On the Implications of the Tachyonic Neutrino Hypothesis for the Dark Energy Problem

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Introduction

There is substantial lore on tachyons [7] suggesting a number of ways in which the possible existence of such faster-than-light particles may be brought into question. Some typical claims are: (1) due to the generic appearance of negative energies, such particles would violate stability of the vacuum, (2) since they are capable of travelling backward in time, they would circumvent our customary notion of causality—messages could be relayed backward in time to the original sender, (3) lack of Lorentz invariance would lead to transgressions of the principles of special relativity, or (4) requiring strict local commutativity (field operators commute at spacelike separation) would demand exponentially growing modes to be brought into the theory, another source of instability different from (1) above. However, a closer examination of the concept of tachyons using the methods of quantum field theory reveals the option of avoiding the worst fears as specified above, while still allowing negative energy (and backward-in-time-moving) particles.

Specifically, we shall demonstrate, through two concrete models defined below, that a framework may be synthesized in which the usual notions of stability, and reasonable replacements of the commonly held notions of causality and Lorentz invariance, may peacefully coexist. The first model describes a scalar tachyonic quantum field theory on Minkowski spacetime, without exponentially growing/decaying modes (which effectively deals with problem (4) above), and yet with a sensible notion of causality. The latter may be expressed as a "no antitelephones" or "chronology protection" property—particles that would be needed to send a message backward in time to the sender via a relay cannot all be created out of the same vacuum. (This is seen in Figs. 1a and 1b which show the allowed energy-momenta of particles that can be created from such a vacuum.) Negative energies are allowed for particles in this model, but such particles are interpreted as progressing backward in time, which, according to the "re-interpretation" principle of [2] for tachyons, is equivalent classically to viewing these as positive energy antiparticles moving forward in time but in the opposite spatial direction. For the present models, however, the "always positive energy" interpretation rule is valid only in a particular frame (the so-called *preferred* or *tachyon* frame [6])—negative energies are allowed in all other frames to preserve Lorentz covariance, but this is not viewed as instability since negative energies are being sent into the past. Thus, for problems (1) and (2), the apparent "sickness" remains, but is found not to be fatal. A simple calculation within this model, involving two body decay with one product a tachyon, yields a reasonable, stable answer [11], in constrast to the nonsensical result obtained by [8] for such a decay.

A criterion for the two-point function of the model, known as the *wave front (or microlocal) spectral condition* [10, 4, 3], is found to be satisfied, suggesting a straightforward inclusion of the free QFT within a renormalizable interacting theory involving other particles (regular or tachyonic) with (self) interactions. The usual notion of strict Lorentz *in*variance is seen *not* to hold for the two-point function of this QFT, reflecting the existence of the special "preferred" inertial frame mentioned above. The principles of relativity are seen to be respected nevertheless, since the speed of light remains constant and the laws of physics (partial differential equations describing classical fields, i.e., before second quantization) remain covariant under Lorentz transformation from one inertial frame to another. Alternatively, the underlying Poincaré symmetry of the spacetime remains intact, while Lorentz symmetry is spontaneously broken in defining the QFT (both field operators and vacuum state) on this spacetime.

Next, the scalar model is extended to the case of Dirac-like tachyons, called *Dirachyons* here, to yield the second model. Due to the requirement of positive energy eigenvalues of the Hamiltonian $\hat{H}$ in the preferred frame for particles and antiparticles, one singles out (with the appropriate overall sign of $\hat{H}$) only the left-handed particles and right-handed antiparticles as being physical. Hence parity breaking occurs naturally and at a fundamental level in the free QFT. This suggests application of this theory to the only known elementary particle which consistently displays this parity breaking in experiments, the neutrino. One immediate consequence of adopting the *tachyonic neutrino hypothesis* of Chodos, Hauser and Kostelecký [5] is that, in the electroweak model, a V-A interaction term would be superfluous; using only V coupling would be more efficient. This is parlayed into a theory of beta decay which exhibits a similar anomaly near the endpoint, similar to what has been seen in tritium beta decay experiments in the past.

Lastly, the stress energy tensor is found to be defined independently of the choice of preferred frame, for both the quantum field theoretic (on Minkowski space) and classical cases (on a general curved spacetime). The case of a perfect tachyonic fluid is treated, and $T^{\mu\nu}$ is found to be $\mathrm{diag}(\rho, p, p, p)$, as for the regular mass and lightlike cases with, however, $\rho < 3p$. For a spatially homogeneous Robertson Walker spacetime, whether the fluid has regular mass ($\rho > 3p$) or tachyonic is shown not to affect $\frac{\ddot{R}}{R}$, but it would affect the sign of $(R^2)\ddot{} \equiv \frac{d^2}{dt^2}(R^2)$ if the spatial sections are flat ($k = 0$).

Scalar quantum field theory of tachyons

In [12] we argue that, in order to maintain any chance of renormalizability of the free theory, Lorentz symmetry should be broken in a reasonable quantum field model satisfying the tachyonic Klein-Gordon equation,

$$(\Box - m^2)u = 0 \, , \tag{1}$$

where $\Box = \partial_t^2 - \nabla^2$. The construction of the QFT is performed first in the preferred frame, and is extended to any other frame using the usual notion of transformation of coordinates under Lorentz transformation. E.g., if a Green's function in the preferred frame is $G(x', y')$, then the same function in the frame obtained by the boost $\Lambda\colon x' \to x$ is the pullback by the inverse Lorentz transformation, namely $G(\Lambda^{-1}x, \Lambda^{-1}y)$. The *oscillatory* mode solutions of Eq.(1) are $u_{\mathbf{k}}(t, \mathbf{x}) = M_{\mathbf{k}} \mathrm{e}^{-i(\omega_{\mathbf{k}} t - \mathbf{k}\cdot\mathbf{x})}$, where $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 - m^2}$ and $|\mathbf{k}| > m$. The normalization factor $M_{\mathbf{k}}$ is chosen so that these modes are normalized with respect to the inner product $(u, v) := i \int_{t=a} u^*(x) \overleftrightarrow{\partial_t} v(x)\, d^3\mathbf{x}$, where $u^*(x) \overleftrightarrow{\partial_t} v(x) = u^*(x)(\partial_t v(x)) - (\partial_t u^*(x))v(x)$. Note that this definition depends crucially on the initial choice of preferred frame (unlike for the massive or light-like cases), but not on a. A basis for the space of *positive energy* or *positive frequency* (oscillatory) solutions is $\{u_{\mathbf{k}}\}$, where $(u_{\mathbf{k}}, u_{\mathbf{l}}) = \delta^{(3)}(\mathbf{k} - \mathbf{l})$, $(u^*_{\mathbf{k}}, u^*_{\mathbf{l}}) = -\delta^{(3)}(\mathbf{k} - \mathbf{l})$, and $(u_{\mathbf{k}}, u^*_{\mathbf{l}}) = (u^*_{\mathbf{k}}, u_{\mathbf{l}}) = 0$. The $M_{\mathbf{k}}$ must be then chosen to be $((2\pi)^3 \cdot 2\omega_{\mathbf{k}})^{-\frac{1}{2}}$. The *negative energy* modes are $u^*_{\mathbf{k}}(t, \mathbf{x}) = \frac{1}{\sqrt{(2\pi)^3 \cdot 2\omega_{\mathbf{k}}}} \mathrm{e}^{i(\omega_{\mathbf{k}} t - \mathbf{k}\cdot\mathbf{x})}$, where $|\mathbf{k}| > m$.

Mode solutions with $|\mathbf{k}| = m$ are ignored since they constitute a set of measure 0 with respect to the entire mass hyperboloid $k^2 = -m^2$ of solutions. Imaginary energy modes (which yield exponentially growing and decaying solutions in time) are deleted from the theory since renormalized observables would generically blow up exponentially in time.

Second quantizing Eq.(1) involves seeking a field operator $\phi(x)$ with $\phi(x) = \int_{|\mathbf{k}|>m} (a_{\mathbf{k}} u_{\mathbf{k}}(x) + a^\dagger_{\mathbf{k}} u^*_{\mathbf{k}}(x))\, d^3\mathbf{k}$, which satisfies the *equal time commutation relations*, modified so as not to include frequencies $\mathbf{k}$ for which $|\mathbf{k}| < m$.

The *vacuum* or *ground state* $|0\rangle$ associated with this particular choice of preferred frame is then defined by $a_{\mathbf{k}} |0\rangle = 0$. The *two-point distribution* is then $\Delta^{(+)}(x, y) = \langle 0|\, \phi(x)\phi(y)\, |0\rangle = \frac{1}{2}\Delta^{(1)}(x, y) + i\frac{1}{2}\Delta(x, y)$. The anti-symmetric part of this two-point distribution is not Lorentz invariant, while the symmetric part is. Further details regarding the renormalizability and causality of the QFT so constructed may be found in [12]. Figures 1a and 1b show (resp. in the preferred frame and in a frame boosted in the x-direction) the energy-momentum vectors allowed for the one-particle spectrum for both particles and antiparticles. (The E axis is vertical.) Note that the hyperplane defining the cutoff in the spectrum also defines (in configuration space) a set of events *simultaneous to* the single event at the origin of Minkowski spacetime. All events are either in, to the future, or to the past of this hyperplane, which thus defines a notion of time ordering between any two events, including spacelike separated ones.

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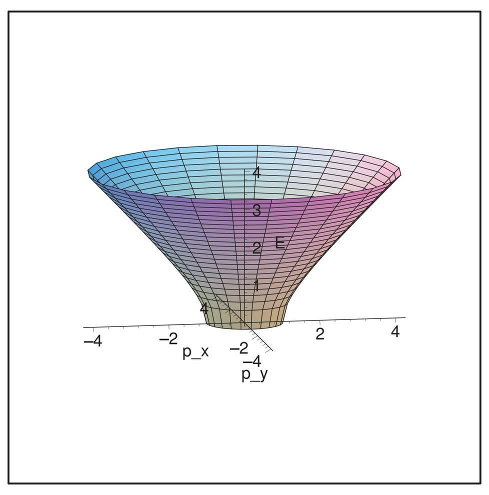

Figure 1a: Single particle spectrum in preferred frame.

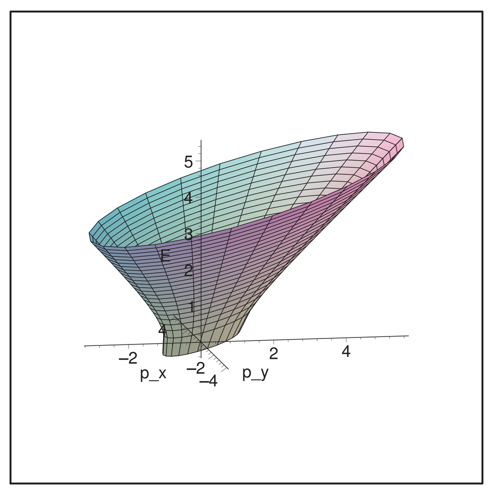

Figure 1b: Single particle spectrum in boosted frame.

Dirachyonic theory

An equation originally due to Tanaka [14] which describes tachyonic Dirac particles (*Dirachyons*) is

$$i\partial\!\!\!/\psi - \gamma_5 m\psi = 0 \, . \tag{2}$$

A representation of the Dirac matrices is chosen so that $\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}$. The σ^i are the Pauli matrices. Here $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. That Eq.(2) describes tachyons is found by squaring the operator $i\partial\!\!\!/$ and using the anticommutativity of the γ_5 with all the γ^i. Furthermore, the Dirachyon equation is covariant under *proper* Lorentz transformations.

As in the scalar case we find all mode solutions in the preferred frame, and extend these to all other frames by Lorentz transformation. If $u_\pm(\mathbf{p})$ are the helicity eigenspinors defined up to phase by $(\mathbf{p}\cdot\sigma)u_\pm(\mathbf{p}) = \pm p u_\pm(\mathbf{p})$, with $p = |\mathbf{p}|$ and $E^2 - p^2 = -m^2$, then we determine the positive energy solutions of Eq.(2) to be (up to normalization constants) $\mathrm{e}^{-i(Et - \mathbf{p}\cdot\mathbf{x})}$ times $\begin{pmatrix} E + p \\ m \end{pmatrix} u_+(\mathbf{p})$ and $\begin{pmatrix} -m \\ E + p \end{pmatrix} u_-(\mathbf{p})$, while the negative energy solutions are (up to normalization) $\mathrm{e}^{i(Et - \mathbf{p}\cdot\mathbf{x})}$ times $\begin{pmatrix} m \\ E + p \end{pmatrix} u_+(-\mathbf{p})$ and $\begin{pmatrix} -(E + p) \\ m \end{pmatrix} u_-(-\mathbf{p})$. The (normalized) solutions without the exponentials are labelled $u_{\mathbf{p},\pm}$ and $v_{\mathbf{p},\pm}$ respectively.

An inner product based on the conserved charge $J^\mu = \bar{\psi}\gamma_5\gamma^\mu\psi$ of Eq.(2) is $(\psi_1, \psi_2) \equiv -\int \psi_1^\dagger \gamma_5 \psi_2\, d^3\mathbf{x}$, which is clearly indefinite. The sign is set so that the modes $u_{\mathbf{p},-}$ and $v_{\mathbf{p},+}$ have positive norms, and $u_{\mathbf{p},+}$ and $v_{\mathbf{p},-}$ have negative norms with respect to this inner product.

Constructing a quantum field theory from these modes, we write $\Psi(x) = \sum_{s=\pm} \int d^3\mathbf{p} \left[a_{\mathbf{p},s} u_{\mathbf{p},s}(x) + b^\dagger_{\mathbf{p},s} v_{\mathbf{p},s}(x)\right]$, where the a's and b's satisfy anticommutation relations $\{a_{\mathbf{p},s}, a^\dagger_{\mathbf{p}',s'}\} = \delta^{(3)}(\mathbf{p} - \mathbf{p}')\delta_{ss'}$ and $\{b_{\mathbf{p},s}, b^\dagger_{\mathbf{p}',s'}\} = \delta^{(3)}(\mathbf{p} - \mathbf{p}')\delta_{ss'}$. The quantized *lepton number operator* $\hat{Q}'$, the quantized version of the conserved charge $Q = \int \psi^\dagger \gamma_5 \psi\, d^3\mathbf{x}$, is $\int d^3\mathbf{p} \left[a^\dagger_{\mathbf{p},-} a_{\mathbf{p},-} - a^\dagger_{\mathbf{p},+} a_{\mathbf{p},+} + b^\dagger_{\mathbf{p},-} b_{\mathbf{p},-} - b^\dagger_{\mathbf{p},+} b_{\mathbf{p},+}\right]$. The second quantized Hamiltonian $\hat{H}$ is $\int d^3\mathbf{p}\, E_{\mathbf{p}} \left[a^\dagger_{\mathbf{p},-} a_{\mathbf{p},-} - a^\dagger_{\mathbf{p},+} a_{\mathbf{p},+} - b^\dagger_{\mathbf{p},-} b_{\mathbf{p},-} + b^\dagger_{\mathbf{p},+} b_{\mathbf{p},+}\right]$. Note that we obtain wrong lepton number and negative energies for the second and third terms, which coincide with the negative normed modes. These modes may be consistently eliminated from the theory in order to obtain a meaningful lepton number operator

$$\hat{Q} = \int d^3\mathbf{p} \left[a^\dagger_{\mathbf{p},-} a_{\mathbf{p},-} - b^\dagger_{\mathbf{p},+} b_{\mathbf{p},+}\right] \, , \tag{3}$$

and positive definite Hamiltonian

$$\hat{H} = \int d^3\mathbf{p}\, E_{\mathbf{p}} \left[a^\dagger_{\mathbf{p},-} a_{\mathbf{p},-} + b^\dagger_{\mathbf{p},+} b_{\mathbf{p},+}\right] \, . \tag{4}$$

Thus the QFT which meaningfully describes a tachyonic Dirac-like particle inherently breaks parity maximally, and it is interesting to investigate how appropriate this model may be in describing the neutrino.

Incorporating a neutrino satisfying the above Dirachyonic QFT into a theory of beta decay, with a V interaction, yields (after much algebra and some approximations) a matrix element which is in accord with that given by [6], but with the final differential decay rate $\frac{d\Gamma}{dE}$ evaluated for more general parameter values than considered by [6]. (The parameters on which the curves depend are the tachyonic mass parameter m, the speed of the earth with respect to the preferred frame β, the angle of the direction of the preferred frame's velocity from the axis of the earth's rotation α, a background level C, and the endpoint energy E.) The final formula is found in [13]. Further work, e.g., possibly in cooperation with KATRIN [9], is necessary to clarify whether the theoretical curves provide an accurate representation of the data. Note that in 2019, KATRIN's first run produced a negative mass squared to one sigma, after rigorous, double-blind, parallel analysis of the data [1]. See Figure 2.

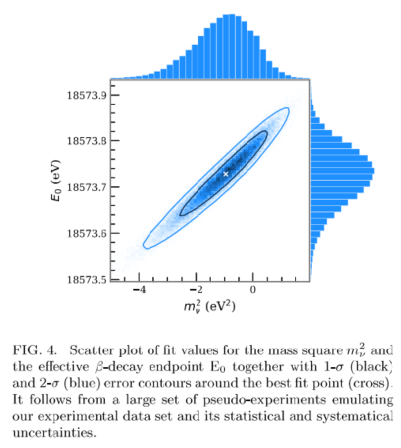

Figure 2: First results from KATRIN, from [1].

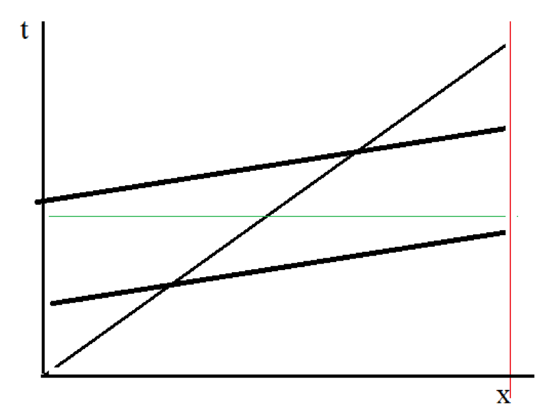

Figure 3: Definition of stress energy tensor components for perfect tachyonic fluid.

Stress Tensor of Perfect Tachyonic Fluid

One redeeming feature of this model is that point-splitting renormalization of observables quadratic in the field yields tensor fields which are independent of the choice of preferred frame (to leading order). In particular this is true of the stress energy tensor $T^{\mu\nu}$, whose classical definition is also independent of the preferred frame. Thus a uniformly distributed collection of space-like particles with uniformly distributed momenta directions, with energy $E = \gamma m v$ and x-momentum $p = \gamma m(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ has stress energy tensor components $T^{00} = \gamma^2 m n v^2$, $T^{0i} = T^{i0} = 0$, $T^{ii} = \gamma^2 m n/3$ for $i = 1, 2, 3$, and $T^{ij} = 0$ for $i \neq j$. Here, m is the magnitude of the momentum of the particle in a frame in which its energy is zero, n is the (say) x-momentum flux through a (time-like) hypersurface of constant x in a frame in which all particle momenta are in the x direction. In Figure 3, the situation is viewed in a frame that is boosted from the zero-energy frame by speed v in the x direction. The above components of $T^{\mu\nu}$ are obtained in this frame. After averaging over a uniform distribution of momentum directions, and using $v < 1$, one averages over some distribution of speeds v. Thus for a perfect tachyonic fluid, with $T^{\mu\nu} = (\rho + p)u^\mu u^\nu - p g^{\mu\nu}$, we have $\rho < 3p$.

Robertson Walker Spacetime with Perfect Tachyonic Fluid

For a spatially homogeneous Robertson Walker spacetime with metric

$$ds^2 = dt^2 - R^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right) \, , \tag{5}$$

and with the conservation equation $T^{\mu\nu}_{\ \ ;\nu} = 0$ holding (only the $\mu = 0$ component is nontrivial), one gets

$$\frac{d}{dt}(\rho R^3) = -p\frac{d}{dt}(R^3) \, . \tag{6}$$

Along with this equation, the only independent component of Einstein's equation $G_{\mu\nu} = 8\pi T_{\mu\nu}$ (without cosmological term $\Lambda g_{\mu\nu}$) is the $\mu = \nu = 0$ component and is

$$\frac{3(\dot{R}^2 + k)}{R^2} = 8\pi\rho \, . \tag{7}$$

From the last two equations,

$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3}(\rho + 3p) \, , \tag{8}$$

showing that tachyonic neutrinos do not solve the Dark Energy Problem, whatever the value of the spatial scalar curvature k. Furthermore, taking the trace of Einstein's equations yields

$$\frac{(R^2)\ddot{}}{R^2} = -\frac{2k}{R^2} + \frac{4\pi}{3}(\rho - 3p) \, , \tag{9}$$

which shows that in a spatially flat Robertson Walker universe, the sign of $(R^2)\ddot{}$ mimicks that of $\rho - 3p$.

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