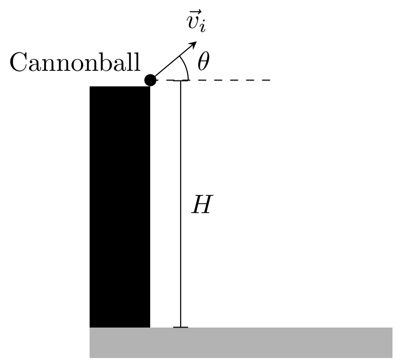
Problem Context

While on a vacation to Kenya, you visit the port city of Mombassa on the Indian Ocean. On the coast, you find an old Portuguese fort probably built in the 16^{th} century. Large stone walls rise vertically from the shore to protect the fort from cannon fire from pirate ships. You wonder how close a pirate ship would have to sail to the fort to be in range of the fort's cannon. Of course you realize that the range depends on the velocity that the cannonball leaves the cannon, as well as the height of the cliff.

1 Setting up the problem

1. Below is a schematic drawing of the fort. Choose an origin and coordinate system that you will use to describe this motion, as well as any forces that are important for this system. To help guide your eye, I have included the initial velocity vector on this schematic $\vec{v}_i$, as well as the launch angle θ which is relative to the horizontal. I've also labeled the cliff's height as H. Explain your choice of origin and the orientation of your coordinate system.



2. Based on your coordinate system, write a vector for the initial position and the initial velocity in terms of the given quantities H, θ and the magnitude of the initial velocity $|\vec{v}_i| = v_i$:

   $\vec{r}_i = (x_i, y_i) =$ $\qquad\qquad$ $\vec{v}_i = (v_{ix}, v_{iy}) =$

3. Draw a free body diagram for a projectile shortly after it has been fired by the cannon. (For today, ignore air resistance).

4. Write an algebraic expression for each of the components of your acceleration.

5. Based on this information, write general equations allowing you to calculate each component of the position and velocity vectors at any time t:

   $x(t) =$
   $y(t) =$
   $v_x(t) =$
   $v_y(t) =$

➪ Check your answers with an instructor before continuing!

2 Measuring the height of the cliff

1. If we assume a cliff height H, launch speed v_i and launch angle θ, write an algebraic expression determining the time T that a projectile will hit the ocean.

> **More information**
>
> In order to determine the height of the cliff, you grab a rock, a stopwatch, and your trusty binoculars and go to the edge of the cliff. You then drop the rock and start your watch simultaneously. Carefully following the fall of the rock in your binoculars, you stop your watch at the instant that you see the splash and find that the rock hit the water 5.0 s later.

2. Using the above information, determine the height of the cliff and the speed of the rock when it hit the ocean.

3 Calculate the Range

> **More information**
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> You have been looking through the archives and find a reference to the muzzle velocity of the cannon, finding a value of $60.0\,\mathrm{m\,s^{-1}}$. You aren't sure how accurate that is, so you want to leave this value as a parameter you can change in the calculations below.

1. Write a code with the following given/constant variables:
   - Gravitational acceleration g
   - Muzzle velocity v_i
   - Launch angle θ

   and the following calculated variables:
   - Time-in-flight T
   - Cliff height H

   That allows you to calculate the position vector (x, y), velocity vector (v_x, v_y), acceleration vector (a_x, a_y), and speed $v = \sqrt{v_x^2 + v_y^2}$ at 100 instants that occur while the object is in flight.
2. From your code, create the following plots:
   - x vs. t and y vs. t
   - v_x vs. t and v_y vs. t
   - a_x vs. t and a_y vs. t
3. Using your code, and the given numerical values, determine to the nearest degree the launch angle that will give the maximum range of the cannon.

On Friday, you need this code, we'll start with it as a baseline when we talk about adding in air resistance. And we'll bring back our friend the Euler method.