Satılmış Furkan Kahraman
22002075
12/10/2023
Report 2

EE 321 Lab 2

Part 1

```
function [xs] = SUMCS(t, A, omega)

    M = length(A);
    N = length(t);
    xs = zeros(1,N);

    for i = 1:M
        xs = xs + A(i) * exp(j * omega(i) * t);
    end
end
```

SUMCS function as MATLAB code. The function is defined at first. Then, to create a for loop which is able to add "xs" values on itself, M number is defined. Then, the summation is given at lab report is applied and function is ended. Finally, folder name has changed with function name. Then, main code part is started to writing.

At this part, we define the matrices which are mentioned at the lab. The time, and the component of the summation of complex numbers. The variables are determined by the function rand(). Then, function is called with determined variables and the output is selected as real and imaginary part. Finally, the output of these process is used to draw graph. Graphs are below at the figure 1.

```
t = 0:0.001:1;
n = 40;

A = 3 * (rand(1,n) + i * rand(1,n));
omega = pi * rand(1, n);

xs = SUMCS(t, A, omega);

real_part = real(xs);
imaginary_part = imag(xs);
magnitude = abs(xs);
phase = angle(xs);

subplot(2,2,1);
plot(t,real_part);
title('Re(xs(t))');
xlabel('t');
ylabel('Real(xs)');

subplot(2,2,2);
plot(t,imaginary_part);
title('Im(xs(t))');
xlabel('t');
ylabel('Imag(xs)');
```

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```
subplot(2,2,3);
plot(t,magnitude);
title('abs(xs(t))');
xlabel('t');
ylabel('abs(xs)');

subplot(2,2,4);
plot(t,phase);
title('angle(xs(t))');
xlabel('t');
ylabel('Phase(xs)');
```

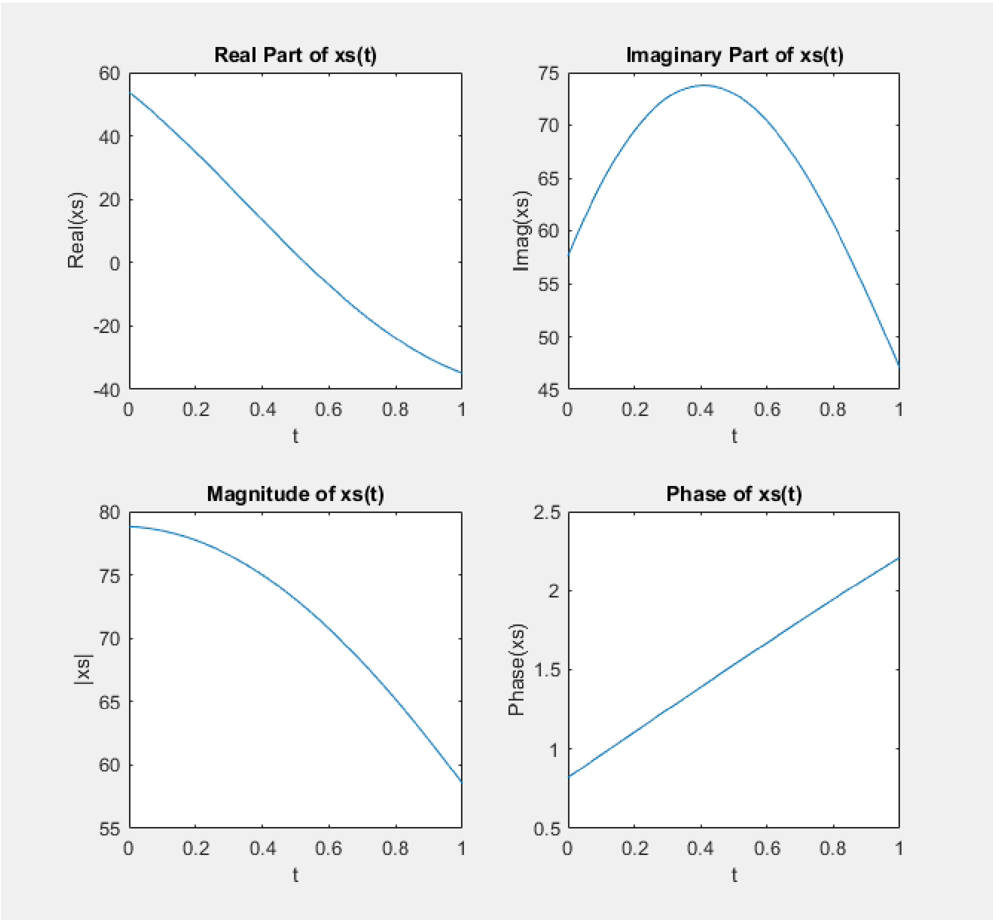

Figure 1 – Graphs

Part 2

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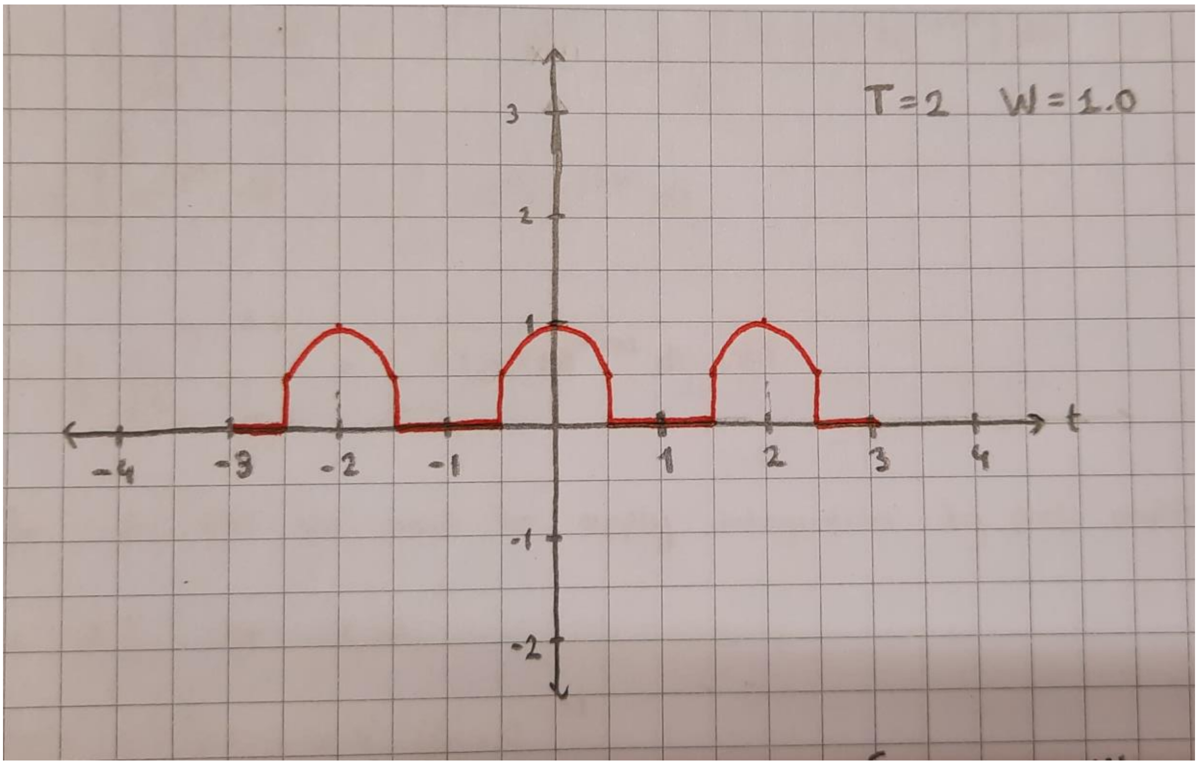

Hand drawn x(t) graph

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$$X_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\cdot e^{-j\frac{2\pi kt}{T}}\,dt \qquad x(t) = 1-2t^2 \quad \text{if } -\frac{W}{2} < t < \frac{W}{2}$$

$$X_k = \frac{1}{T}\int_{-W/2}^{W/2} (1-2t^2)\cdot e^{-j\frac{2\pi kt}{T}}\,dt$$

Integration by parts $\int u\cdot dv = u\cdot v - \int v\cdot du$

$$u = (1-2t^2) \qquad dv = e^{-j\frac{2\pi kt}{T}}\,dt \qquad du = -4t\,dt \qquad v = \left(\frac{-T}{2\pi jk}\right)\cdot e^{-j\frac{2\pi kt}{T}}$$

$$X_k = \frac{1}{T}\left[(1-2t^2)\left(\frac{-T}{2\pi jk}\right)\left(e^{-j\frac{2\pi kt}{T}}\right) - \int (-T/2\pi jk)\, e^{-j\frac{2\pi kt}{T}}\cdot(-4t)\,dt\right.$$

$$X_k = \frac{1}{T}\left[(-T/2\pi jk)\, e^{-j\frac{2\pi kt}{T}}(1-2t^2) + (2T/\pi jk)\int t\cdot e^{-j\frac{2\pi kt}{T}}\,dt\right.$$

$$\int t\cdot e^{-j\frac{2\pi kt}{T}}\,dt = \left[\left(\frac{-T}{2\pi jk}\right)\cdot t\cdot e^{-j\frac{2\pi kt}{T}} - \left(\frac{-T}{2\pi jk}\right)\cdot\int e^{-j\frac{2\pi kt}{T}}\,dt\right.$$

$$X_k = \frac{1}{T}\left[(-T/2\pi jk)\, e^{-j\frac{2\pi kt}{T}}(1-2t^2) + \left(\frac{2T}{\pi jk}\right)\left[\left(\frac{-T}{2\pi jk}\right)\cdot t\cdot e^{-j\frac{2\pi kt}{T}} + \frac{T^2}{4\pi^2k^2}\, e^{-j\frac{2\pi kt}{T}}\right.\right.$$

$$X_k = \left[(-1/2\pi jk)\cdot e^{-j\frac{2\pi kt}{T}}(1-2t^2) + e^{-j\frac{2\pi kt}{T}}\cdot\left(\frac{T}{4\pi^2k^2}\right) - \left(\frac{T}{2\pi jk}\right)\cdot t\cdot e^{-j\frac{2\pi kt}{T}}\right]$$

limits are substituted $-W/2$ to $W/2$ (-0.5 to 0.5)

$$X_k = \left[(-1/2jk\pi)\cdot e^{-j\pi k}\cdot(1-2(0.5)^2) + (T/4\pi^2k^2)\cdot e^{-j\pi k} - (T/2jk\pi)\cdot(0.5)\cdot e^{-j\pi k}\right] - \cdots$$

$$\cdots \left[(-1/2jk\pi)\, e^{-j\pi k}\cdot(1-2(-0.5)^2) + (T/4\pi^2k^2)\cdot e^{j\pi k} - (T/2jk\pi)(-0.5)\cdot e^{j\pi k}\right]$$

$$X_k = \left[(-1/2jk\pi)\, e^{-j\pi k}(1-0.5) + (T/4\pi^2k^2)\, e^{-j\pi k} - (T/4jk\pi)\right] - \left[(-1/2j\pi k)\, e^{-j\pi k}(1+0.5) + \right.$$

$$\left.(T/4\pi^2k^2)\, e^{-j\pi k} + T/4jk\pi\right]$$

$$X_k = (-1/4j\pi k)\cdot e^{-j\pi k} - (T/2kj\pi)(1+0.5)\cdot e^{-j\pi k} - (T/2jk\pi)(1-0.5)\, e^{-j\pi k}$$

Fourier Series Expansion Coefficient calculation Part 1

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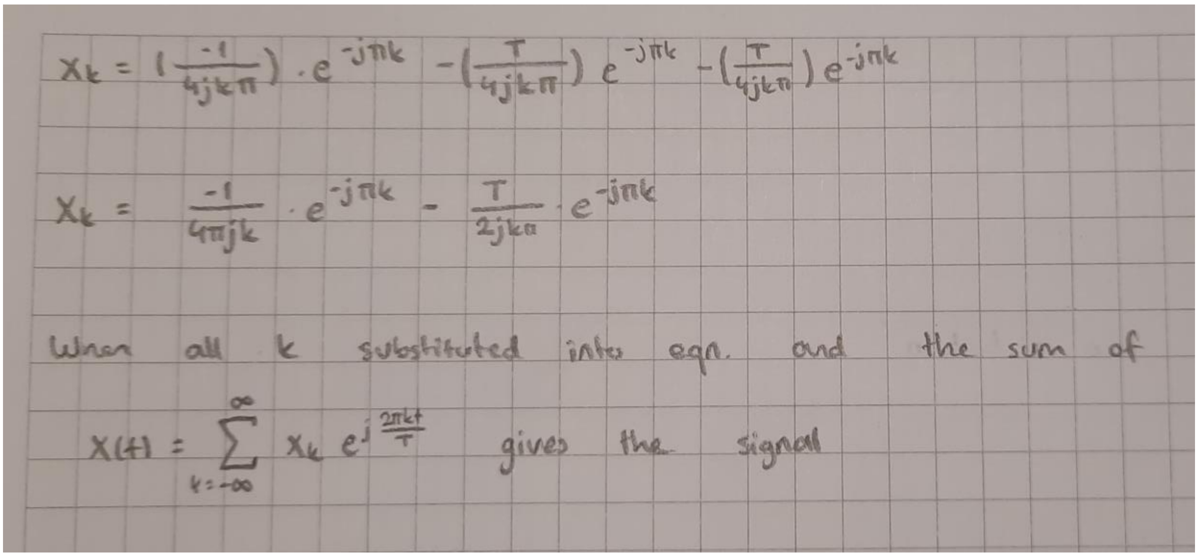

$$X_k = \left(\frac{-1}{4jk\pi}\right) \cdot e^{-j\pi k} - \left(\frac{T}{4jk\pi}\right) e^{-j\pi k} - \left(\frac{T}{4jk\pi}\right) e^{-j\pi k}$$

$$X_k = \frac{-1}{4\pi jk} \cdot e^{-j\pi k} - \frac{T}{2jk\pi} \cdot e^{-j\pi k}$$

When all k substituted into eqn. and the sum of

$$x(t) = \sum_{k=-\infty}^{\infty} X_k \, e^{j\frac{2\pi k t}{T}}$$ gives the signal

Fourier Series Expansion Coefficient calculation Part 2

Part 3

```
function [xt] = FSWave(t, K, T, W)

    xt = zeros(size(t))

    for k = -K:K
        Xk = (1/T) * integral(@(x) (1 - 2*x.^2) .* exp(-j * 2 * pi * k * x /T), -
W/2, W/2);
        xt = xt + Xk* (exp(j * 2 * pi * k * t / T));
```

```
    end
end
```

At this code, at first coefficients are determined by the integral part. Then, with using the determined coefficients Fourier series expansion is used and the signal is constituted.

```
t = -5:0.001:5;
T = 2;
W = 1;
K = 20 + 7;

xt = FSWave(t, K, T, W)

plot(t, xt)
```

At this part of the code, all variables are given and function is called. With this parameters, the function graph is shown at the figure 2.

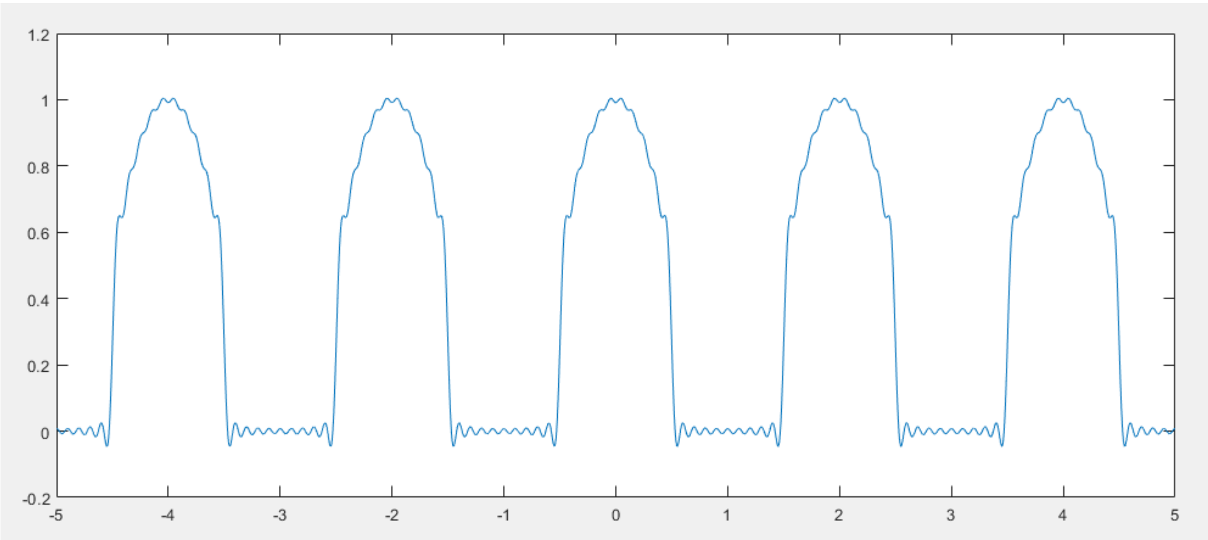

Figure 2 – x(t) periodic signal

```
imagxt = imag(xt)

plot(t, imagxt)
```

At this code, imaginary part of the signal is extracted and plotted. The imaginary part of the signal is so weak. The graph is shown at the figure 3.

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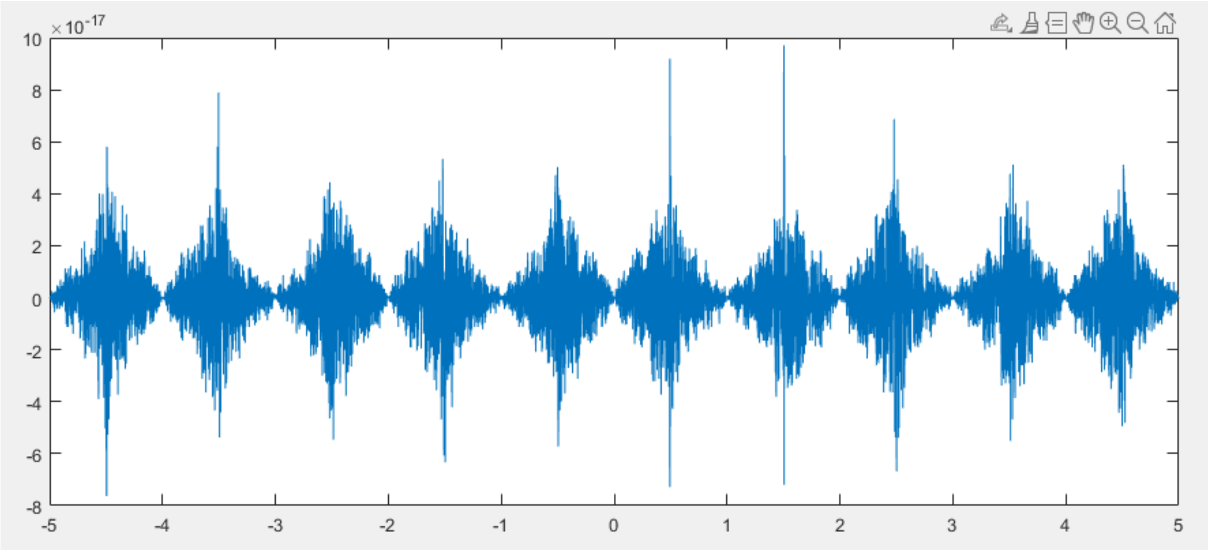

Figure 3 – Imaginary part of the signal

The imaginary part should be zero but MATLAB is using very good approximation for trigonometric functions like Taylor series expansion so the result cannot be precise but it is so close to precise value. At the next part of the Part 3, K values are changed and the graphs change according to changing parameter.

```
t = -5:0.001:5;
T = 2;
W = 1;
K1 = 27;
K2 = 7+0;
K3 = 15+0;
K4 = 50+0;
K5 = 100+0;

xt1 = FSWave(t,K1,T,W);
plot(t,xt) with K1
subplot(3,2,1);
plot(t,xt1);
title('x(t) with K=2');
xlabel('t');
ylabel('Real(xt)');

xt2 = FSWave(t,K2,T,W);
% plot(t,xt) with K2
subplot(3,2,2);
plot(t,xt2);
title('x(t) with K=7');
xlabel('t');
ylabel('Real(xt)');

xt3 = FSWave(t,K3,T,W);
% plot(t,xt) with K3
subplot(3,2,3);
plot(t,xt3);
```

```
title('x(t) with K=15');
xlabel('t');
ylabel('Real(xt)');

xt4 = FSWave(t,K4,T,W);
% plot(t,xt) with K4
subplot(3,2,4);
plot(t,xt4);
title('x(t) with K=50');
xlabel('t');
ylabel('Real(xt)');

xt5 = FSWave(t,K5,T,W);
% plot(t,xt) with K5
subplot(3,2,5);
plot(t,xt5);
title('x(t) with K=100');
xlabel('t');
ylabel('Real(xt)');
```

The graphs are at the figure 4-8.

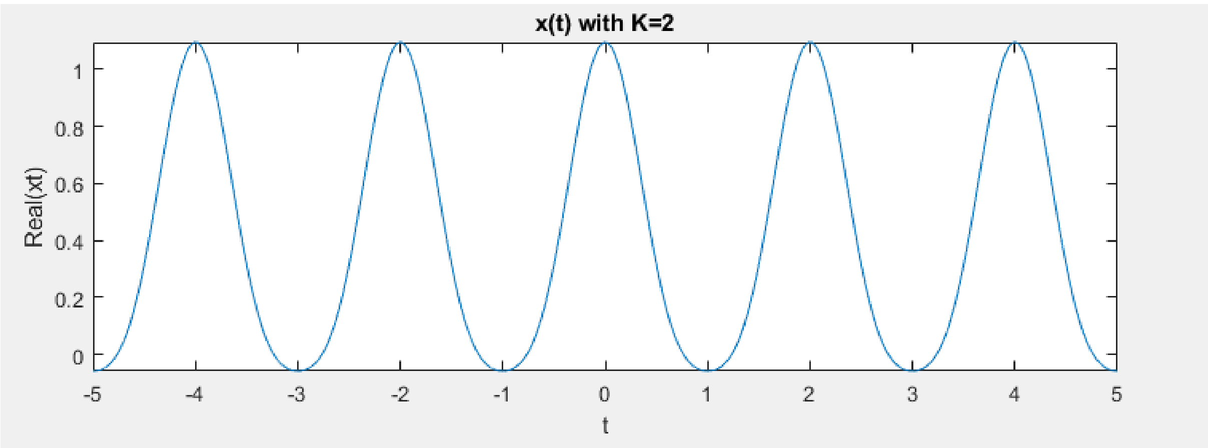

Figure 4 – K=2 graph

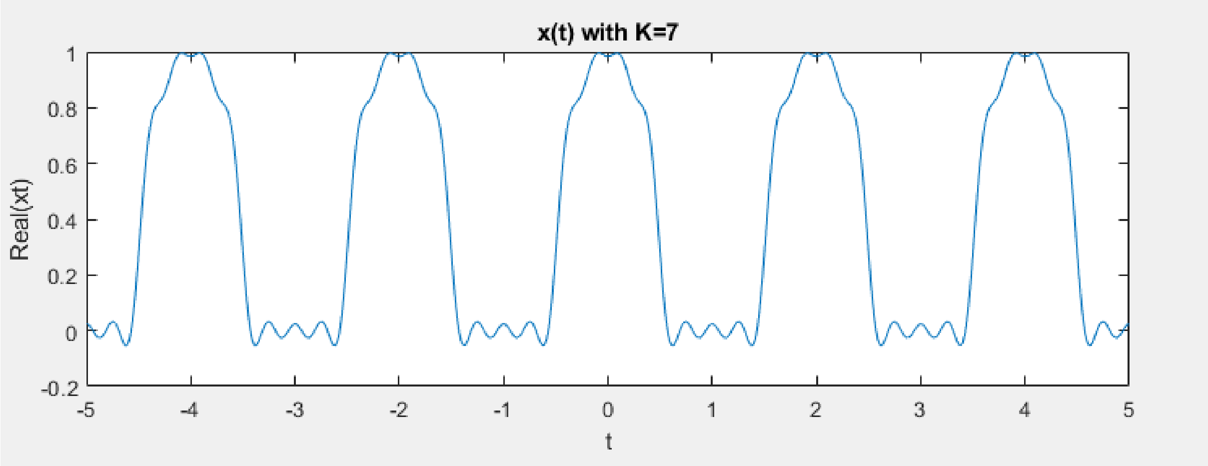

Figure 5 – K=7 graph

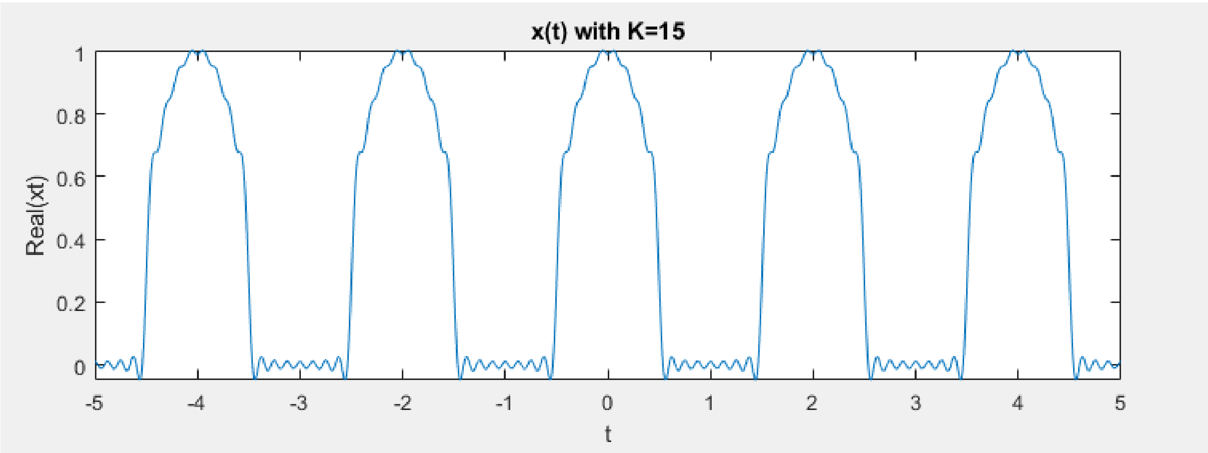

Figure 6 - K=15 graph

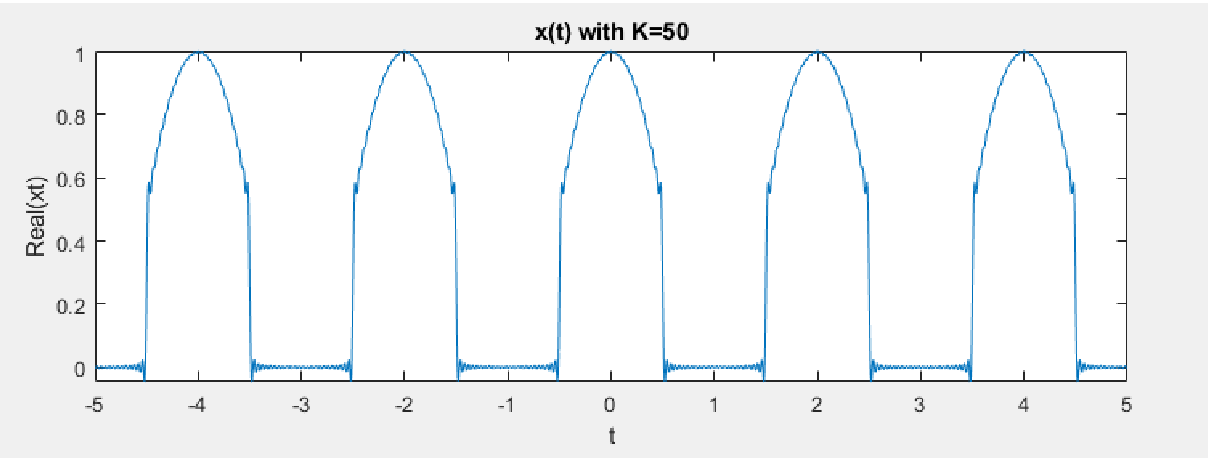

Figure 7 – K = 50 graph

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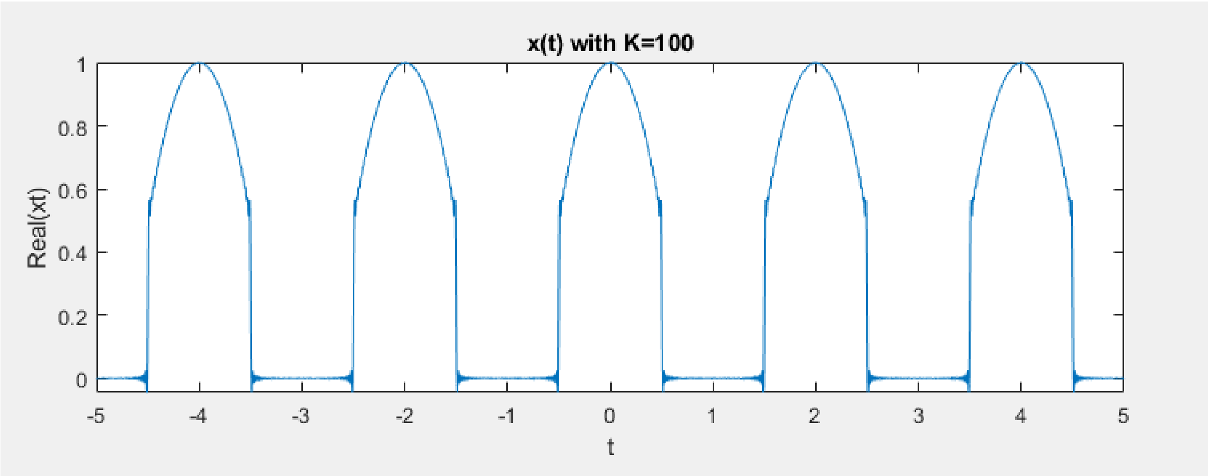

Figure 8 – K = 100 graph

As K is increasing, the processed point on the integral is increasing too and the result is getting closer to the real signal. I observe some irregularities and oscillations within the neighborhood of the discontinuities.

Part 4

a)

```
function [yt] = FSWavePart4(t,K,T,W);

yt = zeros(size(t))

    for k = -K:K
        Xk = (1/T) * integral(@(x) (1 - 2*x.^2) .* exp(-j * 2 * pi * -k * x / T),
-W/2, W/2);
        xt = xt + Xk * exp(j * 2 * pi * k * t / T);
    end
end
```

At this part, k is substituted with its negative magnitude -k. It takes the symmetric of the signal according to x = 0 plane. The result is shown at the figure 9.

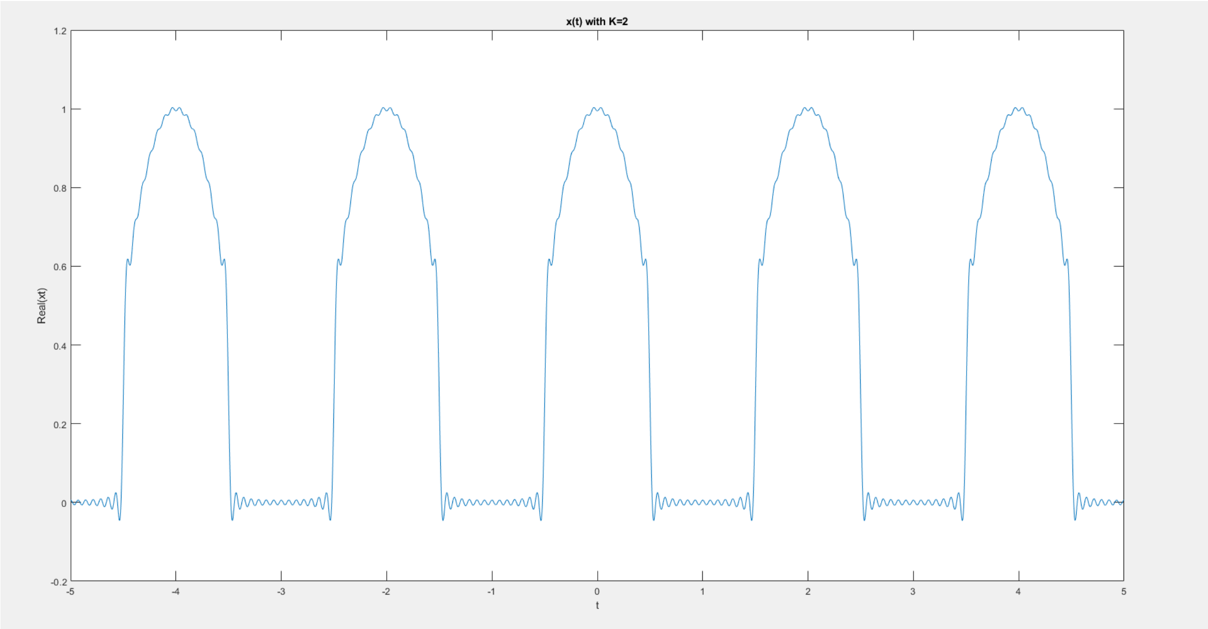

Figure 9 – K =27 and -k signal

b)

At this part, the function has been added to a new multiplier.

```
function [yt] = FSWavePart4(t,K,T,W);

yt = zeros(size(t))

    for k = -K:K
        Xk = (1/T) * integral(@(x) (1 - 2*x.^2) .* exp(-j * 2 * pi * k * x / T), -
W/2, W/2);
        xt = xt + Xk * exp(-j * 2 * pi * k * (0.6) / T) * exp(j * 2 * pi * k * t /
T);
    end
end
```

With this extra multiplier the signal shifts. The new form of the signal is shown at the figure 10.

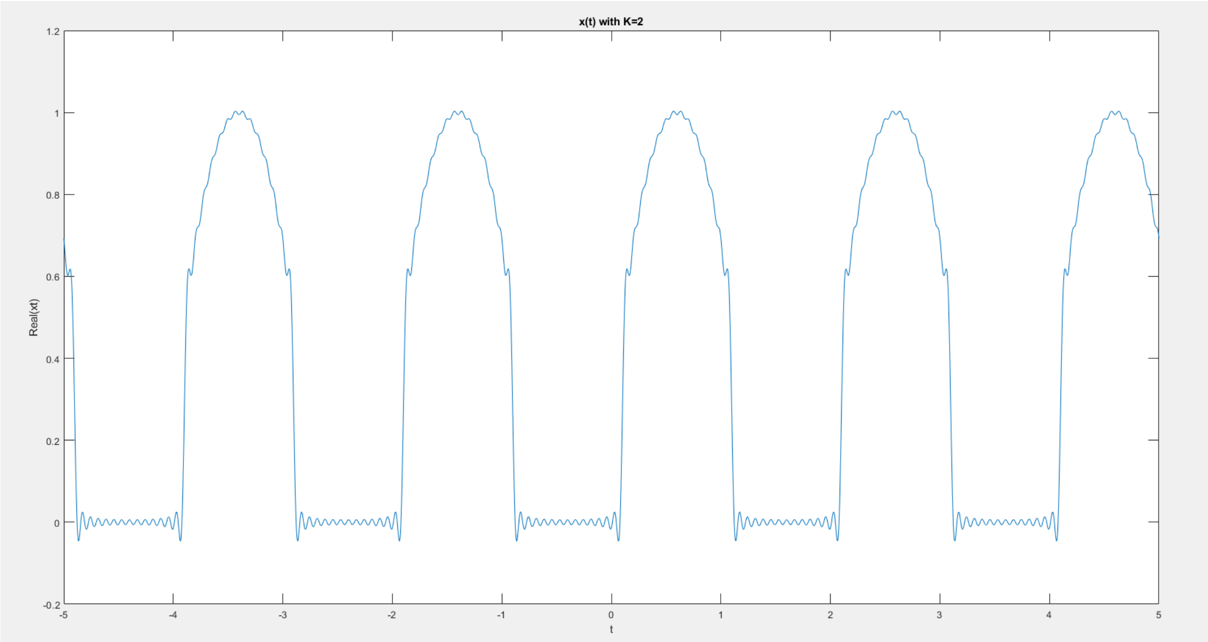

Figure 10 – New graph

When new t0 added to the Fourier series expansion formula the time parameters are added to each other then, the time shifts to the left.

c)

```
function [yt] = FSWavePart4(t,K,T,W);

yt = zeros(size(t))

    for k = -K:K
        Xk = (1/T) * integral(@(x) (1 - 2*x.^2) .* exp(-j * 2 * pi * k * x / T), -
W/2, W/2);
        xt = xt + Xk * (-j * 2 * pi * k / T) * exp(j * 2 * pi * k * t / T);
    end
end
```

At this part, the derivative of the signal is taken so the result show that the derivative of our main signal. The result is given at the figure 11.

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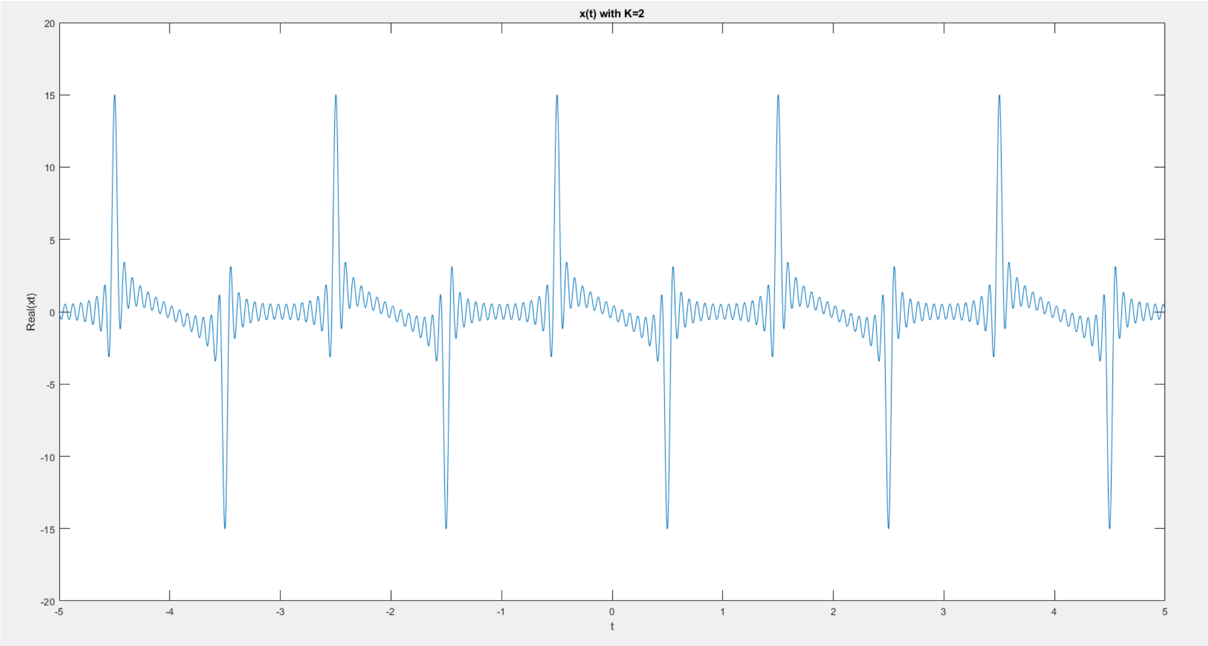

Figure 11 – The derivative of the signal

d)

```
function [xt] = FSWave1(t, K, T, W)

    xt = zeros(1,10001);

    for k = -K:K

        Xk = (1/T) * integral(@(x) (1 - 2*x.^2) .* exp(-j * 2 * pi * k * x / T), -
W/2, W/2);

        if k > 0
            xt = xt + Xk * exp(j * 2 * pi * (K + 1 - k)  * t / T);

        elseif k == 0
            xt = xt + Xk * exp(j * 2 * pi * k * t / T);

        elseif k < 0
            xt = xt + Xk * exp(j * 2 * pi * (-K - 1 - k) * t / T);

        end
    end
end
```

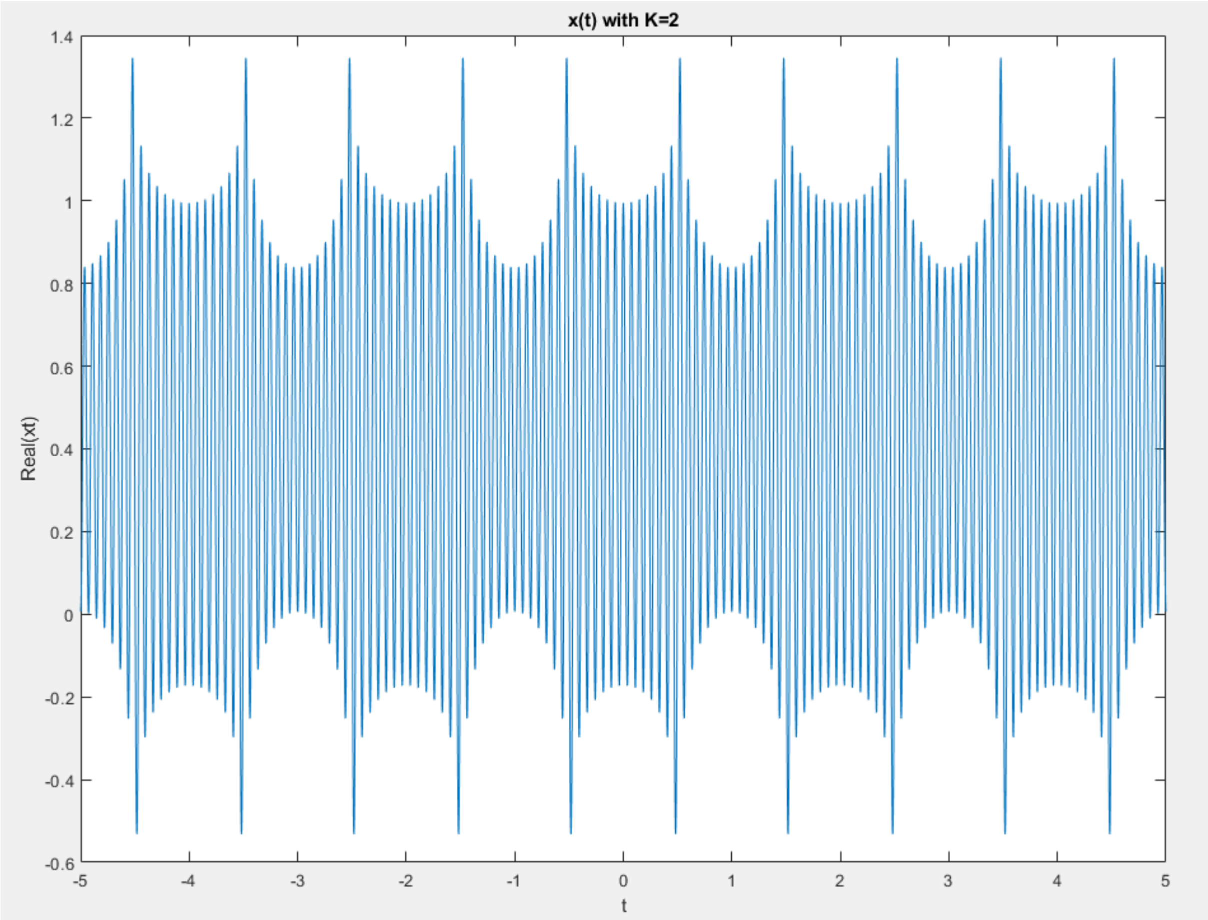

Figure 12 – Signal graph

At this part, the sinusoidal waves and their coefficients are mixed. Coefficients are not matched with true frequency. So, there is a different signal.