Feb 2 Notes :

Let $\{a_n\}$ be a sequence.
Let $\{b_n\}$ be the <u>tail</u> of $\{a_n\}$, i.e.

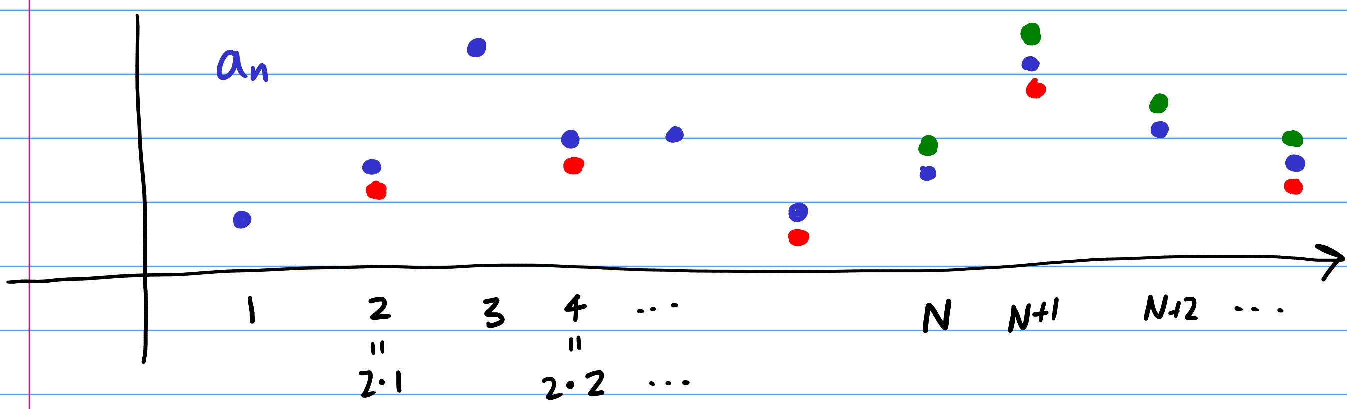
$$\{b_n\}_{n=1}^{\infty} = \{a_n\}_{n=N}^{\infty}$$

for some N.

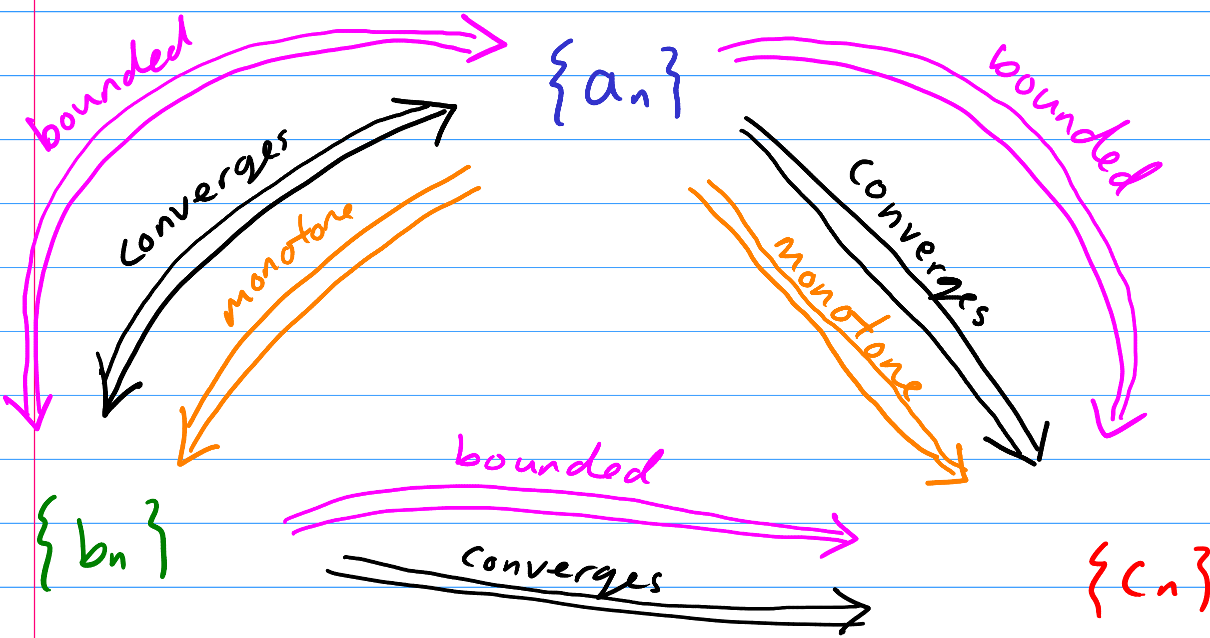
Let $\{c_n\}$ be the even indexed terms in $\{a_n\}$, i.e.

$$\{c_n\}_{n=1}^{\infty} = \{a_{2n}\}_{n=1}^{\infty}$$

They look like this:



Then the following holds:



Some theorems

Squeeze theorem

Let $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ be sequences.
Suppose $a_n \leq b_n \leq c_n$ for all n, and

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L.$$

Then $\lim_{n\to\infty} b_n$ exists and is L.

Use this to deal with pesky alternating things like $(-1)^n$ or $\cos(n)$ if they appear in a sequence. (see Q1 from problem sheet)

Monotone convergence theorem.

Let $\{a_n\}$ be a sequence.
If $\{a_n\}$ is monotone and bounded then $\lim_{n\to\infty} a_n$ exists (a_n is convergent).

Use this to prove convergence of some sequences even if you don't know what the limit is! (see Q2 from problem sheet)