Practice with Bases

Megan Selbach-Allen, Pranav Nuti, Shintaro Fushida-Hardy

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1 Goals

The goals of this activity are:

1. To give students practice working with bases.
2. To give students practice with making effective presentations.

2 Materials

For this activity you will need:

1. White boards (or alternatively, flip chart paper)
2. Markers

3 Instructions

This activity will take approximately 60 minutes.

1. Form groups of 3 to 4 students, and explain the goals of the activity.
2. Give each groups a problem about bases, and give them time to solve it.
3. Ask students to make a five minute presentation of their solution. Ask students to really think about how to communicate math in a way that's engaging and easy to follow. Everyone in the group should be involved in the presentation in some way.
4. Let the entire class listen to all the presentations.

4 Tips

1. Ask groups to make sure they explain: the question, how they found their solution, why their solution works, and what the bigger picture is.
2. Here are some sample problems. Some problems are harder than others, and you can choose which problems you assign to which group.
   (a) One basis for $\mathbb{R}^3$ is the standard basis. Write it down. Come up with 1 other orthonormal basis.

(b) $3x - 2y + 2z = 0$ is the equation of a plane which is also a subspace of $\mathbb{R}^3$. Write down an orthogonal basis for it.

(c) Show that any collection of three vectors in $\mathbb{R}^2$ has redundancy. How does this relate to bases of $\mathbb{R}^2$?

(d) Suppose u is in the span of v and w. Show that $\{u, v, w\}$ cannot be pairwise orthogonal. What does this tell you about three orthogonal vectors in $\mathbb{R}^3$?

(e) Find a 3-vector v which extends $\{(3, 1, 2), (-1, 1, 1)\}$ to an orthogonal basis of $\mathbb{R}^3$. How is this related to descriptions of planes.