

Testing local Lorentz invariance with gravitational waves

- effective field theory searches
- cosmic ray constraints
- interferometer constraints

Jay D. Tasson
Carleton College

underlying theory at Planck scale

options for probing experimentally

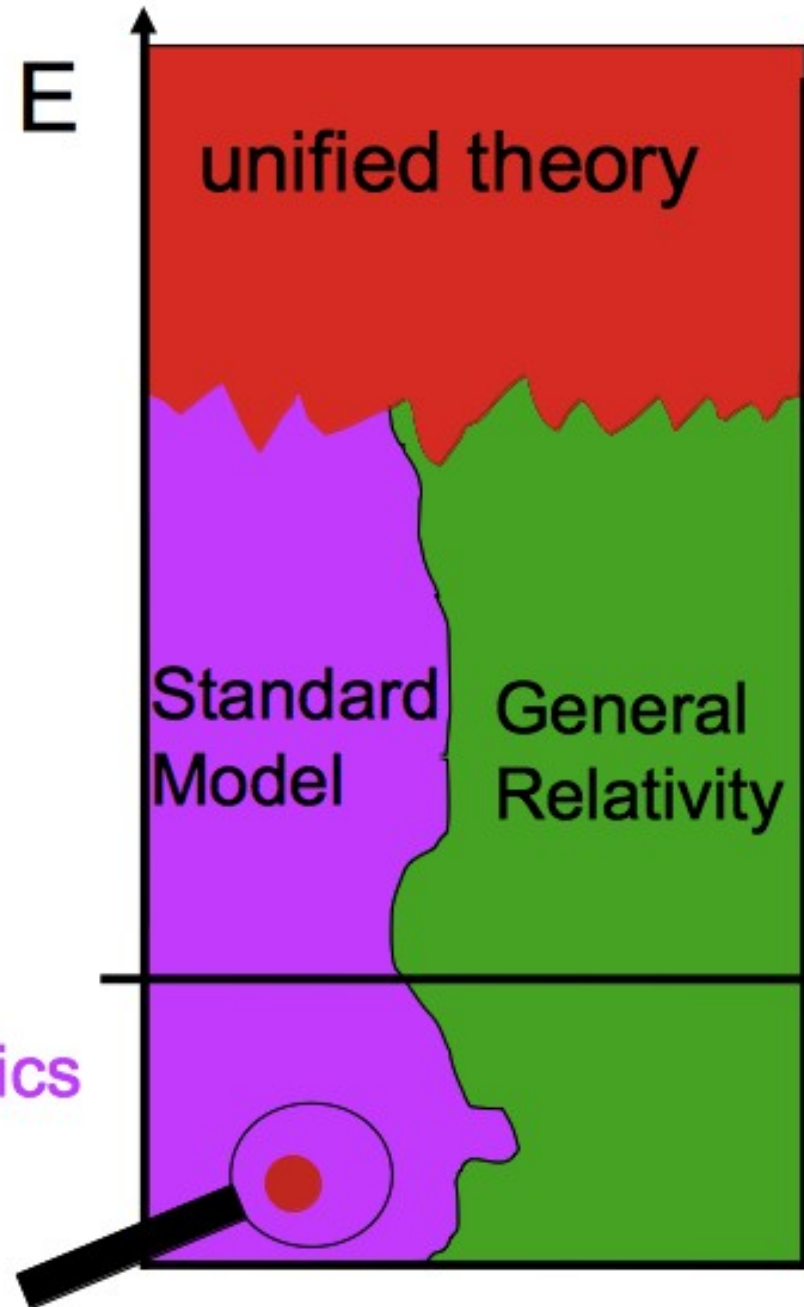
- galaxy-sized accelerator



- suppressed effects in sensitive experiments

CPT and Lorentz violation

- can arise in theories of new physics
- difficult to mimic with conventional effects



effective field theory search for Lorentz violation (SME)

- perform a systematic search via an effective field theory

known physics
SM + GR

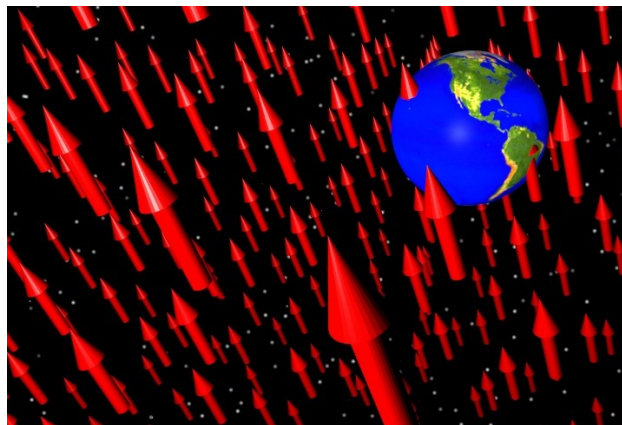
+ ○ + ● + . + ... =

quantum
gravity

standard linearized
gravity

coefficients for
Lorentz violation

$$\mathcal{L} \supset \frac{1}{4} \epsilon^{\mu\rho\alpha\kappa} \epsilon^{\nu\sigma\beta\lambda} h_{\mu\nu} \left(\underset{d=4}{\eta_{\kappa\lambda}} - \underset{d=6}{\bar{s}_{\kappa\lambda}} - \bar{s}_{\kappa\lambda\delta\gamma} \partial^\delta \partial^\gamma \right) \partial_\alpha \partial_\beta h_{\rho\sigma}$$



On-going effort

Data Tables for Lorentz and CPT Violation [arXiv:0801.0287v10](#)
~250 Refs. with sensitivities, including:

gravitational Čerenkov radiation
superconducting gravimeters
short-range gravity devices
gravitational-wave interferometers
lunar laser ranging
binary-pulsar observations
planetary ephemerides
gravity probe B
bound kinetic energy WEP
atom interferometers
comagnetometry
perihelion precession
equivalence-principle pendulum
Solar-spin precession

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atom interferometer
ring-laser gyroscopes
torsion pendula
binary pulsars
short-range gravity
gravitational-wave detectors
space-based WEP tests
antimatter gravity
charged matter WEP
muonium free fall
light bending
time-delay & Doppler tests

constraints via gravitational waves to date

Data Tables for Lorentz and CPT Violation arXiv:0801.0287v10
~250 Refs. with sensitivities, including:

Constraints on Lorentz Violation from Gravitational Cherenkov Radiation,
Kostelecky and Tasson, Phys. Lett. B 749, 551 (2015).

The Standard-Model Extension and Gravitational Tests,
Tasson, Symmetry 8, 111 (2016).

Testing Local Lorentz Invariance with Gravitational Waves,
Kostelecky and Mewes, Phys. Lett. B 757, 510 (2016).

*Theoretical Physics Implications of the Binary Black-Hole Mergers
GW150914 and GW151226.*

Yunes, Yagi, and Pretorius, Phys. Rev. D 94, 084002 (2016).

Searching for Photon-Sector Lorentz Violation using Gravitational-Wave Detectors,
Kostelecky, Melissinos, and Mewes, Phys. Lett. B 761, 1 (2016).

searching for symmetry violation

pure phenomenology

theoretical framework

advantage

- easy parameterization

- quantitative comparisons of very different experiments
eg: lab gravity & atomic
- calculate predictions
- consistent analysis

disadvantage

- not predictive
- unclear assumptions
- no relation between experiments

- more work!

Lorentz-violating effects

- 1) boost violation
- 2) rotation invariance violation
- 3) dispersion relation effects

Lorentz-violation in gravitational waves

- A) effects on generation at the source
- B) effects on the physics of the detectors
- C) effects on vacuum propagation
 - i) dispersion (isotropic or not)
 - ii) birefringence (isotropic or not)
- D) kinematic effects on gravitational wave interactions

Definition – Isotropic Lorentz violation/preferred frame models:
Models such that coordinates can be found in which rotation invariance is preserved and only boost invariance is violated.

A limit of the general field theoretic approach in which coordinates can be found such that only the “0” components for Lorentz-violating tensors are nonzero

$$\mathcal{L} \supset \frac{1}{4} \epsilon^{\mu\rho\alpha\kappa} \epsilon^{\nu\sigma\beta\lambda} h_{\mu\nu} \left(\eta_{\kappa\lambda} - \bar{s}_{\kappa\lambda} - \bar{s}_{\kappa\lambda\delta\gamma} \partial^\delta \partial^\gamma \right) \partial_\alpha \partial_\beta h_{\rho\sigma}$$

effective field theory, linearized gravity

$$\mathcal{L}_{\mathcal{K}^{(d)}} = \frac{1}{4} h_{\mu\nu} \hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} h_{\rho\sigma},$$

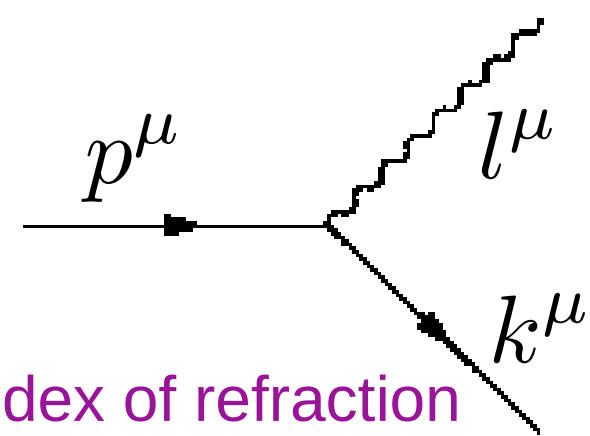
$$\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} = \mathcal{K}^{(d)\mu\nu\rho\sigma\varepsilon_1\varepsilon_2\dots\varepsilon_{d-2}} \partial_{\varepsilon_1} \partial_{\varepsilon_2} \dots \partial_{\varepsilon_{d-2}}$$

$$\equiv \mathcal{K}^{(d)\mu\nu\rho\sigma} \circ^{d-2}$$

- 1) general action-based linearized theory of gravity
based on a symmetric 2-tensor field,
containing the GR limit,
constructed from constant coefficients, derivatives, gravitational field
- 2) 14 classes of operators:
3 of which maintain the usual GR gauge structure
theory questions for the others

Notation: $\mathcal{K}^{(d)\mu\nu\rho\sigma\varepsilon_1\varepsilon_2\dots\varepsilon_{d-2}} \supset k_{(V)jm}^{(d)} \bar{s}_{jm}^{(d)}$

Čerenkov Radiation



effective direction and energy dependent index of refraction

$$n \approx 1 - \frac{1}{2} \sum_d (-1)^{d/2} s^{(d)} |\vec{l}|^{d-4}$$

$$s^{(d)}(\hat{p}) = \sum_{jm} Y_{jm}(\hat{p}) \bar{s}_{jm}^{(d)}$$

power loss to gravitons

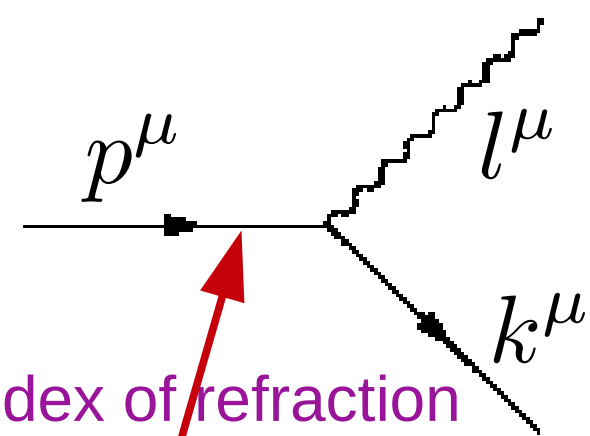
$$\frac{dE}{dt} = - \frac{1}{8|\vec{p}|\sqrt{m_w^2 + \vec{p}^2}} \int \frac{d^3l}{(2\pi)^2 |\vec{l}|} |\mathcal{M}|^2 \delta(\cos\theta - \cos\theta_C)$$

$$\cos\theta_C = \frac{\sqrt{m_w^2 + \vec{p}^2}}{|\vec{p}|} \frac{1}{n(|\vec{l}|)} + \frac{|\vec{l}|}{2|\vec{p}|} \left(1 - \frac{1}{[n(|\vec{l}|)]^2} \right)$$

natural cutoff at $l = p$

$$\frac{dE}{dt} = -F^w(d) G_N (s^{(d)})^2 |\vec{p}|^{2d-4}$$

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natural cutoff at $l = p$

$$\frac{dE}{dt} = -F^w(d) G_N (s^{(d)})^2 |\vec{p}|^{2d-4}$$

scalars, fermions, photons

direction dependence
momentum dependence

Čerenkov Radiation

solve power loss for time of flight

$$t = \frac{\mathcal{F}^w(d)}{G_N(s^{(d)})^2} \left(\frac{1}{E_f^{2d-5}} - \frac{1}{E_i^{2d-5}} \right)$$

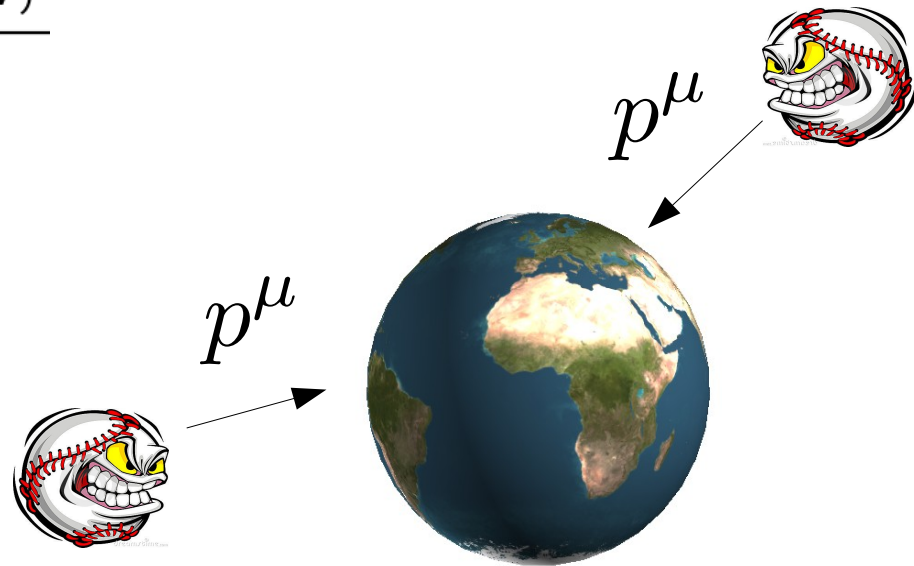
constraints via travel distance and observed energy

$$s^{(d)}(\hat{p}) \equiv (\bar{s}^{(d)})^{\mu\nu\alpha_1\ldots\alpha_{d-4}} \hat{p}_\mu \hat{p}_\nu \hat{p}_{\alpha_1} \cdots \hat{p}_{\alpha_{d-4}} = \sum_{jm} Y_{jm}(\hat{p}) \bar{s}_{jm}^{(d)} < \sqrt{\frac{\mathcal{F}^w(d)}{G_N E_f^{2d-5} L}}$$

OMG particle and friends

we see 'em so they didn't gravi-Čerenkov away all energy

Observatory	Events	E_{\max} (EeV)
AGASA	22	213
Fly's Eye	1	320
Haverah Park	13	159
HiRes	11	127
Pierre Auger	136	127
SUGAR	31	197
Telescope Array	60	162
Volcano Ranch	2	139
Yakutsk	23	160



Čerenkov radiation

constraints

- billion fold improvements on 9 d=4 coefficients
- first-ever constraints on 153 d=6, d=8, d=10 coefficients

Conservative constraints on coefficients $\bar{s}_{jm}^{(8)}$ in GeV^{-4} .

d	j	Lower bound	Coefficient	Upper bound
8	0	$-7 \times 10^{-49} <$	$\bar{s}_{00}^{(8)}$	
8	1	$-1 \times 10^{-45} <$	$\bar{s}_{10}^{(8)}$	$< 1 \times 10^{-45}$
		$-9 \times 10^{-46} <$	$\text{Re } \bar{s}_{11}^{(8)}$	$< 8 \times 10^{-46}$
		$-9 \times 10^{-46} <$	$\text{Im } \bar{s}_{11}^{(8)}$	$< 9 \times 10^{-46}$
8	2	$-9 \times 10^{-46} <$	$\bar{s}_{20}^{(8)}$	$< 1 \times 10^{-45}$
		$-1 \times 10^{-45} <$	$\text{Re } \bar{s}_{21}^{(8)}$	$< 8 \times 10^{-46}$
		$-8 \times 10^{-46} <$	$\text{Im } \bar{s}_{21}^{(8)}$	$< 9 \times 10^{-46}$
		$-1 \times 10^{-45} <$	$\text{Re } \bar{s}_{22}^{(8)}$	$< 9 \times 10^{-46}$
		$-1 \times 10^{-45} <$	$\text{Im } \bar{s}_{22}^{(8)}$	$< 9 \times 10^{-46}$
8	3	$-1 \times 10^{-45} <$	$\bar{s}_{30}^{(8)}$	$< 1 \times 10^{-45}$

effective field theory dispersion relation

$$\omega = \left(1 - \varsigma^0 \pm \sqrt{(\varsigma^1)^2 + (\varsigma^2)^2 + (\varsigma^3)^2} \right) |\mathbf{p}|,$$

$$\varsigma^0 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{\mathbf{n}}) k_{(I)jm}^{(d)},$$

$$\varsigma^1 \mp i\varsigma^2 = \sum_{djm} \omega^{d-4} {}_{\pm 4}Y_{jm}(\hat{\mathbf{n}}) \left(k_{(E)jm}^{(d)} \pm ik_{(B)jm}^{(d)} \right),$$

$$\varsigma^3 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{\mathbf{n}}) k_{(V)jm}^{(d)}.$$

1) anisotropy

convenient spherical harmonic direction dependence

2) birefringence

plus/minus in dispersion

3) dispersion through various powers of frequency

$$\left| \sum_{jm} Y_{jm}(160^\circ, 120^\circ) k_{(V)jm}^{(5)} \right| \lesssim 2 \times 10^{-14} \text{ m}$$

summary

- Effective field theory methods facilitate a broad and organized search for new physics.
- Much work has been done with gravitational systems.
- Much remains to explore, e.g. gravitational waves.