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22002075
15/12/2023
Report 6

EE 321 Lab 6

Part 1

Part 1

$$(1)\quad y[n] = \sum_{\ell=1}^{N} a[\ell]\, y[n-\ell] + \sum_{k=0}^{M} b[k]\, x[n-k] \qquad x[n]=0 \quad n<0 \;,\quad y[n]=0 \quad n<0$$

$$\Rightarrow y[0] = \sum_{\ell=1}^{N} a[\ell]\, y[0-\ell] + \sum_{k=0}^{M} b[k].\, x[0-k]$$

$$y[0] = \Big(a[1].\underbrace{y[-1]}_{0} + a[2]\underbrace{y[-2]}_{0} + \dots\Big) + \Big(b[0].x[0] + b[1].\underbrace{x[-1]}_{0} + b[2].\underbrace{x[-2]}_{0} + \dots\Big)$$

$$\underline{y[0] = b[0].x[0]}$$

$$\Rightarrow y[1] = \sum_{\ell=1}^{N} a[\ell]\, y[1-\ell] + \sum_{k=0}^{M} b[k]\, x[1-k]$$

$$y[1] = \Big(a[1].y[0] + a[2]\underbrace{y[-1]}_{0} + \dots\Big) + \Big(b[0].x[1] + b[1].x[0] + b[2].\underbrace{x[-1]}_{0}\dots\Big)$$

$$y[1] = a[1].y[0] + b[0].x[1] + b[1]x[0] = \underline{a[1].b[0].x[0] + b[0].x[1] + b[1]x[0]}$$

z - transform by using (1)

$$y[n] = \sum_{\ell=1}^{N} a[\ell]\, y[n-\ell] + \sum_{k=0}^{M} b[k].\, x[n-k]$$

$$\Downarrow$$

$$Y(z) = \sum_{\ell=1}^{N} a[\ell].\, z^{-\ell}.\, Y(z) + \sum_{k=0}^{M} b[k]\, z^{-k}\, X(z) \quad \Rightarrow \quad Y(z)\Big[1 - \sum_{\ell=1}^{N} a[\ell]\, z^{-\ell}\Big] = X(z)\Big[\sum_{k=0}^{M} b[k]\, z^{-k}\Big]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b[k]\, z^{-k}}{1 - \sum_{\ell=1}^{N} a[\ell]\, z^{-\ell}} = \frac{\sum_{p=0}^{P} c_n[p]\, z^{-p}}{\sum_{q=0}^{Q} c_d[p]\, z^{-q}} \Rightarrow \begin{array}{l} c_n[p] = b[k] \qquad -k=-p \qquad P=M \\ a[0] = \frac{1}{N} \;,\; a[\ell] = c_d[p] \\ -q = -\ell \qquad Q=N \end{array}$$

Figure 1 – Calculations

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MatLab Codes

DTLTI function

```
function [y] = DTLTI(a,b,x,Ny)

y = zeros(1,Ny);
    for n = 1:size(y,2)

        for k = 1:min(n,size(b,2))
            y(n) = y(n) + x(n-k+1)*b(k);
        end

        for l = 1:min(n,size(a,2))
            y(n) = y(n) + y(n-l+1)*a(l);
        end

    end
end
```

Part 2

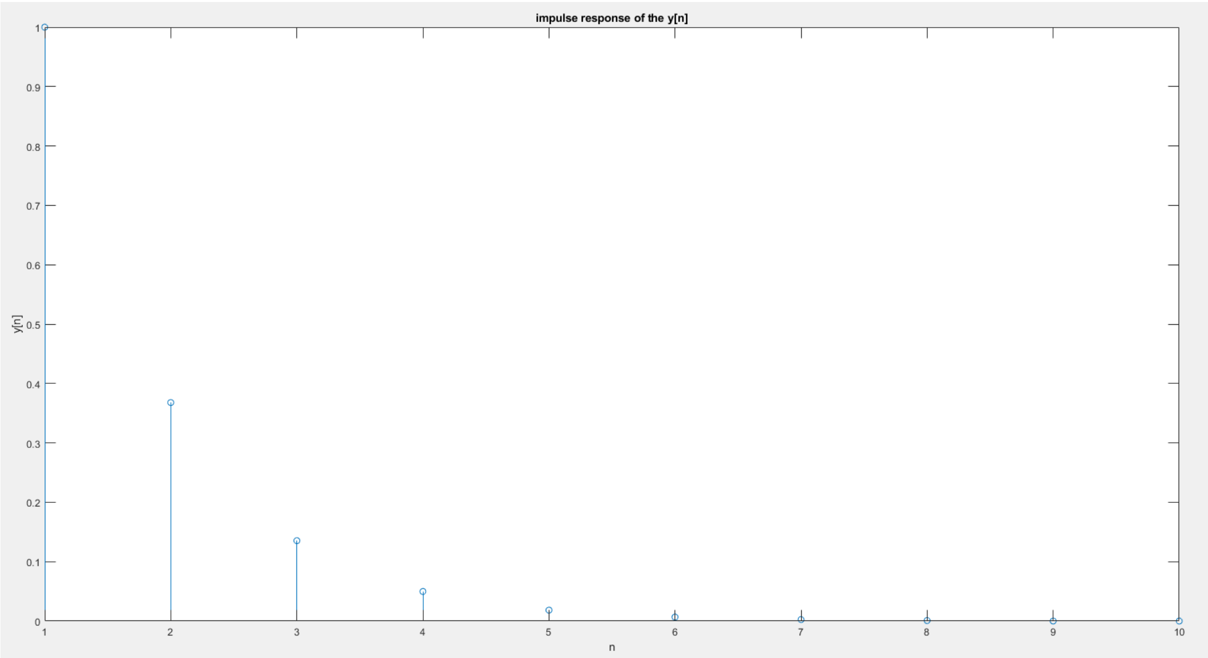

Figure 2 – Impulse response of y[n]

This graph shows that the impulse response of the y[n] when b(k) = exp(-k) and a is all zeros.

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a)

```
ID = 22002075;
D4 = mod(ID,4);
M = 5 + D4;
Ny = 10;
n = 1:1:Ny;
k = 0:1:M-1;

%%% coefficients
a = zeros(1,Ny);
b = exp(-k);

%%% defining impulse function x[n] = delta(n)
x = zeros(1,11);
x(1) = 1;

%%% calling DTLTI
y = DTLTI(a,b,x,Ny)

stem(n,y)
title("impulse response of the y[n]")
xlabel("n")
ylabel("y[n]")
```

b)

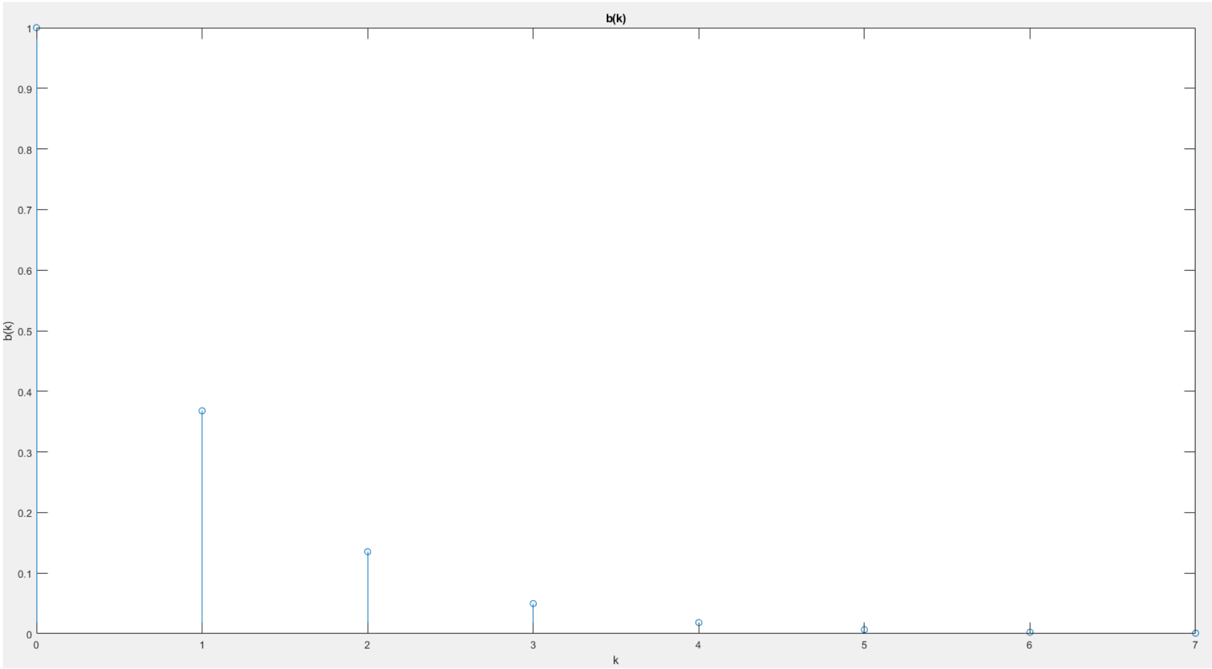

Figure 3 – values of b(k)

It is seemed that b(k) and y[n] (when x[n] is impulse) have the same values. So, it shows that b(k) is the only coefficient which effect the y[n].

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c)

The system has finite impulse response so the system is FIR. The magnitude of the impulse response is directly related with the magnitude of the coefficient b(k). Until k = M-1 there is an impulse response after that value impulse response will be equal to zero.

d)

$$Y[n] = \sum_{k=0}^{M-1} b[k] . x[n-k] \qquad M = 22002075 \bmod 4 + 5 = 8 \qquad M-1 = 7$$

$$Y[n] = \sum_{k=0}^{7} b[k] \, x[n-k] \quad \xleftrightarrow{z} \quad Y(z) = \sum_{k=0}^{7} e^{-k} X(z) z^{-k}$$

$$X(z) \cdot \sum_{k=0}^{7} e^{-k} z^{-k} = X(z) \sum_{k=0}^{7} (e.z)^{-k}$$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{7} (e.z)^{-k} = \frac{1 - (ez)^{-7}}{1 - (ez)^{-1}}$$

$$H(e^{jw}) = \frac{1 - (e.e^{jw})^{-7}}{1 - (e.e^{jw})^{-1}} = \frac{1 - e^{(jw+1)(-8)}}{1 - e^{(jw+1)(-1)}}$$

Figure 4 - Calculations

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e)

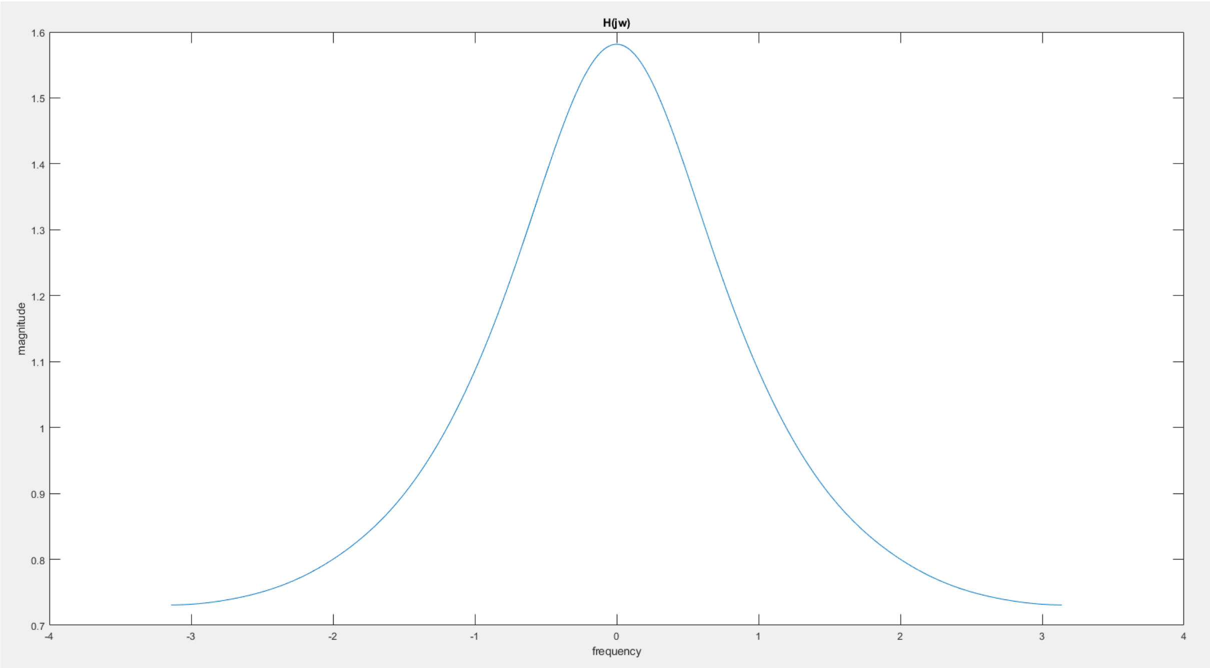

Figure 5 – Plot of the DTFT of the transfer function

The system is low-pass filter because it passes the low content frequencies. The 3 dB means that the filter has not reduced the signal so much and the 3 dB points are called cut-off frequency. Before or beyond this frequency the filter reduced the value of the signal so much. The cut-off frequency of the system is 0.29π.

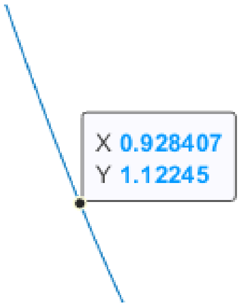

Figure 6 – Y =1.12

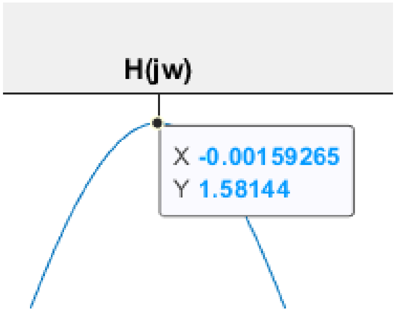

Figure 7 – Y=1.58

The band width of the system is 0.58π.

f)

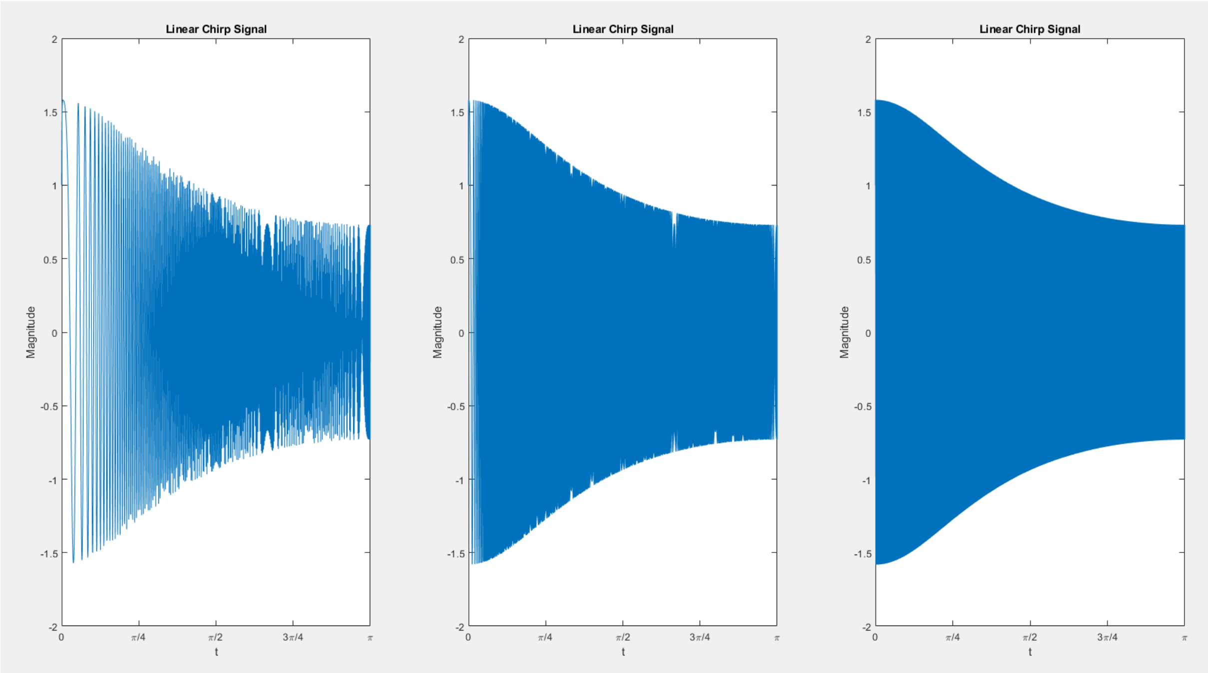

Figure 8 – Output when x(t) = cos(2πϕ(t))

MatLab Codes

```
f0 = 0;
f1 = 700;
Fs = 1400;

t1 = 0:1/Fs:1;
t2 = 0:1/Fs:10;
t3 = 0:1/Fs:1000;

T1 = 1;
T2 = 10;
T3 = 1000;

k1 = (f1 - f0)/T1;
k2 = (f1 - f0)/T2;
k3 = (f1 - f0)/T3;

x1 = cos(2 * pi * (0.5 * k1 * t1.^2));
x2 = cos(2 * pi * (0.5 * k2 * t2.^2));
x3 = cos(2 * pi * (0.5 * k3 * t3.^2));

N1 = length(x1);
N2 = length(x2);
N3 = length(x3);
```

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```
ID = 22002075;
D4 = mod(ID,4);
Ny = 8;
M = 5 + D4;

k = 0:M-1;
a = zeros(1,Ny);
b = exp(-k);

y1 = DTLTI(a,b,x1,N1);
y2 = DTLTI(a,b,x2,N2);
y3 = DTLTI(a,b,x3,N3);

subplot(1,3,1);
plot(t1,y1);
title('Signal');
xlabel('t');
ylabel('Magnitude');
xticks([0 0.25 0.5 0.75 1]);
xticklabels({'0', '\pi/4', '\pi/2', '3\pi/4', '\pi'});
xlim([0, 1]);


subplot(1,3,2);
plot(t2,y2);
title('Signal');
xlabel('t');
ylabel('Magnitude');
xticks([0 2.5 5 7.5 10]);
xticklabels({'0', '\pi/4', '\pi/2', '3\pi/4', '\pi'});
xlim([0, 10]);


subplot(1,3,3);
plot(t3,y3);
title('Signal');
xlabel('t');
ylabel('Magnitude');
xticks([0 250 500 750 1000]);
xticklabels({'0', '\pi/4', '\pi/2', '3\pi/4', '\pi'});
xlim([0, 1000]);
```

f)

There is no sudden jump at the output. The general trend of the signal is decreasing the amplitude when frequency increase. The input signal's frequency is increasing as time move forward. So as the frequency increase, the filter will not pass the high frequency content because of the system is low pass filter.

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Part 3

a,b,c)

a) $$H(z) = \frac{(z - z_1)}{(z-p_1)(z-p_2)} = \frac{z}{z^2}\frac{(1 - z_1 . z^{-1})}{(1-p_1 . z^{-1})(1-p_2 z^{-1})} = \frac{z^{-1} - z_1 z^{-2}}{(1-p_1 z^{-1})(1-p_2 z^{-1})}$$

$$= \frac{z^{-1} - \left(\frac{4+2j}{\sqrt{20}}\right) z^{-2}}{\left(1 - \frac{4+4j}{\sqrt{33}} z^{-1}\right)\left(1 - \frac{5+2j}{\sqrt{26}} z^{-1}\right)}$$

b) $$X(z)\left(z^{-1} - z_1 z^{-2}\right) = Y(z)\left(1 - p_1 z^{-1}\right)\left(1 - p_2 z^{-1}\right)$$

$$x[n-1] - x[n-2].z_1 = Y(z)\left(1 - p_1 z^{-1} - p_2 z^{-1} + p_1 p_2 z^{-2}\right)$$

$$x[n-1] - x[n-2].z_1 = y[n] - y[n-1].p_1 - y[n-1].p_2 + p_1 p_2 \, y[n-2]$$

$$y[n] = y[n-1](p_1 + p_2) - (p_1 . p_2)\, y[n-2] + x[n-1] - x[n-2] z_1$$

$$y[n] = \sum_{\ell=1}^{2} a[\ell]\, y[n-\ell] + \sum_{k=1}^{2} b[k]\, x[n-k]$$

$b[1] = 1 \qquad b[2] = -z_1$

$a[1] = p_1 + p_2 \qquad a[2] = -1(p_1 . p_2)$

c) $$H(z) = \frac{z^{-1}(1 - z_1 z^{-1})}{(1-p_1 z^{-1})(1-p_2 z^{-1})} = \frac{A}{(1-p_1 z^{-1})} + \frac{B}{(1-p_2 z^{-1})}$$

$$A(1 - p_2 z^{-1}) + B(1 - p_1 z^{-1}) = (1 - z_1 z^{-1})$$

$A + B = 1 \qquad -Ap_2 - Bp_1 = -z_1 \Rightarrow Ap_2 + Bp_1 = z_1$

$$A = H(z)(1 - p_1 z^{-1})\Big|_{z=p_1} = \frac{p_1^{-1} - z_1 p_1^{-2}}{1 - p_2 p_1^{-1}} \qquad B = H(z)(1 - p_2 z^{-1})\Big|_{z=p_2} = \frac{p_2^{-1} - z p_2^{-2}}{1 - p_1 p_2^{-1}}$$

$$H(z) = \frac{p_1^{-1} - z_1 p_1^{2}}{(1 - p_2 p_1^{-1})} \cdot \frac{1}{(1 - p_1 z^{-1})} + \frac{p_2^{-1} - z_1 p_2^{-2}}{1 - p_1 p_2^{-1}} \cdot \frac{1}{(1 - p_2 z^{-1})}$$

$$h[n] = \frac{p_1^{-1} - z_1 p_1^{-2}}{(1 - p_2 p_1^{-1})} \cdot p_1^{n} \cdot u[n] + \frac{p_2^{-1} - z_1 p_2^{-2}}{1 - p_1 p_2^{-1}} \cdot p_2^{n} \cdot u[n]$$

$$h[n] = \frac{\left(\frac{4+4j}{\sqrt{33}}\right)^{-1} - \left(\frac{4+2j}{\sqrt{20}}\right)\left(\frac{4+4j}{\sqrt{33}}\right)^{-2}}{\left(1 - \frac{4+4j}{\sqrt{33}} \cdot \left(\frac{5+2j}{\sqrt{26}}\right)^{-1}\right)} \cdot \left(\frac{4+4j}{\sqrt{37}}\right)^{n} \cdot u[n] + \frac{\left(\frac{5+2j}{\sqrt{26}}\right)^{-1} - \left(\frac{4+2j}{\sqrt{20}}\right)\left(\frac{5+2j}{\sqrt{26}}\right)^{-2}}{\left(1 - \left(\frac{4+4j}{\sqrt{33}}\right)^{-1}\left(\frac{5+2j}{\sqrt{26}}\right)\right)} \cdot \left(\frac{5+2j}{\sqrt{26}}\right)^{n} \cdot u[n]$$

Figure 9 - Calculations

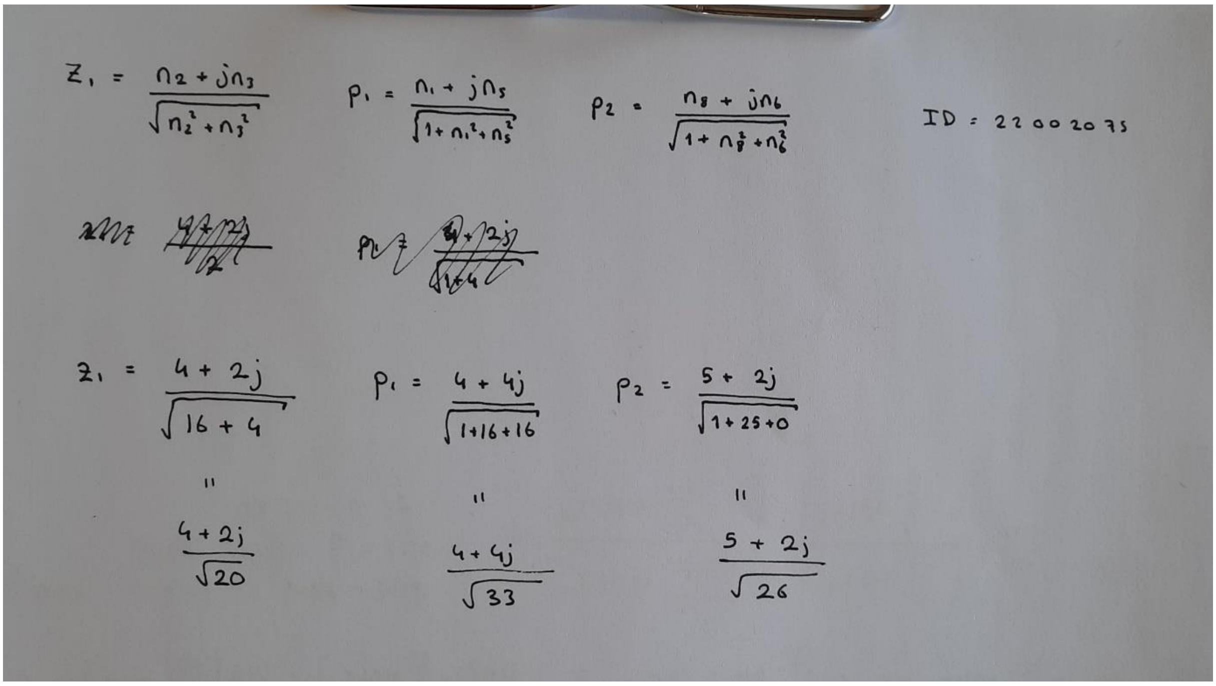

Figure 10 – Zero and poles values

d)

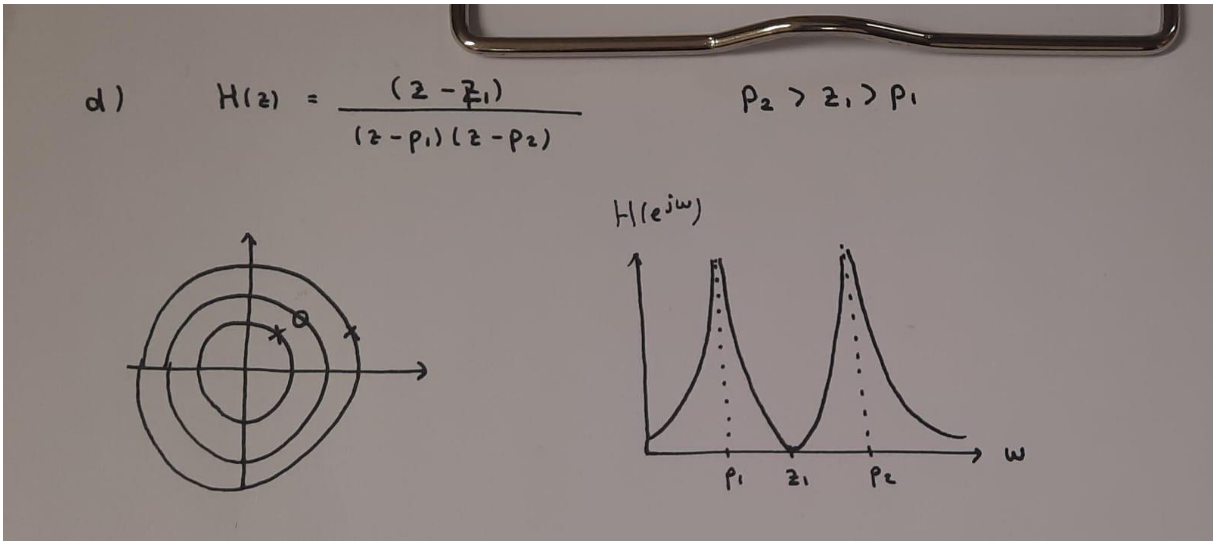

Figure 11 – Approximation of H(e^jw) graph

e)

The system is stable because unit circle in the region of the converges area. If ROC contains unit circle, it means that DTFT exist because z-transform evaluated on unit circle.

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f)

This filter is IIR because there are 2 poles in the transfer function. To obtain FIR system transfer function should contain only zeros.

g)

The system filters as band-pass filters. It passes the frequencies which is in the under the peak part.

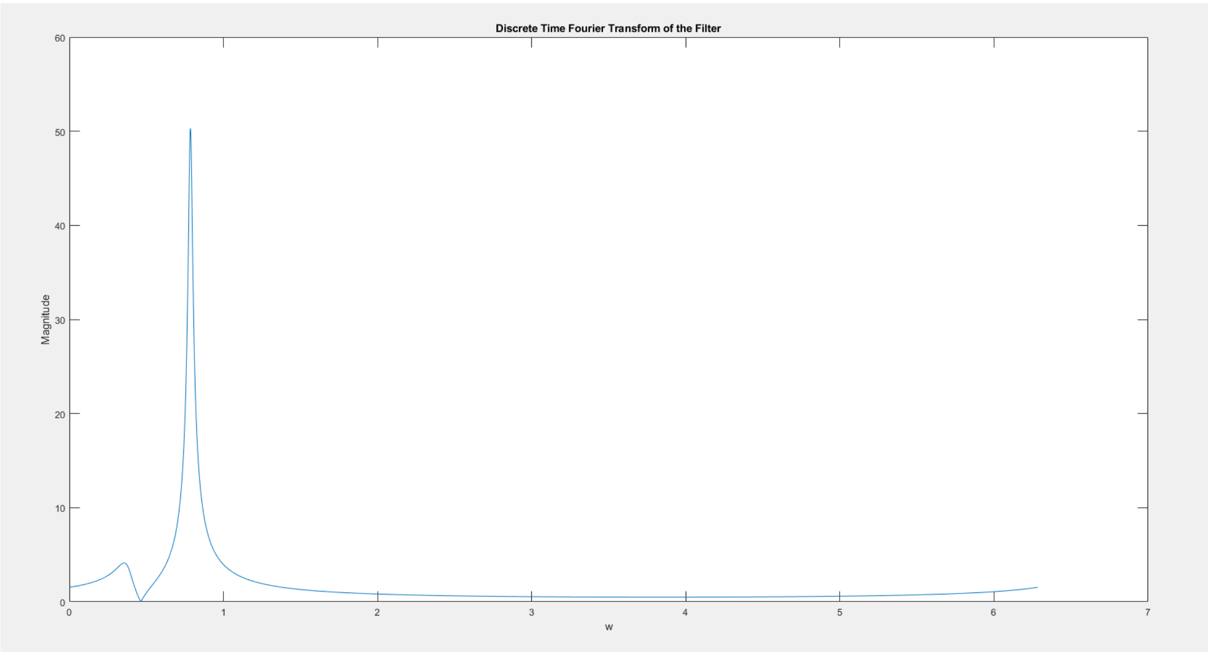

Figure 12 - DTFT of the transfer function

h)

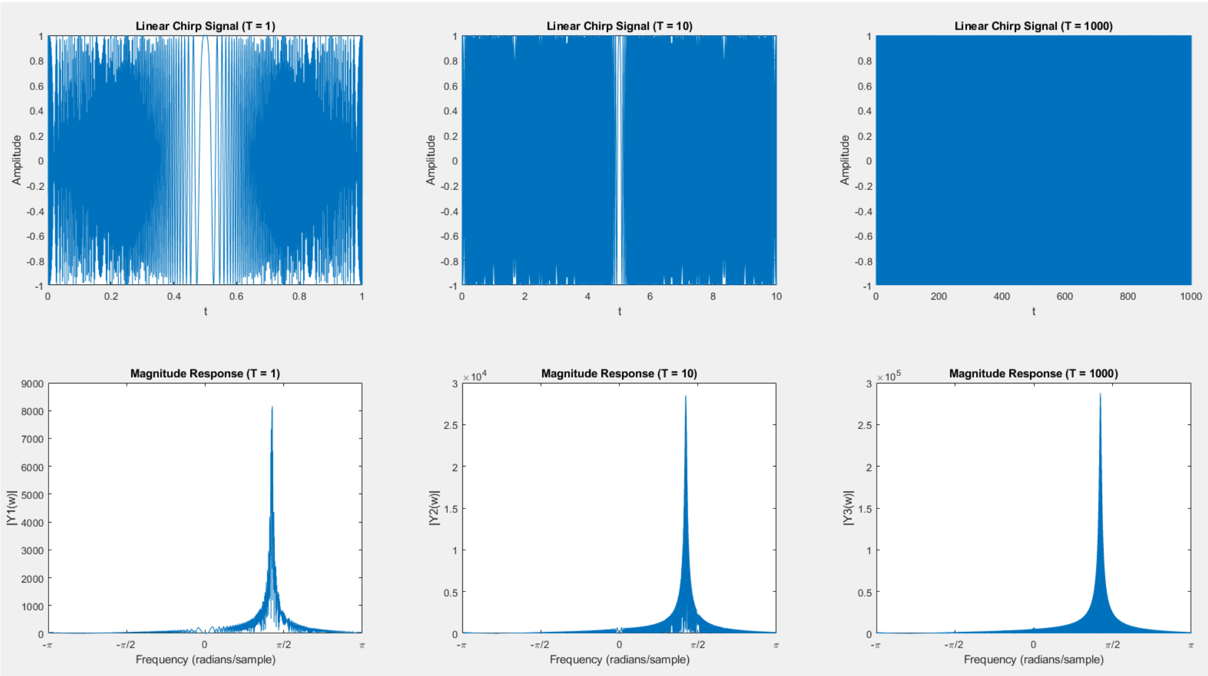

Figure 13 – (-700 Hz– 700 Hz) response

The magnitude response of the system is not symmetric. It does not respond the negative frequencies.

At chirping from -600 Hz to 800 Hz, the sampling frequency was 1400 Hz so by Shannon-Nyquist sampling theorem, there will be a loss of information.

MatLab Code

```
f0 = -700;
f1 = 700;

T1 = 1;
T2 = 10;
T3 = 1000;
Fs = 1400;

k1 = (f1 - f0) / T1;
k2 = (f1 - f0) / T2;
k3 = (f1 - f0) / T3;

t1 = 0:1/Fs:T1;
t2 = 0:1/Fs:T2;
t3 = 0:1/Fs:T3;
```

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```
x1 = exp(1j * 2 * pi * (f0 * t1 + 0.5 * k1 * t1.^2));
x2 = exp(1j * 2 * pi * (f0 * t2 + 0.5 * k2 * t2.^2));
x3 = exp(1j * 2 * pi * (f0 * t3 + 0.5 * k3 * t3.^2));


zero1 = (4 + 2*1i) / sqrt(20);
pole1 = (4 + 4*1i) / sqrt(33);
pole2 = (5 + 2*1i) / sqrt(26);

a = [pole1+pole2,-pole1*pole2];
b = [1,-zero1];

L1 = length(x1);
L2 = length(x2);
L3 = length(x3);

y1 = DTLTI(a, b, real(x1), L1);
y2 = DTLTI(a, b, real(x2), L2);
y3 = DTLTI(a, b, real(x3), L3);


subplot(2,3,1);
plot(t1, abs(x1));
title('Signal, T = 1');
xlabel('t');
ylabel('Amplitude');
xlim([0, 1]);

subplot(2,3,2);
plot(t2, abs(x2));
title('Signal, T = 10');
xlabel('t');
ylabel('Amplitude');
xlim([0, 10]);

subplot(2,3,3);
plot(t3, abs(x3));
title('Signal, T = 1000');
xlabel('t');
ylabel('Amplitude');
xlim([0, 1000]);

freq1 = linspace(-pi, pi, L1);
freq2 = linspace(-pi, pi, L2);
freq3 = linspace(-pi, pi, L3);

subplot(2,3,4);
plot(freq1, abs(fft(y1)));
title('Magnitude Response, T = 1');
xlabel('Frequency');
ylabel('Y1(jw)');
xticks([-pi -pi/2 0 pi/2 pi]);
xticklabels({'-\pi', '-\pi/2', '0', '\pi/2', '\pi'});
xlim([-pi, pi]);
```

```
subplot(2,3,5);
plot(freq2, abs(fft(y2)));
title('Magnitude Response, T = 10');
xlabel('Frequency');
ylabel('Y2(jw)');
xticks([-pi -pi/2 0 pi/2 pi]);
xticklabels({'-\pi', '-\pi/2', '0', '\pi/2', '\pi'});
xlim([-pi, pi]);


subplot(2,3,6);
plot(freq3, abs(fft(y3)));
title('Magnitude Response, T = 1000');
xlabel('Frequency');
ylabel('Y3(jw)');
xticks([-pi -pi/2 0 pi/2 pi]);
xticklabels({'-\pi', '-\pi/2', '0', '\pi/2', '\pi'});
xlim([-pi, pi]);
```