Various Sneaky Mathematicians (VSM)

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This activity builds on IOLA materials on introductory linear algebra, available here: `http://iola.math.vt.edu` and discussed here: `https://www.tandfonline.com/doi/abs/10.1080/10511970.2012.667516`.

1 Goals

The goals of problem 1 in this activity are:

1. To remind students about how to represent a line in parametric form.
2. To provide students an opportunity to understand the utility of parametric form.

The goals of problem 2 in this activity are:

1. To introduce students to the parametric form for planes in $\mathbb{R}^3$ through asking them to select a second mode of transport.
2. To introduce the idea that the set of points that satisfy an equation depends on which vector space the points are a part of.

The goals of problem 3 in this activity are:

1. To introduce students to the parametric form for planes in $\mathbb{R}^3$ in a general setting.
2. To help students see that a plane in $\mathbb{R}^3$ is not all of $\mathbb{R}^3$, and to introduce the equational form of a plane.

The goal of problem 4 in this activity are:

1. To continue the discussion of the equational form of a plane and its relationship to the parametric form for planes.

The goals of problem 5 in this activity are:

1. To introduce the point-normal form of a plane

2 Materials

For this activity you will need:

1. Handouts (for both part 1 and part 2). The problems are listed in section 5 below.
2. White boards (or alternatively, flip chart paper)
3. Markers

3 Instructions

This activity will take approximately 200 minutes.

1. Form groups of 3 to 4 students and give students handouts. Explain the relationship to the Magic Carpet problems, and that we're exploring concepts which we will formalise later.
2. Ask students to work on the first handout. As students work on the problems, visit each group to answer any questions they may have.
3. After part 1, ask students to work on part 2.
4. Assign to each group one of the 5 problems, based on how noteworthy their solution to the problem is. Ask students to start creating a poster to represent their thinking on that problem. Emphasize that the poster should make sense all on its own.
5. When students are done, organize a gallery walk for students to look at each other's work.
6. Afterwards, formally discuss the notion of a plane, including the parametric and equational forms of a plane. In addition, ask students to discuss: any cool strategies they noticed, and any patterns in the different strategies.

4 Tips

1. If one group finishes particularly early, consider additional questions you could ask them
2. Encourage students to make use of graphing software if they think it'll help them.
3. This activity is the first in a series of activities about planes, and students do not need to formally understand planes at the end of this activity.

5 VSM Questions

Question 1

Old Man Gauss is back in $\mathbb{R}^2$ and hidden somewhere along the line $y = 7x - 23$. You have a brand new hoverboard and when you start it for the first time you must set the direction of travel (i.e. vector). Once the direction is set it cannot be changed without great difficulty (so you are stuck with the direction you choose). You also have a dodgy, enchanted portal that can take you to any point in $\mathbb{R}^2$, but you can only be confident it will work once.

1. How should you program your hoverboard and where should you take the portal to ensure you can find Old Man Gauss?
2. Ask your instructor to learn the exact point (x, y) where Old Man Gauss is hiding. Based on the point and direction you chose in (1), when will you reach Old Man Gauss?

Question 2

Unlike Old Man Gauss, Seki Takakazu has managed to hide in $\mathbb{R}^3$. He is at a point (x, y, z) that satisfies the equation $y = 7x - 23$.

1. Can you reach Seki with the hoverboard and portal you had from Question 1? If your answer is yes, show why this is true. If your answer is no, what other transport would you need to reach Seki?

Question 3

You have a dodgy enchanted portal that can take you to the point $(3, 5, -7)$. Right when you step out of the portal you find your hoverboard and magic carpet. Your hoverboard travels along $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and your magic carpet along $\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$.

1. Sunzi has hidden at the point $(36, 36, 0)$. Can you reach him? If yes, then how long do you need on each mode of transport, and if no then show why this is not possible.

2. Where can Sunzi hide so you cannot find him?

Question 4

Maryam Mirzakhani is also in $\mathbb{R}^3$ and is hidden at a point satisfying the equation $3x + 2y - z = 4$. You have a new hoverboard and magic carpet ready to be programmed (and again you can only program each once). You also have a single use portal available to jump anywhere in $\mathbb{R}^3$.

1. How should you program your hoverboard and magic carpet and how should you use your portal to ensure you can find Mirzakhani? (Note: remember we are now in $\mathbb{R}^3$ so you will need to specify 3-dimensions for travel points.)

2. Ask your instructor to learn the exact point (x, y, z) where she is hiding. Based on the point and direction you chose in (1), when will you reach her?

Question 5

Bhāskara II is standing on yet another plane in $\mathbb{R}^3$. He tells us he is perpendicular to the plane with his feet at the point $(26, 22, 12)$ and the tip of his head at $(29, 30, 7)$.

1. Where can Bhāskara walk around?