The Nearest Vector Interpretation and the Projection Formula

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1 Goals

The goals of this activity are:

1. To give students an intuitive way of understanding what a projection is.
2. To guide students through the derivation for the formula for the projection of a vector onto a line.
3. To guide students through the derivation for the formula for the projection of a vector onto a plane.

2 Materials

For this activity you will not need any materials.

3 Instructions

This activity will take approximately 90 minutes.

1. Explain the goals of the activity.
2. Explain that another intuitive way of understanding the projection of a vector v onto a vector w is to think of it as the closest vector along w to the tip of v.
3. To demonstrate what this means, draw 4 vectors in $\mathbb{R}^2$, and ask students to draw the nearest vector to the tip of each vector along $w = (1, 0)$.
4. Ask students to guess what the vector they drew is (for example, students might guess that the vector they drew looks like $1.5w$).
5. Repeat the activity with $w = (1, 1)$. Students typically figure out that to find the nearest point, you have to drop a perpendicular from the tip of v onto w.
6. Tell students that it is possible to find a formula for the projection, not merely guess. Using the fact that we have to drop a perpendicular, set up the equations $v = u + cw, u \cdot w = 0$. c is the number they were previously guessing.
7. Re-explain at this point the notion of 'belonging': since v is a sum of u and cw, cw is the 'portion' of v that 'belongs' to w.

8. Giving students the hint that they should take the dot product of both sides of the equation with w, guide them through the computation of the value of c.
9. Give students some pairs of vectors to practice the formula with.
10. Repeat the activity with projecting vectors onto planes.

4 Tips

1. This activity comes after the activity with the toy train.
2. Because of its length, it is useful to give students a break during the activity.