Position, Velocity, and Acceleration

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1 Zooming in on a position vs. time graph

Suppose an object's motion is described by the following position function:

$$x(t) = t^2 - 6t + 10$$

1. Using a computer,[1] plot this motion between $t = 0$ s and $t = 6$ s. Does this object move with nearly constant velocity or with definitely varying velocity? Explain how you can tell.

2. Now plot this motion between $t = 1.5$ s and $t = 2.5$ s. (This should not take long. If you are struggling ***ask for help***.) Does this object move with nearly constant velocity or with definitely varying velocity? Explain how you can tell.

3. Now plot this motion between $t = 1.95$ s and $t = 2.05$ s. Does this object move with nearly constant velocity or with definitely varying velocity? Explain how you can tell.

4. All three graphs are representations of the same motion

    1. How can you account fo the last graph being so much straighter than the first?
    2. Can you tell from a very small time interval on a graph whether the motion over the whole graph has constant velocity?
    3. Find your average velocity over the small time interval from part 3 above. Show your work
    4. Add a line with slope equal to this average velocity to your plot from part 1 that goes through the point $(2, 2)$. What does this line remind you of from your Calculus I class? How do you think that the slope of this line compares to the instantaneous velocity at $t = 2$ s. Recall the definition of instantaneous velocity:

[1] Use any computational tool that you choose and are comfortable with. If you don't have a computational tool you are comfortable with, then open Excel on your laptop and look at this page for making scatter plots: https://www.excel-easy.com/examples/scatter-plot.html.

$$v = \frac{dx}{dt}$$

2 Predicting future position

Suppose that you know two things about an object:

1. Current location (x_i)
2. Current velocity (v_i)

Given what we know about velocity:

$$v = \frac{dx}{dt} \quad \text{and} \quad \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

As well as the fact that when Δt is small enough,

$$v \approx \bar{v}$$

Write an algebraic expression predicting the future position (x_f) some time δt after the initial time.

$$x_f =$$

3 Defining acceleration

Acceleration is defined as the time derivative of velocity:

$$a = \frac{dv}{dt}$$

Given what we know about the position, and average velocity, write an algebraic expression for the average acceleration

$$\bar{a} =$$

Suppose that you know two things about an object:

1. Current velocity (v_i)
2. Current acceleration (a_i)

Is it reasonable to predict that the velocity a short time δt later can be written as the following? Why or why not?

$$v_f = v_i + a_i \delta t$$

4 Motion of an accelerating object

Suppose you have an object moving in 1D with an acceleration given by the following function of time:

$$a(t) = \left(1 \frac{\mathrm{m}}{\mathrm{s}^2}\right) e^{-t}$$

You also know that at $t = 0$ the position is $x_i = 0$ m and the velocity is $v_i = 0 \frac{\mathrm{m}}{\mathrm{s}}$. We want to know where, and how fast, the object is moving at $t = 5$ s.

1. Carry out the following procedure to make this prediction. You can start by working with paper and pencil, but you will want to transition this calculation to code.

   a. Numerically predict the velocity and position at $t = 1$ s.
   b. Given your previous prediction, now predict the velocity and position at $t = 2$ s.
   c. Given your previous prediction, now predict the velocity and position at $t = 3$ s.
   d. Given your previous prediction, now predict the velocity and position at $t = 4$ s.
   e. Given your previous prediction, now predict the velocity and position at $t = 5$ s.

2. Do you think that this is accurate? Why or why not?
3. Make a plan for making this calculation more accurate, then carry it out.
4. Finally make the following plots from $t = 0$ s to $t = 5$ s.

   - $x(t)$ vs. t
   - $v(t)$ vs. t
   - $a(t)$ vs. t

5. ***Interpret this motion*** what is happening to the acceleration as $t \to \infty$? Do your velocity vs. time and position vs. time graphs show behavior that agrees with this observation about the acceleration?

Include your code and the figures that you generate as a part of your submission.