*Testing Gravity*
SFU Harbour Center
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# Signatures from Inflationary Massive Gravity

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C. Lin & MS, PLB 752, 84 (2016) [arXiv:1504.01373 ]
G. Domenech, T. Hiramatsu, C. Lin, MS, M. Shiraishi, Y. Wang, arXiv:1701.05554

# Introduction

# Inflation: the origin of Big Bang

**Brout, Englert & Gunzig '77, Starobinsky '79, Guth '81, Sato '81, …**

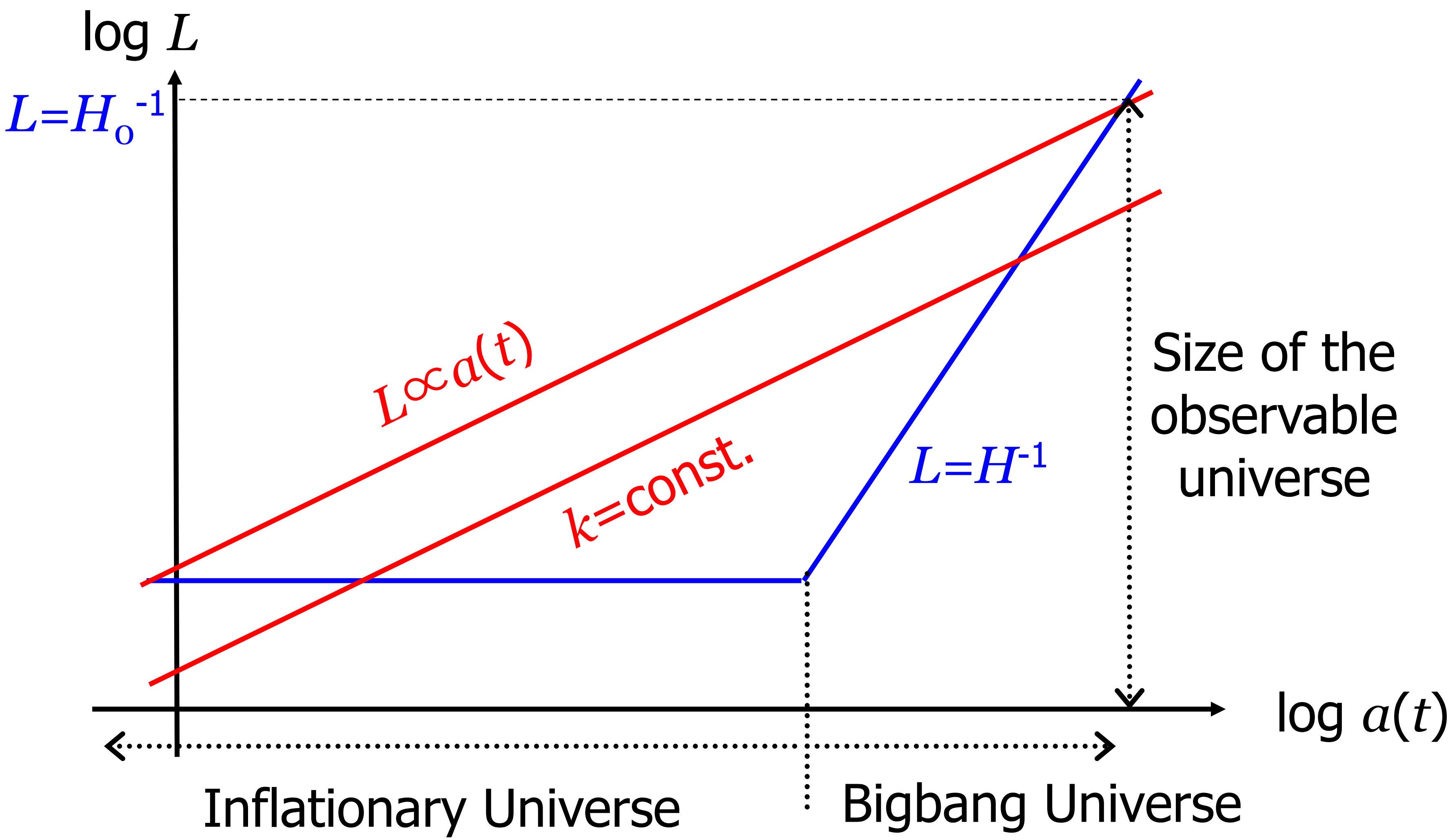
- Inflation is a quasi-exponential expansion of the Universe at its very early stage; perhaps at $t \sim 10^{-36}$ sec.

- It was meant to solve the initial condition (singularity, horizon & flatness, etc.) problems in Big-Bang Cosmology:
  - if any of them can be said to be solved depends on precise definitions of the problems.

- Quantum vacuum fluctuations during inflation turn out to play the most important role. They give the initial condition for all the structures in the Universe.

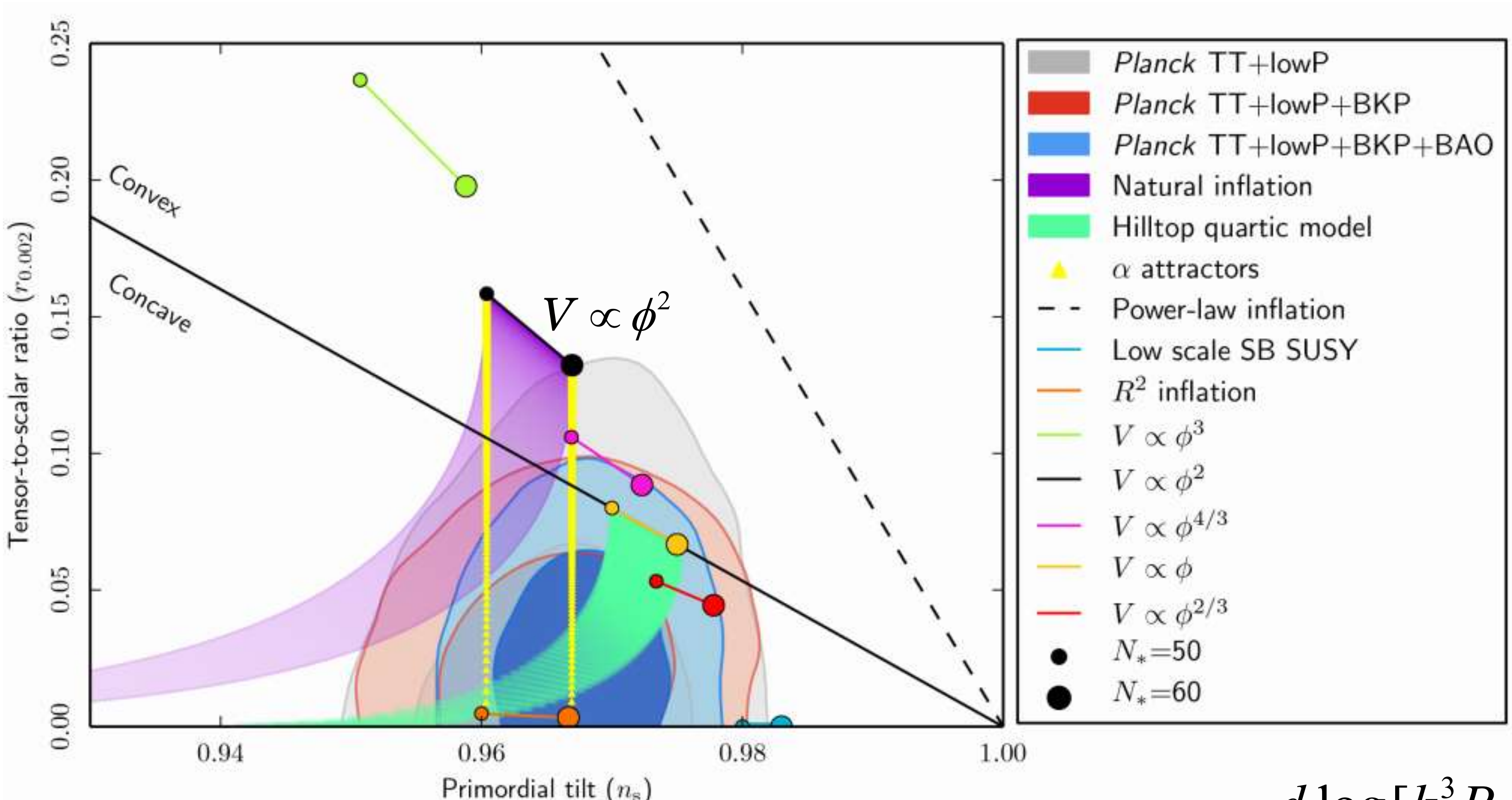
- Cosmic gravitational wave background is also generated.

# length scales of the inflationary universe



# Planck constraints on inflation

**Planck 2015 XX**




$$n_s - 1 \equiv \frac{d \log[k^3 P_S(k)]}{d \log k}$$

$$r \equiv \frac{P_T(k)}{P_S(k)}$$

- scalar spectral index: $n_s$ ~ 0.96
- tensor-to-scalar ratio: $r$ < 0.1
- simplest $V \propto \phi^2$ model is almost excluded

# Current status

- scalar spectral index: $n_s<1$ at ~ 5 $\sigma$
- tensor/scalar ratio: $r < 0.1$ implies $E_{inflation} < 10^{16}$ GeV
- simple, canonical models are on verge of extinction
  ($m^2\phi^2$ model excluded at > 2 $\sigma$)
- $R^2$ (Starobinsky) model seems to fit best. But why?
  (large $R^2$ correction but negligible higher order terms)
- $f_{NL}^{local}$ <O(1) suggests (effectively) single-field slow-roll
  (but non-slow-roll models with $f_{NL}^{local}$ =O(1) not excluded)

some element of non-canonicality is needed

# Massive Gravity?

# The idea of massive gravity

- Gauge theory:

  Higgs VEV spontaneously breaks gauge symmetry

  ⇨ massive gauge field

- Gravity:

  Spontaneous broken general covariance

  ⇨ massive gravitons

Boulware-Deser(BD) ghost
(must be removed)

dRGT gravity

spin 2 = 2+2+1 (+1) dof
(tensor+vector+scalar)

This assumes however Poincare symmetry on flat background.
If $\nexists$ background, covariance should NOT be violated.

Spontaneous broken local Lorentz invariance
= existence of a preferred frame

⇨ massive tensor modes!

# Dubovsky's model

Dubovsky 2004

- 4 scalar (Stuckelberg) fields: $\varphi^a = (\varphi^0, \quad \varphi^i)$

$$ds^2 = \eta_{ab} e_\mu^{(a)} e_\nu^{(b)} dx^\mu dx^\nu : \quad e_\mu^{(a)} \to \Lambda^a{}_b(x) e_\nu^{(b)}, \quad \Lambda^a{}_b \in SO(3,1)$$

- VEV spontaneously breaks local SO(3,1) symmetry

$$e_\mu^{(a)} = \frac{\partial \varphi^a}{\partial x^\mu} : \quad \varphi^a = \delta_\mu^a x^\mu \to e_\mu^{(a)} = \delta_\mu^a$$

- required symmetry (Poincare symmetry is not imposed)

$$\varphi^i \to \Lambda^i_j \varphi^j, \quad \varphi^i \to \varphi^i + \xi^i(\varphi^0); \quad \Lambda^i_j \in \text{global } SO(3)$$

- action: $S = \frac{M_P^2}{2} \int d^4x \left[ R + m_g^2 f(X, Z^{ij}) \right]$

2+1 dof (tensor+scalar)

$$X = g^{\mu\nu} \partial_\mu \varphi^0 \partial_\nu \varphi^0 = g^{00} = N^{-2}$$

Lapse fcn.

$$Z^{ij} = g^{\mu\nu} \partial_\mu \varphi^i \partial_\nu \varphi^j - \frac{g^{\mu\alpha} \partial_\mu \varphi^0 \partial_\alpha \varphi^i g^{\nu\alpha} \partial_\nu \varphi^0 \partial_\beta \varphi^j}{X} = h^{ij}$$

3-metric

only $\varphi^0$ becomes dynamical

# Inflationary massive gravity: minimal model

- Identify $\varphi^0$ with inflaton: $\phi = \varphi^0$

$$S = \int d^4x \left[ \frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{9}{8} M_P^2 m_g^2(\phi) \frac{\delta Z^{ij} \delta Z^{ij}}{Z^2} \right]$$

$$\delta Z^{ij} = Z^{ij} - 3 \frac{Z^{ik} Z^{kj}}{Z}; \quad Z = Z^{ii}$$

assumption: $m_g^2 \ll H^2$ during inflation

$m_g^2 = \frac{1}{2} \lambda \phi^2$ during reheating

- Symmetry:

$$\varphi^i \to \Lambda^i_j \varphi^j, \quad \varphi^i \to \lambda \varphi^i; \quad \Lambda^i_j \in \text{global } SO(3), \ \lambda = const.$$

These symmetries guarantees $\varphi^i$ to be non-dynamical.

$$\because \quad \varphi^i = x^i + \delta\phi^i : \quad \delta\phi^i = w^{ij}(t) x^j + v(t) x^i + O(1), \ w^{ij} = -w^{ji}$$

at leading order in gradient expansion

# massive tensor perturbation

- tensor 2nd order action

$$S_T^{(2)} = \frac{M_P^2}{8}\int d^3x dt a^3 \left[ (\dot{\gamma}_{ij})^2 - \left( \frac{k^2}{a^2} + m_g^2 \right) \gamma_{ij}^2 \right]$$

- quantization

$$\gamma_{ij} = \sum_{s=\pm} \int \frac{d^3k}{(2\pi)^{3/2}} \left[ a(\boldsymbol{k},s) e_{ij}(\boldsymbol{k},s) \gamma_k(t) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + h.c. \right]$$

$a(\boldsymbol{k},s)$ : annihilation operator

$e_{ij}(\boldsymbol{k},s)$ : polarization tensor

$$e_{ij}(\boldsymbol{k},s) e^{ij}(\boldsymbol{k},s') = \delta_{ss'}$$

- eom

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \left( m_g^2 + \frac{k^2}{a^2} \right) \gamma_k = 0$$

# tensor spectrum

- during inflation $0 < m_g^2 \ll H^2$

$$\Rightarrow \quad P_T(k) = \frac{2H^2}{\pi^2 M_P^2}\left(\frac{k}{k_f}\right)^{2m_g^2/3H^2}$$

at the end of inflation
$k_f = a(t_f)H(t_f)$

spectral index: $n_T \approx -2\varepsilon + \frac{2m_g^2}{3H^2}; \quad \varepsilon = -\frac{\dot{H}}{H^2}$

if $\frac{m_g^2}{3H^2} > \varepsilon$ blue-tilted!

- Lyth bound

$\Delta\phi \simeq 15 M_P r^{1/2}$ : distance traveled by $\phi$ during inflation

$$r = \frac{P_T(k)}{P_S(k)} \simeq 16\varepsilon$$ for standard slow-roll inflation

if $r > 0.001$, $\phi$ travels more than Planck distance

beyond validity of QFT?

# Observational Signatures?

# resonant GW amplification

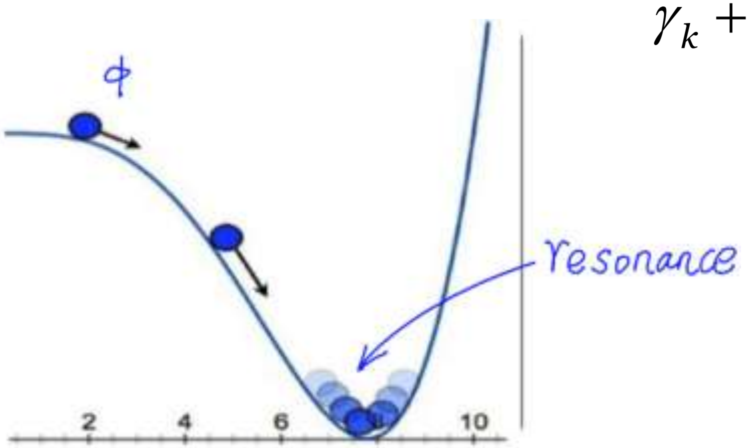
Lin & MS '15

$$m_g^2 = \frac{1}{2}\lambda\phi^2; \quad \phi = \phi_f \left(\frac{a}{a_f}\right)^{-3/2} \sin(Mt+\theta)e^{-\Gamma Mt}$$ after inflation

$M$ : inflaton mass

$\Gamma M$ : decay (reheating) rate

- Mathieu-like eq. for $k \ll aH$



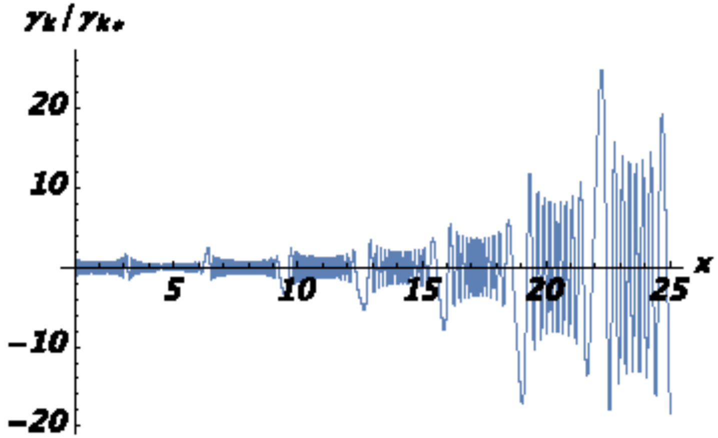

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + m_g^2(\phi)\gamma_k = 0$$

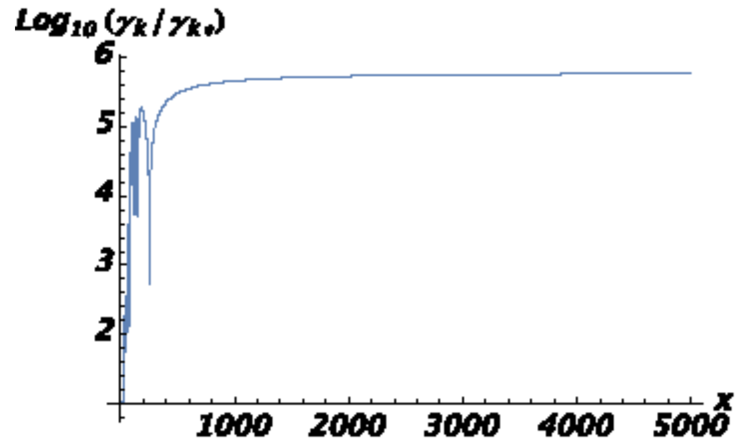
$$\Rightarrow \left[\frac{d^2}{dx^2} + \frac{2}{x}\frac{d}{dx} + \frac{\xi e^{-\Gamma x}}{x^2}\sin^2(x)\right]\gamma_k = 0$$

$$x = Mt, \quad \xi = \frac{\lambda\phi_f^2}{3\pi M^2}$$

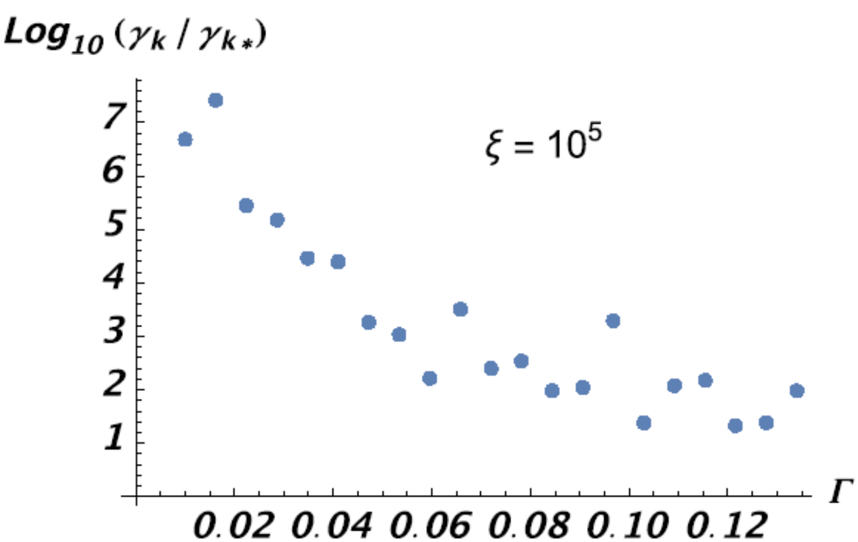
broad parametric resonance for $\frac{\xi e^{-\Gamma x}}{x^2} \gg 1$

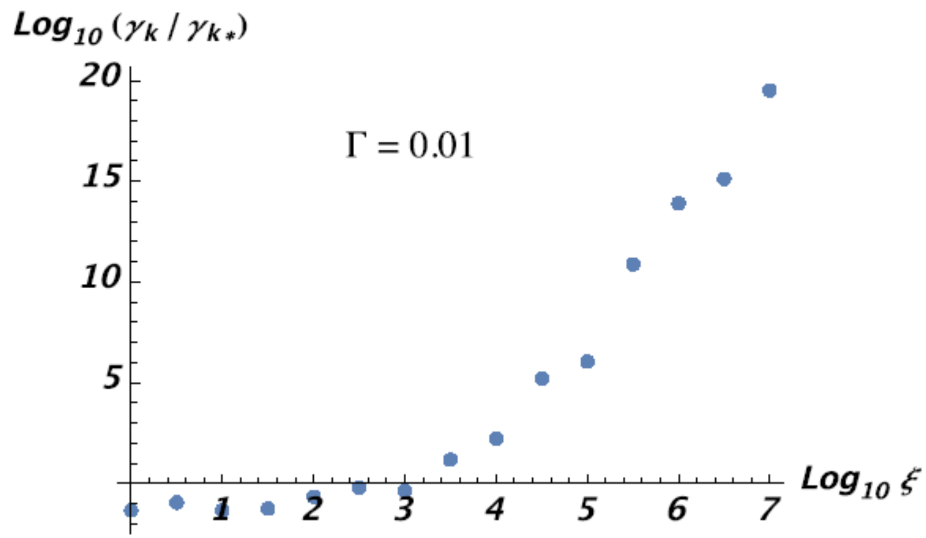
# broad parametric resonance




$$\xi = 10^6,\ \Gamma = 0.05$$




# parameter dependence

# evading Lyth bound

- tensor perturbation can be exponentially amplified by broad parametric resonance:

$$P_T(k) = A\, P_{T,0}(k); \quad A \gg 1$$

- scalar (curvature) perturbation remains the same:

$$P_S(k) = P_{S,0}(k)$$

$$\Rightarrow \quad r = \frac{P_T(k)}{P_S(k)} = 16\varepsilon A$$

Lyth bound is modified as $\Delta\phi \simeq 15 M_P \sqrt{\frac{r}{A}}$

**tensor perturbation can be large enough to be detected without invalidating low-energy EFT**

# non-Gaussianity?

Domenech, Hiramatsu, Lin, MS, Shiraishi & Wang '17

- 3rd order interaction Hamiltonian in on $\delta\phi$ =0 isotropic gauge:

$$H_{\rm int} = -L_{\rm int} \supset \lambda_{TSS} M_P^2 \varepsilon \int d^3x a \gamma_{ij} \partial_i \mathcal{R}_c \partial_j \mathcal{R}_c + \cdots$$

$$\lambda_{TSS} = \frac{m_g^2}{4\varepsilon H^2} + \lambda$$

new coupling term that could appear in low-energy EFT from (unknown) UV physics

3pt fcn:

$$\left\langle \gamma_{\boldsymbol{k}_1}^s \mathcal{R}_{\boldsymbol{k}_2} \mathcal{R}_{\boldsymbol{k}_3} \right\rangle = (2\pi)^3 \delta^{(3)}(\sum \boldsymbol{k}_i) \frac{\lambda_{TSS} H^4}{4\varepsilon M_P^4} \frac{e_{ij}^{-s} k_{2i} k_{3j}}{\prod k_i^3} \left( k_t - \frac{\sum_{i<j} k_i k_j}{k_t} - \frac{k_1 k_2 k_3}{k_t^2} \right)$$

dominates CMB 3-pt fcn if $\lambda_{TSS}$ >>1

# scale-dependent non-Gaussianity

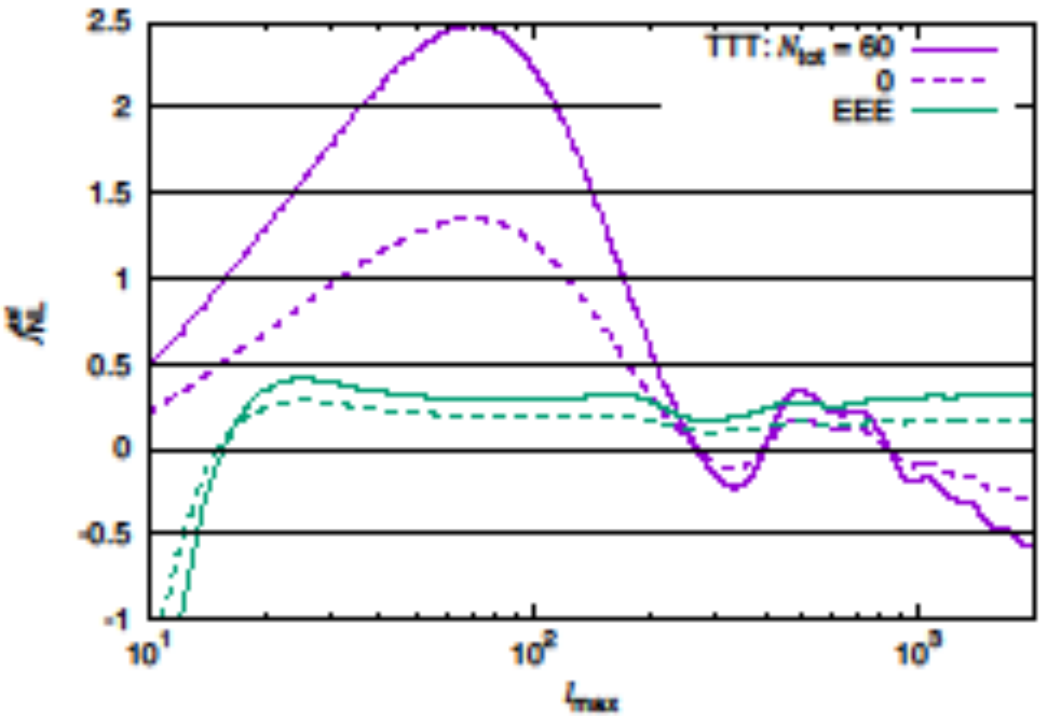
$$\langle \delta T \delta T \delta T \rangle \sim \langle \gamma_{ij} \partial_i \mathcal{R}_c \partial_j \mathcal{R}_c \rangle$$

more details in Domenech's talk/poster

$\gamma_{ij} \propto a^{-1}$ after it re-enters horizon

small scale modes re-enter horizon earlier

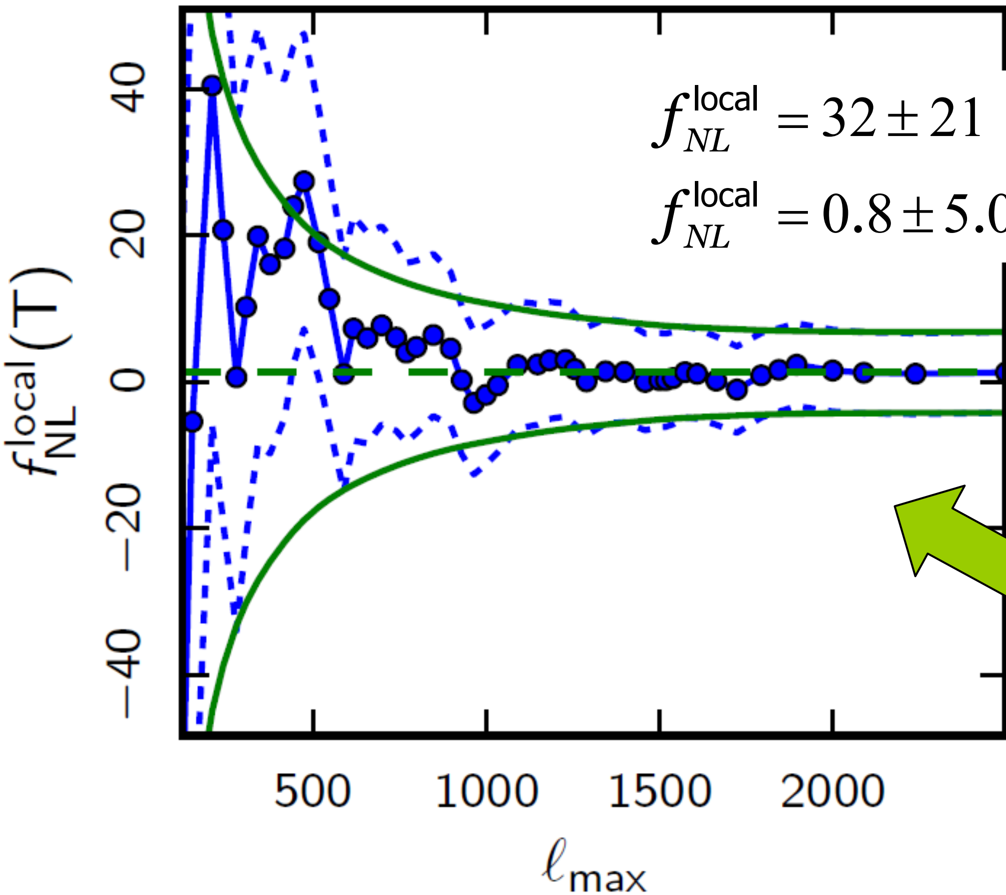
➡ large $\ell$ multipoles are suppressed




$\ell_{max}$ dependence of $f_{NL}^{\text{local}}$

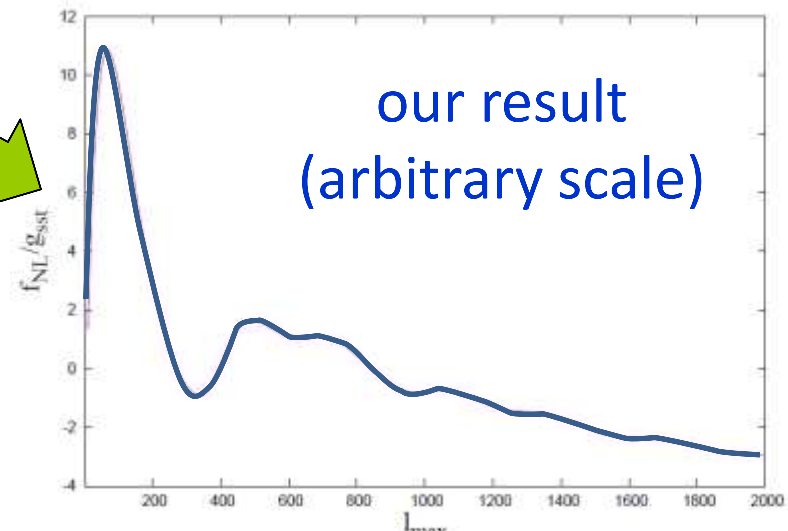
$\lambda_{TSS} = 100, \ \varepsilon = 10^{-2}$

# WMAP2010/Planck 2015




$$f_{NL}^{local} = 32 \pm 21 \quad \text{for } \ell_{max} \simeq 500 \qquad \text{WMAP 2010}$$

$$f_{NL}^{local} = 0.8 \pm 5.0 \quad \text{for } \ell_{max} \simeq 2500 \qquad \text{Planck 2015}$$




our result
(arbitrary scale)

a hint of scale-dep non-Gaussianity due to TSS

# Summary

- Inflation is a natural platform for modified gravity
  - Inflation = scalar-tensor theory
- GW (tensor mode) can become massive without encountering BD ghost problem
  - symmetry: $\varphi^i \to \Lambda^i_j \varphi^j, \quad \varphi^i \to \lambda\varphi^i; \quad \Lambda^i_j \in SO(3), \ \lambda = const.$

- GW can be parametrically amplified during reheating
  - evading Lyth bound even if r >0.001
- GW spectrum may be blue-tildted
  - primordial GW may be detectable by LIGO/Virgo/KAGRA…
- 3rd order interaction can give rise to sizable scale-dependent non-Gaussianity
  - $\exists$ already a hint in WMAP/Planck data