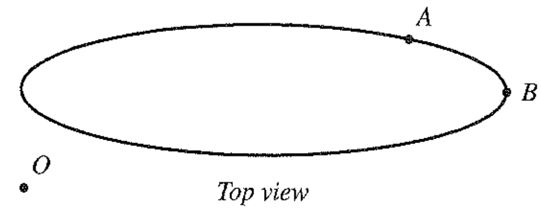
I. Velocity

An object is moving clockwise around an oval track as shown at right. Sketch the trajectory on a large sheet of paper. (Make your diagram *large.*) Point O is the origin of the coordinate system.



Top view

A. Draw $\vec{r}_A$ and $\vec{r}_B$, the position vectors for the object when it is at points A and B.

B. Draw the vector that represents the displacement of the object (*i.e.*, the change in position) from A to B.

Describe how to use the displacement vector to determine the direction of the average velocity of the object between A and B. Draw a vector to represent the average velocity.

C. Choose a point on the oval between points A and B, and label that point B'.

As point B' is chosen to lie closer and closer to point A, does the direction of the average velocity over the interval AB' change? If so, how?

What happens to the magnitude of the displacement as point B' is chosen to lie closer and closer to point A? Does this magnitude approach a limiting value? If so, what is that value?

Must the magnitude of the average velocity change in the same way? Explain.

D. Describe the direction of the (instantaneous) velocity of the object at point A.

How would you characterize the direction of the (instantaneous) velocity at *any* point on the trajectory?

II. Acceleration for motion with constant speed

Suppose that the object in section I is moving clockwise around the track at *constant speed.*

Top view

A. On your large diagram, draw vectors to represent the velocities at points *A* and *B*.

1. Did the *speed* of the object change? Explain how you can tell from the vectors that you have drawn.

2. Did the *velocity* of the object change? Explain how you can tell from the vectors that you have drawn.

B. On a *separate* part of your paper, copy the velocity vectors $\vec{v}_A$ and $\vec{v}_B$ so that their "tails" are at the same point.

$\vec{v}_A$, $\vec{v}_B$, and $\Delta\vec{v}$

1. From your diagram, determine the *change-in-velocity vector,* $\Delta\vec{v}$.

2. Describe how to use the change-in-velocity vector to determine the direction of the average acceleration of the object between *A* and *B*. Draw a vector to represent the average acceleration between points *A* and *B*.

3. On your diagram, label the angle θ between the "head" of $\vec{v}_A$ and the "tail" of $\Delta\vec{v}$. Is this angle *greater than, less than,* or *equal to* 90°?

   As point *B* is chosen to lie closer and closer to point *A*, does the angle θ *increase, decrease,* or *remain the same?* Explain how you can tell.

   Does the angle θ approach a *limiting* value? If so, what is that value?

4. As point *B* is chosen to lie closer and closer to point *A*, the magnitude of $\Delta\vec{v}$ approaches zero. Does the magnitude of the average acceleration also approach zero? Explain.

5. Determine the direction of the (instantaneous) acceleration at point *A*.

   Draw a diagram that shows the velocity and acceleration at point *A*. Place both vectors with their tails at point *A*. Is the angle between the acceleration and the velocity *greater than, less than,* or *equal to* 90°?

➪ Check your reasoning for part B with a tutorial instructor.

C. Suppose you were to choose a new point on the trajectory where the curve is tighter than at point *A (e.g.,* point *B).*

1. Is the magnitude of the acceleration at the new point *greater than, less than,* or *equal to* the magnitude of the acceleration at point *A?* Explain your reasoning.

2. Describe the direction of the acceleration at the new point.

3. At each of the indicated points on the diagram at right, sketch a vector that represents the acceleration of the object.

   Is the acceleration directed toward the "center" of the oval at every point on the trajectory?

Top view

Acceleration vectors for constant speed

III. Acceleration for motion with increasing speed

Suppose that the object in section II is *speeding up* as it moves clockwise around the oval track.

A. Draw vectors to represent the velocity at two points on the track that are relatively close together. Label the two points *C* and *D*.

B. On a *separate* part of your paper, copy the velocity vectors $\vec{v}_C$ and $\vec{v}_D$ so that they are tail-to-tail. Use the same procedure as in previous sections to determine the change in velocity, $\Delta\vec{v}$.

Label the angle θ between the head of $\vec{v}_C$ and the tail of $\Delta\vec{v}$. Is this angle *greater than, less than,* or *equal to* 90°?

Determine the direction of the average acceleration of the object between *C* and *D*.

C. Imagine that point *D* is chosen to lie closer and closer to point *C*.

Consider how θ changes as point *D* is taken to be closer to point *C*.

What does this suggest about the possible value or range of values for angle θ for an object that is speeding up? Explain.

What happens to the magnitude of $\Delta\vec{v}$ as point *D* is chosen to be closer and closer to point *C*?

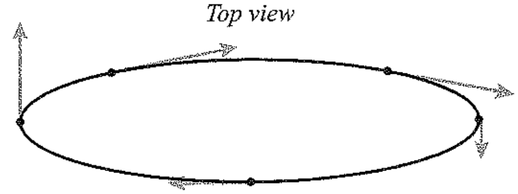
D. Describe how you would determine the acceleration of the object at point *C*.

Draw a diagram that shows both $\vec{v}_C$ and $\vec{a}_C$ with their tails at point *C*. Is the angle between them *greater than, less than,* or *equal to* 90°?

E. Based on your answers above, indicate a possible direction for the acceleration vector at each marked point.

➪ Discuss section III with a tutorial instructor.

Top view



Acceleration vectors for speeding up (velocity vectors shown in grey)