UNIVERSIDAD TECNICA
FEDERICO SANTA MARIA

DEPARTAMENTO DE
FÍSICA

FÍSICA COMPUTACIONAL

Homework #4

Due to January 11

Please solve the following problems and email your solutions to both Prof. Florez and Prof. Carquín. Comprehensive instructions to handle your software are expected as well as a detailed description of your programs' structure.

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Problem #1: Poisson distributions

A Poisson distribution has a mean of 3.7. Calculate "by hand" the probability that it will give two events or less. Then calculate the same result using `ppois` in R or `poisson_cdf` in ROOT, or the equivalent in your favorite math program.

Problem #2: Monte Carlo methods

Generate data using the Monte Carlo method according to an exponential distribution convolved with a Gaussian distribution.

a) Carry out the convolution analytically and plot it, using the value $\tau = 3$ and $\sigma = 2$.
b) Generate two random variables, one drawn from an exponential distribution with $\tau = 3$ and one from a Gaussian distribution with $\sigma = 2$. Make a histogram of the sum of these two variables and compare with the analytic convolution from part (a).
c) Use the rejection method to generate data distributed according to the analytic convolution of part (a).

Problem #3: Unbinned fit

Ten events of the type $e^+e^- \to \mu^+\mu^-$ are observed. The measured values of $cos\theta$ (where θ is the scattering angle) are $-0.5, -0.25, -0.1, -0.05, 0.0, 0.04, 0.11, 0.14, 0.24, 0.6$.

a) Assuming the scattering angle distribution to be $(1 + \lambda cos\theta)$, obtain λ and its error using the maximum-likelihood method.
b) Is the assumed theoretical distribution compatible with the measurement?
c) Answer the same questions when the measured values are 0.05, 0.15, 0.25, 0.35, 0,45, 0.55, 0.65, 0.75, 0.85, 0.95.

Problem #4: Extended Ising model

Create a program using C++ which calculates the specific heat and magnetization curves for an Ising model in a square optical lattice of ½ local spin sites. The program should implement a Monte Carlo method, which follows a Metropolis algorithm. In addition to the exchange interaction consider the Dzyaloshinskii-Moriya interaction. Use first nearest neighbors for your calculations and feel free to use local or cluster updates during the MC steps.

Exchange interaction: $J_{ij}\langle S_i^z S_j^z\rangle$ DM interaction: $\mathbf{D}_{ij} \cdot [\mathbf{S}_i \times \mathbf{S}_j]$

Problem #5: ALPS for Extended Ising model

Using the ALPS libraries create a simulation that can crosscheck your Problem #4' results. Plot the main thermal observables from your MC calculations vs ALPS simulations. Show that in the limit $D_{ij} = 0$, the thermal observables predict the famous Ising model phase transition at a temperature:

$$T_c = 2/\ln(1 + \sqrt{(2)})$$