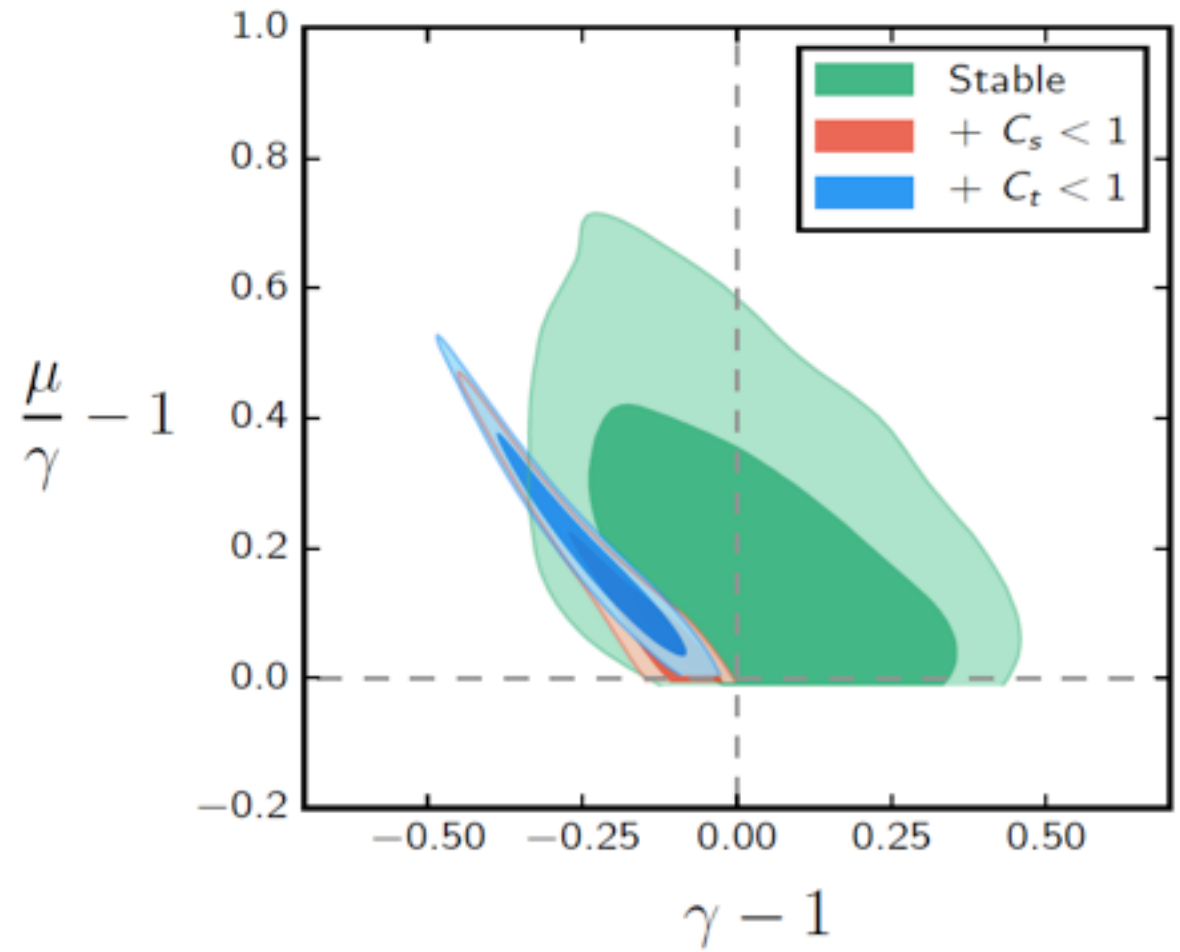
The Challenge

Priors

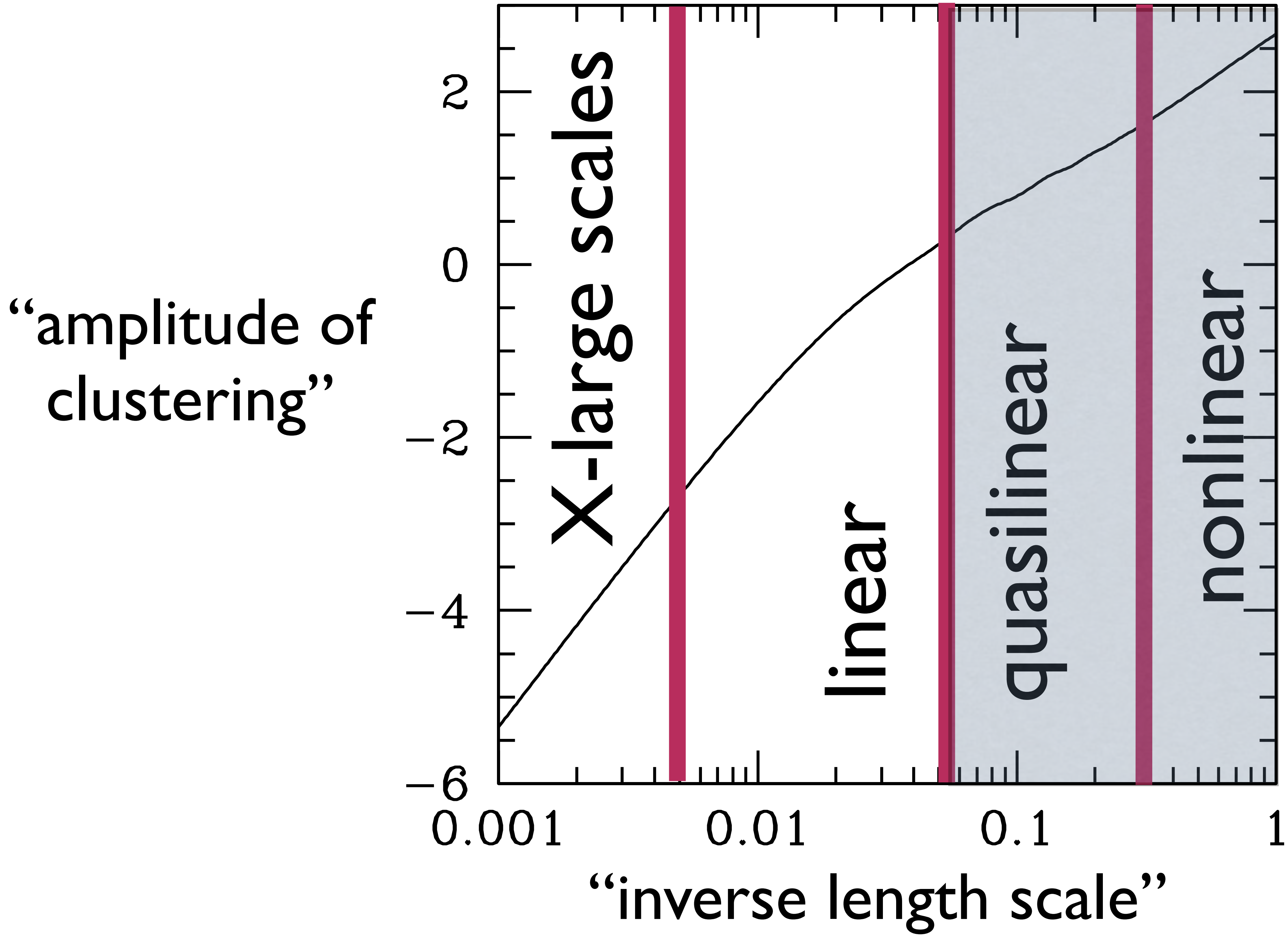
without priors



with priors



Salvatelli et al 2016

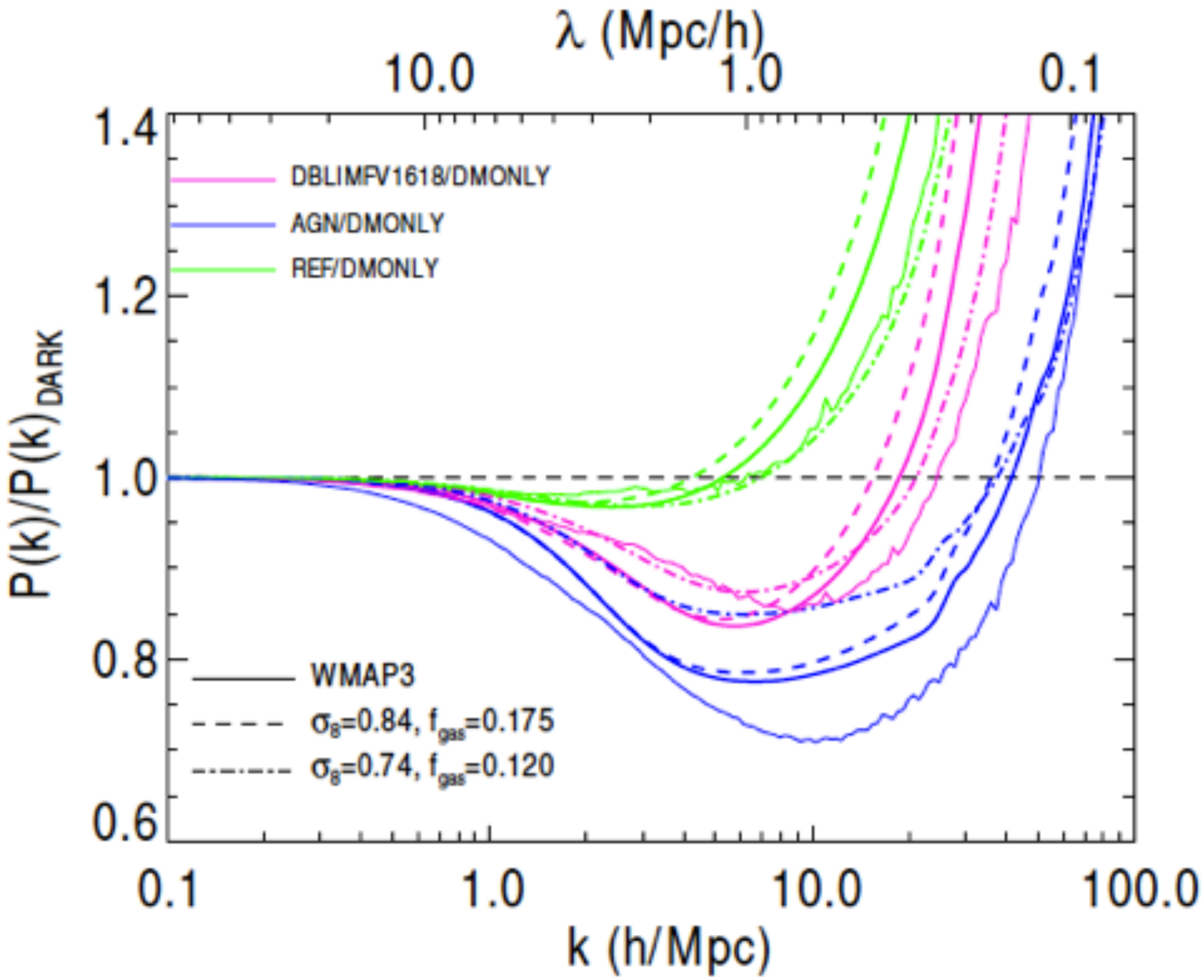


$$N(k) \propto k^3$$

More statistical power

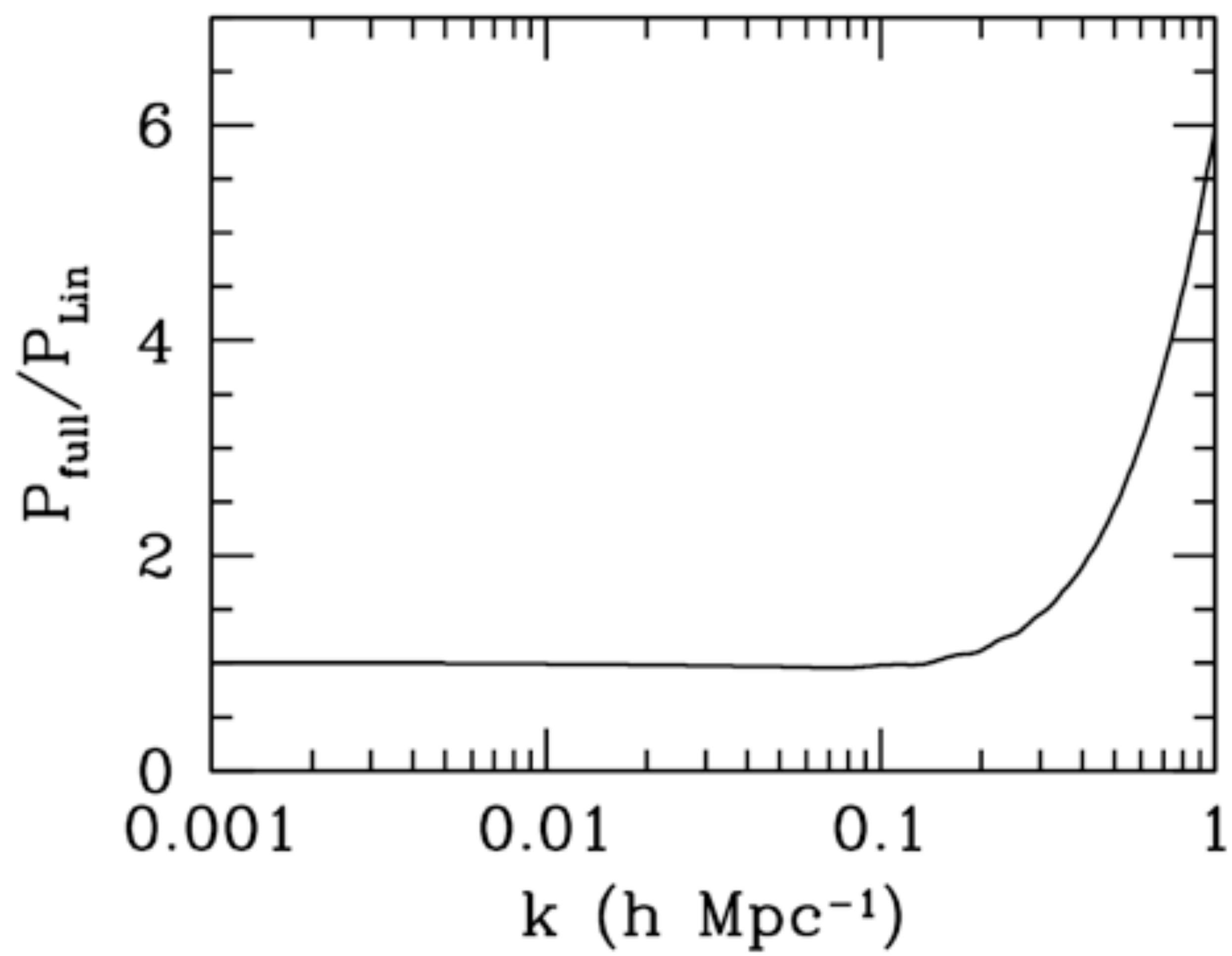
Non-linear physics

baryonic feedback



Sembloni et al 2012

non-linear growth



Screening

$\boxed{\text{Newtonian potential}}$

$$\Phi = -\frac{GM}{r}$$

$$\ddot{\vec{r}} = -\nabla[\Phi + \phi]$$

$\boxed{\text{Fifth force}}$

$$\phi = -\frac{\tilde{G}M}{r}e^{-mr}$$

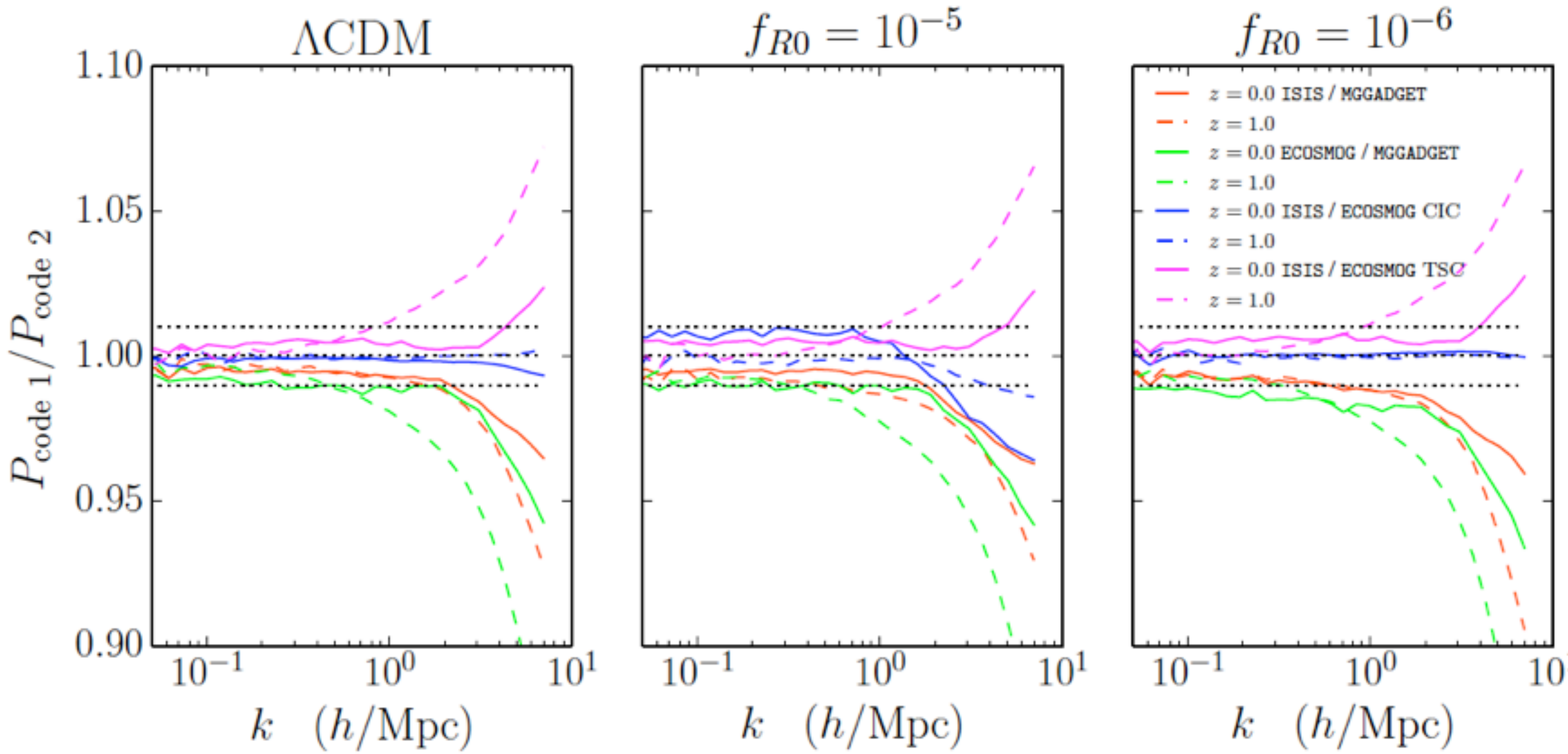
Chameleon: $m = m(\rho)$ $m \to \infty$ when $\rho \gg \rho_S$

Vainshtein: $\tilde{G} \to 0$ when $r \ll r_V$

$\boxed{\text{Non-linear effect...}}$

Non-linear physics

Comparison of N-body codes: 1% out to k~1

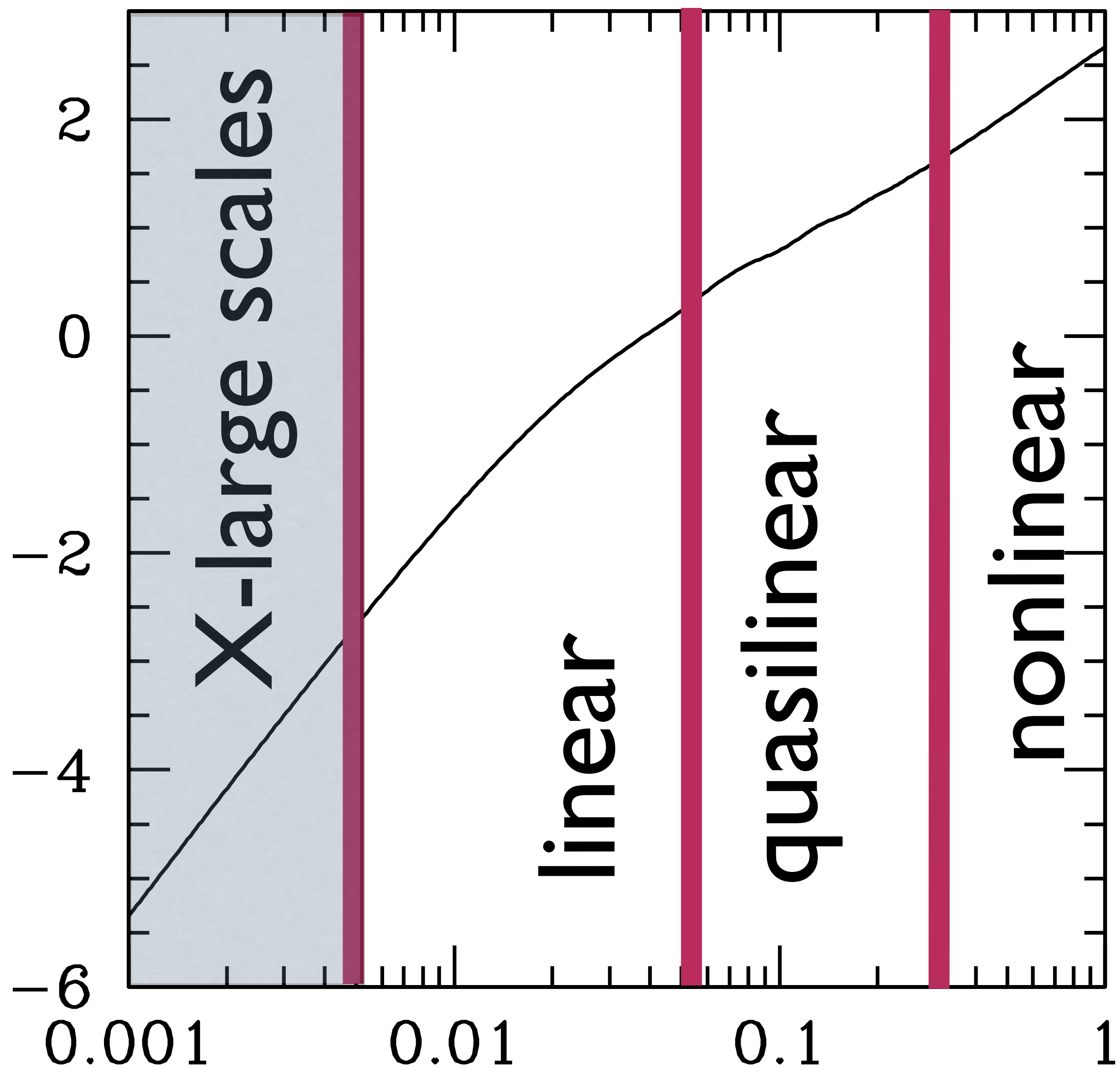


Winther et al 2015

Model specific → Model independent?

Ultra-large scales

“amplitude of clustering”

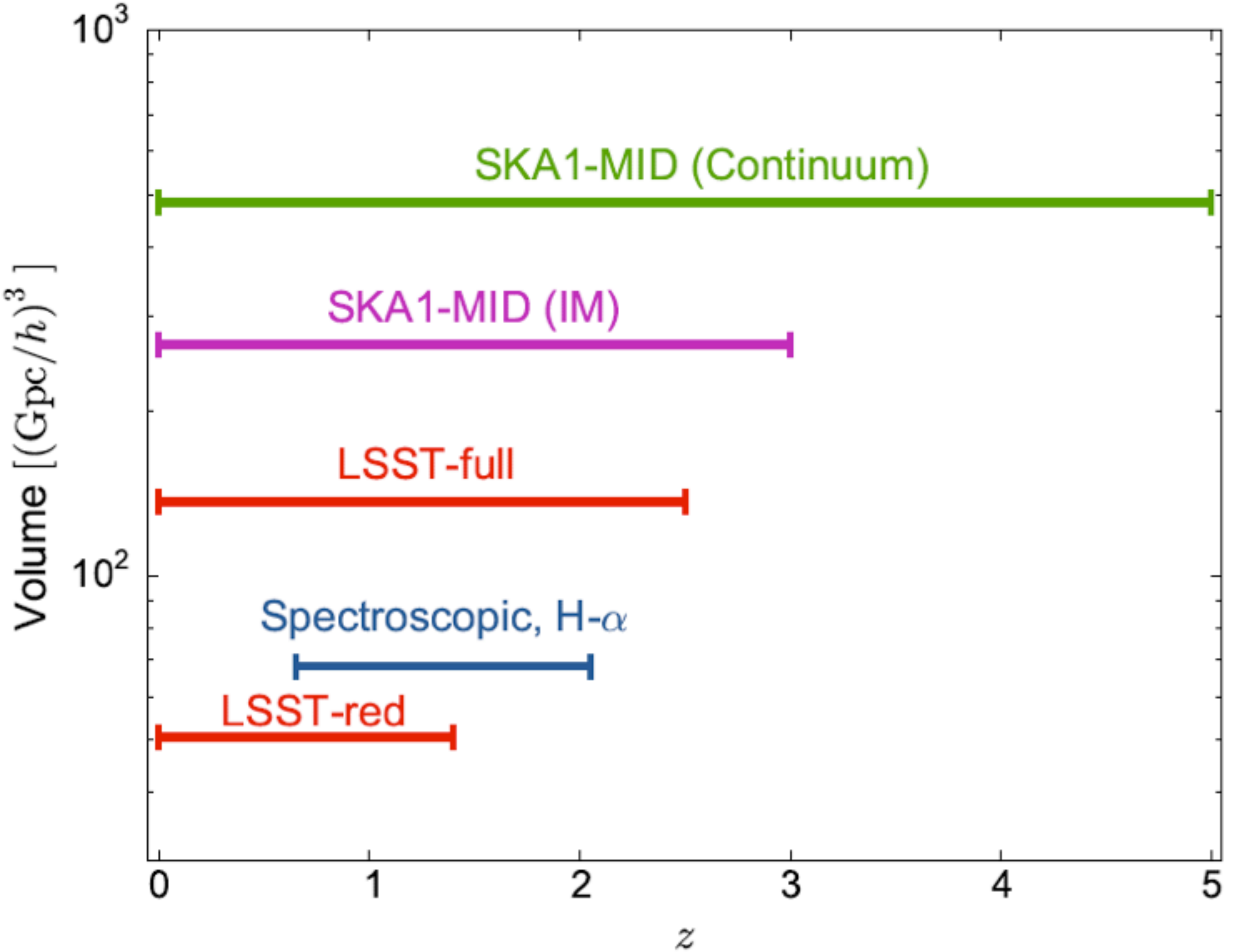


“inverse length scale”

$$N(k) \propto k^3$$

More statistical power

Ultra-large scales

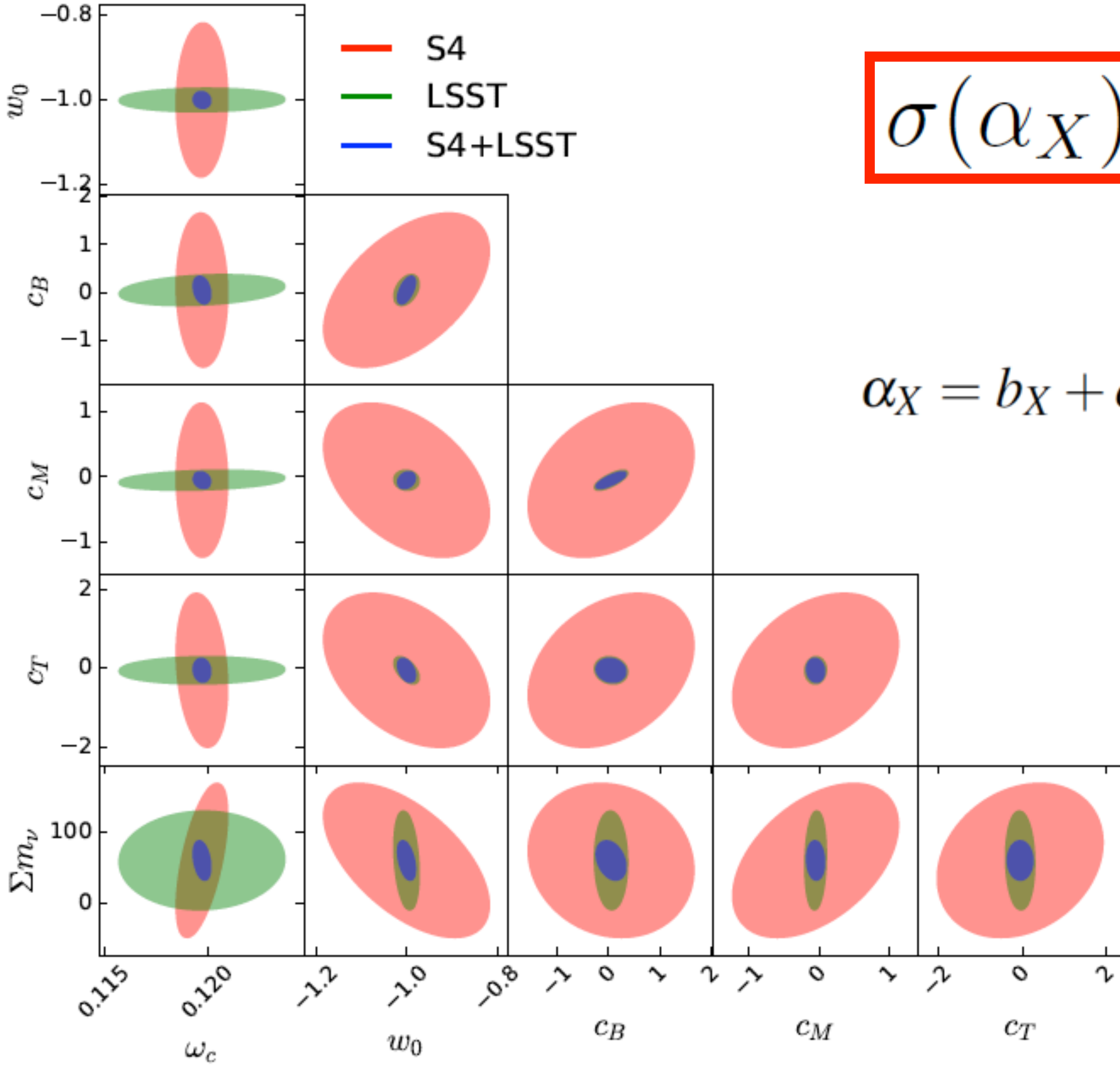


Plot by D. Alonso

The Future

The Future is now

Data Type	Now	Soon	Future
Photo-z:LSS (weak lensing)	DES, RCS, KIDS	HSC	LSST, Euclid, SKA, WFIRST
Spectro-z (BAO, RSD, ...)	BOSS	DESI,PFS,HETDEX, Weave	Euclid, SKA
SN Ia	HST, Pan-STARRS, SCP, SDSS, SNLS	DES, J-PAS	JWST,LSST
CMB/ISW	WMAP, Planck	AdvACT	Simons Array, Stage IV, LiteBird
sub-mm, small scale lensing, SZ	ACT, SPT,Planck, ACTPol,SPTPol,	PolarBear,Spider, Vista	CCAT, SKA
X-Ray clusters	ROSAT, XMM, Chandra	XMM, XCS, eRosita	
HI Tomography	GBT	Meerkat, Baobab, Chime, Kat 7	SKA



$$\boxed{\sigma(\alpha_X) \sim 0.1}$$

$$\alpha_X = b_X + c_X \frac{\Omega_{\rm DE}(z)}{\Omega_{\rm DE}(z=0)}$$

Alonso et al 2016

Jordan-Brans-Dicke Theory

One free parameter

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Cassini (Bertotti et al 2003)	$\omega_{\rm BD}$	$>$	$40,000$
Planck (Avilez & Skordis 2015)	$\omega_{\rm BD}$	$>$	$1,000$
LSST+SKA+S4 (Alonso et al 2016)	$\omega_{\rm BD}$	$>$	$20,000$

“An important contribution of the general theory of relativity to cosmology has been to keep out theologians by a straightforward application of tensor analysis.”

E. Schucking

D. Alonso, L. Amendola, M. Amin, T. Baker, R.Bean, E. Bellini, C. Blake, P. Brax, P. Bull, C. Burrage, S. Daniels, A. Davies, P. G. Ferreira, J. Gleyzes, G. Gubitosi, W. Hu, L. Hui, C, Heymans, S. Joudaki, B. Jain, J. Khoury, K. Koyama, M. Kunz, M. Lagos, D. Langlois, D. Leonard, E. Linder, L. Lombrisier, D. Mota, A. Narimani, J. Noller, J. Peacock, W. Percival, F. Piazza, L. Pogosian, D. Sapone, D. Scott, I. Sawicki, A. Silvestri, F. Simpson, A. Taylor, F. Vernizzi, H. Winther, J. Zuntz, ...