Basis Ladder

Megan Selbach-Allen, Pranav Nuti, Shintaro Fushida-Hardy

SSEA 2022

1 Goals

The goals of this activity are:

1. To give students practice with checking if a set of vectors has a redundancy.
2. To reiterate the definition of a basis, and to give students practice working with bases.

2 Materials

For this activity you will need:

1. Handouts

3 Instructions

Thing activity will take approximately 30 minutes.

1. Explain to students the classic game of *Word Ladder*.
2. Form groups of 3 to 4 students, give students handouts, and explain the goals of the activity.
3. Give students time to ask questions about the rules of the activity.
4. Before they start the activity, ask students to check that $\mathbb{R}^3$ has dimension 3, and make sure that students understand that to check whether the sets in the questions are bases, they only need to check that they have no redundancies.
5. Give students 30 minutes to think about the problems. As students work on the problems, visit each group to answer any questions they may have.
6. At the end, talk about how they might have observed that the activity is tedious. Talk also about why it should even be possible to go from one basis to another this way.

4 Tips

1. This activity comes after the introduction of the concept of a basis, i.e., a spanning set with no redundancies, and an explanation of the theorem that a set of vectors with size $\dim(V)$ and no redundancies is a basis.

2. This activity aims, firstly, to show students that checking the "no redundancies" property can be tedious, and to thus further stress the usefulness of the orthogonality criterion[1].

3. Secondly, this activity can be used to introduce the *basis-exchange property* in a more advanced linear algebra class, explain how spanning trees in a graph have the same property, and to talk about the benefits of abstraction.

[1] But the time complexity of checking orthogonality and non-singularity seem to be the same actually, so maybe there's a better way to talk about this.