

# Numerically calculating Position, Velocity, and Acceleration

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## 1 Euler's method for determining position and velocity given acceleration

Givens numerical values for: - An initial time:  $t_0$  (usually, we pick 0 sec) - An initial position:  $x_0$  (Depends on the problem, will *always* be given) - An initial velocity:  $v_0$  (Depends on the problem, will *always* be given) - A known acceleration function  $a(t)$  (Depends on the problem, will be given for now)

In general, our goal is to make a table full of numbers that allows you to fill in the table below:

$t$	$x$	$v$	$a$
$t_0$	$x_0$	$v_0$	$a_0$
$t_1$	$x_1$	$v_1$	$a_1$
$t_2$	$x_2$	$v_2$	$a_2$
$t_3$	$x_3$	$v_3$	$a_3$
$t_4$	$x_4$	$v_4$	$a_4$
...	...	...	...

### Step 1: Choose an appropriate time step $\delta t$

Literally, just pick a numerical value (at least to start out).

**Step 2: Fill in the first line of the table above.**

You'll need to plug the initial time into your acceleration function to get the initial acceleration.

$$a(t_0) = a_0$$

**Step 3: Calculate the *future* time, position, velocity, and acceleration:**

$$t_1 =$$

$$x_1 =$$

$$v_1 =$$

$$a_1 =$$

**Step 4: Repeat step 3 until you are done**

Usually, you are looking for a certain final time, or are given a condition when you should stop calculating.

In general, you calculate the next element based off of the previous elements

$$t_{j+1} =$$

$$x_{j+1} =$$

$$v_{j+1} =$$

$$a_{j+1} =$$

This lets you determine the next row of your table based on the previous row of the table.

## 2 Motion of an accelerating object

Suppose you have an object moving in 1D with an acceleration given by the following function of time:

$$a(t) = \left(1 \frac{\text{m}}{\text{s}^2}\right) e^{-t}$$

You also know that at  $t = 0$  the position is  $x_i = 0$  m and the velocity is  $v_i = 0 \frac{\text{m}}{\text{s}}$ . We want to know where, and how fast, the object is moving at  $t = 5$  s.

1. Carry out the following procedure to make this prediction. You can start by working with paper and pencil, but you will want to transition this calculation to code.
  - a. Numerically predict the velocity and position at  $t = 1$  s.
  - b. Given your previous prediction, now predict the velocity and position at  $t = 2$  s.
  - c. Given your previous prediction, now predict the velocity and position at  $t = 3$  s.
  - d. Given your previous prediction, now predict the velocity and position at  $t = 4$  s.
  - e. Given your previous prediction, now predict the velocity and position at  $t = 5$  s.
2. Do you think that this is accurate? Why or why not?
3. Make a plan for making this calculation more accurate, then carry it out.
4. Finally make the following plots from  $t = 0$  s to  $t = 5$  s using a step size that is appropriate given the class presentation.
  - $x(t)$  vs.  $t$
  - $v(t)$  vs.  $t$
  - $a(t)$  vs.  $t$
5. **Interpret this motion** what is happening to the acceleration as  $t \rightarrow \infty$ ? Do your velocity vs. time and position vs. time graphs show behavior that agrees with this observation about the acceleration?

Include your code and the figures that you generate as a part of your submission.

### 3 Appendix: The analytic method for solving this problem:

$$a(t) = \frac{dv}{dt} = e^{-t} \quad v(0) = 0 \quad x(0) = 0$$

**Step 1: Integrate the acceleration to get the velocity**

**Step 2: Apply the condition  $v(t = 0) = 0$**

**Step 3: Integrate the velocity to find the position**

**Step 4: Apply the condition  $x(t = 0) = 0$**

#### Differential Equations lingo

I don't want you to focus on learning this for this class, but I am attempting to put what you are learning here in a context that will be useful for future coursework so that when it comes up again, it hopefully looks/feels a little familiar.

The equation

$$a(t) = e^{-t}$$

can be framed in one of two ways.

1. A 2nd order differential equation (second order because it has a 2nd derivative in it):

$$\frac{d^2x}{dt^2} = e^{-t}$$

2. Two, 1st order differential equations:

$$\frac{dv}{dt} = e^{-t} \quad \frac{dx}{dt} = v(t)$$

Either way, the system has a *generalized solution* (that is, one where we don't have any initial values or *boundary conditions*). If we want to have a *specific solution*, we need to know the value of the functions  $x(t)$  and  $v(t)$  for some time value. (It need not be zero, like we had in this example).