TESTING GRAVITY 2017

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Canonical Exorcism for Cosmological Ghosts

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WE HAVE JOBS!!!

2. **2017-01-24:** String Theory (Prague, Inst. Phys. - Europe) **[Deadline: 2017-02-15]**
   Postdoc - hep-th
   Detailed record

   1 postdoc position

3. **2017-01-23:** Senior Postdoc in Cosmology and Gravitation (Prague, Inst. Phys. - Europe) **[Deadline: 2017-02-15]**
   Postdoc - astro-ph, gr-qc, hep-th
   Detailed record

   1 senior researcher position

4. **2017-01-23:** Cosmology and Gravitation (Prague, Inst. Phys. - Europe) **[Deadline: 2017-02-15]**
   Postdoc - astro-ph, gr-qc, hep-th
   Detailed record

   3 postdoc positions

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Christos Charmousis (LPT Orsay)
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Andrei Frolov (SFU)
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YITP-

Canonical Exorcism for Cosmological Ghosts

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Cosmological Ghosts

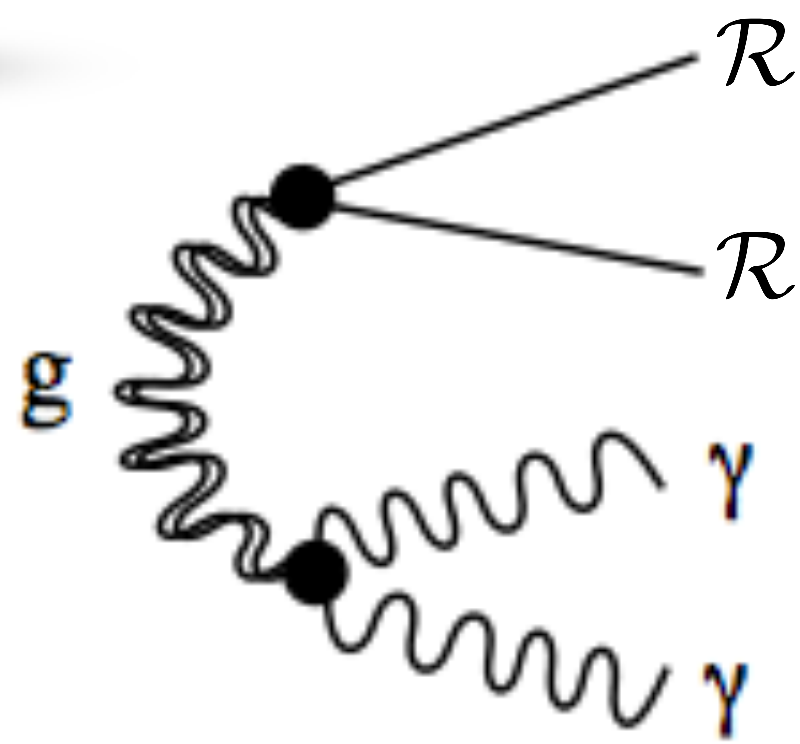
$$S = \frac{1}{2}\int d\eta d^3x\, Z\left(\left(\mathcal{R}'\right)^2 - c_{\rm S}^2\left(\partial_i \mathcal{R}\right)^2\right)\,, \qquad \mathcal{R} = \Phi + H\frac{\delta\varphi}{\dot{\varphi}}\,,$$

Ghost $Z(t) < 0$

$$H_{\mathbf{k}} = \frac{\left|P_{\mathbf{k}}\right|^2}{2Z} + \frac{Z c_S^2 k^2 \left|\mathcal{R}_{\mathbf{k}}\right|^2}{2} < 0$$

ghosts - modes (oscillators)

with the ***negative mass***



$$\Gamma_{0\to 2\gamma 2\phi} \sim \frac{\Lambda^8}{M_{\rm Pl}^4}$$

Cline, Jeon, Moore, (2003)

Can one change
the sign of the Hamiltonian
by a canonical transformation?

Canonical Transformations

$$(q, p, H) \to (\theta, \pi, \mathcal{H})$$

preserve Poincaré-Cartan integral invariant:

$$I = \oint pdq - Hdt = \oint \pi d\theta - \mathcal{H}dt$$

generating function: $pdq - Hdt - (\pi d\theta - \mathcal{H}dt) = dF$

$$p = \frac{\partial F}{\partial q}, \quad \pi = -\frac{\partial F}{\partial \theta}, \quad \mathcal{H} = H + \frac{\partial F}{\partial t}$$

preserve Poisson brackets: $\{q, p\} = \{\theta, \pi\} = 1$

Motion -canonical transformation

$$(q, p, H) \to (Q_0, P_0, 0)$$

with the generating function which is the on-shell action- i.e. Hamilton principal function $S\left(q, Q_0\right)$

$$\frac{\partial S}{\partial t} + H\left(\frac{\partial S}{\partial q}, q, t\right) = 0$$

Hamilton-Jacobi equation

Transformation in the Heisenberg picture

$$\hat{\mathcal{O}}' = \hat{U}\hat{\mathcal{O}}\hat{U}^\dagger$$

unitary time-dependent transformation of the ***canonical variables***

$$\hat{H}'(\hat{\mathcal{O}}', t) = \hat{U}^\dagger\, \hat{H}(\hat{\mathcal{O}}', t)\, \hat{U} - i\hat{U}^\dagger \left(\frac{\partial \hat{U}}{\partial t}\right)_{\mathcal{O}'}$$

the Hamiltonian transforms as a
connection in the non-Abelian field theories!

Time-Dependent Linear **Bogolyubov** Transformations

$$\widehat{q} = \alpha\widehat{\theta} + \beta\widehat{\pi}\,,$$
$$\widehat{p} = \gamma\widehat{\theta} + \delta\widehat{\pi}\,,$$

$$\alpha\left(t\right),\ \beta\left(t\right),\ \gamma\left(t\right),\ \delta\left(t\right)$$

canonical: $\alpha\delta - \beta\gamma = 1$

$$F\left(q,\theta,t\right) = -\frac{1}{\beta}\left(\theta q - \frac{\delta}{2}q^2 - \frac{\alpha}{2}\theta^2\right)$$

Change in Hamiltonian for an Oscillator

$$H_{-} = -\frac{1}{2}\left(p^2 + \omega^2 q^2\right) \quad \Rightarrow \quad H_n = \frac{1}{2}\pi^2 A + \pi\theta C + \frac{1}{2}\theta^2 B$$

$$A = \beta^2 \left(\frac{d}{dt}\left(\frac{\delta}{\beta}\right) - \left(\frac{\delta}{\beta}\right)^2 - \omega^2 \right)$$

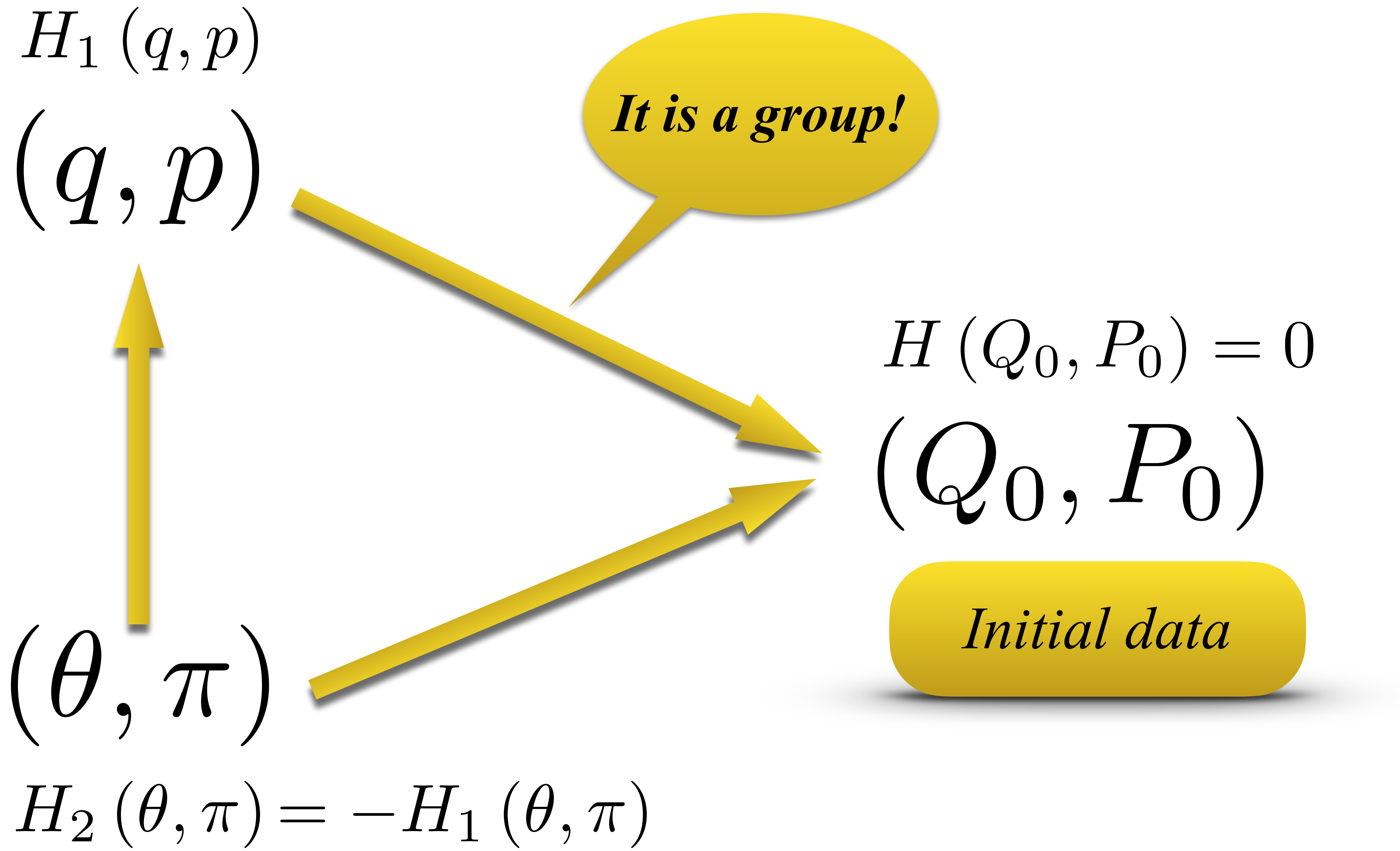
$$C = \frac{1}{\beta}\left[\frac{d\beta}{dt} + \delta + \alpha A\right]$$

$$B = \alpha^2 \left[\frac{d}{dt}\left(\frac{\gamma}{\alpha}\right) - \left(\frac{\gamma}{\alpha}\right)^2 - \omega^2 \right],$$

Strong time dependence

Can one always solve these nonlinear equations and change the sign of energy?

Canonical map through zero Hamiltonian



Harmonic Oscillator Example

$$H_- = -\frac{1}{2}\left(p^2 + \omega^2 q^2\right)$$

$$H_+ = \frac{1}{2}\left(\pi^2 + \omega^2 \theta^2\right)$$

$$q\left(t\right) = q_0 \cos\omega t - \frac{p_0}{\omega}\sin\omega t$$

$$p\left(t\right) = q_0 \omega \sin\omega t + p_0 \cos\omega t\,,$$

$$p = -\dot{q}$$

$$\theta\left(t\right) = q_0 \cos\omega t + \frac{p_0}{\omega}\sin\omega t$$

$$\pi\left(t\right) = -q_0 \omega \sin\omega t + p_0 \cos\omega t\,,$$

$$\pi = \dot{\theta}$$

$$\boxed{\begin{aligned} q &= \theta \cos 2\omega t - \frac{\pi}{\omega}\sin 2\omega t\,, \\ p &= \theta\omega \sin 2\omega t + \pi \cos 2\omega t\,. \end{aligned}}$$

$$\{q, p\} = \{\theta, \pi\} = 1$$

Conclusions

- There are many *canonical* variables for cosmological perturbations
- Time-dependent linear canonical, unitary (at least mode by mode) transformations always allow to find such *canonical* variables which are *not ghosty*

Thanks a lot for attention!