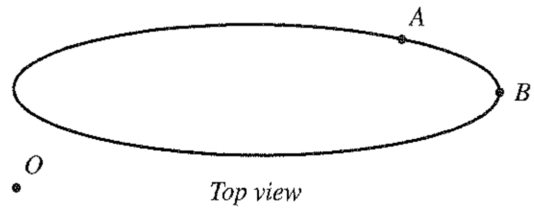
I. Velocity

An object is moving clockwise around an oval track as shown at right. Sketch the trajectory on a large sheet of paper. (Make your diagram *large.*) Point O is the origin of the coordinate system.



Top view

A. Draw $\vec{r}_A$ and $\vec{r}_B$, the position vectors for the object when it is at points A and B.

B. Draw the vector that represents the displacement of the object (*i.e.*, the change in position) from A to B.

Describe how to use the displacement vector to determine the direction of the average velocity of the object between A and B. Draw a vector to represent the average velocity.

C. Choose a point on the oval between points A and B, and label that point B'.

As point B' is chosen to lie closer and closer to point A, does the direction of the average velocity over the interval AB' change? If so, how?

What happens to the magnitude of the displacement as point B' is chosen to lie closer and closer to point A? Does this magnitude approach a limiting value? If so, what is that value?

Must the magnitude of the average velocity change in the same way? Explain.

D. Describe the direction of the (instantaneous) velocity of the object at point A.

How would you characterize the direction of the (instantaneous) velocity at *any* point on the trajectory?

II. Acceleration for motion with constant speed

Suppose that the object in section I is moving clockwise around the track at *constant speed.*

Top view

A. On your large diagram, draw vectors to represent the velocities at points A and B.

1. Did the *speed* of the object change? Explain how you can tell from the vectors that you have drawn.

2. Did the *velocity* of the object change? Explain how you can tell from the vectors that you have drawn.

B. On a *separate* part of your paper, copy the velocity vectors $\vec{v}_A$ and $\vec{v}_B$ so that their "tails" are at the same point.

$\vec{v}_A$, $\vec{v}_B$, and $\Delta\vec{v}$

1. From your diagram, determine the *change-in-velocity vector,* $\Delta\vec{v}$.

2. Describe how to use the change-in-velocity vector to determine the direction of the average acceleration of the object between A and B. Draw a vector to represent the average acceleration between points A and B.

3. On your diagram, label the angle θ between the "head" of $\vec{v}_A$ and the "tail" of $\Delta\vec{v}$. Is this angle *greater than, less than,* or *equal to* 90°?

   As point B is chosen to lie closer and closer to point A, does the angle θ *increase, decrease,* or *remain the same?* Explain how you can tell.

   Does the angle θ approach a *limiting* value? If so, what is that value?

4. As point B is chosen to lie closer and closer to point A, the magnitude of $\Delta\vec{v}$ approaches zero. Does the magnitude of the average acceleration also approach zero? Explain.

5. Determine the direction of the (instantaneous) acceleration at point *A*.

   Draw a diagram that shows the velocity and acceleration at point *A*. Place both vectors with their tails at point *A*. Is the angle between the acceleration and the velocity *greater than, less than,* or *equal to* 90°?

➪ Check your reasoning for part B with a tutorial instructor.

C. Suppose you were to choose a new point on the trajectory where the curve is tighter than at point *A (e.g.,* point *B).*

1. Is the magnitude of the acceleration at the new point *greater than, less than,* or *equal to* the magnitude of the acceleration at point *A?* Explain your reasoning.

2. Describe the direction of the acceleration at the new point.

3. At each of the indicated points on the diagram at right, sketch a vector that represents the acceleration of the object.

   Is the acceleration directed toward the "center" of the oval at every point on the trajectory?

Top view

Acceleration vectors for constant speed

III. Acceleration for motion with increasing speed

Suppose that the object in section II is *speeding up* as it moves clockwise around the oval track.

A. Draw vectors to represent the velocity at two points on the track that are relatively close together. Label the two points *C* and *D*.

B. On a *separate* part of your paper, copy the velocity vectors $\vec{v}_C$ and $\vec{v}_D$ so that they are tail-to-tail. Use the same procedure as in previous sections to determine the change in velocity, $\Delta\vec{v}$.

Label the angle θ between the head of $\vec{v}_C$ and the tail of $\Delta\vec{v}$. Is this angle *greater than, less than,* or *equal to* 90°?

Determine the direction of the average acceleration of the object between *C* and *D*.

C. Imagine that point *D* is chosen to lie closer and closer to point *C*.

Consider how θ changes as point *D* is taken to be closer to point *C*.

What does this suggest about the possible value or range of values for angle θ for an object that is speeding up? Explain.

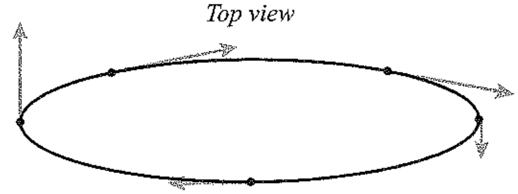
What happens to the magnitude of $\Delta\vec{v}$ as point *D* is chosen to be closer and closer to point *C?*

D. Describe how you would determine the acceleration of the object at point *C*.

Draw a diagram that shows both $\vec{v}_C$ and $\vec{a}_C$ with their tails at point *C*. Is the angle between them *greater than, less than,* or *equal to* 90°?

E. Based on your answers above, indicate a possible direction for the acceleration vector at each marked point.

Top view

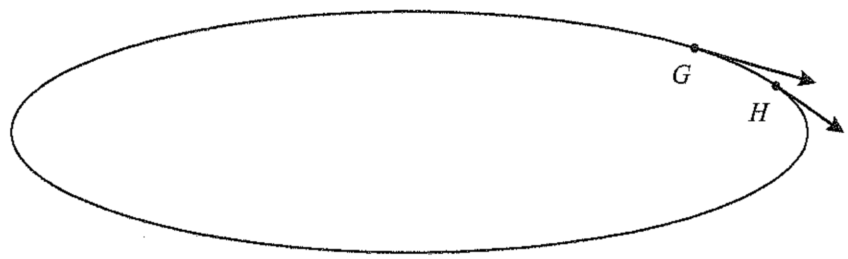


Acceleration vectors for speeding up (velocity vectors shown in grey)

➪ Discuss section III with a tutorial instructor.

1. An object moves clockwise with *decreasing speed* around an oval track. The velocity vectors at points G and H are shown below.

Top view



a. Copy the vectors $\vec{v}_G$ and $\vec{v}_H$ in the space at right, and determine the change-in-velocity vector, $\Delta\vec{v}$.

b. If point H were chosen to lie closer to point G, describe how $\Delta\vec{v}$ would change.

$\vec{v}_G$, $\vec{v}_H$, and $\Delta\vec{v}$

c. Describe how you would determine the acceleration at point G (both magnitude and direction). In the space at right, indicate the direction of the acceleration of the object at point G. (Your drawing need only be qualitatively correct.)

Direction of $\vec{a}_G$

d. Copy $\vec{v}_G$ and $\vec{a}_G$ (placed "tail-to-tail") in the space at right. Is the angle between the acceleration and the velocity *greater than, less than,* or *equal to* 90°?

$\vec{v}_G$ and $\vec{a}_G$ (placed "tail-to-tail")

e. Generalize your results above and from tutorial to answer the following question:

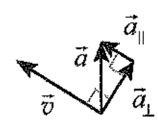
For an object moving along a curved trajectory, how does the angle between the acceleration and the velocity compare to 90° if the object moves with (i) constant speed, (ii) increasing speed, and (iii) decreasing speed?

2. Each diagram below shows the velocity and acceleration for an object at a certain instant in time.

	Instant *1*	Instant *2*	Instant *3*	Instant *4*
Acceleration	↑	↑	↑	→
Velocity	↖	←	↙	→

a. For each instant, state whether the object is *speeding up, slowing down,* or *neither (e.g., moving with constant speed).* Base your answers on what you have done in tutorial and problem 1.

b. The diagram at right illustrates how the acceleration at instant *1* can be treated as having two components: one parallel to the velocity ($\vec{a}_{\parallel}$) and one perpendicular to the velocity ($\vec{a}_{\perp}$).



For each of the other instants, draw a diagram similar to the one given for instant *1*. Label the parallel and perpendicular components of the acceleration relative to the velocity. If either component is zero, state so explicitly.

Instant *2*	Instant *3*	Instant *4*

c. For each of the instants *1–4,* compare your descriptions of the motion in part a with the components of the acceleration in part b. Then answer the following:

i. Give a general rule that describes how the component of the acceleration *parallel* to the velocity affects the motion of an object.

ii. Give a general rule that describes how the component of the acceleration *perpendicular* to the velocity affects the motion of an object.