Root 2 and Beyond

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1 Conventions

When we write a^{b^c}, does that mean $a^{(b^c)}$? Or does it mean $(a^b)^c$? Both of these are reasonable interpretations, and the notation doesn't inform which choice is "correct". To remove the ambiguity, we make a *convention*. In other words, we make an agreed-upon rule about what the notation means, even though the notation doesn't hint towards it.

With iterated exponents, the most common *convention* is that

$$a^{b^{c^{d^{\cdots}}}} = a^{(b^{(c^{(d^{\cdots})})})}.$$

In other words, we start calculating powers from the top down, rather than from the bottom up. An excerpt from wikipedia shows that different calculators have different conventions:

> Calculators may associate exponents to the left or to the right. For example, the expression `a^b^c` is interpreted as $a^{(b^c)}$ on the TI-92 and the TI-30XS MultiView in "Mathprint mode", whereas it is interpreted as $(a^b)^c$ on the TI-30XII and the TI-30XS MultiView in "Classic mode".

A more familiar convention is brackets, exponents, division, multiplication, addition, subtraction (BEDMAS): Given an expression such as $4 \times 2 + 5$, changing the order of operations changes the final coutcome. Moreover, the notation itself doesn't make it obvious what the order should be. The most common convention is "BEDMAS" in which we would calculate the multiplication before addition, so that the result is 13. Again, different calculators have different conventions in this context too!

2 Iterated roots

In your groups, choose one of the following problems to work on for the next 30 minutes! (You'll spend 25 minutes working together to explore the problem and creating a poster, and in the last five minutes we'll walk around and see what everyone else did.)

1. Show that $(a^b)^c = a^{(b \times c)}$. (Maybe you can do this visually?) How does this help to compute or conceptualise fractional exponents, like $5^{1/3}$?
2. What is $\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdots}}}}$?
3. Can you find examples of values of a, b, and c so that $a^{(b^c)} = (a^b)^c$?