

Dot Product

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1 Goals

The goals of this activity are:

1. To give students some sample usages of various math words used repeatedly in mathematics textbooks, like *theorem* and *lemma*.
2. To give students an introduction to what proof means in mathematics classes.
3. To give students a full understanding of why the dot product of two vectors being zero means that the vectors are perpendicular.

2 Materials

For this activity you will need:

1. Handouts

3 Instructions

This activity will take approximately 90 minutes.

1. Explain the goals of the activity.
2. Present the definition of the dot product, an example calculation, and the conjecture that $u \cdot v = 0$ exactly when the vectors u and v are perpendicular, recalling why the conjecture makes sense for 2 dimensional vectors.
3. Form 5 groups of 4 students, and give each group a part of a lemma to prove regarding the properties of the dot product. Stress that the students should use the definition and nothing else in their proofs, and that it is helpful to try concrete examples.
4. After 20 minutes, students form new groups: 5 groups of 4 people each. Make sure that each of the 4 people in the new group worked on a different part of the lemma. Ask students to explain the part of the lemma they worked on to the members of their new group.
5. Give students handouts, and ask students to work on the part of the handout after the lemma.
6. As students work on the handout, visit each group to answer any questions they may have.
7. Discuss the similarities of the series of activities culminating in this handout to the mathematical process in general.

4 Tips

1. The four parts of the lemma are

$$u \cdot v = v \cdot u, v \cdot v = \|v\|^2, c(v \cdot u) = v \cdot (cu), v \cdot (w + u) = v \cdot w + v \cdot u$$

These are present on the handout, but the handout is only given to students after they finish proving the lemma.

2. This activity comes after the activity about math words, the activity about patterns in perpendicular vectors, and the activity about the cosine law.
3. Because of its length, it is useful to give students a break during the activity.