# Problems for Section

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SSEA 2022

1. For each of the following sets of vectors, prove or disprove that it is a subspace.

   (a) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, z = 2x - y \right\}$

   (b) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, z = 1 + 2x - y \right\}$

   (c) $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \,\middle|\, y = x^2 \right\}$

   (d) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, 3x - y + z = 0, x + y - 4z = 0 \right\}$

2. Consider the 3-vectors $\mathbf{v} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$. Find scalars $a, b, c \in \mathbb{R}$ such that

$$\operatorname{span}(\mathbf{v}, \mathbf{w}) = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, ax + by + cz = 0 \right\}.$$

   Interpret this in terms of descriptions of planes.

3. Find a pair of 3-vectors $\mathbf{v}, \mathbf{w}$ such that $\operatorname{span}(\mathbf{v}, \mathbf{w}) = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, 2x - 3y + 2z = 0 \right\}$.

4. Consider the 4-vectors $\mathbf{v_1} = \begin{bmatrix} -2 \\ 2 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 1 \end{bmatrix}$. Show that the set of vectors

$$V = \{\mathbf{w} \in \mathbb{R}^4 \mid \mathbf{w} \cdot \mathbf{v_1} = 0, \mathbf{w} \cdot \mathbf{v_2} = 0\}$$

   is a subspace in each of the following ways:

   - Write $V$ as a span of some vectors. (Hint: write $\mathbf{w}$ as the vector $(w_1, w_2, w_3, w_4)$ and try to solve for $w_1$ and $w_2$.)

- Show that $V$ contains 0, and is closed under vector addition and scalar multiplication.

5. Consider the 3-vectors $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v}' = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \mathbf{w}' = \begin{bmatrix} -2 \\ -3 \\ 4 \end{bmatrix}$. Show that $\operatorname{span}(\mathbf{v}, \mathbf{w}) = \operatorname{span}(\mathbf{v}', \mathbf{w}')$. (For example, show that $\mathbf{v}$ and $\mathbf{w}$ are contained in the span of $\mathbf{v}'$ and $\mathbf{w}'$ and vice versa.)