Position, Velocity, and Acceleration

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[1]:
```
%matplotlib inline

import matplotlib.pyplot as plt
#from matplotlib.patches import Rectangle
import pandas as pd
import numpy as np
from thisActivity import *
from great_tables import GT, md, style, loc

plt.rcParams.update({'font.size':18})
```

Something to review from last time

$$\bar{v}_{AC} = \frac{1}{2}\left(\bar{v}_{AB} + \bar{v}_{BC}\right)$$

- How long does it take to go from A to B?
- How long does it take to go from B to C?

Can we fix this? Yes, if we take a time-weighted average:

$$\bar{v}_{AC} = \frac{1}{t_{AB} + t_{BC}}\left(t_{AB}\bar{v}_{AB} + t_{BC}\bar{v}_{BC}\right)$$

Aside: Weighted Average from statistics

Find the average of the set $\{1, 1, 1, 2, 2, 3\}$:

6th grade method:

$$\bar{x} = \frac{1+1+1+2+2+3}{6} = \frac{10}{6} \approx 1.67$$

Another way of thinking of it:

$$\bar{x} = \frac{(\text{num of 1's})1 + (\text{num of 2's})2 + (\text{num of 3's})3}{(\text{num of 1's}) + (\text{num of 2's}) + (\text{num of 3's})}$$

So, if the number x_i appears in our set n_i times:

$$\bar{x} = \frac{\sum_i n_i x_i}{\sum_i n_i} = \sum_i p(x_i)x_i$$

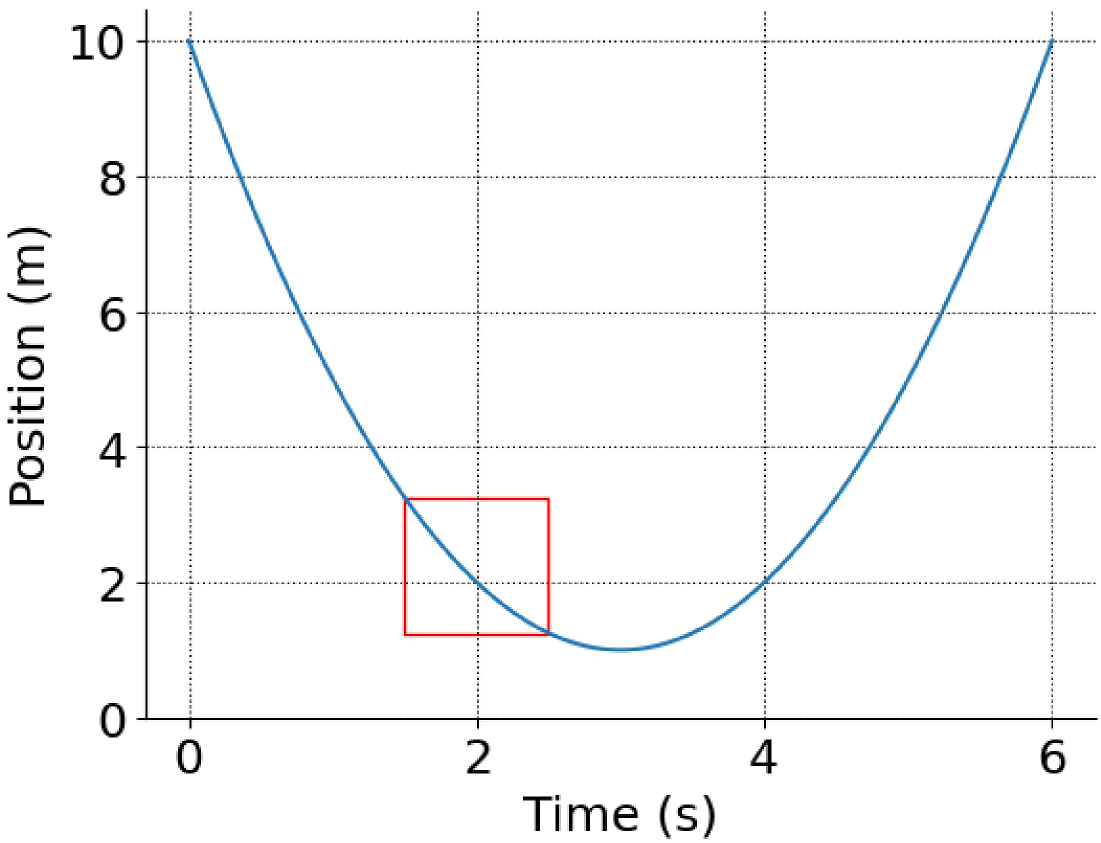
where $p(x_i) = \frac{n_i}{N}$ and $N = \sum_i n_i$ is the count of elements in our set.

Zooming in on a position vs. time graph

$$x(t) = t^2 - 6t + 10$$

1. Plot between $t = 0$ s and $t = 6$ s.

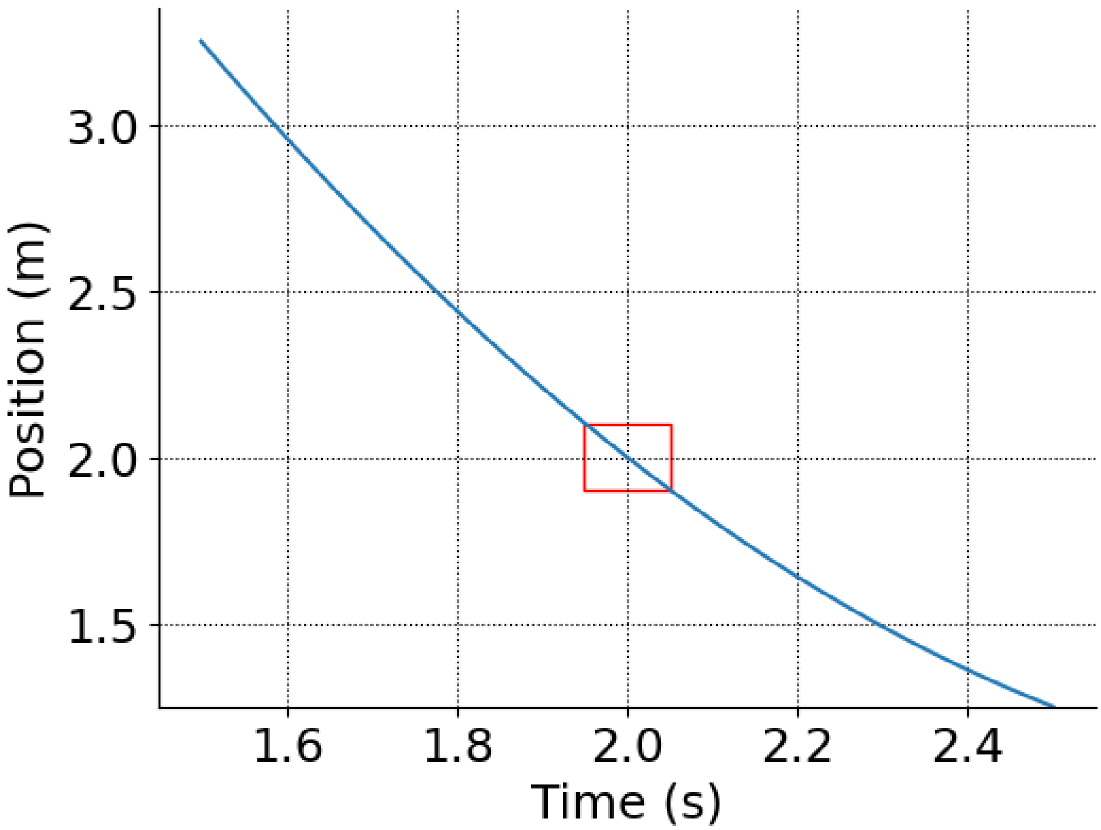
[2]:
```
fig, ax = xvtPlt(0,6)
p = plt.Rectangle((1.5,pos(1.5)),width=1,height=(pos(2.5)-pos(1.5)),
             ec='r',fill=False)
ax.add_patch(p)
plt.show()
```



2. Plot between $t = 1.5$ s and $t = 2.5$ s. (Zoom in on red box above)

$$x(t) = t^2 - 6t + 10$$

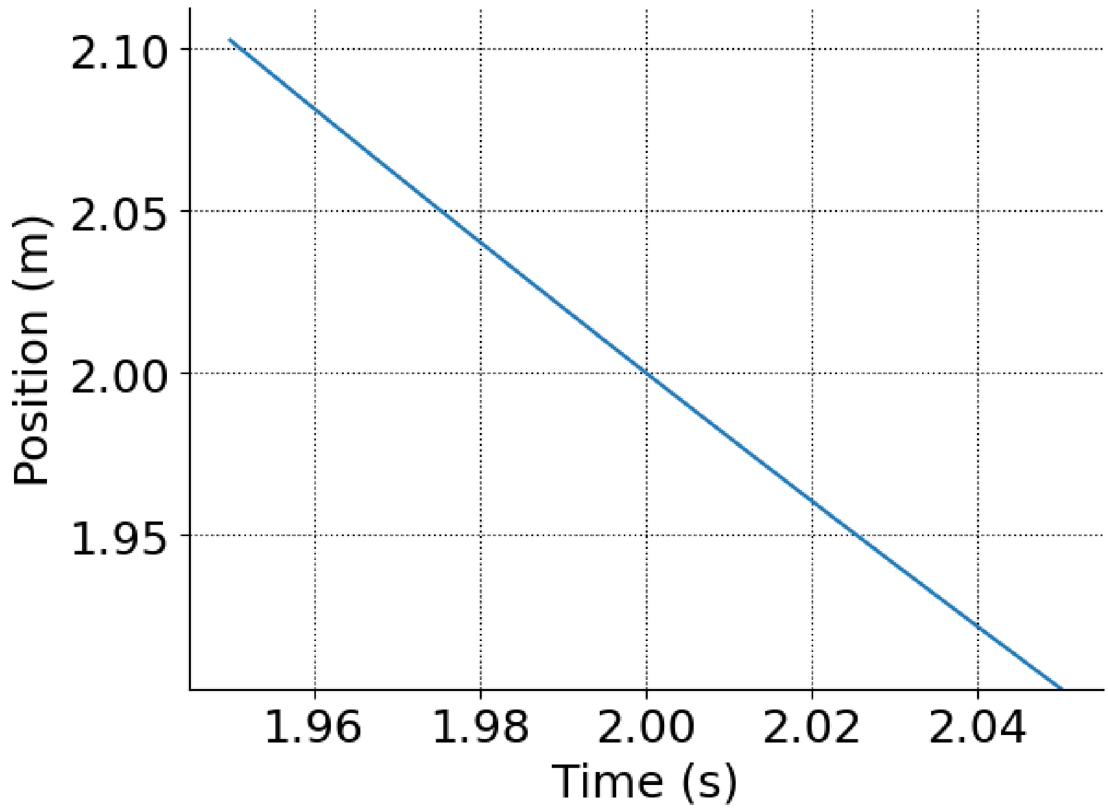
[3]:
```
fig, ax = xvtPlt(1.5,2.5)
p = plt.Rectangle((1.95,pos(1.95)),width=.1,height=(pos(2.05)-pos(1.95)),
             ec='r',fill=False)
ax.add_patch(p)
plt.show()
```



3. Plot between $t = 1.95$ s and $t = 2.05$ s. (Zoom in on red box above)

$$x(t) = t^2 - 6t + 10$$

[4]:
```
xvtPlt(1.95,2.05)
plt.show()
```



4. All three graphs are representations of the same motion
   1. How can you account for the last graph being so much straighter than the first?
   2. Can you tell from a very small time interval on a graph whether the motion over the whole graph has constant velocity?
   3. Find your average velocity over the small time interval from part 3 above. Show your work
   4. Add a line with slope equal to this average velocity to your plot from part 1 that goes through the point $(2, 2)$. What does this line remind you of from your Calculus I class? How do you think that the slope of this line compares to the instantaneous velocity at $t = 2$ s. Recall the definition of instantaneous velocity:

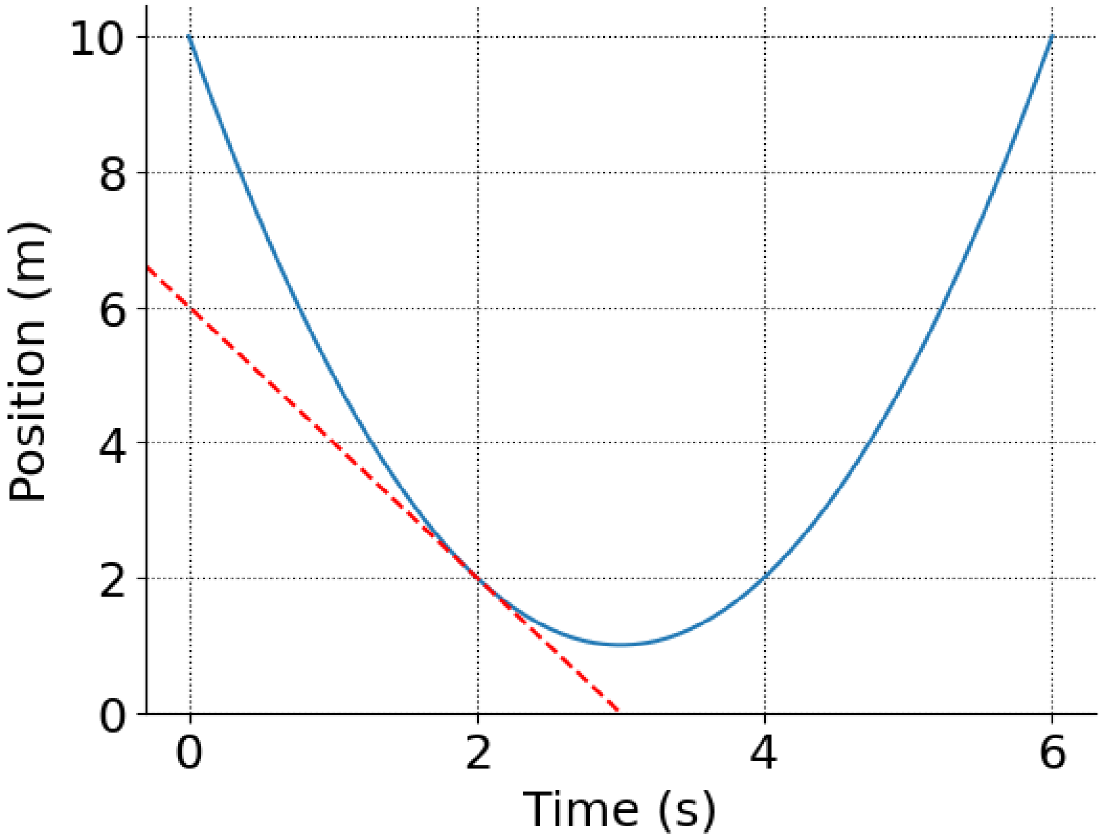
$$v = \frac{dx}{dt}$$

[5]:
```
# Behind the scenes, I have created a function called `pos(t)` which calculates␣
↪t^2-6t+10
avgSpd = (pos(2.05)-pos(1.95))/(2.05-1.95)
print(f"The average speed between 1.95 s and 2.05 s is {avgSpd} m/s")

# This line creates the plot
fig, ax = xvtPlt(0,6)
```

```
ax.axline((2, 2), slope=avgSpd, color='r',ls='--')  # adds a red, dashed line␣
↪going through
                                                     # (2,2) with slope = avgSpd
plt.show()
```

```
The average speed between 1.95 s and 2.05 s is -2.0000000000000133 m/s
```



Predicting future position

Suppose that you know three things about an object: 1. Time right now (t_i) 2. Current location (x_i) 2. Current velocity (v_i)

Given what we know about velocity:

$$v = \frac{dx}{dt} \quad \text{and} \quad \bar{v} = \frac{\Delta x}{\Delta t}$$

As well as the fact that when Δt is small enough,

$$v \approx \bar{v}$$

Write an algebraic expression predicting the future position (x_f) some time δt after the initial time.

$$x_f = v_i \delta t + x_i$$

Defining acceleration

Acceleration is defined as the time derivative of velocity:

$$a = \frac{dv}{dt}$$

Given what we know about the position, and average velocity, write an algebraic expression for the average acceleration:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

Discuss in Groups

Suppose that you know two things about an object: 1. Current velocity (v_i) 2. Current acceleration (a_i)

Is it reasonable to predict that the velocity a short time δt later can be written as the following? Why or why not?

$$v_f = v_i + a_i \delta t$$

FYI - This method for integrating differential equations is called *Euler's Method.*

https://www.youtube.com/embed/v-pbGAts_Fg

(Pause at 1:11)