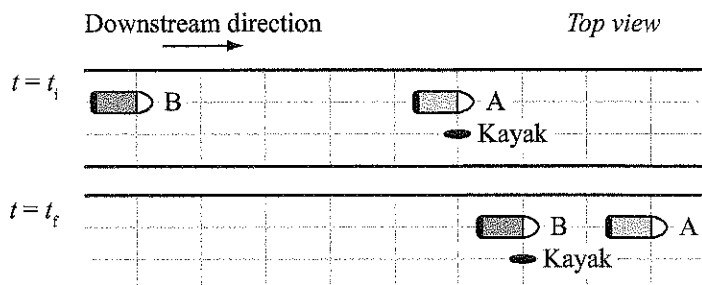


1. Two riverboats, A and B, move downstream along a straight section of river as shown. At time  $t_i$ , boat A passes a kayak. At time  $t_f$ , boat B passes the kayak.

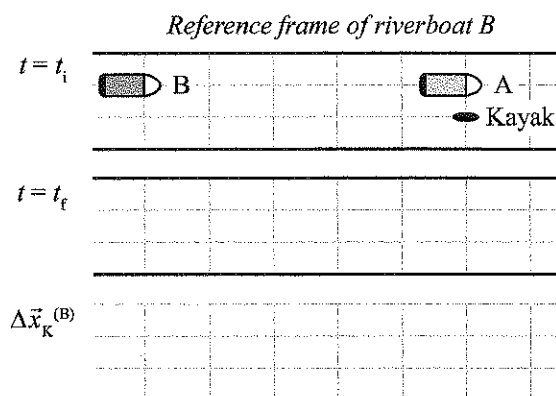


- a. Reference frame of boat B:

Complete the middle diagram at right by drawing and labeling the boats and the kayak at their positions at time  $t_f$  as measured in the reference frame of boat B.

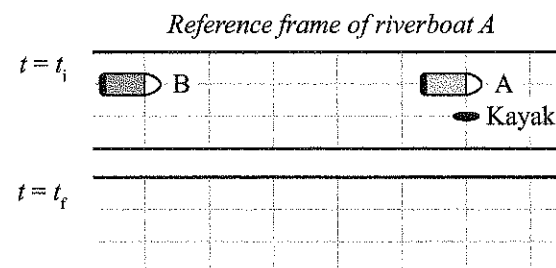
In the space at lower right, draw  $\Delta \vec{x}_K^{(B)}$ , the displacement of the kayak in the reference frame of boat B.

In the reference frame of boat B, is the speed of the kayak *greater than*, *less than*, or *equal to* the speed of boat A? Explain.



- b. Reference frame of boat A:

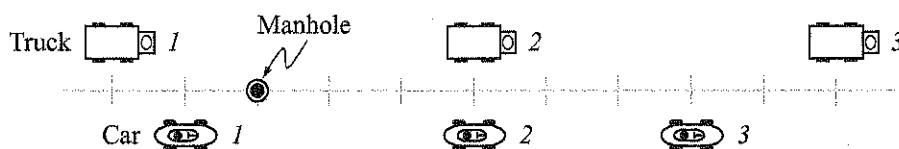
Complete the diagram at right by drawing and labeling the boats and the kayak at their positions at time  $t_f$  as measured in the reference frame of boat A.



- c. Is the speed of the kayak in the reference frame of boat A *greater than*, *less than*, or *equal to* the speed of the kayak in the reference frame of boat B? Explain.
- d. Rank the following quantities from largest to smallest: (i)  $|\Delta \vec{x}_K^{(A)}|$ , (ii)  $|\Delta \vec{x}_K^{(B)}|$ , (iii) the distance between boats A and B at time  $t_i$ , and (iv) the distance between boats A and B at time  $t_f$ . Explain your reasoning.

- e. A third riverboat, boat C, moves downstream so as to remain a fixed distance behind boat B at all times. The displacement of the kayak from time  $t_i$  to time  $t_f$  is measured in the frame of boat A, in the frame of boat B, and in the frame of boat C. Rank these three displacements according to magnitude, from largest to smallest. Explain.

2. A car and a truck are near a manhole on a straight road. Their positions are shown at instants 1–3, separated by equal time intervals.



- a. In the spaces provided, draw the following displacements over the interval from instant 1 to instant 3:
- the displacement of the car *in the frame of the manhole* and *in the frame of the truck*
  - the displacement of the truck *in the frame of the manhole*

$\Delta \vec{x}_C^{(M)}$

$\Delta \vec{x}_C^{(T)}$

$\Delta \vec{x}_T^{(M)}$

- b. In the space below, draw and label a vector diagram to show which of these three displacement vectors is the sum of the other two.

Express the relationship between the three vectors as an algebraic equation.

- c. The relationship  $\vec{v}_C^{(M)} = \vec{v}_C^{(T)} + \vec{v}_T^{(M)}$  is known as *the Galilean transformation of velocities*. Explain how this relation is consistent with your result above for the displacements.

Does this relationship apply to the *instantaneous* velocities at instant 2? at instant 3? Explain.

3. Car P moves to the west with constant velocity  $\vec{v}_P^{(R)}$  along a straight road. Car Q starts from rest at instant 1, and moves to the west with increasing speed. At instant 5, car Q has velocity  $\vec{v}_Q^{(R)}$  relative to the road ( $\vec{v}_Q^{(R)} < \vec{v}_P^{(R)}$ ). Instants 1–5 are separated by equal time intervals.

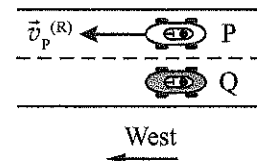
At instant 3, cars P and Q are adjacent to one another (*i.e.*, they have the same position).

- a. In the reference frame of the road, at instant 3 is the speed of car Q *greater than*, *less than*, or *equal to* that of car P? Explain.

- b. Complete the sketch at right by drawing a qualitatively correct velocity vector for car Q in the frame of the road at instant 3. Make sure the completed sketch is consistent with your answer to part a.

In this situation, which car is passing the other? Explain.

Sketch of cars P and Q  
at instant 3



- c. In the space at right, sketch and label a vector diagram illustrating the Galilean transformation of velocities that relates  $\vec{v}_P^{(R)}$ ,  $\vec{v}_Q^{(R)}$ , and  $\vec{v}_Q^{(P)}$  at instant 3.

In the frame of car P, at instant 3 is car Q *moving to the west*, *moving to the east*, or *at rest*? Explain.

$\vec{v}_P^{(R)}$ ,  $\vec{v}_Q^{(R)}$ , and  $\vec{v}_Q^{(P)}$   
at instant 3

- d. Repeat the application of the Galilean transformation to sketch the velocity vectors of car Q in the frame of car P at instants 2, 3, and 4. Explain.

In the frame of car P, is car Q *speeding up, slowing down, or moving with constant speed*? Explain.

$\vec{v}_Q^{(P)}$  at instant 2






$\vec{v}_Q^{(P)}$  at instant 3

$\vec{v}_Q^{(P)}$  at instant 4

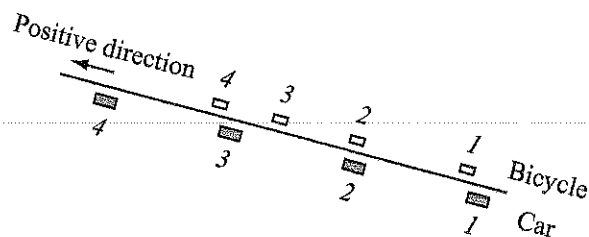
- e. Complete the diagram at right by drawing car Q at its position at instants 1–5 as measured in the frame of car P.

Explain how your completed diagram is consistent with your velocity vectors from part d above.

*Diagram for the reference frame of car P*

Instant 1	 Car P
Instant 2	
Instant 3	
Instant 4	
Instant 5	

4. A bicycle coasts up a hill while a car drives up the hill at constant speed. The strobe diagram shows their positions at instants 1–4, separated by equal time intervals. The bicycle comes to rest relative to the road at instant 4.



- a. *Reference frame of the road:*

As measured in the reference frame of the road:

- is the acceleration of the bicycle *in the positive direction, in the negative direction, or is its magnitude zero*? Explain.
- are the velocity and acceleration of the bicycle in *the same or opposite* directions?

## b. Reference frame of the car:

- i. Sketch velocity vectors of the bicycle *in the reference frame of the car* at instants 2 and 3. Explain your reasoning.

 $\vec{v}_B^{(C)}$  at instant 2

- ii. In the frame of the car, is the bicycle *moving in the positive direction, moving in the negative direction, or at rest*:

 $\vec{v}_B^{(C)}$  at instant 3

- at instant 2?

- at instant 3?

- iii. In the frame of the car, is the bicycle *speeding up, slowing down, or neither*:

- at instant 2?

- at instant 3?

- iv. As measured in the reference frame of the car:

- is the acceleration of the bicycle *in the positive direction, in the negative direction, or is its magnitude zero*? Explain how your answer is consistent with your velocity vectors in part b.i.
- in the frame of the car, are the velocity and acceleration of the bicycle *in the same or in opposite* directions? Explain how your answer is consistent with your answers to parts b.i and b.ii.

The frame of the road and the frame of the car are examples of *inertial reference frames*. The direction (and magnitude) of the acceleration of an object is the same in all inertial frames.

## c. Consider the following statement:

"The acceleration of the bicycle must be the same in all inertial frames. Since the bicycle is slowing down in the frame of the road, it must be slowing down in the frame of the car as well."

Do you agree or disagree? Explain.